**(a)**

When plotting a circle starting from $P(x_p, y_p)$ and moving y by 1,
we can pick from two possible pixels:
$$N = P(x_p, y_p + 1) \text{ or } NW = P(x_p - 1, y_p + 1).$$

Using the midpoint($M$ between $N$ and $NW$) as the decision parameter,
we can decide which pixel to proceed.
$$M = (x_p - 1/2, y_p + 1)$$
$$D = F(M) = F(x_p - 1/2, y_p + 1)$$
$$= (x_p - 1/2)^2 + (y_p + 1)^2 - r^2$$
If $D < 0, M$ is inside the circle and we can proceed to $N$.
Otherwise, if $D \geq 0, M$ is outside/on the circle and we can proceed to $NW$.

To find out $D_{new}$, we should consider two scenarios:
  1) $D_{new}$ after proceeding with N
  2) $D_{new}$ after proceeding with NW

1) $D_{new}$ after proceeding with N $(x_p, y_p + 1)$:
$$\begin{aligned}
D_{new} &= (x_p - 1/2)^2 + (y_{p+1} + 1)^2 - r^2 \\
D_{old} &= (x_p - 1/2)^2 + (y_p + 1)^2 - r^2 \\
D_{new} - D_{old} &= (x_p - 1/2)^2 + (y_{p+1} + 1)^2 - r^2 - [(x_p - 1/2)^2 + (y_p + 1)^2 - r^2] \\
&= 2(y_p + 1) + 1 \\
\therefore D_{new} &= D_{old} + 2(y_p + 1) + 1
\end{aligned}$$

2) $D_{new}$ after proceeding with NW $(x_p - 1, y_p + 1)$:
$$\begin{aligned}
D_{new} &= (x_{p-1} - 1/2)^2 + (y_{p+1} + 1)^2 - r^2 \\
D_{old} &= (x_p - 1/2)^2 + (y_p + 1)^2 - r^2 \\
D_{new} - D_{old} &= (x_{p-1} - 1/2)^2 + (y_{p+1} + 1)^2 - r^2 - [(x_p - 1/2)^2 + (y_p + 1)^2 - r^2] \\
&= 2(y_p + 1) - 2(x_p + 1) + 1 \\
\therefore D_{new} &= D_{old} + 2(y_p + 1) - 2(x_p + 1) + 1
\end{aligned}$$

We can derive $D_{start}$ by plugging in the initial coordinates (r, 0).
$$\begin{aligned}
D_{start} &= F(r - 1/2, 1) \\
&= (r - 1/2)^2 + 1 - r^2 \\
&= 5/4 - r \\
&\cong 1 - r \qquad \text{(r is an integer)}
\end{aligned}$$

Lastly, we can derive coordinates in 8 symmetric regions by considering eight symmetric points at:
(x,y), (-x,y), (x,-y), (-x,-y), (y,x), (-y,x), (y,-x), (-y,-x) on a circle.


**(b)**

OpenGL uses right-handed coordinate system, where positive x-axis is to viewer's right, positive y-axis is up. Origin $(0, 0)$ is located at the bottom-left corner of the window. Therefore, if any of the x or y is less 0, the coordinate won't be shown in the window.

**How to compile:**
1) Unzip the Assignment1.zip
2) Open terminal and go to the folder.
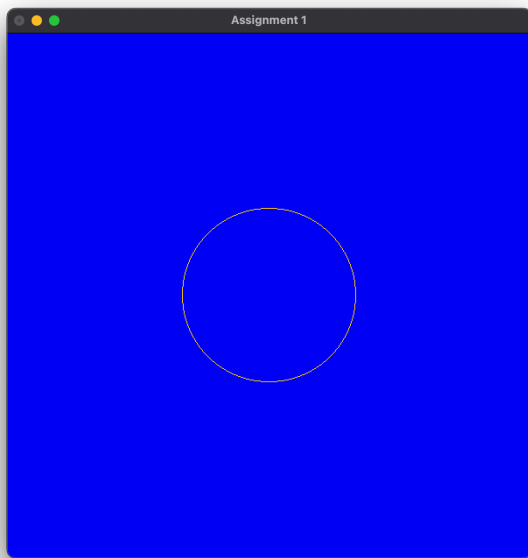3) Run below commands to go to build folder and compile the program.
   $ cd build
   $ cmake ..
   $ make
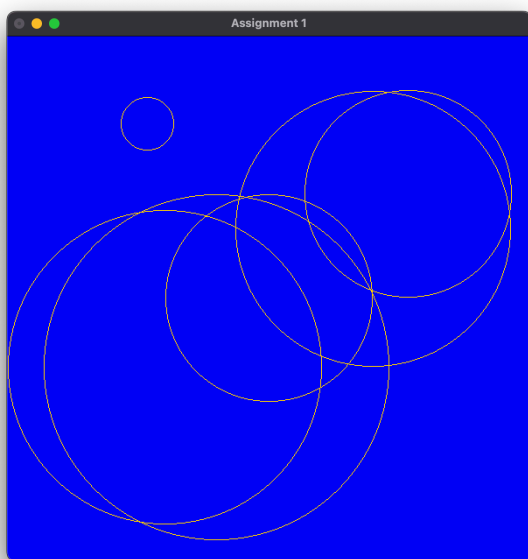4) Run Assignment1 with below command.
   $ ./Assignment1

**How to run:**
1) Draw a circle with 3 integers (x, y, radius)



Example:

```
ahhyunmoon@ahhyuns-mbp build % ./Assignment1
Draw a circle from integers or a file? (i/f):  i
X-coordinate: 300
Y-coordinate: 300
Radius: 100
```

2) Draw circles with an input file (with/without animation)



Example:

No Animation:

```
ahhyunmoon@ahhyuns-mbp build % ./Assignment1
Draw a circle from integers or a file? (i/f):  f
Enter your file name:  input_circles.txt
Add animation (y/n):   n
```

With Animation:

```
ahhyunmoon@ahhyuns-mbp build % ./Assignment1
Draw a circle from integers or a file? (i/f):  f
Enter your file name:  input_circles.txt
Add animation (y/n):   y
```