SYSTEM OF EQUATIONS

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ABSTRACT:

This thesis covers the essential and very basic principles of systems of equations, with a primary focus on system of linear equations .This research aims to provide a comprehensive understanding of how equations interact within a systematic framework. Through a detailed exploration of the fundamental principles governing linear equations within systems, this study illuminates mechanisms that govern these mathematical constructs and their real-world applications.

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CHAPTER:1

INTRODUCTION TO SYSTEM OF EQUATIONS

1.1 DEFINITION:

“ A system of equations is a collection of equations that involves some variables and have some relationship according to a particular situation .”

EXPLANATION AND EXAMPLES:

A system of equations involves a set of two or more equations that share common variables and must be solved simultaneously to find the values of those variables that satisfy all the equations. These systems are commonly used in various fields such as mathematics, physics, economics, and engineering to model real-world situations and find solutions.

For example, consider a system of equations representing the relationship between the number of apples(x) and oranges(y) in a fruit basket:

$2x + 3y = 10$ ....(1)

$x - y = 2$ ....(2)

In this system,(x) represents the number of apples and (y) represents the number of oranges. By solving this system, we can determine the specific values of " x" and "y" that satisfy both equations and provide the solution to the problem at hand.

1.2 Overview:

A system of equations is a collection of two or more equations that share common variables and must be solved simultaneously to find the values of those variables that satisfy all the equations.

These systems are fundamental in mathematics and have wide applications in various fields such as science, engineering, economics, and social sciences. Solving a system of equations involves finding the values of the variables that make all the equations true at the same time. There are different methods to solve systems of equations, including substitution, elimination, and matrix methods. The solutions to a system of equations can be unique, infinite, or non-existent, depending on the relationships between the equations. Overall, understanding and systems of equations are essential skills in problem-solving and critical thinking across different disciplines.

For example consider a case example of such systems of equations....

$X^2 + y^2 = 25$ ....(A)

$2x – y = 1$ ....(B)

In this system, (x) and (y) represent the variables in two quadratic equations. Solving this system would involve finding the values of (x) and (y) that satisfy both equations simultaneously.

This is an example of graphical scenario of two functions of different degrees which are widely used in engineering and fields of electronics.

(<u>NOTE:</u> Solution of such systems of equations are discussed in next section.)

1.3 HISTORICAL BACKGROUND:

- System of linear equations arose in Europe with the introduction in **1637** by **René Descartes** of coordinates in geometry. In fact, in this new geometry, now called Cartesian geometry, lines and planes are represented by linear equations, and computing their intersections amounts to solving systems of linear equations.

- In ancient Babylon, around 1800 BCE, clay tablets have been discovered containing mathematical problems that required solving systems of linear equations. The Babylonians used geometric methods and algebraic techniques to solve these problems related to trade, taxation, and construction.

- The ancient Egyptians also developed methods for solving systems of linear equations to solve practical problems related to land surveying, construction projects, and distribution of resources. The Rhind Mathematical Papyrus, dating back to around 1650 BCE, contains examples of linear equations and their solutions.

- In ancient Greece, mathematicians such as Euclid and Diophantus made significant advancements in solving systems of equations. Euclid's "Elements" introduced methods for solving simultaneous linear equations using geometric interpretations, while Diophantus's work focused on solving systems of linear and quadratic equations.

- ❖ During the Islamic Golden Age, mathematicians such as Al-Khwarizmi and Omar Khayyam further developed algebraic methods for solving systems of equations. Al-Khwarizmi's book "Al-Kitab al-Mukhtasar fi Hisab al-Jabr wal-Muqabala" (The Compendious Book on Calculation by Completion and Balancing) introduced systematic methods for solving linear and quadratic equations, laying the foundation for algebra as a discipline.

- ❖ In the Renaissance period, European mathematicians like François Viète and René Descartes made significant contributions to the development of algebraic notation and methods for solving systems of equations. Descartes's introduction of coordinate geometry provided a powerful tool for solving systems of equations geometrically.

- ✓ Overall, the historical development of systems of equations reflects the evolution of algebraic methods and problem-solving techniques across different civilizations and time periods, leading to the sophisticated mathematical tools and concepts used in modern mathematics.

CHAPTER NO : 2

TYPES Of SYSTEM Of EQUATIONS

2.1 Simultaneous equations:

Two or more linear equations that all contain the same unknown variables are called a system of simultaneous linear equations.

Explanation with Example:

Solving such a system means finding values for the unknown variables which satisfy all the equations at the same time.

$2a - 3b + c = 9, \quad a + b + c = 2, \quad a - b - c = 9$

$3x - y = 5, \quad x - y = 4$

$a + b = 9, \quad a - b2 = 16$

We can solve such a set of equations using different methods.

There are further different types of simultaneous equations on the basis of degree types of solutions and methods of solution.

Some types of simultaneous equations are given below:

- ✓ System of Linear equations.
- ✓ System of Non linear equations.

System of Linear equations:

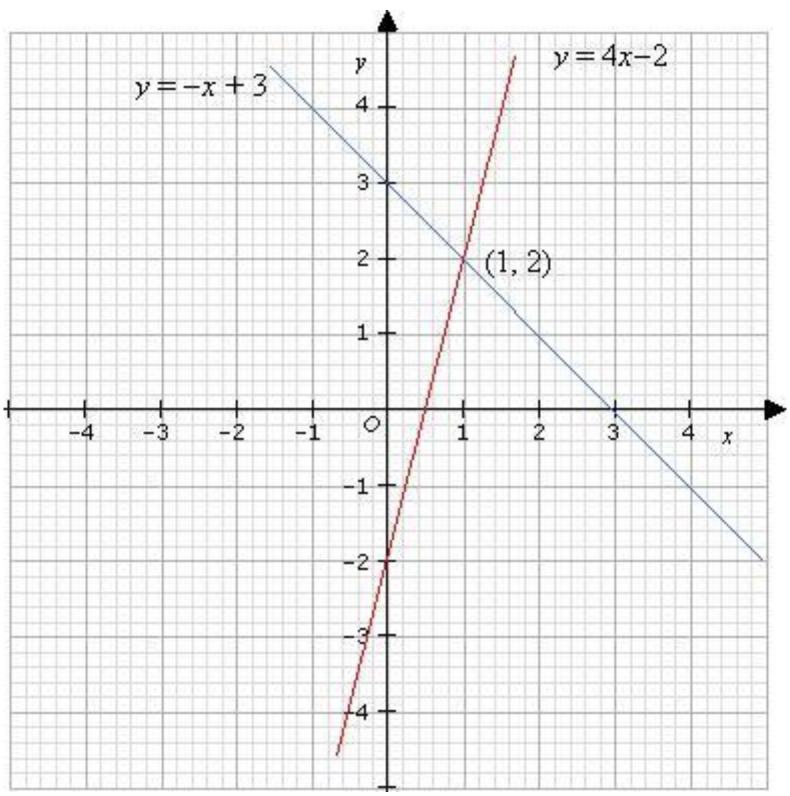
Those equation which have degree equal to 1 are called linear equations.The graphs of such systems of linear equations are straight lines and the point of their intersection is actually the solution of the given system of equations.

EXAMPLE:1

Consider the following system of linear equations....

y =-x + 3..................(BLUE LINE)

y = 4x – 2.................(RED LINE)



From the graph it is clear that the point of intersection of of these two lines is

(1,2) ,so we can say that x = 1 and y = 2 satisfies both equations.

(NOTE: Graphical method for the solution of system of equations is explained in CHAPTER 3)

System of non linear equations:

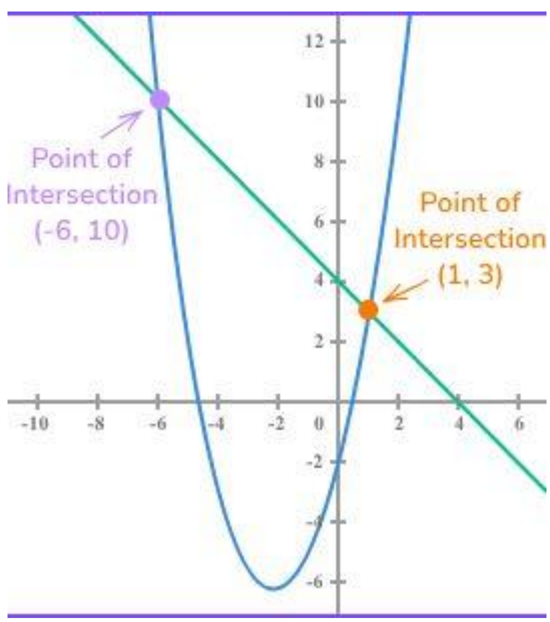
Those system of simultaneous equations in which any one of the equation does not have degree equal to one are called system of non linear equations.

The graph of a non linear equations are not straight line but a curve.

<u>EXAMPLE:2</u>

$x + y = 4$.................(GREEN LINE)

$x^2 + y^2 = -27$..............(BLUE CURVE)



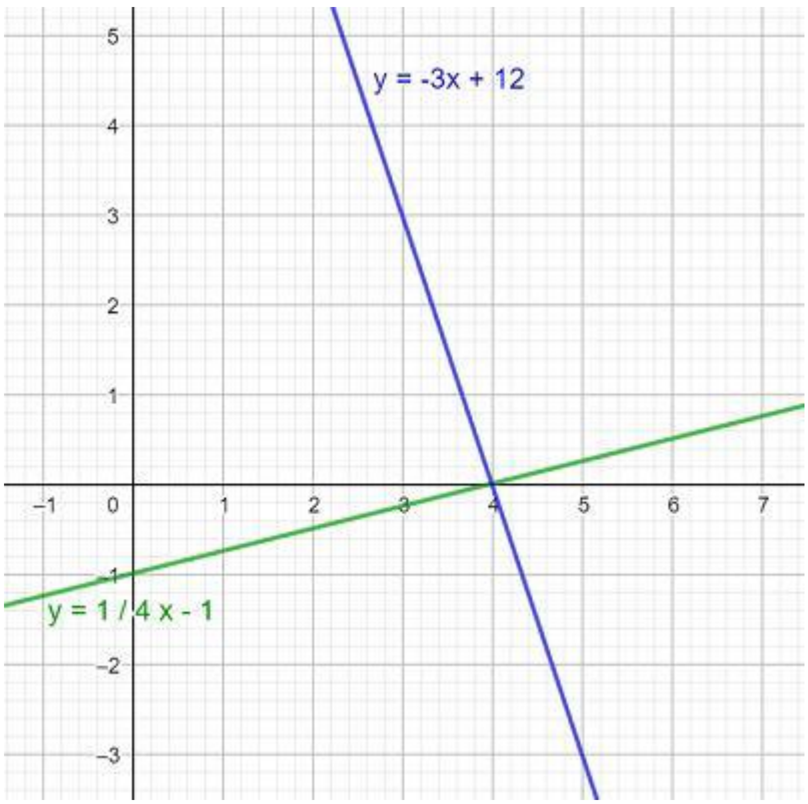
The solution of this given system of equations is (-6,10) and (1,3).

<u>2.2 INDEPENDENT SYSTEM OF EQUATIONS:</u>

An independent system of equations is a system with exactly one solution. This is a point in the form which indicates the intersection of the two lines.

Y = -3 x + 12..............(BLUE LINE)

Y = 1/4x – 1...............(GREEN LINE)



So the solution of this Independent system is (4,0).

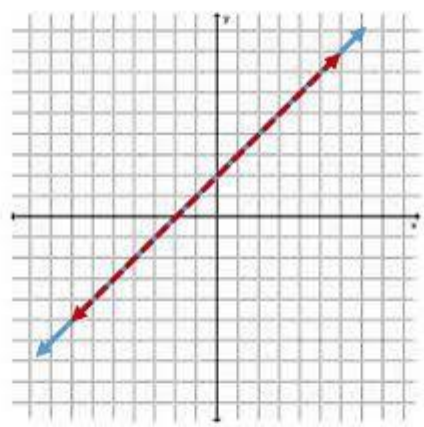
A system of equations is when two or more equations are solved at the same time to determine a solution that fits both equations. The equations can be of any form (linear, quadratic, cubic, etc.), but the solution must fit both equations. In a system of equations, there are two main ways of determining the solution; graphically or algebraically. When a solution is determined graphically, the two equations must intersect at one or more points. If the two equations do not intersect at one or more points, then the systems of equations are said to have no solution. When a solution is determined algebraically, the x-value and y-value that are true for the first equation must ALSO be true for the second equation and all other equations.

2.3 DEPENDENT SYSTEM OF EQUATIONS:

A dependent system of linear equations is a set of equations that has an infinite number of solutions. The equations in that system represent the same line when plotted on a graph.

EXAMPLE:2

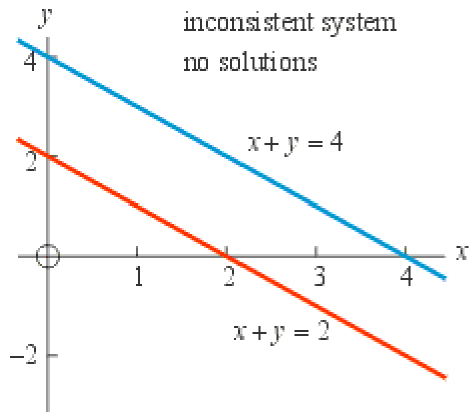
$$y = x + 2$$
$$-3x + 3y = 6$$



These equations have the graph lines which are overlapping on each other, so we can say that the have infinitesimally many solutions because infinite ordered pairs lie on both the lines .In other words the point of intersection of these equations are infinite. .Such types of equations have same slope and y _intercept.

2.4 INCONSISTENT SYSTEM OF EQUATIONS:

A system of linear equations is inconsistent if it has no solutions. When this is the case, the graphs of the lines in the system do not intersect, meaning they are parallel.



X + y = 4.............(Blue line)

X + y = 2.............(Red line)

As these lines do not intersect each other so the given system does not have a solution. Such type of systems are called inconsistent system of equations.

These are some types of system of equations on the basis of number of solutions. There are also other types of solution on the basis of number of variables, number of equations in the System and some other types.

CHAPTER NO: 3

METHODS OF SOLVING SYSTEM OF EQUATIONS

3.1 GRAPHICAL METHOD:

One method to solve a given system of equations is by mean of plotting a graph of all the equations on Cartesian plane. This Method is a very general method because it can be applied to all types of systems. This method i.e. by using graph can help us to understand the relation and behavior of equations relative to each

Following steps should be followed to solve a system of equations by graphing:

1) Graph the first equation.
2) Graph the second equation on the same rectangular coordinate system.
3) Determine whether the lines intersect, are parallel, or are the same line.
4) Identify the solution to the system.

- If the lines intersect, identify the point of intersection. Check to make sure it is a solution to both equations. This is the solution to the system.
- If the lines are parallel, the system has no solution.
- If the lines are the same, the system has an infinite number of solutions.

Explanation with Example:

Consider we have to solve the system by graphing:

y=2x+1................(A)

y=2x-1................(B)

First find the slope and y_intercept of equation (A)....

y=2x+1

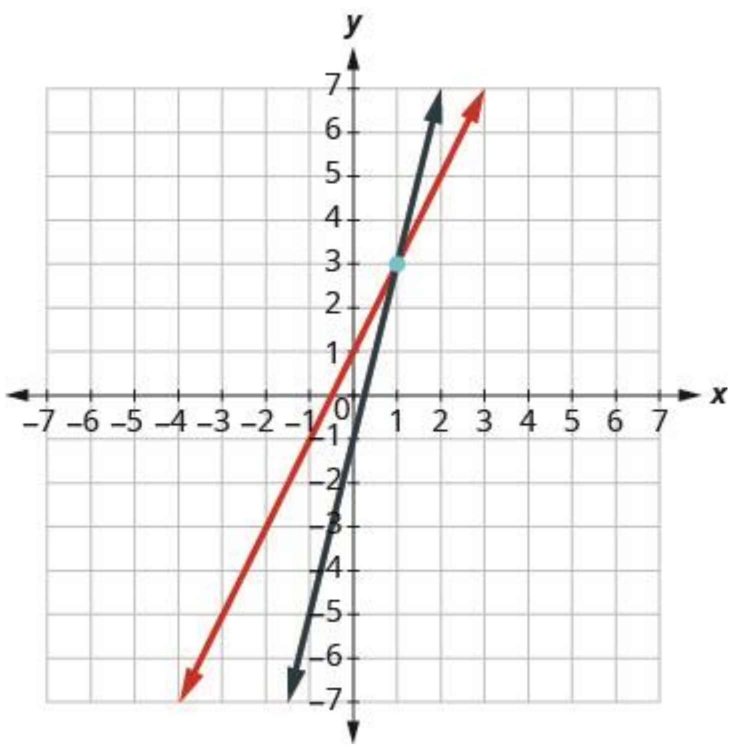
m=2 ; c=1

Now find the slope and y_intercept of equation (B)

Y=2x-1

m=2; c=-1

STEP :1: Graph the lines ...



Here Red line represents equation (A) while black line represents equation (B).

STEP:2: Now find a point of intersection between these two given lines...

From the graph it is clear that these two lines intersect at the point **(1,3)**. So this point may be the solution of the given system of equations.

STEP:3: Now Check this point for both equations...

Check the solution in both equations.

$$\begin{array}{ll} y = 2x + 1 & y = 4x - 1 \\ 3 \stackrel{?}{=} 2 \cdot 1 + 1 & 3 \stackrel{?}{=} 4 \cdot 1 - 1 \\ 3 = 3\checkmark & 3 = 3\checkmark \end{array}$$

Hence the point (1,3) satisfies both of the equations so it is the solution of the given system of equations.

3.2: ALGEBRICAL METHODS:

Algebrical methods are also used to find a solution of given system of equations. Algebrical methods include following techniques…

- By addition method
- By substitution method
- By elimination method
- By matrices method

3.2.1: BY ADDITION METHOD:

- Choose a variable to eliminate.
- Multiply one or both equations so that the coefficients of this variable are the LCM of the coefficients with opposite signs.
- Add the equations together, then solve.
- Substitute the value into one of the original equations to find the remaining variable.

<u>3.2.2:BY SUBSTITUTION METHOD:</u>

The substitution method works by substituting one y-value with the other. To put it simply, the method involves finding the value of the x-variable in terms of the y-variable. After this is done, we then end up substituting the value of x-variable in the second equation.

FOR EXAMPLE:

$x-2y=8$ ....(1)

$x+y=5$.............. (2)

From equation (2),

$5-y=x$

Substitute this value in equation (1),

$x-2y=8$

$5-y-2y=8$

$5-3y=8$

$-3y=8-5$

$-3y = 3$

$Y=-1$

Put this value in equation (2),

$x-1=5$

$x=5+1$

$x=6$

$x=6$ and $y = -1$

So the solution set is ; $\{(6,-1)\}$

3.2.3: BY ELIMINATION METHOD:

The elimination method is an algebraic method to solve system equations. It requires that the coefficients of the same variable in the equations be opposite so that, when the equations are added together, that variable will be eliminated.

EXAMPLE WITH SOLUTION:

Elimination Method with Three Equation

$3x-y+2z = 5$……………..(1)

$4x+2y-z = 6$ ……………..(2)

$5x-3y + z = 1$ ……………(3)

Add equation (2) & (3) to eliminate Z,

$4x + 2y – 4 = 6$

$5x – 3y + 2 = 1$

$\boxed{9x\ y = 7 \text{ ……(P)}}$

Multiply equation (3) by 2,

$10x-6y+2z = 2$

Subtract equation (1) from this equation,

$10x-6y+2z =2$

$3x-y + 2z =5$

$\boxed{7x-5y=3\text{…(Q)}}$

Solve equation (P) & (Q) using elimination,

$(9x – y = 7) \times (-5) \rightarrow \quad -45x + 5y = -35$

$7x – y = -3$

-38x = -38

X = 1

Now using previous concept we can find the values of y and z.

3.2.4: BY METHOD OF MATRICES:

Following steps should be followed to solve a system of equations by matrices method.

1. Write the augmented matrix for the system of equations.
2. Using row operations get the entry in row 1, column 1 to be 1.
3. Using row operations, get zeros in column 1 below the 1.
4. Using row operations, get the entry in row 2, column 2 to be 1.
5. Continue the process until the matrix is in row-echelon form.
6. Write the corresponding system of equations.
7. Use substitution to find the remaining variables.
8. Write the solution as an ordered pair or triple.

Use an inverse matrix to solve the simultaneous equations:

$$\begin{aligned} -x + 6y - 2z &= 21 \\ 6x - 2y - z &= -16 \\ -2x + 3y + 5z &= 24 \end{aligned}$$

Write the system of equations using matrices:

$$\begin{pmatrix} -1 & 6 & -2 \\ 6 & -2 & -1 \\ -2 & 3 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 21 \\ -16 \\ 24 \end{pmatrix}$$

Find the inverse of the left-hand matrix:

$$\frac{1}{189}\begin{pmatrix} 7 & 36 & 10 \\ 28 & 9 & 13 \\ -14 & 9 & 34 \end{pmatrix}$$

Left-multiply the right-hand matrix by this inverse:

$$\frac{1}{189}\begin{pmatrix} 7 & 36 & 10 \\ 28 & 9 & 13 \\ -14 & 9 & 34 \end{pmatrix}\begin{pmatrix} 21 \\ -16 \\ 24 \end{pmatrix} = \frac{1}{189}\begin{pmatrix} -189 \\ 756 \\ 378 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix}$$

Hence $x = -1$, $y = 4$ and $z = 2$.

You can confirm that this is equivalent to the original equations by multiplying out the left-hand side:

$$\begin{pmatrix} -1 & 6 & -2 \\ 6 & -2 & -1 \\ -2 & 3 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -x + 6y - 2z \\ 6x - 2y - z \\ -2x + 3y + 5z \end{pmatrix}$$

Use your calculator to find the inverse matrix and then to multiply the matrices.

Problem-solving

Once you have found the inverse matrix, you could use it to solve a similar system of equations with a different answer vector.

CHAPTER: 4

Applications for system of equations

Systems of equations have a wide range of applications across various fields such as mathematics, physics, engineering, economics, and more. They are used to model and solve real-world problems that involve multiple unknowns and relationships between different quantities.

4.1: APPLICATIONS IN BUSINESS AND ECONOMY:

In business and economy, systems of equations are used to solve problems involving different variables and relationships. Here are some simplified examples to illustrate their applications:

- **Profit Analysis:**

Suppose a company sells two types of products, Product A and Product B. The company's total profit depends on the number of units sold for each product and the prices. By setting up a system of equations representing the revenue and cost for each product, the company can determine how many units of each product need to be sold to maximize profit.

❖ **Supply and Demand:**

A business needs to determine the selling prices for its products based on supply and demand. The quantity demanded by customers and the quantity supplied by the business can be represented by a system of equations. By solving this system, the business can find the equilibrium price where supply equals demand, helping in pricing decisions.

❖ **Cost Analysis:**

A manufacturing company incurs fixed costs and variable costs in producing goods. By setting up a system of equations representing the total cost based on the number of units produced and the cost per unit, the company can analyze different production scenarios and determine the most cost-effective production levels.

- ✓ In conclusion, systems of equations play a crucial role in business and economy by providing a mathematical framework to analyze and solve various real-world problems. Whether it’s optimizing profits, determining investment strategies, setting prices based on supply and demand, or analyzing costs, systems of equations help businesses make informed decisions and improve efficiency.

4.2: APPLICATIONS IN ENGINEERING:

In engineering, systems of equations are widely used to model and solve problems that involve multiple variables and relationships. Here are some simplified examples of applications of systems of equations in engineering:

❖ **Structural Analysis:**

Engineers use systems of equations to analyze the forces and stresses in structural components such as beams, trusses, and frames. By setting up equilibrium equations for each component and considering the interactions between them, engineers can determine the internal forces and deformations to ensure the structural integrity and safety of the design.

❖ **Electrical Circuits:**

In electrical engineering, systems of equations are used to analyze and solve circuits with multiple components such as resistors, capacitors, and inductors. By applying Kirchhoff's laws and Ohm's law, engineers can set up a system of equations to determine voltages, currents, and power dissipation in the circuit.

❖ **Control Systems:**

Engineers use systems of differential equations, which can be represented as a system of equations, to model and analyze dynamic systems such as control systems in robotics, aerospace, and automotive industries. By solving these equations, engineers can design controllers to regulate system behavior and achieve desired performance.

❖ **Thermodynamics and Fluid Dynamics:**

In fields like mechanical and chemical engineering, systems of equations are used to analyze heat transfer, fluid flow and thermodynamic processes. Engineers can model complex systems using conservation laws such as mass, energy, and momentum equations to predict temperature distributions, fluid velocities, and pressure changes.

✓ In engineering, systems of equations serve as a fundamental tool for problem-solving, design optimization, and performance analysis across various disciplines. By leveraging mathematical models to represent physical phenomena and relationships, engineers can simulate, predict, and control complex systems to meet design requirements and address engineering challenges effectively.

4.3: APPLICATIONS IN PHYSICS:

In physics, systems of equations are essential for modeling and solving a wide range of phenomena and problems that involve multiple interacting variables. Here are some simplified examples to illustrate the applications of systems of equations in physics:

- ❖ **Projectile Motion:**

When an object is launched into the air, its motion can be described by a system of equations that relate the object's position, velocity, and acceleration in both the horizontal and vertical directions. By solving these equations, physicists can analyze the trajectory, maximum height, and range of the projectile.

- ❖ **Newton's Laws of Motion:**

Newton's second law, which relates force, mass, and acceleration, can be expressed as a system of equations for objects experiencing multiple forces in different directions. Physicists use these equations to predict the motion of objects under the influence of various forces and accelerations.

- ❖ **Electricity and Magnetism:**

In electromagnetism, systems of equations are used to describe the behavior of electric and magnetic fields. Maxwell's equations, which form a system of partial differential equations, govern the propagation of electromagnetic waves and interactions between electric charges and currents.

- ❖ **Thermodynamics**:

In thermophysics, systems of equations are employed to analyze heat transfer, energy conversion, and thermal processes. Physicists use equations representing the laws of thermodynamics to study the behavior of systems undergoing heating, cooling, and phase transitions.

❖ **Quantum Mechanics:**

In quantum physics, systems of equations such as the Schrödinger equation are used to describe the behavior of quantum systems, including particles and waves. Physicists solve these equations to predict the energy levels, wave functions, and probabilities associated with quantum phenomena.

- ✓ In physics, systems of equations serve as a powerful tool for modeling, analyzing, and predicting the behavior of physical systems across different scales and domains. By formulating mathematical relationships between variables and applying fundamental principles, physicists can gain insights into the underlying mechanisms of natural phenomena and make predictions about the behavior of complex systems.

4.4: APPLICATIONS IN OUR DAILY LIFE:

Systems of equations have numerous applications in our daily lives, often without us even realizing it. Here are some common scenarios where systems of equations are used:

❖ **Budgeting:**

When managing personal finances, individuals often need to solve systems of equations to balance income, expenses, savings, and investments. By setting up equations that represent different sources of income and various expenses, people can make informed decisions about budgeting and financial planning.

❖ **Travel and Commuting**:

When planning a route or mode of transportation, people may need to solve systems of equations to compare travel times, distances, costs, and environmental impacts. By considering different factors simultaneously, individuals can make efficient and sustainable travel choices.

❖ **<u>Home Improvement</u>**:

In tasks like painting a room, laying tiles, or designing furniture layouts, systems of equations can help calculate material quantities, costs, and dimensions. By formulating equations that relate area, volume, and budget constraints, homeowners can make informed decisions about home improvement projects.

❖ **<u>Time Management</u>**:

Balancing work, school, family, and social activities requires juggling multiple responsibilities and schedules. Systems of equations can be used to optimize time allocation, prioritize tasks, and maximize productivity by considering constraints and priorities simultaneously.

✓ In our daily lives, systems of equations provide a structured approach to solving problems that involve multiple variables, constraints, and objectives. By applying mathematical models to real-world situations, individuals can make better decisions, optimize resources, and achieve desired outcomes efficiently. The versatility and practicality of systems of equations make them a valuable tool for problem-solving and decision-making in various aspects of daily life.

CONCLUSIONS AND KEY FACTS

Conclusions:

In conclusion, the study of systems of equations plays a crucial role in problem-solving and decision-making across a wide range of disciplines and applications. By formulating mathematical models that represent relationships between multiple variables, engineers, physicists, and individuals in daily life can analyze complex systems, predict outcomes, and optimize solutions effectively. The versatility and applicability of systems of equations make them indispensable for addressing real-world challenges, innovating technological solutions, and enhancing our understanding of natural phenomena. As a fundamental tool in mathematics and science, systems of equations empower us to explore, interpret, and manipulate the interconnected variables that shape our world.

Key Facts:

- Systems of equations involve multiple equations with multiple variables that are interconnected and must be solved simultaneously.
- They are used in engineering for design optimization, performance analysis, and problem-solving.
- In physics, systems of equations model and predict various phenomena such as projectile motion, electromagnetism, and thermodynamics.
- In daily life, systems of equations are applied in budgeting, cooking, fitness planning, travel decisions, home improvement, and time management.
- Systems of equations provide a structured approach to solving problems with multiple objectives, and variables.

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