TO BE SORTED
RANDOM

ALEC HEWITT

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(Analysis) Thm 9.5.10
proof
recall: $f(x) = \sum_{n=0}^{n} \frac{d^n f(a) h^n}{n!} + R_n$; $R_n \frac{1}{(n+1)!} d^{n+1} f(c) h^{n+1}$
$df(a) = 0, n = 1$
$\implies f(x) = f(a) + \frac{1}{2} h d^2 f(c) h$; $x = a + h$
$\exists m > 0$ s.t. if $||d^2 f(c)|| < \frac{m}{2}$
$\implies d^2 f(c)$ pos def
2^{nd} order partial derivative is continuous at a and
$||c - a|| \leq || \implies ||d^2 f(a) - d^2 f(c)|| < \frac{m}{2} if ||h|| \sim$ small $\implies d^2 f(c)$ pos def $\implies f(a+h) = f(a) + \frac{1}{2} h \cdot d^2 f(c) h > f(a)$
$\implies f$ has local min at a
Thm 9.5.11
Corollary 9.5.12
Thm 9.5.15
Defn 9.6.1

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Peano's Axioms
N1. There is an element $1 \in \mathbb{N}$
N2. For each $n \in \mathbb{N}$ there is a successor element $s(n) \in \mathbb{N}$
N3. 1 is not the successor of an element of $\mathbb{N}$
N4. If two elements of $\mathbb{N}$ have the same successor, then they are equal.
N5. If s subset A of $\mathbb{N}$ contains 1 and is closed under succession (meaning $s(n) \in A$whenever $n \in A$), then $A = \mathbb{N}$.

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Thm 1.2.1 Suppose $\{P_n\}$ is a sequence of statements, one for each $n \in \mathbb{N}$.
These statements are all true provided

(1) P_1 is true (base case);

(2) whenever P_n is true for some $n \in \mathbb{N}$, then $P_{s(n)}$ is also true, then P_n is true $\forall n \in \mathbb{N}$

<u>Proof</u>
Let $A \subset \mathbb{N}$ s.t. $n \in Aand(P_n istrue)$
pt (1) $\implies 1 \in A$
pt (2) $\implies s(n) \in Awhenn \in A$
by N5 $A = \mathbb{N} \implies P_n true \forall n$
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<u>Thm 9.6.2</u>
B inverse for $dF(a)$ at a (constant matrix)
$\implies d(BF)(a) = dB \cdot F(a) + B \cdot dF(a) = B \cdot dF(a) = I$
$\implies G(x) = BF(x)w/dG(a) = I,\ u \cdot dG(a) \cdot u = u \cdot Iu$
$= ||u||^2 = 1 \implies posdef$
<u>recall:</u> Lemma 9.5.9 $\implies \exists m > 0 s.t. ||dG(x) - dG(a)|| > \frac{m}{2}$
$\implies dG(x)$ pos def.
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(Comp Anal.)
<u>Thm 3.3.7</u>
Assume no such sequence
$\implies \exists \epsilon > 0 and \delta > 0$ s.t. $|f(z) - W| > \epsilon \forall z \in U$
$= \{z \in \mathbb{C} | 0 < |z - z_0| < \delta\}. Since f(z) \neq w$ in U
$g(z) \equiv \frac{1}{f(z)-w}$ analytic on $Uw/|g(z)| < \frac{1}{\epsilon}$
<u>recall:</u> 3.3.4 (1) a) f bounded in deleted neighborhood of $z_0 \implies z_0$ removable discontinuity.
$g \neq 0$ constantly since f is not constantly infinite

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<u>Thm 3.3.7</u>
<u>Proof</u>
Spose $\nexists$ such sequence $\implies \exists \epsilon > 0,\ \delta > 0$
s.t. $|f(z) - w| > \epsilon,\ \forall z \in U = \{z \in \mathbb{C} | 0 < |z - z_0| < \delta\}$
$\implies f(z) \neq w \implies g(z) = \frac{1}{f(z)-w}$ analytic
$\implies |g(z)| < \frac{1}{\epsilon} \implies z_0$ removable by Thm 3.3.4 (a) g not constantly zero since f is not constantly infinite (isolated singularity) by Corollary 3.2.8 g has a convergent power series and if g is 0 at z_0 then $g(z) = (z - z_0)^k \phi(z)$
$\implies f(z) = w + \frac{1}{g(z)}$ is analytic if $k = 0$ or pole of order $k \implies z_0$ is not essential
$\implies$ contradiction

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Prop 4.1.1 If $g(z)$ and $h(z)$ are analytic and have zeros at z_0 of the same order, then $f(z) = g(z)/h(z)$ has a removable singularity at z_0.

Proof Prop 3.3.4 $\implies$ $g(z) = (z - z_0)^k \tilde{g}(z),\ \tilde{g}(z_0) \neq 0 h(z) = (z - z_0)^k \tilde{h}(z), \tilde{h}(z_0) \neq 0$. $\tilde{g} and \tilde{h}$ analytic and nonzero at z_0. $\implies$ $f(z) = \tilde{g}(z)/\tilde{h}(z)$analytic at z_0.

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Prop 4.1.2 Let g and h be analytic at z_0 and assume $g(z_0 \neq 0$, $h(z_0) = 0$ and $h'(z_0) \neq 0$. then $f(z) = g(z)/h(z)$ has a simple pole at z_0 and $\text{Res}(f; z_0) = \frac{g(z_0)}{h'(z_0)}$

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Prop 4.1.2

Proof $h(z_0) = 0; h'(z_0) \neq 0 \implies h'(z_0) = \lim_{z \to z_0} \frac{h(z) - h(z_0)}{z - z_0}$

$= \lim_{z \to z_0} \frac{h(z)}{z - z_0} \neq 0$

recall Prop 3.3.4: $\lim_{z \to z_0} (z - z_0) f(z) = Res(f; z_0) if \lim \neq 0$

$\therefore \lim_{z \to z_0} (z - z_0) \frac{g(z)}{h(z)} = \frac{g(z_0)}{h'(z_0)} = Res(\frac{g(z)}{h(z)}; z_0)$