Theory of Computation

Assignment

Sub Code: CSE-223

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CSE – 3

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TOC - Assignment

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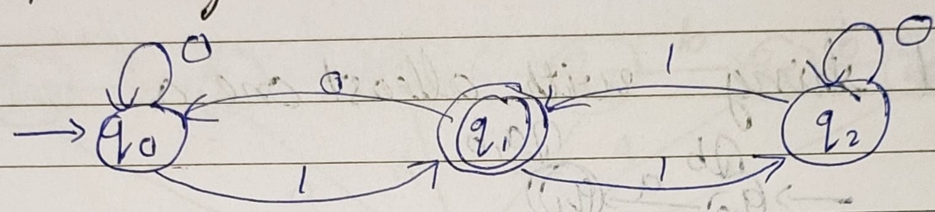
Q1. Which strings 0001, 01001, 0000110 are accepted by DFA



01001 is accepted

$\to q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1$

final state so accepted

0,1001 is accepted

$q_0 \xrightarrow{0} q_0 \xrightarrow{[illegible]} q_{[illegible]} \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1$

final state so accepted

0000110 is NOT accepted

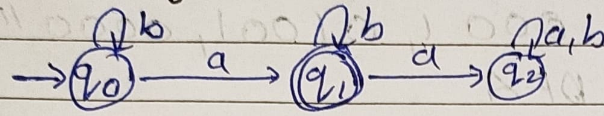
$\to q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_2 \xrightarrow{0} q_2$

Non-final state so not accepted

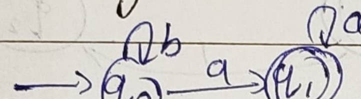
Therefore 0001, 01001 are accepted.

2. For $\Sigma = \{a, b\}$, contruct dfa's that accept the sets consisting of

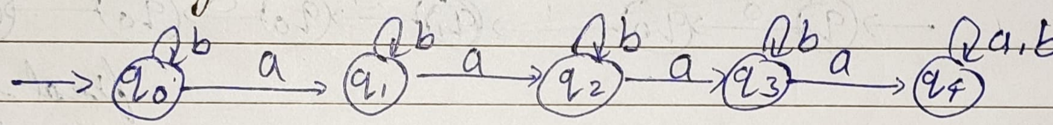
a. all strings with exactly one a



b. all string with atleast one a

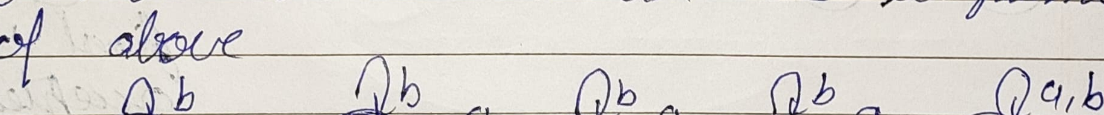


c. all string with no more than 3a's

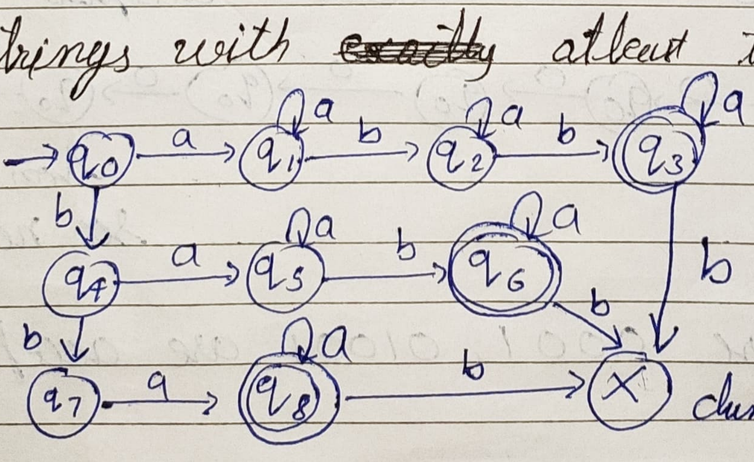


DFA accepting strings with more than 3a's

So, for required DFA accepting all string with no more than 3 a's will be complement of above



d. all strings with ~~exactly~~ atleast two b's & exactly one a.

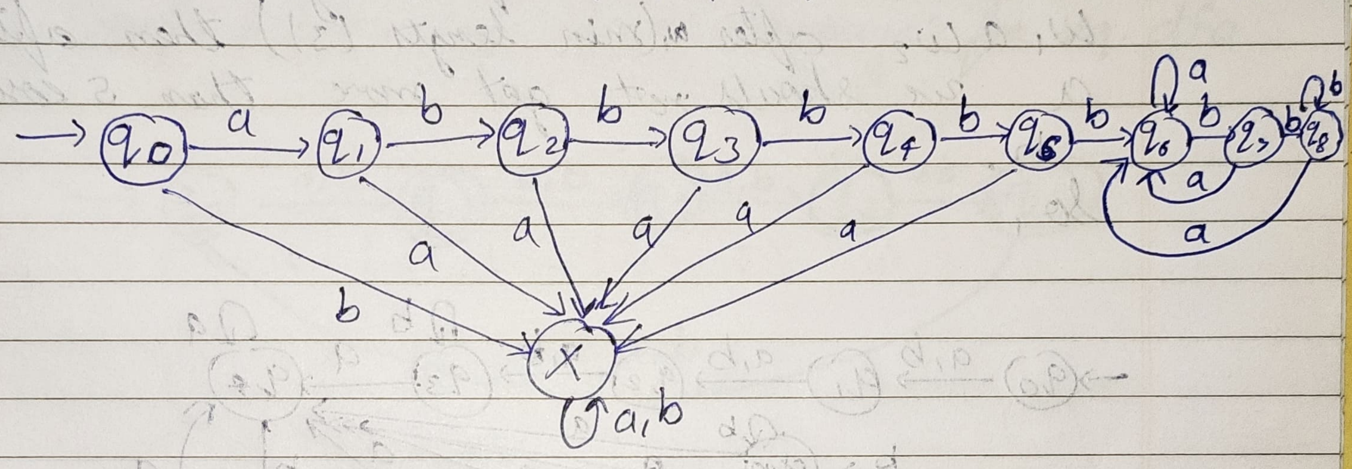


dummy dead state

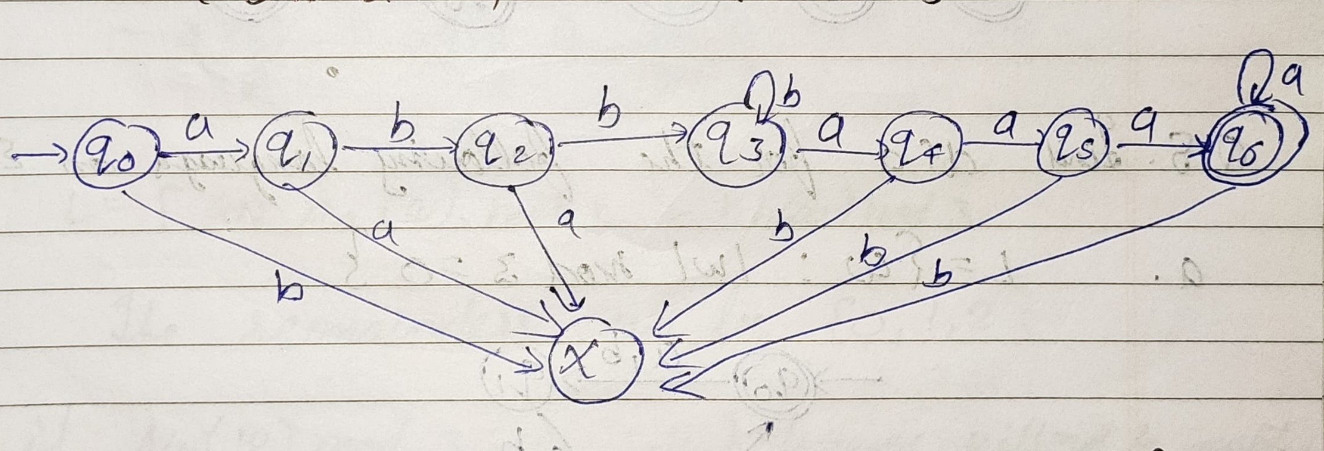
Q3. Give dfa's for the language

a) $L = \{ab^5 w b^2 ; w \in \{a,b\}^*\}$

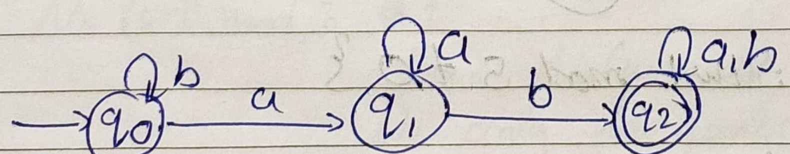
$w = \{a,b\} = \{\varepsilon, a, b, aa, bb, ab, ba, ----\}$



b) $L = \{ab^n a^m ; n \geq 2 ; m \geq 3\}$



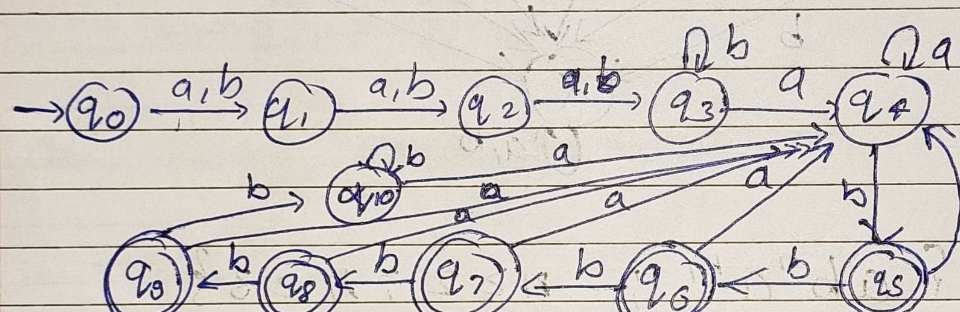
c) $L = \{w_1 ab w_2 : w_1 = \{a,b\}^*, w_2 \{a,b\}^*\}$



Q4. With $\Sigma = \{a, b\}$, give a dfa for $L = w_1 a w_2$ $|w_1| \geq 3$, $|w_2| \leq 5$.

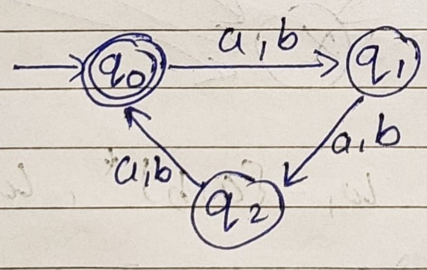
$\Sigma = \{a, b\}$ for $L = w_1 a w_2$ $|w_1| \geq 3$; $|w_2| \leq 5$
Here it means that if we get the a in $w_1 a w_2$ after w_1(min length (3)) then after that a we should not get more than 5 consecutive b.

So,



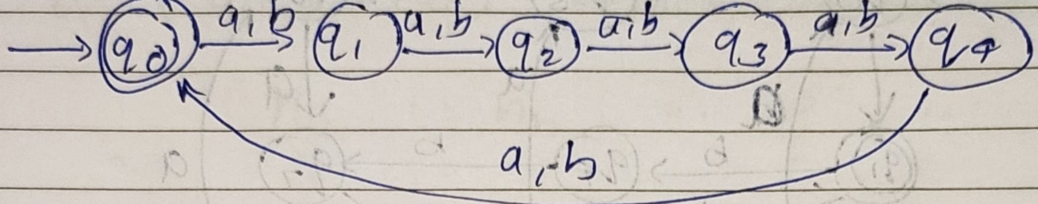
Q5. Find dfa for the following language on $\Sigma = \{a, b\}$

a. $L = \{w : |w| \bmod 3 = 0\}$

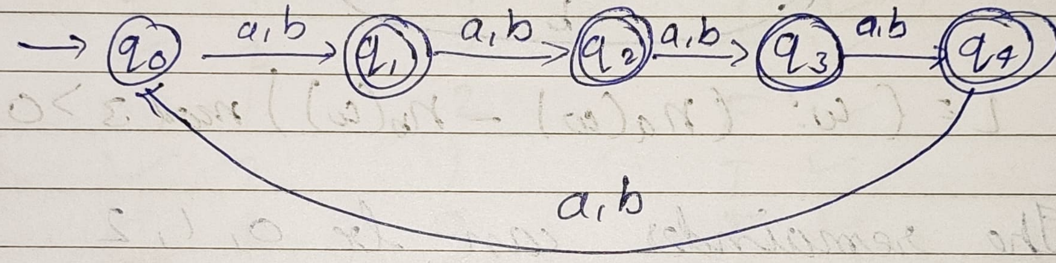


b. $L = \{w : |w| \bmod 5 \neq 0\}$

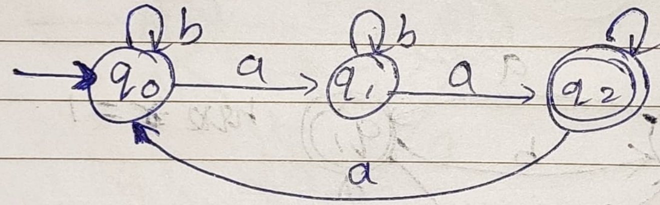
first find L such that $|w| \bmod 5 = 0$



Complement of above language for req. dfa



c. $L = \{ w : n_a(w) \bmod 3 > 1$



d. $L = \{ w : n_a(w) \bmod 3 > n_b(w) \bmod 3$

The remainders can be $\{0, 1, 2\}$

if $N_a(w) \bmod 3 = 0$ such language will not be accepted

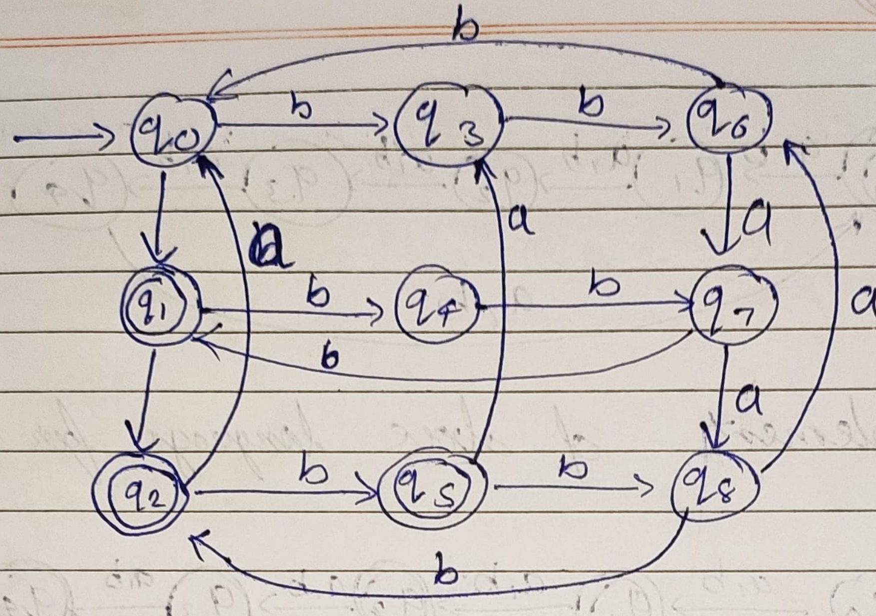
if $N_a(w) \bmod 3 = 1$

then $N_b(w) \bmod 3 = 0$

then only language will be accepted

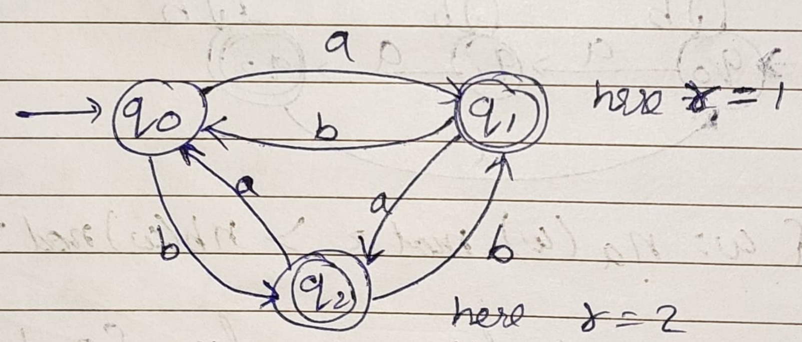
if $N_a(w) \bmod 3 = 2$

then $N_b(w) \bmod 3 = 1, 0$



e. $L = \{ w : (n_a(w) - n_b(w)) \bmod 3 > 0 \}$

The remainder can be 0, 1, 2
but it should not be 0.
so dfa will be



DFA accepting above language

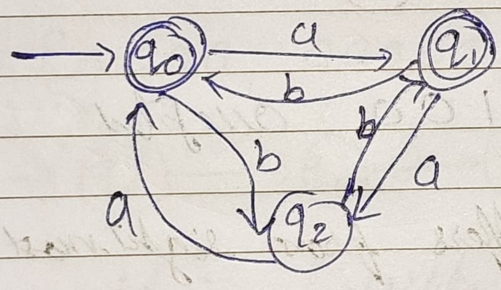
~~f. $L = \{ w : n_a(w) + 2n_b(w)) \bmod 3 < 2 \}$~~

n_a n_b r

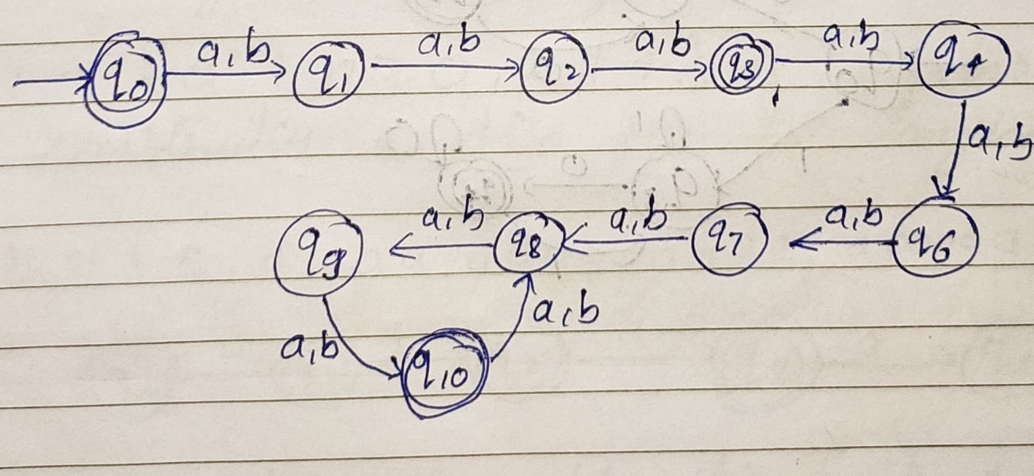
0
0
1
1
0
0
2
1
2
1
2
0

f. $L = \{ w : (n_a(w) + 2n_b(w)) \bmod 3 < 2 \}$

	n_a	n_b	R	
✓	0	1	F	
✓	1	0	T	✓
✓	0	2	T	✓
✓	2	0	F	✓
✓	1	1	T	✓
✓	2	2	T	✓
✓	1	2	F	✓
✓	2	1	T	✓
✓	0	0	T	✓
	3	0	T	✓

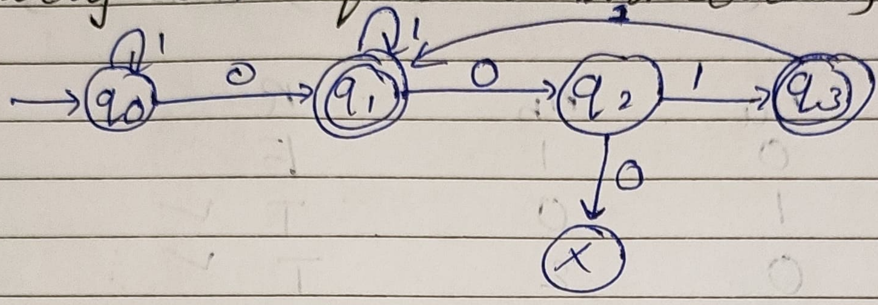


g. $L = \{ w : |w| \bmod 3 = 0,\ |w| \neq 6$

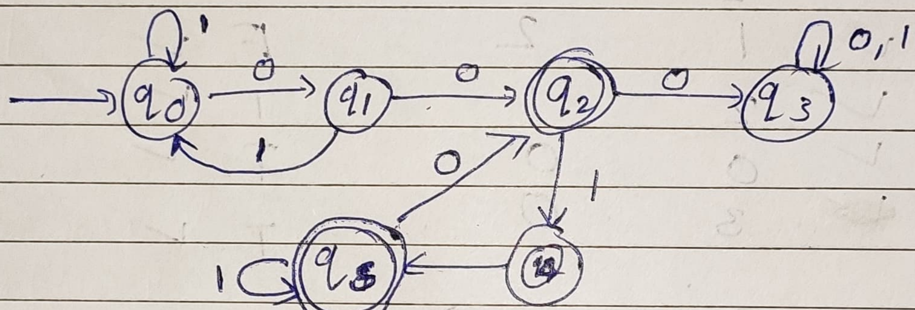


6.

a. Every 00 is followed immediately by 1

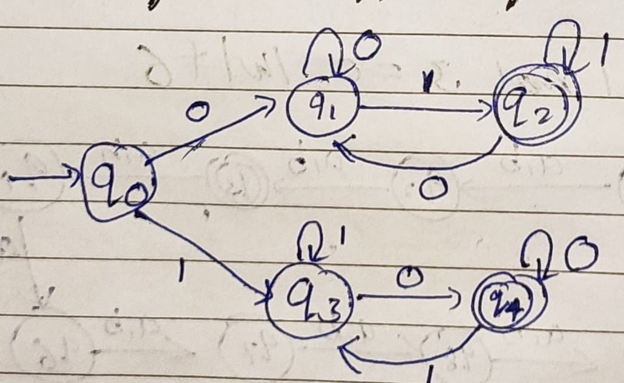


b. all strings containing 00 but not 000

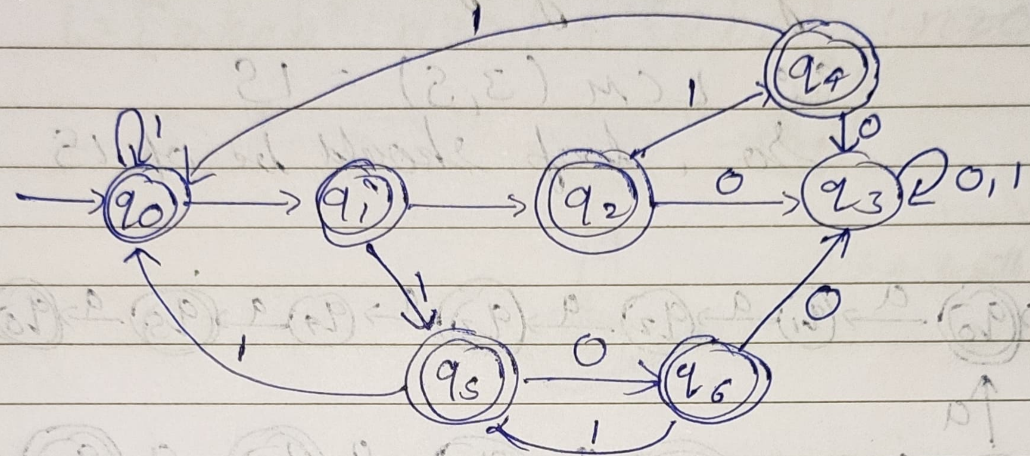


1010100111100 accepted

c. Leftmost symbol differs from rightmost symbol

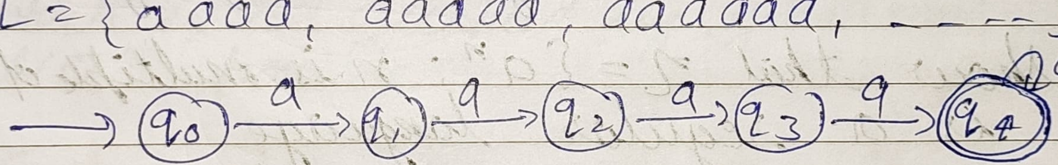


d. Every substring of four symbols has at most two 0's



7. Show that $L = \{a^n ; n \geq 4\}$ is regular by contructin dfa for language

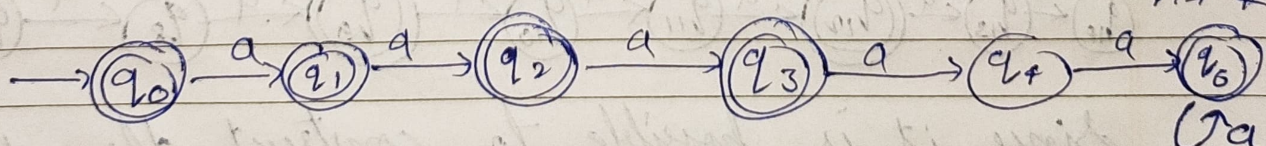
$L = \{aaaa, aaaaa, aaaaaa, ----\}$



Since it is possible to contruct DFA.
∴ language is regular.

8. $L = \{a^n ; n \geq 0, n \neq 4\}$ is regular by constructin a dfa for the language

$L = \{\varepsilon, a, aa, aaa, aaaaa, aoooa \;--\}$
$n \neq 4$

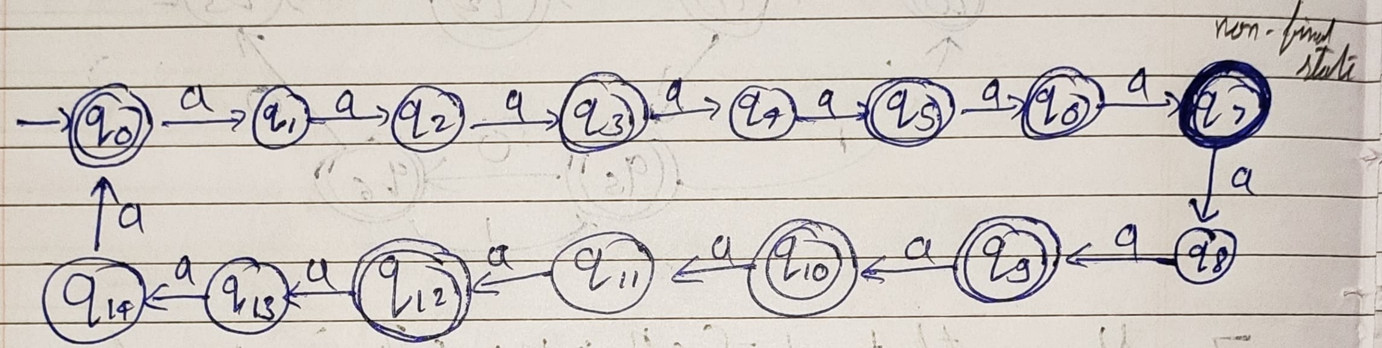


Since it is possible to contruct dfa,
so, it is a regular language.

9. Show that $L = \{a^n : n \text{ is either multiple of } 3 \text{ or } 5\}$ is a regular language

$\because$ LCM (3, 5) = 15

So, loop should be of 15

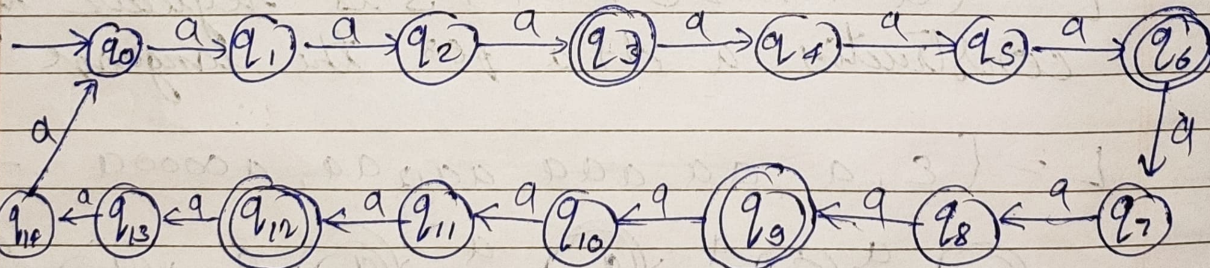


Since it is possible to construct dfa, so it is a regular language.

10. Show that $L = \{a^n : n \text{ is multiple of } 3 \text{ but not } 5\}$ is a regular language

$n = 3, 6, 9, 12, 18, 21, 24, 27, \ldots$

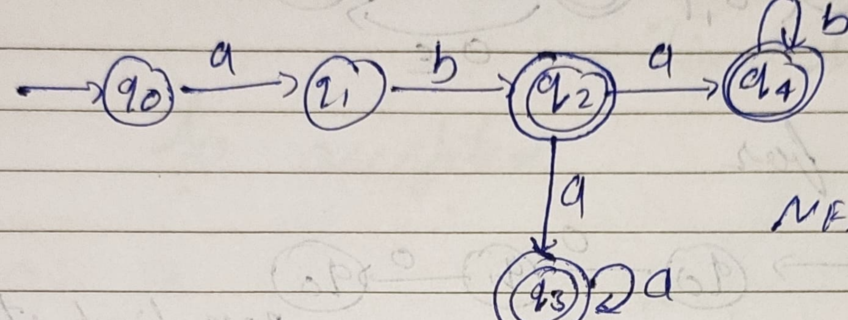
$n \neq 0, 15, 30, \ldots$



Since it is possible to construct dfa, it is a regular language.

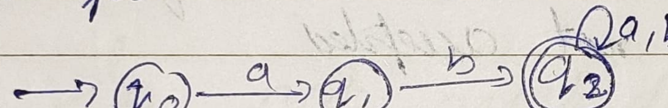
11. Design a nfa with no more than five states

$L = \{abab^n ; n \geq 0\} \cup \{aba^n : n \geq 0\}$



NFA with 5-states

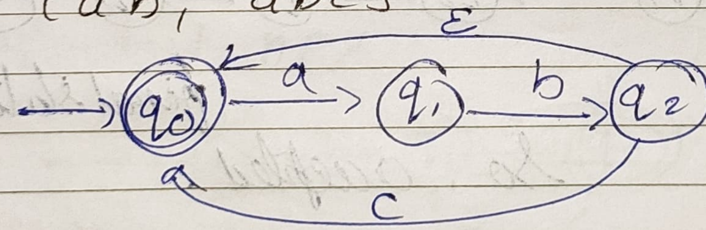
It is possible to draw 3 states



The string abb^n is accepted by this machine which is not accepted by grammar.

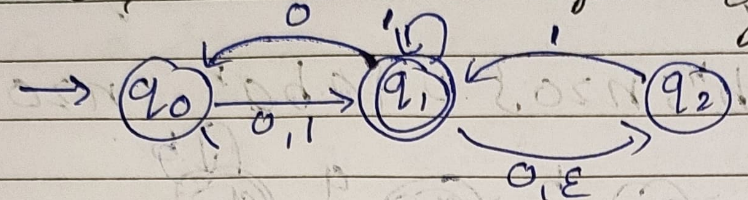
→ NOT possible.

12. Construct nfa with 3 states that accepts language $\{ab, abc\}^*$



NFA with three states

Q13. Which of the strings 00, 01001, 10010, 000, 0000 are accepted by following nfa?



Check for

00

$\rightarrow q_0 \xrightarrow{0} q_1 \xrightarrow{0} q_0$

non-final state

$\rightarrow q_0 \xrightarrow{0} q_1 \xrightarrow{0} q_2$

non-final state

So 00 not accepted

01001

$\rightarrow q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{0} \phi$

not accepted in this case

$\rightarrow q_0 \xrightarrow{0} q_1 \xrightarrow{\varepsilon} q_2 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \xrightarrow{0} q_1 \xrightarrow{1} q_1$

final state

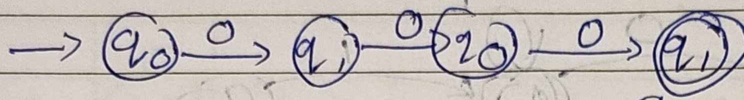
So accepted.

10010

$\rightarrow q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_1 \xrightarrow{0} q_0$

NOT accepted

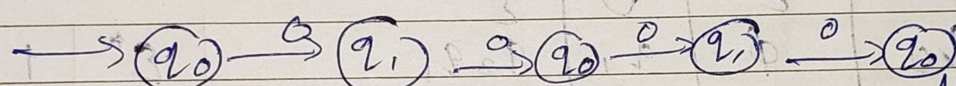
0 0 0 0



Final state

So accepted

0 0 0 0

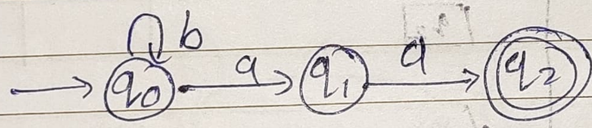


Non-final state

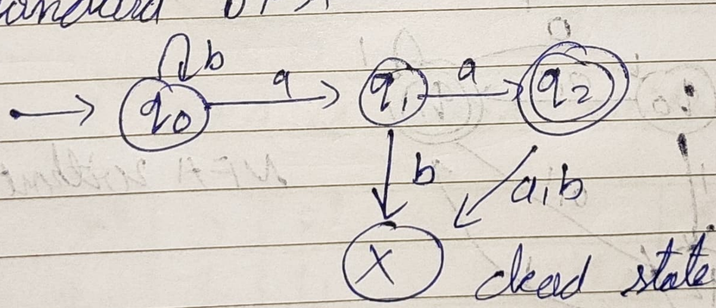
So not accepted

Therefor only 01001 & 000 are accepted.

14.

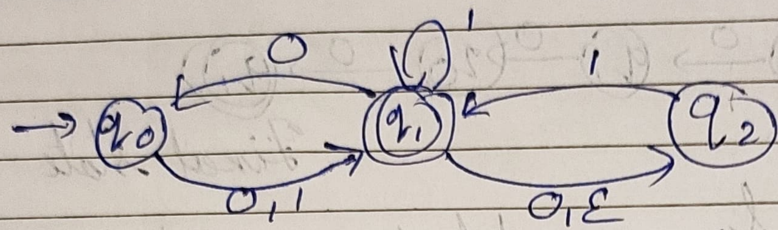


Standard DFA



15. Convert NFA to equivalent DFA

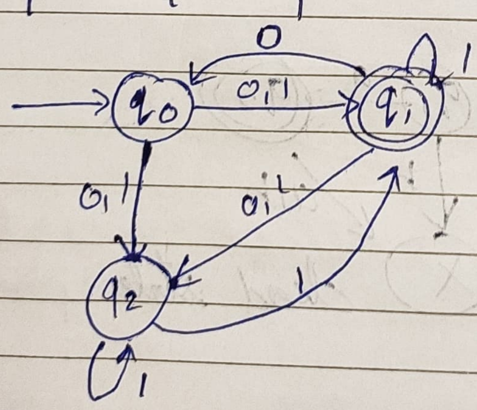
i.



NFA to NFA without ε transition

	ε^*	0	ε^*
q_0	ϕ	q_1	$q_2 q_1$
	q_0	q_1	q_1, q_2
			$\boxed{q_1 q_2}$
q_1	$q_1 q_2$		
	q_1	$q_0 q_2$	$q_0 q_2$
	q_2	ϕ	ϕ
			$\boxed{q_2}$
q_2	q_2	ϕ	ϕ

	ε^*	1	ε^*
q_0	q_0	q_1	q_1, q_2
q_1	q_1	q_1	q_1, q_2
	q_2	q_1	q_1, q_2
q_2	q_2	q_1	q_1, q_2

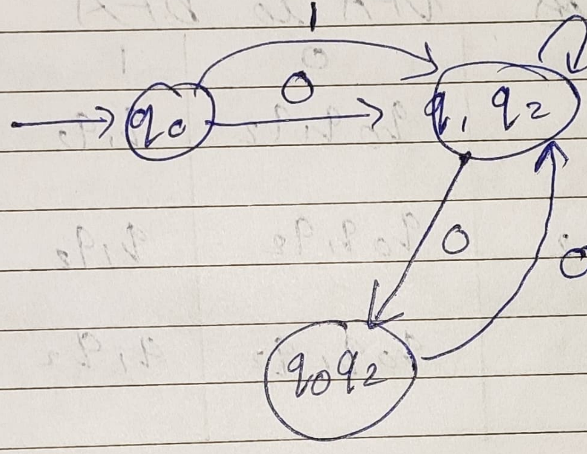


NFA without ε transition

NFA to DFA

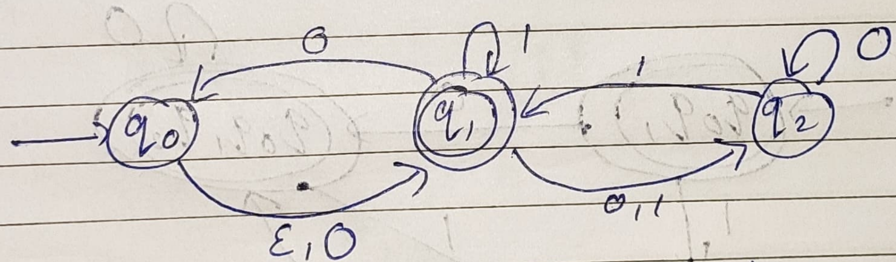
	0	1
→ q_0	$\{q_1, q_2\}$	$\{q_1, q_2\}$
q_1	$\{q_2\ q_3\}$	$\{q_2\ q_1\}$
q_2	ϕ	$\{q_1, q_2\}$

	0	1
→ q_0	$\{q_1\ q_2\}$	$\{q_1, q_2\}$
$\{q_1, q_2\}$	$\{q_0 q_2\}$	$\{q_1, q_2\}$
$\{q_0 q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$



Required DFA

ii



Convert NFA to DFA without ε transitions

	ε^*	0	ε^*
q_0	q_0	q_1	q_1
	q_1	q_2	q_2
q_1	q_1	$q_0 q_2$	q_0, q_2, q_1
q_2	q_2	q_2	q_2

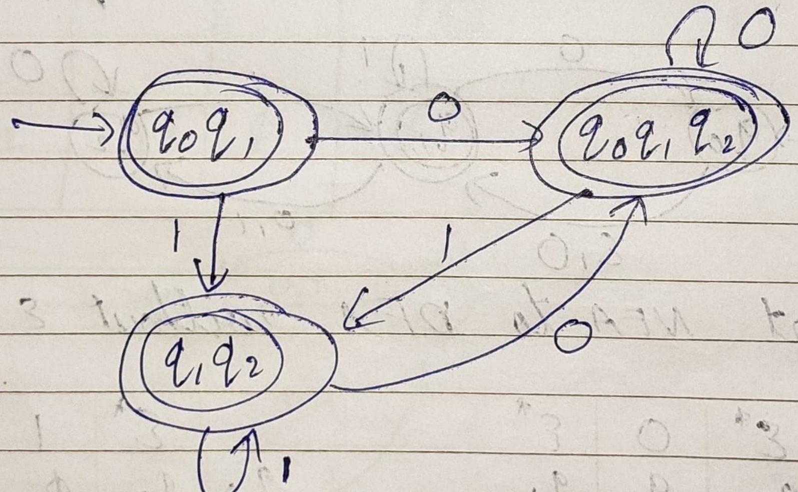
	ε^*	1	ε^*
q_0	q_0	ϕ	ϕ
	q_1	q_1 q_2	q_1, q_2
q_1	q_1	q_1, q_2	q_1, q_2
q_2	q_2	q_1	q_1

	0	1
$\rightarrow q_0$	q_1, q_2	$q_1 q_2$
q_1	$q_0 q_2 q_1$	q_1, q_2
q_2	q_2	q_1

Here ε closure of q_0 gives. So $q_0 q_1$ is initial state

Convert this NFA to DFA

	0	1
$\rightarrow q_0 q_1$	$q_0 q_1 q_2$	$q_1 q_2$
$q_0 q_1 q_2$	$q_0 q_1 q_2$	$q_1 q_2$
$q_1 q_2$	$q_0 q_1 q_2$	q_1, q_2

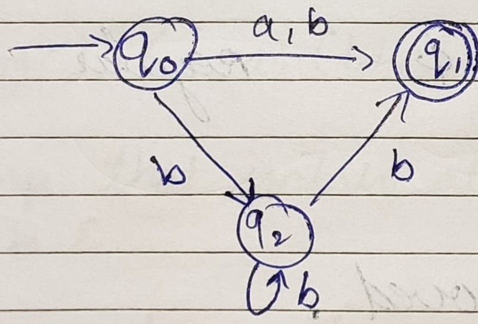


16. Prove that for every nfa with arbitary number of final states there is an equivalent nfa with only one final state. Can we claim this for dfa?

Sol. We can create one final state q_{final} & connect nfa's final states to q_{final} & set them to non-final state

$\therefore$ ε transition does not exist in dfa, therefore we cannot always find corresponding dfa with only one final state

17. Find a nfa without ε-transitions and with a single final state that accepts the set $\{a\} \cup \{b^n \; n \geq 1\}$



Q18. Prove that all finite languages are regular

Proof: If L is an empty set then it is defined by the regular expression ϕ & so it is regular.

If L is finite language composed of strings $s_1, s_2, \ldots s_n$, $n \geq 0$.

then it is defined by Regular language
$s_1 \cup s_2 \cup s_3 \cup --- s_n$
So it is also regular

let $L = L_1 \cap L_2$
$L_1 = \{a^n b^n \; n \geq 0\}$
$L_2 = \{b^n a^n \; n \geq 0\}$

$L_1 \rightarrow$ infinite $\quad L_2 \rightarrow$ infinite

$L_1 \cap L_2 = \phi \quad$ Regular

$L = \{\varepsilon\}$

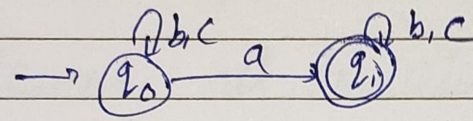
Hence proved.

19. Give RE for following languages on $\Sigma = \{a, b, c\}$

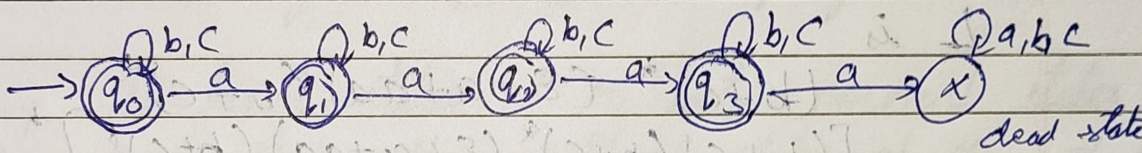
a. all string containing exactly one a.

$L = \{a, abc, ac, ab, ba, ca, \ldots\}$

$L = (b+c)^* \, a \, (b+c)^*$



b. all strings with no more than 3 a's



$L = (b+c)^*(\lambda+a)(b+c)^*(\lambda+a)(b+c)^*(\lambda+a)(b+c)^*$

c. all strings that contain at least one occurrence of each symbol in Σ.

$\rightarrow abc + bca + bac + cab + cba + acb$

$XaXbXcX + XbXcXaX + XbXaXcX + XcXaXbX$
$+ XcXbXaX + XaXcXbX$

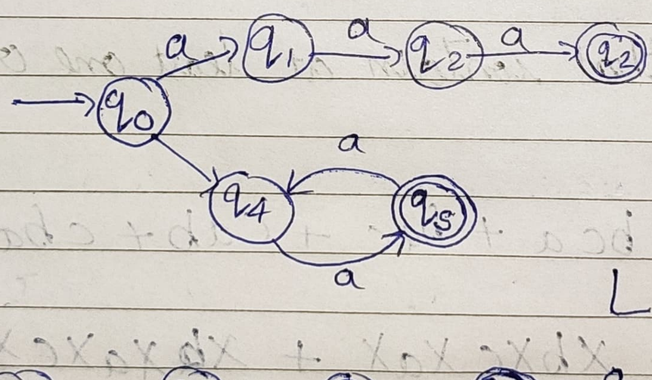
L is

$(a+b+c)^* \; a \; (a+b+c)^* \; b \; (a+b+c)^* \; c \; (a+b+c)^* \; +$
$(a+b+c)^* \; b \; (a+b+c)^* \; c \; (a+b+c)^* \; a \; (a+b+c)^* \; +$
$(a+b+c)^* \; b \; (a+b+c)^* \; a \; (a+b+c)^* \; c \; (a+b+c)^* \; +$
$(a+b+c)^* \; c \; (a+b+c)^* \; a \; (a+b+c)^* \; b \; (a+b+c)^* \; +$
$(a+b+c)^* \; c \; (a+b+c)^* \; b \; (a+b+c)^* \; a \; (a+b+c)^* \; +$
$(a+b+c)^* \; a \; (a+b+c)^* \; c \; (a+b+c)^* \; b \; (a+b+c)^*$

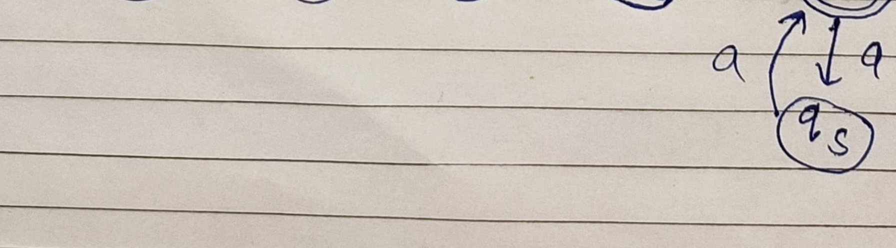
d. All strings that contain no run of a's of length greater than 2.

L is
$$(b+c)^* (\varepsilon + a + aa)(b+c)^*$$
$$[(b+c)(b+c)^* (\varepsilon + a + aa)(b+c)^*]^*$$

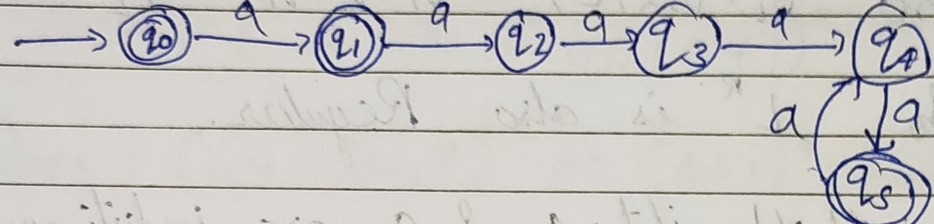
20. Find dfa that accepts language defined by given nfa



L is $a^3 + (aa)^+$



Q21. Find a dfa that accepts the complement of the language defined by the nfa



22. Show that if L is regular, so is L^R.

Reverse all transitions in M_L

Add a new initial state q_s and generate ε transitions from q_s to each of final states in M_L

Turn all final states of M_L into normal states of M_{L^R} and turn the initial states of M_L into final state of M_{L^R}

$M_L = (Q_L, \Sigma, \delta_L, q_0, F)$ be a NFA that accepts L.

NFA M_{L^R} defined below accepts L^R

a. $M_{L^R} = (Q_L \cup \{q_0\}, \Sigma, \delta_{L^R}, q_s \{q_0\})$ & $q_s \notin Q_L$

b. $\delta_L(q_i, a) = q_j \Leftrightarrow \delta_{L^R}(q_j, a) = q_i$

c. $\delta_{L^R}(q_s, \varepsilon) = q \quad \forall q \in F_L$

if $w \in L(M_L)$, $\delta_L^*(q_0, w) = q_i \in F_L \Leftrightarrow \delta_L^*(q_i, w) = q_0$

Also $\delta_{L^R}(q_s, \varepsilon) = q_i \ \forall \ q_i \in F_L$

So L^R is also Regular.

23. Prove: If the states q_a & q_b are indistinguishable, and if q_a & q_c are distinguishable, then q_b & q_c must be distinguishable.

$\because$ q_a & q_c are distinguishable, $\exists \ w \in \Sigma^*$ such that

$$\delta^*(q_a, w) \in F$$

$$\delta^*(q_c, w) \notin F$$

q_a & q_b are indistinguishable, then,

$$\delta^*(q_a, w) \in F$$
$$\delta^*(q_b, w) \in F.$$

So, q_b & q_c are distinguishable by w.

as $\delta^*(q_b, w) \in F$

$$\delta^*(q_c, w) \notin F$$

Hence proved.

24. For $\Sigma = \{0, 1\}$ give a RE r such $L(r) = \{w \in \Sigma^* : w$ has atleast one pair of consecutive zeroes?

$$L \to (0+1)^* 00 (0+1)^*$$

Does $((0+1)(0+1)^*)^* 00 (0+1)^*$ denote language L

$$(0+1)(0+1)^* \to (0+1)^+$$

$$((0+1)^+)^* \to (0+1)^*$$

So $((0+1)(0+1)^*)^* 00 (0+1)^* = (0+1)^* 00 (0+1)^*$

So this RE represents the above language L string that has atleast one pair of consecutive zeroes.

25. Give RE for the following languages

a. $L1 = \{a^n b^m : n \geq 4, m \leq 3\}$

$$aaaaa^* + aaaaa^*b + aaaaa^*bb + aaaaa^*bbb$$

$$= aaaaa^*(\varepsilon + b + bb + bbb)$$

b. complement of the above language L_1

$$L = \{a^n b^m \;;\; n \geq 4 \;\; m \leq 3\}$$

$$L' = \bar{L} = \{a^n b^m \;,\; n < 4 \text{ or } m > 3\} \cup$$ strings in which b is followed by a

So,

L' is

$$b^* + ab^* + aab^* + aaab^* + a^* bbbbb^* + (a+b)^* ba (a+b)^*$$

$$= (\varepsilon + a + aa + aaa) b^* + a^* bbbbb^* + (a+b)^* ba (a+b)^*$$

c. $L = \{w : |w| \bmod 3 = 0\}$

$$((a+b)(a+b)(a+b))^*$$

26. Give RE for $L = \{a^n b^m \;;\; n \geq 1 \;\; m \geq 1, \; nm \geq 3\}$

aabb, aaab, abbb, aaaab, abbbb, aaabb, aabbb, ---

$$aa^*b^*b^3 + a^2a^*b^*b^2 + a^3a^*b^*b$$

27. Find RE for $L = \{vwv \;;\; v, w \in \{a,b\}^*, |v| = 2\}$

$$aa(a+b)^*aa + bb(a+b)^*bb + ab(a+b)^*ab + ba(a+b)^*ba$$

28.

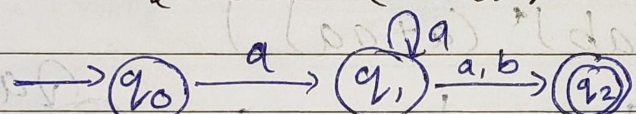
a. $(r_1^*)^* \equiv r_1^*$ True

b. $r_1^*(r_1 + r_2)^* \equiv (r_1 + r_2)^*$ True

c. $(r_1 + r_2)^* \equiv (r_1^* r_2^*)^*$ True

d. $(r_1 r_2)^* \equiv r_1^* r_2^*$ False

29. Find a nfa that accepts the language
$L = (aa^*(a+b))$ where $\Sigma = \{a, b\}$



30. Find a RE that denotes all bit strings whose value, when interpreted as a binary integer, is greater than or equal to 40.

$\bar{6}\ \bar{5}\ \bar{4}\ \bar{3}\ \bar{2}\ \bar{1}\ \bar{0}$

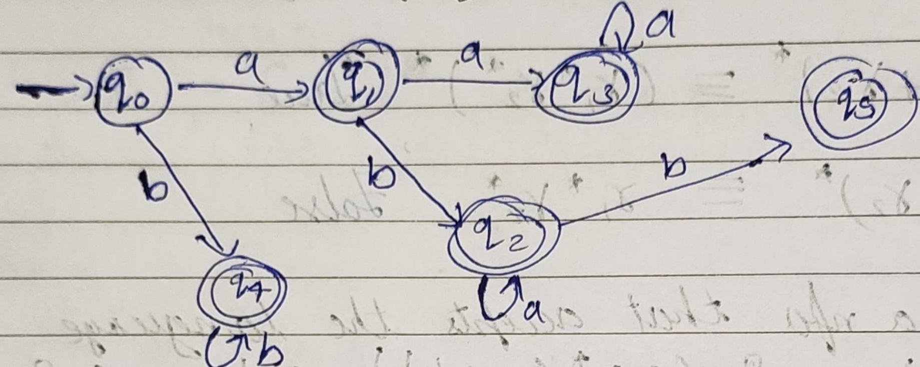
40 to 63 $\rightarrow 0^*1(0+1)1(0+1)(0+1)(0+1)$

64 & above
$(0+1)^* . 1(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)$

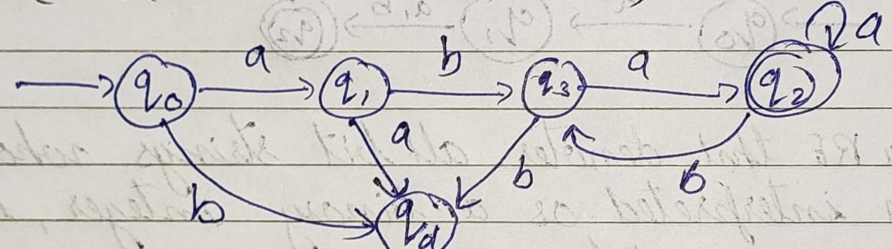
So
$RE = 0^*1(0+1)1(0+1)(0+1)(0+1)$
$+ (0+1)^* 1(0+1)(0+1)(0+1)(0+1)(0+1)(0+1)$

31. Find dfa's that accept languages where $\Sigma = \{a, b\}$

a. $L(aa^* + aba^*b^*)$

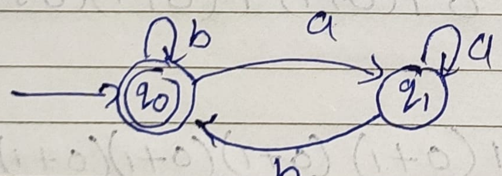


b. $L(ab(a+ab)^*(a+aa))$

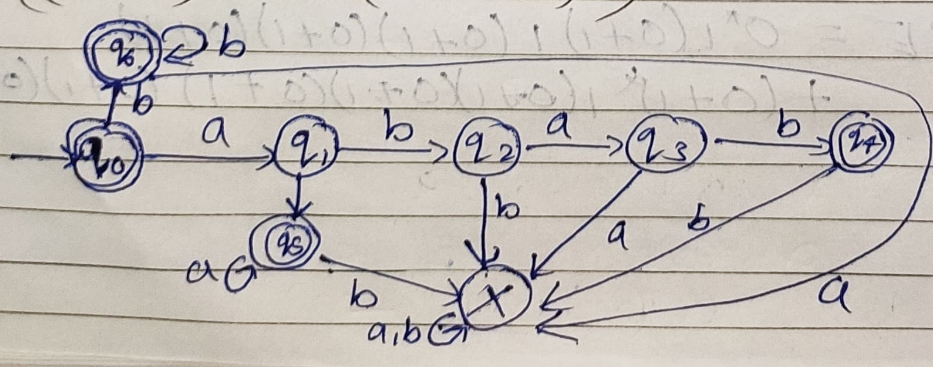


d. $L((aa^*)^*b)^*$

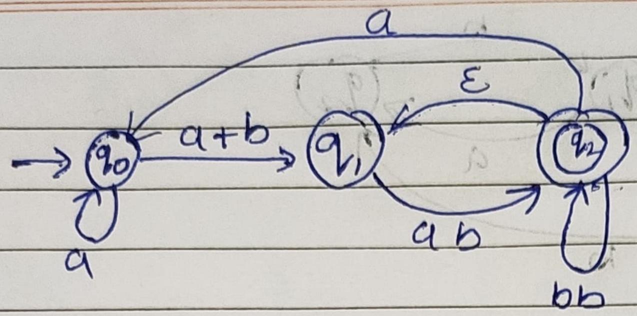
$= L((a^*b)^*)$



c. $L((abab)^* + (aaa^*b)^*)$

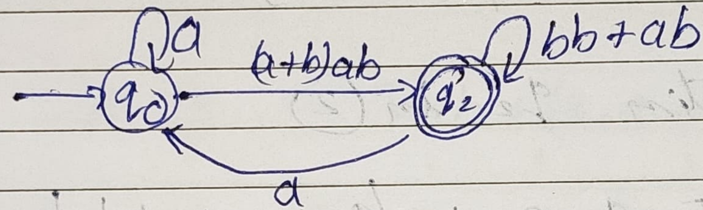


32.



$L = \{(a+b)ab,\ (a+b)abbb,\ (a+b)abb(a+b)ab\}$

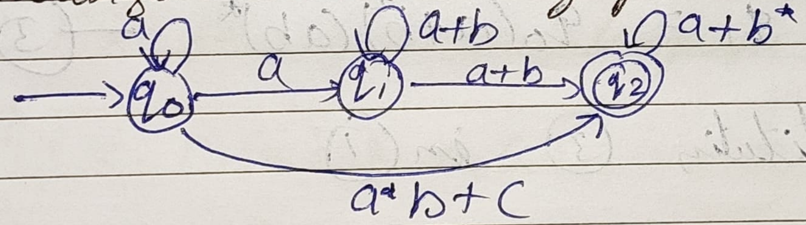
a) So,



b) So,

$r = a^*(a+b)ab(ab+bb+aa^*(a+b)ab)^*$

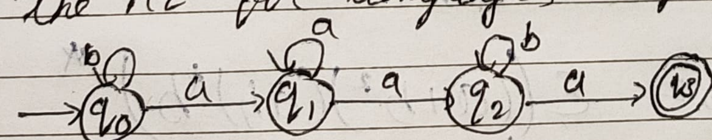
33. What language is accepted by the following generalized transition graph



$r = (a^*a(a+b)^*(a+b) + (a^*b+c))(a+b^*)^*$

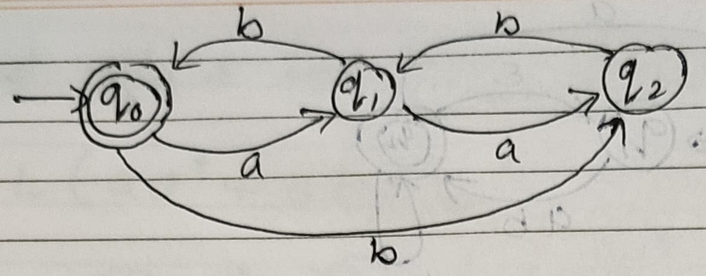
34. Find the RE for languages accepted by the following automata

i.



$r = b^*aa^*ab^*a$

ii.



Here,

$$q_0 = \varepsilon + q_1 b \quad \text{— (1)}$$
$$q_1 = q_0 a + q_2 b \quad \text{— (2)}$$
$$q_2 = q_1 a + q_0 b$$

Substitution q_2 in (2)

$$q_1 = q_0 a + (q_1 a + q_0 b) b$$
$$= q_0 a + q_1 ab + q_0 b^2$$
$$q_1 = q_0 a + q_0 b^2 + q_1 ab$$

By Arden's Theorem

$$R = Q + RP$$
$$\Rightarrow R = QP^*$$

So $q_1 = q_0 (a+b^2)(ab)^*$ — (3)

Substitution (3) in (1)

$$q_0 = \varepsilon + q_0 (a+b^2)(ab)^* b$$

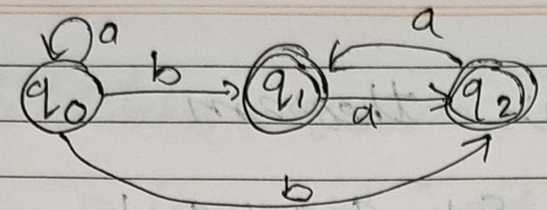
$$R = Q + RP$$
$$\Rightarrow R = QP^*$$

So

$$q_0 = \varepsilon\left((a+b^2)(ab)^* b\right)^*$$
$$= \left((a+b^2)(ab)^* b\right)^*$$
$$= \left(a(ab)^* b + b^2 (ab)^* b\right)^*$$

Ans

iii.



Here, $q_0 = \varepsilon + q_0 a$

So, $q_0 = \varepsilon a^* = a^*$

$q_1 = q_0 b + q_2 a$

$q_2 = q_1 a + q_0 b$

So $q_1 = q_0 b + (q_1 a + q_0 b) a$

$= q_0 b + q_1 aa + q_0 ba$

$q_1 = q_0 (b + ba) + q_1 aa$

$\Rightarrow q_1 = q_0 (b + ba)(aa)^*$

So $\boxed{q_1 = a^*(b+ba)(aa)^*}$

35. Find RE for following languages on $\{a, b\}$

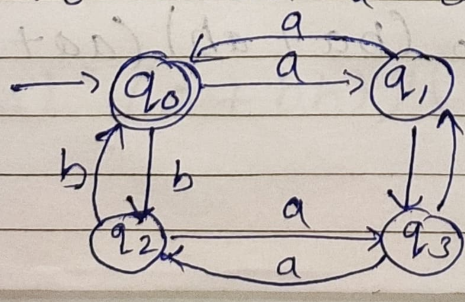
a. $L = \{w : n_a(w)$ & $n_b(w)$ both are even

$q_0 \rightarrow$ even a even b

$q_1 \rightarrow$ even a odd a

$q_2 \rightarrow$ odd b even a

$q_3 \rightarrow$ odd a odd b



By Arden's theorem

$$q_0 = \varepsilon + q_1 a + q_2 b \quad \text{--- (1)}$$

$$q_1 = q_0 a + q_3 b$$

$$q_2 = q_0 b + q_3 a$$

$$q_3 = q_2 a + q_1 b$$

Substituting q_1 & q_2 in (1)

$$q_0 = \varepsilon + (q_0 a + q_3 b) a + (q_0 b + q_3 a) b$$

$$= \varepsilon + q_3 ba + q_3 ab + q_0 (aa + bb)$$

$$\Rightarrow q_0 = (\varepsilon + q_3 ba + q_3 ab)(aa + bb)^*$$

$$q_0 = (aa + bb)^* + q_3 (ba + ab (aa + bb)^* \quad \text{--- (2)}$$

$$q_3 = q_2 a + q_1 b$$

$$= (q_0 b + q_3 a) a + (q_0 a + q_3 b) b$$

$$= q_0 ba + q_0 ab + q_3 (aa + bb)$$

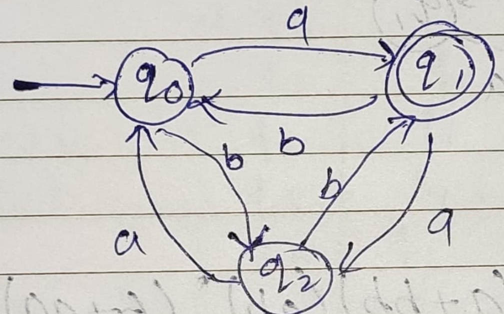
$$q_3 = q_0 (ba + ab)(aa + bb)^*$$

Substituting q_3 in (2)

$$q_0 = (aa+bb)^* + q_0(ba+ab)(aa+bb)^*(ba+ab)(aa+bb)^*$$

$$\Rightarrow q_0 = (aa+bb)^*\left((ab+ba)(aa+bb)^*(ba+ab)(aa+bb)^*\right)^*$$

b. $L = \{w : (n_a(w) - n_b(w)) \bmod 3 = 1\}$



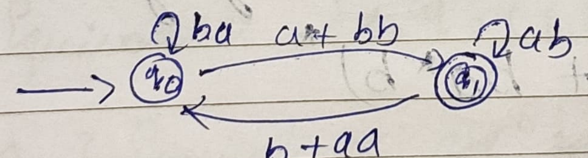
DFA accepting above language

Here q_1 is final state

$$q_1 = q_0 a + q_2 b$$

$$q_0 = \varepsilon + q_1 b + q_2 a$$

$$q_2 = q_1 a + q_0 b$$

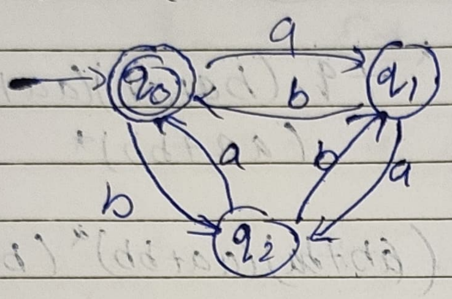


nfa for above dfa

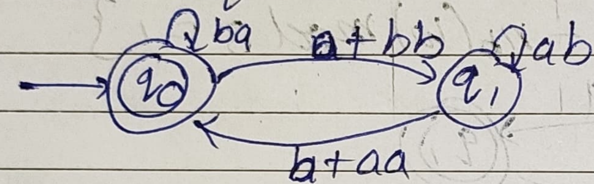
So RE is

$$\left((ba)^*(a+bb)\right)\left((ab)^* + (b+aa)(ba)^*(a+bb)\right)^*$$

c. $L = \{w : (n_a(w) - n_b(w)) \bmod 3 = 0\}$



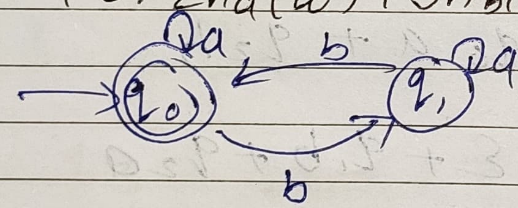
NFA for above DFA



So RE is

$$((ba)^* + (a+bb)(ab)^*(b+aa))^*$$

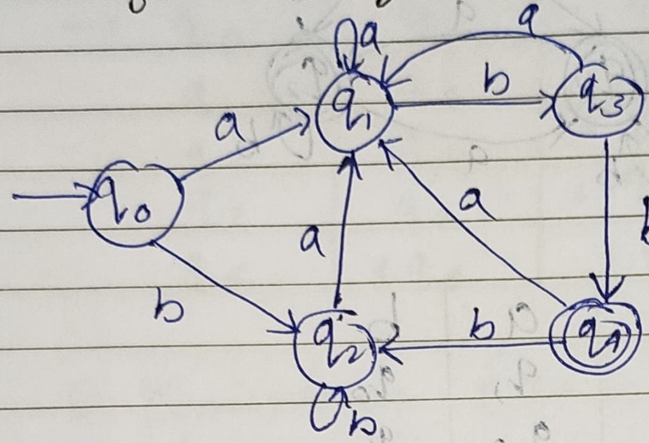
d. $L = \{w : 2n_a(w) + 3n_b(w) \text{ is even}\}$



So RE for above language is

$$(a + ba^*b)^*$$

36. Minimize the given DFA



	a	b
$\rightarrow q_0$	q_1	q_2
q_1	q_1	q_3
q_2	q_1	q_2
q_3	q_1	q_4
q_4	q_1	q_2

0 Equivalence $\{q_0 q_1 q_2 q_3\}$ $\{q_4\}$

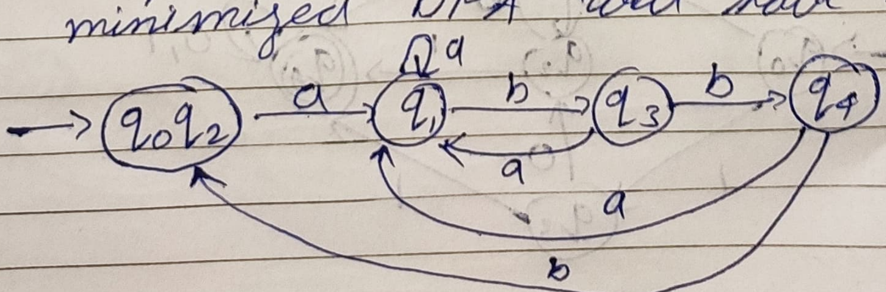
1 Equivalence $\{q_0 q_1 q_2\}$ $\{q_3\}$ $\{q_4\}$

2 Equivalence $\{q_0 q_2\}$ $\{q_1\}$ $\{q_3\}$ $\{q_4\}$

3 Equivalence $\{q_0 q_2\}$ $\{q_1\}$ $\{q_3\}$ $\{q_4\}$

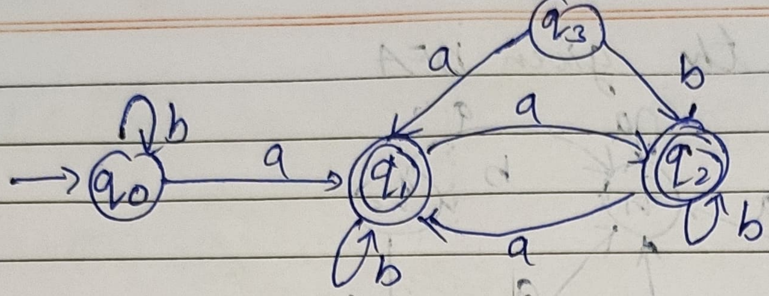
2 & 3 Equivalence are same. So we can stop

The minimized DFA will have 4 states



Minimized DFA

ii.



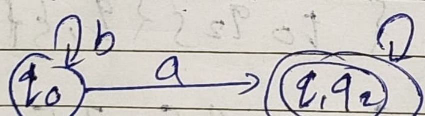
	a	b
→ q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_1	q_2
q_3	q_1	q_2

Here state q_3 is inaccessible from initial state. So remove all edges with it

0 Equivalence $\{q_0\}$ $\{q_1\ q_2\}$

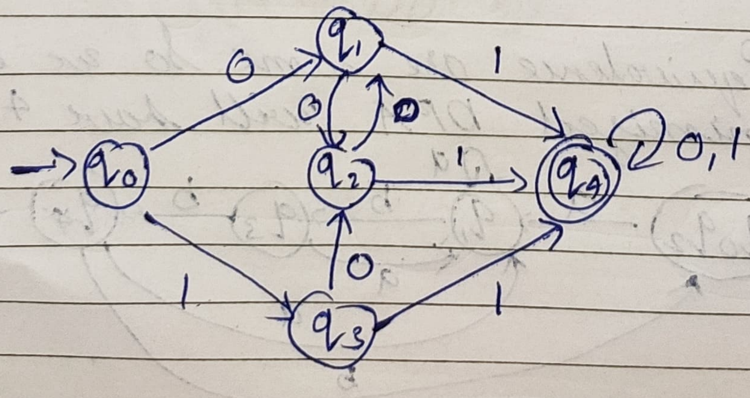
1 Equivalence $\{q_0\}$ $\{q_1\ q_2\}$

∴ 0 and 1 equivalence are same
So dfa can be minimized to 2 states



Minimized DFA

iii.



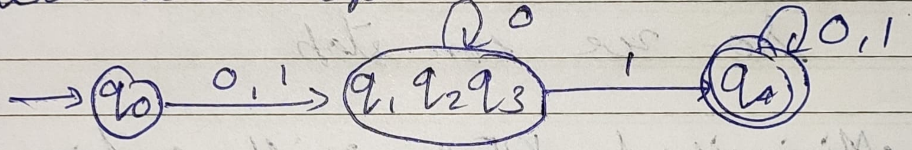
	0	1
$\rightarrow q_0$	q_1	q_3
q_1	q_2	$*q_4$
q_2	q_1	$*q_4$
q_3	q_2	$*q_4$
q_4	$*q_4$	$*q_4$

0 Equivalence $\{q_0\, q_1\, q_2\, q_3\}\ \{q_4\}$

1 Equivalence $\{q_0\}\ \{q_1\, q_2\, q_3\}\ \{q_4\}$

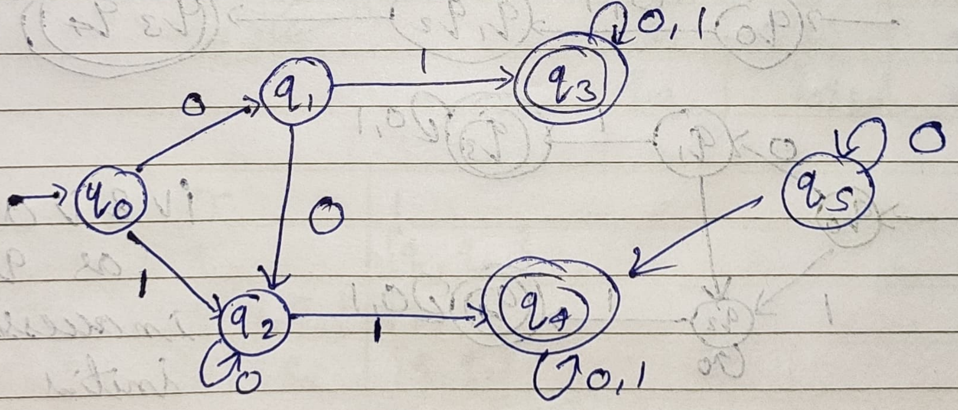
2 Equivalence $\{q_0\}\ \{q_1\, q_2\, q_3\}\ \{q_4\}$

Here we stop



Minimized DFA

iV.



State q_5 is inaccessible from initial state so remove it as well as associate edges.

	0	1
$\rightarrow q_0$	q_1	q_2
q_1	q_2	$*q_3$
q_2	q_2	$*q_4$
q_3	$*q_3$	$*q_3$
q_4	$*q_4$	$*q_4$

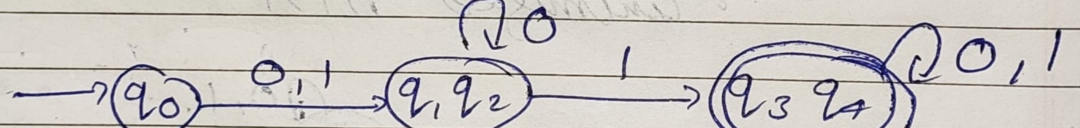
0 Equivalence $\{q_0\ q_1\ q_2\}$ $\{q_3\ q_4\}$

1 Equivalence $\{q_0\}$ $\{q_1\ q_2\}$ $\{q_3\ q_4\}$

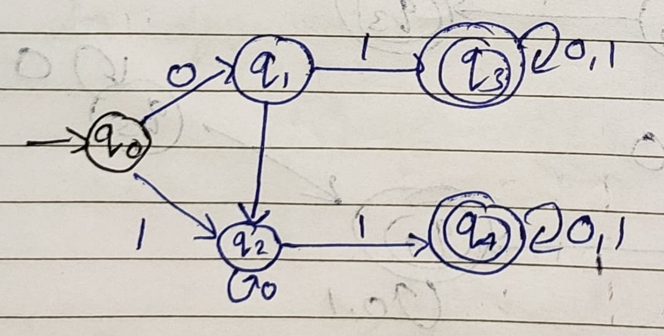
2 Equivalence $\{q_0\}$ $\{q_1\ q_2\}$ $\{q_3\ q_4\}$

So we can stop

Minimized DFA will contain 3 states

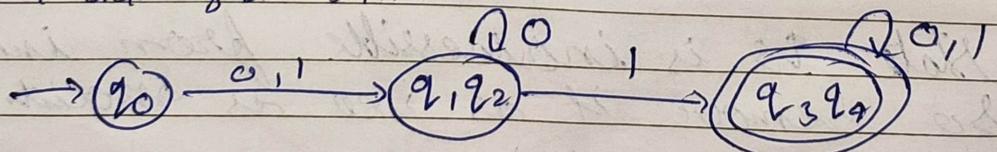


V.



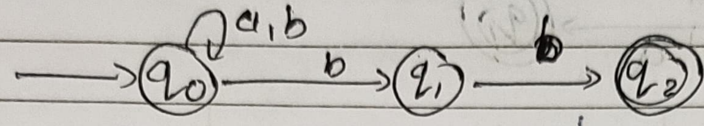
iv & v are same as q5 was inaccessible from initial state in iv

∴ Minimized DFA



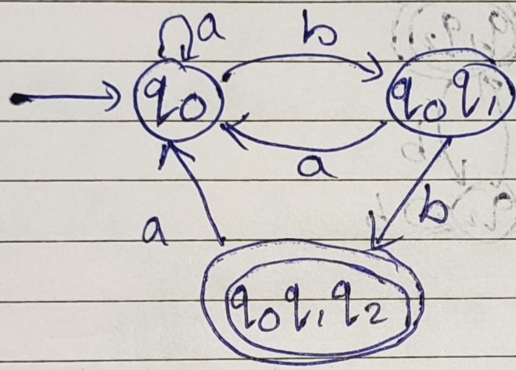
37. NFA to DFA

i.



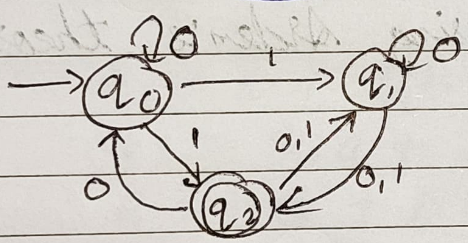
	a	b
$\rightarrow q_0$	q_0	$q_0 q_1$
q_1	ϕ	q_2
q_2	ϕ	ϕ

	a	b
$\rightarrow q_0$	q_0	$\{q_0 q_1\}$
$\{q_0 q_1\}$	q_0	$*\{q_0 q_1 q_2\}$
$*\{q_0 q_1 q_2\}$	q_0	$*\{q_0 q_1 q_2\}$



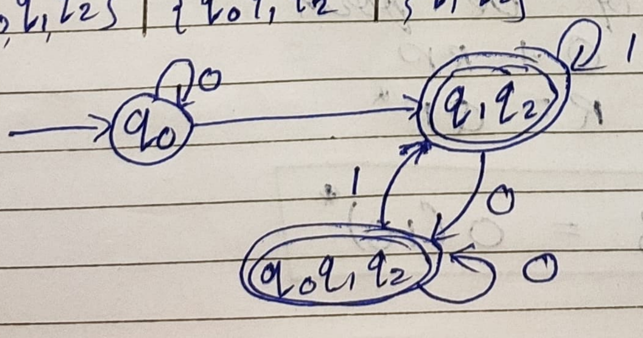
DFA

ii



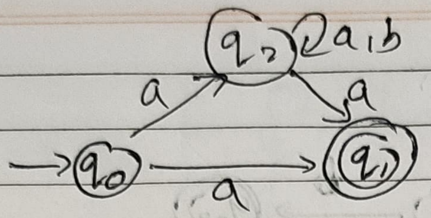
	0	1
$\rightarrow q_0$	q_0	q_1, q_2
q_1	q_1, q_2	q_2
q_2	$q_0 q_1$	q_1

	0	1
$\rightarrow q_0$	q_0	$\{q_1, q_2\}$
$*\{q_1, q_2\}$	$\{q_0 q_1 q_2\}$	$\{q_1, q_2\}$
$*\{q_0 q_1 q_2\}$	$\{q_0 q_1 q_2\}$	$\{q_1, q_2\}$



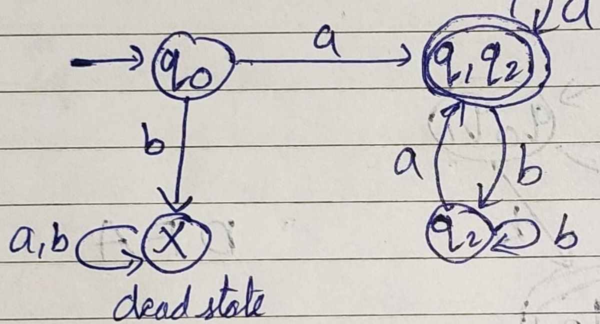
DFA

iii



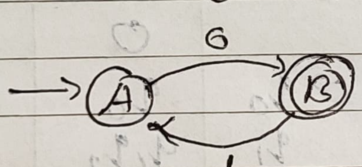
	a	b
$\rightarrow q_0$	q_1, q_2	ϕ
q_1	ϕ	ϕ
q_2	q_1, q_2	q_2

	a	b
$\rightarrow q_0$	$\{q_1 q_2\}$	ϕ
$*\{q_1 q_2\}$	$\{q_1 q_2\}$	q_2
q_2	$\{q_1 q_2\}$	q_2



38. DFA to RE using Arden's theorem

i.



here, $B = A0$

$A = B1 + \varepsilon$

$A = \varepsilon + B1$

So $B = (\varepsilon + B1)0$

$B = 0 + B10$

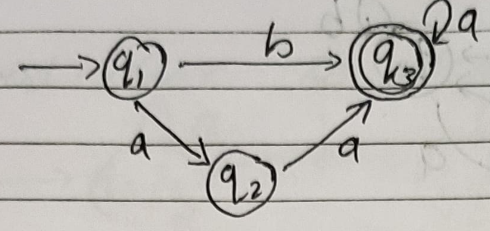
This is of the form

$R = Q + RP$

$\rightarrow R = QP^*$

So $\boxed{B = 0(10)^*}$

ii.



$q_1 = \varepsilon$

$q_2 = q_1 a \Rightarrow q_2 = \varepsilon a \Rightarrow q_2 = a$

$q_3 = q_1 b + q_2 a + q_3 a$

$q_3 = \varepsilon b + aa + q_3 a$

$q_3 = b + aa + q_3 a$

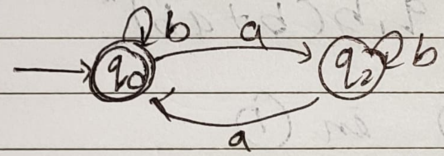
$R = Q + RP$

By Arden's Theorem

$R = QP^*$

$\therefore \quad q_3 = (b + aa) a^*$

iii.



$q_1 = \varepsilon + q_1 b + q_2 a$

$q_2 = q_1 a + q_2 b$

$R = Q + RP$

By Arden Theorem $R = QP^*$

$q_2 = q_1 a (b)^*$

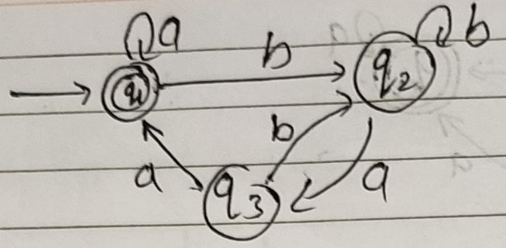
So $q_1 = \varepsilon + q_1 b + q_1 a b^* a$

By Arden's Theorem

$q_1 = \varepsilon (b + a b^* a)^*$

$= (b + a b^* a)^*$

iv.



$q_1 = \varepsilon + q_1 a + q_3 a$ — (1)

$q_2 = q_1 b + q_2 b + q_3 b$ — (2)

$q_3 = q_2 a$ — (3)

Substituting q_3 in (2)

$q_2 = q_1 b + q_2 b + q_2 ab$

$R = Q + RP$

By Arden's Theorem $R = QP^*$

So $q_2 = q_1 b(b+ab)^*$ — (4)

Substituting (3) in (1)

$q_1 = \varepsilon + q_1 a + q_2 aa$

$= \varepsilon + q_1 a + q_1 b(b+ab)^* aa$

$R = Q + RP$

So By Arden's theorem

$q_1 = \varepsilon(a + b(b+ab)^* aa)^*$

$R = Q + RP$

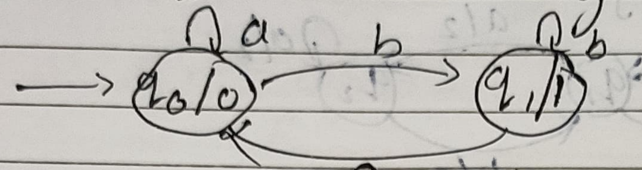
So, by Arden's theorem

$q_1 = \varepsilon(a + b(b+ab)^* aa)^*$

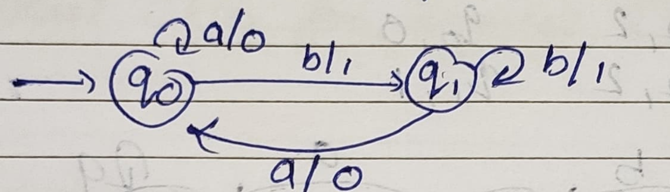
$q_1 = (a + b(b+ab)^* aa)^*$

39. Convert Moore to Mealy Machine

i.

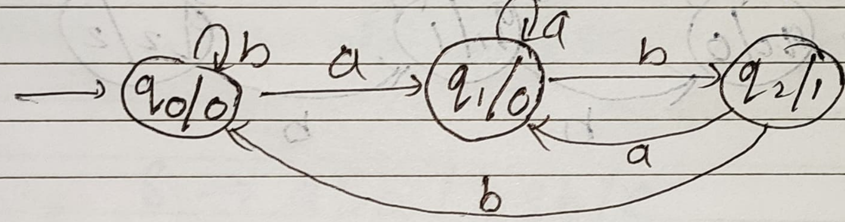


	a	b	Output
$\rightarrow q_0$	$q_0, 0$	$q_1, 1$	0
q_1	$q_0, 0$	$q_1, 1$	1

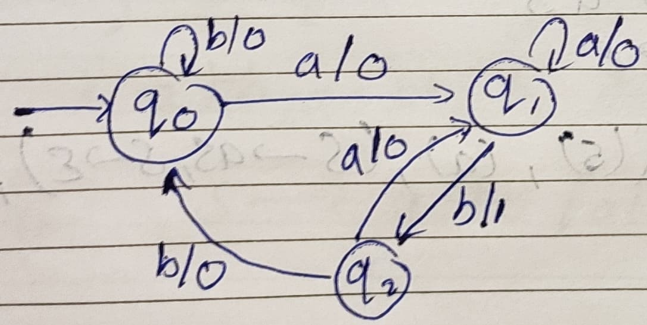


Mealy Machine

ii.

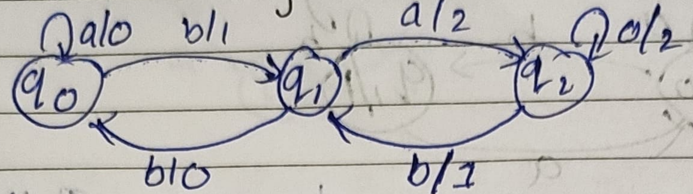


	a	b	Output
$\rightarrow q_0$	$q_1/0$	$q_0/0$	0
q_1	$q_1/0$	$q_2/1$	0
q_2	$q_1/0$	$q_0/0$	1

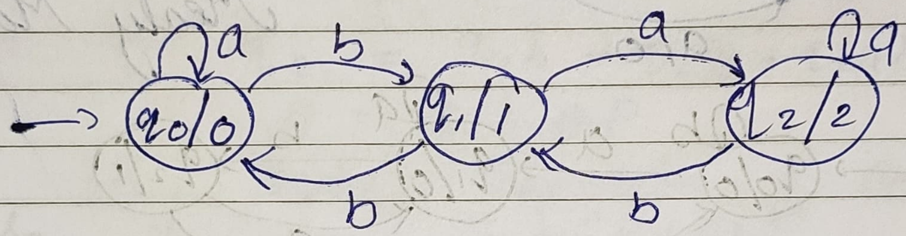


Mealy Machine

40 Convert Mealy to Moore Machine



	a	b
$\rightarrow q_0$	$q_0, 0$	$q_1, 1$
q_1	$q_2, 2$	$q_0, 0$
q_2	$q_2, 2$	$q_1, 1$



41.

a. Construct CFG for language having any no. of a's over the set $\Sigma = \{a\}$

$RE = a^*$

$S \rightarrow aS$

$S \rightarrow \varepsilon$

So $G = \{(S), (a), (S \rightarrow aS, S \rightarrow \varepsilon), S\}$

b. Contruct CFG for language $(0+1)^*$

$S \rightarrow 0S \mid 1S$
$S \rightarrow \varepsilon$

$G = \{(S), (0, 1), (S \rightarrow 0S \mid 1S, S \rightarrow \varepsilon), S\}$

c. Contruct CFG for language $L = \{wcw^R$ where $w \in (a, b)^*$ & $\Sigma = \{a, b, c\}\}$

string generated as $\{aacaa, bcb, abcba, bacab, \ldots\}$

$S \rightarrow aSa \mid bSb \mid c$

d. Contruct CFG for language $L = \{a^n b^{2n}$ $n >= 1$, $\Sigma = \{a, b\}$

String accepted by language is
$\{abb, aabbbb, aaabbbbbb, \ldots\}$

So, $S \rightarrow aSbb \mid abb$