Q3. Singular Value Decomposition (SVD): ¶

Perform Singular Value Decomposition on the matrix A obtained in Question 2. Separate and print matrices U, Σ , and V $\dot{\circ}$. Verify that A equals the product of U, Σ , and V $\dot{\circ}$. Additionally, find the rank 2 and rank 3 approximations of matrix A.

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In [5]: import numpy as np
       import pandas as pd
       import matplotlib.pyplot as plt
       import seaborn as sns
In [6]: A = np.random.randint(0, 10, size=(5, 5))
       print("Original matrix A:")
       print(A)
       Original matrix A:
       [[1 7 4 5 5]
        [0 7 4 9 7]
        [0 0 8 4 8]
        [5 8 2 1 0]
        [5 4 3 6 9]]
In [7]: #Q 3
       # Perform Singular Value Decomposition (SVD)
       U, Sigma, V_transpose = np.linalg.svd(A)
       Sigma_mat = np.zeros_like(A, dtype=float)
       Sigma_mat[:min(A.shape[0], A.shape[1]), :min(A.shape[0], A.shape[1])] = np.
       print("Matrix U:")
       print(U)
       print("\nMatrix Sigma:")
       print(Sigma_mat)
       print("\nMatrix V:")
       print(V_transpose.T)
       Matrix U:
       [[-0.43668603 0.19289904 0.25467238 -0.23815702 -0.80654726]
        [-0.41201734 -0.63414725 -0.27164311 -0.57461949 0.15531079]
        [-0.5116254 -0.04286924 -0.55292861 0.6486873 -0.09938022]]
       Matrix Sigma:
       [[23.91825206 0.
                              0.
                                         0.
                                                   0.
        [ 0.
                   9.74520972 0.
                                         0.
                                                   0.
                             5.32873449 0.
        [ 0.
                   0.
                                                   0.
                                                            ]
        [ 0.
                                        4.46507646 0.
                    0.
                              0.
        [ 0.
                    0.
                              0.
                                         0.
                                                   0.784721
       Matrix V:
       [-0.46023572 0.75474543 0.25136467 -0.20790066 -0.33486643]
        [-0.38976549 -0.28954515 -0.16782382 -0.81634393 0.26393981]
        [-0.5109187 -0.08461727 0.45476037 0.37993813 0.61696193]
        [-0.58661697 -0.44063346 -0.24049561 0.30565987 -0.55718632]]
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In [8]: # Now we show that A equals the product of U, Sigma, and V
A_reconstructed = np.dot(U, np.dot(Sigma_mat, V_transpose))
    print("\nVerification that A equals the product of U, Sigma, and V:")
    print(np.allclose(A, A_reconstructed))

Verification that A equals the product of U, Sigma, and V:
    True
In []:
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