Probabilistic Graphical Models Koller The Bayesian Networks

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1 Basics

Why to use independency Using random variable's independencies is effective in reducing the size of joint probability distribution over them.

Chain Rule The chain rule of conditional probabilities:

$$P(X_1, X_2, \cdots, X_n) = P(x_1) \cdot P(X_2 | X_1) \cdot P(X_3 | X_1, X_2) \cdots P(X_n | X_{n-1}, X_{n-2}, \cdots, X_1)$$
(1)

Factorization Splitting the overall joint probability distribution into several conditional probability distributions (CPDs) where multiplications of CPDs reconstructs joint probability distribution.

2 Naive/Idiot Bayes

The model includes a set of classes C, some number of features X_1, \dots, X_n and assumes that features are conditionally independent given the instance's class:

$$P(C, X_1, \dots, X_n) = P(C) \cdot \prod_{i=1}^n P(X_i|C)$$
 (2)

Factors in this model:

- a. A prior distribution P(C)
- b. A set of CPDs $P(X_j|C)$

3 Bayesian Networks

DAG Representation The core element of representation in bayesian networks is a *directed acyclic graph (DAG)*, in which random variables are nodes and edges correspond to direct influence of random variables on each other.

This DAG can be viewed as:

- a. A data structure that provides the skeleton for representing a joint distribution compactly in a factorized way.
- b. A compact representation for a set of conditional independence assumptions about a distribution.
- Local Probability Models A set of local probability models that represent the nature of the dependence of each variable on its parents, containing probability distributions of single random variables along with CPDs in model.
- **Reasoning Patterns** A joint distribution P_B specifies the probability $P_B(Y = y|E = e)$ of any event y given any observations e. We condition the joint distribution on the event E = e by eliminating the entries in the joint inconsistent with our observation e, and renormalizing the resulting entries to sum to 1; we compute the probability of the event y by summing the probabilities of all of the entries in the resulting posterior distribution that are consistent with y.
 - Causal Reasoning is a top-down flow of probability computations in DAG from causes to symptoms.
 - **Evidential Reasoning** is a bottom-up flow of probability computations in DAG from a symptom to its causes which yields the probability of each cause to be happened.
 - **Intercausal Reasoning** is a cross flow of probability computations in DAG from a cause to another cause passing collidors in the path.

Basic Independencies in Bayesian Networks