

```
disp("Independent work 08 - interpolation 03.")
```

Independent work 08 - interpolation 03.

```
disp("Question 01.")
```

Question 01.

```
% Given data points
```

```
x = [3, 4, 5, 6]
```

```
x = 1×4  
    3    4    5    6
```

```
y = [1.8353, 1.9633, 1.9453, 2.2797]
```

```
y = 1×4  
    1.8353    1.9633    1.9453    2.2797
```

```
% Number of intervals
```

```
n = length(x) - 1;
```

```
% Step 1: Compute h (interval lengths)
```

```
h = diff(x); % h(i) = x(i+1) - x(i)
```

```
% Step 2: Create the matrix A and vector B for the system A * c = B
```

```
A = zeros(n+1, n+1);
```

```
B = zeros(n+1, 1);
```

```
% Step 3: Set up the system of equations
```

```
disp('System of equations: A * c = B')
```

System of equations: A \* c = B

```
% First row (boundary condition: c1 = 0)
```

```
A(1, 1) = 1;
```

```
% Intermediate rows (for cubic spline conditions)
```

```
for i = 2:n
```

```
    A(i, i-1) = h(i-1);
```

```
    A(i, i) = 2 * (h(i-1) + h(i));
```

```
    A(i, i+1) = h(i);
```

```
    B(i) = 3 * ((y(i+1) - y(i)) / h(i) - (y(i) - y(i-1)) / h(i-1));
```

```
end
```

```
% Last row (boundary condition: cn = 0)
```

```
A(n+1, n) = h(n);
```

```
A(n+1, n+1) = 1;
```

```
% Display the system of equations in matrix form
```

```
disp('Matrix A:')
```

Matrix A:

```
disp(A)
```

```
1    0    0    0
1    4    1    0
0    1    4    1
0    0    1    1
```

```
disp('Vector B:')
```

Vector B:

```
disp(B)
```

```
0
-0.4380
1.0572
0
```

```
% Display the system of equations in string form
```

```
disp('The system of equations for c (second derivatives):')
```

The system of equations for c (second derivatives):

```
for i = 2:n
    fprintf('Equation %d: %.4f * c(%d) + %.4f * c(%d) = %.4f\n', ...
           i, A(i,i-1), i-1, A(i,i), i, B(i));
end
```

Equation 2: 1.0000 \* c(1) + 4.0000 \* c(2) = -0.4380

Equation 3: 1.0000 \* c(2) + 4.0000 \* c(3) = 1.0572

```
% Step 4: Solve the system using matrix inversion (X = A^(-1) * B)
```

```
c = inv(A) * B;
```

```
% Display the second derivatives (c)
```

```
disp('Second derivatives at the interpolation points (c):')
```

Second derivatives at the interpolation points (c):

```
disp(c)
```

```
0
-0.2156
0.4243
-0.4243
```

```
% Step 5: Compute other coefficients b, d, and a
```

```
% Initialize arrays for b, d, and a
```

```

b = zeros(n, 1);
d = zeros(n, 1);
a = y(1:n);

% Compute b and d coefficients
for i = 1:n
    b(i) = (y(i+1) - y(i)) / h(i) - h(i) * (2 * c(i) + c(i+1)) / 3;
    d(i) = (c(i+1) - c(i)) / (3 * h(i));
end

% Display the coefficients a, b, c, and d
disp('Spline coefficients:')

```

Spline coefficients:

```
disp('a = ')
```

a =

```
disp(a)
```

```

1.8353    1.9633    1.9453

```

```
disp('b = ')
```

b =

```
disp(b)
```

```

0.1999
-0.0157
0.1930

```

```
disp('c = ')
```

c =

```
disp(c)
```

```

0
-0.2156
0.4243
-0.4243

```

```
disp('d = ')
```

d =

```
disp(d)
```

```

-0.0719
0.2133
-0.2828

```

```

% Step 6: Display the spline equations for each interval
disp('Spline equations for each interval:')

```

Spline equations for each interval:

```
for i = 1:n
    fprintf('Interval %d: P%d(x) = %.4f + %.4f*(x - %.4f) + %.4f*(x - %.4f)^2 +
%.4f*(x - %.4f)^3\n', ...
        i, i, a(i), b(i), x(i), c(i), x(i), d(i), x(i));
end
```

```
Interval 1: P1(x) = 1.8353 + 0.1999*(x - 3.0000) + 0.0000*(x - 3.0000)^2 + -0.0719*(x - 3.0000)^3
Interval 2: P2(x) = 1.9633 + -0.0157*(x - 4.0000) + -0.2156*(x - 4.0000)^2 + 0.2133*(x - 4.0000)^3
Interval 3: P3(x) = 1.9453 + 0.1930*(x - 5.0000) + 0.4243*(x - 5.0000)^2 + -0.2828*(x - 5.0000)^3
```

```
% Define the spline polynomial function
P = @(i, xx) a(i) + b(i) * (xx - x(i)) + c(i) * (xx - x(i)).^2 + d(i) * (xx -
x(i)).^3;

% Step 7: Plotting the spline interpolation with different colors
plot(x, y, 'r*', 'LineWidth', 2, 'DisplayName', 'Original Points')
hold on

% Plot interpolation polynomial with different colors for each section
colors = ['b', 'g', 'm', 'c'];
for i = 1:n
    xx = linspace(x(i), x(i+1), 100);
    yy = P(i, xx);
    plot(xx, yy, 'Color', colors(i), 'LineWidth', 1.5)
end

% Customize plot
title('Cubic Spline Interpolation with Multi-Color Sections')
xlabel('x')
ylabel('y')
title('Cubic Spline Interpolation');
legend('Data Points', 'Spline Segment 1', 'Spline Segment 2', 'Spline Segment 3',
'Location', 'Best');
grid on
legend show
hold off
```

