14.5 Principles of teleportation

The starship *Enterprize* in the television series *Star Trek* seemed to possess a teleportation machine capable of 'beaming' (i.e. teleporting) human beings from one place to another. Unfortunately, machines of this complexity are restricted to the realms of science fiction.

The demonstration of quantum non-locality by violation of Bell's inequality lays the foundation for quantum teleportation. The basic idea of teleportation is to transfer the quantum state of one photon to another that is physically separated from it. In principle we can also use other particles such as electrons, atoms, or nuclei, but so far most of the demonstrations have been done with photons, and so we shall restrict our discussion here to the case of photon teleportation.

Figure 14.10 illustrates the basic operation of a quantum teleportation machine. The idea is to send quantum information from one place to another without direct exchange of qubits. As was the case with quantum cryptography, we refer to the sender and recipient of the quantum information as Alice and Bob respectively. The machine has an input in Alice's laboratory and an output in Bob's. A photon is fed into the input in an unknown quantum state $|\psi\rangle$, and Bob produces another photon in the same quantum state $|\psi\rangle$ at the output. One possible long-term application of teleportation is in the transfer of quantum information (i.e. qubits) between the different nodes of a quantum network consisting of quantum computers at different locations. (See Section 13.7.)

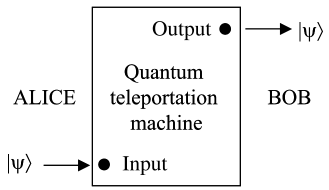

Fig. 14.10 Schematic diagram of the operation of a quantum teleportation machine. A photon in an unknown quantum state $|\psi\rangle$ is fed into the input of the machine and another photon in the same quantum state emerges from the output somewhere else.

Before delving into the details of how such a machine might work, we can first lay down some general principles of its operation.

1. The **quantum no-cloning theorem** says that it is not possible to clone the original photon. The input photon must therefore either be destroyed or lose its initial state in an irretrievable way.
2. The general theory of quantum measurement implies that the fidelity between the output and input wave functions is degraded in proportion to the amount of information gleaned about $|\psi\rangle$ within the teleportation machine. Perfect fidelity can only be achieved when the machine retains no information whatsoever about the unknown quantum state.
3. No *matter* is teleported between the input and output, only *quantum information.*
4. Relativity tells us that we cannot transmit information faster than the speed of light. Therefore, teleportation cannot be used for superluminal information exchange.

See C. H. Bennett *et al.*, *Phys. Rev. Lett.* **70**, 1895 (1993).

With these ideas in mind, let us see how teleportation works in practice. We shall work through a scheme for photon teleportation originally devised by Bennett *et al.* in 1993. Experiments to implement this scheme in the laboratory will then be described in Section 14.6 below.

Figure 14.11 shows the arrangement required for the teleportation of photon polarization. Three photons are required. Photon 1 is the input photon, which is presumed to be in an unknown arbitrary polarization

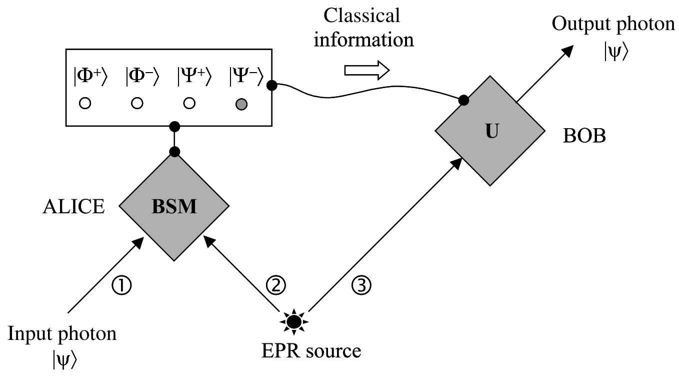

Fig. 14.11 Schematic diagram of an arrangement for photon teleportation. Photon 1 is the input photon whose quantum polarization state $|\psi\rangle$ is to be teleported. Photons 2 and 3 comprise a correlated pair from an EPR source. Alice receives photons 1 and 2 and makes a Bell state measurement (BSM) on them. Bob receives photon 3 and makes a unitary operation (U) on it according to the result of Alice's measurement, which is communicated via a classical channel. Photon 3 then emerges in the same quantum state $|\psi\rangle$ as photon 1.

state given by:

$$|\psi\rangle_1 = C_0|0\rangle_1 + C_1|1\rangle_1, \tag{14.15}$$

where $|C_0|^2 + |C_1|^2 = 1$, and $|0\rangle$ and $|1\rangle$ correspond to the horizontal and vertical polarization states $|\leftrightarrow\rangle$ and $|\updownarrow\rangle$, respectively. Photons 2 and 3 form a correlated photon pair emitted by an EPR source. In general, these two photons could be in any of the four Bell states given by eqns 14.1 and 14.2. We consider here the specific case in which they are in the state:

$$|\Psi^-\rangle_{23} = \frac{1}{\sqrt{2}}\left(|0\rangle_2|1\rangle_3 - |1\rangle_2|0\rangle_3\right). \tag{14.16}$$

This state is readily produced by type II down-conversion (see eqn 14.6 with $\phi = \pi$), and has been employed in the experimental demonstrations of teleportation described in the next section.

The teleportation protocol proceeds by sending photons 1 and 2 to Alice and photon 3 to Bob. Alice performs a 'Bell-state measurement' (BSM) on her two photons giving one of four possible results. She communicates this result to Bob by a classical channel, and Bob then performs a unitary operation $\hat{U}$ to photon 3 depending on the information he has received from Alice. Then, *hey presto*, the output state of photon 3 becomes

$$|\psi\rangle_3 = C_0|0\rangle_3 + C_1|1\rangle_3, \tag{14.17}$$

which is identical to that of the original photon (cf. eqn 14.15).

To see how this works in detail, we need to consider the full wave function for the three particle system, namely:

$$
\begin{aligned}
|\Psi\rangle_{123} &= \frac{1}{\sqrt{2}}(C_0|0\rangle_1 + C_1|1\rangle_1)\,(|0\rangle_2|1\rangle_3 - |1\rangle_2|0\rangle_3) \\
&= \frac{1}{\sqrt{2}}\,(C_0|0\rangle_1|0\rangle_2|1\rangle_3 - C_0|0\rangle_1|1\rangle_2|0\rangle_3 \\
&\quad + C_1|1\rangle_1|0\rangle_2|1\rangle_3 - C_1|1\rangle_1|1\rangle_2|0\rangle_3)\,.
\end{aligned} \tag{14.18}
$$

With the following notation for the four Bell states for particles 1 and 2: (cf. eqns 14.1 and 14.2)

$$|\Phi^+\rangle_{12} = \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2), \tag{14.19}$$

$$|\Phi^-\rangle_{12} = \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2 - |1\rangle_1|1\rangle_2), \tag{14.20}$$

$$|\Psi^+\rangle_{12} = \frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2), \tag{14.21}$$

$$|\Psi^-\rangle_{12} = \frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2 - |1\rangle_1|0\rangle_2), \tag{14.22}$$

we can rewrite eqn 14.18 as:

$$
\begin{aligned}
|\Psi\rangle_{123} = \frac{1}{2}\,\big(&|\Phi^+\rangle_{12}(C_0|1\rangle_3 - C_1|0\rangle_3) \\
+\ &|\Phi^-\rangle_{12}(C_0|1\rangle_3 + C_1|0\rangle_3) \\
+\ &|\Psi^+\rangle_{12}(-C_0|0\rangle_3 + C_1|1\rangle_3) \\
-\ &|\Psi^-\rangle_{12}(C_0|0\rangle_3 + C_1|1\rangle_3)\big)\,.
\end{aligned} \tag{14.23}
$$

Alice's BSM device may be considered to be a black box with four lights on it and inputs for photons 1 and 2, as illustrated schematically in Fig. 14.11. When the two input photons are in the Bell state $|\Phi^+\rangle_{12}$, the first bulb lights up. If they are in the state $|\Phi^-\rangle_{12}$, the second one lights up, etc.

The teleportation works by the non-local correlations intrinsic to the entangled state given by eqn 14.23. The measurement by Alice instantly determines the state of photon 3 for Bob. Thus, for example, if Alice's first bulb lights up, then Bob knows that photon 3 must be in the state

$$|\psi\rangle_3 = C_0|1\rangle_3 - C_1|0\rangle_3. \tag{14.24}$$

Therefore, if Alice tells Bob that she has measured the state $|\Phi^+\rangle_{12}$, Bob then knows the state of his photon without needing to carry out any measurements on it. He can then produce the desired output state, namely $(C_0|0\rangle_3 + C_1|1\rangle_3)$, by applying a simple unitary operator to photon 3. (See Example 14.2 below.) If Alice obtains other results, all she has to do is tell Bob the result she has obtained, and Bob then knows which unitary operator to use to complete the teleportation process.

In the case of teleportation of photon polarization, Bob's unitary operations are mere polarization rotations that can be performed very easily with a half wave plate.

Two points are worth emphasizing here. First, the protocol can only work after Alice transmits the result of her measurement to Bob by a classical channel. This is what ensures that no information is transferred faster than the speed of light. Second, photon 1 ends up entangled with photon 2, and neither Alice nor Bob acquire any information about C_0 and C_1. The teleportation process thus clearly adheres to the general principles of quantum measurement and quantum no-cloning.

Example 14.2 Show that a photon in the state $(-C_1|0\rangle + C_0|1\rangle)$ can be transformed to the state $(C_0|0\rangle + C_1|1\rangle)$ by a simple unitary operator.

Solution
We make use of the techniques for manipulating the state of single qubits developed in Section 13.3.2. The input state is written in the form

$$q = \begin{pmatrix} -C_1 \\ C_0 \end{pmatrix},$$

and the output is given by

$$q' = \hat{U} \cdot q.$$

It is apparent that the transformation can be performed if $\hat{U}$ takes the form:

$$\hat{U} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

so that:

$$q' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -C_1 \\ C_0 \end{pmatrix} = \begin{pmatrix} C_0 \\ C_1 \end{pmatrix}.$$

On noting that

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

and remembering that qubit operators are applied from right to left, we see from Table 13.4 that $\hat{U}$ consists of an X gate followed by a Z gate.

14.6 Experimental demonstration of teleportation

The first two experimental demonstrations of quantum teleportation were completed in 1997–8. In this section we describe one of these, namely that of Bouwmeester *et al.* The reader is referred to the reference for details of the experiment by Boschi *et al.*

See D. Bouwmeester, *et al.*, *Nature* **390**, 575 (1997) and D. Boschi, *et al.*, *Phys. Rev. Lett.* **80**, 1121 (1998).

Figure 14.12 shows the experimental arrangement, which included two EPR sources producing a total of four photons. Both EPR sources consisted of a nonlinear crystal of the type shown in Fig. 14.5 pumped by