# Optical Properties of Passive and Active Photonic Resonators


*by*

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CIIT/FA15-BPH-019/ISB


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Optical Properties of Passive and Active Photonic Resonators

A Thesis Presented to

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In partial fulfillment

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by

Ahmad Bilal
CUI/FA15-BPH-019/ISB

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# Optical Properties of Passive and Active Photonic Resonators

A Under Graduate Thesis submitted to the Department of Physics as partial fulfillment of the requirement for the award of Degree of BS (Physics).



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# Final Approval

This thesis titled

## Optical Properties of Passive and Active Photonic Resonators

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# Declaration

I **Ahmad Bilal** (CIIT/FA15-BPH-019/ISB) hereby declare that this project neither as a whole nor as a part there of has been copied out from any source. It is further declared that I have developed this thesis and the accompanied report entirely on the basis of my personal efforts made under the sincere guidance of my supervisors. No portion of the work presented in this report has been submitted in support of any other degree of qualification of this or any other University or Institute of learning, if found I shall stand responsible.

Date:

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# Certificate

It is certified that Ahmad Bilal (Registration No. CIIT/FA156-BPH-019/ISB) has carried out all the work related to this thesis under my supervision at the Department of Physics, COMSATS University Islamabad and the work fulfills the requirement for award of BS degree.

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## Dedication

This thesis is dedicated to my mother who brought me up all by herself, motivated me to always persue my dreams and made me the gentleman I am today.

# Abstract

Since long, electronic integrated circuits have dominated our modern technology. Now with the dawn of photonics, which is basically using integrated circuits made up using optics, its not far that our modern technology takes a new toll and slide into a new generation of digital devices. Basically, Photonics is the technology of generating and harnessing light, and other form of radiant energy whose quantum unit is a photon. These can be used in multiple applications , to explore the vastness of the Universe, cure serious and unknown diseases and even help in forensics to solve difficult crime cases.

In this project, we extended the research on optical ring resonators for such mediums in which there is gain. First we studied normally the optical properties of passive resonators and measured the effects of EIT and EIA in them (details later discussed). Then we moved over focus on active resonators varrying different parameters to acheive EIT and EIA in gain incorporating photonic resonators which have extensive amount of applications. The main focus for this project was to model the characteristics and properties of active resonators and compare it with the results of passive resonators. Due to the gain property of active resonators, similar effects can be seen here as in passive resonators but without losses involved. The main idea was to establish a photonic device that could work efficiently as passive resonators and also have more output.

Light is a beautiful thing, it makes us see things, but what if it also starts to help us organize our data, compute our equations, play our music, record our documents and basically do everything what a modern digital device, built on electronics, is capable of doing? I guess we'll find out!

*Indeed, in the creation of the heavens,*
*and the earth and the alternation of*
*the night and the day, are signs for*
*those of understanding.*

*The Nobel Quran [3:190]*

## Ackowledgement

In the name of Allah, who is the most beneficient and merciful. I would start off this extensive documentation with a quote from Carl Sagan, one of the greatest science educator, who created enough enthusiasm and curiosity in me to persue my career in Physics. He said, *"Somewhere, something incredible is waiting to be known".* This is one of the reason I chose to be a student of physics, it inspires me to search for the unknown clues that are hidden in the very fabric of reality. Physics gave mankind the power to dominate their world and use the best of nature for their benefit.

Since childhood, I had always been fascinated by computers and gadgets. Having the background of engineers in my family, I almost ended up joining the computer engineering in High School. But the curiosity inside me had made me a star gazer. So I had questions about how they get where they are, and what are they made of? These questions were those which made me switch my field to Physics which is a science of never-ending curiosity. In this process, a lot of people are included some directly and some indirectly, most of which is my family, because their never-ending support had made me persue my dreams.

I would personally like to thank my supervisor in BS project, Dr. A. Naweed, who helped me through thick and thin to complete this project and also kept me motivated enough to continue my research in field of photonics. I would like to thank my batch counselor Dr. A. H. Mujtaba, who's support and teachings made us all work harder and harder for the progression of science. Also there is a big role of Ms. Zarqa in my motivation for this project. She not only recomended me to Dr. Naweed, but she is also my mental health counselor when I am in dire need of help. I would like to thank my peers in this, because the support and love I get from them

is unmeasureable. Then again I would like to thank my family and especially my mother, who never asked me about my GPA and anything and always said, "if you love what you are studying, only then you can get true learning."

In the end, it is important to know that Knowledge is a never ending process, and Physics is such a beautiful field that every time I learn a new concept about the universe, it feels like I have been born again.

**Ahmad B. Yousafzai**
**Islamabad, Jan 2019**

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# Chapter 1

# Introduction

Since the dawn of modern technology, the integrated circuits on which today our every electronic device operates, we have progressed a lot in communication systems and developing faster and smaller computing devices. Decades have passed since electric circuits became integrated on microchips which are also called ICs, this technology has no stop but the field of optical research which generated a great amount of research progress make rise to a new form of technology on which we can operate our computing circuits is called Photonics. Now is the time that we integrate photonic crystals and photonic structures on circuits and make use of them in communication, signal processing, biochemical sensing, slow and fast light structures, optical filters, optical buffers, wavelength division-multiplexed (WDM) and on chip optical interconnects.[1] Every phenomenon mentioned here is made possible by confining light in a very small volume. Microresonators can be used to support spectrum of optical modes with required polarization frequency and field patterns. These research phenomenons will revolution the digital technology as we know today, with every hand-held device to corporate machines, all running on circuits made using photonic crystals and optical microresonators.

On a basic level, there are so far two settled components of light control and direction inside the volume of an optical microresonator. The first is the ordinary system of total internal reflection (TIR) and the presence of evanescent waves, where the directing medium must be optically denser, i.e., have a higher refractive index, than the encompassing one so as to accomplish light constrainment. The second is the photonic bandgap (PBG) found in artificial optical media having a spatial periodicity in one, two, or three measurements, named photonic crystals (PC), which is a consequence of the phenomenon of Bragg reflection causing the arrangement of frequency bands where propagation of light is restricted by the destructive interference of field harmonics inside the crystal. Exceptionally bound optical modes can be accomplished in these bands when certain deformities are presented in the generally flawlessly intermittent crystal. With PC defect modes, the light can be bound in a size similar to its wavelength ($\lambda/n$), where $\lambda$ is the vacuum wavelength and $n$ is the medium refractive index.[1]

These topics require a detailed study, which is what we are going to do in this Thesis. The scope of this thesis is not limitted to a certain and most applicable type of optical resonator which are known as Whispering Gallery resonators or (WG), but we are also going to extend this research on to different possible and quite promissing arrangements and geometeries of optical resonators known as Microring resonators. In which we mainly focus on the ring shaped resonators introducing coupling and different modes in single and composite system of resonators. This will allow us to collectively measure and observe the combined effects of such resonators by the help their optical properties and their use in optocommunicating systems. Coupling effects have been observed in detail and have made possible to observed effects like Electromagnetically Induced Transparency and Electromagnetically Induced Absorption in coupled resonator systems which are called Coupled Resonator Induced Transparency and Coupled Resonator Induced Absorption[2].

This documentation is divided into different sections, compiling the work of 1 year long BS final year project. First, we will increase the understanding of the reader of what interferometers, resonators, optical resonators, and microring resonators are, their underlying physics and the phenomenons that are followed by the regimes of these optical systems and what outcome could be acheived by using these optical systems and their applications in photonics. Then we will focus on the systems that we used in this research process and their basic physical explanations. After that, I will show the results of what I have collected by modeling these systems in different conditions (parameters). This extensive documentation will be useful for anyone trying to get started in this field of research because I have written it in a fashion that a newbie in the field of photonics can easily grasp the ideas and can learn from it.

## 1.1 Resonators

A resonator is a device that exhibits resonant behavior naturally (or artificially) on some resonant frequencies, that is, it oscillates at those frequencies with higher amplitudes than others. These frequencies are called its resonant frequencies. These oscillations can either be electromagnetic waves or mechanical waves as well. There are different uses of resonators, they can be used to filter some specific frequencies or can also be used to generate a specific frequency of the wave. A resonator in which the waves exists in hallow space is called a cavity resonator, which is used in electronics and radio signal processing, known as microwave cavities, to generate, transmit and receive electromagnetic signals. Acoustic cavity resonators, in which sound is produced by air vibrating in a cavity with one opening, are known as Helmholtz resonators.

### 1.1.1 Explaination

The term resonator is most often used for a homogeneous object in which vibrations travel as waves, at an approximately constant velocity, bouncing back and forth between the sides of the resonator. The material of the resonator, through which the waves flow, can be viewed as being made of millions of coupled moving parts (such as atoms). Therefore, they can have millions of resonant frequencies, although only a few may be used in practical resonators. The oppositely moving waves interfere with each other, and at its resonant frequencies reinforce each other to create a pattern of standing waves in the resonator. If the distance between the sides is $d$, the length of a round trip is $2d$. To cause resonance, the phase of a sinusoidal wave after a round trip must be equal to the initial phase so the waves self-reinforce. The condition for resonance in a resonator is that the round trip distance, $2d$, is equal to an integer number of wavelengths $\lambda$ of the wave:

$$2d = N\lambda, \qquad N \in \{1, 2, 3, \dots\}$$

If the velocity of a wave is $c$, the frequency is $f = c/\lambda$ so the resonant frequencies are:

$$f = \frac{Nc}{2d} \qquad N \in \{1, 2, 3, \dots\}$$

So the resonant frequencies of resonators, called normal modes, are equally spaced multiples (harmonics) of a lowest frequency called the fundamental frequency. The above analysis assumes the medium inside the resonator is homogeneous, so the waves travel at a constant speed, and that the shape of the resonator is rectilinear. If the resonator is inhomogeneous or has a nonrectilinear shape, like a circular drumhead or a cylindrical microwave cavity, the resonant frequencies may not occur at equally spaced multiples of the fundamental frequency. They are then called overtones instead of harmonics. There may be several such series of resonant frequencies in a single resonator, corresponding to different modes of vibration. [3]

## 1.2 Optical Resonators

An optical resonator, also known as optical cavity, is usually composed of two highly reflecting mirror held infront of each other parallely inside a vacum so that the system exhibits resonant behavior which allows standing wave modes to exist with almost no loss. Thus optical resonantor is a cavity with walls that are highly reflected for electromagnetic waves (i-e light).

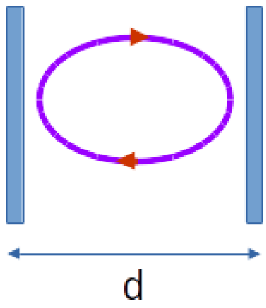


Figure 1.1: Illustration of a basic optical cavity.

## 1.3 Different Types of Optical Resonators

### 1.3.1 Fabry-Perot Resonator

A system of two mirrors held parallel to each other and both having high reflectivities shows a resonant behavior at some frequencies of incident light. If both the mirrors have high reflectance, the incident light is still observed to have pass through them without any decrease in the intensity and is detected, which occurs due to phenoemnonas similar to quantum tunneling effects.

### 1.3.2 Gires-Tournois

It is basically a lossless Fabry-Perot resonator which have a 100% reflecting rear mirror, that means it reflects 100% at all frequencies. Still, some resonant frequencies stays between the mirrors for a longer period of time and thus descript resonant behavior and lead to ultra slow group velocities. This simple device is known for storing spectral power of light which is reflected from it while modifying its phase. That is why it is sometimes referred to as a ”phase only” filter.

## 1.4 Micro Resonators

Microresonators are special type of resonators made from different type of materials which exhibits optical properties while being fabricated on a chip. These kind of resonators are actually useful in observing the effects of optical resonators on a device.

### 1.4.1 Different Geometeries

There are many type of microresonators from which microring-resonators are very useful in making photonic devices and have wide variety of application. Other kind of resonators are also useful for different kind of applications and all have distinct optical properties based on their geometery. (See figure 1.2)

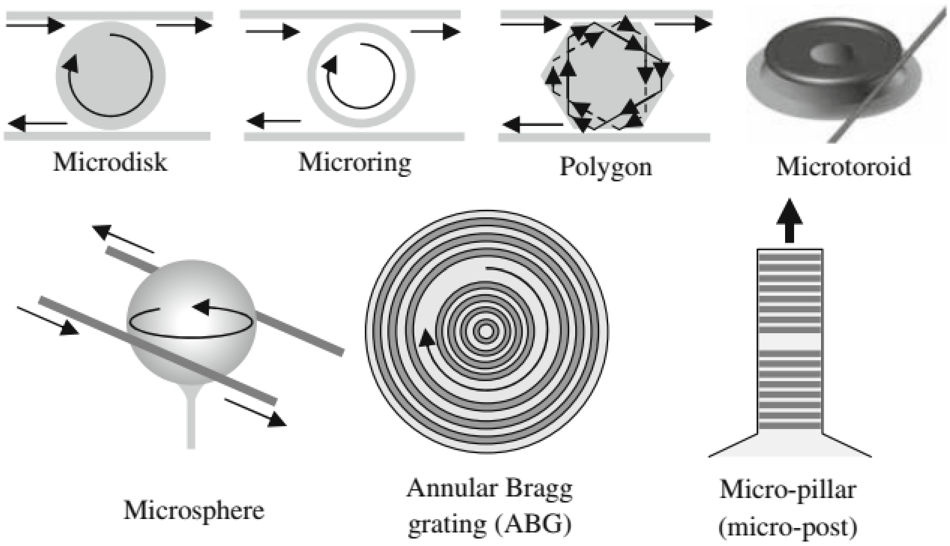


Figure 1.2: Different geometeries of microresonators.[3]

## 1.5 Electromagnetically Induced Transparency and Induced Absorption (EIT and EIA)

Electromagnetically Induced Transparency (EIT), is a coherent optical nonlinearity which makes a medium transparent to some narrow bandwidth of frequencies which were otherwise opaque to the incident radiation. This window leads to slow light at resonant frequencies in an optical resonant system usually involving coupled system. This is observed due to the destructive quantum interference effects of the incident radiation in atomic levels. [5]

Similarly, Electromagnetically Induced Absorption (EIA), is a similar phenomenon to EIT but in this nonlinearity the medium becomes highly opaque to some bandwidth of frequencies at resonance. Thus blocking off completely the resonant frequency radiation and causing a dip in the transmitted field. The quantum interference of light here is descructive and the atomic levels absorb the extra photons at such particular frequencies.

## 1.6 Aim and Objective

This thesis is a detailed study of such phenomenons dealing optical resonators. We will also deeply study the changing behavior of active and passive resonators. Active resonators are those resonators which are made from some gain medium and they also descript EIT and EIA like behavior in similar and distinct fashion. Then we will model the systems using different scientific tools and computation methods to predict their behavior in different circumstances and parameters.

## References

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[2] Induced transparency and absorption in coupled whispering gallery microresonators, PHYSICAL REVIEW A 71, 043804, published 5 April 2005

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# Chapter 2

# Fundamental Characteristics of Optical Resonators

## 2.1 The Fabry-Perot Interferometer

Optical resonators were utilized as helpful gadgets as early as 1899, when Fabry and Perot depicted the utilization of a parallel-plate resonator as a multipass interferometer. Part of the incident light on this Fabry– Perot resonator is transmitted and another part is reflected, with power divisions that rely upon numerous factors. A simple illustration of the basic Fabry-Perot is shown in Figure 2.1, here $r_1 t_1$ are the reflectivity constant and transmitivity constant of the mirror 1 respectively and $r_2 t_2$ are the reflectivity and transmitivity constants of the mirror two respectively. Also, $E_i$ is the incident Electromagnetic energy, $E_t$ is the transmitted energy and $E_r$ is the reflected energy. This is an asymmetric Fabry-Perot resonator:

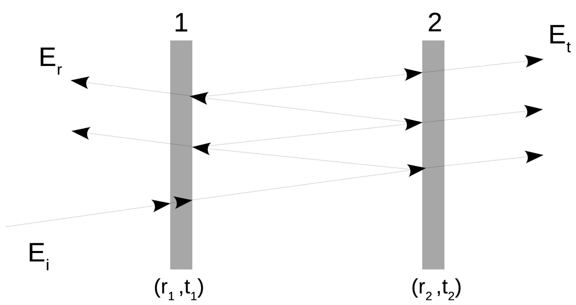


Figure 2.1: Illustrated energy diagram of a simple Fabry-Perot resonator

### 2.1.1 Theory of Fabry-Perot interferometer

If the incident energy is in the form of white coherent light then at that point the transmission and reflection coefficients depend just on the mirror reflectivities. The total reflected power comprises of the power reflected from the principal mirror in addition to all the different reflections between the mirrors that add to the reflectivity in general. In summation, the equations are[1]:

$$\mathcal{R} = R_1 + T_1^2 R_2 \sum_{m=1}^{\infty} (R_1 R_2)^{m-1} = \frac{R_1 - 2R_1R_2 + R_2}{1 - R_1R_2} \xrightarrow[R_1=R_2\equiv R]{} \frac{2R}{1+R} \tag{2.1}$$

Similarly, the transmitted energy in summation is:

$$\mathcal{T} = T_1 T_2 \sum_{m=1}^{\infty} (R_1 R_2)^{m-1} = \frac{T_1 T_2}{1 - R_1 R_2} \xrightarrow[R_1=R_2\equiv R]{} \frac{T^2}{1-R^2} = \frac{1-R}{1+R} \tag{2.2}$$

Assuming, be that as it may, the incident light comprises of a transiently lucid (monochromatic) plane wave, at that point the reflected power will be relative to the square of the reasonable total of every reflected field. Since the fields convey phase information with amplitudes added, the division of reflected and transmitted light depends not just on the mirror reflectivities, but in addition on the mirror separation and excitation wavelength. The rational

total of fields is amplified when every one of the fields interfere constructively (in phase) and limited when they interfere destructively (out of phase).

Phase gathers with propogation separation as $\phi(z) = \beta z$ and may likewise be gained upon communication with the mirrors. The sound forms of

Eqs. 2.1 and 2.2 incorporate an aggregated stage factor for each round-trip that can be translated as a standardized detuning $\phi = T_R\omega$, where $T_R$ is the cavity travel time, $T_R = n_{eff}L/c$ for the circumference, L and effective index $n_{eff}$. Presently, $\tilde{r}$ speaks to the complex reflectivity:

$$
\begin{aligned}
\tilde{r} &= r_1 - t_1^2 r_2 \exp\{(im\phi)\} \sum_{m=1}^{\infty} (r_1 r_2 \exp\{(im\phi)\})^{m-1} \\
&= \frac{r_1 - r_2 \exp\{(i\phi)\}}{1 - r_1 r_2 \exp\{(i\phi)\}} \xrightarrow[r_1 = r_2 \equiv r]{} \frac{r(1 - \exp\{(+i\phi)\})}{1 - r^2 \exp\{(+i\phi)\}}
\end{aligned} \tag{2.3}
$$

and $\tilde{t}$ represents the complex transmittivity:

$$
\begin{aligned}
\tilde{t} &= -t_1 t_2 \exp\{(im\phi/2)\} \sum_{m=1}^{\infty} (r_1 r_2 \exp\{(im\phi)\})^{m-1} \\
&= \frac{-t_1 t_2 \exp\{(im\phi/2)\}}{1 - r_1 r_2} \xrightarrow[r_1 = r_2 \equiv r]{} \frac{-(1 - r^2) \exp\{(im\phi/2)\}}{1 - r^2}
\end{aligned} \tag{2.4}
$$

The square modulus of these perplexing amounts gives the reflection $\mathcal{R}$ and transmission $\mathcal{T}$ coefficients (showin in Fig. 2.2). Antiresonant wavelengths are more emphatically reflected than in the ambiguous case, while thunderous wavelengths are transmitted 100% for adjusted reflectors ($r_1 = r_2$). For a fixed reflect dispersing, the transmission and reflection spectra in this manner show intermittent pinnacles and valleys. Figure 2.2 presenting the transmission and reflection spectra for a lossless, adjusted Fabry– Perot resonator. The part of reflected and transmitted power for mixed up excitation is identical to the separate frightfully arrived at the midpoint of reflection and transmission over a time of the spectrum range.

The values of reflectivity coefficients $r_1$, $r_2$ and the transmitivity coefficients $t_1$, $t_2$ are mentioned in the figure. The plot is of intensity of the Fabry- Perot resonator versus the round trip phase of the system. This displays a 100% transmission and 0% reflection on the resonant frequencies. Meaning all the incident light is detected on the other side of the resonantor of these specific frequencies.

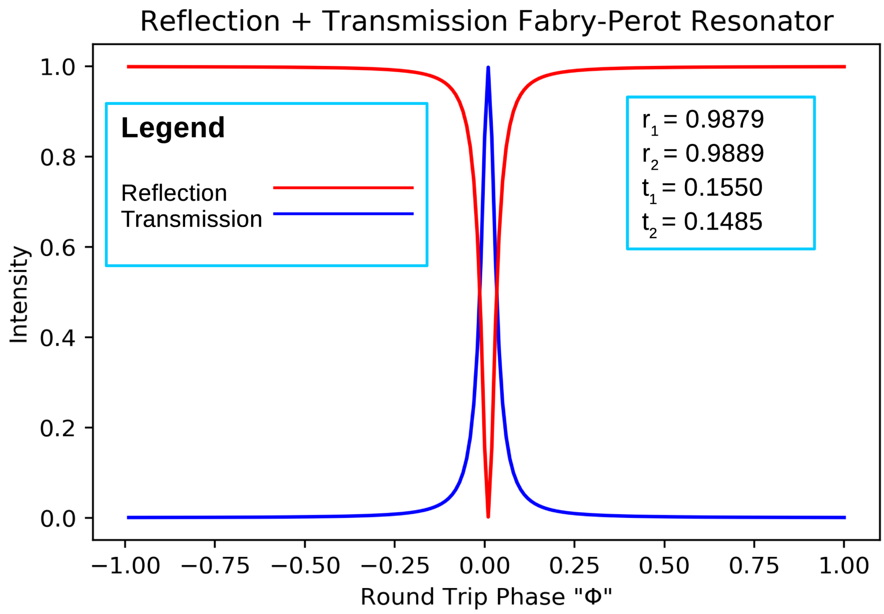


Figure 2.2: Transmitted and reflected field of an asymmetric Fabry-Perot resonator

### 2.1.2 Effective Phase

Now lets look at the phase details of the transmission and the refelction spectra of the asymmetric Fabry-Perot resonator. The phase gives us a lot of details about the travelling light inside the resonator and give other details about dispersion, group delay and group index. Fig. 2.3 shows phases of both transmission and reflection of an asymmetric Fabry-Perot resonator.

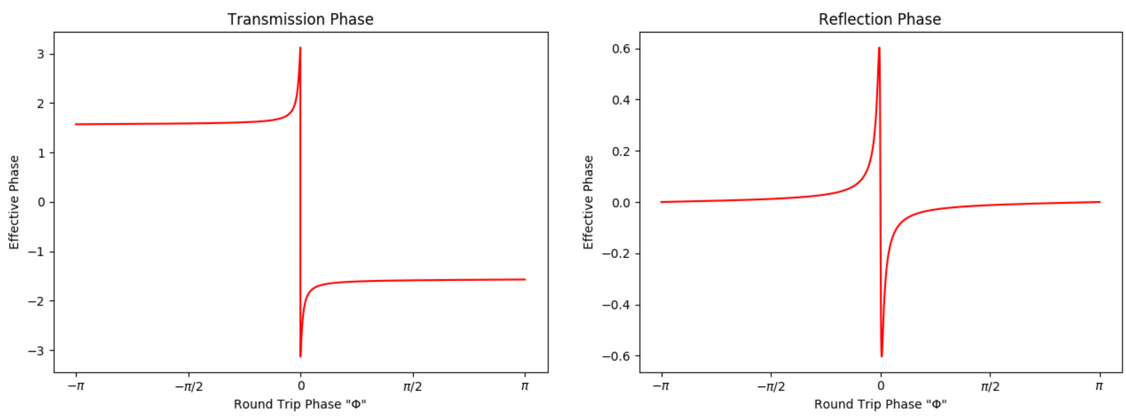


Figure 2.3: Transmission and Reflection phase vs normalized detuning of an asymmetric Fabry-Perot resonator critically coupled.

### 2.1.3 Phasor plots

Phaser plots are another useful way to study the behavior of light inside the optical cavity. The phaser plots are the complex plots between Real and Imaginary parts of the complex reflectivity and transmitivty (equation 2.3 and 2.4 respectively). Figure 2.4 shows the phaser plots of both transmitivity and reflectivity of an asymmetric Fabry-Perot resonator over the detuning period of 0 to $2\pi$ radians.

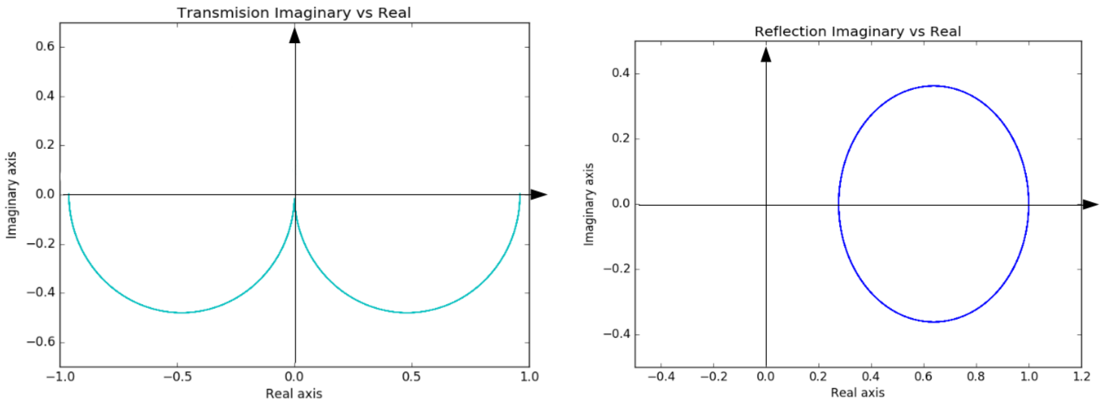


Figure 2.4: Phaser plots of complex Transmitivity and Reflectivity of an asymmetric Fabry-Perot resonator from 0 to $2\pi$

### 2.1.4 Finese, Q-factor

The resonance condition is fulfilled when the (compelling) circumference of the ring, or for the most part the round-trip length, is equivalent to a whole number numerous of the optical wavelength inside the medium. This means a progression of Lorentzian-molded transmission bends equally dispersed in recurrence by the FSR (Free Spectral Range), with the resonance linewidth portraying the capacity time of photons inside the cavity. The photon lifetime can be standardized to one optical cycle, known as the quality factor ($\mathcal{Q}$), or the cavity round-trip time, known as the cavity Finesse ($\mathcal{F}$). The most extreme reachable Q-factor is characterized as $\mathcal{Q}_{int}$, which is the intrinsic loss of the cavity. At the point when the resonator is coupled to the outer world, the Q-factor further decreases because of the loss imported by the coupler ($\mathcal{Q}_{ext}$). Thus the last quality factor $\mathcal{Q}_{load}$ is comprised of these two parts: $\mathcal{Q}_{load}^{-1} = \mathcal{Q}_{int}^{-1} + \mathcal{Q}_{ext}^{-1}$.

$$\mathcal{F}inese = \frac{FSR}{FWHM}$$

$$\mathcal{F}inese = \frac{2\pi}{2ra\cos\left(\frac{2ra}{1+a^2r^2}\right)}$$

If $ra = 1$ then,

$$\mathcal{F}inese = \frac{\pi}{1 - ra} \tag{2.5}$$

Similarly,

$$\mathcal{Q}_{factor} = \frac{\lambda_{res}}{FWHM}$$

$$\mathcal{Q}_{factor} = \frac{nLf}{\lambda}$$

$$\mathcal{Q}_{factor} = mf \tag{2.6}$$

## 2.2 Gain incorporation in Resonators

Light, when travels through a medium, loses its intensity exponentially. This law is called the $Beer'slaw$ for electromagnetic intensity. But some mediums, whose refractive index is such as they oppose the exponential decay of the light and rather increase the intensity in the propogation through the medium, are called natural gain medium. Also, there can be artificial source to activate gain in a certain system. This is done by pumping energy or external light source i-e. Lasers, to excite the atoms inside the cavity. This makes the stimulated emmission releases of the photons increase exponentially and we see increase in the incident intensity of the input light. We can use these gain mediums and build microresonators from them and observe different quantum optical phenomenons. First I will explain a bit about how gain works.

### 2.2.1 Beer's Law

The simple radiation law follows the beer's law in absorption of any kind of radiation inside a medium. This tells us that the initially intensity of the light source depends on the variables of the medium it is passing through. For electromagnetic radiation, we can write this law as,

$$I(z) = I_o \exp(-\alpha z) \tag{2.7}$$

Here, $I_o$ is the initial intensity of the radiation, $\alpha$ is the attenuation constant of the medium, $z$ is the amount of distance traveled through the medium and $I(z)$ is the intensity of light after traveling the distance $z$.

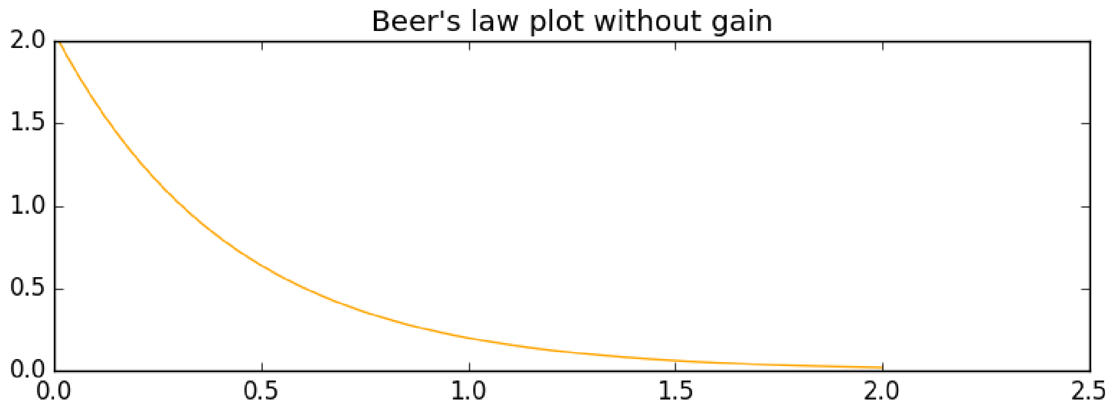


Figure 2.5: Beer's law plot with attenuation 0.01/cm: y-axis shows the intensity of light and x-axis shows the distance traveled in meters.

### 2.2.2 Beer's law study as gain

In a gain medium, the intensity of the light will not decrease but it will gradually increase. This means that the attenuation $\alpha$ is negative or we can introduce a new coefficient for such medium say $g$ such that $-\alpha \to +g$ where $g$ is some positive real number. This means that the intensity function now grows exponentially rather than decaying exponential.

$$I(z) = I_o \exp(+gz) \tag{2.8}$$

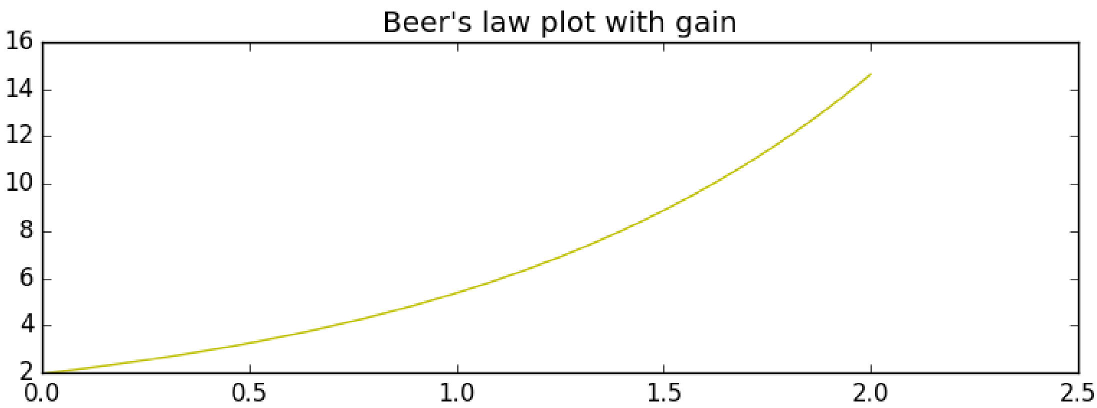


Figure 2.6: Beer's law plot with gain value 0.01/cm: y-axis shows the intensity of light and x-axis shows the distance traveled in meters.

### 2.2.3 Gain medium

The active laser medium also called gain medium or lasing medium is the source of optical increase inside a laser. The gain is the result of stimulated emmision of electronic or sub-atomic changes to a lower energy state from a higher energy state recently populated by a pump source. This gain in optical systems is usually used for amplification purposes and hence make optical amplifiers.

## 2.3 Ring Geometry Resonators

In this section, I will discuss different kinds of ring shaped resonators whose principle is pretty much similar to the Fabry-Perot resonator and are more simple to make. Basically, a ring resonator is a simple waveguide which is turned in the shape of a ring. This allows it to exhibit resonant behavior on very specific frequencies. The light is coupled inside the ring due to the phenomenon of total internal reflection and interference. This kind of behaviour is noticed in all kind of classical waves, such as sound waves, which was observed inside a large cathedral's halls, thus it was named whispering galleries. Also, these resonators can be made using different material but in this thesis, we used semi-conductor silicon as the primary material.

### 2.3.1 Evanescent Coupling

This optical system experiences passage of light through both of the rings through evanescent coupling, a classical phenomenon with quantum like properties. This evanescent coupling allow the light propagation through the both rings of the resonator making it a composite of resonant wavelength systems. This is the power transfer of the wave which is dependant on the proximity of the optical resonator and the waveguide also the length or area that has been exposed to the coupler also plays an important role i-e how much part of the waveguide/resonator overlaps; shown in Fig. 2.17.

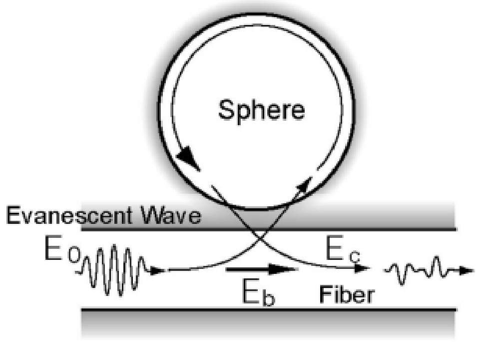


Figure 2.7: Schematic illustration of the microsphere-fiber-taper system.[2]

Coupling strength of an ideal coupler is dependant on the interaction length between the two optical modes which means power will transfer more efficiently when the two modes are mathced on the basis of their respective phases. This allow us to observe distinct behaviors as well as multiple resonances in transmittance and reflectance.

### 2.3.2 All-Pass Ringresonator

A straightforward ring resonator is made by taking one yield of a conventional directional coupler and bolstering it once again into one input. Such a device displays periodic cavity resonance (reverberation) when light navigating the ring procures a phase move relating to a number numerous of $2\pi$ radians. The resonator is numerically defined from two parts: a coupling quality and an input way. In opposition to the limitless entirety inferences performed before for the Fabry– Perot and Gires– Tournois, in which we expected steadystate task and coordinating fields and derived basic spectral properties. Although, both strategies are similarly substantial, the field-coordinating technique has the benefit of simplicity.

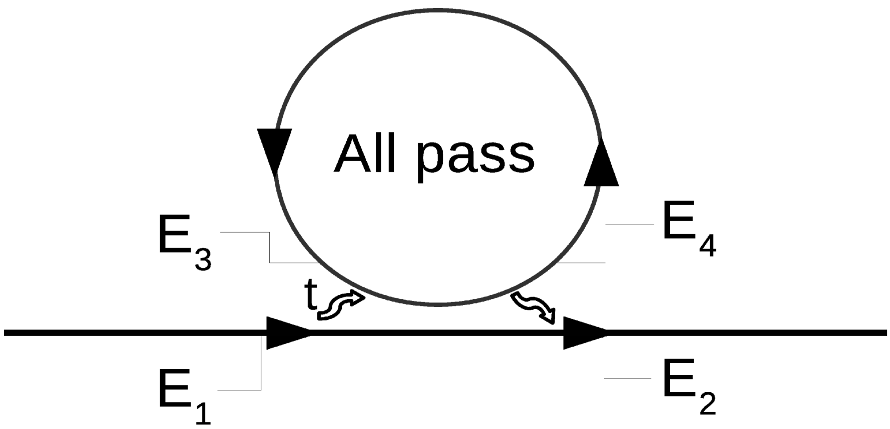


Figure 2.8: Illustrated fields of an all pass resonator

**Transmission and Reflection**

Let us now look at some reflection and transmission spectra of a passive All-pass ring resonator. Fig. 2.6 shows that the transmission and reflection peaks are flipped as in case of an symmetric Fabry-Perot resonator.

**Transmission and Reflection with gain**

Now we introduce gain into the system and observe that the tranmission dip also shifts into a peak and go way above the 1 mark meaning that it is greater than the initial intensity and the reflection peak is also above 1 mark meaning a lot of incident light is being reflected. We will study the transmission of some other different geometries of ring resonators with gain.

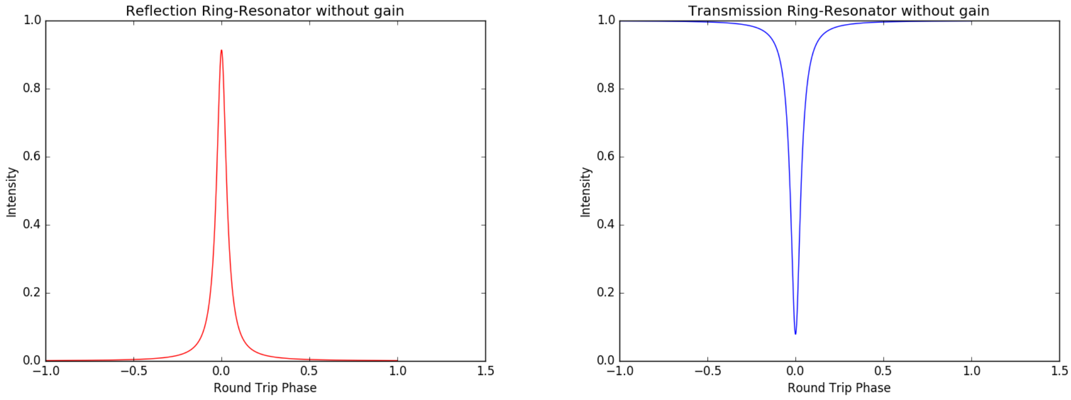


Figure 2.9: Reflection and Transmission spectra of a passive All-pass ring resonator

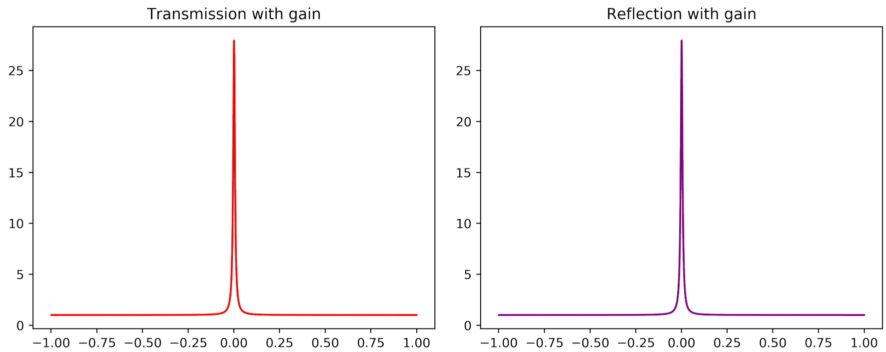


Figure 2.10: Gain introduced into an all-pass resonator: we see clear difference in the intensities.

#### Effective Phase

The phase of the All-pass ring resonator is shown in Figure 2.7. We can easily observe from this that with changing the values of the coupling $r$, the shapre of the graph changes as that of a function of $ArcTan\phi$. The relation for phase is given by,

$$\Phi_{eff} = \pi + \phi + 2\tan^{-1}\frac{r\sin\phi}{1 - r\cos\phi} \tag{2.9}$$

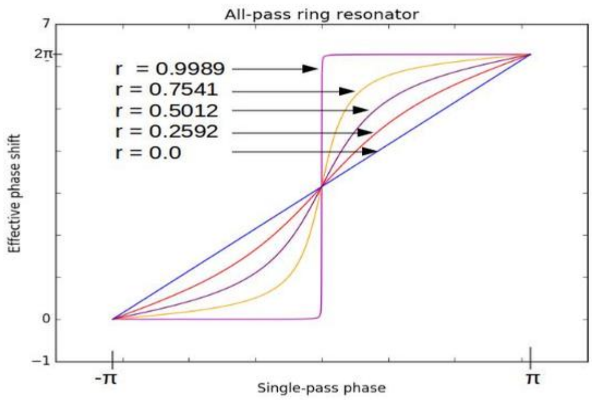


Figure 2.11: Phase diagram of an All-Pass ring resonator from 0 to $\pi$ where $r$ is the coupling parameter.

**Phasor Plots**

Now looking into some complex refractivity and transmitivity of an All-pass ringresonator (Fig. 2.7). These plots are plotted over the complex plain from the detuning limits of 0 to $2\pi$. The transmission loop does not go to negative real axis and touches the origin but the reflection curve does not even form a loop.

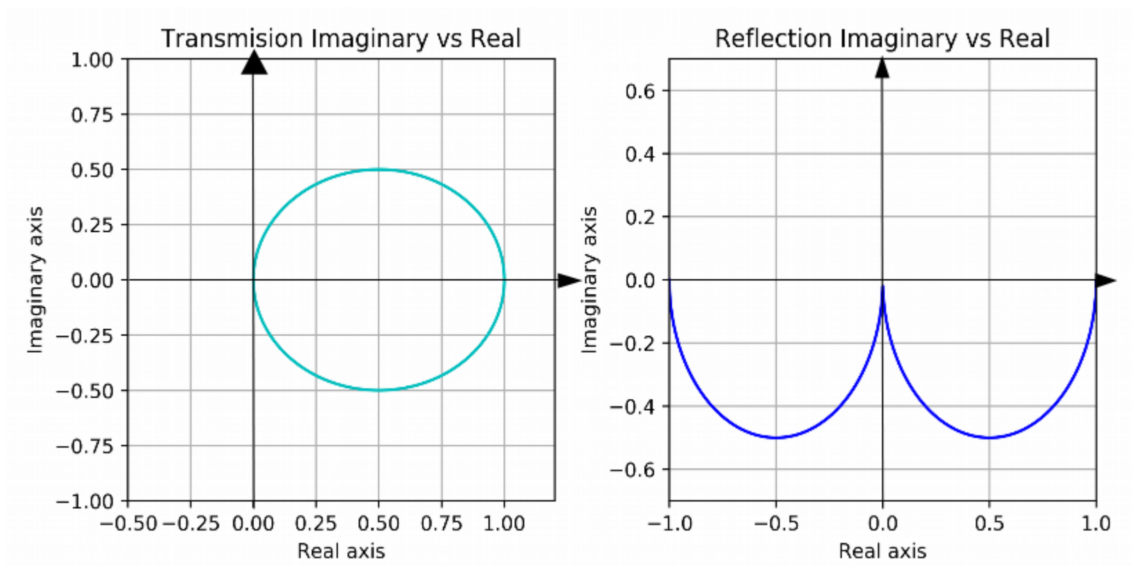


Figure 2.12: Phaser plots of complex Transmitivity and Reflectivity of an All-pass ring resonator

### 2.3.3 Add-Drop Ringresonator

The immediate waveguide similarity of a free-space Fabry– Perot is gotten by including a second guide that side-couples to the resonator as in Fig. 1.4. Since this setup acts as a tight band abundancy channel that can include or drop a recurrence band from an approaching sign, it is regularly named as an add– drop filter. Fig. 2.8 shows the basic geometry of the add-drop ring resonator with its associated fields labeled accordingly. This resonator has an input, through and drop interfaces where $t_1$ is add and $t_2$ is drop coefficients.Input field is labeled as $E_1$ while the through field is labeled as $E_2$. The drop field is on the left top corner lableled as $E_5$. The ratio of these fields to the incident/input field defines the total transmitivity and total reflectivity of the filter.

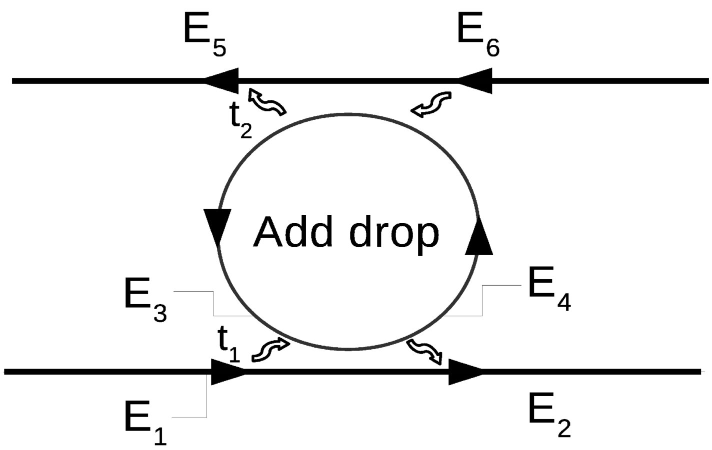


Figure 2.13: Illustrated fields of an add drop resonator

#### Transmission and Reflection

Let us now look at some reflection and transmission spectra of a passive Add- drop filter. Fig. 2.10 shows that the transmission and reflection peaks are flipped as in case of an asymmetric Fabry-Perot resonator and the transmission phase is a direct function of the detuning. Reflection phase is also shown in Fig. 2.11.

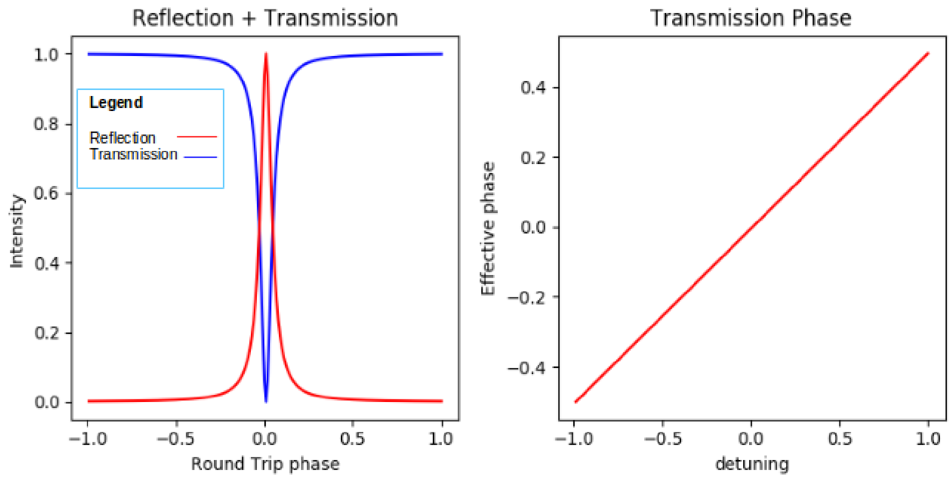


Figure 2.14: Reflection and Transmission spectra along with transmission phase

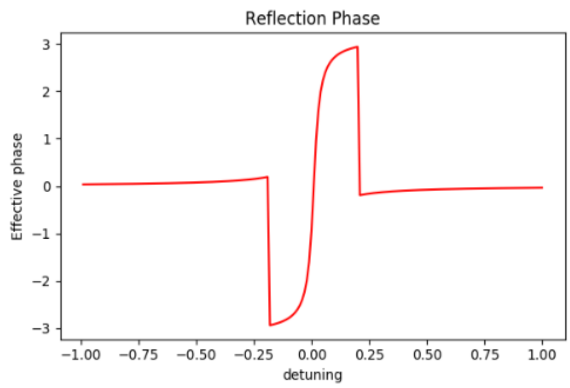


Figure 2.15: Reflection phase of the all-pass ring resonator.

### Transmission and Reflection with gain

Now we introduce gain into the system and observe that the tranmission dip also shifts into a peak which above the 1 mark meaning that it is greater than the initial intensity and the reflection peak is almost near zero meaning most of the incident light is being transmitted. We will study the transmission of some other different geometries of ring resonators with gain.

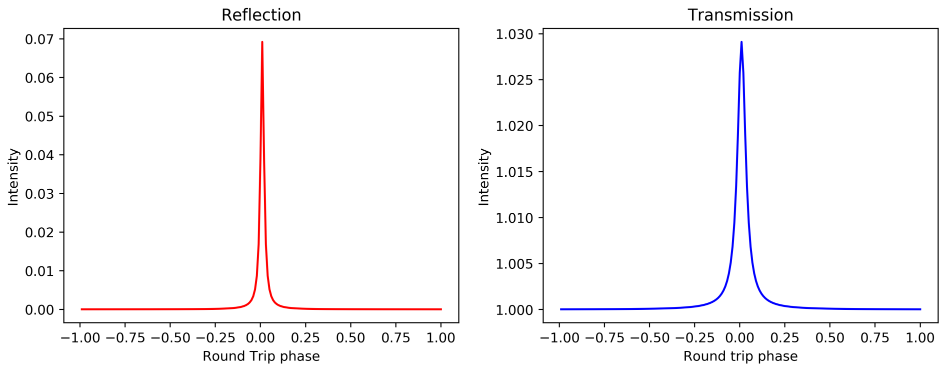


Figure 2.16: Gain introduced into an all-pass resonator: we see clear difference in the intensities.

### Phasor Plots

Now let us see how complex plots of Add drop is different from the All-pass resonator. Fig. 2.11 shows that the loop goes towards the negative real axis as the phase is increased. This tells a lot about the distinct behavior.

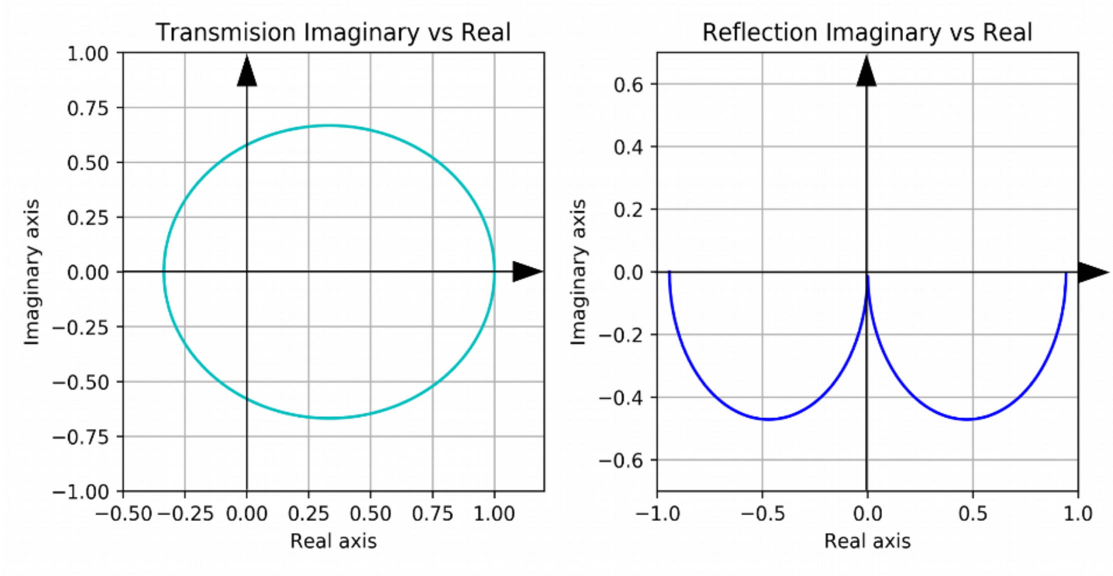


Figure 2.17: Phaser plots of complex Transmitivity and Reflectivity of an All-pass ring resonator from 0 to $2\pi$

## 2.4 Coupled Ring Resonator

Now we turn another optical waveguide into a ring shape and install it on the top of the all-pass ring resonator such that now we have dual ring geometry and a wave guide coupler. This geometry does allow resonant behaviors and the spectra varies largely from an all-pass resonator. In this arrangement, coupling between the two resonators (rings) also play an important role in the spectra of the light that passes through the resonator. Fig. 2.16 displays the basic geometry of the couple ring system we are going to discuss along with their energies.

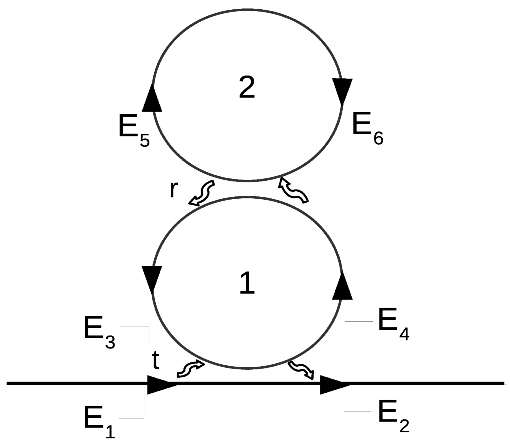


Figure 2.18: Illustrated fields and geometry of a coupled ring resonator

### 2.4.1 Coupled resonator induced transparency and induced absorption

Coupled resonators, like the one above, also shows electromagnetically induced absorption and induced transparency known as CRIT and CRIA [3,4]. These kind of effects are common in atomic systems but we have observed these effects in a ring resonator system which we will discuss in detail in coming chapters.

## References

[1] Optical Microresonators, Theory, Fabrication, and Applications, John Heebner, DOI: 10.1007/978-0-387-73068-4

[2] Dynamics of fast and slow pulse propagation through a microsphere–optical-fiber system, PHYSICAL REVIEW E 75, 016610, (2007)

[3] Coupled-resonator-induced transparency, PHYSICAL REVIEW A 69, 063804 (2004)

[4] Induced transparency and absorption in coupled whispering-gallery microresonators, PHYSICAL REVIEW A 71, 043804 (2005)

# Chapter 3

# Coupled Resonator Induced Transparecy and Absorption

## 3.1 Electromagnetically Induced Transparency

Electromagnetically Induced Tranparency is a well known phenomenon in atomic physics but its all-optical analogue has generated a lot of interest in this beautiful natural phenomenon. Basically, EIT is a transparency window in transmission and absorption spectrum. This transparency window is the result of fano interference amoung different transition pathways. There is another similar concept which is known as Autler-Townes Splitting ATS, which also shows a transparency window but it is the result of strong field-driven interactions which causes the energy levels to split.

EIT also enables us to hold control over the optical response of the medium. Basically, EIT is the result of having a strong connection between the light and the matter. Amplitudes of different pathways interfere due to quantum interference effects. These can be used in applications such as all-optical switching, slow light, optical sensing, light storage and quantum information processing.

In photonics, EIT is said to be observed in plasmonic structures, photonic crystals, whispering gallery mode micro cavities and coupled ring micro resonators. These devices can be summed up under one name, photonic devices and by seeing such effects we can say that we can get control of how information and energy travel through our device.

### 3.1.1 EIT in Atoms

For EIT to happen classically, one may assume that all the oscillating atoms in the medium have came to a hault just to neutralize the incoming field effect and thus these electrons does not contribute in the dielectric of the material. But atoms are small and must be treated quantum mechanically, in which we deal with probability amplitudes and expected value of electron's position. For EIT to occur, we must have a three-level atomic system which we will discuss below.

### 3.1.2 Three level Atoms

In a three level system, what really happens quantum mechanically, without disrupting the escence of classical phenomenons, The probability amplitudes of level $|3\rangle$ is driven by two terms in the system. One is being the probability amplitude of the ground state $|1\rangle$ and the other is the oppositely phased and is the probability amplitude of the state $|2\rangle$. These both driving forces are opposite in signs but equal in magnitudes and have a frequecy $\omega_p$ and are so balanced that probabilty amplitude of state $|3\rangle$ and the expected value of the amplitude of the sinosoidal motion at every frequency that has been applied is zero.

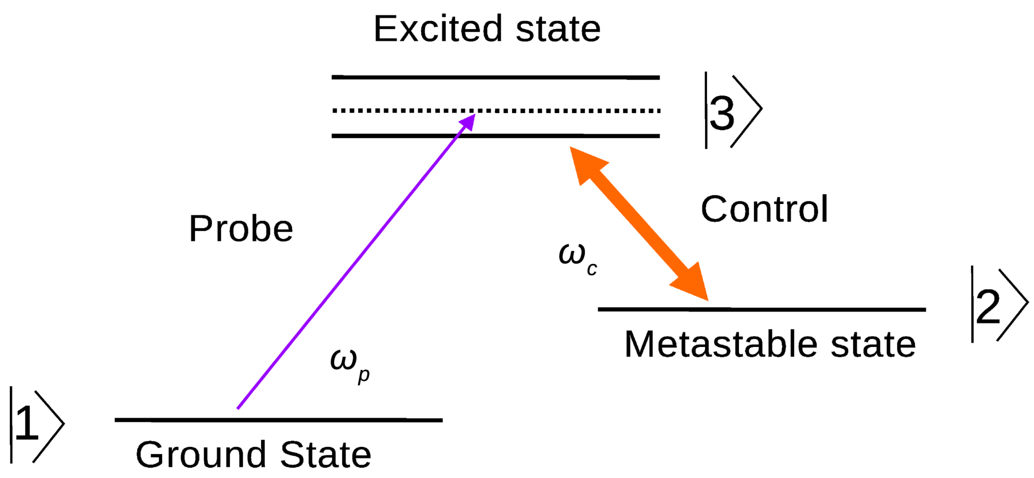


Figure 3.1: A three-level system where level 3 splits due to the much stronger field of control laser.

One may ask how that opposite phase for transition from the coherent states $|1\rangle \rightarrow |2\rangle$ along with the applied field $\omega_c$, makes absolute cancellation? Because, we use the laser pulses that generates fast enough laser photons that the phase of transitions is maintain and is the correct phase for cancellation.

## 3.2 Coupled Resonator Induced Transparency (CRIT)

We can observe EIT in coupled resonator systems as well as in other optical systems like whispering gallery resonators but the scope of this thesis is limited to ring resonators systems only. This kind of geometry (that we discussed in section 2.4) has been promising since a long time in the field of photonics. EIT can be observed in this system by mostly the explaination of classical wave travel and quantum fluctuations. The traveling photon is coupled inside the first ring through evanescent wave and travels inside the ring and acquires a phase shift equal to the round trip inside the optical cavity. When the light source and the phase shifted intracavity field matches so as that the constructive interference is amplified i-e their phases matches perfectly, then at those frequency there is a transparency window in the absorption spectrum i.e a narrow dip, or we see a sharp peak in the transmission spectrum. [2]

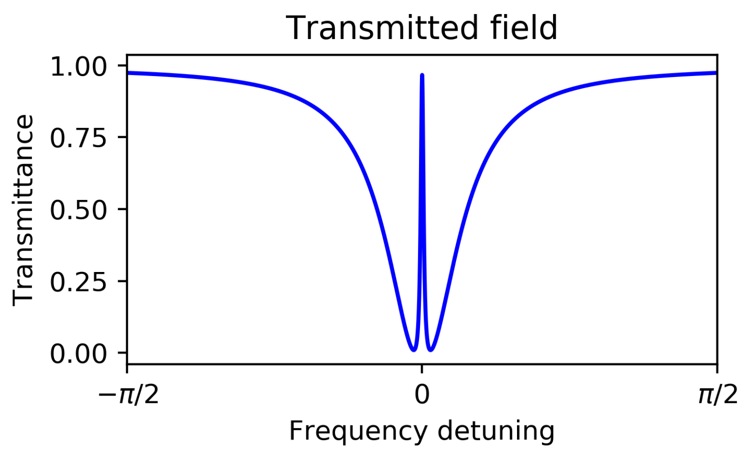


Figure 3.2: Electromagnetically Induced Transparency observed in a 2 ring resonator system.

### Transmittance

Figure 3.2 displays the plot of transmitted intensity vs frequency detuning in a coupled resonator system as shown in fig. 2.16. The parameters used here are couplings $r_1 = 0.9$ and $r_2 = 0.999$ and attenuations $a_1 = 0.88$ and $a_2 = 0.9999$ for ring 1 and 2 respectively. Reproduced from the original work on *Coupled resonator induced transparency* [2] from 2004.

### Effective Phase

Now let us look at the phase response of such coupled resonator system. Figure 3.3 shows effective phase of the system in red and Figure 3.4 shows the coupling phase which is the phase between the two coupled rings, in yellow.

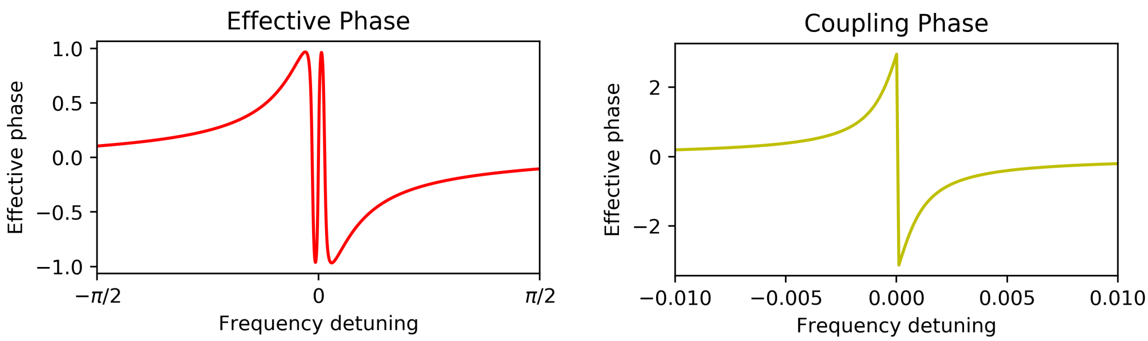


Figure 3.3: Effective phase of the system in red and coupling phase shown in yellow vs frequency detuning.

### Effective Phase derivative

Figure 3.4 shows the derivative of the phase of the system which gives us great information about the group index and group velocity of the system.

This value is directly related to the group index of the system. From the graph, we can see that there are negative values for off resonances and positive values on resonances. Which tells us that we have superluminal light off resonance and subluminal on resonance.

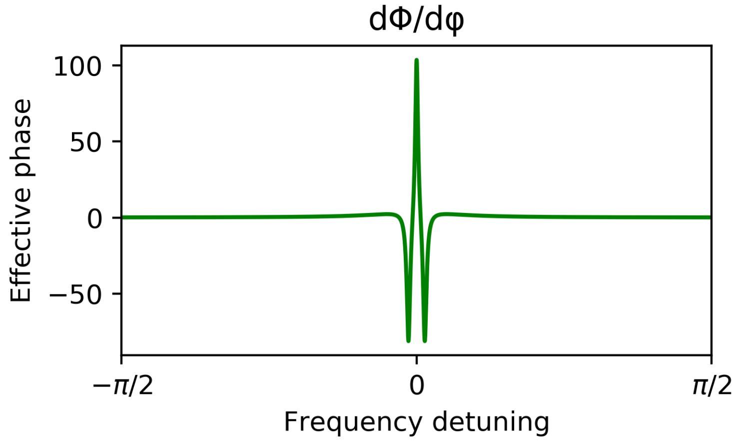


Figure 3.4: Derivative of the phase of the system vs frequency detuning.

### 3.2.1 CRIT with gain

As before, now we are going to observe what changes does the system has when we introduce gain in it. This can be introduced by pumping some monochromatic light source or a laser, in either one of the rings which will drastically incompensate the losses inside the resonator and will increase the overall output transmission of the system even above the incident light source.

### 3.2.2 Results

We observe EIT in a coupled two resonator system, the transmission and effective phase of the system is shown displaying normal dispersion meaning slow light in the system.

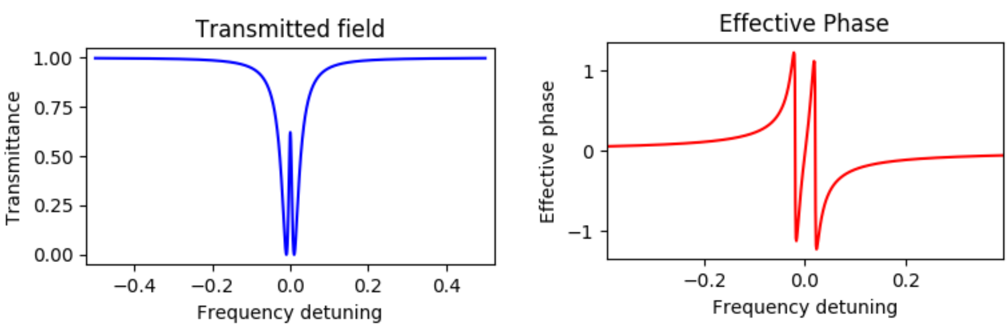


Figure 3.5: Coupled Resonator Induced Transparency with its effective phase in a passive resonator system.

### Introducing gain in resonator 2

Now we will activate gain in the resonator 2, which has high Quality-factor (shown in red in fig. 3.6). The transmission peak of the EIT starts to rise up gradually as $g$ , the gain coefficient, is increased. The peak rises towards 1 mark up till $g \rightarrow \alpha$, where alpha is the attenuation constant.

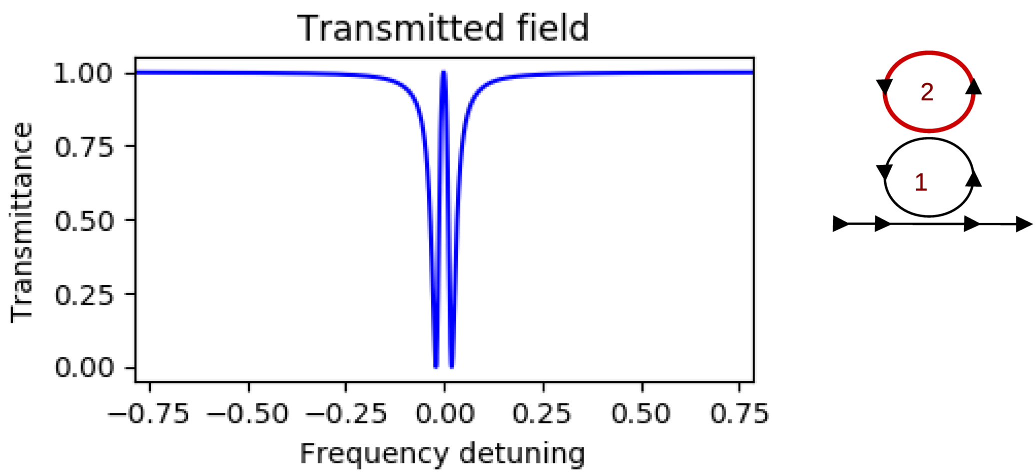


Figure 3.6: Coupled Resonator Induced Transparency with in an active resonator system.

When $g = \alpha$ , the peak of the transmission touches the 1 mark on the graph meaning now all of the incident intensity is being detected on the other side i.e it has become completely transparent. This means that we have now compensated for all the losses inside the system which can be intrinsic, coupling or bending losses. When this happens, we see an abrupt change in thae effective phase of the system. This system now gives us anamolous dispersion on resonance meaning we get fast light in EIT.

To get a brief idea of what is really happenning, I have also calculated the group index of the system. In this case, we receive negative values for the group index $n_g$ on resonance. Fig. 3.8 displays the group index for this particular case.

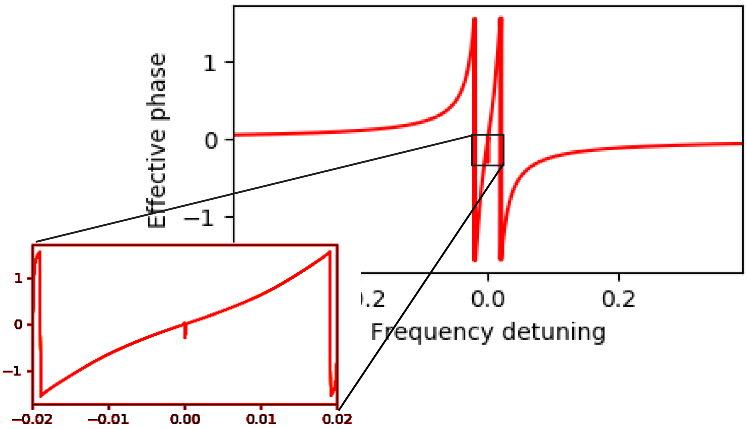


Figure 3.7: Effective phase of Coupled Resonator Induced Transparency in an active resonator system.

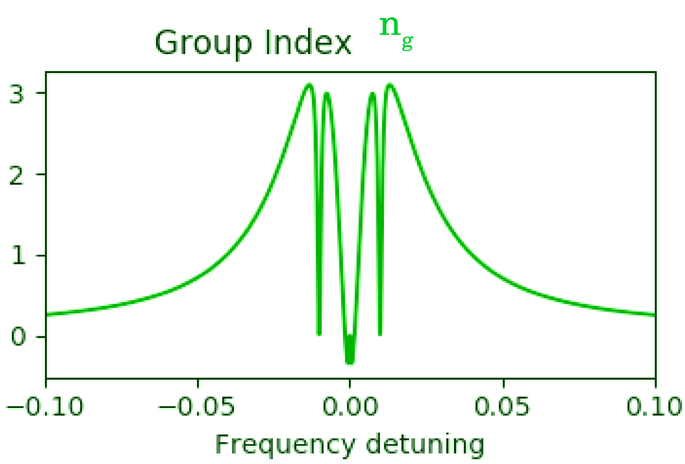


Figure 3.8: Group index of Coupled Resonator Induced Transparency in an active resonator system showing negative on resonant frequencies.

**Introducing gain in resonator 1**

Now we will activate gain in resonator 1 (shown in red), which has lower Quality-factor in comparison to the resonator 2. The spectrum of the EIT starts to rise up and displays a hanging EIT closer to the 1 mark on the graph. (Figure 3.9)

The effective phase of the system remains the same as of a passive resonator, displayin normal dispersion meaning slow light. As we can also see from the group index. The index value on resonance calculated to be about $\approx 76.2$ which displays a very less reduction in the speed of light.

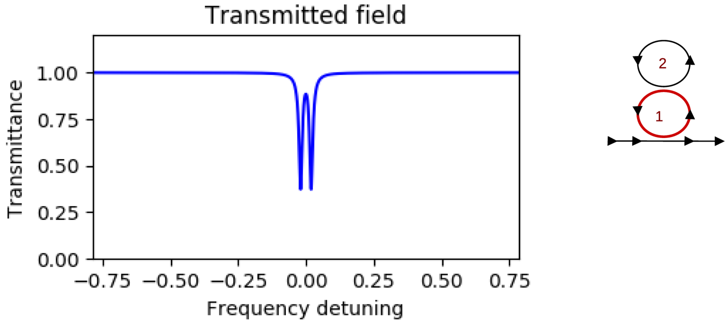


Figure 3.9: CRIT of the 2 resonator system with gain activated in resonantor 1.

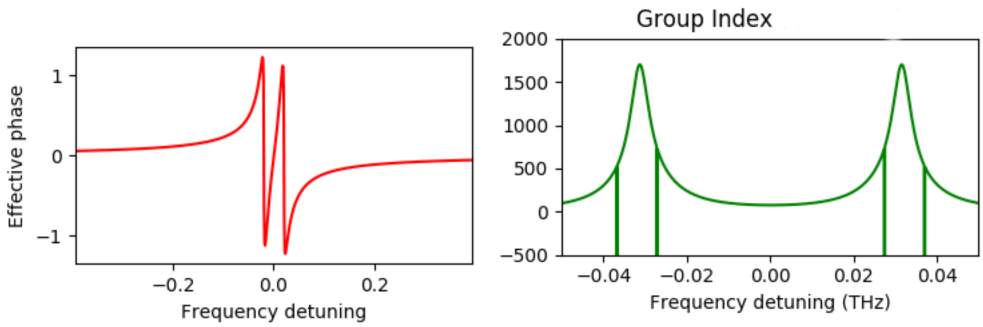


Figure 3.10: Effective phase shows normal dispersion (in red) and group index $n_g$ shown in green.

**Introducing gain in both resonators**

Now we will activate gain in both of the resonators simultaneously such that the ratio of the gain coefficients, $g_1$ and $g_2$, are the same. We observe that the peak of the EIT as well as the whole transmission starts to rise up towards the 1 mark as $g \to \alpha$ Fig. 3.11. The EIT transmission window also narrows down gradually.

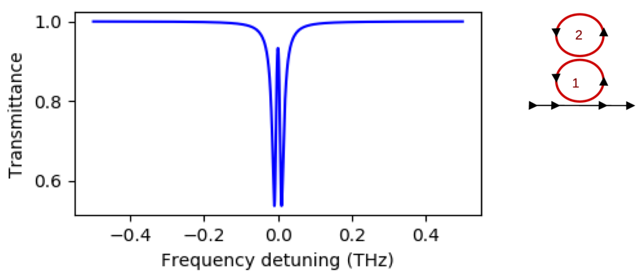


Figure 3.11: Transmission graph of two resonator system with gain activated in both (shown in red).

The effective phase of the system shows a rather distinct curves which are basically due to the artifacts in the system of computation errors. The on resonance information tells us that we have normal dispersion and positive group index about $\approx 766.5$ Fig. 3.12.

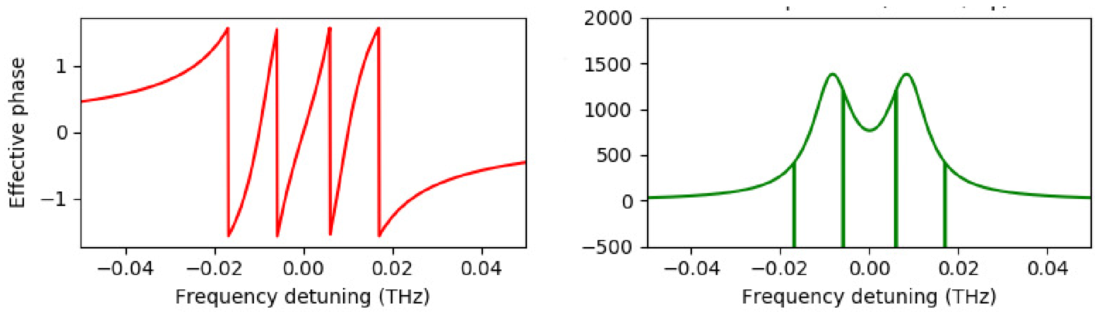


Figure 3.12: Phase and Group index of a resonator system with gain in both resonators.

When the gain coefficient $g$ becomes greater than $\alpha$, then the whole transmission graph flips about the x-axis and we now see a distinct spectrum with an reversed EIT shown in figure 3.11.

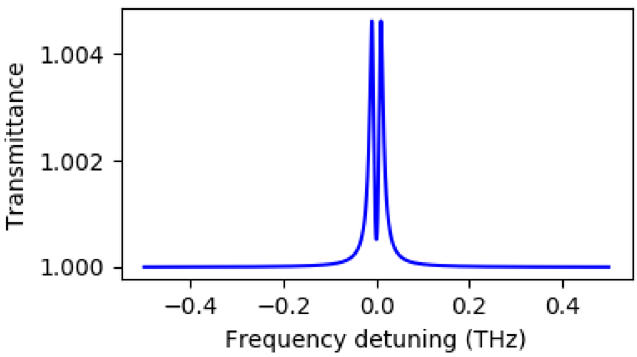


Figure 3.13: Flipping of the EIT spectrum when gain coefficient is bigger than the attenuation coefficient.

This flipping of the transmission does not affect the dispersion of the medium which means the effective phase and the group index of the system remains the same in this case as well. However, we now have a transmission peak which is above the 1 mark on the graph meaning we have compensated for all the losses in the system and also we have generated extra light.

## 3.3 Electromagnetically Induced Absorption

A similar phenomenon in which quantum interferences causes a narrow dip in the transmission spectrum of the system is known as Electromagnetically Induced Transparency (EIA). EIA is believed to occur when the probability amplitudes of the exciting electrons from a three level system, one coming from the groud state due to the probe laser and the other coming from the metastable state due to control laser of high intensity, interferes constructively thus enhancing the absorption in the resonants frequencies and no light is transmitted in that narrow bandwidth of the spectrum. As a whole, EIA is not a very well understood phenomenon and its classical explaination lacks the true esscence.

## 3.4 Coupled Resonator Induced Absorption

Coupled resonator system as discussed above also displays electromagnetically induced absorption. The optical analogue of EIA is known as CRIA. CRIA can give us both fast light and slow light, most of the light in the resonant bandwidth is absorbed thus its applications are limited.

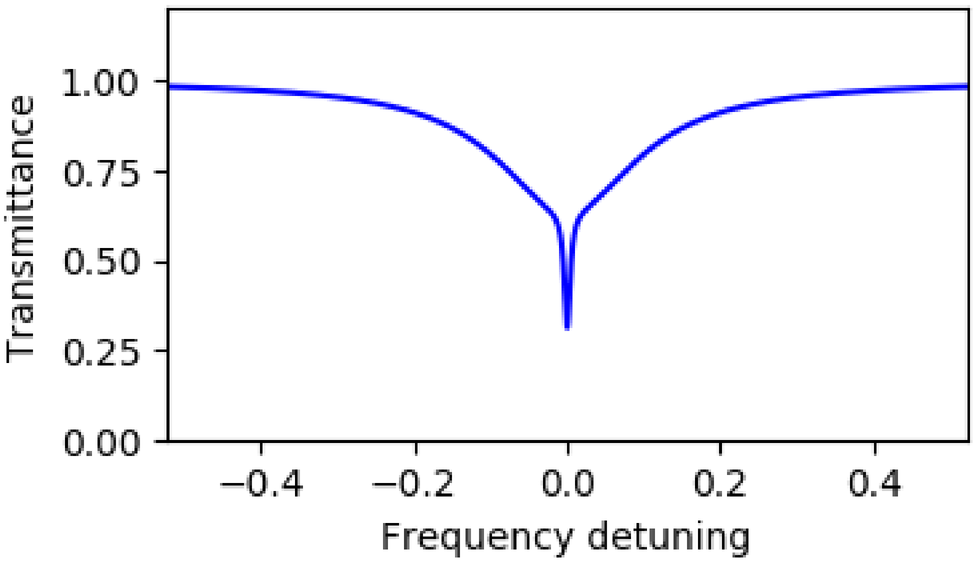


Figure 3.14: Coupled Resonator Induced Absorption in a coupled resonator system.

## 3.5 CRIA with gain

Now we will observe the behavior of the transmission spectrum of CRIA when we goin to introduce gain in it. Similarly, first we are going to activate gain in resonator 2, then into the resonator 1 and in last we activate and increase gain in both resonators simultaneously.

### 3.5.1 CRIA with slow light

We wil first see the response of the CRIA (fig. 3.14) in active medium with having normal dispersion. shown in fig. 15, with its group index.

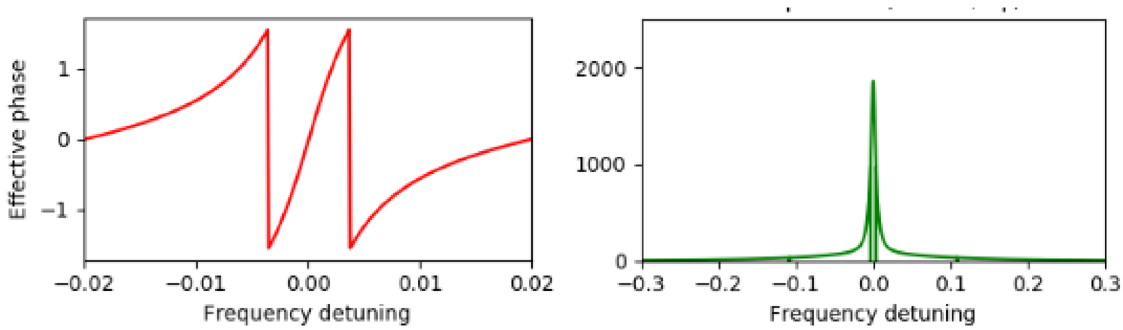


Figure 3.15: Phase and group index of CRIA.

#### Introducing gain in resonator 2

Now we activate gain in the second resonator such that $g \leqslant \alpha$ . When the gain is closer to the value of $\alpha$ ( $g \to \alpha$) makes the EIA change into EIT type transmission.

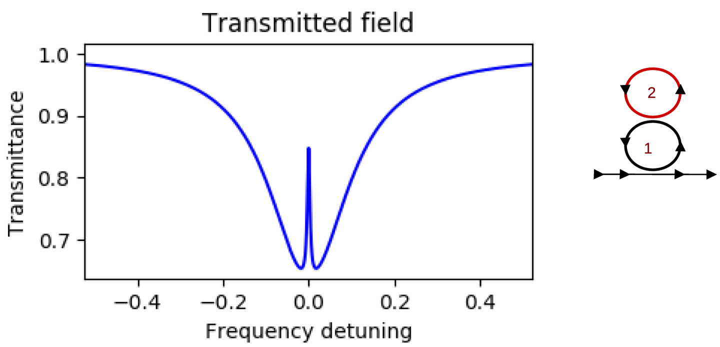


Figure 3.16: EIA dip changes into an EIT type transmission.

The effective phase of the transmission changes from normal dispersion to anamolous dispersion. The group index displays negative value of $n_g \approx -4505$, meaning negative group delay and superluminal light.

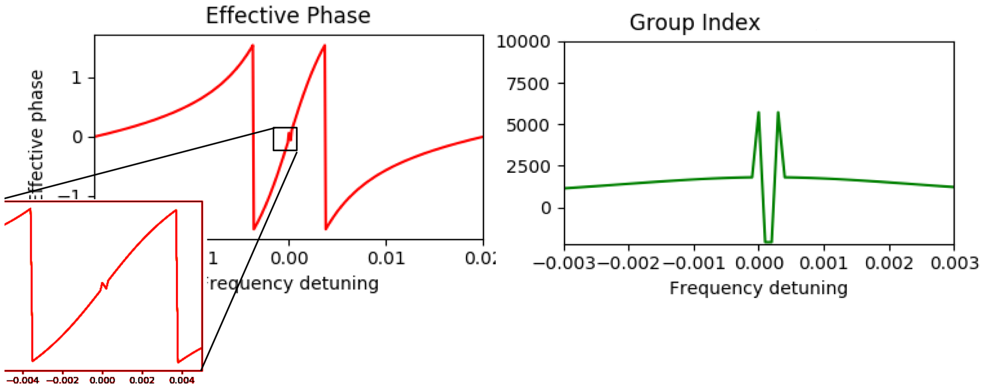


Figure 3.17: Phase of the system shown in red and group index in green.

**Introducing gain in resonator 1**

Now we activate gain in the first resonator shown in red and we see that the EIA resonance narrows down and becomes a sharp transmission dip.

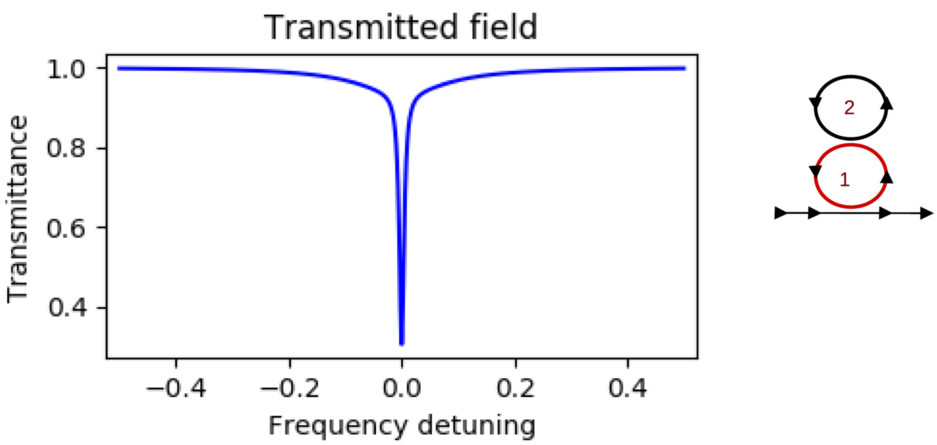


Figure 3.18: CRIA with gain activated in resonator 1.

Phase and group index gives us slow light and normal dispersion.

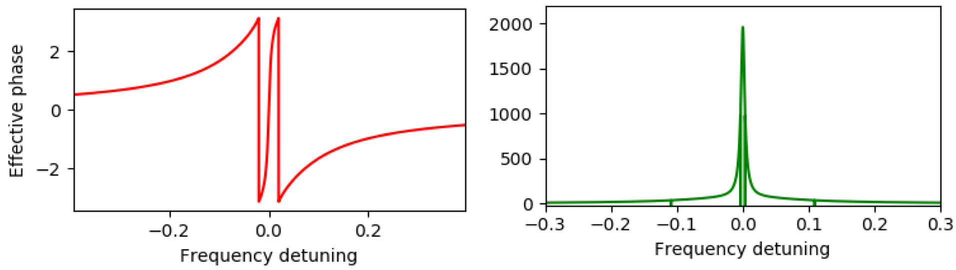


Figure 3.19: CRIA phase and group index.

**Introducing gain in both resonators**

Now we will activate gain in both of the resonators simultanously. We see no clear difference in the transmission spectrum when $g_1 and g_2 < \alpha$.

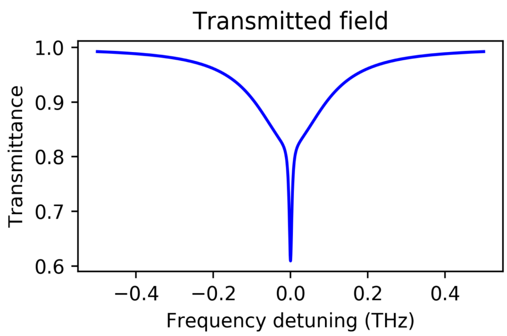


Figure 3.20: CRIA with gain in both resonators.

The phase and group index is also shown

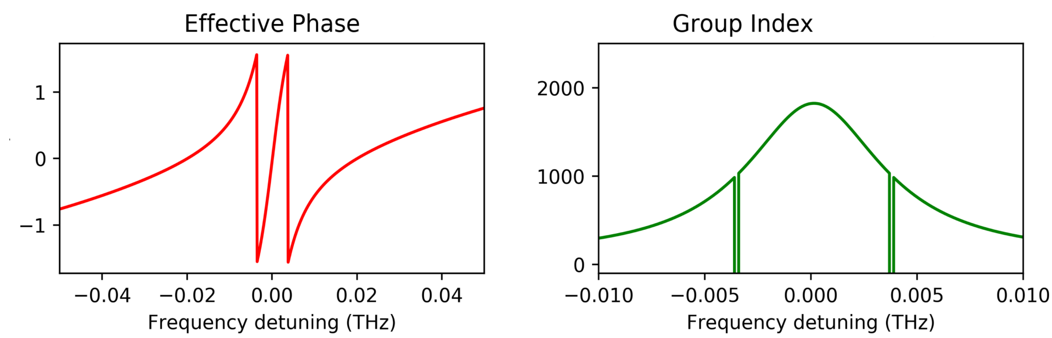


Figure 3.21: CRIA phase in red and group index in green.

Again when $g \rightarrow \alpha$, the transmission spectrum values are very near to 1 now and we see anamolous dispersion in the effective phase of the system and negative group index of about $-4550$.

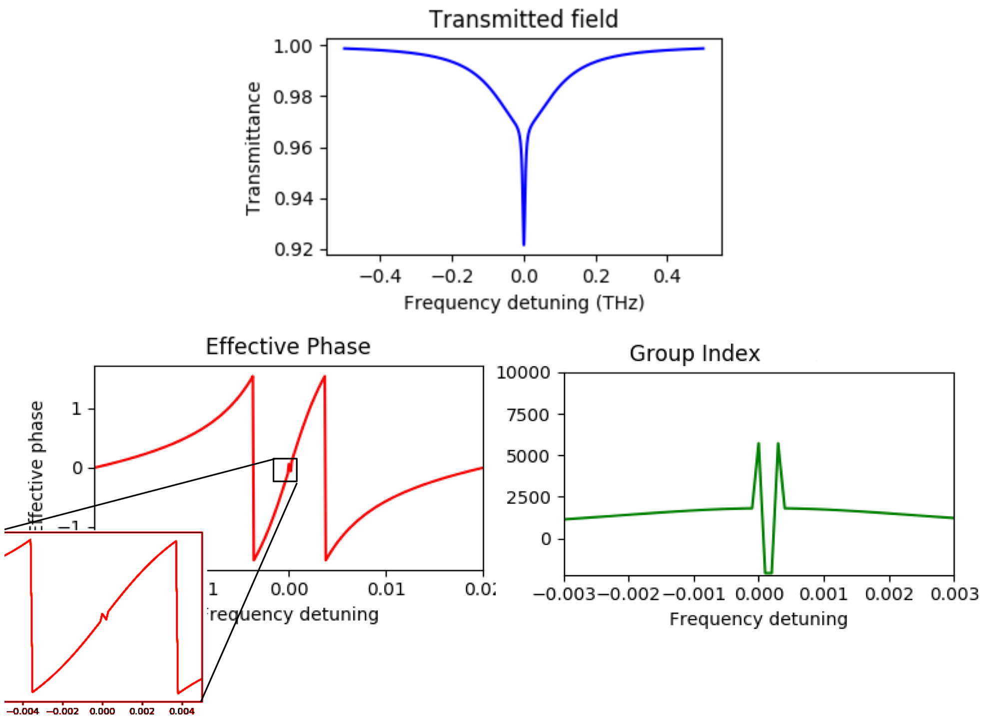


Figure 3.22: Phase of the system shown in red and group index in green.

Further increasing the gain we observe the spectrum flips over the horizontal axis and we see that our EIA has now become an EIT like transmission. The dispersion remains anamolous uptil few values of $g > \alpha$, but after significant introduction of gain in the system, we again see a transition from fast light to slow light and normal dispersion.

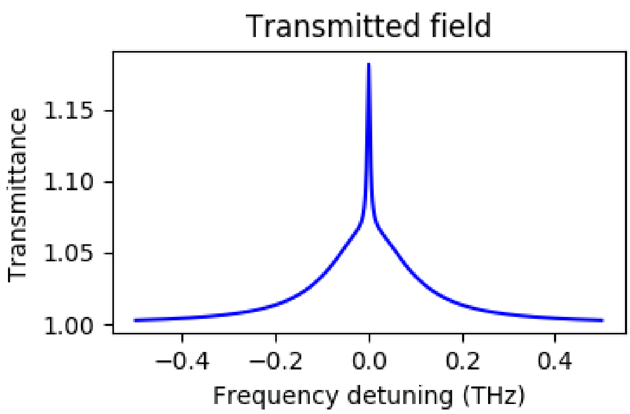


Figure 3.23: Transmission of the system

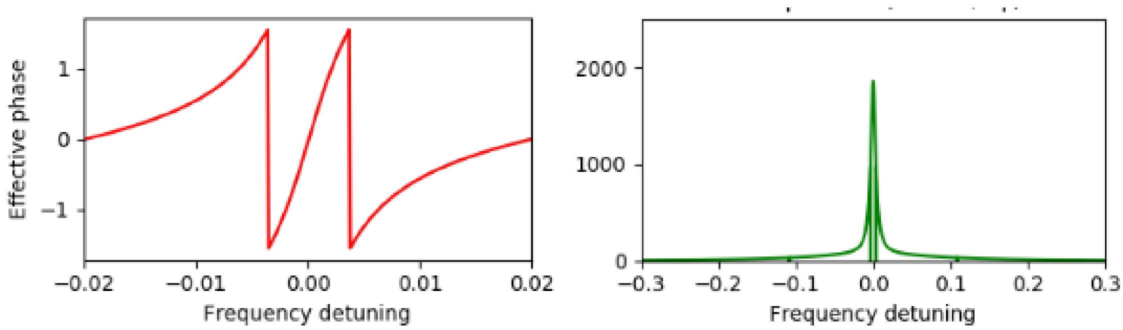


Figure 3.24: Phase and group index of the system.

### 3.5.2 CRIA with fast light

Now we will study CRIA which shows fast light with a passive resonator system and study the behavior when gain is introduced into it simultanously.

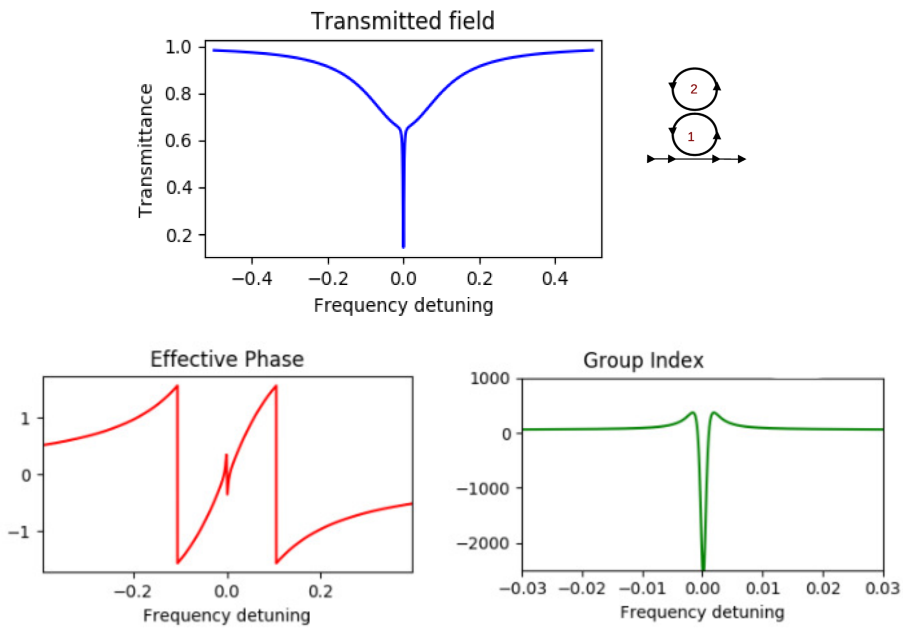


Figure 3.25: CRIA observed in a passive two resonator system.

#### Introducing gain in resonator 2

Now we activate gain in the second resonator shown in red and we see that the EIA resonance narrows down and becomes a sharp. Also the dispersion of the system changes as $g \approx \alpha$ and we see normal dispersion and a positive group index.

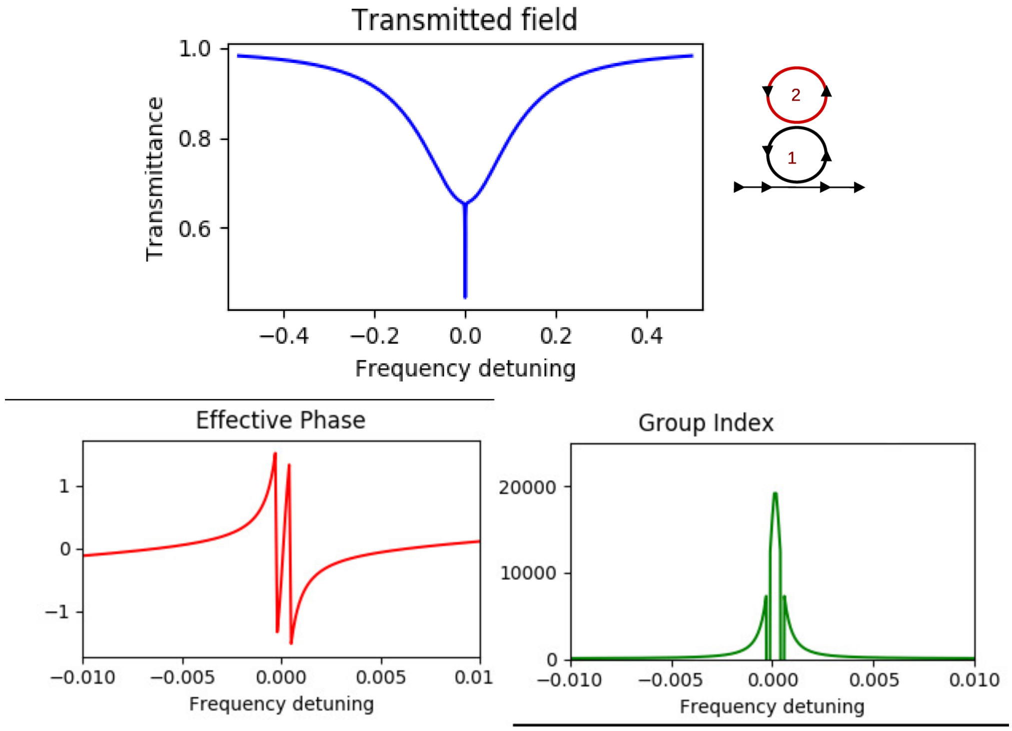


Figure 3.26: Transition from fast to slow light in CRIA.

When gain of the system is higher than the losses, such that $g > \alpha$, then we see a change in transmission that the EIA dip has changed into an EIT peak with normal dispersion and positive group index.

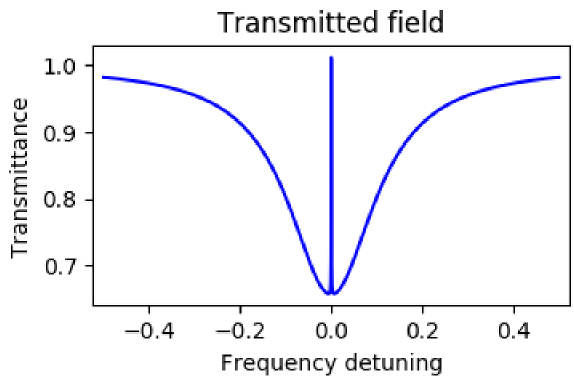


Figure 3.27: Transmission dip transforming into an tranmission peak.

However, the effective phase and the group index remains the same even for this tranmission, but now we have more output light and less of our input signal is absorbed thus increasing its essence.

### Introducing gain in resonator 1

Now we activate gain in the first resonator. We see that the EIA resonance narrows down and becomes a sharp dip. Also we see two off resnonces starts to appear. As $g \approx \alpha$, the dip is narrow and touches the zero in the graph meaning almost all of the light is absorbed. We still see fast light and negative group index from here but most of the light is absorbed. The transmission spectrum is shown for $g > \alpha$.

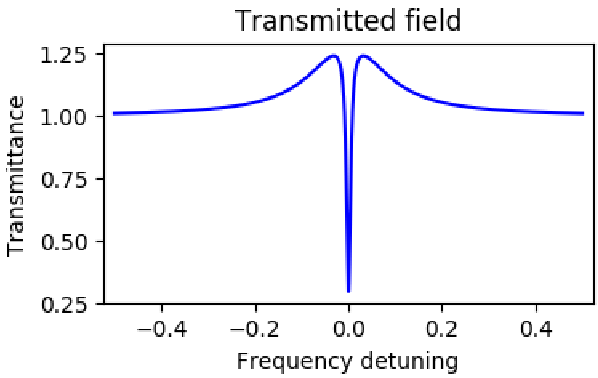


Figure 3.28: Transmission dip of CRIA with gain in resonator 1.

### Introducing gain in both resonator

When we introduce gain in both of the resonator simultaneously, then we see similar effects as with gain activated in resonator 2. But the transmission dip changes into a peak when $g > \alpha$ meaning all the losses are compensated and we have normal dispersion with an high amount of transmission.

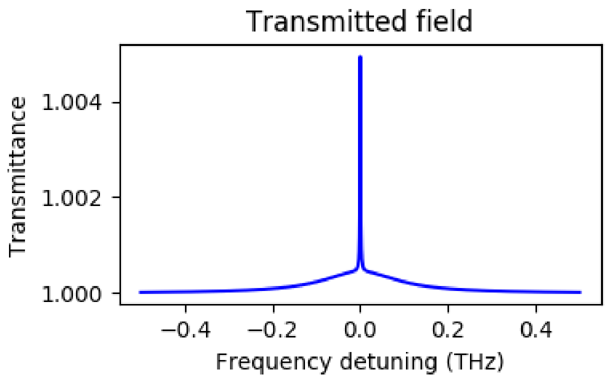


Figure 3.29: Transmission dip transforming into an tranmission peak.

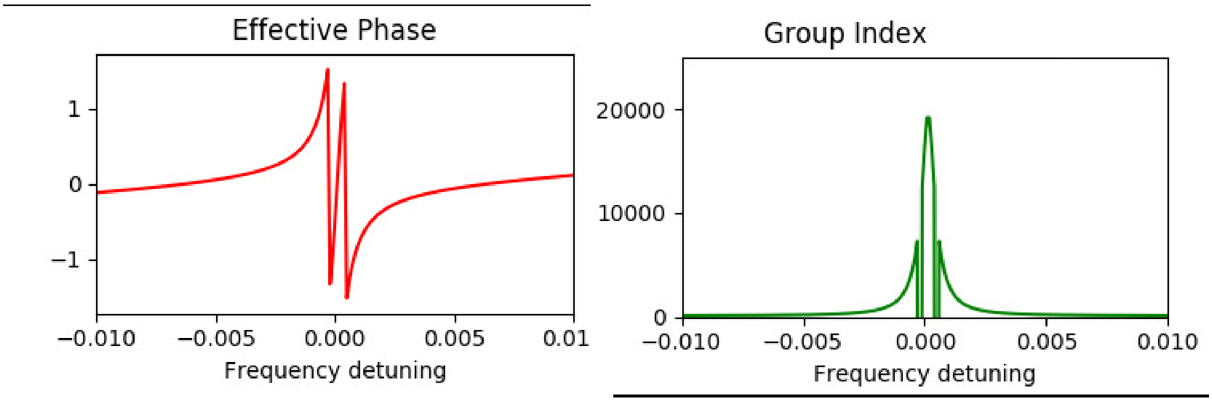


Figure 3.30: Respective phase and group index of the system.

## 3.6 Conclusion

We have clearly observed by studying the properties of different cases of a coupled resonator system. This observation told us that when we introduce gain into the system, we can see drastic changes in the transmission and phase spectrum of the system thus affecting the group delay and dispersion of the system. This allowed acheive gain tunability in these system which means that we can tune between fast and slow light by simply introducing gain inside the system.

## References

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[2] Coupled-resonator-induced transparency, PHYSICAL REVIEW A 69, 063804 (2004)
[3] What is and what is not electromagnetically induced transparency in whispering-gallery microcavities, DOI:10.1038/ncomms6082, Published 24 Oct 2014

[4] Induced transparency and absorption in coupled whispering gallery microresonators, PHYSICAL REVIEW A 71, 043804, published 5 April 2005

[5]

# Chapter 4

# Composite Resonator Systems

## 4.1 Coupled resontaor with Gain medium

Most of the time, using mpmath is simply a matter of setting the desired precision and entering a formula. For verification purposes, a quite (but not always!) reliable technique is to calculate the same thing a second time at a higher precision and verifying that the results agree.

### 4.1.1 Gain element

To perform more advanced calculations, it is important to have some understanding of how mpmath works internally and what the possible sources of error are. This section gives an overview of arbitrary-precision binary floating-point arithmetic and some concepts from numerical analysis.Most of the time, using mpmath is simply a matter of setting the desired precision and entering a formula. For verification purposes, a quite (but not always!) reliable technique is to calculate the same thing a second time at a higher precision and verifying that the results agree.

## 4.2 Calculation/Equations

To perform more advanced calculations, it is important to have some understanding of how mpmath works internally and what the possible sources of error are. This section gives an overview of arbitrary-precision binary floating-point arithmetic and some concepts from numerical analysis.Most of the time, using mpmath is simply a matter of setting the desired precision and entering a formula. For verification purposes, a quite (but not always!) reliable technique is to calculate the same thing a second time at a higher precision and verifying that the results agree.

### 4.2.1 For single

To perform more advanced calculations, it is important to have some understanding of how mpmath works internally and what the possible sources of error are. This section gives an overview of arbitrary-precision binary floating-point arithmetic and some concepts from numerical analysis.Most of the time, using mpmath is simply a matter of setting the desired precision and entering a formula. For verification purposes, a quite (but not always!) reliable technique is to calculate the same thing a second time at a higher precision and verifying that the results agree.

### 4.2.2 For coupled

To perform more advanced calculations, it is important to have some understanding of how mpmath works internally and what the possible sources of error are. This section gives an overview of arbitrary-precision binary floating-point arithmetic and some concepts from numerical analysis.Most of the time, using mpmath is simply a matter of setting the desired precision and entering a formula. For verification purposes, a quite (but not always!) reliable technique is to calculate the same thing a second time at a higher precision and verifying that the results agree.

### 4.2.3 For triple

To perform more advanced calculations, it is important to have some understanding of how mpmath works internally and what the possible sources of error are. This section gives an overview of arbitrary-precision binary floating-point arithmetic and some concepts from numerical analysis.

## 4.3 Coupling Regimes

To perform more advanced calculations, it is important to have some understanding of how mpmath works internally and what the possible sources of error are. This section gives an overview of arbitrary-precision binary floating-point arithmetic and some concepts from numerical analysis.To perform more advanced calculations, it is important to have some understanding of how mpmath works internally and what the possible sources of error are. This section gives an overview of arbitrary-precision binary floating-point arithmetic and some concepts from numerical analysis.

## 4.4 Gain controlled EIT and EIA

# Chapter 5

# Gain Incorporation

# Chapter 6

# Conclusion

Most of the time, using mpmath is simply a matter of setting the desired precision and entering a formula. For verification purposes, a quite (but not always!) reliable technique is to calculate the same thing a second time at a higher precision and verifying that the results agree.

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# Appendix A

# Abrevations

EIT Electromagnetically Induced Transparency

EIA Electromagnetically Induced Absorption

CRIT Coupled Resonator Induced Transparency

CRIA Coupled Resonator Induced Absorption

FSR Free Spectral Range

MRR Micro Ring Resonator

MZI Mach Zehnder Interferometer

FWHM Full width at half maximum

CMT Coupled Mode Theory
Total Internal Reflection