MECH 489 - Mini Project
Measuring the Thermal Performance of a New Plate Heat Exchanger

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March 20, 2023

Abstract

This report details the design process and planning involved to measure the thermal performance of a novel plate heat exchanger. The recommendation this report provides applies appropriate governing thermodynamics and heat transfer theory and includes instrumentation selection, uncertainty analysis, as well as a cost and time budget breakdown. Relevant industry and academic practices were carefully considered and explained for these modules. Measurands in the experiment were temperature and mass flow rates, fixed parameters were heat transfer coefficients and surface areas, mass flow rates and inlet temperatures were independent variables, and accordingly, the outlet temperatures were the dependent variables. The test follows the test matrix compiled by the team and is recommended to be conducted for specific C_{min}/C_{max} ratios. The uncertainties involved come from both the propagation of equations and instruments used, and the dominating uncertainty is considered in post-processing. Overall, the test would span 8 weeks in its entirety and total an estimated \$18,000. The final outcome would be the ε-NTU curves that when compared to analytical curves will give a measure of the efficacy of the given heat exchanger.

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1 Introduction

The purpose of this experiment is to obtain the performance curves (i.e. the ε - NTU curves) for a new plate heat exchanger. This heat exchanger has two modes of operation.

Table 1: Operating conditions of plate heat exchanger

Option 1	**Hot Fluid: Water**	**Cold Fluid: Water**
Flow Rate	1-2 kg/s	0.25-1 kg/s
Inlet Temperature	60-80°C	5-20°C
Option 2	**Hot Fluid: Air**	**Cold Fluid: Water**
Flow Rate	1000-1500 m^3/hr	0.1-0.25 kg/s
Inlet Temperature	100-150ºC	5-20ºC

This report will discuss the experimental strategy that will be implemented by the experimental team to produce the required performance curves. In addition to this, relevant sketches, test matrix, expected results, and uncertainty will be discussed. The overall cost estimation and an overall schedule will also be provided for this testing procedure. Figure 1 shows the schematic for a general plate heat exchanger.

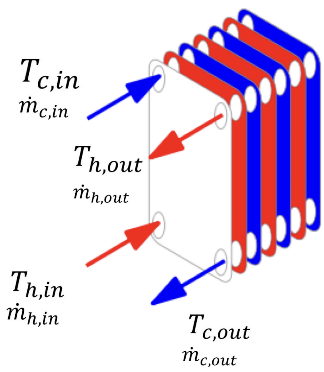


Figure 1: Plate Heat Exchanger [8]

2 Methods

2.1 Governing Equations

A heat exchanger's effectiveness is defined by comparing its actual heat transfer to its theoretical maximum heat transfer. The maximum heat transfer is given by the expression [6, 11].

$$q_{max} = C_{min}(T_{h,i} - T_{c,i}) \tag{1}$$

where,

$$C_{min} = \min(\dot{m}_h c_h, \dot{m}_c c_c) \tag{2a}$$

$$C_{max} = \max(\dot{m}_h c_h, \dot{m}_c c_c) \tag{2b}$$

Therefore, a heat exchanger's effectiveness is defined as:

$$\varepsilon \equiv \frac{q}{q_{max}} = \frac{\dot{m}_h c_h \Delta T_h}{q_{max}} = \frac{\dot{m}_c c_c \Delta T_c}{q_{max}} \tag{3}$$

For any given heat exchanger, it can be shown that the effectiveness is a function of the following parameters [11]

$$\varepsilon = f(\text{NTU}, \frac{C_{min}}{C_{max}}) \tag{4}$$

where

$$\text{NTU} \equiv \frac{UA}{C_{min}} \tag{5}$$

For a given heat exchanger, its overall heat transfer coefficient, surface area, and logarithmic mean temperature difference relate to the overall heat transfer.

$$q = UA\Delta T_{LMTD} = \dot{m}_h c_h \Delta T_h = \dot{m}_c c_c \Delta T_c \tag{6}$$

where

$$\Delta T_{LMTD} = \frac{T_{h,i} - T_{c,o} - (T_{h,o} - T_{c,i})}{\ln \frac{T_{h,i} - T_{c,o}}{T_{h,o} - T_{c,i}}} \tag{7}$$

Rearranging,

$$UA = \frac{\dot{m}_h c_h \Delta T_h}{\Delta T_{LMTD}} \tag{8}$$

$$\text{NTU} = \frac{\dot{m}_h c_h \Delta T_h}{C_{min} \Delta T_{LMTD}} \tag{9}$$

Therefore, both the effectiveness and NTU values depend on the mass flow rates of the cold and hot fluids, the inlet and outlet temperatures of the cold and hot fluids, as well as the specific heat capacity of the two fluids. In the experiment, the mass flow rates and various temperatures will be measurands. The fixed parameters will be the overall heat transfer coefficient and heat transfer surface area specific to this heat exchanger. The independent variables will be the two mass flow rates of the fluids and the inlet temperatures, with the consequent dependent variables being the outlet temperatures. With all of this information, both the NTU and the effectiveness can be calculated for a specific C_{min}/C_{max}. The test matrix for both the options for the heat exchanger is outlined below in Tables 2 and 3. The values of specific heat capacity and density of the working fluids are outlined in Appendix B. The test matrix is created using a combination of temperatures for the working fluids and mass flow rates so roughly get C_{min}/C_{max} values of $0, 0.25, 0.5, 0.75$ and 1.

Table 2: Test Matrix for Option 1

Temperature (Hot)	**Temperature (Cold)**	$\dot{m}$ **(Hot)**	$\dot{m}$ **(Cold)**	C_{min}/C_{max}
68°C	11°C	1 kg/s	0.25 kg/s	0.25
72°C	14°C	2 kg/s	1 kg/s	0.50
72°C	14°C	1 kg/s	0.7 kg/s	0.70
68°C	11°C	1 kg/s	1 kg/s	1.00

Table 3: Test Matrix for Option 2

Temperature (Hot)	**Temperature (Cold)**	$\dot{V}$ **(Hot)**	$\dot{m}$ **(Cold)**	C_{min}/C_{max}
100°C	5°C	1000 m^3/hr	0.25 kg/s	0.25
140°C	17°C	1400 m^3/hr	0.16 kg/s	0.50
120°C	11°C	1200 m^3/hr	0.1 kg/s	0.72
150°C	20°C	1500 m^3/hr	0.1 kg/s	0.85

2.2 Instrumentation and Equipment

The instrumentation needed to collect relevant data relates to the inlet and outlet temperatures of the heat exchanger, along with the mass flow rates. Measuring this will allow the generation of ε-NTU curves according to the background theory discussed earlier.

The three common devices used to measure temperature in the industry are thermocouples, RTDs, and thermistors. For the purposes of this experiment, the team will be proceeding with thermocouples because their ruggedness and robustness make them the best candidate, whereas RTDs and thermistors are expensive, fragile, and often require an external power source. For this application, Type K thermocouples will be used since they offer less error close to 0°C, which will be close to the coldest temperature of water in this heat exchanger. For air, the tip of the thermocouple will be exposed and embedded into the pipe with the airflow, in order to keep the procedure as simple as possible. For water, the tip cannot be exposed so the team recommends using a stainless steel probe with its end welded into the water pipes for accurate readings. The table below summarizes the thermocouples that will be used in this experiment.

Table 4: Thermocouples

Manufacturer	**Temperature Range**	**Application**
Red Lion (Exposed Tip) [18]	2 to 570°C	Air
Fluke 80PT-25 (Stainless Steel Probe) [1]	-196 to 250°C	Water

As for flow measurement, the team recognizes that measuring air flow and water flows are vastly different applications and thus are best achieved using separate instruments. The airflow that needs to be measured is as high as 1500 m^3/hr and so, a hot wire anemometer is best suited for this application as it strikes a good balance between allowing high enough flow rates with relatively accurate measurement. For water, a rotameter can be used which offers great accuracy in the results. A rotameter cannot be used with air because commercially available rotameters are typically not rated for such high flow rates. The table below summarizes the flow measurement devices. The water rotameter contains a scale and flow can be manually adjusted until the desired value in the scale is reached. The hot wire anemometer outputs the flow rate in its own handheld device so the airflow can be manually adjusted in a similar manner to the water to achieve desired flow rates.

Table 5: Flow Measurement

Manufacturer/Model	**Flow Rate Capacity**	**Application**
Brook Instruments Sho-Rate™ Flow Meter 1250A [21]	2.1 L/s	Water
Testo 405i Hot Wire Anemometer [24]	170,000 m^3/hr	Air

The major equipment needed for this experiment is the water chillers and burners to get the water to the required temperatures for this heat exchanger. Similarly, the air also needs to be accordingly heated. According to BC Hydro, readily available tap water is available for use as hot as 60°C [5], which can be further heated to supply the hot water. Tap water can also be available at 20°C which can be further cooled to supply the cold water to the heat exchanger. Looking at Table 2, the maximum heating load for the water occurs at a flow rate of 2 kg/s at a supply temperature of 72°C. The maximum cooling load for the water across both options occurs at a flow rate of 1 kg/s with a temperature of 11°C. Similarly Table 3 shows the max heating load for the air occurs at a temperature of 150°C and a flow rate of 150 m^3/hr (0.5 kg/s at STP conditions). The heating and cooling loads then become

$$Q_{HL,water} = (\dot{m}c_p\Delta T)_{HL,water} \tag{10a}$$

$$Q_{CL,water} = (\dot{m}c_p\Delta T)_{CL,water} \tag{10b}$$

$$Q_{HL,air} = (\dot{m}c_p\Delta T)_{HL,air} \tag{10c}$$

Assuming constant specific heat capacities, the maximum heating and cooling loads for air and water are

$$Q_{HL,water} = (2\frac{\text{kg}}{\text{s}})(4.2\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}})(70-12)^\circ\text{C} = 100 \text{ kW} \tag{11a}$$

$$Q_{CL,water} = (1\frac{\text{kg}}{\text{s}})(4.2\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}})(20-11)^\circ\text{C} = 38 \text{ kW} \tag{11b}$$

$$Q_{HL,air} = (0.5\frac{\text{kg}}{\text{s}})(1\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}})(150-25)^\circ\text{C} = 63 \text{ kW} \tag{11c}$$

Now that the respective heating and cooling loads have been determined, consider table 6 which summarizes the heating and cooling equipment needed for this project. One of the water chillers listed above will be sufficient at providing enough cooling to the supply of water. It also has a built-in tank with enough capacity to allow return cold water to be recirculated into the chiller again. As for the burner, since one burner of 100 kW is not commercially as available and cheap, purchasing three burners

Table 6: Heating and Cooling Equipment

Manufacturer/Model	Capacity	Application
LNEYA FL-3500 Water Chiller [10]	40 kW	Water
AO Smith Commercial Gas Tankless Heater [3]	35 kW	Air and Water

and running them in parallel is the best way to heat up the supply of water. A third of the supply water would go into each burner and reconvene before heading to the heat exchanger. Two of the three burners can also be repurposed to work with the hot supply air.

In addition to this, there is the pipework that is needed along with the appropriate valves and pumps. In order to keep the procedure simple, the use of only a pressure relief valve and a ball valve to control flow rates will be needed. In addition to this, the pipes themselves are required as a storage reservoir for the hot water. One for cold water is not needed since the chiller has an inbuilt storage tank. For air, a pressure relief damper will be used with a blower, along with a manual balancing damper to control the flow rate.

Table 7: Piping and Ductwork

Manufacturer/Model	Specification	Quantity
Centrifugal Pump - 3" Impeller, 1/3HP [22]	33.3 GPM	2
McMaster-Carr Blower (1963K15) [7]	900 CFM	1
iO HVAC Controls Manual Balancing Damper [12]	16" Round	1
TANA 1-1/2 Inch Full Port Ball Valve [23]	1000 PSI	2
Pressure Relief Valve Water [16]	1.5"	2
Model BD-250 Round Pressure Relief Damper [4]	16" Round	1
McMaster-Carr Welded Steel Piping [13]	1.5" [15]	50 m
S-Tl Thermaflex Flexible HVAC Duct [20]	16" Round [14]	25 m
Rheem Commercial Storage Tank w/ Insulation [19]	120 Gal	1
Miscellaneous Pipe and Duct Fittings	-	-

The pipes and ducts have been sized in accordance with the existing guides in the references. Note that the lengths of the pipe and duct needed are mere estimates and the actual length needed will depend on the location where the experiment will take place in. The same goes for the required pipe and duct fittings. Consequently, they have been included in planning for this experiment and the cost they add to the project will be treated as estimates.

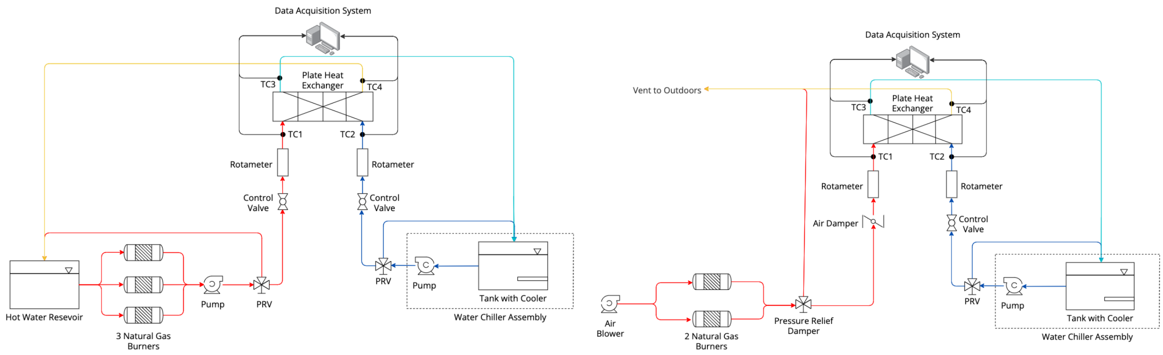
The data acquisition system needed only needs to collect temperature data, so National Instrument's USB-6000 [17] will provide sufficient slots to measure the temperatures.

2.3 Proposed Layout

The proposed layout for our test setup is shown in Figure 2 for both options. The setup comprises the instrumentation outlined in Section 2.2 as appropriate for a water or air system. The schematic for each option is described below.

2.3.1 Option 1: Water - Water System

The entire system can be initially flooded using the hot and cold water supplied from the tap at 60°C or 20°C respectively. The flow rates in the system are achieved using a pump each for the hot and cold water, along with a ball valve to regulate the flow rate to the desired level and a check valve to prevent backflow. The ball valve is placed between the pump and the exchanger to improve its ability to control the flow rates in the heat exchanger, and the check valve is placed before the pump to prevent backflow into the heater/chiller in case of downstream pressure build-up. The outlet from either end of the heat exchanger is discharged back to the supply tank for hot and cold water to prevent excessive wastage of water during testing. The flow rate is measured at the inlet using the rotameter mentioned above and the temperature is measured both at the inlet and outlet using the thermocouples. Both



(a) Option 1 (b) Option 2

Figure 2: Piping and Instrumentation Layout

the data acquisition systems are kept close to the inlet of the heat exchanger to get the most accurate exchanger inlet conditions. Finally, the thermocouples connect to the DAQ module and a PC to store and pre-process the data visually.

2.3.2 Option 2: Water - Air System

The air-water heat exchanger is nearly identical to the one described in Section 2.3.1. The only differences are the use of a blower to feed the hot fluid half of the exchanger and using two heaters instead of three. Furthermore, there is no recovery of the energy from the air at the outlet of the exchanger as it is discharged to the surroundings. In terms of instrumentation, there is the air thermocouple and rotameter for the hot fluid side of the exchanger.

3 Expected Results and Uncertainties

As discussed in Section 2.1, the effectiveness of a heat exchanger has a strong dependence on its operating conditions. Specifically, this dependence is on the value of NTU and the ratio $\frac{C_{min}}{C_{max}}$. Heat transfer textbooks and guides contain experimental and analytical ε-NTU curves [6]. Although this figure is

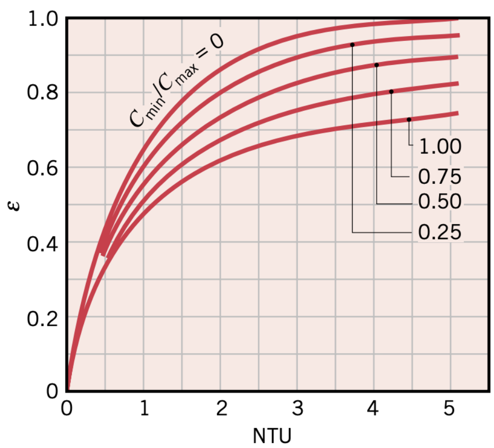


Figure 3: Typical ε-NTU Curve [6]

extracted from a single-pass, cross-flow heat exchanger, the general shape of the curves would hold for the plate heat exchanger being used in this experiment. As the figure shows, slower-moving fluids, characterized by a smaller value of C_{min}, will have higher effectiveness because there is more time for heat transfer to occur. Similarly, a higher NTU value, which is a non-dimensionalization of the overall heat transfer coefficient and surface area, also enhances heat transfer, allowing higher effectiveness to take place. The effect of increasing NTU, however, eventually diminishes in returns of effectiveness because, at this point, heat transfer is governed by the respective flow rates and specific heat capacities.

Table 8: Uncertainty in Equipment

Equipment	Resolution	Accuracy	Repeatability
Red Lion Thermocouple	0.1°C	±0.5%	± 0.25°C
Fluke Thermocouple	0.1°C	max(± 1°C, ± 0.75%)	-
Testo 405i Hot Wire Anemometer	0.2 m^3/hr	± 50 m^3/hr	-
Brook Instruments Flow Meter	0.002 kg/s	±5%	±0.25% FS

Our experiment has two main uncertainties associated with it: uncertainty in temperature and uncertainty in mass flow rate. The thermocouples and flow meters specified for the experiment have their own uncertainties associated with them that are reported in Table 8; the specific heat capacity of water did not have any uncertainty, but here, the uncertainty in the specific heat capacity and density of air as a function of temperature will be neglected. The uncertainties present in our measuring equipment translate to the uncertainty in our ε-NTU values according to Appendix A.1. The team recognizes that the pump, chiller, and heater specified in the experiment have their own internal controllers present and some noise in the heat flux or flow rate due to their internal feedback systems, the team expects that the time average of the result at steady-state will be sufficient to address it. The worst-case total uncertainty in the specified equipment is shown in table 9. The overall uncertainty is limited by the thermocouple reading.

Table 9: Worst Case Total Uncertainty in Equipment

Equipment	Total Uncertainty	Percentage Uncertainty
Red Lion Thermocouple	0.4°C	0.4%
Fluke Thermocouple	1.0°C	20%
Testo 405i Hot Wire Anemometer	50 m^3/hr	5%
Brook Instruments Flow Meter	0.1 kg/s	7%

4 Cost Estimate and Experiment Timeline

The estimated timeline of the testing plan is highlighted below. Staggering tests is of the utmost importance here to obtain four ε-NTU curves for each option and perform three repetitions of our tests. The slowest step in the process is heating up water to the required temperature and filling up the hot water tank each run of the experiment.

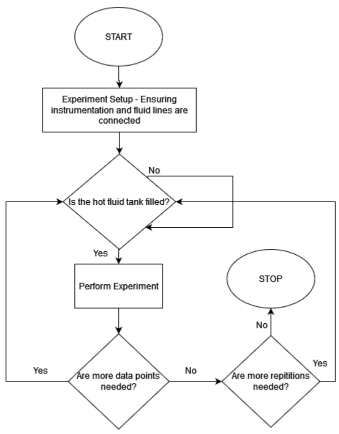


Figure 4: Experiment Flowchart

If the team had duplicated some of the equipment specified above there would have been an option of performing the air-water heat exchanger tests while the hot water tank fills up, but in this timeline calculation, it is assumed that this option does not exist. Figure 4 shows the condensed workflow for the experiment, it must be noted that cold water is not considered in the limiting timestep as the chiller is able to cool the water and maintain the required flow rate continuously. Table 10 outlines the estimated worse-case times needed to run the experiment, the total time per repetition is calculated assuming that 4 curves per option and the total time is calculated assuming three repetitions are performed.

Table 10: Experiment Timeline

Experiment Step	**Time Option 1 (mins)**	**Time Option 2 (mins)**
Experiment Setup	30	30
Heating Hot Fluid	12	N/A
Cooling Cold Fluid	N/A	N/A
Running Test for 1 ε-NTU Curve	60	60
Time per repetition	318	270
Total Time		**29.4 hrs or 4 days**

The equipment, especially the heaters, and chillers, for the project will take between 4 - 6 weeks to arrive after they have been ordered at the end of the planning phase of the project. Once the equipment arrives and is installed the experiments will take around 4 days working 8 hours a day as roughly shown in Table 10 and the post-processing of the information will take roughly a week. Thus, the team expects to be able to deliver the final results regarding the heat exchanger approximately 8 weeks after the end of the planning phase.

Table 11: Bill of Materials

Item	**Unit Cost (C$)**	**Qty**	**Net Price (C$)**
Red Lion Thermocouple for Air	267.00	2	534.00
Fluke 80PT-25 Thermocouple for Water	191.11	4	764.44
Brook Instruments Sho-Rate™ Flow Meter	352.87	2	705.74
Testo 405i Hot Wire Anemometer	181.96	1	181.96
LNEYA FL-3500 Water Chiller	575.99	1	575.99
AO Smith Commercial Gas Tankless Heater	1734.99	3	5204.97
Centrifugal Pump - 3" Impeller, 1/3HP	1,442.00	2	2884.00
McMaster-Carr Blower (1963K15)	786.76	1	786.76
iO HVAC Controls Manual Balancing Damper	175.28	1	175.28
TANA 1-1/2 Inch Full Port Ball Valve	82.61	2	165.22
Pressure Relief Valve Water	406.52	2	813.04
Model BD-250 Round Pressure Relief Damper	77.00	1	77.00
McMaster-Carr Welded Steel Piping	10.12/ft	50 m	1660.10
S-Tl Thermaflex Flexible Hvac Duct	48.13/m	25 m	1203.27
Miscellaneous Pipe and Duct Fittings	-	-	200.00
Rheem Commercial Storage Tank w/ Insulation	1850.00	1	1850.00
NI USB 6000	335.00	1	335.00
Natural Gas (Appendix A.2)	0.40	178 m^3	70.40
Net Total			18,187.17

The table below summarizes the comprehensive list of instrumentation and equipment needed to undertake this project. Not all of these items may require purchasing, depending on the availability of these materials with the client. Still, to be on the side of caution, they have been included in this preliminary budgetary analysis.

5 Conclusions

The purpose of this experiment was to obtain the performance curves (i.e. the ε - NTU curves) for a new plate heat exchanger with two modes of operation. Any given heat exchanger's effectiveness is defined by comparing its actual heat transfer to its theoretical maximum heat transfer. The governing equations were acquired from thermodynamic and heat transfer theory and it was seen that the effectiveness and NTU values depend on the mass flow rates of the cold and hot fluids, the inlet and outlet temperatures of the cold and hot fluids, as well as the specific heat capacity of the two fluids.

In the experiment, the mass flow rates, and various temperatures were the measurements, the overall heat transfer coefficient and heat transfer surface area specific to this heat exchanger were the fixed parameters, and the two mass flow rates of the fluids and the inlet temperatures were the independent variables. Consequently, the dependent variable is the outlet temperatures. With all of this information, both the NTU and the effectiveness can be calculated for a specific C_{min}/C_{max} and a test matrix was obtained for both options to roughly get C_{min}/C_{max} values of 0, 0.25, 0.5, 0.75, and 1.

The set of instruments selected to take temperature and flow rate measurements were thermocouples and an anemometer (airflow)/rotameter (water flow). They were chosen after careful consideration of industry practices, the needs of this specific test apparatus, and the overall robustness of the options across categories like accuracy, price, and durability. The data acquisition instrumentation was also similarly selected. The other equipment specified was for the test setup and included water chillers, burners, air compressors, pipes, air ducts, check valves, ball valves, storage tanks, and an air duct damper, blower, and pump, along with natural gas. The count of components and their specific models have been selected after careful deliberations over the requirements as well as for design scale brevity. All this equipment was laid out in a design that would be adaptable to most test facilities and the test setup was finalized. Overall, the cost of such a test system was kept to a minimum and totaled an estimated $18,187.

The remaining physical work to be performed included filling the tanks and gas storages as needed, getting data points, and reiterating as needed according to control checks. The post-processing included considering the uncertainties in our governing equations and uncertainties with rigorous propagation, taking the higher of the two uncertainties. The critical uncertainty was found to be in the thermocouple. Considering this, our final performance curves were to be generated and compared to analytical ε - NTU curves. This would be our determining result on the efficiency of a given plate heat exchanger. In our preliminary cost and time budget, the team expects that including purchase orders, delivery, setup, the bottleneck of tank filling, data acquisition, and post-processing our experiment to span approximately 8 weeks.

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[26] *Water - Specific Heat vs. Temperature*. URL: https://www.engineeringtoolbox.com/specific-heat-capacity-water-d_660.html.

A Derivations and Calculations

A.1 Uncertainty Calculations

The uncertainty for a variable $R = f(x_1, x_2, x_3, ..., x_n)$ is given by the expression

$$u_R = \left[\sum_{i=1}^{n} \left(\frac{\partial R}{\partial x_i} u_{x_i} \right)^2 \right]^{\frac{1}{2}} \tag{12}$$

Now, consider the following sample calculation that finds the uncertainty in q_{max}.

$$q_{max} = C_{min} \left(T_{h,i} - T_{c,i} \right) \tag{13a}$$

Taking partial derivative

$$\frac{\partial q_{max}}{\partial C_{min}} = T_{h,i} - T_{c,i} \tag{13b}$$

$$\frac{\partial q_{max}}{\partial T_{h,i}} = C_{min} \tag{13c}$$

$$\frac{\partial q_{max}}{\partial T_{c,i}} = -C_{min} \tag{13d}$$

Using uncertainty equation

$$u_{q_{max}} = \sqrt{\left(\frac{\partial q_{max}}{\partial C_{min}} u_{C_{min}} \right)^2 + \left(\frac{\partial q_{max}}{\partial T_{h,i}} u_{T_{h,i}} \right)^2 + \left(\frac{\partial q_{max}}{\partial T_{c,i}} u_{Tc,i} \right)^2} \tag{13e}$$

$$u_{q_{max}} = \sqrt{\left(\left(T_{h,i} - T_{c,i} \right) u_{C_{min}} \right)^2 + \left(C_{min} u_{T_{h,i}} \right)^2 + \left(-C_{min} u_{Tc,i} \right)^2} \tag{13f}$$

The same process has been applied to all the relevant equations from Section 2.1.

$$u_{q_{max}} = \sqrt{\left(\left(T_{h,i} - T_{c,i} \right) \ u_{c_{min}} \right)^2 + \left(C_{min} \ u_{T_{h,i}} \right)^2 + \left(-C_{min} \ u_{T_{c,i}} \right)^2} \tag{14}$$

$$u_{C_{min}} = \sqrt{\left(c u_{\dot{m}} \right)^2 + \left(\dot{m} u_c \right)^2} \tag{15}$$

$$u_{C_{max}} = \sqrt{\left(c u_{\dot{m}} \right)^2 + \left(\dot{m} u_c \right)^2} \tag{16}$$

$$u_\varepsilon \equiv \sqrt{\left(\frac{1}{q_{max}} u_q \right)^2 + \left(-\frac{q}{q_{max}} u_{q_{max}} \right)^2} \tag{17}$$

$$= [(\frac{c_h(T_{h,o} - T_{h,i})}{q_{max}} u_{\dot{m}_h})^2 + (\frac{\dot{m}_h(T_{h,o} - T_{h,i})}{q_{max}} u_{c_h})^2 + (\frac{\dot{m}_h c_h}{q_{max}} u_{T_{h,o}})^2 + ... \tag{18}$$

$$...(\frac{-\dot{m}_h c_h}{q_{max}} u_{T_{h,i}})^2 + (\frac{-\dot{m}_h c_h (T_{h,o} - T_{h,i})}{q_{max}^2} u_{q_{max}})^2]^{\frac{1}{2}} \tag{19}$$

$$= [(\frac{c_h(T_{c,o} - T_{c,i})}{q_{max}} u_{\dot{m}_h})^2 + (\frac{\dot{m}_h(T_{c,o} - T_{c,i})}{q_{max}} u_{c_h})^2 + (\frac{\dot{m}_h c_h}{q_{max}} u_{T_{c,o}})^2 + (\frac{-\dot{m}_h c_h}{q_{max}} u_{T_{c,i}})^2 + ... \tag{20}$$

$$...(\frac{-\dot{m}_h c_h (T_{c,o} - T_{c,i})}{q_{max}^2} u_{q_{max}})^2]^{\frac{1}{2}} \tag{21}$$

$$u_{NTU} = \sqrt{\left(\frac{1}{C_{min}} u_{UA} \right)^2 + \left(\frac{-UA}{C_{min}^2} u_{C_{min}} \right)^2} \tag{22}$$

$$u_q = \sqrt{(\Delta T_{LMTD} u_{UA})^2 + (UA u_{\Delta T_{LMTD}})^2} \tag{23}$$

$$= \sqrt{(c_h(T_{h,o} - T_{h,i})u_{\dot{m}_h})^2 + (\dot{m}_h(T_{h,o} - T_{h,i})u_{c_h})^2 + (\dot{m}_h c_h u_{T_{h,o}})^2 + (\dot{m}_h c_h u_{T_{h,i}})^2} \tag{24}$$

$$= \sqrt{(c_h(T_{c,o} - T_{c,i})u_{\dot{m}_h})^2 + (\dot{m}_h(T_{c,o} - T_{c,i})u_{c_h})^2 + (\dot{m}_h c_h u_{T_{c,o}})^2 + (\dot{m}_h c_h u_{T_{c,i}})^2} \tag{25}$$

$$u_{\Delta T_{LMTD}} = [(\frac{\ln(\frac{T_{h,o}-T_{c,o}}{T_{h,o}-T_{c,i}}) - T_{h,i} + T_{c,o} + T_{h,o} - T_{c,i}T_{h,i} - T_{c,o}}{\ln^2(\frac{T_{h,i}-T_{c,o}}{T_{h,o}-T_{c,i}})} u_{T_{h,i}})^2 + ... \tag{26}$$

$$...(\frac{-\ln(\frac{T_{h,o}-T_{c,o}}{T_{h,o}-T_{c,i}}) + \frac{T_{h,i}-T_{c,o}-T_{h,o}-T_{c,i}}{T_{h,i}-T_{c,o}}}{\ln^2(\frac{T_{h,i}-T_{c,o}}{T_{h,o}-T_{c,i}})} u_{T_{c,o}})^2 + ... \tag{27}$$

$$...(\frac{-\ln(\frac{T_{h,o}-T_{c,o}}{T_{h,o}-T_{c,i}}) + \frac{T_{h,i}-T_{c,o}-T_{h,o}+T_{c,i}}{T_{h,o}-T_{c,i}}}{\ln^2(\frac{T_{h,i}-T_{c,o}}{T_{h,o}-T_{c,i}})} u_{T_{h,o}})^2 + ... \tag{28}$$

$$...(\frac{\ln(\frac{T_{h,o}-T_{c,o}}{T_{h,o}-T_{c,i}}) - \frac{T_{h,i}+T_{c,o}+T_{h,o}+T_{c,i}}{T_{h,o}-T_{c,i}}}{\ln^2(\frac{T_{h,i}-T_{c,o}}{T_{h,o}-T_{c,i}})} u_{T_{c,i}})^2]^{\frac{1}{2}} \tag{29}$$

$$u_{UA} = [(\frac{c_h(T_{h,o} - T_{h,i})}{\Delta T_{LMTD}} u_{\dot{m}_h})^2 + (\frac{\dot{m}_h(T_{h,o} - T_{h,i})}{\Delta T_{LMTD}} u_{c_h})^2 + (\frac{\dot{m}_h c_h}{\Delta T_{LMTD}} u_{T_{h,i}})^2 + ... \tag{30}$$

$$...(\frac{-\dot{m}_h c_h}{\Delta T_{LMTD}} u_{T_{h,i}})^2 + (\frac{-\dot{m}_h c_h (T_{h,i} - T_{h,o})}{\Delta T_{LMTD}^2} u_{\Delta T_{LMTD}})^2]^{\frac{1}{2}} \tag{31}$$

$$u_{NTU} = [(\frac{c_h(T_{h,o} - T_{h,i})}{C_{min}\Delta T_{LMTD}} u_{\dot{m}_h})^2 + (\frac{\dot{m}_h(T_{h,o} - T_{h,i})}{C_{min}\Delta T_{LMTD}} u_{c_h})^2 + (\frac{\dot{m}_h c_h}{C_{min}\Delta T_{LMTD}} u_{T_{h,o}})^2 + ... \tag{32}$$

$$...(\frac{-\dot{m}_h c_h}{C_{min}\Delta T_{LMTD}} u_{T_{h,i}})^2 + (\frac{-\dot{m}_h c_h (T_{h,o} - T_{h,i})}{C_{min}^2 \Delta T_{LMTD}} u_{C_{min}})^2 + ... \tag{33}$$

$$...(\frac{-\dot{m}_h c_h (T_{h,i} - T_{h,o})}{C_{min}\Delta T_{LMTD}^2} u_{\Delta T_{LMTD}})^2]^{\frac{1}{2}} \tag{34}$$

A.2 Natural Gas Cost Estimate

In this section, the cost of the natural gas needed to run the experiments will be discussed. Note that the approximate energy density of natural gas is 40 MJ/m^3 [9] and the current cost of natural gas is 8.23 USD/ft^3 (0.40 CAD/m^3) [25]. Note that for each of the two operating options (Water-Water or Air-Water), there are 4 distinct conditions and there will be 3 trials of each condition. Another assumption is that each trial will run for one hour (3600 s), which the team estimates to be enough time for the heat exchanger to reach a steady state condition. For Option 1 (Water-Water), the heating load was previously determined to be 100 kW. Therefore, the volume of natural gas needed is

$$V_{fuel,1} = \frac{(100 \times 10^3 \text{ W})(3600 \text{ s})(4 \text{ conditions})(3 \text{ trials})}{40 \times 10^6 \text{ J/m}^3} = 108 \text{ m}^3 \tag{35}$$

Similarly, for Option 2 (Air-Water), the heating load is 63 kW, so the volume becomes

$$V_{fuel,2} = \frac{(63 \times 10^3 \text{ W})(3600 \text{ s})(4 \text{ conditions})(3 \text{ trials})}{40 \times 10^6 \text{ J/m}^3} = 68 \text{ m}^3 \tag{36}$$

The net volume of fuel needed becomes

$$V_{fuel,net} = V_{fuel,1} + V_{fuel,2} = 176 \text{ m}^3 \tag{37}$$

Thus, the total cost becomes

$$\text{Natural Gas Cost} = 176 \text{ m}^3 \times 0.40 \text{ CAD/m}^3 = 70.40 \text{ CAD} \tag{38}$$

Note that this cost is an estimate and provides a rough idea of the expected fuel costs that the client can expect for the duration of this project.

B Material Properties

The material properties for water were obtained from reference [26]. Since the uncertainty for these values was not specified, negligible uncertainty will be assumed in the material properties of water.

Table 12: Material Properties of Water vs Temperature

Temperature [°C]	**Specific Heat Capacity** [$kJ/kg.K$]	**Density** [kg/m^3]
5	4.21	999.93
8	4.20	999.80
11	4.19	999.56
14	4.19	999.20
17	4.19	998.74
20	4.18	998.19
60	4.19	983.19
64	4.19	981.07
68	4.19	978.87
72	4.19	976.58
76	4.20	974.21
80	4.20	971.76

The material properties of air were obtained from reference [2]. The source reported an uncertainty of 0.2% for both the specific heat capacity and density. Note that the uncertainties and changes in the specific heat capacities are neglected here because the instrument uncertainty will be dominant.

Table 13: Material Properties of Air vs Temperature

Temperature [°C]	**Specific Heat Capacity** [$kJ/kg.K$]	**Density** [kg/m^3]
100	1.011	0.947
110	1.013	0.922
120	1.014	0.898
130	1.015	0.876
140	1.016	0.854
150	1.018	0.834