Fundamentals of Robotics HW 1

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Abstract

This report is part of Fundamentals of Robotics course for 3d year students at Innopolis University. In this report I am working on KR 10 R1100-2 manipulator designed by KUKA, where I managed to develop kinematic model of the robot , solve forward kinematics problem and solve inverse kinematics problem using Pieper's Solution. I implemented the proposed solution using Matlab and created a test file to check the validity of my solutions.

1 Kinematic Scheme

The kinematic scheme for KR 10 R1100-2 is described in Figure 1.

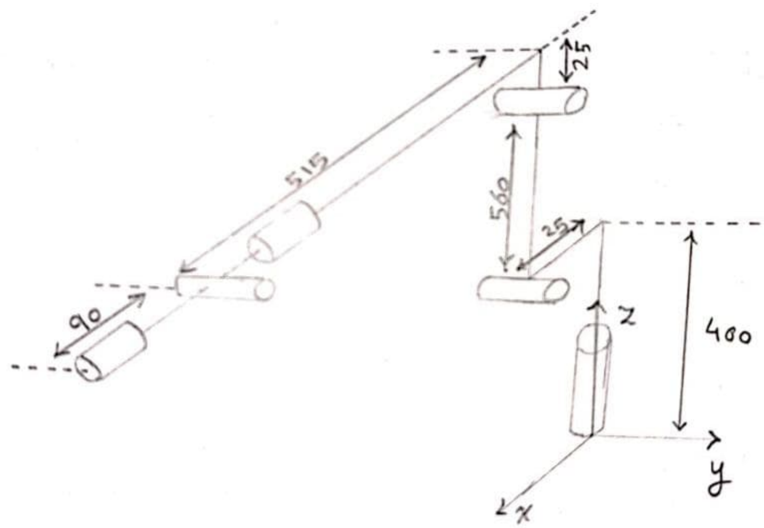

Figure 1: KR 10 R1100-2 Kinematic Scheme

2 Forward Kinematics

Forward kinematics for KR 10 R1100-2 can be described by equation 1

$$T = T_z(0.4)R_z(q_1)T_x(0.025)R_y(q_2)T_z(0.56)R_y(q_3)T_z(0.025)T_x(0.515)R_x(q_4)R_y(q_5)R_x(q_6)T_x(0.09) \tag{1}$$

$$T = \begin{bmatrix} n_x & s_x & a_x & x \\ n_y & s_y & a_y & y \\ n_z & s_z & a_z & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

3 Inverse Kinematics

Inverse kinematics problem can be divided into two sub-problems (Position and Orientation)

3.1 Position

In order to solve the position problem we need to find the center of the spherical wrist, that can be found by $T_n = T.T_x(0.09)^-1$ We are interested in the first three values of the last column of matrix T_n as they describe the position of the center of the spherical wrist.

```
x = cos(q1)/40 + (14*cos(q1)*sin(q2))/25 - (9*sin(q1)*sin(q4)*sin(q5))/100
     + (9*cos(q2 + q3)*cos(q1)*cos(q5))/100 + (103*cos(q1)*cos(q2)*cos(q3))/200
     + (cos(q1)*cos(q2)*sin(q3))/40 + (cos(q1)*cos(q3)*sin(q2))/40
     - (103*cos(q1)*sin(q2)*sin(q3))/200 - (9*cos(q1)*cos(q2)*cos(q4)*sin(q3)*sin(q5))/100
     - (9*cos(q1)*cos(q3)*cos(q4)*sin(q2)*sin(q5))/100

y = sin(q1)/40 + (14*sin(q1)*sin(q2))/25 - (103*sin(q1)*sin(q2)*sin(q3))/200
     + (9*cos(q2 + q3)*cos(q5)*sin(q1))/100 +(103*cos(q2)*cos(q3)*sin(q1))/200
     + (cos(q2)*sin(q1)*sin(q3))/40 + (cos(q3)*sin(q1)*sin(q2))/40 +
     (9*cos(q1)*sin(q4)*sin(q5))/100 - (9*cos(q2)*cos(q4)*sin(q1)*sin(q3)*sin(q5))/100 -
     (9*cos(q3)*cos(q4)*sin(q1)*sin(q2)*sin(q5))/100

z = cos(q2 + q3)/40 - (103*sin(q2 + q3))/200 + (14*cos(q2))/25
    + (9*sin(q4 - q5)*cos(q2 + q3))/200 - (9*cos(q2 + q3)*sin(q4 + q5))/200
    - (9*sin(q2 + q3)*cos(q5))/100 + 2/5


nx = cos(q5)*(cos(q1)*cos(q2)*cos(q3) - cos(q1)*sin(q2)*sin(q3)) - sin(q5)*(sin(q1)*sin(q4)
    + cos(q4)*(cos(q1)*cos(q2)*sin(q3) + cos(q1)*cos(q3)*sin(q2)))


ny = sin(q5)*(cos(q1)*sin(q4) - cos(q4)*(cos(q2)*sin(q1)*sin(q3) + cos(q3)*sin(q1)*sin(q2)))
    - cos(q5)*(sin(q1)*sin(q2)*sin(q3) - cos(q2)*cos(q3)*sin(q1))


nz = - sin(q2 + q3)*cos(q5) - cos(q2 + q3)*cos(q4)*sin(q5)


sx = sin(q6)*(cos(q5)*(sin(q1)*sin(q4) + cos(q4)*(cos(q1)*cos(q2)*sin(q3) + cos(q1)*cos(q3)*sin(q2)))
    + sin(q5)*(cos(q1)*cos(q2)*cos(q3) - cos(q1)*sin(q2)*sin(q3))) - cos(q6)*(cos(q4)*sin(q1)
    - sin(q4)*(cos(q1)*cos(q2)*sin(q3) + cos(q1)*cos(q3)*sin(q2)))


sy = cos(q6)*(cos(q1)*cos(q4) + sin(q4)*(cos(q2)*sin(q1)*sin(q3) + cos(q3)*sin(q1)*sin(q2)))
    - sin(q6)*(cos(q5)*(cos(q1)*sin(q4) - cos(q4)*(cos(q2)*sin(q1)*sin(q3) + cos(q3)*sin(q1)*sin(q2)))
    + sin(q5)*(sin(q1)*sin(q2)*sin(q3) - cos(q2)*cos(q3)*sin(q1)))


sz = cos(q2 + q3)*cos(q6)*sin(q4) - sin(q6)*(sin(q2 + q3)*sin(q5) - cos(q2 + q3)*cos(q4)*cos(q5))


ax = sin(q6)*(cos(q4)*sin(q1) - sin(q4)*(cos(q1)*cos(q2)*sin(q3) + cos(q1)*cos(q3)*sin(q2)))
    + cos(q6)*(cos(q5)*(sin(q1)*sin(q4) + cos(q4)*(cos(q1)*cos(q2)*sin(q3) + cos(q1)*cos(q3)*sin(q2)))
    + sin(q5)*(cos(q1)*cos(q2)*cos(q3) - cos(q1)*sin(q2)*sin(q3)))


ay = - sin(q6)*(cos(q1)*cos(q4) + sin(q4)*(cos(q2)*sin(q1)*sin(q3) + cos(q3)*sin(q1)*sin(q2)))
    - cos(q6)*(cos(q5)*(cos(q1)*sin(q4) - cos(q4)*(cos(q2)*sin(q1)*sin(q3) + cos(q3)*sin(q1)*sin(q2)))
    + sin(q5)*(sin(q1)*sin(q2)*sin(q3) - cos(q2)*cos(q3)*sin(q1)))


az = - cos(q6)*(sin(q2 + q3)*sin(q5) - cos(q2 + q3)*cos(q4)*cos(q5)) - cos(q2 + q3)*sin(q4)*sin(q6)
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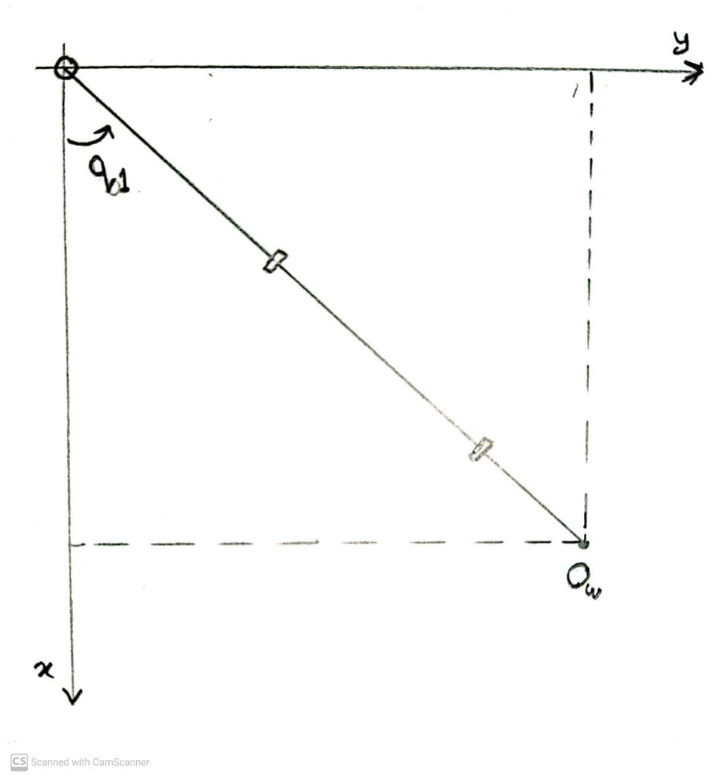

Figure 2: Top View

Noticing the top view of the manipulator in Figure 2, it is possible to describe q_1 by the following equation

$$q_1 = atan2(\frac{y}{x}) \tag{3}$$

It is worth noting that when $x = 0$ and $y = 0$ a singularity occurs because the end-effector intersects with the z axis, which means no matter what value is given to q_1 the end-effector will keep its position.

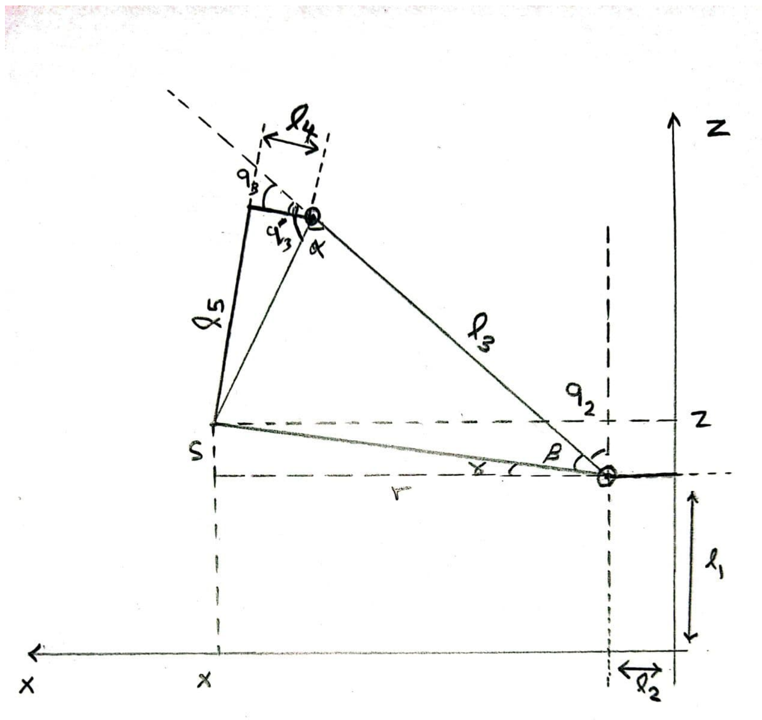

Figure 3: Side View

by looking at the side view of the manipulator and using the geometrical approach it is possible to obtain q_2 and q_3.

By applying the Cosine theorem we get

$$r^2 + s^2 = l_5^2 + l_4^2 + l_3^2 - 2l_3\sqrt{l_5^2 + l_4^2}\cos\alpha \tag{4}$$

We can write the above equation in terms of q_3'

$$r^2 + s^2 = l_5^2 + l_4^2 + l_3^2 + 2l_3\sqrt{l_5^2 + l_4^2}\cos q_3' \tag{5}$$

By simplifying the previous expression we can describe our variable of interest q_3 as follows

$$q_3 = q_3' - atan2(l_5, l_4) \tag{6}$$

Equation 5 can yield a complex solution in some cases, such cases happen when the point is unreachable. In other words such point exists outside the work space of the robot. Such cases are detected in my code by checking a the following flag $|\frac{r^2+s^2-l_5^2-l_4^2-l_3^2}{2l_3\sqrt{l_5^2+l_4^2}}| > 1$

It is worth noting that $q_3 = 0$ is a singularity case as robot loses 1 DOF.

By observing Figure 3 once again and applying the Sine Theorem we get $\frac{\sin\alpha}{\sqrt{r^2+s^2}} = \frac{\sin\beta}{\sqrt{l_5^2+l_4^2}}$ and γ can be found by $\gamma = atan2(s, r)$ finally q_2 can be obtained by

$$q_2 = \frac{\pi}{2} - \gamma - \beta \tag{7}$$

3.2 Orientation

After obtaining q_1, q_2, q_3 it is possible to write

$$T_0^6 = T_0^3 . T_3^6 \tag{8}$$

Where we found $T_0^3$and T_0^6 is given.

$$T_3^6 = {T_0^3}^{-1}.T_0^6 \tag{9}$$

Next, we solve T_3^6 for q_4, q_5, q_6 and get the following matrix

```
T36 =
[          c(q5),                    s(q5)*s(q6),                 c(q6)*s(q5)          ]
[  s(q4)*s(q5), c(q4)*c(q6) - c(q5)*s(q4)*s(q6), - c(q4)*s(q6) - c(q5)*c(q6)*s(q4)]
[ -c(q4)*s(q5), c(q6)*s(q4) + c(q4)*c(q5)*s(q6),   c(q4)*c(q5)*c(q6) - s(q4)*s(q6)]
```

for each joint two solutions are possible

```
q4=atan2(T36(2,1),-T36(3,1))

q6=atan2(T36(1,2),T36(1,3))

q5=atan2(sqrt(T36(1,3)^2+T36(1,2)^2),T36(1,1))

%Second Set of possible solutions

q41=atan2(-T36(2,1),T36(3,1))

q61=atan2(-T36(1,2),-T36(1,3))

q51=atan2(-sqrt(T36(1,3)^2+T36(1,2)^2),T36(1,1))
```