PHY407 Lab 05

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The workload was distributed as followings:

- **Jianbang Lin did question 2**
- **Sang Bum Yi did question 1**

Question 1

a) Newman 7.2

The number of sunspots on the Sun for each month since January 1749 was plotted as a function of time, which can be found in Figure 1. Then, the Fourier transform was performed on this data in order to find the Fourier coefficients. Although the textbook instructs to use Discrete Fourier Transform (DFT), whose formula is Equation 1 below, the Fast Fourier Transform (FFT) was employed using the built-in python function as FFT produced the same result as DFT. Lastly, the magnitude squared of the absolute value of the Fourier coefficients was plotted in Figure 2.

$$c_k = \sum_{n=1}^{N-1} y_n \exp\left(-i\frac{2\pi kn}{N}\right) \quad \text{[Eq 1]}$$

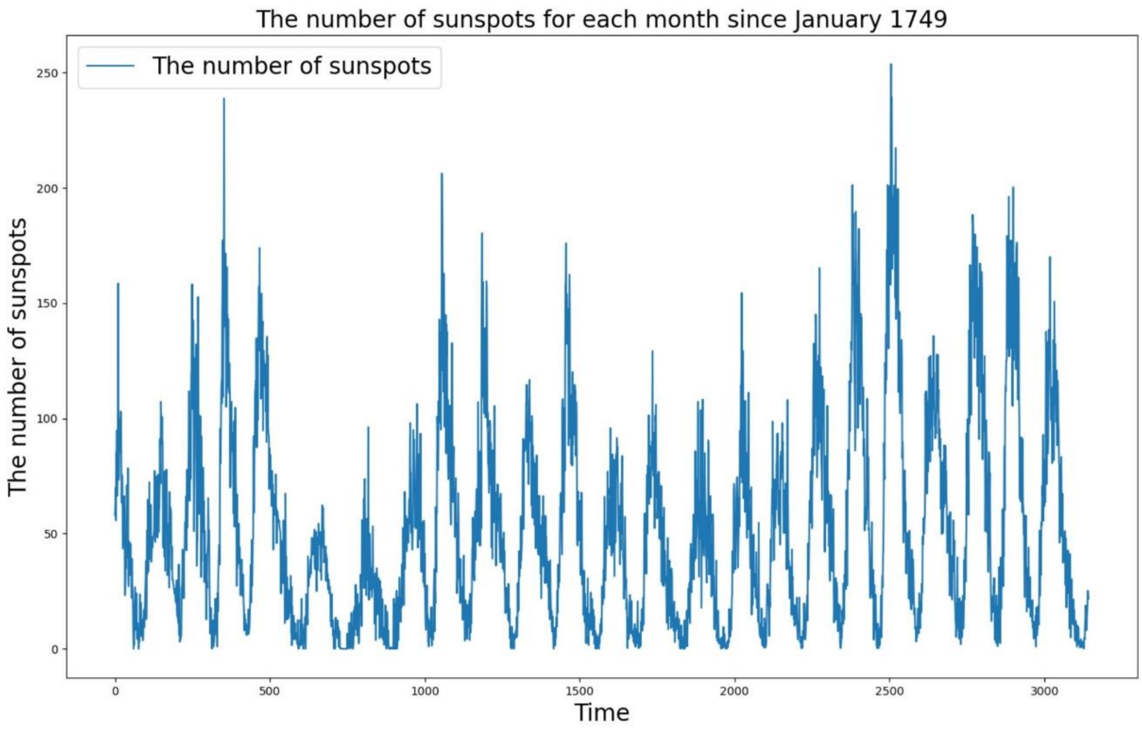

Figure 1: The number of sunspots as a function of time. Note that the total number of samples is 3143.

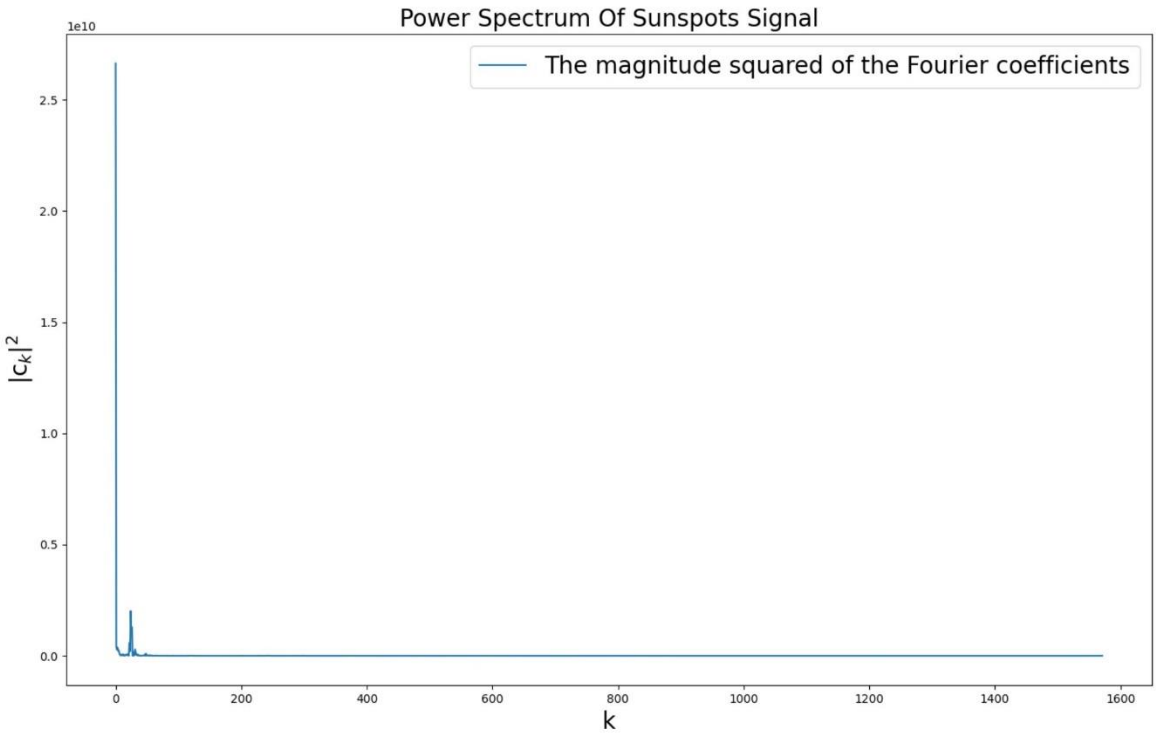

Figure 2: The magnitude squared of the Fourier coefficients of the number of sunspots. Note that this graph is also called "Power Spectrum of Sunspots Signal".

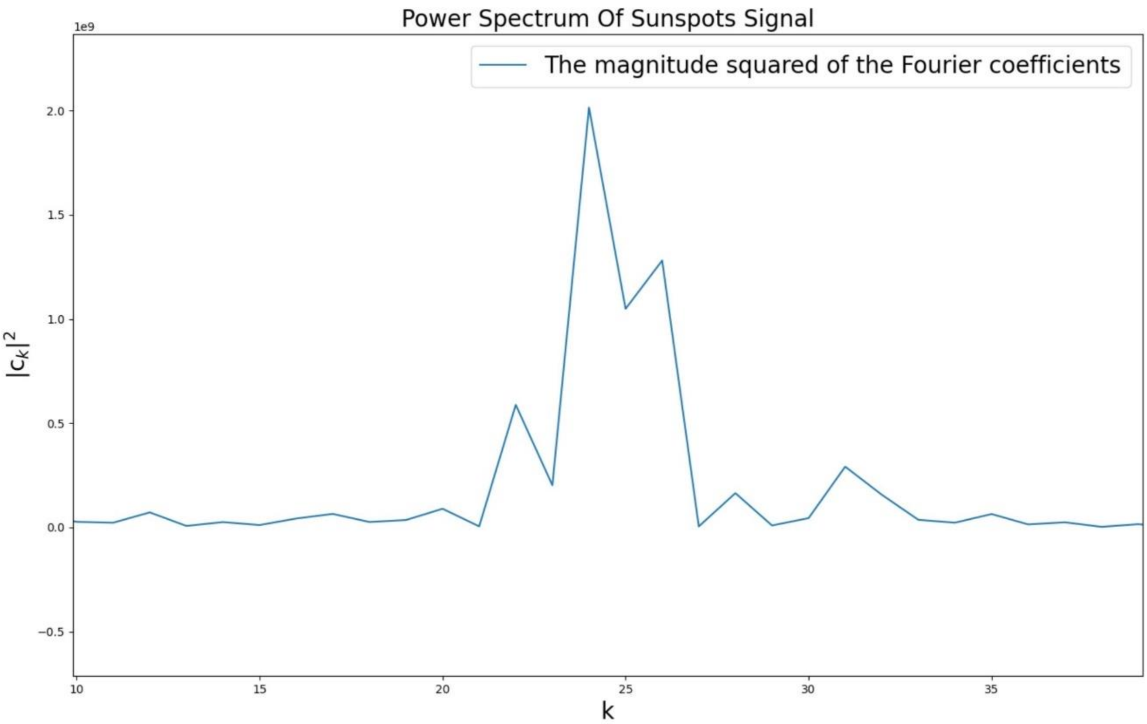

Figure 3: A subplot of the maximum peak of Figure 2.

A close observation of Figure 2 reveals that the maximum peak occurs at k = 24, which was confirmed in a subplot that zoomed in around the peak position. The subplot can be found in Figure 3. Since the total number of samples was 3143, the period of the sine wave can be calculated as

$$\frac{total\ number\ of\ samples}{k_{max}} = \frac{3143}{24} \approx 130.95 \quad (two\ decimal\ points)$$

This periodicity determined by the k-value at which the maximum peak occurs in the Fourier coefficient graph agrees with the estimates of periods shown in Figure 1, which is also approximately 130 months.

b) Newman 7.4

The closing values of the Dow Jones Industrial Average were investigated to study the effect of setting the Fourier coefficients to zero. The Fourier coefficients were calculated using FFT on the closing values of the Dow Jones Industrial Average from late 2006 until the end of 2010. Then, the Fourier coefficients were set to zero except the first 10% of the values and the inverse Fourier transform was applied. Lastly, the same step was repeated by setting all but the first 2% of the Fourier coefficients to zero. As seen in Figure 4 below, the original data and two additional sets of data that were analysed through FFT and Inverse Fourier Transform were plotted in the same graph.

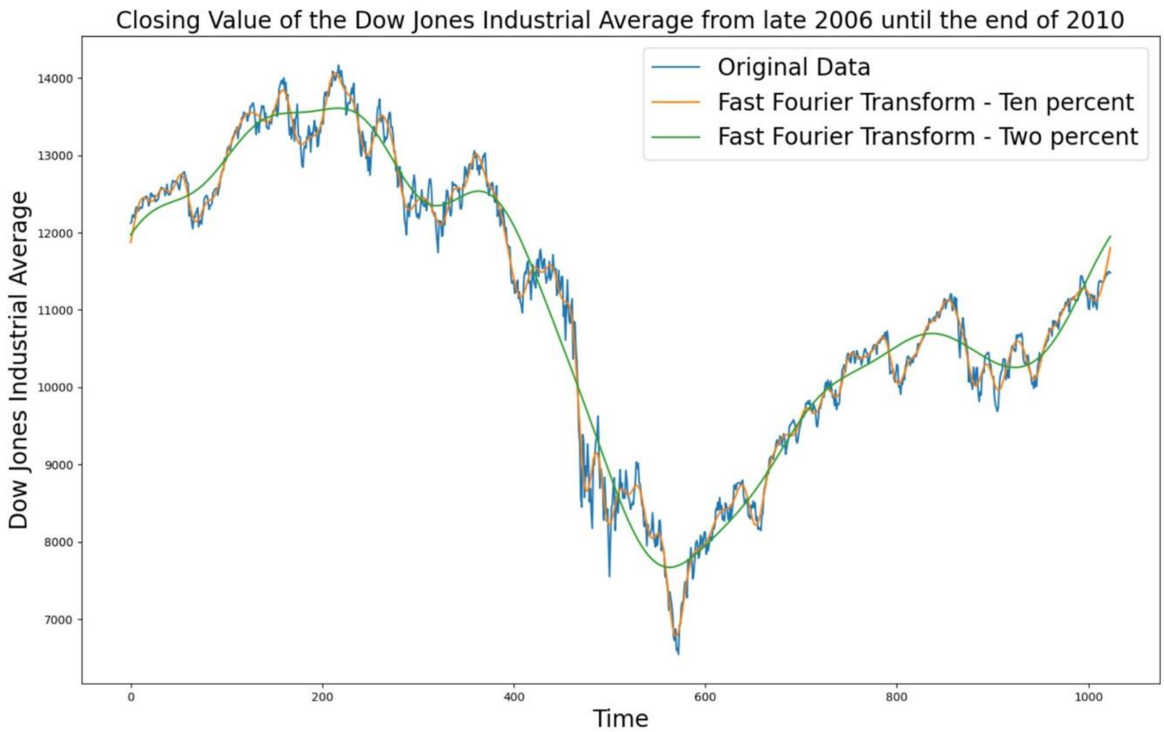

Figure 4: Three sets of data of the Dow Jones Industrial Average are present (Original data, First 10% of Fourier coefficients remained, and First 2% of Fourier coefficients remained)

Note that the noise on the blue line is the greatest, which represents the original data without Fourier Transform applied. After performing Fourier Transform and setting all but first 10% of Fourier coefficients to zero, the noise attenuated and the line graph became smooth, which is an orange line in Figure 4. Finally, after setting all but first 2% of Fourier coefficients to zero, the line graph became even smoother, which is a green line in Figure 4. However, the green line seems to have lost the detailed peaks that were present in both the blue and orange lines as a result of setting 98% of Fourier coefficients to zero.

c) Newman 7.6

The Dow Jones Industrial Average from 2004 until 2008 was investigated to compare the Discrete Fourier Transform (DFT) and Discrete Cosine Transform (DCT). Equation A and B below show the detailed expression for DCT and Inverse DCT.

$$c_k = \sum_{n=0}^{N-1} y_n \cos\left(\frac{\pi k(n+\frac{1}{2})}{N}\right) \qquad \text{[Eq 2]}$$

$$y_n = \frac{1}{N}\left[a_0 + 2\sum_{k=0}^{N-1} a_k \cos\left(\frac{\pi k(n+\frac{1}{2})}{N}\right)\right] \qquad \text{[Eq 3]}$$

Similar to part a, Fast Fourier Transform (FFT) was employed instead of DFT as FFT produced the same result as DFT. Repeating the steps in part b, the FFT was applied to the data and the Fourier coefficients were set to zero except the first two percent of values. In addition, this step was once again repeated using DCT instead of FFT. The resulting values were plotted in Figure 5.

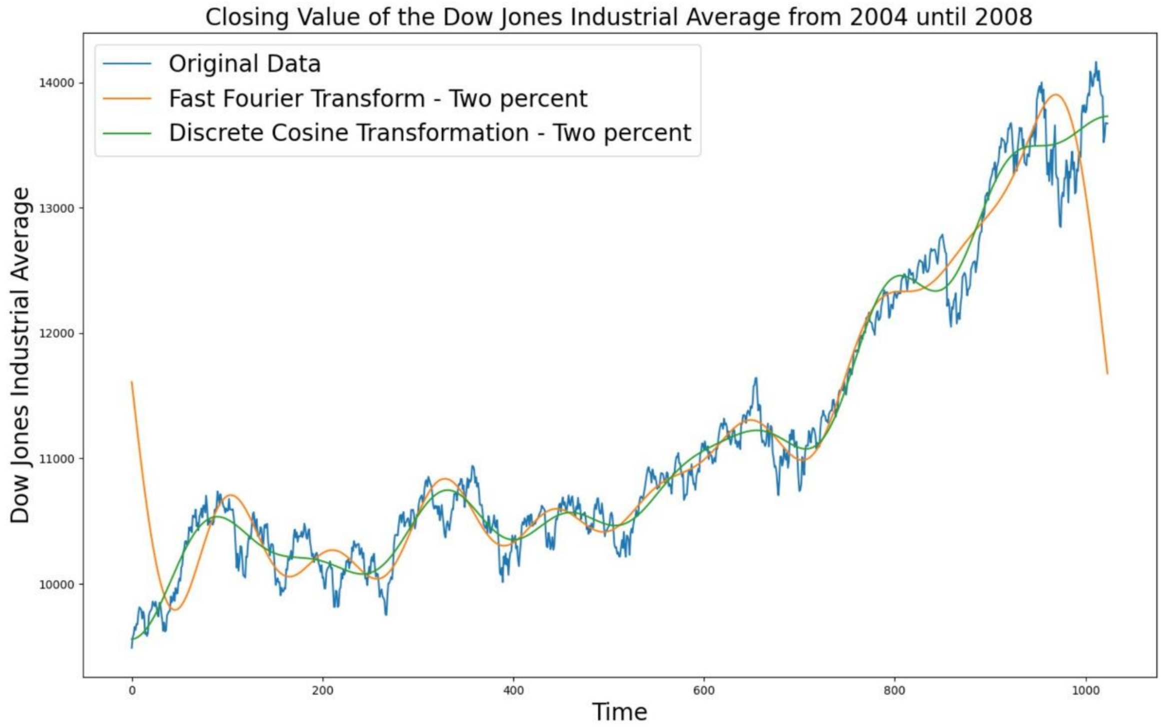

Figure 5: Three sets of data of the Dow Jones Industrial Average are present (Original data, First 2% of Fourier coefficients remained after FFT, and First 2% of Fourier coefficients remained after DCT)

Note that an additional artifact is observed at the beginning and end of the orange line in Figure 5, which represents the data with first two percent of Fourier coefficients remained after FFT. This is because the function has to be periodic, with its first and last values being equal. On the other hand, the DCT succeeded to deal with this, as the green line did not show the same artifact. However, the green line is a lot smoother than the orange line, which means that setting 98% of DCT coefficients lost more information than setting 98% of FFT coefficients.

d) Newman 7.3

The waveforms of the piano and the trumpet were recorded at the industry-standard rate of 44,100 samples per second, and then plotted as seen in Figure 6 and 7 respectively.

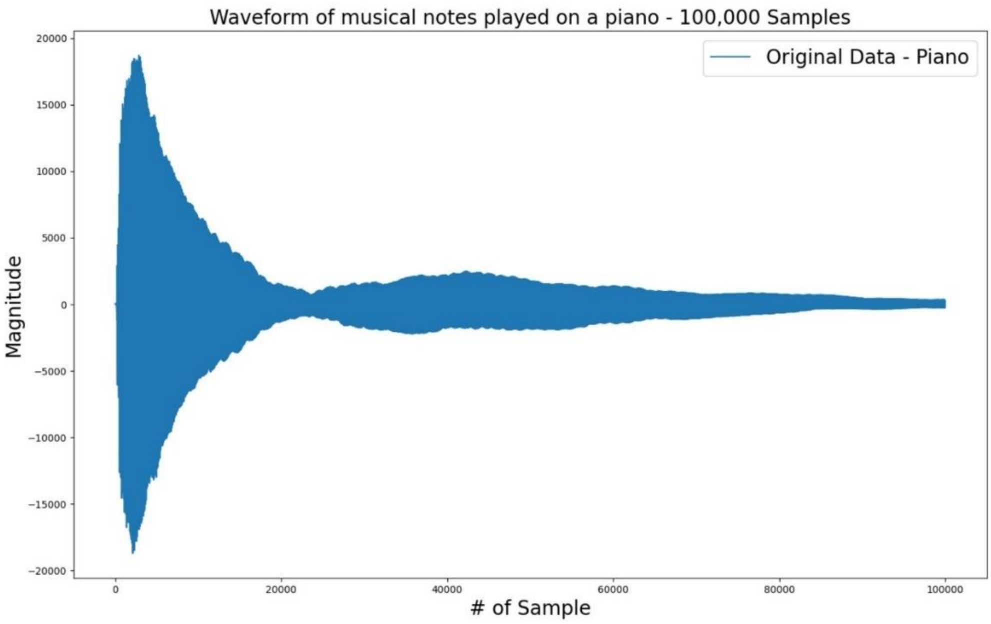

Figure 6: The waveform of the piano. A total of 100,000 samples was taken.

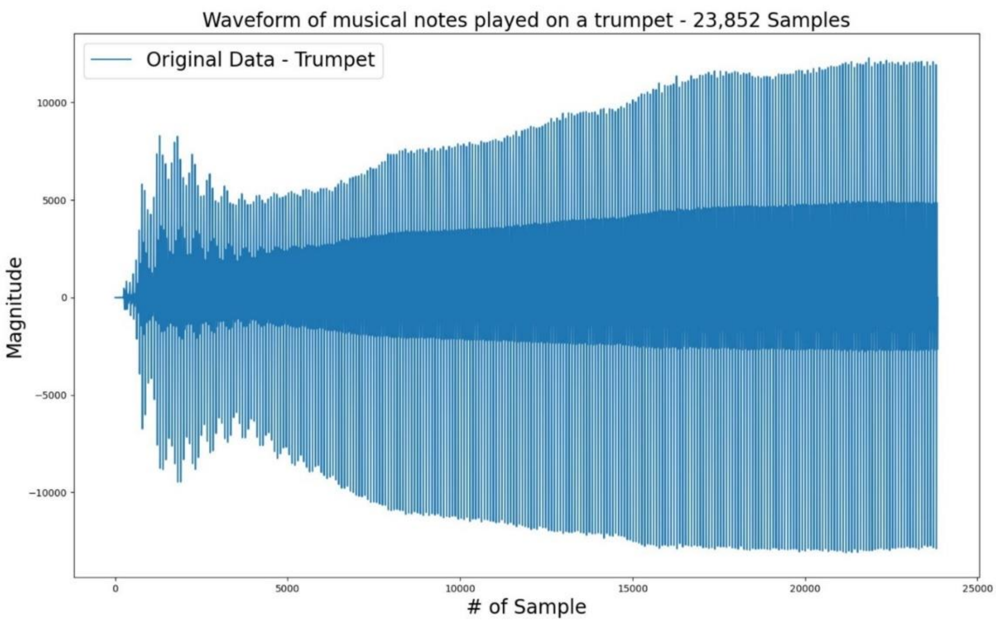

Figure 7: The waveform of the trumpet. A total of 23,852 samples was taken.

Then, the Fourier coefficients were calculated using DFT and the first 10,000 of them were plotted in Figure 8 and 9 below for the piano and the trumpet respectively. Note that the built-in function of Python library that performs Fast Fourier Transform was used instead of writing the custom code, which gives the same result as the Discrete Fourier Transform.

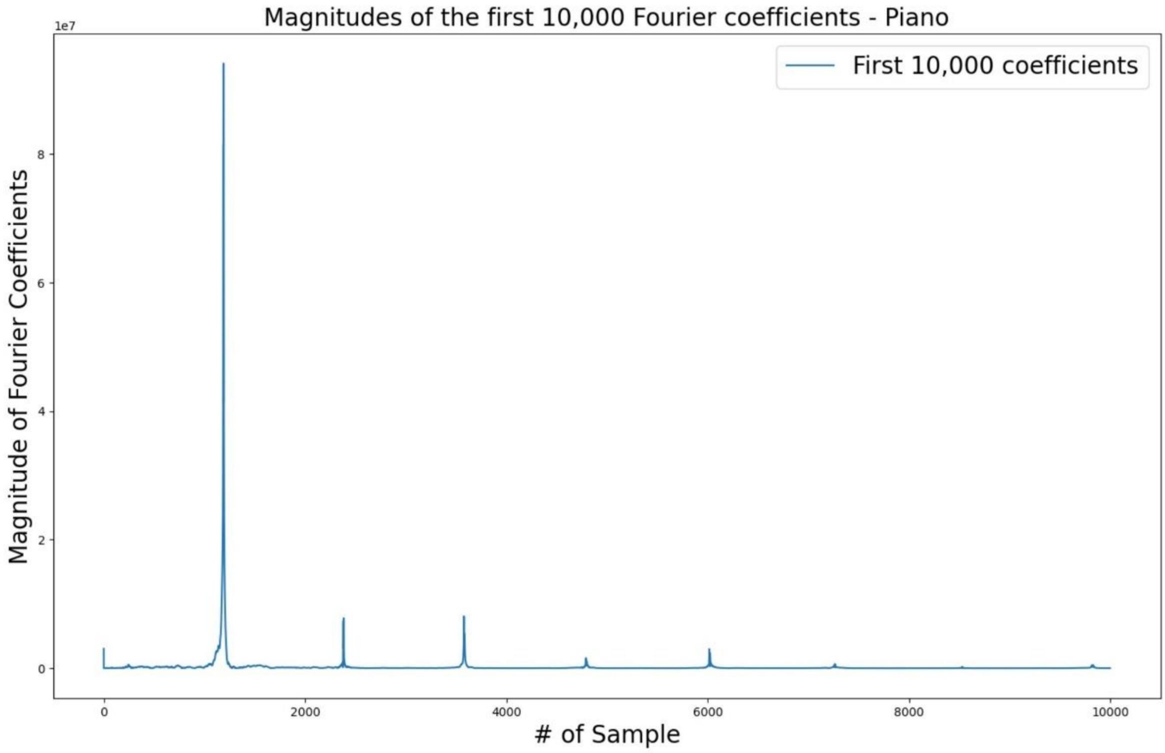

Figure 8: A plot of the magnitude of the first 10,000 Fourier Coefficients for the piano, which were calculated using Fast Fourier Transform method.

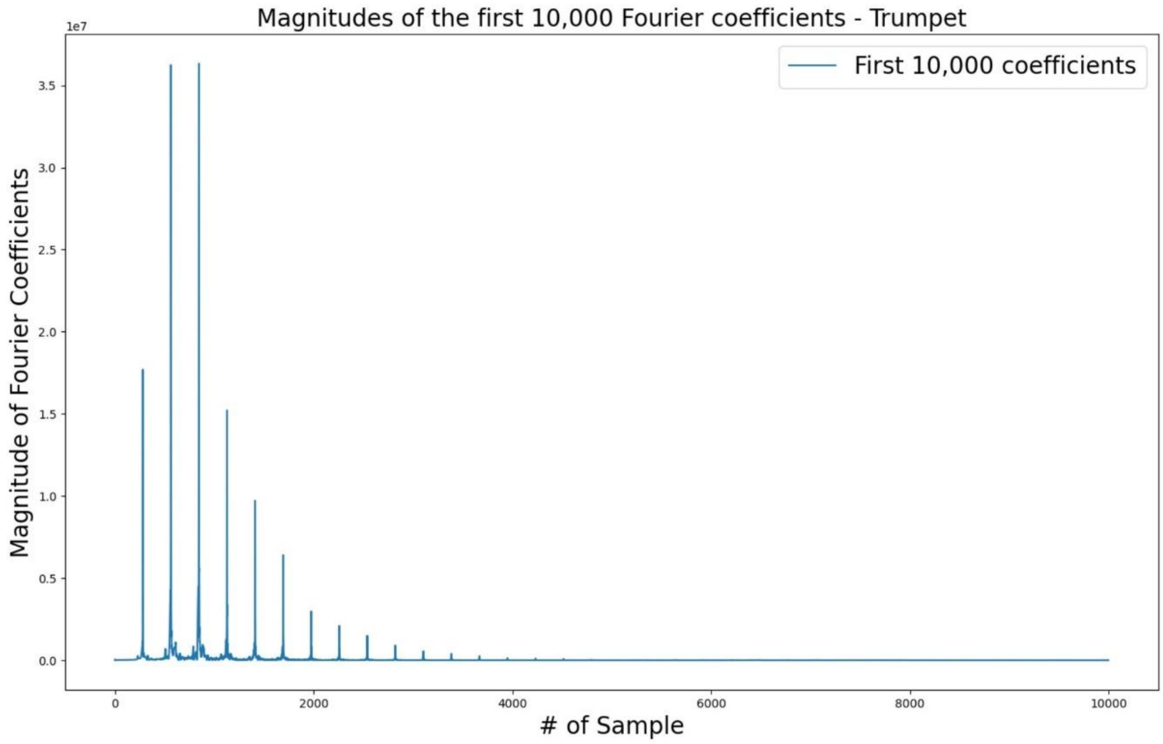

Figure 9: A plot of the magnitude of the first 10,000 Fourier Coefficients for the trumpet, which were calculated using Fast Fourier Transform method.

Based on the results in Figure 8 and 9, the frequencies played by the piano and the trumpet were determined using Equation 4 below and listed in Table 1 and 2, respectively.

$$Frequency = \frac{Position\ of\ a\ peak * Recording\ Rate}{Total\ Number\ of\ Samples} \quad \text{[Eq 4]}$$

Position Of A Peak	Frequency (Unit: Hz)
1190	524
2384	1051
3580	1578
4191	1848
6017	2053
7265	3203
8530	3761
9827	4333

Table 1: The positions of peaks and the corresponding frequencies for the piano. Note that the total number of samples for the piano is 100,000 samples.

Position Of A Peak	Frequency (Unit: Hz)
282	521
564	1042
847	1566
1129	2087
1411	2608
1694	3132
1976	3653
2258	4174
2540	4696
2822	5217
3105	5740
3387	6262
3669	6783

Table 2: The positions of peaks and the corresponding frequencies for the trumpet. Note that the total number of samples for the trumpet is 23,852 samples.

According to the instruction, both instruments were playing the same musical note when the recordings were made. Comparing Table 1 and 2, we can find that the frequency shared by both instruments is the first entry in each table, which corresponds to the musical note of C_5 that has the frequency of 523 Hz (Mottola).

Question 2

This question focus on deblurring image using Fourier transformation, and the following equation is used:

$$\tilde{b}_{kl} = KL\tilde{a}_{kl}\tilde{f}_{kl} \quad \text{[Eq 5]}$$

Where $\tilde{b}_{kl}$ is value of the blurred image at position kl, and K, L are the dimensions of the image, $\tilde{a}_{kl}$ and $\tilde{f}_{kl}$ are values of cleared image and point spread function at position kl.
The point spread function can be written as:

$$f(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}} \quad \text{[Eq 6]}$$

Where value of **σ** is given, and it is 25.

a)

This part of the problem is to change a blur image into a cleared one, the original picture is shown as following:

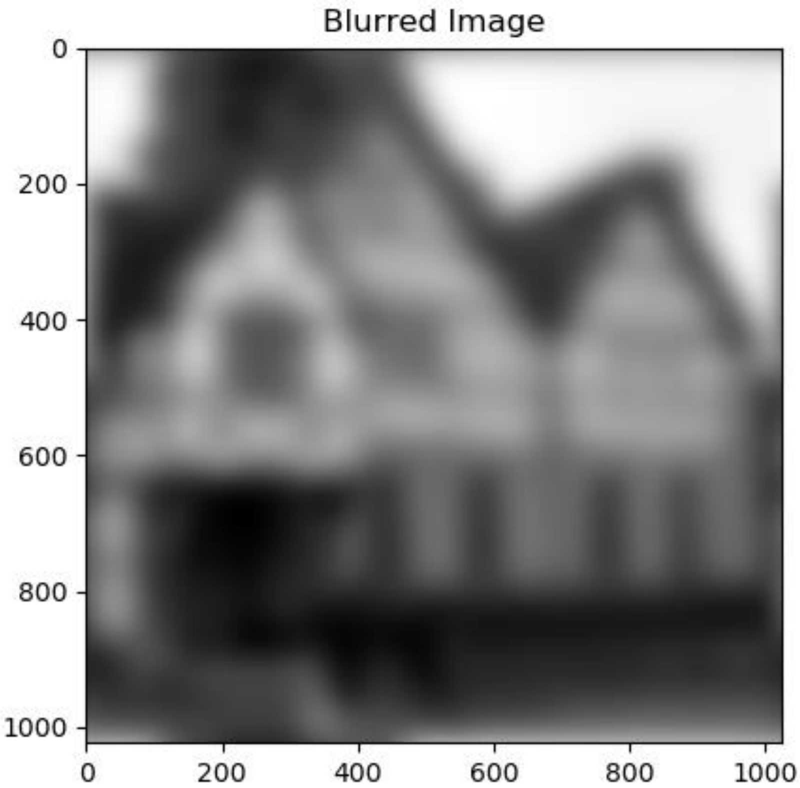

Figure 10: Picture obtained from file blur.txt

A rough shape of the house can be seen, but the details are hardly distinguishable. After applying the method mentioned above, the clear image can be obtained:

Figure 11: Using deconvolution method on data stored in blur.txt

Compared with the blurred image, they both have the same size; 1024x1024. The clear image has only black white, and the blur image has more color. However, details are more distinguishable in the clear picture, such as lines of the house, and people in front of the house.

For a better visualization of the point spread function on blur.txt, the density plot is made, and it is shown as follows:

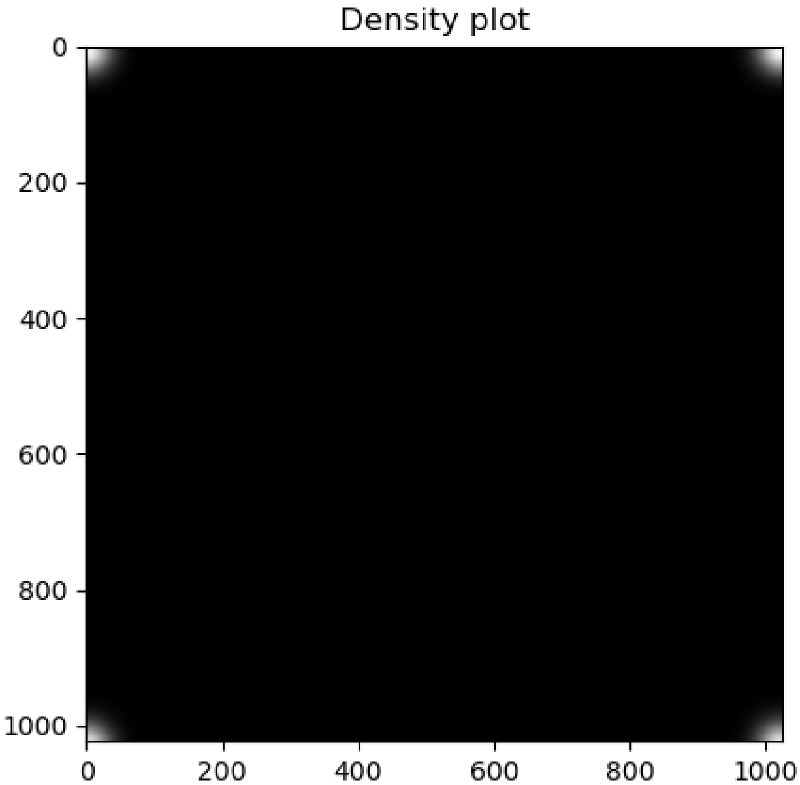

Figure 12: Density plot of blur.txt point spread function

This graph shows a black picture with four corners white. This is because the point spread function can be decomposed into functions of sin and cos. Therefore, point spread function is periodic, and the period of each oscillation is around 1000 pixies.

Works Cited

Mottola, R.M. Liutaio Mottola Lutherie Information Website. 9 January 2020. https://www.liutaiomottola.com/formulae/freqtab.htm. Accessed 16 Oct 2020.