第二章: 基本概念

1. 均值函数，方差函数

$$\mu_t = E(X_t), \sigma_t^2 = Var(X_t).$$

2. 自协方差函数

$$\gamma_{t,s} = Cov(X_t, X_s) = E(X_t, X_s) - \mu_t \mu_s.$$

3. 自相关函数

$$\rho_{t,s} = Corr(X_t, X_s) = \frac{\gamma_{t,s}}{\sqrt{\gamma_{t,t}\gamma_{s,s}}}.$$

4. 严(强)平稳性

任意有限维分布都与初始时间点无关，只与时间间隔相关。***如果期望和方差都存在的话***，那么我们就有

$$X_t \overset{d}{=} X_s. \qquad \mu_t = \mu_s \equiv \mu, \qquad \sigma_t^2 = \sigma_s^2 \equiv \sigma^2$$

$$(X_{t_1}, X_{t_2}) \overset{d}{=} (X_{t_1+k}, X_{t_2+k}). \qquad \gamma_{t,s} = \gamma_{0,|t-s|}$$

$$(X_{t_1}, X_{t_2}, \cdots, X_{t_n}) \overset{d}{=} (X_{t_1+k}, X_{t_2+k}, \cdots, X_{t_n+k}).$$

Let $\gamma_k = Cov(X_t, X_{t+k}), \quad \text{then} \quad \rho_k = Corr(X_t, X_{t+k}) = \frac{\gamma_{t,t+k}}{\sqrt{\gamma_{t,t}\gamma_{t+k,t+k}}} = \frac{\gamma_k}{\gamma_0}.$

5. **宽(弱)平稳性**

均值函数和方差函数都是常数: $\mu_t \equiv \mu, \qquad \sigma_t^2 \equiv \sigma^2$

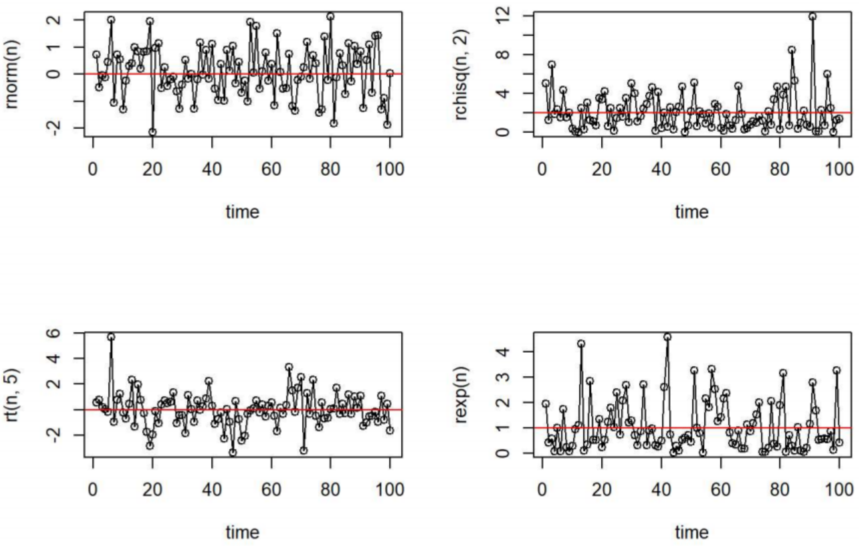
协方差仅依赖于时间的滞后: $\gamma_k = \gamma_{t,t+k} = \gamma_{0,k}.$

Let $\gamma_k = Cov(X_t, X_{t+k}), \quad \text{then} \quad \rho_k = Corr(X_t, X_{t+k}) = \frac{\gamma_{t,t+k}}{\sqrt{\gamma_{t,t}\gamma_{t+k,t+k}}} = \frac{\gamma_k}{\gamma_0}.$

例1. **白噪声序列** $\{e_t, i.i.d.\}$, 平稳的

$$\gamma_k = \begin{cases} \sigma_e^2, & \text{if } k = 0 \\ 0, & \text{if } k > 0 \end{cases} \qquad\qquad \rho_k = \begin{cases} 1, & \text{if } k = 0 \\ 0, & \text{if } k > 0 \end{cases}$$

```
n=100;
opar=par(mfrow=c(2,2))
plot(rnorm(n),xlab='time',type='o');abline(h=0,col='red')
plot(rchisq(n,2),xlab='time',type='o');abline(h=2,col='red')
plot(rt(n,5),xlab='time',type='o');abline(h=0,col='red')
plot(rexp(n),xlab='time',type='o');abline(h=1,col='red')
```




```
par(opar)
```

例2. **滑动平均序列** $\{X_t = \frac{e_t + e_{t-1}}{2}\}$, 平稳的

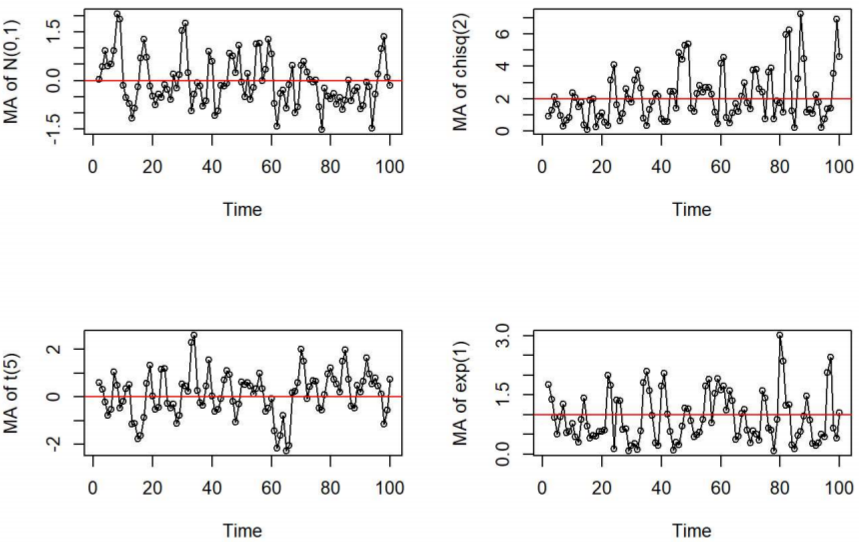
$$\gamma_k = \begin{cases} 0.5 \cdot \sigma_e^2, & \text{if } k = 0 \\ 0.25 \cdot \sigma_e^2, & \text{if } k = 1 \\ 0, & \text{if } k > 1 \end{cases} \qquad \rho_k = \begin{cases} 1, & \text{if } k = 0 \\ 0.5, & \text{if } k = 1 \\ 0, & \text{if } k > 1 \end{cases}$$

```
n=100;
opar=par(mfrow=c(2,2))
tsnorm=rnorm(n)
tsMA1=ts((tsnorm+zlag(tsnorm))/2.0,freq=1,start=1)
plot(tsMA1,type='o',ylab='MA of N(0,1)');
abline(h=0,col='red')

tschisq=rchisq(n,2)
tsMA2=ts((tschisq+zlag(tschisq))/2.0,freq=1,start=1)
plot(tsMA2,type='o',ylab='MA of chisq(2)');
abline(h=2,col='red')

tst=rt(n,5)
tsMA3=ts((tst+zlag(tst))/2.0,freq=1,start=1)
plot(tsMA3,type='o',ylab='MA of t(5)');
abline(h=0,col='red')

tsexp=rexp(n)
tsMA4=ts((tsexp+zlag(tsexp))/2.0,freq=1,start=1)
plot(tsMA4,type='o',ylab='MA of exp(1)');
abline(h=1,col='red')
```



```
par(opar)
```

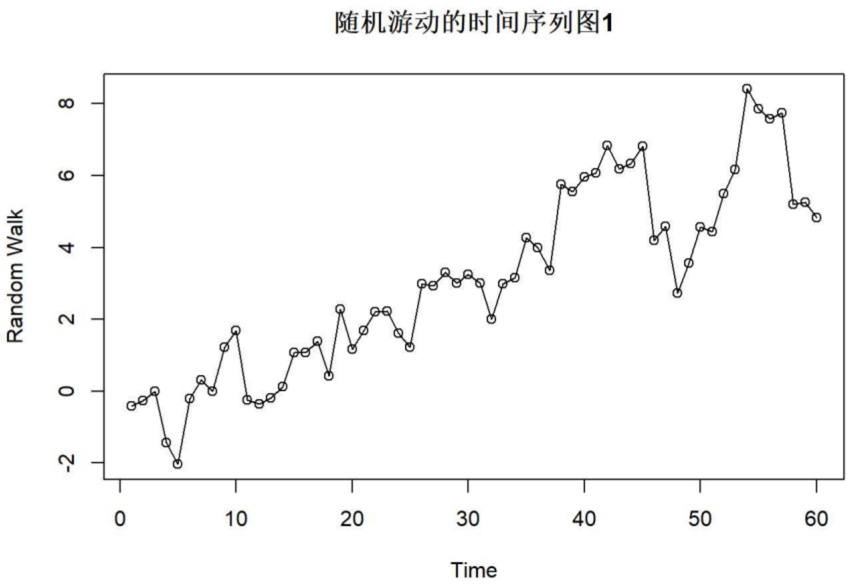
例3. **随机游动序列** $\{Y_t = e_1 + e_2 + \cdots + e_t\}$, ***非平稳的***

$$\mu_t = t \cdot \mu, \qquad \sigma_t^2 = t \cdot \sigma_e^2.$$

$$\begin{aligned}\gamma_{t,s} &= Cov(e_1 + e_2 + \cdots + e_t, e_1 + e_2 + \cdots + e_s) \\ &= Var(e_1 + e_2 + \cdots + e_{min(t,s)}) \\ &= min(t,s) \cdot \sigma_e^2\end{aligned}$$

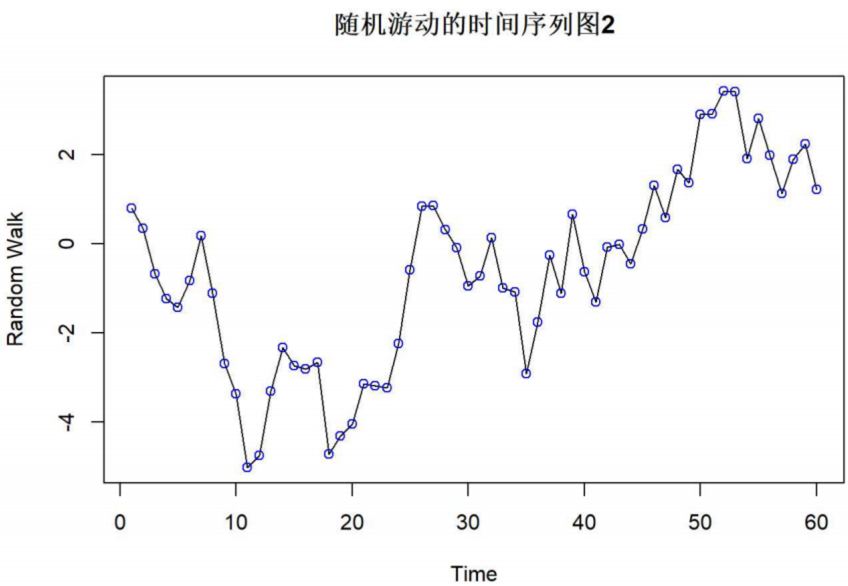
$$\rho_{t,s} = \frac{\gamma_{t,s}}{\sqrt{\gamma_{t,t}\gamma_{s,s}}} = \frac{min(t,s) \cdot \sigma_e^2}{\sqrt{t\sigma_e^2 \cdot s\sigma_e^2}} = \frac{\sqrt{min(t,s)}}{\sqrt{max(t,s)}}$$

```
data(rwalk)
n<-length(rwalk)
plot(rwalk,type='o',ylab='Random Walk', main = '随机游动的时间序列图1')
```

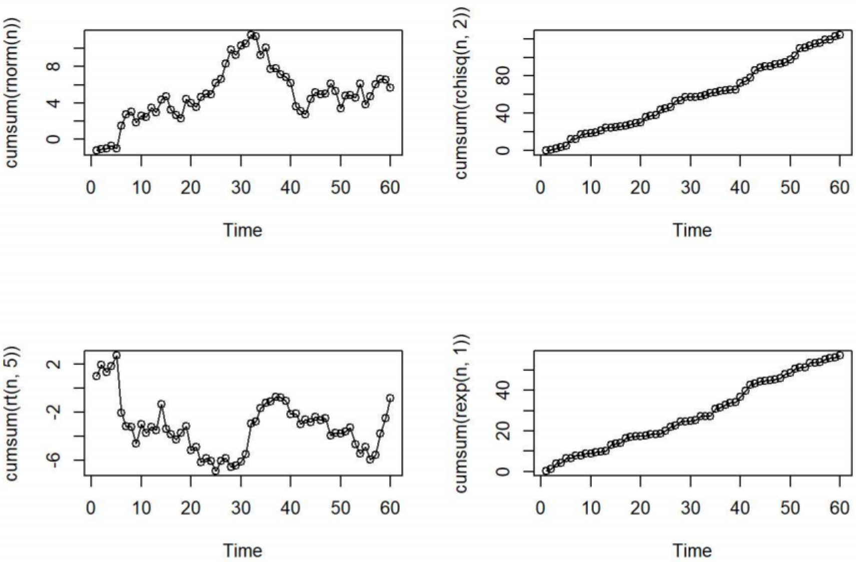



```
###生成随机游动过程方法1
Rwalk <- vector()
Rwalk[1] <- rnorm(1)
for (i in 2:n){
  set.seed(123+i*456)
  Rwalk[i] <- Rwalk[i-1] + rnorm(1)
}
ts_Rwalk <- ts(Rwalk,freq=1,start=1)

plot(ts_Rwalk,type='l',ylab='Random Walk',xlab='Time',main='随机游动的时间序列图2')
points(ts_Rwalk,col='blue')
```



```
###生成随机游动过程方法2：cumsum(rnorm(n))
opar=par(mfrow=c(2,2))
plot(cumsum(rnorm(n)),type='o',xlab='Time');
plot(cumsum(rchisq(n,2)),type='o',xlab='Time');
plot(cumsum(rt(n,5)),type='o',xlab='Time');
plot(cumsum(rexp(n,1)),type='o',xlab='Time');
```




```
par(opar)
```