

(a)

When plotting a circle starting from  $P(x_p, y_p)$  and moving y by 1,  
we can pick from two possible pixels:

$$N = P(x_p, y_p + 1) \text{ or } NW = P(x_p - 1, y_p + 1).$$

Using the midpoint ( $M$  between  $N$  and  $NW$ ) as the decision parameter,  
we can decide which pixel to proceed.

$$\begin{aligned} M &= (x_p - 1/2, y_p + 1) \\ D &= F(M) = F(x_p - 1/2, y_p + 1) \\ &= (x_p - 1/2)^2 + (y_p + 1)^2 - r^2 \end{aligned}$$

If  $D < 0$ ,  $M$  is inside the circle and we can proceed to  $N$ .

Otherwise, if  $D \geq 0$ ,  $M$  is outside/on the circle and we can proceed to  $NW$ .

To find out  $D_{new}$ , we should consider two scenarios:

- 1)  $D_{new}$  after proceeding with N
- 2)  $D_{new}$  after proceeding with NW

1)  $D_{new}$  after proceeding with N ( $x_p, y_p + 1$ ):

$$\begin{aligned} D_{new} &= (x_p - 1/2)^2 + (y_{p+1} + 1)^2 - r^2 \\ D_{old} &= (x_p - 1/2)^2 + (y_p + 1)^2 - r^2 \\ D_{new} - D_{old} &= (x_p - 1/2)^2 + (y_{p+1} + 1)^2 - r^2 - [(x_p - 1/2)^2 + (y_p + 1)^2 - r^2] \\ &= 2(y_p + 1) + 1 \\ \therefore D_{new} &= D_{old} + 2(y_p + 1) + 1 \end{aligned}$$

2)  $D_{new}$  after proceeding with NW ( $x_p - 1, y_p + 1$ ):

$$\begin{aligned} D_{new} &= (x_{p-1} - 1/2)^2 + (y_{p+1} + 1)^2 - r^2 \\ D_{old} &= (x_p - 1/2)^2 + (y_p + 1)^2 - r^2 \\ D_{new} - D_{old} &= (x_{p-1} - 1/2)^2 + (y_{p+1} + 1)^2 - r^2 - [(x_p - 1/2)^2 + (y_p + 1)^2 - r^2] \\ &= 2(y_p + 1) - 2(x_p + 1) + 1 \\ \therefore D_{new} &= D_{old} + 2(y_p + 1) - 2(x_p + 1) + 1 \end{aligned}$$

We can derive  $D_{start}$  by plugging in the initial coordinates ( $r, 0$ ).

$$\begin{aligned} D_{start} &= F(r - 1/2, 1) \\ &= (r - 1/2)^2 + 1 - r^2 \\ &= 5/4 - r \\ &\cong 1 - r \quad (r \text{ is an integer}) \end{aligned}$$

Lastly, we can derive coordinates in 8 symmetric regions by considering eight symmetric points at:  
(x,y), (-x,y), (x,-y), (-x,-y), (y,x), (-y,x), (y,-x), (-y,-x) on a circle.

(b)

OpenGL uses right-handed coordinate system, where positive x-axis is to viewer's right, positive y-axis is up. Origin (0, 0) is located at the bottom-left corner of the window. Therefore, if any of the x and y is less 0, the coordinate won't be shown in the window.