DEPARTMENT OF COMPUTER ENGINEERING AND IT

AMIRKABIR UNIVERSITY OF TECHNOLOGY

COMPUTER ENGINEERING && IT DEPARTMENT

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Statistical Pattern Recognition

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1 Plotting conditional Parzen window based density estimates

In the Parzen window formula, variable V denotes the hypercube volume which given $h = 1$ is equal to 1. So the final formula is rewritten as (1) in which ω_i denotes the ith class.

$$\hat{p}(x|\omega_i) = \frac{1}{n}\Sigma_{j=1}^{n_i}\phi(x - x_j) \tag{1}$$

According to (1), the first class conditional density estimate of first class based on Parzen window is expressed in (2).

$$\hat{p}(x|\omega_1) = \frac{1}{5}(\phi(x-x_1)+\phi(x-x_2)+\phi(x-x_3)) = \frac{1}{5}(\phi(x-4)+\phi(x-1)+\phi(x-5)) \tag{2}$$

And that of the second class is expressed in (3).

$$\hat{p}(x|\omega_2) = \frac{1}{5}(\phi(x - x_4) + \phi(x - x_5)) = \frac{1}{5}(\phi(x - 3) + \phi(x - 2)) \tag{3}$$

Figure 1 displays both Parzen window estimated likelihood as a function of x for both classes.

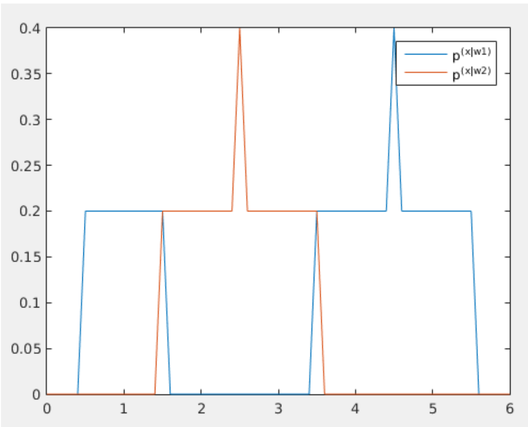


Figure 1: Estimated likelihood of class conditional densities using Parzen window.

2 Plotting Parzen window estimate using Gaussian window function

Equation (4) represents the normal kernel.

$$\frac{1}{h^2}\phi(\frac{X-X_i}{h}) = (2\pi)^{-\frac{n}{2}}h^{-n}|\Sigma|^{\frac{1}{2}}exp[-\frac{1}{2}h^{-2}(X-X_i)^T\Sigma^{-1}(X-X_i)] \quad (4)$$

According to $\phi \sim N(0,1)$ the covariance matrix is $\Sigma = I$ and the mean vector is $\mu = 0$. The variable n denoting the number of dimensions is equal to 2. So the normal kernel represented in (4) is rewritten as equation (5).

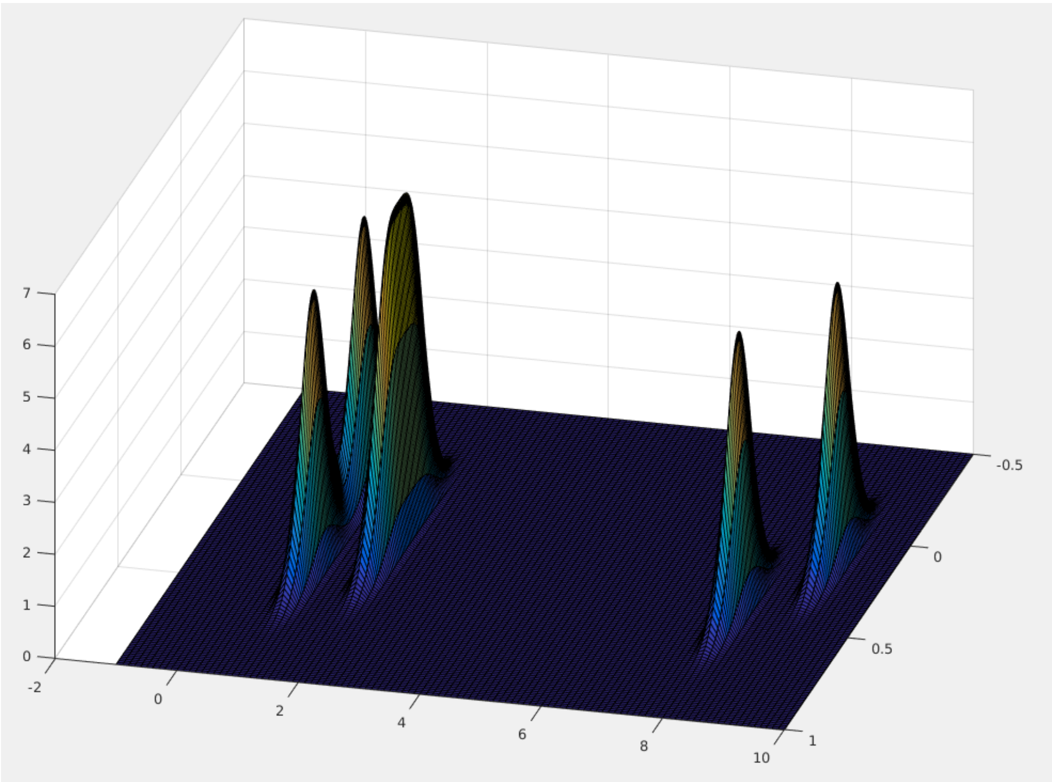
$$\frac{1}{h^2}\phi(\frac{X-X_i}{h}) = (2\pi)^{-1}h^{-2}exp[-\frac{1}{2}h^{-2}(X-X_i)^T(X-X_i)] \quad (5)$$

The Parzen estimate is represented in (6) in normal kernel case.

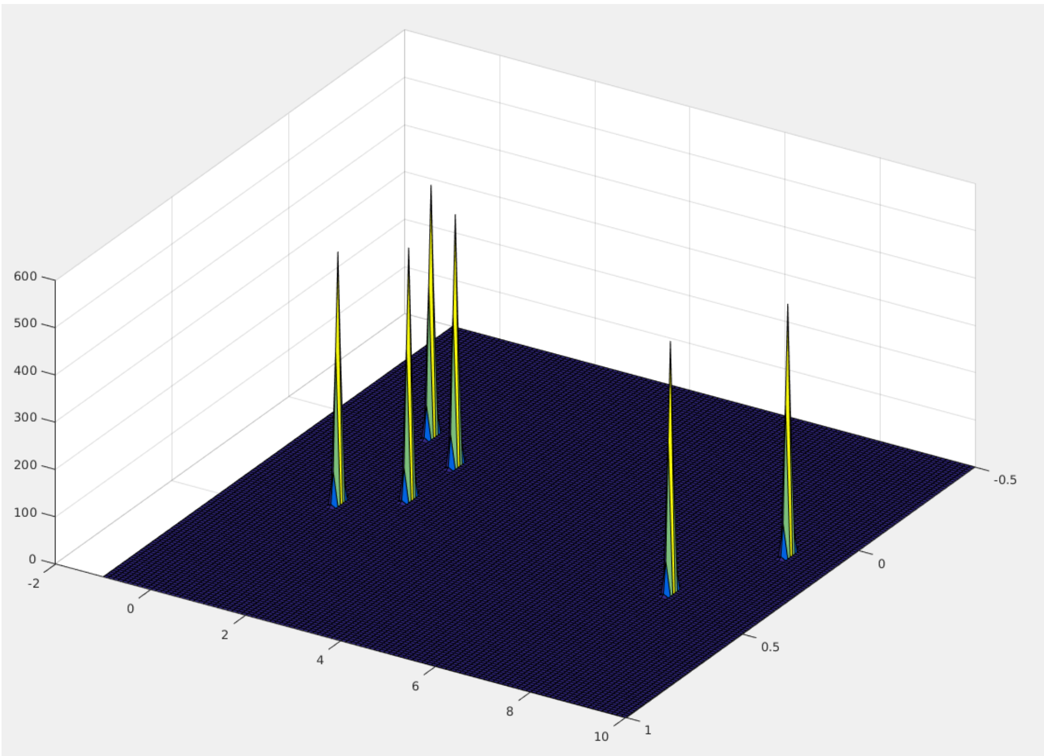
$$\hat{P} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{h^2}\phi(\frac{X-X_i}{h}) = \frac{1}{2N\pi h^2}\sum_{i=1}^{N}exp[-\frac{1}{2}h^{-2}(X-X_i)^T(X-X_i)] \quad (6)$$

Using given dataset $x = \{< 0.0, 1 >; < 0.1, 2 >; < 0.1, 9 >; < 0.3, 2 >; < 0.4, 1 >; < 0.4; 8 >\}$ the Parzen estimate is computable. Figure 2a displays the estimated density using $h = 0.1$ and figure 2b displays estimated density in case of $h = 0.01$.

As it is clearly obvious, decreasing h strongly makes model finer. The estimated density function is coarser when $h = 0.1$ rather than when $h = 0.01$.



(a) Estimated density with $h = 0.1$



(b) Estimated density with $h = 0.01$

Figure 2: Estimated density functions using Parzen Gaussian window