

## MA331 Homework02

**Problem 1.** Assume that  $N \sim \mathcal{B}(n, p)$ , a Binomial distribution with number of trials  $n$  and probability of success  $p$ . Set  $p = 0.4$ .

- (i) For  $n = 20, 30, 50, 75, 100$ , accurately compute  $P(N \leq 8.25)$  by using R function.
- (ii) For  $n = 20, 30, 50, 75, 100$ , approximate  $P(N \leq 8.25)$  by using Laplace theorem.
- (iii) Evaluate errors of all approximations of (ii), i.e., the absolute difference between the accurate computation and the Laplace approximation, and the scatter plot the errors.
- (iv) What do you perceive based on the scatter plot of (iii).

**Problem 2.** By using R function generate a simple and random sample of size  $n$  for the population  $X \sim \mathcal{N}(2, 3^2)$ .

- (i) For  $n = 20$ , simulate  $\frac{\bar{X}-2}{\sqrt{3^2/n}}$  and  $\frac{(n-1)S^2}{3^2}$  for 100 times, and then scatter plot all observations of  $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ ,  $\frac{(n-1)S^2}{3^2}$  and  $\left(\frac{\bar{X}-2}{\sqrt{3^2/n}}, \frac{(n-1)S^2}{3^2}\right)$ , respectively.
- (ii) For  $n = 30$ , simulate  $\frac{\bar{X}-2}{\sqrt{3^2/n}}$  and  $\frac{(n-1)S^2}{3^2}$  for 100 times, and then scatter plot all observations of  $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ ,  $\frac{(n-1)S^2}{3^2}$  and  $\left(\frac{\bar{X}-2}{\sqrt{3^2/n}}, \frac{(n-1)S^2}{3^2}\right)$ , respectively.
- (iii) For  $n = 50$ , simulate  $\frac{\bar{X}-2}{\sqrt{3^2/n}}$  and  $\frac{(n-1)S^2}{3^2}$  for 100 times, and then scatter plot all observations of  $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ ,  $\frac{(n-1)S^2}{3^2}$  and  $\left(\frac{\bar{X}-2}{\sqrt{3^2/n}}, \frac{(n-1)S^2}{3^2}\right)$ , respectively.
- (iv) For  $n = 75$ , simulate  $\frac{\bar{X}-2}{\sqrt{3^2/n}}$  and  $\frac{(n-1)S^2}{3^2}$  for 100 times, and then scatter plot all observations of  $\frac{\bar{X}-2}{\sqrt{3^2/n}}$ ,  $\frac{(n-1)S^2}{3^2}$  and  $\left(\frac{\bar{X}-2}{\sqrt{3^2/n}}, \frac{(n-1)S^2}{3^2}\right)$ , respectively.
- (v) Based on the plots of  $\frac{\bar{X}-2}{\sqrt{3^2/n}}$  and  $\frac{(n-1)S^2}{3^2}$  in (i) - (iv) describe your findings on probability distributions of  $\frac{\bar{X}-2}{\sqrt{3^2/n}}$  and  $\frac{(n-1)S^2}{3^2}$ , respectively.
- (vi) Based on the plots of  $\left(\frac{\bar{X}-2}{\sqrt{3^2/n}}, \frac{(n-1)S^2}{3^2}\right)$  in (i) - (iv) describe your findings on the statistical association between  $\frac{\bar{X}-2}{\sqrt{3^2/n}}$  and  $\frac{(n-1)S^2}{3^2}$ .