Fundamentals of Robotics HW 1

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Abstract

This report is part of Fundamentals of Robotics course for 3d year students at Innopolis University. In this report I am working on KR 10 R1100-2 manipulator designed by KUKA, where I managed to develop kinematic model of the robot , solve both forward and inverse kinematics problems, the latter has been solved using Pieper's Solution. I implemented the proposed solution using Matlab and created a test file to check the validity of my solutions.

1 Kinematic Scheme

The kinematic scheme for KR 10 R1100-2 is described in Figure 1.

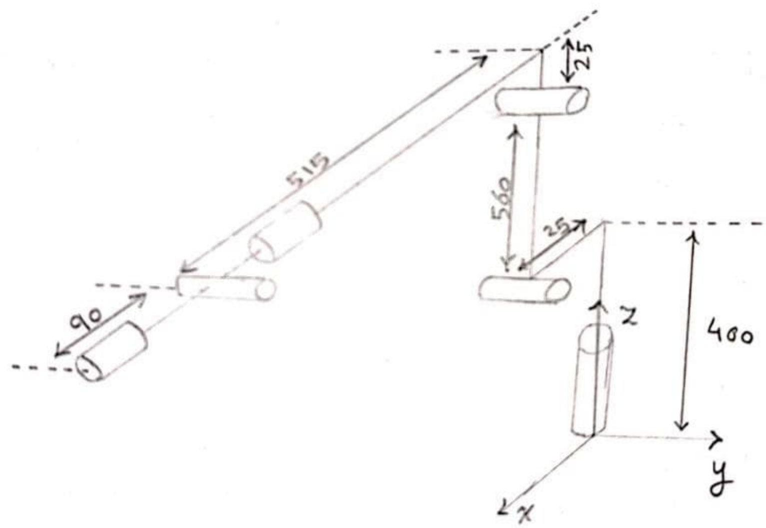

Figure 1: KR 10 R1100-2 Kinematic Scheme

2 Forward Kinematics

Forward kinematics for KR 10 R1100-2 can be described by Equation 1

$$T = T_z(0.4)R_z(q_1)T_x(0.025)R_y(q_2)T_z(0.56)R_y(q_3)T_z(0.025)T_x(0.515)R_x(q_4)R_y(q_5)R_x(q_6)T_x(0.09) \tag{1}$$

$$T = \begin{bmatrix} n_x & s_x & a_x & x \\ n_y & s_y & a_y & y \\ n_z & s_z & a_z & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Formulas of forward kinematics solution are presented in details in the next page.

```
x = cos(q1)/40 + (14*cos(q1)*sin(q2))/25 - (9*sin(q1)*sin(q4)*sin(q5))/100
     + (9*cos(q2 + q3)*cos(q1)*cos(q5))/100 + (103*cos(q1)*cos(q2)*cos(q3))/200
     + (cos(q1)*cos(q2)*sin(q3))/40 + (cos(q1)*cos(q3)*sin(q2))/40
     - (103*cos(q1)*sin(q2)*sin(q3))/200 - (9*cos(q1)*cos(q2)*cos(q4)*sin(q3)*sin(q5))/100
     - (9*cos(q1)*cos(q3)*cos(q4)*sin(q2)*sin(q5))/100

y = sin(q1)/40 + (14*sin(q1)*sin(q2))/25 - (103*sin(q1)*sin(q2)*sin(q3))/200
     + (9*cos(q2 + q3)*cos(q5)*sin(q1))/100 +(103*cos(q2)*cos(q3)*sin(q1))/200
     + (cos(q2)*sin(q1)*sin(q3))/40 + (cos(q3)*sin(q1)*sin(q2))/40 +
     (9*cos(q1)*sin(q4)*sin(q5))/100 - (9*cos(q2)*cos(q4)*sin(q1)*sin(q3)*sin(q5))/100 -
     (9*cos(q3)*cos(q4)*sin(q1)*sin(q2)*sin(q5))/100

z = cos(q2 + q3)/40 - (103*sin(q2 + q3))/200 + (14*cos(q2))/25
    + (9*sin(q4 - q5)*cos(q2 + q3))/200 - (9*cos(q2 + q3)*sin(q4 + q5))/200
    - (9*sin(q2 + q3)*cos(q5))/100 + 2/5


nx = cos(q5)*(cos(q1)*cos(q2)*cos(q3) - cos(q1)*sin(q2)*sin(q3)) - sin(q5)*(sin(q1)*sin(q4)
    + cos(q4)*(cos(q1)*cos(q2)*sin(q3) + cos(q1)*cos(q3)*sin(q2)))


ny = sin(q5)*(cos(q1)*sin(q4) - cos(q4)*(cos(q2)*sin(q1)*sin(q3) + cos(q3)*sin(q1)*sin(q2)))
    - cos(q5)*(sin(q1)*sin(q2)*sin(q3) - cos(q2)*cos(q3)*sin(q1))


nz = - sin(q2 + q3)*cos(q5) - cos(q2 + q3)*cos(q4)*sin(q5)


sx = sin(q6)*(cos(q5)*(sin(q1)*sin(q4) + cos(q4)*(cos(q1)*cos(q2)*sin(q3) + cos(q1)*cos(q3)*sin(q2)))
    + sin(q5)*(cos(q1)*cos(q2)*cos(q3) - cos(q1)*sin(q2)*sin(q3))) - cos(q6)*(cos(q4)*sin(q1)
    - sin(q4)*(cos(q1)*cos(q2)*sin(q3) + cos(q1)*cos(q3)*sin(q2)))


sy = cos(q6)*(cos(q1)*cos(q4) + sin(q4)*(cos(q2)*sin(q1)*sin(q3) + cos(q3)*sin(q1)*sin(q2)))
    - sin(q6)*(cos(q5)*(cos(q1)*sin(q4) - cos(q4)*(cos(q2)*sin(q1)*sin(q3) + cos(q3)*sin(q1)*sin(q2)))
    + sin(q5)*(sin(q1)*sin(q2)*sin(q3) - cos(q2)*cos(q3)*sin(q1)))


sz = cos(q2 + q3)*cos(q6)*sin(q4) - sin(q6)*(sin(q2 + q3)*sin(q5) - cos(q2 + q3)*cos(q4)*cos(q5))


ax = sin(q6)*(cos(q4)*sin(q1) - sin(q4)*(cos(q1)*cos(q2)*sin(q3) + cos(q1)*cos(q3)*sin(q2)))
    + cos(q6)*(cos(q5)*(sin(q1)*sin(q4) + cos(q4)*(cos(q1)*cos(q2)*sin(q3) + cos(q1)*cos(q3)*sin(q2)))
    + sin(q5)*(cos(q1)*cos(q2)*cos(q3) - cos(q1)*sin(q2)*sin(q3)))


ay = - sin(q6)*(cos(q1)*cos(q4) + sin(q4)*(cos(q2)*sin(q1)*sin(q3) + cos(q3)*sin(q1)*sin(q2)))
    - cos(q6)*(cos(q5)*(cos(q1)*sin(q4) - cos(q4)*(cos(q2)*sin(q1)*sin(q3) + cos(q3)*sin(q1)*sin(q2)))
    + sin(q5)*(sin(q1)*sin(q2)*sin(q3) - cos(q2)*cos(q3)*sin(q1)))


az = - cos(q6)*(sin(q2 + q3)*sin(q5) - cos(q2 + q3)*cos(q4)*cos(q5)) - cos(q2 + q3)*sin(q4)*sin(q6)
```

3 Inverse Kinematics

Inverse kinematics problem can be divided into two sub-problems (Position and Orientation)

3.1 Position

In order to solve the position problem we need to find the center of the spherical wrist, that can be found by

$$T_n = T.T_x(0.09)^{-1}$$

We are interested in the first three values of the last column of matrix T_n as they describe the position of the center of the spherical wrist.

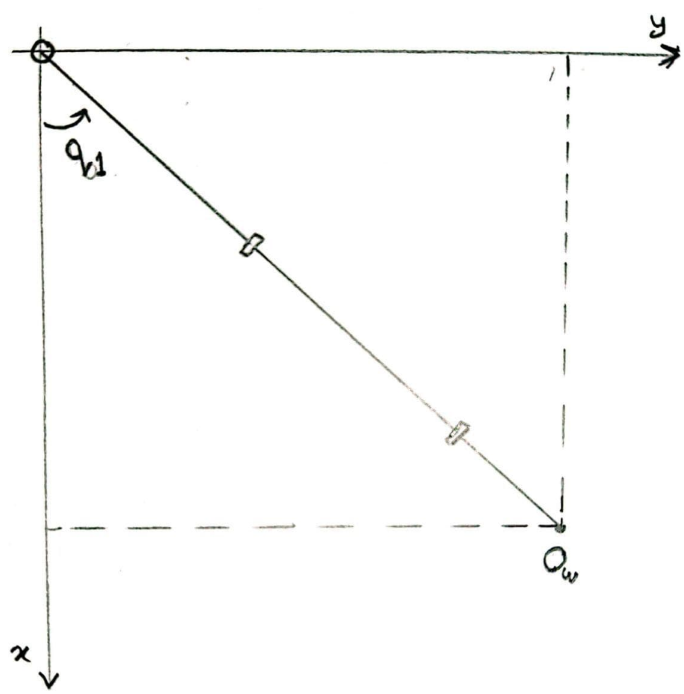

Figure 2: Top View

By observing the top view of the manipulator in Figure 2, it is possible to describe q_1 by the following equation

$$q_1 = atan2(\frac{y}{x}) \tag{3}$$

It is worth noting that when $x = 0$ and $y = 0$ a singularity occurs because the end-effector intersects with the z axis, which means no matter what value is given to q_1 the end-effector will keep its position.

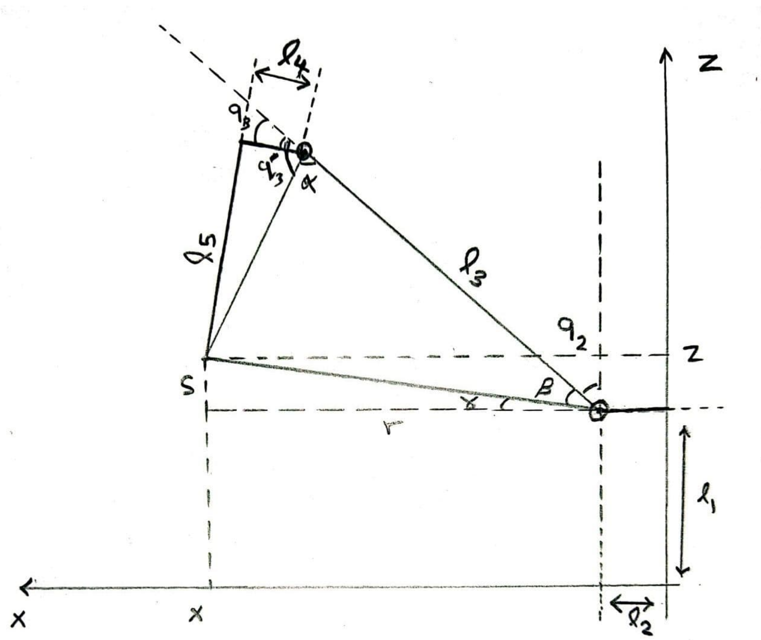

Figure 3: Side View

By studying the side view of the manipulator and using the geometrical approach it is possible to obtain q_2 and q_3.
r is calculated from the top view as follows

$$r = \sqrt{x^2 + y^2} - l_2$$

s is calculated from the top view as follows

$$s = z - l_1$$

By applying the Cosine theorem we get

$$r^2 + s^2 = l_5^2 + l_4^2 + l_3^2 - 2l_3\sqrt{l_5^2 + l_4^2}\cos\alpha \tag{4}$$

We can write the above equation in terms of q_3'

$$r^2 + s^2 = l_5^2 + l_4^2 + l_3^2 + 2l_3\sqrt{l_5^2 + l_4^2}\cos q_3' \tag{5}$$

By simplifying the previous expression we can describe our variable of interest q_3 as follows

$$q_3 = q_3' - atan2(l_5, l_4) \tag{6}$$

Equation 5 can yield a complex solution in some cases, such cases happen when the point is unreachable. In other words such point exists outside the work space of the robot.
During the implementation phase I detected these cases by checking a the following flag

$$|\frac{r^2 + s^2 - l_5^2 - l_4^2 - l_3^2}{2l_3\sqrt{l_5^2 + l_4^2}}| > 1$$

It is worth noting that $q_3 = 0$ is a singularity case as robot loses 1 DOF.

By observing Figure 3 once again and applying the Sine Theorem we get

$$\frac{\sin\alpha}{\sqrt{r^2 + s^2}} = \frac{\sin\beta}{\sqrt{l_5^2 + l_4^2}}$$

and γ can be found by $\gamma = atan2(s, r)$ finally q_2 can be obtained by

$$q_2 = \frac{\pi}{2} - \gamma - \beta \tag{7}$$

3.2 Orientation

After obtaining q_1, q_2, q_3 it is possible to write

$$T_0^6 = T_0^3 . T_3^6 \tag{8}$$

Where we found $T_0^3$and T_0^6 is given.

$$T_3^6 = {T_0^3}^{-1} . T_0^6 \tag{9}$$

Next, we solve T_3^6 for q_4, q_5, q_6

```
T36 =
[          c(q5),                  s(q5)*s(q6),                 c(q6)*s(q5)          ]
[  s(q4)*s(q5), c(q4)*c(q6) - c(q5)*s(q4)*s(q6), - c(q4)*s(q6) - c(q5)*c(q6)*s(q4)]
[ -c(q4)*s(q5), c(q6)*s(q4) + c(q4)*c(q5)*s(q6),   c(q4)*c(q5)*c(q6) - s(q4)*s(q6)]
```

for each joint two solutions are possible

```
q4=atan2(T36(2,1),-T36(3,1))

q6=atan2(T36(1,2),T36(1,3))

q5=atan2(sqrt(T36(1,3)^2+T36(1,2)^2),T36(1,1))

%Second Set of possible solutions

q41=atan2(-T36(2,1),T36(3,1))

q61=atan2(-T36(1,2),-T36(1,3))

q51=atan2(-sqrt(T36(1,3)^2+T36(1,2)^2),T36(1,1))
```

4 GitHub

All files can be found here