Let $Y_t = 5 + \varepsilon_t - \frac{1}{2}\varepsilon_{t-1} + \frac{1}{4}\varepsilon_{t-2}$ , where $\varepsilon_t$ is white noise

$E(\varepsilon_t) = 0$;

$E(\varepsilon_t\ \varepsilon_{t+k}) = 0$ for $k \neq 0$

$E(\varepsilon_t\ \varepsilon_{t+k}) = \sigma^2$ , $k = 0$

and $t$ is time, $t = 1, 2, 3 \ldots$

## Problem 1(a)

$$E(Y_t) = E\left(5 + \varepsilon_t - \frac{1}{2}\varepsilon_{T-1} + \frac{1}{4}\varepsilon_{T-2}\right)$$

→ linearity of expectations.

$$E(Y_t) = E(5) + E(\varepsilon_t) - \frac{1}{2}E(\varepsilon_{t-1}) + \frac{1}{4}E(\varepsilon_{T-2})$$

→ Since $\varepsilon_t$ is white noise all expectations of $\varepsilon_t, \varepsilon_{t-1}$ & $\varepsilon_{t-2} = 0$.

hence $E(Y_t) = 5$

## Problem 1(b)

$$Var(Y_t) = Cov(Y_t, Y_t) = \overset{1}{E(Y_t^2)} - E(Y_t)^2$$

→ $E(Y_t)$ already known.

For $E(Y_t^2)$

$$Y_t^2 = \left(5 + \varepsilon_t - \frac{1}{2}\varepsilon_{t-1} + \frac{1}{4}\varepsilon_{t-2}\right)^2$$

$$Y_t^2 = 5^2 + \left(\varepsilon_t - \frac{1}{2}\varepsilon_{t-1} + \frac{1}{4}\varepsilon_{t-2}\right)^2 + 2 \times 5 \times \left(\varepsilon_t - \frac{1}{2}\varepsilon_{t-1} + \frac{1}{4}\varepsilon_{t-2}\right)$$

Since $E(\varepsilon_t) = 0$

$$E(Y_t^2) = 5^2 + E\left(\left(\varepsilon_t - \frac{1}{2}\varepsilon_{T-1} + \frac{1}{4}\varepsilon_{T-2}\right)^2\right)$$

$$= E(\varepsilon_t^2) + \left(\frac{1}{2}\right)^2 E(\varepsilon_{t-1}^2) + \left(\frac{1}{4}\right)^2 E(\varepsilon_{t-2}^2)$$

Since prerersence of white noise.

$$= \sigma^2 + \frac{1}{4}\sigma^2 + \frac{1}{16}\sigma^2$$

$$= \sigma^2\left(1 + \frac{1}{4} + \frac{1}{16}\right)$$

$$= \sigma^2\left(\frac{21}{16}\right)$$

$$E(Y_t^2) = 25 + \frac{21}{16}\sigma^2$$

Plugging in 1

$$Var(Y_t) = E(Y_t^2) - E(Y_t)^2 = 25 + \frac{21}{16}\sigma^2 - 25$$

$$= \frac{21}{16}\sigma^2$$

## Problem 1 c

Autocovariance for $k = 1, 2, 3 \ldots$

$$Cov(Y_t, Y_{t+k}) = E(Y_t Y_{t+k}) - E(Y_t)(Y_{t+k})$$

→ Since $E(Y_t) = 5$

$$Cov(Y_t, Y_{t+k}) = E(Y_t Y_{t+k}) - 25$$

→ $\varepsilon_t$ is white noise, so $E(e_t e_{t+k}) = 0$ for $K \neq 0$. Analyzing $k=1$.

$$Y_t = 5 + \varepsilon_t - \frac{1}{2}\varepsilon_{t-1} + \frac{1}{4}\varepsilon_{t-2}$$

$$Y_{t+1} = 5 + \varepsilon_{t+1} - \frac{1}{2}\varepsilon_t + \frac{1}{4}\varepsilon_{t-1}$$

So

$$E(Y_t Y_{t+1}) = E\left(\left(5+\varepsilon_t - \frac{1}{2}\varepsilon_{t-1} + \frac{1}{4}\varepsilon_{t-2}\right)\right) + \left(\left(5+\varepsilon_{T+1} - \frac{1}{2}\varepsilon_T + \frac{1}{4}\varepsilon_{T-1}\right)\right)$$

Considering white noise.

$$E(Y_t Y_{t+1}) = 25 + 0 - \frac{1}{2} E(\varepsilon_t^2) + 0$$

$$E(Y_t Y_{t+1}) = 25 - \frac{1}{2}\sigma^2$$

For $\underline{\underline{k=1}}$

$$Cov(Y_t, Y_{t+1}) = E(Y_t Y_{t+1}) - 25 = -\frac{1}{2}\sigma^2$$

For $\underline{\underline{k=2}}$

$$Y_{t+2} = 5 + \varepsilon_{t+2} - \frac{1}{2}\varepsilon_{T+1} + \frac{1}{4}\varepsilon_T$$

$$E(Y_t Y_{t+2}) = 25 + \frac{1}{4}\sigma^2$$

$$Cov(Y_t Y_{t+2}) = \frac{1}{4}\sigma^2$$

For higher values $k \geq 3$ autocovariance is zero since

$\varepsilon_t$ & $\varepsilon_{t+k}$ become independent.

**Problem 1 (D)**

$$Corr(Y_t, Y_{t+k}) = \frac{Cov(Y_t, Y_{t+k})}{Var(Y_t)}$$

Since variance of $Y_t = \frac{21}{16}\sigma^2$

For $k=0$

$$Corr(Y_t, Y_{t+k}) = 1$$

For $k=1$

$$Cov(Y_t, Y_{t+1}) = \frac{-1}{2}\sigma^2$$

hence

$$Corr(Y_t\ Y_{t+1}) = \frac{-\frac{1}{2}\sigma^2}{\frac{21}{16}\sigma^2} = \frac{-\frac{1}{2}}{\frac{21}{16}} = \frac{-8}{21}$$

For $k=2$

$$Cov(Y_t, Y_{t+2}) = \frac{1}{4}\sigma^2$$

$$Corr(Y_t, Y_{t+2}) = \frac{\frac{1}{4}\sigma^2}{\frac{21}{16}\sigma^2} = \frac{\frac{1}{4}}{\frac{21}{6}} = \frac{4}{21}$$

for $k \geq 3$ autocovariance is zero.

## Q2(A)

In [36]:
```
import pandas as pd
```

In [37]:
```
measurement_data = pd.read_csv('/content/Measurement_Q1 (1).csv')
```

In [38]:
```
measurement_data.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 150 entries, 0 to 149
Data columns (total 2 columns):
 #   Column       Non-Null Count  Dtype
---  ------       --------------  -----
 0   Time         150 non-null    int64
 1   Measurement  150 non-null    float64
dtypes: float64(1), int64(1)
memory usage: 2.5 KB
```

In [39]:
```
measurement_data.head()
```

Out[39]:

| | Time | Measurement |
|---|---|---|
| **0** | 1 | 1.84 |
| **1** | 2 | 3.93 |
| **2** | 3 | 4.00 |
| **3** | 4 | 5.42 |
| **4** | 5 | 6.89 |

In [40]:
```
from statsmodels.tsa.arima.model import ARIMA
```

In [41]:
```
model = ARIMA(measurement_data["Measurement"], order=(0,1,1))
```

In [42]:
```
model_fit = model.fit()
```

In [43]:
```
print(model_fit.summary())
```

```
                                SARIMAX Results
==============================================================================
==
Dep. Variable:            Measurement   No. Observations:                    1
50
Model:                 ARIMA(0, 1, 1)   Log Likelihood                  -202.6
09
Date:                Fri, 04 Oct 2024   AIC                              409.2
17
Time:                        22:40:25   BIC                              415.2
25
Sample:                             0   HQIC                             411.6
58
                                - 150
Covariance Type:                  opg
==============================================================================
==
                 coef    std err          z      P>|z|      [0.025      0.97
5]
------------------------------------------------------------------------------
--
ma.L1          0.7531      0.060     12.573      0.000       0.636       0.8
71
sigma2         0.8834      0.109      8.080      0.000       0.669       1.0
98
==============================================================================
=======
Ljung-Box (L1) (Q):                   0.26   Jarque-Bera (JB):
1.10
Prob(Q):                              0.61   Prob(JB):
0.58
Heteroskedasticity (H):               1.00   Skew:
-0.21
Prob(H) (two-sided):                  0.99   Kurtosis:
2.98
==============================================================================
=======

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (compl
ex-step).
```

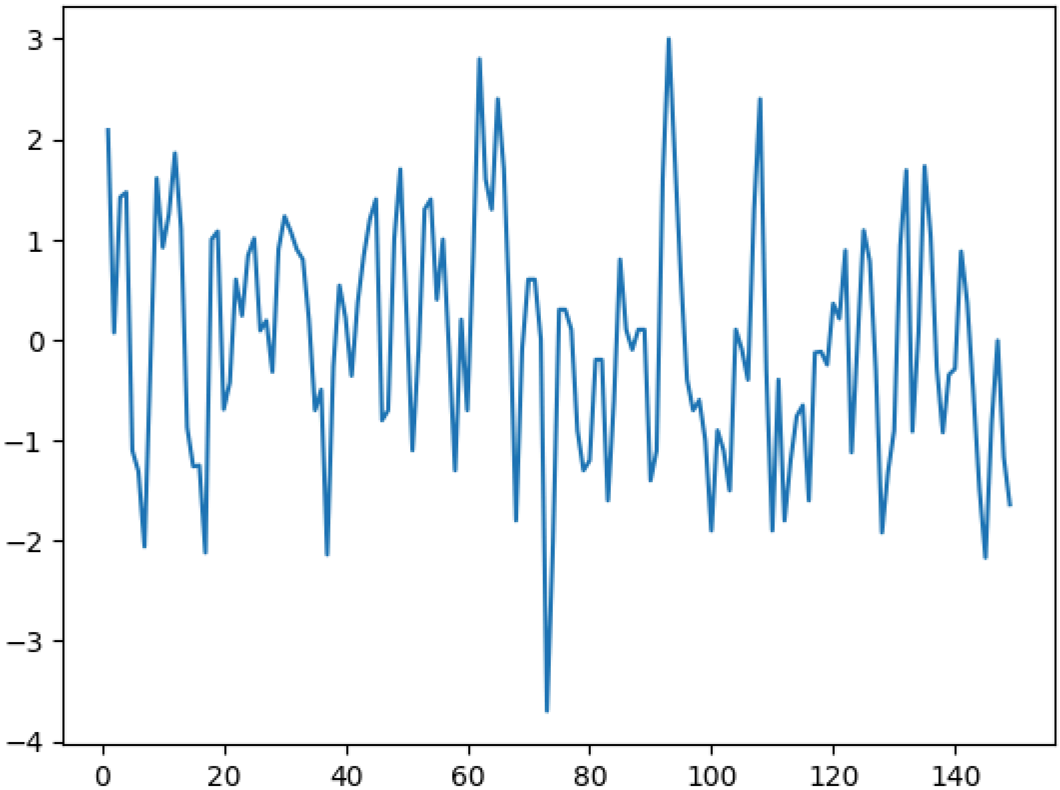
# Q2(B)

In [44]:
```
measurement_data_2 = measurement_data.copy()
```

In [45]:
```
measurement_data_2 = measurement_data_2["Measurement"].diff()
```

In [46]:
```
measurement_data_2.plot()
```

Out[46]: <Axes: >




## Q2 (C)

Both models are the same thing as IMA integrates differencing, where as MA data is manually differenced. Both models ma.L1 coefficient (0.7531) are the same. This coefficient tells how much correlation in the the previous lag and the current term, and how much of it has it captured in the model.

In [47]:
```
model_2 = ARIMA(measurement_data_2, order=(0,0,1))
```

In [48]:
```
model_fit_2 = model_2.fit()
```

In [49]:
```
print(model_fit_2.summary())
```

```
                                   SARIMAX Results
==============================================================================
==
Dep. Variable:            Measurement   No. Observations:                    1
50
Model:                 ARIMA(0, 0, 1)   Log Likelihood                  -202.6
07
Date:                Fri, 04 Oct 2024   AIC                              411.2
15
Time:                        22:40:26   BIC                              420.2
47
Sample:                             0   HQIC                             414.8
84
                                - 150
Covariance Type:                  opg
==============================================================================
==
                 coef    std err          z      P>|z|      [0.025      0.97
5]
------------------------------------------------------------------------------
--
const         -0.0064      0.137     -0.047      0.962      -0.274       0.2
61
ma.L1          0.7531      0.061     12.392      0.000       0.634       0.8
72
sigma2         0.8834      0.110      8.052      0.000       0.668       1.0
98
==============================================================================
=======
Ljung-Box (L1) (Q):                   0.26   Jarque-Bera (JB):
1.11
Prob(Q):                              0.61   Prob(JB):
0.57
Heteroskedasticity (H):               1.00   Skew:
-0.21
Prob(H) (two-sided):                  1.00   Kurtosis:
3.00
==============================================================================
=======

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (compl
ex-step).
```

## Q3 (A)

In [51]:
```
passengers_data = pd.read_csv('/content/ch1passengers_HW3.csv')
```

In [52]:
```
passengers_data.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 201 entries, 0 to 200
Data columns (total 6 columns):
 #   Column         Non-Null Count  Dtype 
---  ------         --------------  ----- 
 0   Date           201 non-null    object
 1   Year           201 non-null    int64 
 2   Month          201 non-null    int64 
 3   DOMESTIC       201 non-null    object
 4   INTERNATIONAL  201 non-null    object
 5   TOTAL          201 non-null    object
dtypes: int64(2), object(4)
memory usage: 9.5+ KB
```

In [53]:
```
passengers_data.head()
```

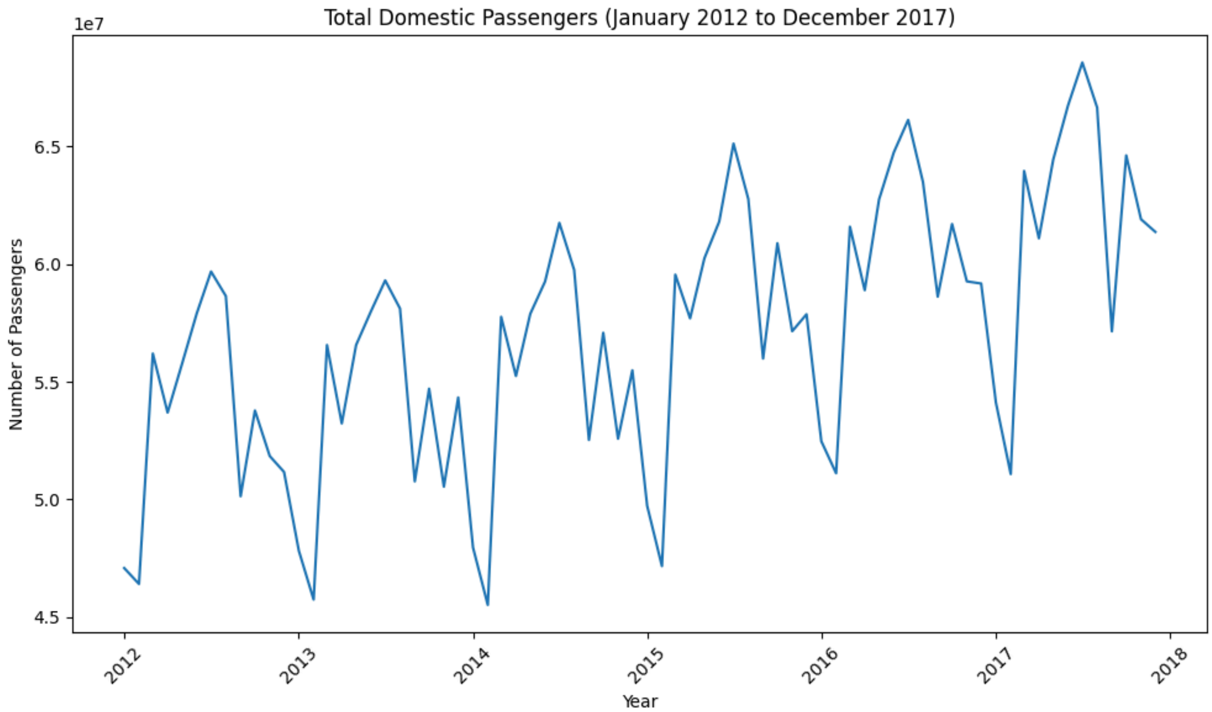
Out[53]:

| | Date | Year | Month | DOMESTIC | INTERNATIONAL | TOTAL |
|---|---|---|---|---|---|---|
| **0** | 2002-10-01 | 2002 | 10 | 48,054,917 | 9,578,435 | 57,633,352 |
| **1** | 2002-11-01 | 2002 | 11 | 44,850,246 | 9,016,535 | 53,866,781 |
| **2** | 2002-12-01 | 2002 | 12 | 49,684,353 | 10,038,794 | 59,723,147 |
| **3** | 2003-1-01 | 2003 | 1 | 43,032,450 | 9,726,436 | 52,758,886 |
| **4** | 2003-2-01 | 2003 | 2 | 41,166,780 | 8,283,372 | 49,450,152 |

## Q3 (A)

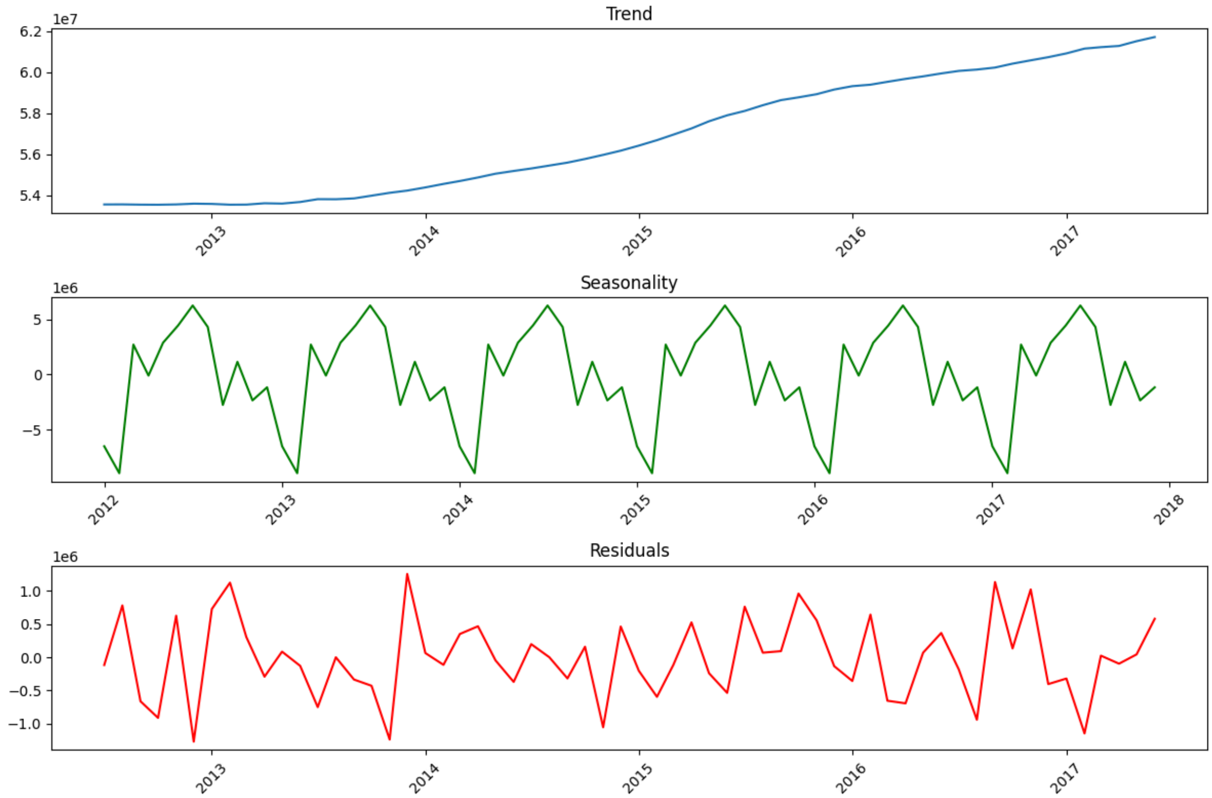
In [90]:
```
data_passengers = passengers_data.copy()
```

In [91]:
```
data_passengers['Date'] = pd.to_datetime(data_passengers['Date'], errors='co

filtered_data = data_passengers[(data_passengers['Date'] >= '2012-01-01') &

import matplotlib.pyplot as plt

plt.figure(figsize=(10,6))
plt.plot(filtered_data['Date'], filtered_data['DOMESTIC'].str.replace(",", "
plt.xlabel('Year')
plt.ylabel('Number of Passengers')
plt.title('Total Domestic Passengers (January 2012 to December 2017)')
plt.xticks(rotation=45)
plt.tight_layout()
plt.show()
```




# Q3 (B)

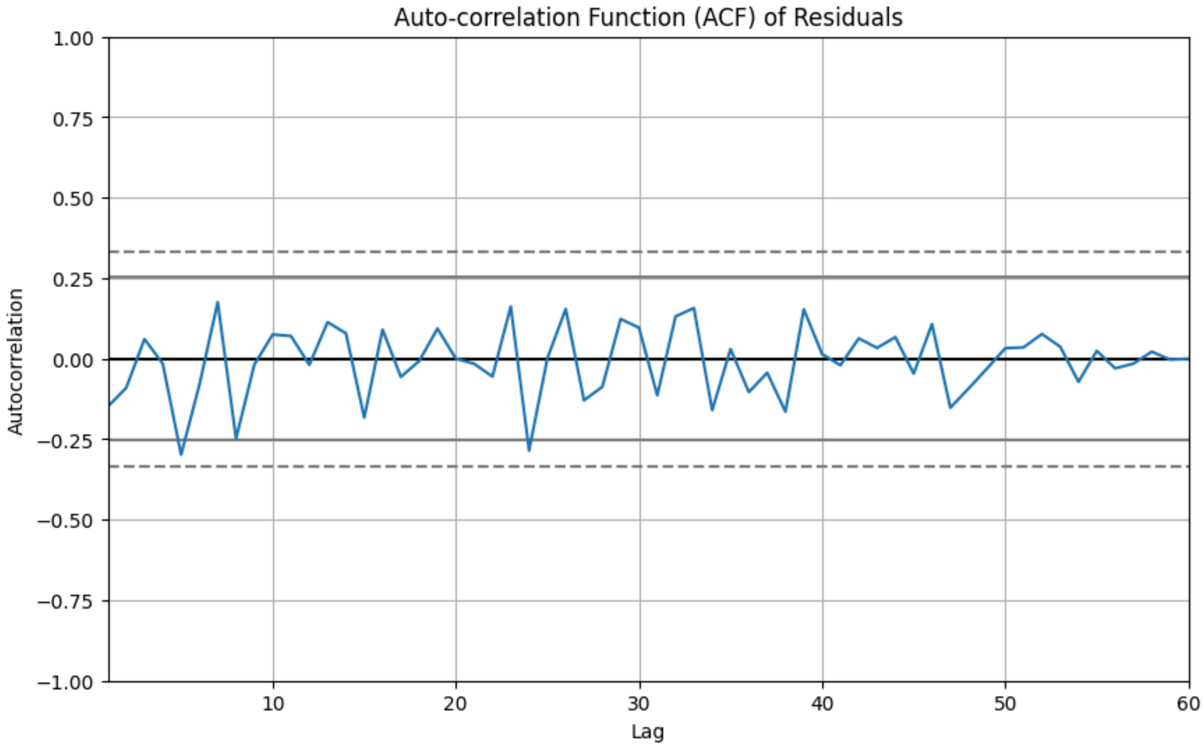
In [93]:
```
from statsmodels.tsa.seasonal import seasonal_decompose

domestic_passengers = filtered_data['DOMESTIC'].str.replace(",", "").astype(


decomposition = seasonal_decompose(domestic_passengers, model='additive', pe

plt.figure(figsize=(12, 8))

# trend
plt.subplot(3, 1, 1)
plt.plot(filtered_data['Date'], decomposition.trend, label='Trend')
plt.title('Trend')
plt.xticks(rotation=45)

# seasonal component
plt.subplot(3, 1, 2)
plt.plot(filtered_data['Date'], decomposition.seasonal, label='Seasonality',
plt.title('Seasonality')
plt.xticks(rotation=45)

# residuals
plt.subplot(3, 1, 3)
plt.plot(filtered_data['Date'], decomposition.resid, label='Residuals', colo
plt.title('Residuals')
plt.xticks(rotation=45)

plt.tight_layout()
plt.show()
```




# Q3 (C)

In [95]:

```
from pandas.plotting import autocorrelation_plot

residuals = decomposition.resid.dropna()

plt.figure(figsize=(10, 6))
autocorrelation_plot(residuals)
plt.title('Auto-correlation Function (ACF) of Residuals')
plt.show()
```




## Q3 (D)

The confidence interval formula is +- 1.96/SQRT(N) @ 95% confidence. The width of the confidence interval is 2 times 1.96/SQRT(N) i.e., 0.462. And the autocorrelation values not statistically correlated will be within range of +- 0.231.

## Q3 (E)

No lag in the plot is above or below the confidence interval meaning the lag is statistically insignificant. Hence the residual is white noise i.e., random.

```
In [ ]:
```