PHY407 Lab 03

Sang Bum Yi, 1004597714
Jianbang Lin, 1004970720

The workload was distributed as followings:

- **Jianbang Lin did question 2**
- **Sang Bum Yi did questions 1 and 3**

Question 1

a.i)

In the previous lab, three methods of evaluating an integral were investigated: Trapezoidal Rule, Simpson's Rule, and python's built-in function (*scipy.special.dawsn*). The equation 1 below, called "Dawson function", was thus evaluated using all three methods, changing the number of slices from 8 up to 2048 and setting the x-value to 4.

$$D(x) = e^{-x^2} \int_0^x \exp(t^2) dt \qquad \text{[Eq 1]}$$

The trapezoidal rule and the Simpson's rule provide an approximation to the real value of the integral, whereas scipy.special.dawsn function gives the exact value. Since the accuracy of approximations by the two methods gets higher as the number of slices increases, the values evaluated at N = 2048 were compared to the exact value given by scipy.special.dawsn function in table 1 below.

Method	Value
Trapezoidal Rule (N = 2048)	0.12935054435619742
Simpson's Rule (N = 2048)	0.12934800128127624
Scipy.special.dawsn	0.12934800123600510

Table 1: The values of D(4) evaluated at N=2048 using the trapezoidal and Simpson's rule and the exact value given by scipy.special.dawsn

The evaluated value of the trapezoidal and Simpson's rule differed from that of scipy.special.dawsn only by 2.54 * 10^{-6} and 4.52 * 10^{-11}, respectively. As expected in the textbook, the Simpson's rule showed a better approximation to the exact value than the trapezoidal rule for the same number of slices. The detailed output, which shows the evaluated value as well as the number of slices and the method used, can be found in Appendix A.

a.ii)

Since both trapezoidal and Simpson's rules are approximations, there exists the relative error, which is the difference between the approximated value and the exact value. However, it was shown in the textbook that the relative error could be obtained even if the exact value is not known, by taking the difference between the approximated values at $N_1 = N$ and $N_2 = 2N$. In other words, the relative error at N number of slices is obtained by taking the approximation with twice more number of slices and subtracting the original approximation from it. The equation 2 below shows such relative error.

$$\epsilon_N = I_{2N} - I_N \quad \text{[Eq 2]}$$

where ϵ_N is the relative error at N number of slices and I refers to the approximated value, with the subscript indicating the number of slices taken.

Using the equation 2, the relative errors were obtained for the same slices used in the previous part, which can be found in Appendix B. In addition, the number of slices and the magnitude of the relative errors were plotted in the log-log graph below.

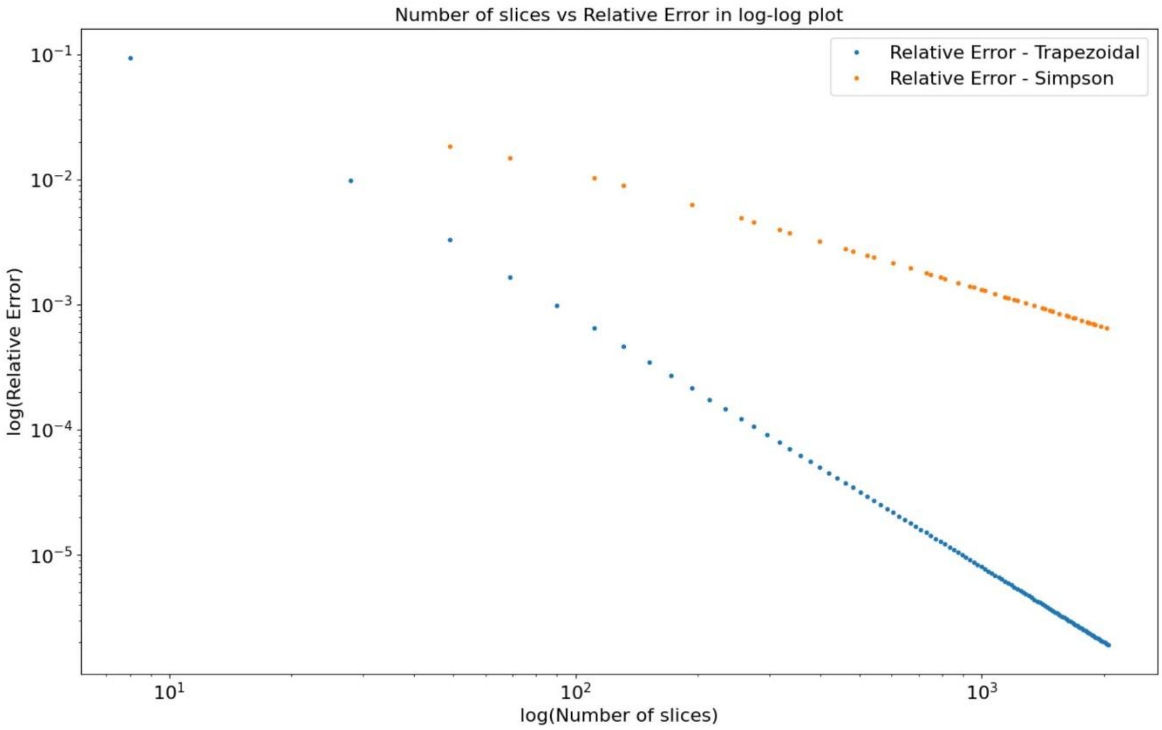

Figure 1: The log-log plot of number of slices versus relative error

Note that the relative errors of the trapezoidal rule and Simpson's rule show straight lines with a negative slope in a log-log plot, which means it follows a power law with a negative exponent. In other words, the relative error of the trapezoidal rule exponentially drops as the number of slices increases.

b)

In addition to the trapezoidal and Simpson's rules, another approximation method called "Gaussian quadrature" can be used to evaluate an integral computationally. Instead of summing up the area under the curve in each slice as the previous two methods did, the Gaussian quadrature sums up the values of the function evaluated at each sample points with weights.

Using the gaussian quadrature, the probability of "blowing snow", which refers to snow that is lifted to a significant height above the surface, in the Canadian Prairies was computed with 100 slices. The formula for the blowing snow is given in equations 3, 4, and 5 below.

$$P(u_{10}, T_a, t_h) = \frac{1}{\sqrt{2\pi}\delta} \int_0^{u_{10}} \exp\left[-\frac{(\bar{u}-u)^2}{2\delta^2}\right] du \qquad \text{[Eq 3]}$$

where u_{10} is the average hourly windspeed at a height of 10m, T_a is the average hourly temperature in °C, and t_h is the snow surface age in hours.

$$\bar{u} = 11.2 + 0.365T_a + 0.00706T_a^2 + 0.9\ln(t_h) \qquad \text{[Eq 4]}$$

$$\delta = 4.3 + 0.145T_a + 0.00196T_a^2 \qquad \text{[Eq 5]}$$

where $\bar{u}$ refers to the mean wind speed and δ refers to the standard deviation of the wind speed.

The initial conditions were given as u_{10} = (6, 8 ,10) m/s and t_h = (24, 48, 72) hours, which gave a total of 9 cases: (u_{10}, t_h) = (6, 24), (6, 48), (6, 72), (8, 24), (8, 48), (8, 72), (10, 24), (10, 48), and (10, 72). With the initial conditions, the equation 3 becomes a function of T_a, which was taken from -100 °C to 100 °C for this evaluation.

Finally, the probability of blowing snow $P(u_{10}, T_a, t_h)$ was plotted against the average hourly temperature (T_a), which can be found in Figure 2.

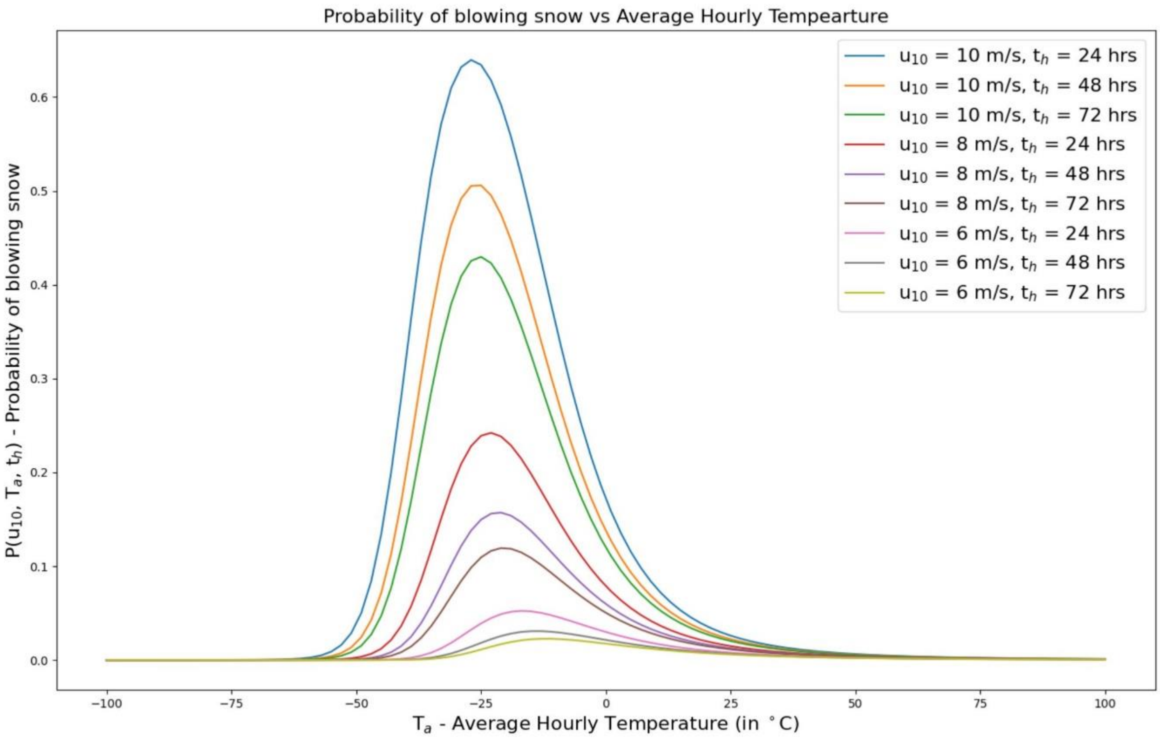

Figure 2: The plot of probability of snow versus average hourly temperature.

As seen in figure 2, the probability of blowing snow tended to increase as the age of the snow was shorter at the same strength of the wind. Similarly, the probability of blowing snow also increased as the strength of the wind got stronger, given the same age of the snow. In short, the probability of blowing snow is proportional to the strength of the wind and inversely proportional to the age of the snow.

In Figure 2, the temperature at which blowing snow is most likely to occur was studied at the same age of the snow. For example, the highest probabilities of blowing snow at t_h = 24 hrs occurred at T_a = -27℃, -23℃, -17℃ when u_{10} = 10 m/s, 8 m/s, 6 m/s, respectively. Therefore, blowing snow is most likely to occur at a lower temperature if the strength of the wind gets stronger.

Question 2

Question 2 studies the behaviour of a spineless point particle in a quadratic potential well, the wave function is given as:

$$\psi_n(x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} e^{\frac{-x^2}{2}} H_n(x)$$ [Eq 6]

Where x is the position of interest, n is the energy level, Hn(x) is the Hermite polynomial:

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$$ [Eq 7]

a)

To understand the relation between wave function and energy level, wave function is plot with different energy level (n=0, 1, 2, 3) on the x range:

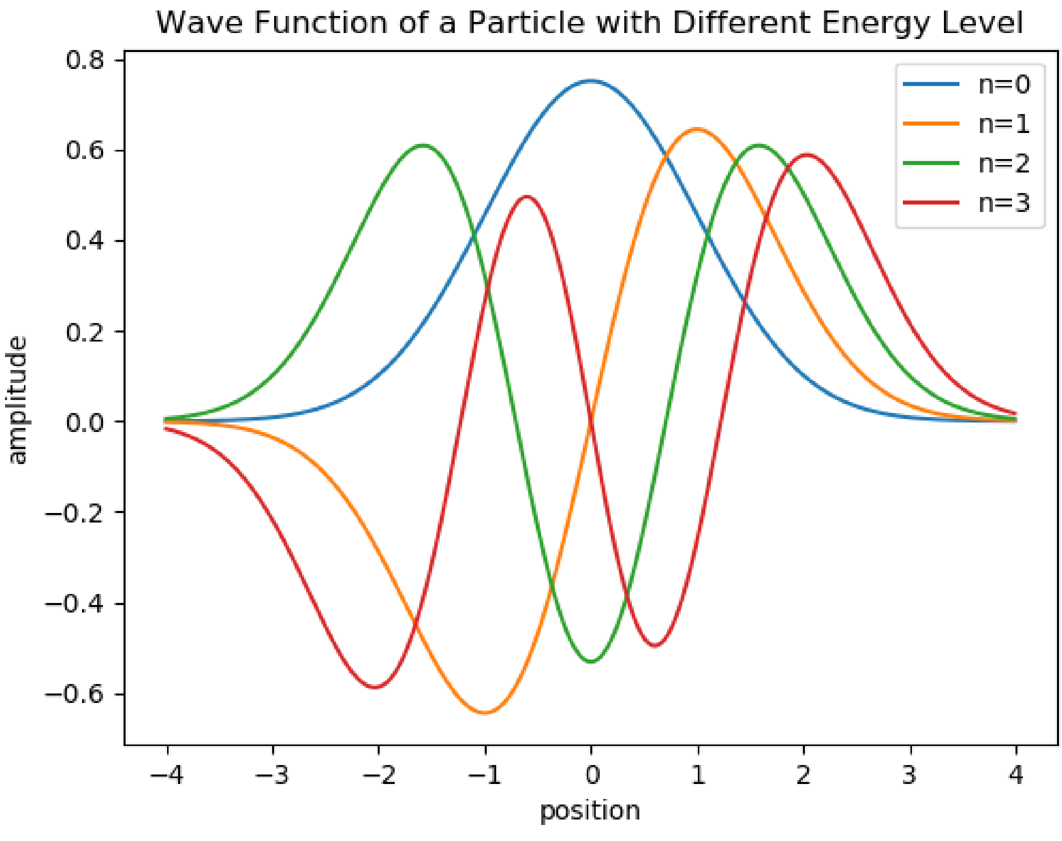

Figure 3: Wave function of a spineless point particle in a quadratic potential well with energy level n =0,1,2,3 ranging from x = -4 to x = 4.

This graph shows that when the energy level (n) of the wave function is even, the wave function will behave like an even function, and when the energy level (n) is odd, the wave function will behave like an add function. Also, energy level (n) decides the number of nodes the wave function has, the number of nodes is equal to n. It's noticeable that as energy level (n) increases, the maximum amplitude decreases.

b)

As mentioned earlier, the number of nodes is equal to the energy level, and the following graph shows the wave function with energy level of 30:

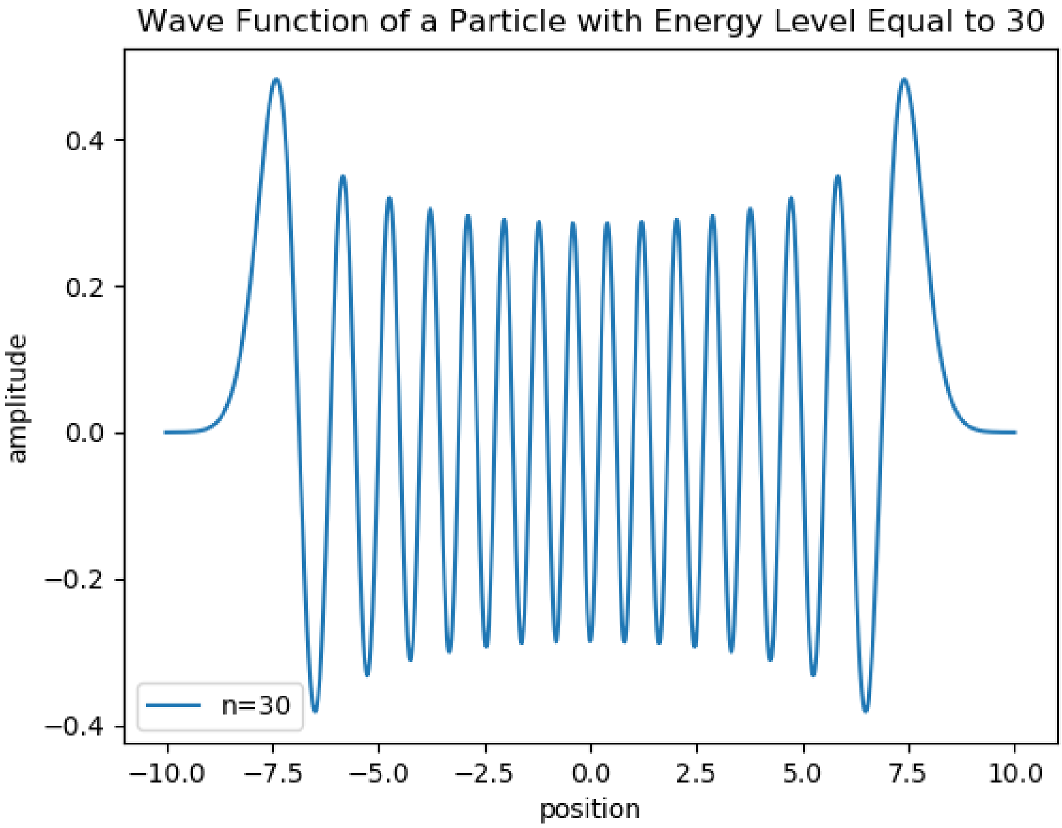

Figure 4: Wave function of a spineless point particle in a quadratic potential well with energy level n =30 ranging from x = -10 to x = 10.

The wave function for n=30 is mostly spread between x = -10 and x =10, it shows that as n increases, the wave function will spread wider. Its amplitude proves the previous observation; the wave amplitude will decrease with energy level.

c)

The mean square position of this particle is given as:

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\psi_n(x)|^2 \mathrm{d}x$$

[Eq 8]

The mean square momentum is given as:

$$\langle p^2 \rangle = \int_{-\infty}^{\infty} \left| \frac{d\psi_n(x)}{dx} \right|^2 \mathrm{d}x$$

[Eq 9]

Where derivative of psi is:

$$\frac{d\psi_n(x)}{dx} = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} e^{\frac{-x^2}{2}} \left(-xH_n(x) + 2nH_{n-1}(x) \right)$$

[Eq 10]

Total energy of particle can be found by combining equations 8 and 9:

$$E = \frac{1}{2}\left(\langle x^2 \rangle + \langle p^2 \rangle \right)$$

[Eq 11]

Calculation on an improper integral is required when finding the mean square position/momentum. However, it is difficult to perform the calculation on an infinite number of points. So change of variable is needed to modify the integrating range, making it finite, we create variable z, such that:

$$x = \frac{z}{1 - z^2}$$

[Eq 12]

Then mean square position and momentum will become:

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\psi_n(x)|^2 \mathrm{d}x = \int_{-1}^{1} \left(\frac{z}{1-z^2} \right)^2 \cdot \left| \psi_n\left(\frac{z}{1-z^2} \right) \right|^2 \cdot \frac{z^2+1}{\left(1-z^2\right)^2} \mathrm{d}z$$

[Eq 13]

$$\langle p^2 \rangle = \int_{-\infty}^{\infty} \left| \frac{d\psi_n(x)}{dx} \right|^2 \mathrm{d}x = \int_{-1}^{1} \left| \frac{d\psi_n\left(\frac{z}{1-z^2} \right)}{dz} \right|^2 \cdot \frac{z^2+1}{\left(1-z^2\right)^2} \mathrm{d}z$$

[Eq 14]

By using equations 11, 13, 14, energy, mean square position, momentum can be evaluated, at n=5, E=5.5, <x2>=5.5, <p2>=5.5, and √<x2>=2.35 as described in the textbook. The relation between energy level and energy can be found by plotting total energy of a particle by using various energy level:

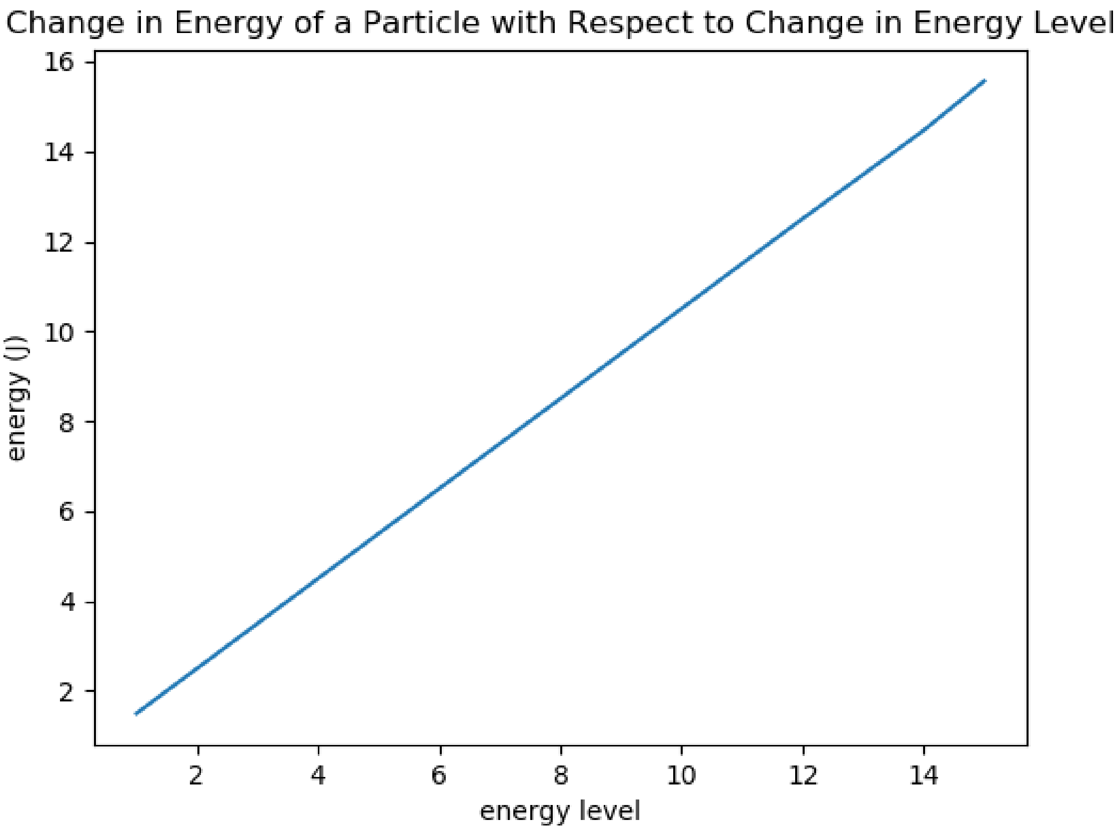

Figure 5: Relation between energy and energy level for a spineless point particle in a quadratic potential well.

In this graph energy level ranges from n=1 to n =15, n=0 is excluded because the mean square potential is not defined when n=0. This graph shows that there exists a linear relation between energy level (n) and the energy, as energy level increases, total energy will increase as expected.

The uncertainty in position and momentum are square root of the mean square position and momentum, the relation between uncertainty in position and momentum is shown as following:

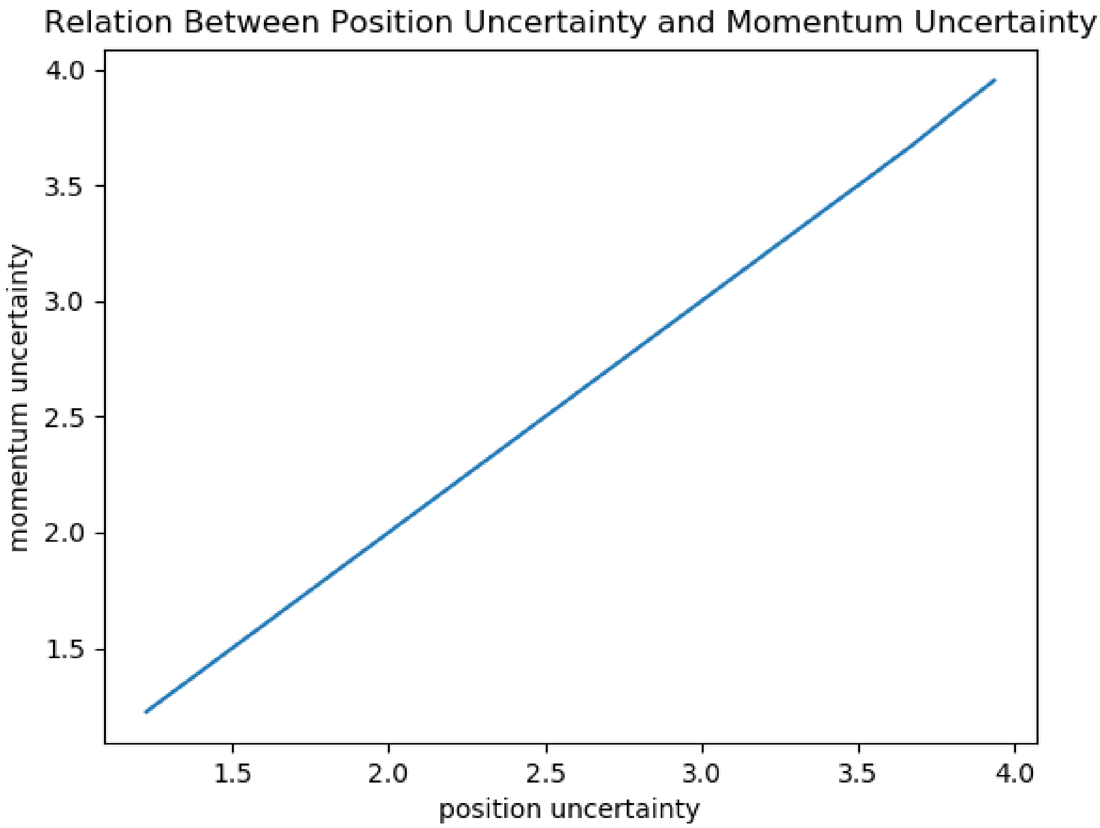

Figure 6: Relation between uncertainty in position and momentum for a spineless point particle in a quadratic potential well.

This graph shows that there exists a linear relation between position uncertainty and momentum uncertainty, as position uncertainty increases, momentum uncertainty increases as well. Also position uncertainty approximately equal to momentum uncertainty in every point.

Question 3

a)

The elevation of the Earth's surface was recorded by The NASA's SRTM (Space Shutter Radar Topography Mission) and made available at the US Geological Survey website. Therefore, using this elevation data, a relief map was created for the area where Lake Geneva is located.

In order to find the appropriate file to download, the coordinate of Lake Geneva was determined, which was 46.44°N 6.53°E. The downloaded file was then moved to the working directory of python program and read into a two-dimensional array w(x,y). The two-dimensional array w(x,y) was used to compute the gradient at each point, which was done by taking a partial derivative with forward/backward/central difference methods. To be specific, the forward difference method was used for points that don't have a preceding point, the backward difference method for points that don't have a following point, and central difference for interior points. The expressions for forward/backward/central difference are equation 6, 7, and 8 respectively.

$$\frac{df}{dx} \cong \frac{f(x+h)-f(x)}{h} \quad \text{[Eq 15]}$$

$$\frac{df}{dx} \cong \frac{f(x)-f(x-h)}{h} \quad \text{[Eq 16]}$$

$$\frac{df}{dx} \cong \frac{f(x+h)-f(x-h)}{2h} \quad \text{[Eq 17]}$$

Using the gradient and the angle of incident light, which was π/6, the intensity of illumination of the surface of the mountains was obtained using the equation 9. The detailed derivation of equation 9 can be found on the textbook, p212.

$$I = -\frac{\cos(\phi)(\partial w/\partial x)+\sin(\phi)(\partial w/\partial y)}{\sqrt{(\partial w/\partial x)^2+(\partial w/\partial y)^2+1}} \quad \text{[Eq 18]}$$

Prior to translating the computations mentioned above, a pseudocode was made as follows.

```
# Pseudo code
# File Read
#   Find the location of Lake Geneva - 46'44N 6.53E
#   Download the dataset from http://dds.cr.usgs.gov/srtm/version2_1/SRTM3
#   Place the downloaded file into the same folder
#   Import struct, numpy, matplotlib
#   Create a 1201x1201 array to store the height data
#   Read signed 2 byte integers from the file using struct.unpack('>h',
    buf)[0] and store it to the array
# Calculate Gradient
#   Create a 1201x1201 array to store the gradient data (one for dw/dx, one
    for dw/dy)
#   Use central difference for interior points, forward difference for the
    first point, and backward difference for the last point to calculate
    the derivative
#   Store the calculated value to the 1201x1201 array
# Calculate the illumination
#   Create a 1201x1201 array to store the illumination data
#   Define the angle (pi/6)
#   Define the distance between grid points
#   Use for loop to compute the illumination at each point and store it to
    the 1201x1201 array
# Plot
#   Use imshow - cmap="gray", extent=(6, 7, 46, 47)
#   Adjust vmin and vmax to get informative plot
#   Plot w and I
"""
```

b)

The intensity of illumination was calculated using equation 9, with the distance between grid points and the angle of light incidence being 83m and π/6 respectively. Finally, the height and the intensity of illumination were plotted as follows.

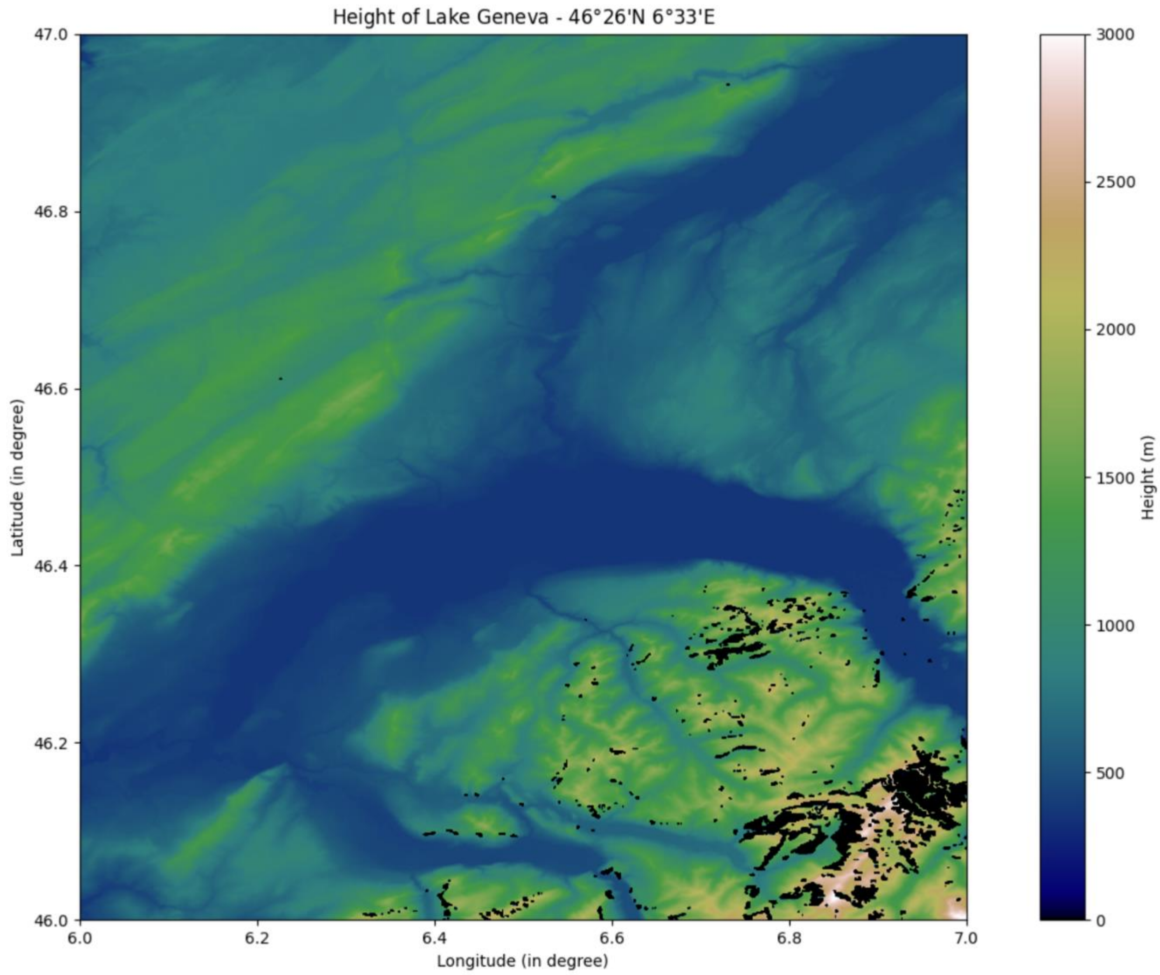

Figure 7: The plot of Lake Geneva based on elevation of Earth's surface

Note that the black regions represent missing values in the file, which were large negative numbers. In order to account for them, vmin parameter of imshow function in python was set to 0 to get an informative graph. Furthermore, the intensity of illumination on the Earth's surface was calculated using equation 9 and plotted in figure 4 below.

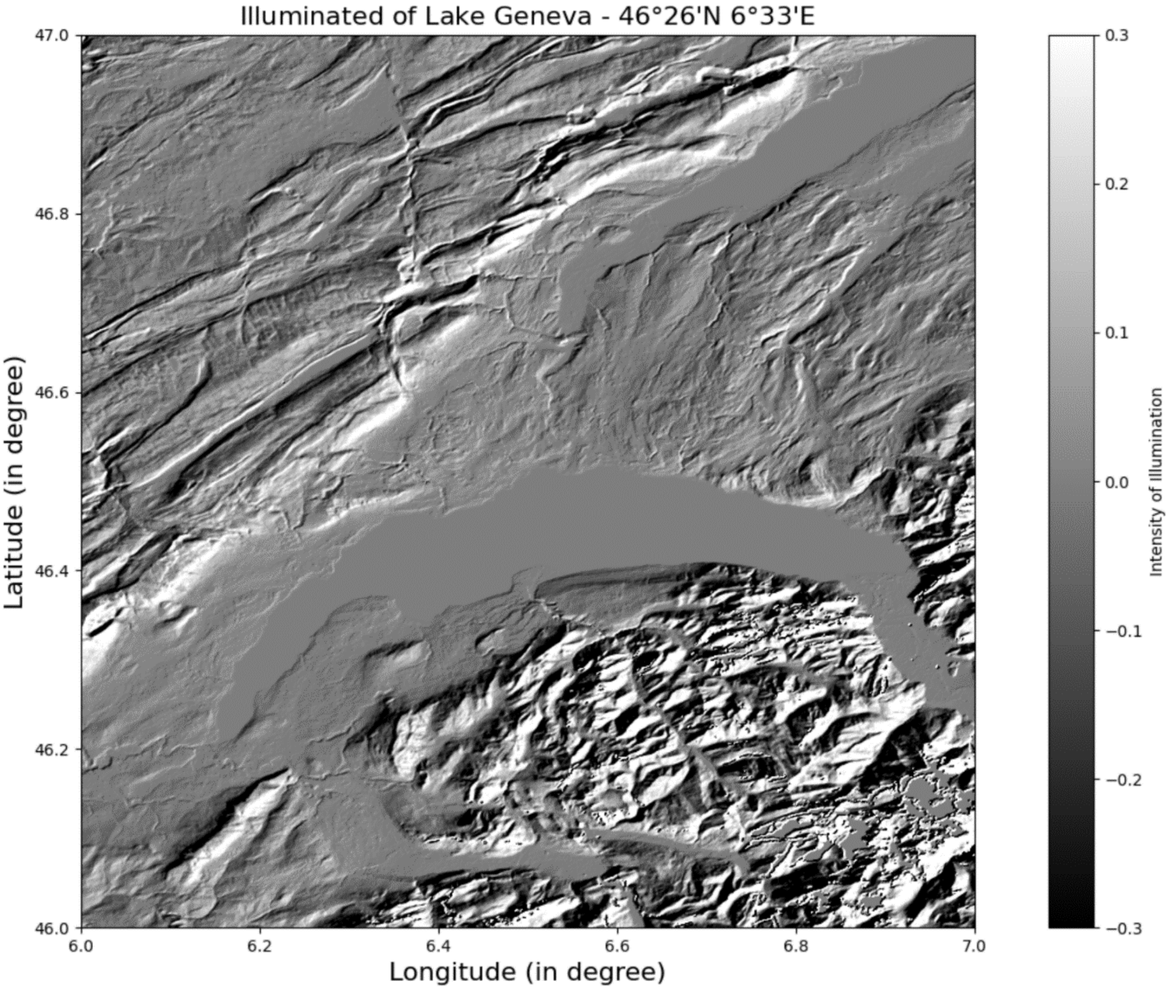

Figure 8: The plot of Lake Geneva based on the intensity of illumination. The distance between grid points and the angle of light incidence were set to 83m and π/6, respectively.

Lake Geneva is a meaningful place for physicists especially, because the world's largest particle accelerator is located at the bottom-left corner of the lake. Figure 5 below shows the zoomed-in image of where CERN is located.

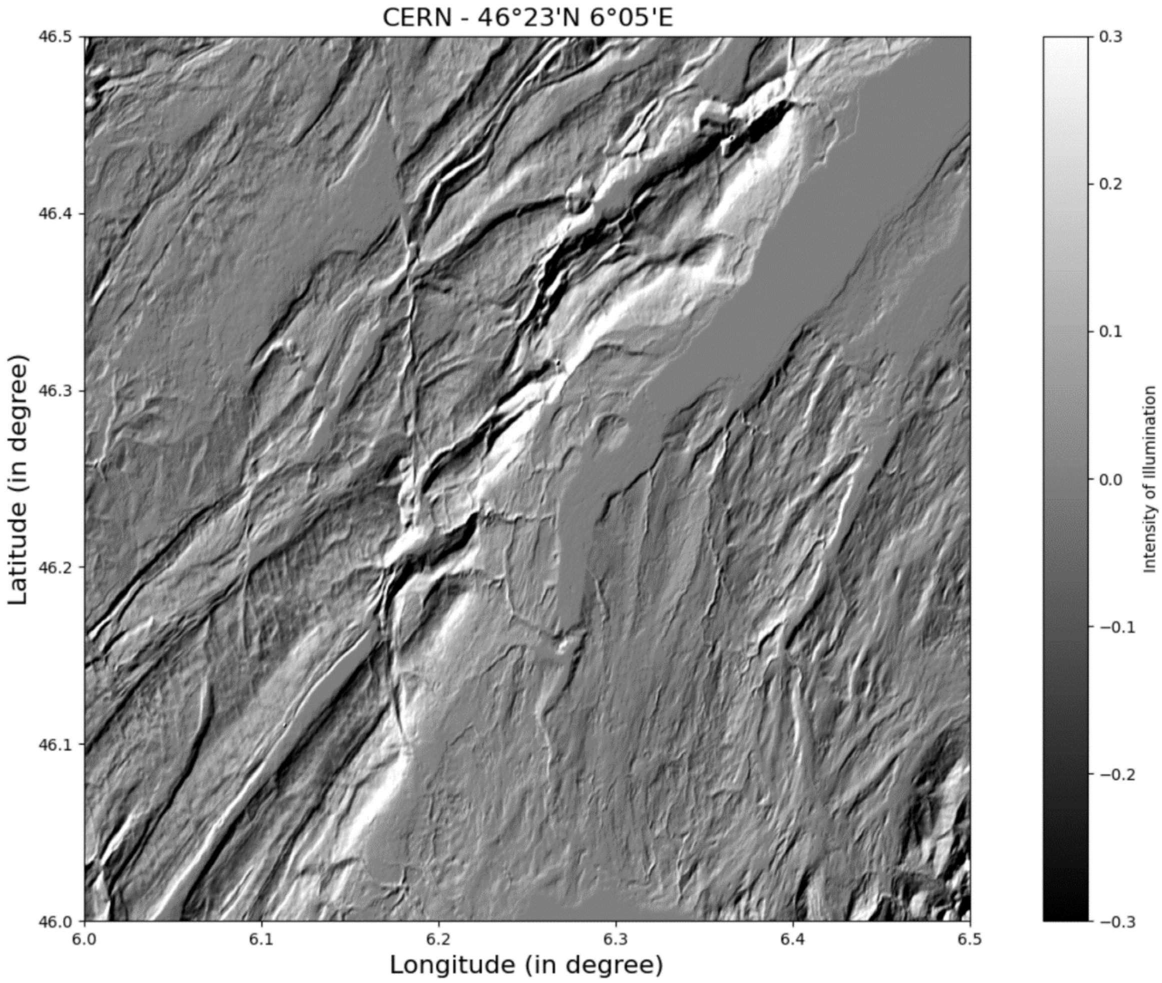

Figure 9: The plot of the town of Geneva and CERN based on the intensity of illumination. The distance between grid points and the angle of light incidence were set to 83m and π/6, respectively.

The main difference between the height plot and the illumination plot is the angle at which the light is incident. The height plot shows Lake Geneva viewed straight down vertically, which therefore makes no shaded area that is hidden from the view. However, the illumination plot has the light shining from the south-west, making some places shaded and look different than their actual height.

Appendix A – The evaluated values of Dawson function

Part A.i - Trapezoidal rule - 8 number of slices : 0.26224782053479523

Part A.i - Trapezoidal rule - 28 number of slices : 0.14264122191781292

Part A.i - Trapezoidal rule - 49 number of slices : 0.13375647512484615

Part A.i - Trapezoidal rule - 69 number of slices : 0.13157969697504585

Part A.i - Trapezoidal rule - 90 number of slices : 0.13066184993385596

Part A.i - Trapezoidal rule - 111 number of slices : 0.13021242336551103

Part A.i - Trapezoidal rule - 131 number of slices : 0.1299688907000902

Part A.i - Trapezoidal rule - 152 number of slices : 0.12980930921943476

Part A.i - Trapezoidal rule - 172 number of slices : 0.12970832921817352

Part A.i - Trapezoidal rule - 193 number of slices : 0.1296342190002396

Part A.i - Trapezoidal rule - 214 number of slices : 0.12958082335342352

Part A.i - Trapezoidal rule - 234 number of slices : 0.12954273847568648

Part A.i - Trapezoidal rule - 255 number of slices : 0.1295119936387565

Part A.i - Trapezoidal rule - 275 number of slices : 0.1294890132667087

Part A.i - Trapezoidal rule - 296 number of slices : 0.1294697186722143

Part A.i - Trapezoidal rule - 317 number of slices : 0.12945412912888715

Part A.i - Trapezoidal rule - 337 number of slices : 0.12944190814639572

Part A.i - Trapezoidal rule - 358 number of slices : 0.1294312158162316

Part A.i - Trapezoidal rule - 378 number of slices : 0.12942264411721846

Part A.i - Trapezoidal rule - 399 number of slices : 0.12941499463560008

Part A.i - Trapezoidal rule - 420 number of slices : 0.1294084634701786

Part A.i - Trapezoidal rule - 440 number of slices : 0.12940309234322248

Part A.i - Trapezoidal rule - 461 number of slices : 0.1293981879464771

Part A.i - Trapezoidal rule - 481 number of slices : 0.12939410151292652

Part A.i - Trapezoidal rule - 502 number of slices : 0.12939032547152715

Part A.i - Trapezoidal rule - 523 number of slices : 0.12938699504949894

Part A.i - Trapezoidal rule - 543 number of slices : 0.12938417565557536

Part A.i - Trapezoidal rule - 564 number of slices : 0.12938153212221004

Part A.i - Trapezoidal rule - 584 number of slices : 0.1293792749356602

Part A.i - Trapezoidal rule - 605 number of slices : 0.1293771416573652

Part A.i - Trapezoidal rule - 626 number of slices : 0.12937521943412836

Part A.i - Trapezoidal rule - 646 number of slices : 0.12937356025980834

Part A.i - Trapezoidal rule - 667 number of slices : 0.1293719762480734

Part A.i - Trapezoidal rule - 688 number of slices : 0.12937053505083138

Part A.i - Trapezoidal rule - 708 number of slices : 0.12936927998227865

Part A.i - Trapezoidal rule - 729 number of slices : 0.1293680717462305

Part A.i - Trapezoidal rule - 749 number of slices : 0.12936701423556163

Part A.i - Trapezoidal rule - 770 number of slices : 0.12936599133696258

Part A.i - Trapezoidal rule - 791 number of slices : 0.12936504881848065

Part A.i - Trapezoidal rule - 811 number of slices : 0.12936421839183787

Part A.i - Trapezoidal rule - 832 number of slices : 0.1293634100907102

Part A.i - Trapezoidal rule - 852 number of slices : 0.12936269517953475

Part A.i - Trapezoidal rule - 873 number of slices : 0.1293619967740577

Part A.i - Trapezoidal rule - 894 number of slices : 0.129361347003149

Part A.i - Trapezoidal rule - 914 number of slices : 0.12936076934637622

Part A.i - Trapezoidal rule - 935 number of slices : 0.1293602022585304

Part A.i - Trapezoidal rule - 955 number of slices : 0.12935969658246654

Part A.i - Trapezoidal rule - 976 number of slices : 0.12935919872329169

Part A.i - Trapezoidal rule - 997 number of slices : 0.12935873199031564

Part A.i - Trapezoidal rule - 1017 number of slices : 0.12935831409262663

Part A.i - Trapezoidal rule - 1038 number of slices : 0.12935790103763953

Part A.i - Trapezoidal rule - 1058 number of slices : 0.1293575302978587

Part A.i - Trapezoidal rule - 1079 number of slices : 0.12935716299516745

Part A.i - Trapezoidal rule - 1100 number of slices : 0.12935681652701012

Part A.i - Trapezoidal rule - 1120 number of slices : 0.12935650451082983

Part A.i - Trapezoidal rule - 1141 number of slices : 0.12935619439169987

Part A.i - Trapezoidal rule - 1161 number of slices : 0.12935591454765524

Part A.i - Trapezoidal rule - 1182 number of slices : 0.12935563586554014

Part A.i - Trapezoidal rule - 1203 number of slices : 0.12935537164973718

Part A.i - Trapezoidal rule - 1223 number of slices : 0.1293551325636385

Part A.i - Trapezoidal rule - 1244 number of slices : 0.1293548938304299

Part A.i - Trapezoidal rule - 1264 number of slices : 0.1293546774385148

Part A.i - Trapezoidal rule - 1285 number of slices : 0.1293544610134947

Part A.i - Trapezoidal rule - 1306 number of slices : 0.12935425494425928

Part A.i - Trapezoidal rule - 1326 number of slices : 0.1293540677201217

Part A.i - Trapezoidal rule - 1347 number of slices : 0.12935388004113074

Part A.i - Trapezoidal rule - 1368 number of slices : 0.12935370093862322

Part A.i - Trapezoidal rule - 1388 number of slices : 0.12935353786704823

Part A.i - Trapezoidal rule - 1409 number of slices : 0.12935337406049432

Part A.i - Trapezoidal rule - 1429 number of slices : 0.12935322472033356

Part A.i - Trapezoidal rule - 1450 number of slices : 0.12935307451636915

Part A.i - Trapezoidal rule - 1471 number of slices : 0.12935293069923737

Part A.i - Trapezoidal rule - 1491 number of slices : 0.1293527993414733

Part A.i - Trapezoidal rule - 1512 number of slices : 0.12935266698740683

Part A.i - Trapezoidal rule - 1532 number of slices : 0.12935254596236165

Part A.i - Trapezoidal rule - 1553 number of slices : 0.1293524238847765

Part A.i - Trapezoidal rule - 1574 number of slices : 0.12935230666067904

Part A.i - Trapezoidal rule - 1594 number of slices : 0.12935219929849412

Part A.i - Trapezoidal rule - 1615 number of slices : 0.12935209083346264

Part A.i - Trapezoidal rule - 1635 number of slices : 0.12935199139477713

Part A.i - Trapezoidal rule - 1656 number of slices : 0.12935189083744733

Part A.i - Trapezoidal rule - 1677 number of slices : 0.12935179403402675

Part A.i - Trapezoidal rule - 1697 number of slices : 0.12935170516134414

Part A.i - Trapezoidal rule - 1718 number of slices : 0.12935161516535368

Part A.i - Trapezoidal rule - 1738 number of slices : 0.12935153246999054

Part A.i - Trapezoidal rule - 1759 number of slices : 0.1293514486578129

Part A.i - Trapezoidal rule - 1780 number of slices : 0.12935136779446546

Part A.i - Trapezoidal rule - 1800 number of slices : 0.1293512933981079

Part A.i - Trapezoidal rule - 1821 number of slices : 0.12935121790510615

Part A.i - Trapezoidal rule - 1841 number of slices : 0.12935114839551143

Part A.i - Trapezoidal rule - 1862 number of slices : 0.12935107780764285

Part A.i - Trapezoidal rule - 1883 number of slices : 0.12935100956823903

Part A.i - Trapezoidal rule - 1903 number of slices : 0.12935094666737867

Part A.i - Trapezoidal rule - 1924 number of slices : 0.12935088272123818

Part A.i - Trapezoidal rule - 1944 number of slices : 0.1293508237366988

Part A.i - Trapezoidal rule - 1965 number of slices : 0.1293507637310941

Part A.i - Trapezoidal rule - 1986 number of slices : 0.12935070561889705

Part A.i - Trapezoidal rule - 2006 number of slices : 0.12935065196208848

Part A.i - Trapezoidal rule - 2027 number of slices : 0.1293505973230695

Part A.i - Trapezoidal rule - 2048 number of slices : 0.12935054435619742

Part A.i - Simpson's rule - 8 number of slices : 0.18269096459712167

Part A.i - Simpson's rule - 28 number of slices : 0.1304366936564012

Part A.i - Simpson's rule - 49 number of slices : 0.1108239433107295

Part A.i - Simpson's rule - 69 number of slices : 0.1144536445848963

Part A.i - Simpson's rule - 90 number of slices : 0.12935992246451478

Part A.i - Simpson's rule - 111 number of slices : 0.11905964932119194

Part A.i - Simpson's rule - 131 number of slices : 0.12040898183789493

Part A.i - Simpson's rule - 152 number of slices : 0.12934948378446415

Part A.i - Simpson's rule - 172 number of slices : 0.12934890670853927

Part A.i - Simpson's rule - 193 number of slices : 0.12301140214925256

Part A.i - Simpson's rule - 214 number of slices : 0.12934837976552158

Part A.i - Simpson's rule - 234 number of slices : 0.1293482661594062

Part A.i - Simpson's rule - 255 number of slices : 0.12444703993648204

Part A.i - Simpson's rule - 275 number of slices : 0.12478140146133652

Part A.i - Simpson's rule - 296 number of slices : 0.12934810481126183

Part A.i - Simpson's rule - 317 number of slices : 0.12535407942318091

Part A.i - Simpson's rule - 337 number of slices : 0.12557927465333815

Part A.i - Simpson's rule - 358 number of slices : 0.1293480496668914

Part A.i - Simpson's rule - 378 number of slices : 0.1293480402067678

Part A.i - Simpson's rule - 399 number of slices : 0.12614026908797582

Part A.i - Simpson's rule - 420 number of slices : 0.12934802680974833

Part A.i - Simpson's rule - 440 number of slices : 0.12934802246908902

Part A.i - Simpson's rule - 461 number of slices : 0.12655609383922486

Part A.i - Simpson's rule - 481 number of slices : 0.12666818414830258

Part A.i - Simpson's rule - 502 number of slices : 0.1293480137698919

Part A.i - Simpson's rule - 523 number of slices : 0.12687658370486962

Part A.i - Simpson's rule - 543 number of slices : 0.12696484675025757

Part A.i - Simpson's rule - 564 number of slices : 0.12934800910351255

Part A.i - Simpson's rule - 584 number of slices : 0.12934800808009603

Part A.i - Simpson's rule - 605 number of slices : 0.1272024193870601

Part A.i - Simpson's rule - 626 number of slices : 0.12934800642035427

Part A.i - Simpson's rule - 646 number of slices : 0.12934800580763972

Part A.i - Simpson's rule - 667 number of slices : 0.12739694673778296

Part A.i - Simpson's rule - 688 number of slices : 0.1293480047895846

Part A.i - Simpson's rule - 708 number of slices : 0.1293480044048014

Part A.i - Simpson's rule - 729 number of slices : 0.12755914995395456

Part A.i - Simpson's rule - 749 number of slices : 0.12760587339229615

Part A.i - Simpson's rule - 770 number of slices : 0.12934800350108877

Part A.i - Simpson's rule - 791 number of slices : 0.1276964643627608

Part A.i - Simpson's rule - 811 number of slices : 0.12773637287675635

Part A.i - Simpson's rule - 832 number of slices : 0.1293480028977694

Part A.i - Simpson's rule - 852 number of slices : 0.12934800274715882

Part A.i - Simpson's rule - 873 number of slices : 0.12784869044481084

Part A.i - Simpson's rule - 894 number of slices : 0.12934800248259898

Part A.i - Simpson's rule - 914 number of slices : 0.1293480023770264

Part A.i - Simpson's rule - 935 number of slices : 0.12794637728369596

Part A.i - Simpson's rule - 955 number of slices : 0.12797523041311165

Part A.i - Simpson's rule - 976 number of slices : 0.12934800211358927

Part A.i - Simpson's rule - 997 number of slices : 0.12803211656923036

Part A.i - Simpson's rule - 1017 number of slices : 0.12805758064600933

Part A.i - Simpson's rule - 1038 number of slices : 0.12934800192197712

Part A.i - Simpson's rule - 1058 number of slices : 0.12934800187156345

Part A.i - Simpson's rule - 1079 number of slices : 0.12813061208773513

Part A.i - Simpson's rule - 1100 number of slices : 0.129348001779921

Part A.i - Simpson's rule - 1120 number of slices : 0.1293480017421002

Part A.i - Simpson's rule - 1141 number of slices : 0.12819582149179656

Part A.i - Simpson's rule - 1161 number of slices : 0.1282153921640353

Part A.i - Simpson's rule - 1182 number of slices : 0.12934800164398577

Part A.i - Simpson's rule - 1203 number of slices : 0.1282544014227772

Part A.i - Simpson's rule - 1223 number of slices : 0.128272048217782

Part A.i - Simpson's rule - 1244 number of slices : 0.12934800156853626

Part A.i - Simpson's rule - 1264 number of slices : 0.1293480015479853

Part A.i - Simpson's rule - 1285 number of slices : 0.12832330700992717

Part A.i - Simpson's rule - 1306 number of slices : 0.12934800150974954

Part A.i - Simpson's rule - 1326 number of slices : 0.12934800149360462

Part A.i - Simpson's rule - 1347 number of slices : 0.12836990460781358

Part A.i - Simpson's rule - 1368 number of slices : 0.12934800146339767

Part A.i - Simpson's rule - 1388 number of slices : 0.12934800145057238

Part A.i - Simpson's rule - 1409 number of slices : 0.12841244907232519

Part A.i - Simpson's rule - 1429 number of slices : 0.12842539459637106

Part A.i - Simpson's rule - 1450 number of slices : 0.1293480014161626

Part A.i - Simpson's rule - 1471 number of slices : 0.12845144712131848

Part A.i - Simpson's rule - 1491 number of slices : 0.1284633428704156

Part A.i - Simpson's rule - 1512 number of slices : 0.1293480013883822

Part A.i - Simpson's rule - 1532 number of slices : 0.12934800138058017

Part A.i - Simpson's rule - 1553 number of slices : 0.12849829305900937

Part A.i - Simpson's rule - 1574 number of slices : 0.12934800136575642

Part A.i - Simpson's rule - 1594 number of slices : 0.12934800135936547

Part A.i - Simpson's rule - 1615 number of slices : 0.1285305869185981

Part A.i - Simpson's rule - 1635 number of slices : 0.12854048704862814

Part A.i - Simpson's rule - 1656 number of slices : 0.12934800134190358

Part A.i - Simpson's rule - 1677 number of slices : 0.12856051616884281

Part A.i - Simpson's rule - 1697 number of slices : 0.12856970873530077

Part A.i - Simpson's rule - 1718 number of slices : 0.1293480013274249

Part A.i - Simpson's rule - 1738 number of slices : 0.12934800132328925

Part A.i - Simpson's rule - 1759 number of slices : 0.1285968895533759

Part A.i - Simpson's rule - 1780 number of slices : 0.1293480013153385

Part A.i - Simpson's rule - 1800 number of slices : 0.1293480013118709

Part A.i - Simpson's rule - 1821 number of slices : 0.1286222360760076

Part A.i - Simpson's rule - 1841 number of slices : 0.1286300514154365

Part A.i - Simpson's rule - 1862 number of slices : 0.1293480013022601

Part A.i - Simpson's rule - 1883 number of slices : 0.1286459279097042

Part A.i - Simpson's rule - 1903 number of slices : 0.1286532439385331

Part A.i - Simpson's rule - 1924 number of slices : 0.12934800129412424

Part A.i - Simpson's rule - 1944 number of slices : 0.12934800129176952

Part A.i - Simpson's rule - 1965 number of slices : 0.1286749850362086

Part A.i - Simpson's rule - 1986 number of slices : 0.1293480012871993

Part A.i - Simpson's rule - 2006 number of slices : 0.12934800128518767

Part A.i - Simpson's rule - 2027 number of slices : 0.12869540681469213

Part A.i - Simpson's rule - 2048 number of slices : 0.12934800128127624

Part A.i - Scipy.special.dawsn : 0.1293480012360051

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Appendix B – The relative errors of the Dawson function

Trapezoidal - Relative error at N = 8 : -0.09396100157895806

Trapezoidal - Relative error at N = 28 : -0.009911917383189833

Trapezoidal - Relative error at N = 49 : -0.0032999776163211147

Trapezoidal - Relative error at N = 69 : -0.0016721374760925722

Trapezoidal - Relative error at N = 90 : -0.0009848200908338378

Trapezoidal - Relative error at N = 111 : -0.0006480714068775317

Trapezoidal - Relative error at N = 131 : -0.0004655406020976638

Trapezoidal - Relative error at N = 152 : -0.00034591115986012944

Trapezoidal - Relative error at N = 172 : -0.0002702033836017026

Trapezoidal - Relative error at N = 193 : -0.0002146364427231262

Trapezoidal - Relative error at N = 214 : -0.00017459880153852803

Trapezoidal - Relative error at N = 234 : -0.00014604048633015676

Trapezoidal - Relative error at N = 255 : -0.00012298547758343714

Trapezoidal - Relative error at N = 275 : -0.00010575249843094991

Trapezoidal - Relative error at N = 296 : -9.128321592125066e-05

Trapezoidal - Relative error at N = 317 : -7.959222393424947e-05

Trapezoidal - Relative error at N = 337 : -7.042728921094477e-05

Trapezoidal - Relative error at N = 358 : -6.240866300735726e-05

Trapezoidal - Relative error at N = 378 : -5.598033273762626e-05

Trapezoidal - Relative error at N = 399 : -5.0243577030151254e-05

Trapezoidal - Relative error at N = 420 : -4.534547610685058e-05

Trapezoidal - Relative error at N = 440 : -4.13173345421014e-05

Trapezoidal - Relative error at N = 461 : -3.763920640215135e-05

Trapezoidal - Relative error at N = 481 : -3.4574510346913057e-05

Trapezoidal - Relative error at N = 502 : -3.1742588856992215e-05

Trapezoidal - Relative error at N = 523 : -2.9244861200011796e-05

Trapezoidal - Relative error at N = 543 : -2.7130385296364468e-05

Trapezoidal - Relative error at N = 564 : -2.5147795735758205e-05

Trapezoidal - Relative error at N = 584 : -2.3454953820362112e-05

Trapezoidal - Relative error at N = 605 : -2.1855037387885634e-05

Trapezoidal - Relative error at N = 626 : -2.041340550745785e-05

Trapezoidal - Relative error at N = 646 : -1.916905349985676e-05

Trapezoidal - Relative error at N = 667 : -1.7981070444539338e-05

Trapezoidal - Relative error at N = 688 : -1.690019450670266e-05

Trapezoidal - Relative error at N = 708 : -1.5958911134955622e-05

Trapezoidal - Relative error at N = 729 : -1.5052750492156042e-05

Trapezoidal - Relative error at N = 749 : -1.4259631052199184e-05

Trapezoidal - Relative error at N = 770 : -1.3492469522480288e-05

Trapezoidal - Relative error at N = 791 : -1.2785591496616844e-05

Trapezoidal - Relative error at N = 811 : -1.2162780578833798e-05

Trapezoidal - Relative error at N = 832 : -1.1556563120934671e-05

Trapezoidal - Relative error at N = 852 : -1.1020386800980608e-05

Trapezoidal - Relative error at N = 873 : -1.049658926779995e-05

Trapezoidal - Relative error at N = 894 : -1.0009266915717019e-05

Trapezoidal - Relative error at N = 914 : -9.57602928594592e-06

Trapezoidal - Relative error at N = 935 : -9.150718047579165e-06

Trapezoidal - Relative error at N = 955 : -8.77146496441883e-06

Trapezoidal - Relative error at N = 976 : -8.398074323179294e-06

Trapezoidal - Relative error at N = 997 : -8.048027949514314e-06

Trapezoidal - Relative error at N = 1017 : -7.734607568199436e-06

Trapezoidal - Relative error at N = 1038 : -7.424819067597399e-06

Trapezoidal - Relative error at N = 1058 : -7.14676659535618e-06

Trapezoidal - Relative error at N = 1079 : -6.8712918296354175e-06

Trapezoidal - Relative error at N = 1100 : -6.611442755377617e-06

Trapezoidal - Relative error at N = 1120 : -6.377432393206428e-06

Trapezoidal - Relative error at N = 1141 : -6.14484474462107e-06

Trapezoidal - Relative error at N = 1161 : -5.9349631903182765e-06

Trapezoidal - Relative error at N = 1182 : -5.725953025753627e-06

Trapezoidal - Relative error at N = 1203 : -5.527792474285809e-06

Trapezoidal - Relative error at N = 1223 : -5.348479037881049e-06

Trapezoidal - Relative error at N = 1244 : -5.169430230073102e-06

Trapezoidal - Relative error at N = 1264 : -5.007137257206917e-06

Trapezoidal - Relative error at N = 1285 : -4.844819424643099e-06

Trapezoidal - Relative error at N = 1306 : -4.6902683578931015e-06

Trapezoidal - Relative error at N = 1326 : -4.549851011614869e-06

Trapezoidal - Relative error at N = 1347 : -4.409092504209511e-06

Trapezoidal - Relative error at N = 1368 : -4.274766304046551e-06

Trapezoidal - Relative error at N = 1388 : -4.152463224049718e-06

Trapezoidal - Relative error at N = 1409 : -4.02960889472892e-06

Trapezoidal - Relative error at N = 1429 : -3.917604293718346e-06

Trapezoidal - Relative error at N = 1450 : -3.804951828001446e-06

Trapezoidal - Relative error at N = 1471 : -3.6970894510268693e-06

Trapezoidal - Relative error at N = 1491 : -3.5985715471364976e-06

Trapezoidal - Relative error at N = 1512 : -3.4993064083888026e-06

Trapezoidal - Relative error at N = 1532 : -3.4085379900516966e-06

Trapezoidal - Relative error at N = 1553 : -3.316980160567029e-06

Trapezoidal - Relative error at N = 1574 : -3.2290624228936693e-06

Trapezoidal - Relative error at N = 1594 : -3.1485410841858297e-06

Trapezoidal - Relative error at N = 1615 : -3.067192605149538e-06

Trapezoidal - Relative error at N = 1635 : -2.9926138550018244e-06

Trapezoidal - Relative error at N = 1656 : -2.9171961176965855e-06

Trapezoidal - Relative error at N = 1677 : -2.8445937961252277e-06

Trapezoidal - Relative error at N = 1697 : -2.777939502790483e-06

Trapezoidal - Relative error at N = 1718 : -2.7104427259838637e-06

Trapezoidal - Relative error at N = 1738 : -2.648421397438705e-06

Trapezoidal - Relative error at N = 1759 : -2.5855624563031476e-06

Trapezoidal - Relative error at N = 1780 : -2.524915126173033e-06

Trapezoidal - Relative error at N = 1800 : -2.469118020764771e-06

Trapezoidal - Relative error at N = 1821 : -2.4124984306372887e-06

Trapezoidal - Relative error at N = 1841 : -2.360366380099732e-06

Trapezoidal - Relative error at N = 1862 : -2.307425622599668e-06

Trapezoidal - Relative error at N = 1883 : -2.256246206117085e-06

Trapezoidal - Relative error at N = 1903 : -2.209070683656389e-06

Trapezoidal - Relative error at N = 1924 : -2.1611112002994926e-06

Trapezoidal - Relative error at N = 1944 : -2.116872905966183e-06

Trapezoidal - Relative error at N = 1965 : -2.0718688127085994e-06

Trapezoidal - Relative error at N = 1986 : -2.028284769167721e-06

Trapezoidal - Relative error at N = 2006 : -1.9880422573170797e-06

Trapezoidal - Relative error at N = 2027 : -1.947063086593648e-06

Trapezoidal - Relative error at N = 2048 : -1.9073380220779867e-06

Simpson - Relative error at N = 8 : -0.04572447950093722

Simpson - Relative error at N = 28 : -0.001011361582841458

Simpson - Relative error at N = 49 : 0.018532561659021868

Simpson - Relative error at N = 69 : 0.01489653575535936

Simpson - Relative error at N = 90 : -1.1165985103950282e-05

Simpson - Relative error at N = 111 : 0.010288678835149043

Simpson - Relative error at N = 131 : 0.008939188059398431

Simpson - Relative error at N = 152 : -1.3894448428475314e-06

Simpson - Relative error at N = 172 : -8.486685013631146e-07

Simpson - Relative error at N = 193 : 0.006336634927356213

Simpson - Relative error at N = 214 : -3.54814149422511e-07

Simpson - Relative error at N = 234 : -2.483321599466848e-07

Simpson - Relative error at N = 255 : 0.0049009730654964445

Simpson - Relative error at N = 275 : 0.00456660847413097

Simpson - Relative error at N = 296 : -9.709360929144317e-08

Simpson - Relative error at N = 317 : 0.003993926740460535

Simpson - Relative error at N = 337 : 0.00376873044077633

Simpson - Relative error at N = 358 : -4.540133630159815e-08

Simpson - Relative error at N = 378 : -3.653319965990498e-08

Simpson - Relative error at N = 399 : 0.0032077341115838987

Simpson - Relative error at N = 420 : -2.397437876000552e-08

Simpson - Relative error at N = 440 : -1.9905256026797602e-08

Simpson - Relative error at N = 461 : 0.0027919084987162723

Simpson - Relative error at N = 481 : 0.002679818017494756

Simpson - Relative error at N = 502 : -1.1750173944946596e-08

Simpson - Relative error at N = 523 : 0.0024714181963624293

Simpson - Relative error at N = 543 : 0.0023831550582559258

Simpson - Relative error at N = 564 : -7.3756168950112055e-09

Simpson - Relative error at N = 584 : -6.4161962964881525e-09

Simpson - Relative error at N = 605 : 0.0021455822204546893

Simpson - Relative error at N = 626 : -4.860236019199604e-09

Simpson - Relative error at N = 646 : -4.285831245454119e-09

Simpson - Relative error at N = 667 : 0.0019510547496977282

Simpson - Relative error at N = 688 : -3.331428855002372e-09

Simpson - Relative error at N = 708 : -2.9707028781356115e-09

Simpson - Relative error at N = 729 : 0.0017888514582865833

Simpson - Relative error at N = 749 : 0.0017421280018627416

Simpson - Relative error at N = 770 : -2.123489495353681e-09

Simpson - Relative error at N = 791 : 0.0016515370003907792

Simpson - Relative error at N = 811 : 0.0016116284743095677

Simpson - Relative error at N = 832 : -1.5578872081256634e-09

Simpson - Relative error at N = 852 : -1.4166920969671537e-09

Simpson - Relative error at N = 873 : 0.0014993108768898145

Simpson - Relative error at N = 894 : -1.1686711320901821e-09

Simpson - Relative error at N = 914 : -1.0696981356250745e-09

Simpson - Relative error at N = 935 : 0.0014016240174375971

Simpson - Relative error at N = 955 : 0.0013727708827357965

Simpson - Relative error at N = 976 : -8.227283854189693e-10

Simpson - Relative error at N = 997 : 0.0013158847171527155

Simpson - Relative error at N = 1017 : 0.001290420636526346

Simpson - Relative error at N = 1038 : -6.430943833013458e-10

Simpson - Relative error at N = 1058 : -5.958316617871162e-10

Simpson - Relative error at N = 1079 : 0.0012173891849928398

Simpson - Relative error at N = 1100 : -5.099179134493426e-10

Simpson - Relative error at N = 1120 : -4.744616921570355e-10

Simpson - Relative error at N = 1141 : 0.0011521797735773442

Simpson - Relative error at N = 1161 : 0.0011326090993662996

Simpson - Relative error at N = 1182 : -3.8248001987817304e-10

Simpson - Relative error at N = 1203 : 0.0010935998369942246

Simpson - Relative error at N = 1223 : 0.0010759530404726714

Simpson - Relative error at N = 1244 : -3.1174637871167477e-10

Simpson - Relative error at N = 1264 : -2.9247998467596403e-10

Simpson - Relative error at N = 1285 : 0.0010246942443345453

Simpson - Relative error at N = 1306 : -2.5663390856855983e-10

Simpson - Relative error at N = 1326 : -2.414982935849963e-10

Simpson - Relative error at N = 1347 : 0.00097809664331161

Simpson - Relative error at N = 1368 : -2.1317969611800436e-10

Simpson - Relative error at N = 1388 : -2.0115609178361638e-10

Simpson - Relative error at N = 1409 : 0.0009355521763095287

Simpson - Relative error at N = 1429 : 0.0009226066515708453

Simpson - Relative error at N = 1450 : -1.6889750709125906e-10

Simpson - Relative error at N = 1471 : 0.0008965541253176113

Simpson - Relative error at N = 1491 : 0.0008846583756615956

Simpson - Relative error at N = 1512 : -1.4285334026808982e-10

Simpson - Relative error at N = 1532 : -1.355386636259226e-10

Simpson - Relative error at N = 1553 : 0.000849708185553244

Simpson - Relative error at N = 1574 : -1.216414469151772e-10

Simpson - Relative error at N = 1594 : -1.156504059185437e-10

Simpson - Relative error at N = 1615 : 0.0008174143247244581

Simpson - Relative error at N = 1635 : 0.0008075141943423569

Simpson - Relative error at N = 1656 : -9.927961230893345e-11

Simpson - Relative error at N = 1677 : 0.0007874850734554129

Simpson - Relative error at N = 1697 : 0.0007782925067063295

Simpson - Relative error at N = 1718 : -8.570591458756383e-11

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Simpson - Relative error at N = 1759 : 0.0007511116878285984

Simpson - Relative error at N = 1780 : -7.437445104230278e-11

Simpson - Relative error at N = 1800 : -7.112391231522963e-11

Simpson - Relative error at N = 1821 : 0.0007257651645243834

Simpson - Relative error at N = 1841 : 0.0007179498249013372

Simpson - Relative error at N = 1862 : -6.211417491464033e-11

Simpson - Relative error at N = 1883 : 0.0007020733302599558

Simpson - Relative error at N = 1903 : 0.0006947573012675845

Simpson - Relative error at N = 1924 : -5.448647089068004e-11

Simpson - Relative error at N = 1944 : -5.227895893966661e-11

Simpson - Relative error at N = 1965 : 0.0006730162031351461

Simpson - Relative error at N = 1986 : -4.799441399860882e-11

Simpson - Relative error at N = 2006 : -4.6108616924556145e-11

Simpson - Relative error at N = 2027 : 0.0006525944242620485

Simpson - Relative error at N = 2048 : -4.244168905209733e-11