14 Entangled states and quantum teleportation

The third branch of quantum information processing is *quantum teleportation*. This is a very new subject, and the aim of researchers working in the field at present is to achieve proof-of-principle demonstrations at the few-particle level. As we shall see, teleportation relies heavily on the properties of *entangled states*. We therefore begin by describing the concept of entangled photon states and explaining how they are generated in the laboratory. This will enable us to describe some recent experiments testing fundamental ideas of interference at the single-photon level. We shall then discuss the Einstein–Podolsky–Rosen (EPR) paradox and Bell's theorem, which will allow us to explain the principles of teleportation, and describe how they have been demonstrated in the laboratory. Finally, we shall briefly discuss a few of the wider issues that arise from the EPR paradox and Bell's theorem.

14.1 Entangled states

Entanglement is one of the most counter-intuitive aspects of the quantum world. The concept is linked to two famous papers in the historical development of quantum theory, and has come to the fore in recent years with the advent of quantum information science. In 1935 Einstein, Podolsky and Rosen published the 'EPR' paper on the properties of an entangled two-particle system formed from the decay of a radioactive source. Soon afterwards, Schrödinger coined the term 'entanglement' in his cat paradox paper that has fuelled the imagination of students and teachers alike for many years.

See A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47**, 777 (1935), and E. Schrödinger, *Die Naturwissenschaften* **23**, 807, 823, 844 (1935). An English translation of the latter is available in *Proc. Am. Philos. Soc.* **124**, 323 (1980). Bohm's variant on the EPR experiment was originally developed in his book *Quantum Theory*, published in 1951 by Prentice-Hall, New Jersey. Bohm actually proposed to make spin measurements on pairs of atoms, but the version we present here is the optical equivalent involving polarization measurements on pairs of photons.

Let us first consider the EPR paper. We will present the argument in the 'EPRB' form introduced by David Bohm in 1951. The scheme for an optical EPRB experiment is shown in Fig. 14.1. A source S emits a pair of photons arbitrarily labelled 1 and 2, with photon 1 going one way and photon 2 going another. The polarization of each photon is measured with a beam-splitter/detector arrangement similar to the one presented in Fig. 12.2. We designate the polarization states $|\updownarrow\rangle$ and $|\leftrightarrow\rangle$ as $|1\rangle$ and $|0\rangle$, respectively, according to the BB84 scheme in the $\oplus$ basis given in Table 12.1.

The subtlety in the experiment occurs when we use a source that emits **correlated photon pairs**. Correlated photon pairs have the following

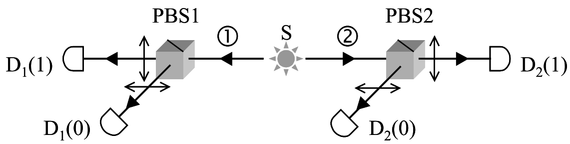

Fig. 14.1 Apparatus for an EPRB experiment. The source S emits two correlated photons arbitrarily labelled 1 and 2 towards polarization detectors involving a polarizing beam splitter (PBS) and single-photon detectors D. The detectors are given a subscript 1 and 2 to identify the photon and the results are designated 0 and 1 according to the scheme presented in Table 12.1 for the $\oplus$ basis.

properties:

1. The polarization of either photon 1 or photon 2 measured independently of the other is random.
2. The polarization of the pair of photons is perfectly correlated; that is, if $D_1(0)$ fires, then $D_2(0)$ always fires, and if $D_1(1)$ fires, then $D_2(1)$ always fires. Alternatively if $D_1(0)$ fires, then $D_2(1)$ always fires, and vice versa.

The second property follows from internal conservation laws of the source that will be discussed in Section 14.2.

A multi-particle system is described as being in an **entangled state** if its wave function cannot be factorized into a product of the wave functions of the individual particles. The mutual dependence of the results of the polarization measurements on the correlated photon pair means that the wave function has to be written in the form:

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|0_1, 0_2\rangle \pm |1_1, 1_2\rangle\right), \tag{14.1}$$

for the case of perfect positive correlation, and

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|0_1, 1_2\rangle \pm |1_1, 0_2\rangle\right), \tag{14.2}$$

for perfect negative correlation, with the subscripts referring to the individual photons. The wave functions in eqns 14.1 and 14.2 are thus examples of entangled states. They are also called **Bell states** for reasons that will become clear in Section 14.4.

The entangled form of the wave functions in eqns 14.1 and 14.2 implies that a measurement of the polarization of one photon determines the result of a polarization measurement on the other. Thus for the wave function given in eqn 14.1 we will obtain either the result (0,0) or (1,1), each with equal probability. Similarly, eqn 14.2 implies results of (0,1) or (1,0) each with 50% probability. In both cases a measurement on one photon allows us to predict the result of the measurement on the other with 100% certainty.

The **Schrödinger cat paradox** illustrates the concept of entangled states in a graphic way by considering the state of a live cat put into a sealed box containing a radioactive atom as shown Fig. 14.2. The box

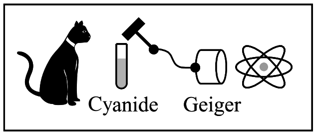

Fig. 14.2 Schrödinger's cat. A live cat is put into a sealed box containing a radioactive atom. The radiation emitted by the decay of the atom is detected by a Geiger counter, which activates a relay on registering a count. The relay is connected to a hammer which smashes a sealed flask of cyanide, and hence kills the cat.

also contains a devious mechanism such that the decay of the atom triggers a device to smash a sealed flask of poison, thereby killing the cat. The state of the cat is therefore entangled with the state of the atom. If we wait for a time such that the probability of the atom decaying is equal to 50%, then we can write the wave function of the system in the form:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|\text{live}, 1\rangle + |\text{dead}, 2\rangle \right), \tag{14.3}$$

where $|1\rangle$ and $|2\rangle$ represent the state of the undecayed and decayed atom, respectively. This seems to imply that we have a state inside the box where the cat is both dead and alive at the same time, in clear contrast to our common experience. On opening the box, we would, of course, find the cat dead or alive with probability equal to 50%.

Much to the relief of cat-lovers, there is no need to perform the Schrödinger cat experiment in the laboratory. Paradoxes of this type are not found in the macroscopic world, because large systems consisting of many particles lose their quantum coherence through interactions with the noisy macroscopic environment. (See Section 13.4.) Things are different, however, at the microscopic level of isolated atoms and photons in a well-controlled environment. Entangled photon states of the type required for the EPRB experiment can readily be generated in the laboratory, and photon Schrödinger cat states have been demonstrated.

Quantum entanglement is not restricted to the case of two-particle polarization that we have considered here. Two-particle photon states with time or momentum entanglement can also be generated, and entangled states involving three or more particles have many interesting properties. However, we shall restrict our attention exclusively to two-particle polarization states for simplicity's sake. The reader is referred to the bibliography for details of other types of entangled states.

14.2 Generation of entangled photon pairs

Many of the early optical experiments on entangled states employed atomic cascades in calcium to generate the correlated photon pairs. The experiment consists of a pair of detectors arranged to collect the photons emitted in an atomic cascade from the $4p^2\ {}^1S_0$ excited state of calcium as shown in Fig. 14.3(a). Figure 14.3(b) shows the corresponding level scheme for the transitions involved. The cascade occurs by allowed transitions at 551.3 and 422.7 nm via the $4p4s\ {}^1P_1$ intermediate level. Narrow-band interference filters F1 and F2 in front of the photomultiplier tube (PMT) detectors selected these photon wavelengths from others produced by alternative decay routes. In the initial experiment by Kocher and Commins in 1967, the calcium atoms were excited to the $4p^2\ {}^1S_0$ level by absorption of ultraviolet photons from a hydrogen arc lamp. Photons at 227.5 nm from the lamp first excited the atoms from the $4s^2\ {}^1S_0$ ground state to the $3d4p\ {}^1P_1$ level, and the atoms then

See C. A. Kocher, and E. G. Commins, *Phys. Rev. Lett.* **18**, 575 (1967).