COMPUTATIONAL PROJECT D: TIDAL TAILS OF INTERACTING GALAXIES

AG612

Abstract. The interaction between two passing galaxies was modelled using the Verlet integration method. The galaxies were modelled as a point mass with test masses in orbit around it, and a second point mass in a parabolic orbit around the first. Parabolic orbits with a range of semi-major axes were tested, in both prograde and retrograde motion with respect to the orbit of the test masses. The simulations found that the prograde orbits tended to form spiral galaxies while retrograde orbits tended to have a minimal effect on the galaxy.

1. Introduction

The objective of this project is to model the interaction between two galaxies. The two galaxies are modelled in separate ways: Galaxy 1 is modelled as a centre particle of mass M_1 with many test particles orbiting around it; Galaxy 2 is modelled simply as a particle, M_2, with the same mass as M_1.

The interaction is set up as follows: M_1 begins stationary at the origin; M_2 begins some distance away follows a parabolic orbit around the M_1.

Modelling this problem involves solving the equation of motion for each particle. The equation of motion for the i^{th} particle is given by Newton's Law of Gravitation

$$\frac{d^2\mathbf{r}_i}{dt^2} = -\sum_{j\neq i} \frac{Gm_i m_j}{r_{ij}}$$

where we are including the contribution from each of the other $j \neq i$ particles. These equations will be solved using the *Verlet Integration Method*. The *Verlet Method* is a second order integration method with an advantage of being straightforward to implement. This method requires three initial conditions: the initial position, $\mathbf{r}_0$, the initial velocity, $\mathbf{v}_0$, and the position at the next step, $\mathbf{r}_1$. In this program, the initial position and velocity are supplied and $\mathbf{r}_1$ is estimated using a single Euler step. The error of the method is h^2 where h is the time step.

To simplify the problem the test masses are not coupled to each other and their motion is determined only by M_1 and M_2. Similarly, M_1 and M_2 do not feel the gravitational pull of the test masses and only interact with each other, forming a two body problem.

This report will contain an analysis of the galactic interaction as well as an analysis of the stability and accuracy of the *Verlet Method* used.

2. Analysis

This project can be considered in two stages. The first stage is setting up the interaction between the test masses and the centre mass of one galaxy. The second stage involves including the second galactic centre mass and ensuring that it performs a parabolic orbit around the first mass.

Before those two stages are discussed, it is worth addressing the scaling of the problem.

2.1. **Scaling.** It is desirable to utilise scaled units for a couple of reasons.

- Computationally, the best precision in numbers is achieved if the values are close to ± 1, where the density of representable numbers is the highest. Numbers are represented less precisely as the number gets very large or very small.
- Choosing units requires choosing a specific mass for the galactic centre, as well as specific distances for the test masses. By using scaled units it allows the model to remain general. If one chooses the results can be scaled back to the appropriate real units.

In this model, $G = 1$,$M = 1$ where $M = M_1 = M_2$. The units of time are defined such that it takes 2π time units to orbit M_1 at a radius of 1 distance unit. Therefore, by defining the distance unit in terms of SI units, it is possible to convert between the scaled system and SI units. The time conversion factor is given by

$$\tilde{t} = t\sqrt{\frac{\alpha^3}{GM}}$$

Where G and M are the Gravitational constant and the relevant central mass respectively, $\tilde{t}$ and t are the time measured in SI and scaled units respectively. α is the ratio between the distance in SI units and the scaled distance unit such that

$$\frac{\tilde{r}}{r} = \alpha$$

$\tilde{r}$ and r are the units expressed in SI and scaled units respectively.

2.2. **Single Galaxy.** It is necessary to ensure that the test masses have the expected motion around M_1 without M_2. The test masses will initially undergo circular motion around M_1. Without M_2 disturbing their orbits, they will remain in circular motion.

The test masses are uncoupled so only one is required for testing. The test mass will begin with circular motion so

$$\frac{GMm}{r^2} = \frac{mv_\perp^2}{r}$$

$$\Rightarrow v_\perp = \sqrt{\frac{GM}{r}}$$

where $v_\perp$ is the tangential speed of the test mass, $M = M_1$, and r is the distance between the test mass and M_1. Using the scaled units introduced earlier

$$v_\perp = \frac{1}{\sqrt{r}}$$

So a test mass placed at $r = 1$ with a tangential velocity of $v = 1$ should undergo circular motion. That was one of the system properties that was tested for stability.

2.3. **Galactic Interaction.** The next stage involves adding a second mass, M_2, that interacts with the test masses and the centre mass of the first galaxy,M_1. The interaction between the two galactic masses is a two body problem. Therefore it can solved analytically.

We begin with the equations of motion for the two masses[1]

$$m_1\ddot{\mathbf{r}}_1 = -\frac{GM_1M_2}{|\mathbf{r}_{12}|^2}\hat{\mathbf{r}}_{12} \tag{2.1}$$

$$m_2\ddot{\mathbf{r}}_2 = -\frac{GM_1M_2}{|\mathbf{r}_{21}|^2}\hat{\mathbf{r}}_{21} \tag{2.2}$$

Combining these two we obtain the equation of motion for the separation between the two masses.

$$\frac{M_1M_2}{M_1+M_2}\ddot{\mathbf{r}} = -\frac{GM_1M_2}{r^2}\hat{\mathbf{r}}$$

where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$. The energy of the system is

$$E = \frac{1}{2}\frac{M_1M_2}{M_1+M_2}v^2 - \frac{GM_1M_2}{r}$$

Where $v = |\dot{\mathbf{r}}|$. For a parabolic orbit as required by the problem, $E = 0$. So

$$v^2 = \frac{2G(M_1+M_2)}{r}$$

Introducing the scaled units and with $M_1 = M_2 = 1$

$$v^2 = \frac{4}{r} \tag{2.3}$$

Another useful result is the equation of a parabolic orbit in Cartesian coordinates.

$$y = \frac{h^2}{4} - \left(\frac{x}{h}\right)^2 \tag{2.4}$$

$$h = |\mathbf{h}| = \mathbf{r}\times\mathbf{v} \tag{2.5}$$

Equations (2.3), (2.5) and (2.4) provide all the information we need to set up the parabolic orbit of the second mass. (2.4) provides a suitable initial position for a given value of h. Differentiating (2.4) with respect to t and using (2.3) gives (after some manipulation)

$$v_{x0}^2 = \frac{4h^4}{h^4+4x^2}\frac{1}{\sqrt{x^2+y^2}} \tag{2.6}$$

$$v_{y0}^2 = \frac{16x^2}{h^4+4x^2}\frac{1}{\sqrt{x^2+y^2}} \tag{2.7}$$

(2.6) and (2.7) give the initial velocity of M_2 in the rest frame of M_1.

3. Implement ion

In this program there are three things to consider: the motion of M_1 and M_2, the test masses, and the integration method. I decided to develop two classes. A *Mass* class to handle the individual particle properties, and a *Verlet Integrator* class which is responsible for performing the integration steps.

[1] A more detailed treatment of the two body problem can be found in *Mechanics, 2nd Ed. by Symon*

3.1. **Mass Class.** There will be many masses and so a suitable way of tracking their positions, velocities and interactions will be needed. The simplest way would probably be to have a set of three arrays; one for the position, velocity, and mass of each object.

I decided instead to take advantage of classes within C++. I created a *Mass* class to hold the position, velocity and mass information for each object. This has several advantages: It holds the relevant particle information in one manageable object; it allows for reusable and specific functions to be made for the class; and it also hides the unnecessary object information from the user, preventing the user from using the object in unintended ways. The mass class is defined in the *Mass.cc file* and linked using the *Mass.h* file.

3.2. **Verlet Class.** The purpose of the *Verlet_Int* class is to neatly handle the integration process. Each *Verlet_Int* object is associated with one particle and contains a pointer to the associated *Mass* object, allowing direct access to the particle data. Calling the *step()* method performs a single step of the *Verlet Integration*.

The step method iterates the position and velocity according to

$$\mathbf{r}_{i+1} = 2\mathbf{r}_i - \mathbf{r}_{i-1} + \frac{\mathbf{F}_i}{m}h^2$$

$$\mathbf{v}_i = \frac{\mathbf{r}_{i+i} - \mathbf{r}_{i-1}}{2h}$$

A second step method was added to handle the interaction between the two centre masses, *simul_step*. For all the test masses, one can just iterate through them, performing the step function on each. However, for the two centre masses, M_1 and M_2, this process will add an additional error. To see how consider the masses at time t_0 with initial positions $\mathbf{r}_1(0)$, $\mathbf{r}_2(0)$. Ideally we would like the motion of the masses to change based on their mutual force $F(\mathbf{r}_1 - \mathbf{r}_2)$. If, however, the step is performed on M_1 first, then the force acting on M_2 when the step is performed will be $F((\mathbf{r}_1 + \Delta\mathbf{r}_1) - \mathbf{r}_2)$. So there is an error associated with performing the step function on M_1 and M_2 sequentially. The *simul_step* method accounts for this by calculating where M_1 should be without updating M_1, updating M_2, then updating M_1 at the end with the precalculated values.

3.3. **Overall Program.** The main structure of the program is as follows: The test masses; large masses; test mass integrators; and large mass integrators are all stored in separate vectors. Once those have been initialised the main loop starts and the integrators are stepped through each cycle. An additional loop controls how often the program outputs data. The output is put into two data files **cmass.txt** and **tmass.txt** which contain the data for the large masses and test masses respectively.

Output data files are processed using Gnuplot.

4. Results

4.1. **Stability and Errors.**

4.1.1. *Test Mass in Circular Motion.* The first tests were to check whether the position and energy of the test mass remained stable while in circular motion (See Fig. 4.1 and Fig. 4.2). The energy can be seen to oscillate but the average energy remains constant. Smaller time steps have the effect of decreasing the amplitude of the oscillation.

The predicted motion is given by $x = cos(t)$, $y = sin(t)$. Over long time periods it is found that the modelled particle lags behind the position it should have. Fig. 4.2 shows the lag for a time step of $\Delta t = 0.1$.

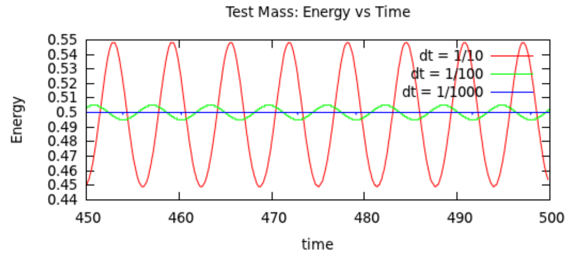

FIGURE 4.1. Test Mass Energy vs Time

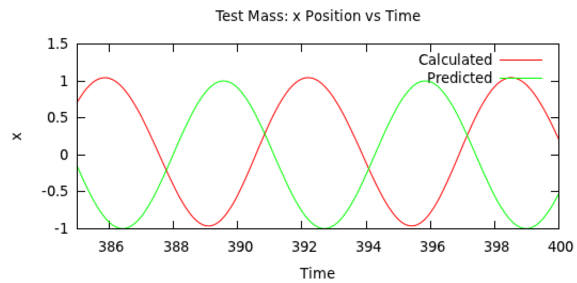

FIGURE 4.2. Test Mass x Position

4.1.2. *Centre Mass Interaction: Circular Orbit.* To determine the stability of the interaction between M_1 and M_2, the system was set such that in the rest frame of M_1, M_2 was in circular motion. Simulations were run with timesteps $\Delta t_1 = 0.1$, $\Delta t_2 = 0.01$ and $\Delta t_3 = 0.001$, for a time of $t = 100$. In all cases a decaying orbit was found and there was a phase difference which developed between the calculated position and the theoretical position (See Fig. 4.3) . The time step Δt_1 was found to be unstable

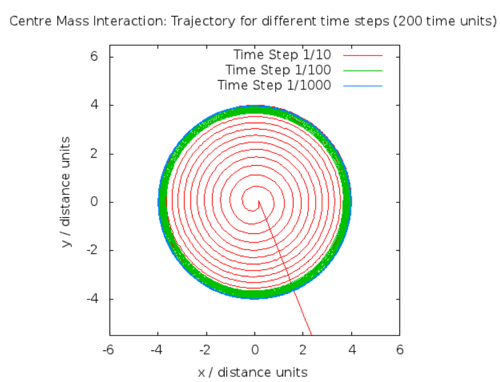

FIGURE 4.3. Trajectory of test mass

after short lengths of time, so is unusable for the simulations. Δt_2 and Δt_3 performed similarly when comparing energy changes over time. Looking at the trajectories shows that over 500 units of time, the trajectory for Δt_2 decays towards the centre. Comparing the radial distance error between to the two time steps over time shows that Δt_3 is significantly more stable than the other time steps (See Fig. 4.4) . Though it is worth bearing in mind that for a parabolic orbit, the time during which the masses will

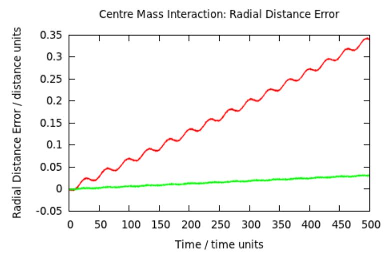

FIGURE 4.4. Radial distance error. (Red) $\Delta t_2 = 0.01$, (Green) $\Delta t_3 = 0.001$

be interacting will be short so errors will not be able to accumulate as they do for circular orbits.

4.1.3. *Centre Mass Interaction: Parabolic Orbit.* To test the parabolic orbits between M_1 and M_2, three time steps are considered: $\Delta t_1 = 0.1$, $\Delta t_2 = 0.01$, and $\Delta t_3 = 0.001$.

In this problem the closest parabolic orbits will have a distance of closest approach of $r = 9$. An initial position of $\mathbf{r}_0 = (-36, -27)$ was chosen with corresponding initial velocity. For each time step the errors relative to the predicted trajectory (2.4) are compared (See Fig. 4.5). Δt_1 performed worst when compared to the other two time steps. The fly-by was enough to significantly change its trajectory. Both Δt_2 and Δt_3 performed well and there was little difference between the performance between the two.

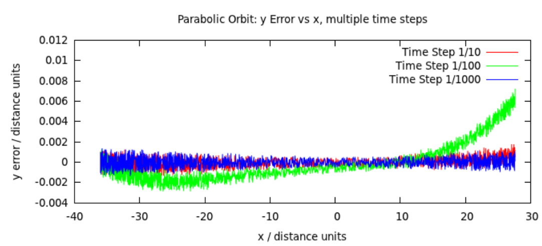

FIGURE 4.5. y position error for different time steps

It seems that both Δt_2 and Δt_3 would be suitable time steps for this problem. Both have good accuracy for parabolic orbits and the test mass circular orbits.Δt_3 would be used instead of Δt_2 if the two centre masses had a closed orbit, or if the parabolic orbit had a small distance of closest approach. However, the fly-by distances will be large enough to use $\Delta t_2 = 0.01$.

4.2. **Galactic Interactions.** The galactic interactions involved sending a lone mass, M_2 around a galaxy which contained test masses in circular orbit around a central mass $M_1 = M_2$. Parabolic orbits with distances of closest approach: 9, 12, and 16 distance units were chosen. For each distance the lone mass, M_2, was moving such that it was orbiting around the galactic mass in the same sense as the test masses (prograde), and then again in the opposite sense (retrograde). Figures presented are in the rest frame of M_1.

Included also are histograms of the semi-major axis of the test masses for selected cases. The value for the semi-major axis was calculated under the assumption that the potential energy from the passing mass was negligible. This is the case for large values of t when lone mass is far away. For $t = 0$, the passing mass starts close enough to contribute a non-negligible amount of potential energy. In the case of the closest approach being $r = 16$ units the potential energy contribution from the passing mass due to its initial position is an additional $\approx 18\%$. In this case I would use the histograms as a qualitative indication of the behaviour of the system.

For future reference, the semi-major axis, a, and specific energy, ϵ, are related by

$$\frac{v^2}{2} - \frac{GM}{r} = \epsilon = -\frac{GM}{2a} \tag{4.1}$$

$$a = \frac{GMr}{2GM - v^2 r} \tag{4.2}$$

4.2.1. *Closest Approach 9 units: Retrograde & Prograde.* The retrograde orbit of mass M_2 increases the eccentricity of the test mass orbits as it passes by, ss well as causing a small amount of clumping of the test masses in the outer rings (See Fig.(4.6)).

For the prograde orbit a significantly different outcome is produced as compared to the retrograde orbit (See Fig. (4.7)). The test masses are pulled out and many are ejected from the orbit. The reason for this difference is that the test masses are subjected to the force of M_2 for a longer duration of time. Many of the masses finish with orbits closer to M_1. However a number of test masses came too close to M_2 and the integrator became unstable. This caused those mass to be ejected from the system.

The results of the prograde orbit are reflected in the semi-major axis histogram (See Fig. (4.8)). The test masses that were ejected from the system end up with negative semi-major axes that can be seen. The histogram also shows that most of the masses end up with a reduced semi-major axis along with a few that have a greater semi major axis. Though one should remember that many of the ejected masses should actually have larger semi-major axes.

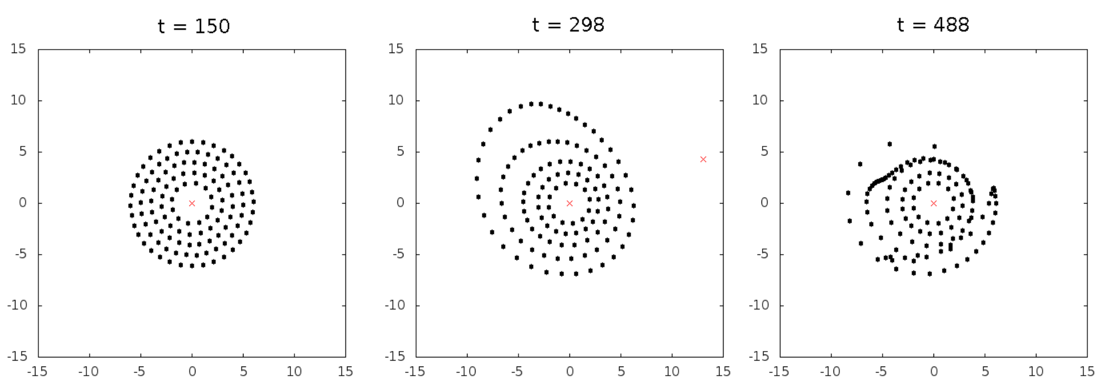

FIGURE 4.6. Mass Positions at t=150, 298, 488

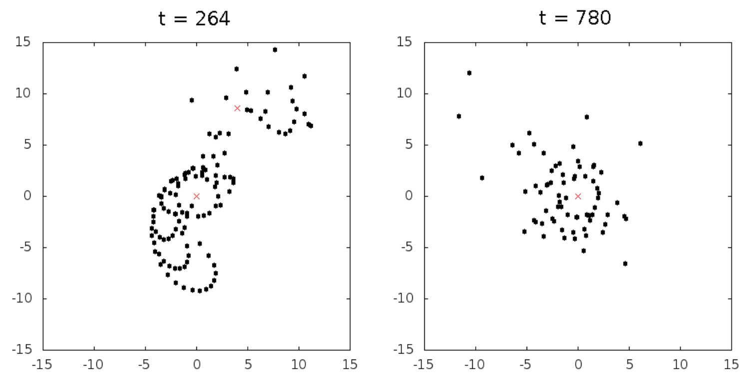

FIGURE 4.7. Masses at t=264, 780

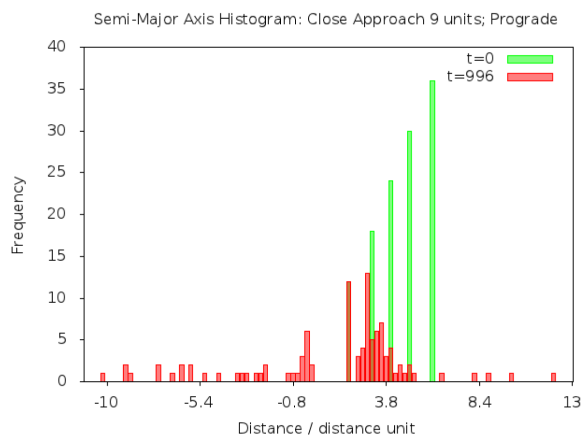

FIGURE 4.8. Semi-Major Axis Histogram, Periapsis 9 units, Prograde

4.2.2. *Closest Approach 12 units: Retrograde & Prograde.* Similar to the case above, the retrograde motion of M_2 causes small changes to the motion of the test masses in the outer rings, as well as clumping of est masses in the outer ring (See Fig. 4.9). The histogram show a small perturbation to the orbit of masses in the outer rings (See Fig. 4.10).

In prograde motion M_2 stretches out the galaxy and causes spiral arms immediately after the fly-by (See Fig. 4.11). The spiral arms quickly lose shape and most of the test masses finish in an orbit close to M_1. Many of the test masses in the outer rings are thrown out of the system or are given orbits with a large semi-major axes. Test masses which come too close to M_1 or M_2 become unstable and tend to be ejected from the system.

The histogram reflects these results. Most of the disturbed masses have a semi-major axis distributed around 4.2 units around M_1 while a few have large semi-major axes (See Fig. 4.12).

4.2.3. *Closest Approach 16 units: Retrograde & Prograde.* For retrograde motion M_2 causes a slight change to the eccentricity of some of the masses but the orbits remain approximately circular. These

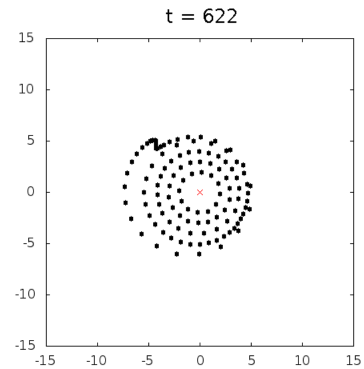

FIGURE 4.9. Masses at t=622

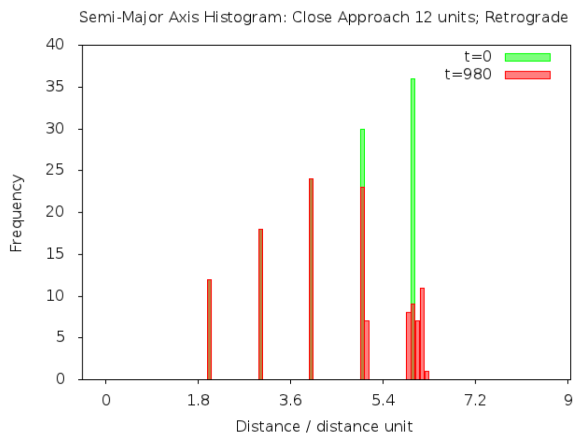

FIGURE 4.10. Semi-Major Axis Histogram, Periapsis 12 units, Retrograde

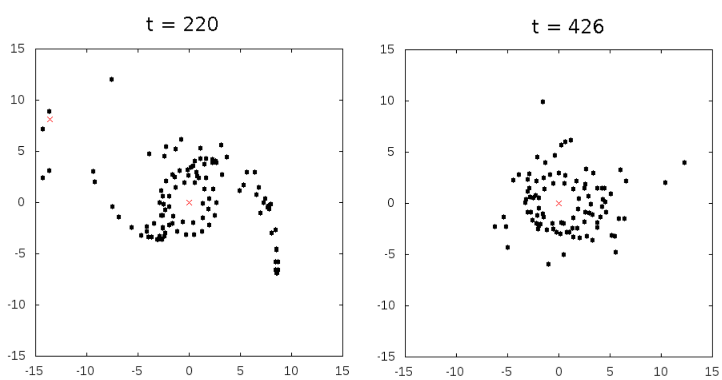

FIGURE 4.11. Masses at t = 220, 426

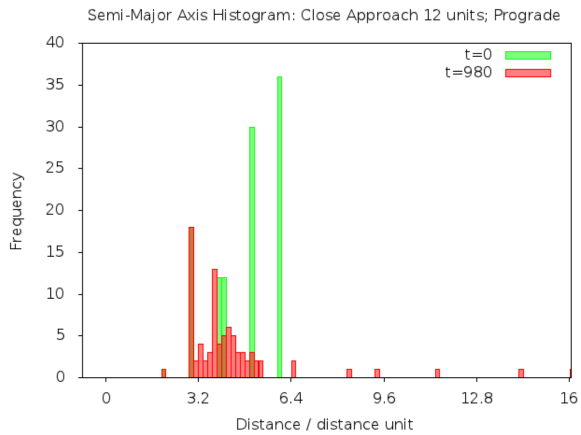

FIGURE 4.12. Semi-Major Axis Histogram, Periapsis 12 units, Prograde

small effects are expected, the maximum for exerted by M_2 on any test mass is

$$F_{max} \approx -\frac{1}{(16-6)^2} = -\frac{1}{100}$$

and force is felt for a short amount of time due to the retrograde motion.

For prograde motion the galaxy takes a spiral form for a short period of time after M_2 goes by (See Fig. 4.13). The formation of the spiral is a result of the tidal forces across the galaxy. The masses closest to M_2 experience a larger attraction than those on the far side of the galaxy. This causes an acceleration gradient which elongate the galaxy. The rotation of the test masses then turns this into a spiral.

The histogram shows that only the outer two rings are disturbed by M_2, causing these test masses to have a semi-major axes distributed around 5 units around M_1 (See Fig. 4.14).

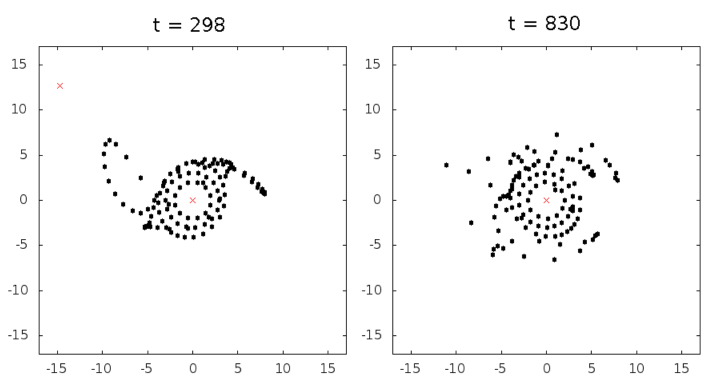

FIGURE 4.13. Masses at t = 298, 830

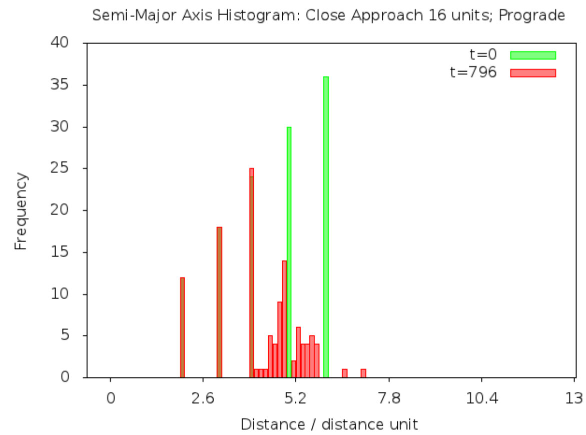

FIGURE 4.14. Semi-Major Axis Histogram, Periapsis 16 units, Prograde

5. Conclusions

A lot of insight into the properties of galactic interaction can be gained from the results of these simulations. An important process that could be seen was the formation of spiral galaxies. The spiral galaxies formed in these simulations had two distinct arms. It is not clear whether most types of spiral galaxies can be formed in a process like this. Though a conclusion I would draw is that a spiral structure is most likely to be created if the object disturbing the galaxy has prograde motion around the galaxy. Similarly, the results of this simulation show that an object in retrograde motion will not significantly change the overall structure of the galaxy.

The spiral galaxies created by the simulations lasted for a short amount of time relative to the timescale of the simulation. If interacting galaxies were the main cause of spiral galaxies then given the short lifetime of the spiral structure, one would expect most spiral galaxies to have partner galaxy nearby, probably also a spiral galaxy. An important caveat for this conclusion is that it is unclear how the interaction would change for different masses. It is possible that a smaller galaxy could still produce the spiral effect.

A final conclusion relating to spiral galaxies is that they will eventually lose their spiral structure a will become elliptical galaxies.

A very general phenomenon to be noted was that a galaxy passing by another galaxy will cause matter within the galaxies to clump together. This could produce dense gas clouds which could in turn produce stars. I would therefore expect there to be regions where stars are being created within galaxies that have just passed by each other.

The results of these simulations also provide motivation for developing the model further. Currently there are some significant limitations of the model: the masses all exist in the same plane; the test masses do not interact with each other; there were a limited number of masses included in the simulations.

Regarding increasing the number of test masses. Preliminary tests were performed with an increased number of test masses. The increased number of masses slowed down the computing process and little additional insight was gained. Therefore I decided not to include the results from those tests.

It would be interesting to see how the structure of a disk changed as a galaxy passed by it from outside of its plane. From a computational perspective this would not be too hard to implement though it would be difficult to create visualisations of the results.

Allowing the test masses to interact with each other would be a challenging task though. The main cause of instability in the simulations was masses getting too close to each other. This would be practically unavoidable for a large system of interacting masses. The time steps in the *Verlet Integration* would have to very small which would increase the length it takes to run the simulations. A possible way of handling interacting masses is to give each mass a finite size. It would then be possible for masses to coalesce or collide elastically.