Practical Physics

Wave Phenomena

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Introduction:

Snell's Law predicts that incidence and reflection angles of wave are the same:

$$\theta = \theta' \tag{1a}$$

Also, the relation between velocities of wave in two different media can be described as:

$$\frac{\sin\theta_i}{v_1} = \frac{\sin\theta_r}{v_2} \tag{1b}$$

To describe diffraction:

$$a\sin\theta = \lambda \tag{2}$$

Wave speed could be described as:

$$v = f\lambda \tag{3}$$

Where f is frequency, and λ is wavelength. It can also be written as:

$$\lambda = \frac{v}{f} = vT \tag{4}$$

Where T is the period.

Wave speed of shallow water gravity waves depends on the depth of the water and its gravitational acceleration:

$$v = \sqrt{g \cdot d} \tag{5}$$

Where d is the depth.

If there are two sources of waves with the same frequency and amplitude, the crest will have patterns of destructive interference when it overlaps with the trough of the wave, and constructive interference when it overlap with another crest. Superposition of these two waves will create lines of destructive interference and constructive interference, positioned radially outward. The resulting wave can be described by:

$$\psi(t) = a\left(\cos\left(\omega t - \frac{\omega r_1}{v}\right) + \cos\left(\omega t - \frac{\omega r_2}{v}\right)\right) = 2a\cos(\omega t)\cos\left(\frac{\pi\left(r_2 - r_1\right)}{\lambda}\right) \tag{6}$$

where its amplitude is:

$$A(\theta) = 2a\cos\left(\frac{\pi d\sin\theta}{\lambda}\right) \tag{7}$$

Where a is the amplitude of one wave, d is the separation between dippers, θ is angle between observed point and normal line. The condition where amplitude is at its maximum:

$$\frac{\pi d\sin\theta}{\lambda} = 0 + m\pi \tag{8}$$

Where m is the order of maxima away from center, when the amplitude is at its minimum:

$$\frac{\pi d\sin\theta}{\lambda} = \pi\left(\frac{1}{2} + m\right) \tag{9}$$

<u>**Materials:**</u> Ripple generator, rod with attached light source capable of strobing, reflector mirror panel, glass surface panel, dippers, adapters and actuators, barriers, refractors, projection screen, labVIEW program.

<u>**Purpose:**</u> To observe, identify, measure, and verify patterns of water in various situations of a wave interferences, including reflection, wave speed, refraction, diffraction, and multi-wave interference.

Procedure: The ripple wave tank was filled and the sponge perimeters were pushed so that they were entirely soaked in the water, in order to ensure that the depth of the water stayed consistent throughout the experiment. For the reflection exercise, a long straight barrier was placed at approximately a 45 degree angle to the vertical barriers of the wave pool, in the middle of the wave pool (*image 1*), with the water at a depth of approximately half of the height of the barrier. The ripple generator was set to the center of the tank, and the plane wave dipper was connected to the ripple generator so that it made contact with the surface of the water while the ripple generator was at rest. The frequency of the wave generator was set to 20Hz and the amplitude set to a position less than half of its maximum amplitude. The light source was set to strobe and the labVIEW program was launched in order to capture the patterns on screen. The angles of incidence and reflection were measured on screen and an image of the pattern was captured. Then, a curved barrier was replaced with the straight barrier and adjusted so it was concave towards the ripple generator (*image 2*). The reflected wave pattern was measured on screen using labVIEW and the focal distance and radius of the curved barrier was estimated, and an image of the pattern was captured. This process was repeated with the curved barrier turned 180 degrees so that it was convex towards the ripple generator.

For wave speed and frequency, the same setup was used as prior, but with no barrier, so the curved barrier was removed from the water pool and the frequency was lowered to 5Hz. The onscreen wavelengths were measured and recorded and an image was captured of the wavelengths (*image 3*). This was repeated by increasing the frequency in increments to complete five different recordings of wavelength with five difference frequencies, starting at the initial 5Hz. The water depth was then altered with a set frequency, and five different water depths were created with five measurements of wave frequency.

To measure refraction with a straight barrier, a trapezoidal refractor piece was placed under the surface of the water by about 1mm, positioned so the triangular piece points toward the plane wave dipper. The light source was kept on strobe and the frequency was adjusted to 15Hz. The refraction pattern was captured as an image (*image 4*) and the angles of incidence and refraction were recorded using the labVIEW program.

For diffraction, two straight barriers, parallel to each other on the save axis were separated tips by 3cm to create a slit. The generator was set to 20Hz and the amplitude adjusted to about half of its maximum. The light source remained on strobe, and a diffraction pattern appeared through the slit of the barriers (*image 5*). An image of the pattern was captured and the angular spread of the circular waves past the slit was recorded. The slit opening width was then adjusted and the recordings were repeated five times.

For interference, the ripple generator was powered off in order to replace the plane wave dipper with two standard sized round dippers, in order to create two waves. The dippers were adjusted to touch the surface of the water, the generator was set to 20Hz and the amplitude set to about half of it maximum setting, with the light source remaining on strobe. The pattern image was captured (*image 6*) and the angles corresponding to the first and second interference lines were recorded. The distance between the standard round dippers was altered using the adjustment capabilities of the plane wave dipper, and the varying images were captured (*image 7*) and the interference pattern angles were recorded.

Data:

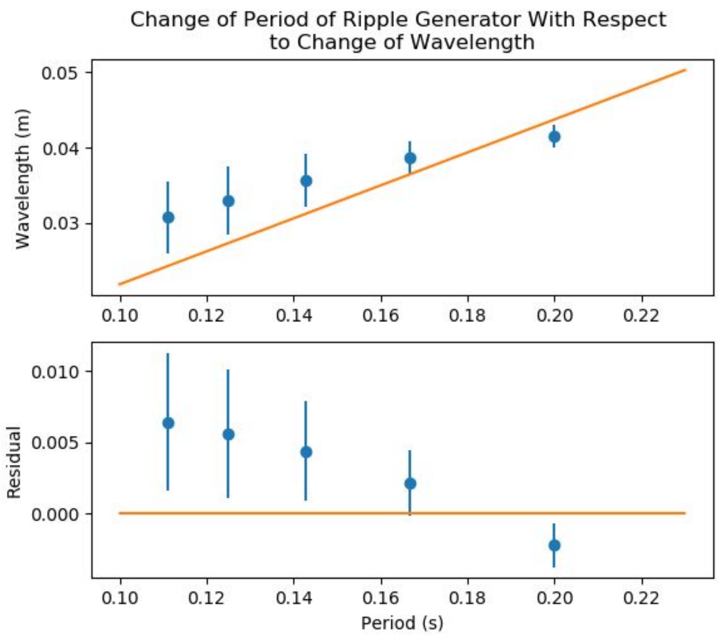


Figure 1: 'Change of period of ripple generator with respect to change of wavelength'. Equation for line of best fit $y = 0.21865 \pm 0.00004\, x$

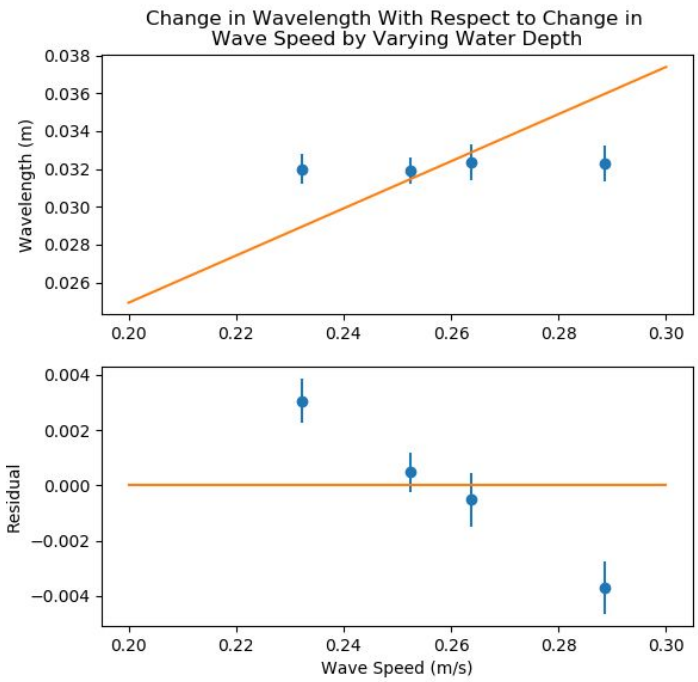


Figure 2: 'Change in wavelength with respect to change in wave speed by varying the water depth'. Wave speed is obtained by substituting water depth into equation 5.

Equation for line of best fit

$y = 0.124628 \pm 0.000003\, x$

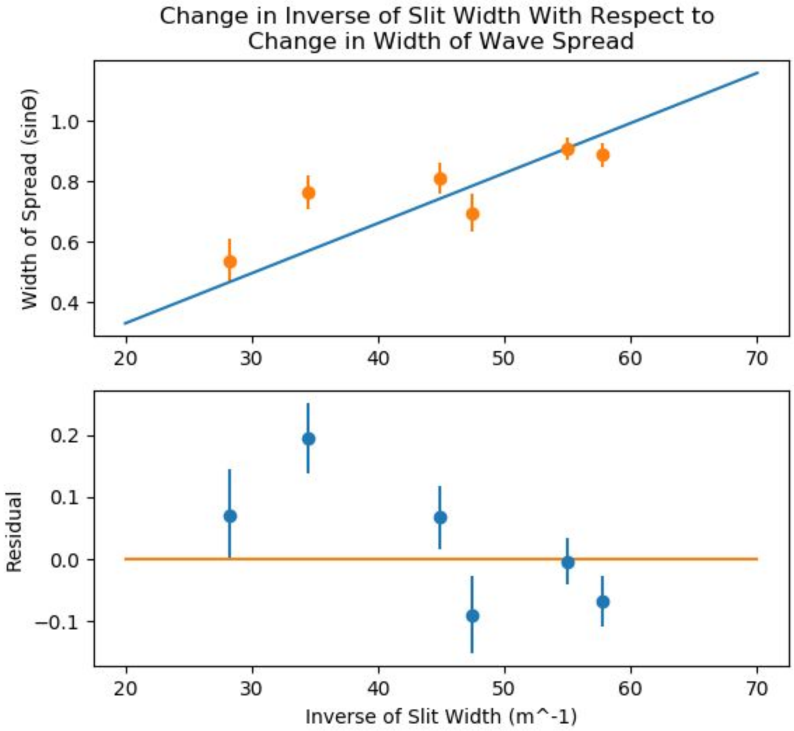


Figure 3: ‘Change in Inverse of slit width with respect to change in width of wave spread’. Equation for line of best fit $y = 0.01654 \pm 0.00002\,x$.

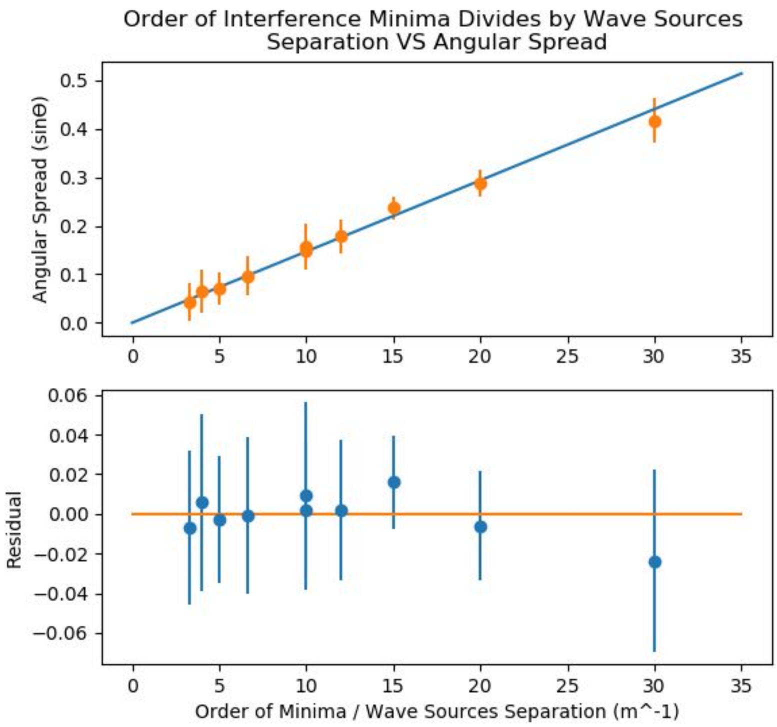


Figure 4: ‘Order of interference minima divides by wave sources separation vs angular spread’.Equation for line of best fit $y = 0.01468 \pm 0.00006\,x$

Discussion:

With the straight barrier reflection (*image 1*), the pattern was seen to be where the angle of incidence (red) was equal to the angle of reflection (green) from the barrier (blue), though the captured image is hard to tell due to human error. The angle of incidence relative to the angled plane barrier is approximately 45 degrees, and the angle of reflection relative to the angled plane barrier is also approximately 45 degrees. This confirms the angle of incidence being equal to that of the angle of reflection in waves. With the curved barrier (*image 2*) for the reflection testing, the focal distance and radius were clearly at the center of the concave and convex curve, with the radii equal on both of the tests. The focal distance was calculated by averaging the three measurements, and it was $3.27 \pm 0.08\ cm$, and radius of the curve barrier is double of focal distance, which is $6.5 \pm 0.2\ cm$.

The wave speed tests were captured and the various wave frequencies (*image 3)* and depths were recorded in order to verify that different wave frequencies and water depths correlate with a variant of wavelength.

To verify the wavelength is inversely proportional to frequency, five trails were taken with different frequency, and the relation between wavelength and period is shown in figure 1. The coefficient of line of best fit is wave speed, in this case is $0.21865 \pm 0.00004\ m/s$. χ_{red}= 2.604, it is close to 1, which suggests that linear model is a good fit for these data. Equation 4 is used as the model function of this graph, it is a linear model that passes through origin, the residual plot also exhibits linear behavior, so a linear model is a good fit. The major contributors to the error were human error when measuring the wavelength, it was taken to be 0.1 cm, and reading error of frequency of ripple generator which was taken to be 0.05 hz.

To confirm wave speed of shallow water gravity wave is as suggested in equation 5, 4 trials were taken, measuring change in wavelength with respect to change in water depth while keeping the frequency constant at 7.0 hz. The result is as shown in figure 2, the coefficient for line of best fit is the period, therefore the experimental frequency was calculated to be $8.024 \pm 0.002\ hz$, it is 12% larger than the actual value. $\chi_{red} =$ 14.938, it was larger than 1, this suggested either that the equation 5 was not a good fitting model for these data or the uncertainties were underestimated. The model function used in this graph was a linear function that passed through the origin, and residual plot exhibits linear behavior as well. However, a linear function that does not pass through the origin would be a better fit for these data,

To track refraction, a barrier that allowed for the surface of the water to pass over it was submerged, and the plane dipper was started. In the image it can be observed that the pattern (*image 4*) the wave pattern of the water once it passes over the trapezoid (seen as a dark shadow in the images, where in the second sub-image, the length of the side of the trapezoid is highlighted by the green arrow) is shifted upward (red arrow), because the depth of the water has changed by a sharp incline from the trapezoid, dragging the surface tension of the water in an alternate direction. And image 4 showed that as the wave enter from medium of fast traveling speed to slower traveling speed, the angle of incident became smaller than the angle of refraction, which verify the prediction made by equation 1b

For diffraction , two of the straight barriers were set with a slit between them, and as seen in the captured image (*image 5*). With this, it can be observed that the wave is steady as it approaches the barriers and their slit, however when the wave passes through the slit between the barriers, the wave is created into a circularly outward wave with patterns of interference, as shown in the image with the red and blue arrows, the first wave of interference can be seen where the wave strength drops off the first time. The angles of the interference meet exactly at the center of the slit where the initial wave was disrupted, where the angles from both sides of the slit are equal to each other, as they should be, as this wave pattern matches with the pattern strength for interference of particles and waves.

Equation 2 could be used to describe diffraction, and to verify it, 6 trials were taken with different slit widths while keeping frequency constant at 20.0 hz and water depth at 9.2 ± 0.5 mm. Figure 3 showed the result, and the coefficient is the experimental wavelength, which was 0.01654 ± 0.00002 m. The theoretical wavelength could be calculated from equations 4 and 5, and it was 0.0151 ± 0.0009 m, which was 9% lower than the experimental value. $\chi_{red} = 4.881$, which suggested the uncertainties were underestimated. Residual plot has random scattering, and it does not contradict the linear assumption. The major contributors to the uncertainties for the experimental value were angle measurement and slit width, they were taken to be 5 degrees and 0.1 cm respectively. The uncertainties for theoretical wavelength calculation came from water depth and frequency of ripple generator, and they were taken to be 0.03 cm and 0.05 hz respectively.

For interference, the patterns captured were variant with the change in the distance between the two circular dippers (*image 6*). With this interference, it is clear that as the dippers were further apart, the interference lines appeared closer together, and vice versa. This makes sense as the initial waves as they are further apart would allow for a greater amount of destructive interference patterns to be displayed on screen, while a closer set of dippers would create less opportunity for interference on screen. An oppositional pattern of wave dippers was also observed, where the dipping of the dippers was set to alternate, while one went down, the other went up, which appeared to create a similar interference pattern as the synchronized circular dippers (*image 7*).

To verify equation 9, equation that described behaviour of line of destructive interference, 10 measurements were taken, by varying separation between dippers while keeping the frequency constant at 20.0 hz and water depth at 9.2 ± 0.5 mm.

Rearranging equation 9:

$$\sin\theta = \lambda \frac{1/2 + m}{d} \tag{10}$$

Figure 4 shows the relation between $\sin\theta$ and $\frac{1/2 + m}{d}$, and the coefficient was $0.01468 \pm 0.00006\ m$, which was the experimental wavelength, and by applying equations 4 and 5, the theoretical wavelength is calculated to be 0.0151 ± 0.0009 m, which was 3.4% higher than the experimental value. $\chi_{red} =$ 0.293, it is close to 1 and it suggested that linear model is a good fit for these data. Residual plot has random scattering, and it does not contradict the linear assumption. The major contributor of uncertainties were angle measurement, and ripplers separation, they were taken to be 2 degrees and 0.3 cm respectively.

The dipper in this experiment was to set in the same phase, if the dipper was opposition of phase, the patterns were still the same as when they were in the same phase. This could be explained by reformulating equation 6:

$$\begin{aligned} \psi(t) &= a\left(\cos\left(\omega t - \frac{\omega r_1}{v}\right) - \cos\left(\omega t - \frac{\omega r_2}{v}\right)\right) \\ &= a\left(\cos\left(\omega t - \frac{\omega r_1}{v}\right) + \cos\left(\omega t - \frac{\omega\left(r_2 + \frac{\pi v}{\omega}\right)}{v}\right)\right) \\ &= 2a\cos(\omega t)\cos\left(\frac{\pi\left(r_2 - r_1\right)}{\lambda} + \frac{1}{2}\right) \end{aligned} \tag{11}$$

The final derivation showed that the relation between the inverse of separation and angular spread were still linear, therefore the same pattern.

Conclusion:

From this experiment it was observed that water does behave as expected when waves are created, and that depth and wavelength of the wave, as well as the origin of the waves are factors that the outcoming wave pattern is dependent on. In a situation such as diffraction, reflection, or refraction, an outside object is needed in order to alter the wave, such as a barrier, a change in water depth, or a difference in frequency of the output wave. However for interference, all that is needed is another wave to create the interference pattern from constructive and destructive interference amongst two or more waves.

Appendix:

Image 1 Reflection with straight barrier

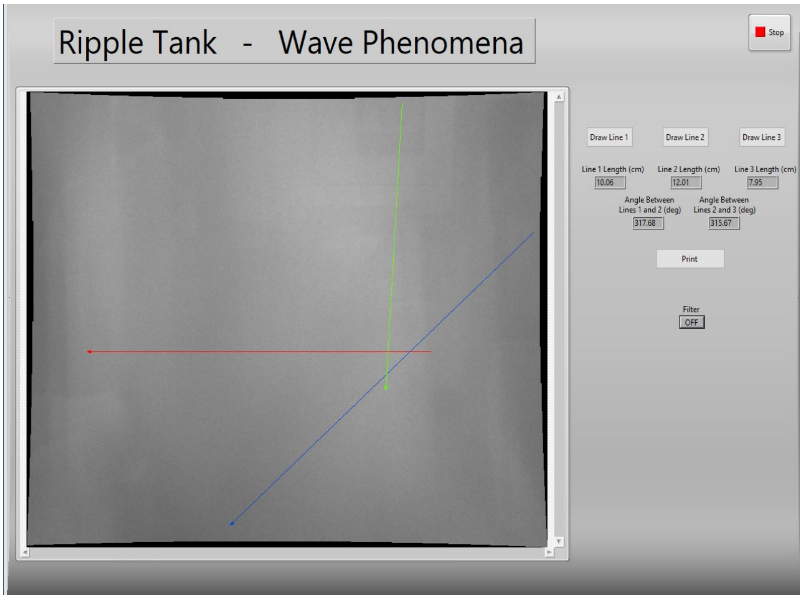


Image 2 Reflection with curved barrier

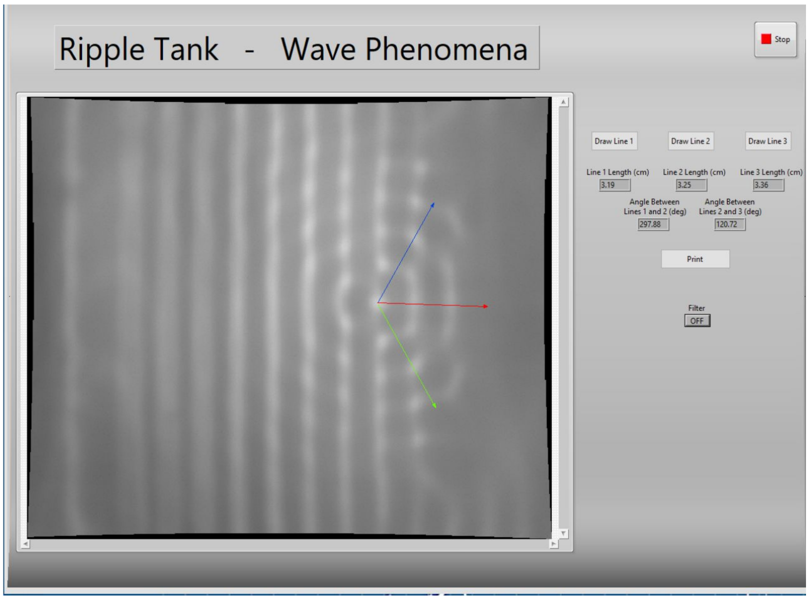
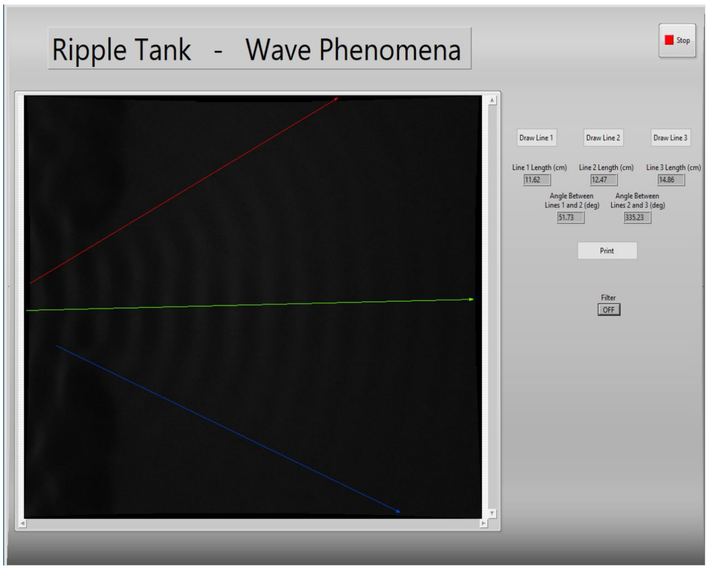


Image 3 Wavelength



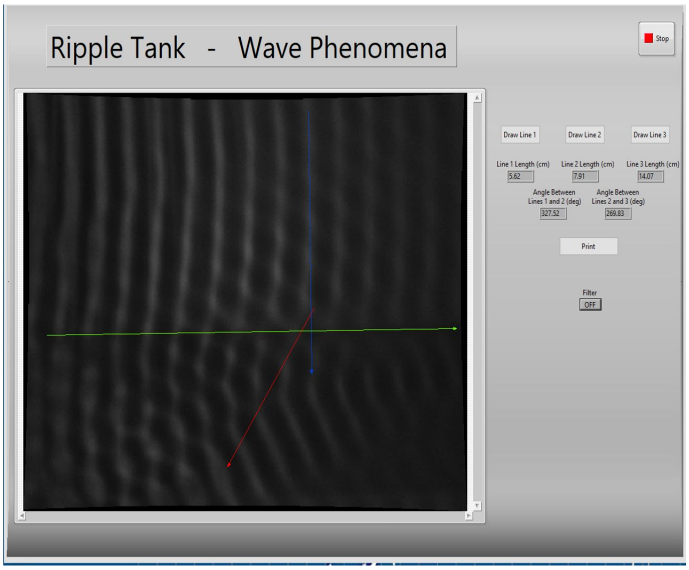


Image 5 Slit diffraction

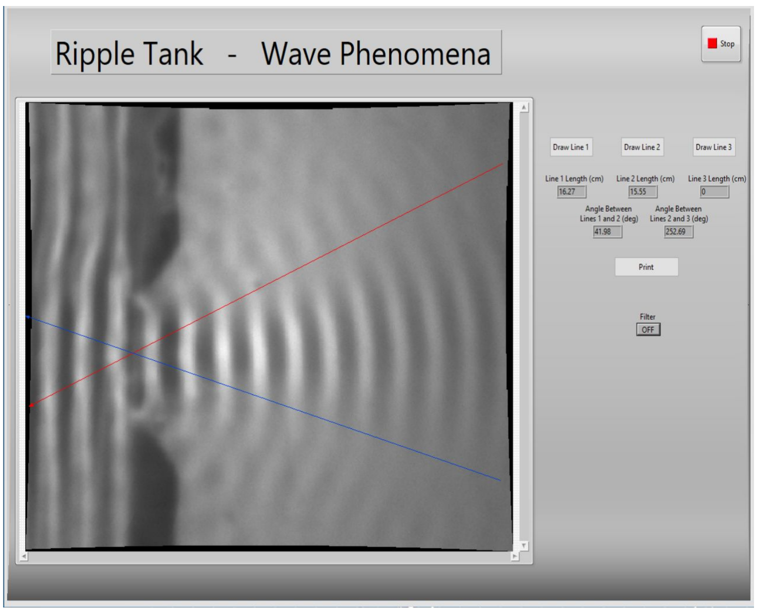


Image 6 Dual dipper interference

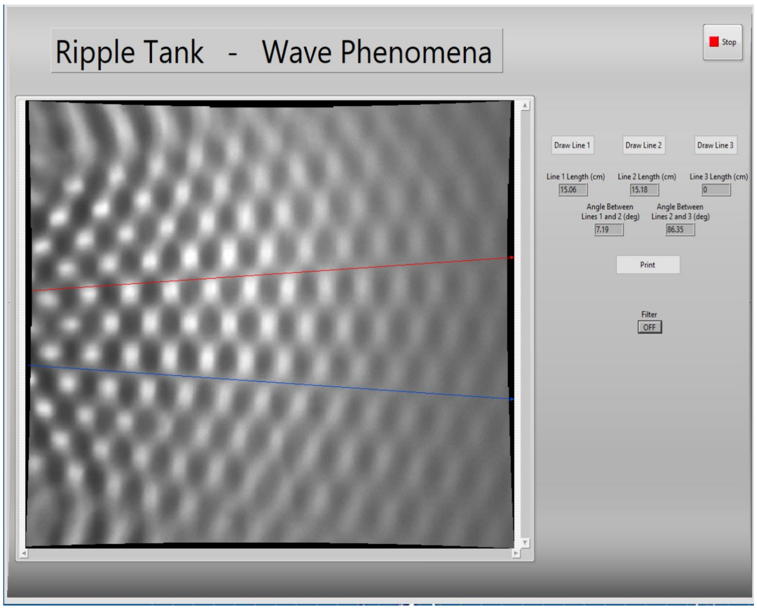


Image 7 Oppositional interference