Exercise I.

1) 2023: 200 million
2024: 240 million

increase $240 - 200 = 40$ mill.

% increase: $\frac{40}{200} \times 100 = 20\%$.

2) $\bar{X}$ = mean for X

$$= \frac{\sum x_i}{N} = \frac{1+2+3+4+5+6}{6} = 3.5$$

$N \rightarrow$ how many X values there are

$\bar{y}$ = mean for Y

$$= \frac{\sum y_i}{N} = \frac{80+100+\cdots}{6} = \frac{890}{6} = 148.30$$

$N \rightarrow$ how many Y values, same as X.

3) $r = \frac{cov(X,Y)}{\sigma_X \sigma_Y}$

$cov(X,Y) = mean(XY) - \bar{X}\bar{Y}$.

$$mean(XY) = \frac{\sum x_i y_i}{N}$$

so, $\text{mean}(xy) = \dfrac{1\times 80 + 2\times 100 + 3\times 120 + \cdots}{6}$

$= \dfrac{3680}{6} = 613.30$

$\Rightarrow \text{cov}(x,y) = 613.3 - 3.5 \times 148.3$
$= 94.25$

$\text{var}(x) = \text{mean}(x^2) - \bar{x}^2$

$\text{mean}(x^2) = \dfrac{\sum x_i^2}{N} = \dfrac{1^2 + 2^2 + 3^2 + \cdots}{6}$

$= \dfrac{91}{6} = 15.17$

$\Rightarrow \text{Var}(x) = 15.17 - 3.5^2$
$= 2.92$

$\Rightarrow \sigma_x = \sqrt{2.92} = 1.71$

$\text{var}(y) = \text{mean}(y^2) - \bar{y}^2$

$\text{mean}(y^2) = \dfrac{\sum y_i^2}{N} = \dfrac{80^2 + 100^2 + \cdots}{6}$

$= \dfrac{150900}{6} = 25150$

$\Rightarrow \text{Var}(y) = 25150 - 148.3^2$

$= 3157.11$

$$\Rightarrow \sigma_y = \sqrt{3157.11}$$
$$= 56.19$$

Therefore,

$$r = \frac{94.25}{1.71 \times 56.19} = 0.98$$

interpretation: very strong positive correlation

4) $(D_{y/x})$: $y = ax + b$,

where:

$$a = r \times \frac{\sigma_y}{\sigma_x} = 0.98 \times \frac{56.19}{1.71}$$

$$\Rightarrow a = 32.20$$

and: $b = \bar{y} - a\bar{x}$

$$= 148.3 - 32.2 \times 3.5$$
$$= 35.6$$

$$y = 32.2x + 35.6$$

(answers might differ a bit according to approximation techniques, but they would still be correct).

5) In $y = 32.2x + 35.6$,

the y is the no. of users (in mill)

Lebanon $= 1\%$ of y

$= 0.01y$.

we want $0.01y \geq 3$

$\Leftrightarrow y \geq 300$.

$\Leftrightarrow 32.2x + 35.6 \geq 300$

$32.2x \geq 300 - 35.6$

$32.2x \geq 264.4$

$$x \geq \frac{264.4}{32.2}$$

$x \geq 8.21$

$\Leftrightarrow x = 9$

so, it's year 2027

Exercise II

1) a) $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to \infty} (x + e^{-x+1})$

$= \infty + e^{-\infty} = \infty + 0 = \infty$

b) $\lim_{x\to\infty}(f(x) - y)$

$= \lim_{x\to\infty} (x + e^{-x+1} - x)$

$= \lim_{x\to\infty} e^{-x+1} = e^{-\infty} = 0.$

c) $f(x) - y = e^{-x+1}$ for all x.

But $e^{u} > 0$ always (for all u)

$\Rightarrow f(x) - y > 0$ for all x.

so (C) is always above (d)

2) a) $f(x) = x + e^{-x+1}$

$f'(x) = 1 - e^{-x+1}$

b) $f'(x) = 0 \iff$

$1 - e^{-x+1} = 0 \iff$

$e^{-x+1} = 1 \iff$

$-x+1 = 0 \iff x = 1$

c) $y - f(1) = f'(1)(x-1)$

but $f(1) = 1 + e^{0} = 2$

and $f'(1) = 0$ (by part b))

so: $(T): y = 2$

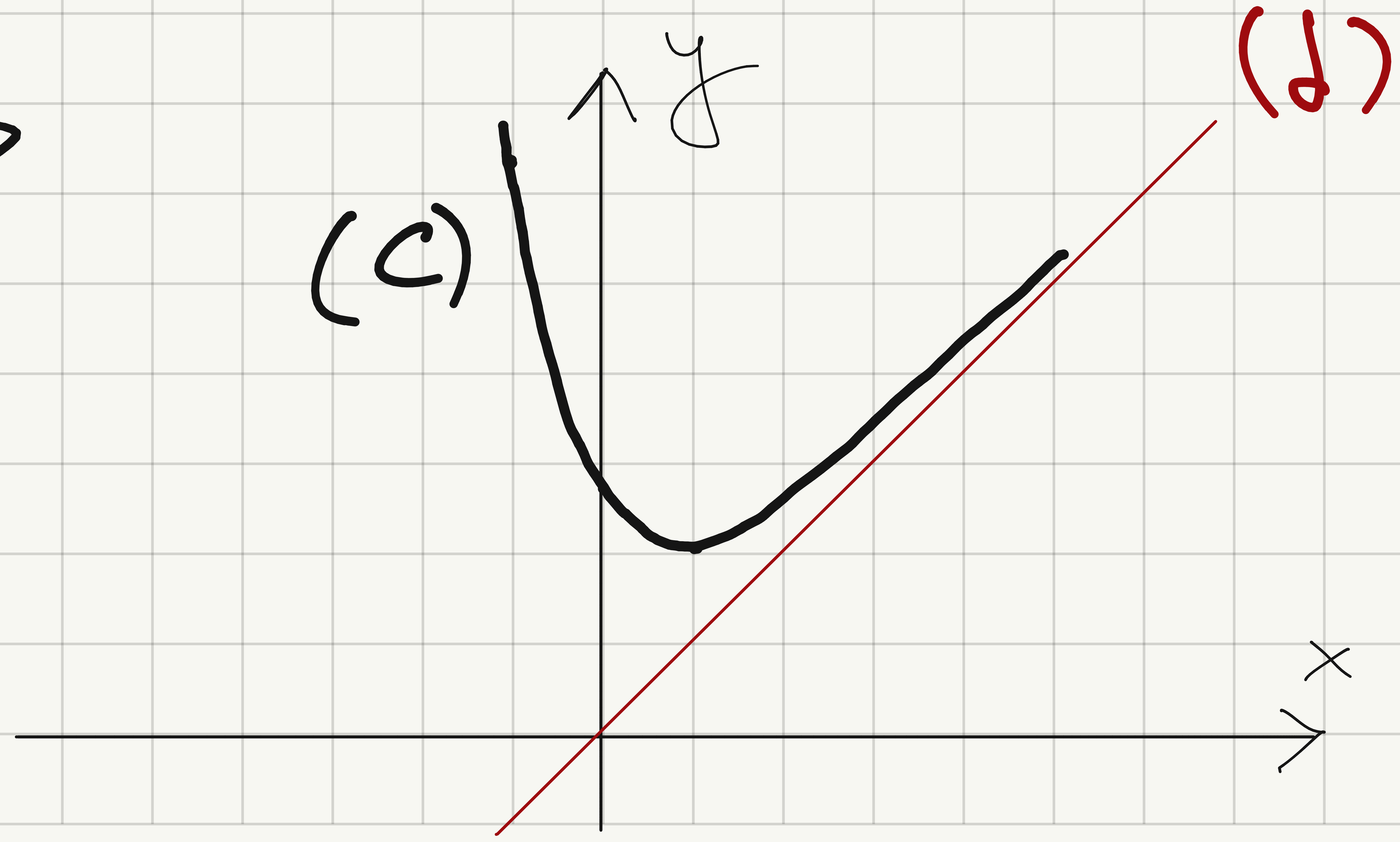
d)

x	$-\infty$		1		$+\infty$
$f'(x)$		$-$	0	$+$	
$f(x)$	$+\infty$	$\searrow$	2	$\nearrow$	$+\infty$

3) a) $f(-1) = -1 + e^{2}$

$f(0) = e$

b)



Exercise III

1) a) $C(0) = 3$

$\Rightarrow$ fixed cost $= 3 \times 10{,}000{,}000$ L.L.

$= 30{,}000{,}000$ L.L.

b) 980 produced

$\Rightarrow x = \frac{980}{1000} = 0.98$

But:

$(0.98, 4.90) \in (C)$

$\Rightarrow C(0.98) = 4.90$

$\Rightarrow$ total cost $= 4.9 \times 10^7$ L.L.

$= 49{,}000{,}000$ L.L.

average cost $= \frac{49{,}000{,}000}{980}$

$= 50{,}000$ L.L.

c) At the point A, the revenue equals the total cost, so the company covers all the costs but doesn't make any revenues.

In other words, we have:

loss = win = 0 L.L.

2) a) $P(x) = R(x) - C(x)$

$\Rightarrow P(x) = 5x - x^2 + e^{-3x} - 4$.

b) $P(0{,}5) = 5(0.5) - 0{,}5^2 + e^{-1.5} - 4$

$= -1.53$ (nearest 10^{-2})

$P(0.5) < 0$, so the company doesn't realize any gains, but it realizes a loss of -1.53×10^7 L.L

i.e. loss $= 15{,}300{,}000$ L.L.

not needed, just extra explanation

Exercise IV

1) $\ln(3a) - \ln\left(\frac{3}{e}\right)$

$= \ln 3 + \ln a - (\ln 3 - \ln e)$

$= \ln 3 + \ln a - \ln 3 + 1$

$= \ln a + 1$ so $\boxed{a}$

2) $x-1>0$ and $-x+3>0$

so $x>1$ and $x<3$

$\Rightarrow D_f =]1,3[$. $\boxed{b}$

3) $c'(x) = 2x+5e^{-5x}$

$\Rightarrow$ marginal cost (for 6 objects)

is $C'(6) = 12+5e^{-30}$. $\boxed{b}$

4) Demand is unit elastic $\Rightarrow E(x)=1 \Rightarrow$

$\frac{1}{2-\ln x} = 1 \Rightarrow 2-\ln x = 1$

$\Rightarrow \ln x = 1 \Rightarrow x = e$ $\boxed{c}$

Exercise V

1) unit price = 300 million L.L.

$\Rightarrow x=3$

$f(3) = (3+12)e^{-3+1} = 15e^{-2}$

$\Rightarrow$ demand $= 15e^{-2} \times 1000$

$= 2030.03$ (nearest 10^{-2})

$\Rightarrow$ no// demanded units = 2030 (approx)

2) supply = 14,000 units

$\Rightarrow g(x) = 14.$

$\Rightarrow \quad x+12 = 14 \Rightarrow x = 2$

so unit price = 200 000 000 L.L.

3) a) Equilibrium: demand = supply

so $f(x) = g(x)$

$(x+12)e^{-x+1} = x+12$

i.e. $e^{-x+1} = 1$

(can divide both sides by $x+12$ since $x>0$ so $x+12 \neq 0$)

so $-x+1 = 0$

i.e. $x = 1$

in hundred million L.L.

$\Rightarrow$ equilibrium price is $1 \times 100\,000\,000$ L.L.

i.e. 100 000 000 L.L.

b) no// of units = $f(1) = g(1)$

$= 13$

4) a) We have: $E(x) = \left| \frac{x}{f(x)} f'(x) \right|$

$$\Rightarrow E(x) = \left| \frac{x}{(x+12)\cancel{e^{-x+1}}} (-x-11)\cancel{e^{-x+1}} \right|$$

$$\Rightarrow E(x) = \left| - \frac{x(x+11)}{x+12} \right|$$

$$\Rightarrow E(x) = \frac{x^2+11x}{x+12}$$

b) $E(3) = \frac{3^2+11(3)}{11+12} = \frac{42}{23}$

$\Rightarrow E(3) = 1.83$ (nearest 10^{-2})

we have $E(3) > 1$

$\Rightarrow$ the demand is elastic

c) new unit price $= \left(1+\frac{1}{100}\right) 300\,000\,000$

$= 303\,000\,000$ L.L.

$$\Rightarrow x = \frac{303\,000\,000}{100\,000\,000}$$

$\Rightarrow x = 3.03$.

$f(3.03) = (3.03+12)e^{-3.03+1}$

$= 15.03 e^{-2.03}$

no/. of demanded units is:

$15.03\, e^{-2.03} \times 1000$

$= 1973.97$ (nearest 10^{-2})

so 1974 (approx.)

Exercise VI

1) a) $P(L \cap H)$

$= P(H|L)\,P(L)$ (Bayes' rule)

$= 0.4 \times 0.3$

$= 0.12$

$P(G \cap H) = P(H|G)\,P(G)$ (Bayes)

$= 0.3 \times 0.2 = 0.06.$

b) $P(E \cap H) = P(H|E) . P(E)$ (Bayes)

$= 0.8 \times \underbrace{0.5}_{} = 0.4$

(since 50% are in ES)

c) $P(H) = P(H \cap L) + P(H \cap G) + P(H \cap E)$

$= 0.12 + 0.06 + 0.4$

$\Rightarrow P(H) = 0.58$

2) $P(E|H) = \dfrac{P(E\cap H)}{P(H)}$

$= \dfrac{0.4}{0.58} = 0.69$ (nearest 10^{-2})

3)

	GS	LS	ES	Total
chose History	12	24	80	116
Didn't choose History	28	36	20	84
Total	40	60	100	200

$P(G\cap H) \times 200 = 0.06 \times 200$
$= 12$

etc...

$P(G) = P(G\cap H) + P(G\cap \overline{H})$

$\Rightarrow P(G\cap \overline{H}) = P(G) - P(G\cap H)$

$= 0.2 - 0.06$

$= 0.14$

$P(G \cap \overline{H}) \times 200 = 0.14 \times 200$
$= 28$

etc ....

b) total possibilities: $200C3$
$= 1313400$

no// students in ES = 100.
no// students not in ES and who didn't choose history $= 28 + 36$
$= 64$

$100C1 \times 64C2$
$= 100 \times 2016 = 201600.$

prob. $= \frac{201600}{1313400} = 0.1535$
(nearest 10^{-4}).

For inquiries/suggestions, you may contact:

ahh75@mail.aub.edu