In[2]:=
$$r12 := \frac{r2 - a2\ \mathrm{Exp}[I\,\phi 2]}{1 - r2\ \ a2\ \mathrm{Exp}[I\,\phi 2]}$$

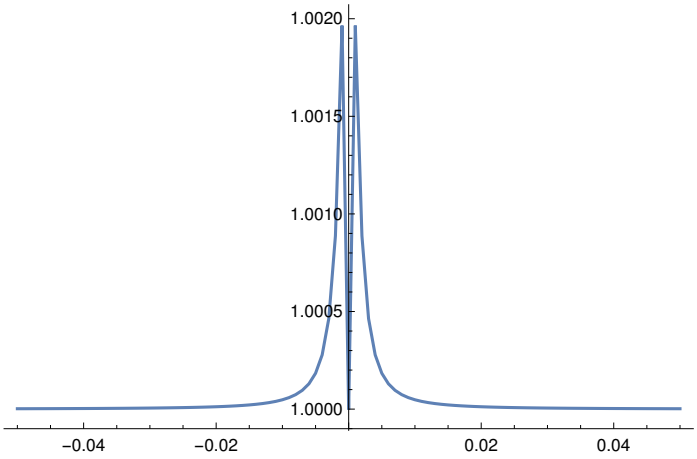
$$trans := \frac{r1 - r12\,a1\ \mathrm{Exp}[I\,\phi 1]}{1 - r1\ r12\ a1\ \mathrm{Exp}[I\,\phi 1]}$$

In[862]:=
```
n := 3.45;
c := 299 792 458;
R1 := 10 × 10^(-6);
R2 := 10 × 10^(-6);
d1 := 2 Pi R1
d2 := 2 Pi R2
λ := 1550 × 10^(-9); (* resonant wavelength *)
Q1 := 1 × 10^5;
Q2 := 1 × 10^6;
g1 := 0.01
g2 := 1500
α1 := 2 Pi n / (λ Q1)
α2 := 2 Pi n / (λ Q2)
a1 := Exp[g1 - α1 d1]
a2 := Exp[g2 - α2 d2]
t1 := √(1 - r1^2)
t2 := √(1 - r2^2)
t3 := √(1 - r3^2)
r1 := 0.999999
r2 := 0.999999
```

In[24]:=
```
ϕ2 := ϕ1
```

In[882]:=
```
ϕ1min := -0.05;
ϕ1max := 0.05;
transdata := Table[{ϕ1, Abs[trans]^2}, {ϕ1, ϕ1min, ϕ1max, 0.001}]
ListLinePlot[transdata, PlotRange → All]
```

Out[885]=



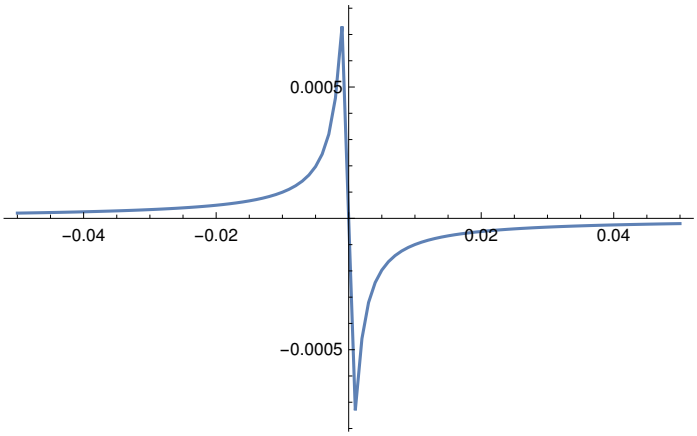
In[780]:=
$$\phi 12 := -\text{ArcTan}\left[\frac{a2\ \text{Sin}[\phi 1]}{r2 - a2\ \text{Cos}[\phi 1]}\right] + \text{ArcTan}\left[\frac{r2\ a2\ \text{Sin}[\phi 1]}{1 - r2\ a2\ \text{Cos}[\phi 1]}\right]$$

$$\phi eff := -\text{ArcTan}\left[\frac{a1\ \text{Abs}[r12]\ \text{Sin}[\phi 1 + \phi 12]}{r1 - a1\ \text{Abs}[r12]\ \text{Cos}[\phi 1 + \phi 12]}\right] + \text{ArcTan}\left[\frac{r1\ a1\ \text{Abs}[r12]\ \text{Sin}[\phi 1 + \phi 12]}{1 - r1\ a1\ \text{Abs}[r12]\ \text{Cos}[\phi 1 + \phi 12]}\right]$$

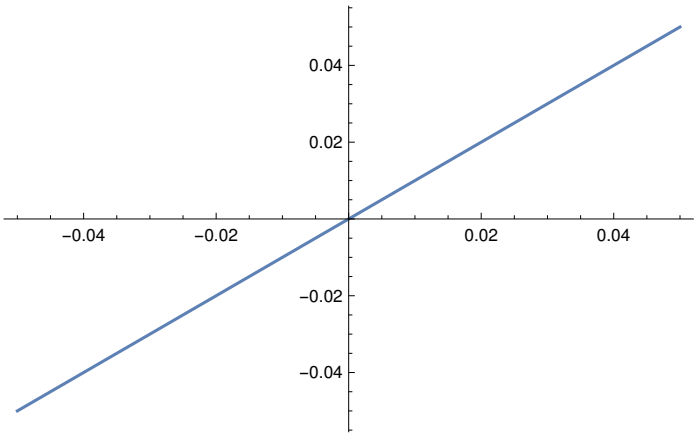
$$vg := (n / c)\ \left(\left(a1\ \left(-1 + r1^2\right)\ \text{Abs}[r12]\right.\right.$$
$$\left.\left(-a1\ \left(1 + r1^2\right)\ \text{Abs}[r12] + r1\ \text{Cos}[\phi 1 + \phi 12] + a1^2\ r1\ \text{Abs}[r12]^2\ \text{Cos}[\phi 1 + \phi 12]\right)\right) /$$
$$\left(\left(r1^2 + a1^2\ \text{Abs}[r12]^2 - 2\ a1\ r1\ \text{Abs}[r12]\ \text{Cos}[\phi 1 + \phi 12]\right)\right.$$
$$\left.\left.\left(1 + a1^2\ r1^2\ \text{Abs}[r12]^2 - 2\ a1\ r1\ \text{Abs}[r12]\ \text{Cos}[\phi 1 + \phi 12]\right)\right)\right)$$

In[886]:=
```
ϕ1min := -0.05;
ϕ1max := 0.05;
transdata := Table[{ϕ1, ϕeff}, {ϕ1, ϕ1min, ϕ1max, 0.001}]
ListLinePlot[transdata, PlotRange → All]
transdata := Table[{ϕ1, ϕ12}, {ϕ1, ϕ1min, ϕ1max, 0.001}]
ListLinePlot[transdata, PlotRange → All]
transdata := Table[{ϕ1, c vg}, {ϕ1, -0.5, 0.5, 0.001}]
ListLinePlot[transdata, PlotRange → All]
```

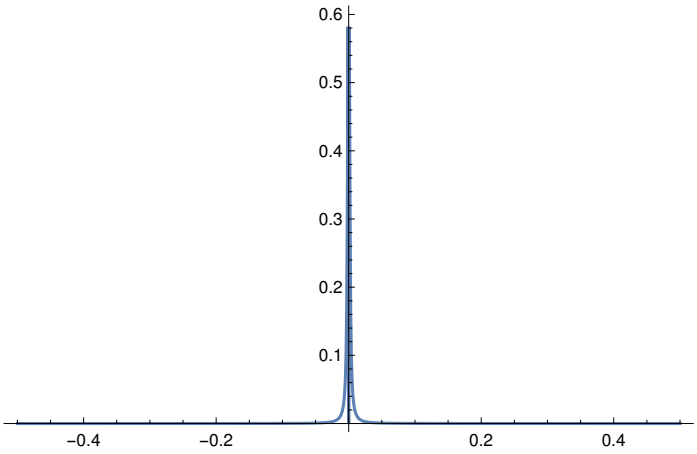
Out[889]=



Out[891]=



Out[893]=



In[894]:=
```
r1 - Abs[r12] a1 /. ϕ1 → 0
r1 - r2 a1
r2 - a2
```

Out[894]= -0.00121561

Out[895]= -0.0012136

Out[896]= $-2.762747754502644 \times 10^{651}$

In[897]:=
```
(1 / vg) /. ϕ1 → 0
```

Out[897]= -6.40198×10^{7}