```
% Clear workspace and initialize
clear all;
close all;
```

Question 01.

```
%% Question 01(a): y' + 2y = ex; y(0) = 1
disp("Question 01(a)");
```

```
Question 01(a)
```

```
syms x y;
equation_1_a = "Dy + 2*y = exp(x)";
equation_1_a
```

```
equation_1_a =
"Dy + 2*y = exp(x)"
```

```
condition_1_a = "y(0) = 1";
condition_1_a
```

```
condition_1_a =
"y(0) = 1"
```

```
% Solve for general and particular solutions
general_1_a = dsolve(equation_1_a, 'x');
```

```
Warning: Support for character vector or string inputs will be removed in a future release. Instead, use
syms to declare variables and replace inputs such as dsolve('Dy = -3*y') with syms y(t); dsolve(diff(y,t) ==
-3*y).
```

```
general_1_a
```

```
general_1_a =
```

$$\frac{e^x}{3} + C_1 e^{-2x}$$

```
particular_1_a = dsolve(equation_1_a, condition_1_a, 'x');
```

```
Warning: Support for character vector or string inputs will be removed in a future release. Instead, use
syms to declare variables and replace inputs such as dsolve('Dy = -3*y') with syms y(t); dsolve(diff(y,t) ==
-3*y).
```

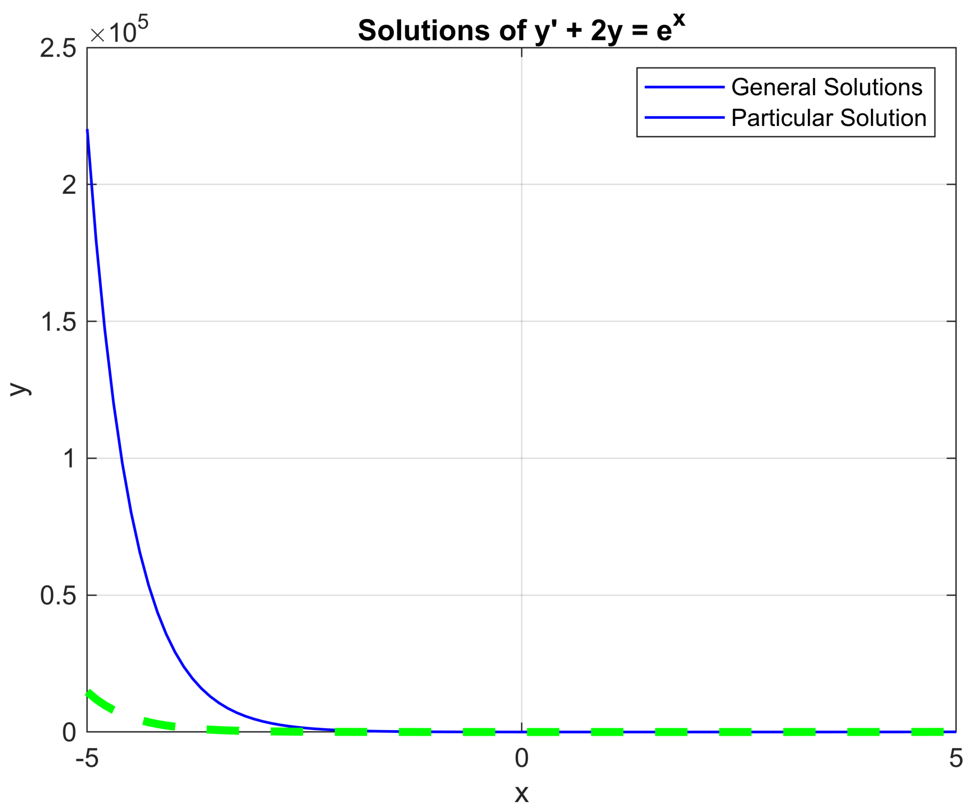
```
particular_1_a
```

```
particular_1_a =
```

$$\frac{2 e^{-2x}}{3} + \frac{e^x}{3}$$

```
% Create figure for question 1(a)
figure('Name', 'Question 1(a) Solutions');
```

```
x = linspace(-5, 5, 100);
C1 = 10;

% Plot general solutions
for C = -3:0.5:3
    f1a = eval(vectorize(subs(general_1_a)));
    plot(x, f1a, 'b-', 'LineWidth', 1);
    hold on;
end

% Plot particular solution
f1a_particular = eval(vectorize(particular_1_a));
plot(x, f1a_particular, 'g--', 'LineWidth', 3);
grid on;
title('Solutions of y'' + 2y = e^x');
xlabel('x');
ylabel('y');
legend('General Solutions', 'Particular Solution');
hold off;
```




```
%% Question 01(b): y'' - 2y' + 10y = 10x^2 + 18x + 6; y(0) = 1, y'(0) = 3.2
disp("Question 01(b)");
```

```
Question 01(b)
```

```
equation_1_b = "D2y - 2*Dy + 10*y = 10*x^2 + 18*x + 6";
```

```
equation_1_b
```

```
equation_1_b =
"D2y - 2*Dy + 10*y = 10*x^2 + 18*x + 6"
```

```
condition_1_b = "y(0) = 1, Dy(0) = 3.2";
condition_1_b
```

```
condition_1_b =
"y(0) = 1, Dy(0) = 3.2"
```

```

% Solve for general and particular solutions
general_1_b = dsolve(equation_1_b, 'x');
```

```
Warning: Support for character vector or string inputs will be removed in a future release. Instead, use
syms to declare variables and replace inputs such as dsolve('Dy = -3*y') with syms y(t); dsolve(diff(y,t) ==
-3*y).
```

```
general_1_b
```

```
general_1_b =
```

$$\frac{11\,x}{5}+x^2+C_1\cos(3\,x)\,\mathrm{e}^x-C_2\sin(3\,x)\,\mathrm{e}^x+\frac{21}{25}$$

```
particular_1_b = dsolve(equation_1_b, condition_1_b, 'x');
```

```
Warning: Support for character vector or string inputs will be removed in a future release. Instead, use
syms to declare variables and replace inputs such as dsolve('Dy = -3*y') with syms y(t); dsolve(diff(y,t) ==
-3*y).
```

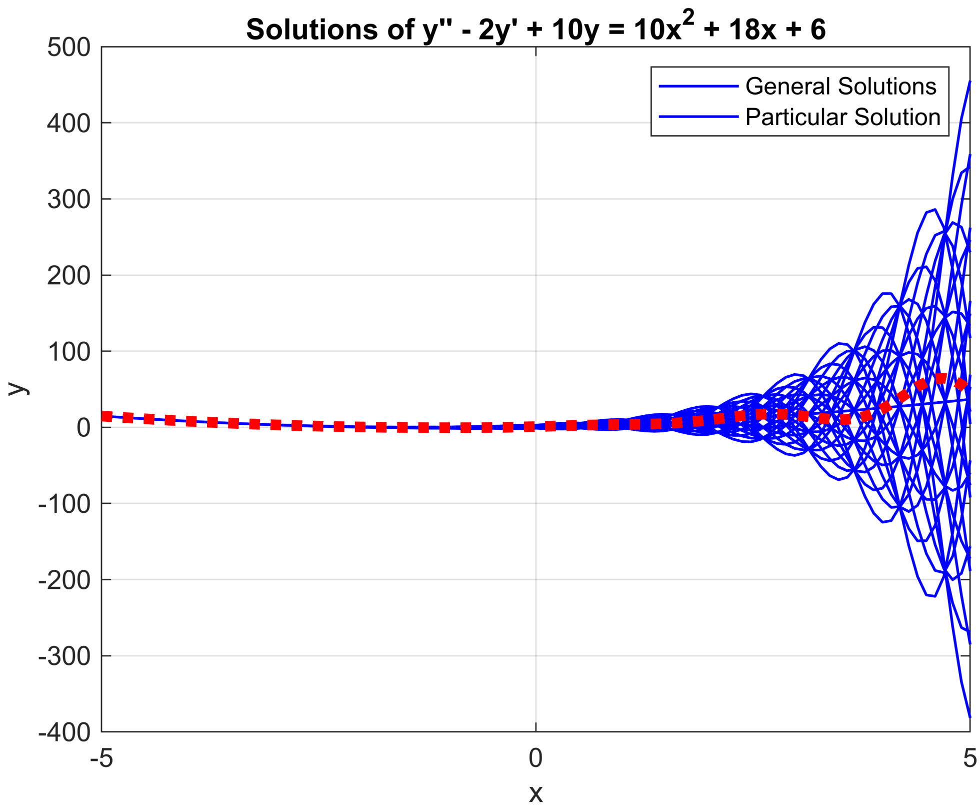
```
particular_1_b
```

```
particular_1_b =
```

$$\frac{11\,x}{5}+\frac{4\cos(3\,x)\,\mathrm{e}^x}{25}+\frac{7\sin(3\,x)\,\mathrm{e}^x}{25}+x^2+\frac{21}{25}$$

```

% Create figure for question 1(b)
figure('Name', 'Question 1(b) Solutions');
x = linspace(-5, 5, 100);

% Plot general solutions
for C1 = -2:1:2
    for C2 = -2:1:2
        f1b = eval(vectorize(subs(general_1_b)));
        plot(x, f1b, 'b-', 'LineWidth', 1);
        hold on;
    end
end

% Plot particular solution
f1b_particular = eval(vectorize(particular_1_b));
plot(x, f1b_particular, 'r:', 'LineWidth', 4);
```

```
grid on;
title('Solutions of y'''' - 2y'' + 10y = 10x^2 + 18x + 6');
xlabel('x');
ylabel('y');
legend('General Solutions', 'Particular Solution');
hold off;
```



Question 02.

```
%% Question 02: y''' + 2y'' + y' + 2e^(-2x) = 0
disp("Question 02");
```

```
Question 02
```

```
% Get number of solutions from user with default value of 5
n = input('Enter the number of solutions to plot (n) [Default=5]: ');
if isempty(n)  % If user just presses Enter
    n = 5;     % Use default value
end

% Get constant value with default value of 1
constant_val = input('Enter a constant value to use as base [Default=1]: ');
if isempty(constant_val)
```

```
    constant_val = 1;
end

% Define symbolic variables
syms y(x);
Dy = diff(y, x);
D2y = diff(y, x, 2);
D3y = diff(y, x, 3);

% Define and solve the ODE
equation_2 = D3y + 2*D2y + Dy + 2*exp(-2*x) == 0;
equation_2
```

equation_2(x) =

$$\frac{\partial^3}{\partial x^3}\, y(x) + 2\,\frac{\partial^2}{\partial x^2}\, y(x) + \frac{\partial}{\partial x}\, y(x) + 2\,e^{-2\,x} = 0$$

```
general_2 = dsolve(equation_2);
general_2
```

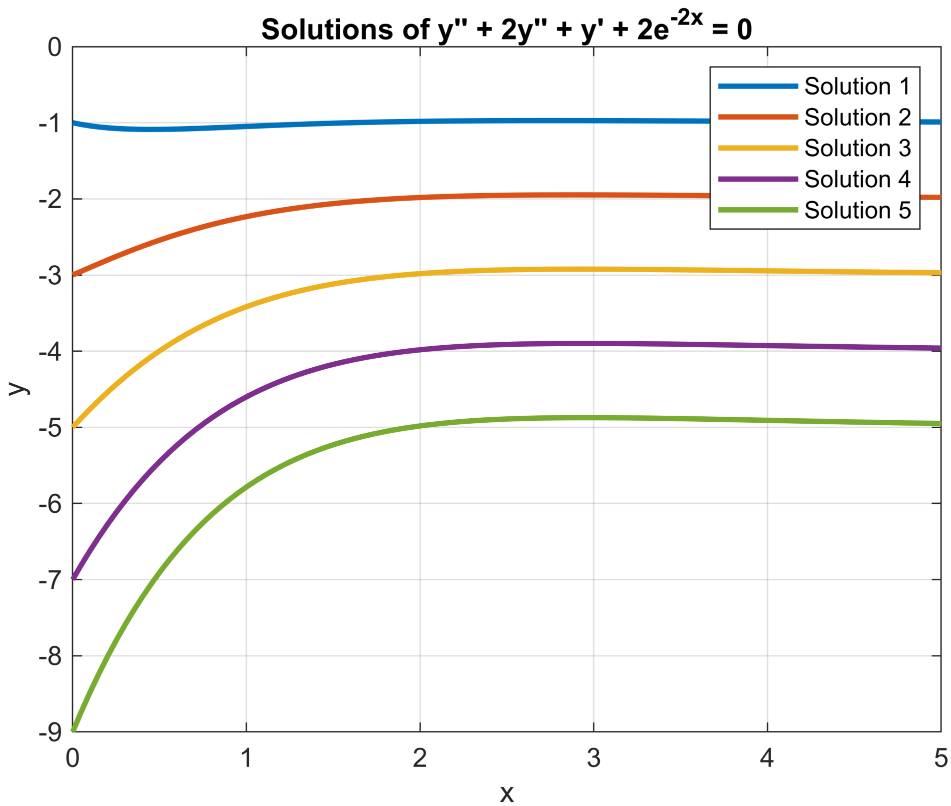
general_2 = $e^{-2\,x}\,\left(x - C_1\,e^{2\,x} + C_1\,x\,e^{2\,x} + 1\right) + C_2\,e^{-x} - x\,e^{-x}\,\left(e^{-x} + C_1\,e^{x}\right) + C_3\,x\,e^{-x}$

```
% Example particular solution
particular_2_example = subs(general_2, 'C1', 1);
particular_2_example
```

particular_2_example = $e^{-2\,x}\,(x - e^{2\,x} + x\,e^{2\,x} + 1) + C_2\,e^{-x} + C_3\,x\,e^{-x} - x\,e^{-x}\,(e^{-x} + e^{x})$

```
% Create figure
figure('Name', 'Question 2 Solutions');
x = linspace(0, 5, 100);

% Plot n particular solutions with different initial conditions
for i = 1:n
    % Use different multiples of the constant value for each solution
    sol = subs(general_2, {'C1', 'C2', 'C3'}, ...
               {constant_val*i, constant_val*(-i), constant_val*(i/2)});
    y_vals = double(subs(sol, x));
    plot(x, y_vals, 'LineWidth', 2);
    hold on;
end

grid on;
title('Solutions of y'''' + 2y'''' + y'' + 2e^{-2x} = 0');
xlabel('x');
ylabel('y');

% Create legend entries based on number of solutions
```

```
legend_entries = cell(1,n);
for i = 1:n
    legend_entries{i} = ['Solution ' num2str(i)];
end
legend(legend_entries);
hold off;
```



Question 03.

```
%% Question 03: System of ODEs
disp("Question 03");
```

```
Question 03
```

```
% x' = 3x + 4y
% y' = 2x + y

% Define symbolic variables
syms x(t) y(t)

% Define the system of ODEs
eq1 = diff(x) == 3*x + 4*y;
eq1
```

```
eq1(t) =
```

$$\frac{\partial}{\partial t}\, x(t) = 3\, x(t) + 4\, y(t)$$

```
eq2 = diff(y) == 2*x + y;
eq2
```

```
eq2(t) =
```

$$\frac{\partial}{\partial t}\, y(t) = 2\, x(t) + y(t)$$

```
% Solve the system using dsolve
[x_sol, y_sol] = dsolve(eq1, eq2);

% Display the general solutions
disp('x(t) = ');
```

```
x(t) =
```

```
pretty(x_sol)
```

```
C2 exp(5 t) 2 - C1 exp(-t)
```

```
disp('y(t) = ');
```

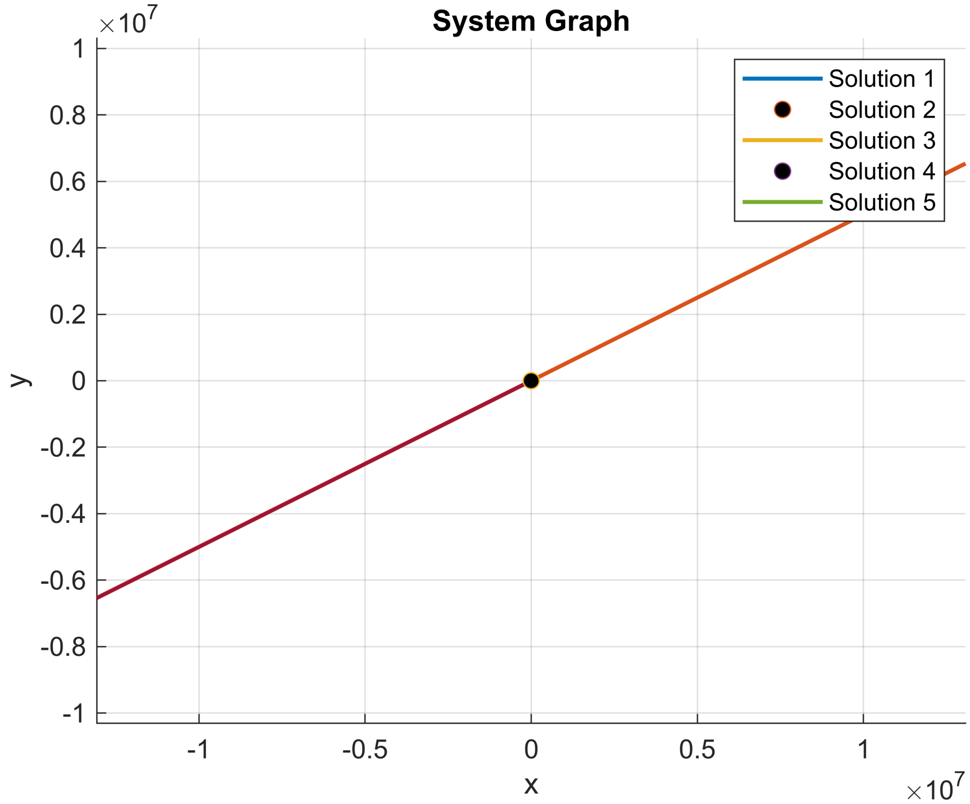
```
y(t) =
```

```
pretty(y_sol)
```

```
C1 exp(-t) + C2 exp(5 t)
```

```
% Create a plot to visualize different particular solutions
figure('Name', 'System Graph')

% Define time range for solutions
t = linspace(0, 3, 100);

% We'll plot 5 different particular solutions with different initial conditions
initial_conditions = [
    [1, 0];     % First set
    [0, 1];     % Second set
    [-1, -1];   % Third set
    [2, -2];    % Fourth set
    [-2, 2]     % Fifth set
];

% Plot each particular solution
hold on
for i = 1:5
    % Substitute initial conditions
```

```
    x0 = initial_conditions(i,1);
    y0 = initial_conditions(i,2);

    % Create particular solutions by substituting constants
    x_particular = subs(x_sol, {'C1', 'C2'}, {x0, y0});
    y_particular = subs(y_sol, {'C1', 'C2'}, {x0, y0});

    % Convert to numerical values
    x_values = double(subs(x_particular, 't', t));
    y_values = double(subs(y_particular, 't', t));

    % Plot the solution trajectory
    plot(x_values, y_values, 'LineWidth', 1.5)

    % Plot starting point
    plot(x0, y0, 'o', 'MarkerFaceColor', 'black')
end

% Add direction field (quiver plot)
[X, Y] = meshgrid(-3:0.5:3, -3:0.5:3);
dX = 3*X + 4*Y;
dY = 2*X + Y;

% Normalize vectors for better visualization
norm = sqrt(dX.^2 + dY.^2);
quiver(X, Y, dX./norm, dY./norm, 0.3, 'Color', [0.7 0.7 0.7])

% Customize the plot
grid on
xlabel('x')
ylabel('y')
title('System Graph')
axis equal
legend('Solution 1', 'Solution 2', 'Solution 3', 'Solution 4', 'Solution 5')
hold off
```


System Graph
Solution 1
Solution 2
Solution 3
Solution 4
Solution 5
y
x
×10^7
×10^7
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1