$$Z_m(t) = \left\langle e^{im\varphi(t)} \right\rangle,$$

矩生成函数：$F(k,t) \equiv \left\langle \exp\left(ke^{i\varphi}\right) \right\rangle \equiv \sum_{m=0}^{\infty} Z_m \frac{k^m}{m!} = Z_0 + Z_1 k + Z_2 \frac{k^2}{2!} + Z_3 \frac{k^3}{3!} + \cdots, Z_m = \left.\frac{\partial^m}{\partial k^m} F\right|_{k=0}.$

累积量生成函数：$\Phi(k,t) \equiv \ln F \equiv \sum_{m=1}^{\infty} K_m \frac{k^m}{m!} = K_1 k + K_2 \frac{k^2}{2!} + K_3 \frac{k^3}{3!} + \cdots, K_m = \left.\frac{\partial^m}{\partial k^m} \Phi\right|_{k=0}.$

$$\sum_{m=1}^{\infty} \frac{K_m}{m!} k^m = \sum_{m=1}^{\infty} \mathcal{K}_m k^m, \mathcal{K}_m = \frac{K_m}{m!} = \frac{1}{m!} \left.\frac{\partial^m}{\partial k^m} \Phi\right|_{k=0}.$$

$\ln F = \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$，代入$x = F - 1 = Z_1 k + Z_2 \frac{k^2}{2!} + Z_3 \frac{k^3}{3!} + \cdots,$

对比$K_1 k + K_2 \frac{k^2}{2!} + K_3 \frac{k^3}{3!} + \cdots,$

$$\# : K_1 = Z_1, \quad K_2 = Z_2 - Z_1^2, \quad K_3 = Z_3 - 3Z_2 Z_1 + 2Z_1^3,$$

$$K_4 = Z_4 - 4Z_3 Z_1 - 3Z_2^2 + 12 Z_2 Z_1^2 - 6Z_1^4, \cdots.$$

圆累积量生成函数：$\Psi(k,t) \equiv k\frac{\partial}{\partial k} \ln F \equiv \sum_{m=1}^{\infty} \kappa_m k^m = \kappa_1 k + \kappa_2 k^2 + \kappa_3 k^3 + \cdots, \kappa_m = \frac{1}{m!} \left.\frac{\partial^m}{\partial k^m} \Psi\right|_{k=0}.$

$$k\frac{\partial}{\partial k}\Phi = k\frac{\partial}{\partial k}\left(\sum_{m=1}^{\infty} K_m \frac{k^m}{m!}\right) = \sum_{m=1}^{\infty} \frac{K_m}{(m-1)!} k^m, \kappa_m = \frac{K_m}{(m-1)!},$$

$$\# : \kappa_1 = Z_1, \quad \kappa_2 = Z_2 - Z_1^2, \quad \kappa_3 = \frac{1}{2}\left(Z_3 - 3Z_2 Z_1 + 2Z_1^3\right), \quad \cdots.$$

$$\mathrm{OA} : Z_m = Z_1^m.$$

$$\sum_{m=0}^{\infty} Z_m \frac{s^m}{m!} = \exp\left(\sum_{m=1}^{\infty} \frac{\kappa_m}{m} s^m\right) = \exp\left(\sum_{m=1}^{\infty} (m-1)! \kappa_m \frac{s^m}{m!}\right)$$

$$\kappa_m = \frac{Z_m}{(m-1)!} - \sum_{j=1}^{m-1} \kappa_{m-j} \frac{Z_j}{j!}, m \geqslant 1.$$