Fundamentals of Robotics HW 4

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Abstract

This report is part of Fundamentals of Robotics course for 3d year students at Innopolis University. In this report I am working on RRR arm, where I managed to solve both forward and inverse kinematics problems, I calculated the Jacobian using the geometrical approach (Skew Theory). A joint trajectory (Polynomial and trapezoidal) has been built in the joint space. In the final task a joint trajectory (trapezoidal) has been built to follow a line given two points in the task space.

1 Jacobian

The kinematic scheme for RRR arm is described in Figure 1.

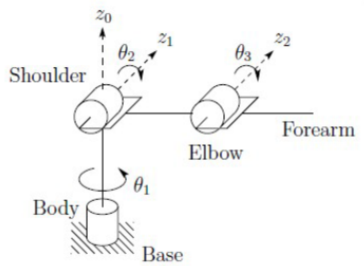

Figure 1: RRR Kinematic Scheme

After calculating the FK we can obtain the jacobian of this robot using Skew Theory and we got the following.

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[ -sin(q1)*(cos(q2 + q3) + cos(q2)), -cos(q1)*(sin(q2 + q3) + sin(q2)), -sin(q2 + q3)*cos(q1)]
[  cos(q1)*(cos(q2 + q3) + cos(q2)), -sin(q1)*(sin(q2 + q3) + sin(q2)), -sin(q2 + q3)*sin(q1)]
[                                 0,          - cos(q2 + q3) - cos(q2),         -cos(q2 + q3)]
[                                 0,                          -sin(q1),              -sin(q1)]
[                                 0,                           cos(q1),               cos(q1)]
[                                 1,                                 0,                     0]
```

2 Joint Trajectory - Polynomial

In this task we are given 6 constraints for each joint

$$B = \begin{bmatrix} q_{i0} \\ \dot{q}_{i0} \\ \ddot{q}_{i0} \\ q_{if} \\ \dot{q}_{if} \\ \ddot{q}_{if} \end{bmatrix}$$

We solve the system $Ax = B$ where

$$A = \begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 & t_0^4 & t_0^5 \\ 0 & 1 & 2t_0 & 3t_0^2 & 4t_0^3 & 5t_0^4 \\ 0 & 0 & 2 & 6t_0 & 12t_0^2 & 20t_0^3 \\ 1 & t_f & t_f^2 & t_f^3 & t_f^4 & t_f^5 \\ 0 & 1 & 2t_f & 3t_f^2 & 4t_f^3 & 5t_f^4 \\ 0 & 0 & 2 & 6t_f & 12t_f^2 & 20t_f^3 \end{bmatrix}$$

After finding all the required coefficients we can plot position, velocity and acceleration for each joint.

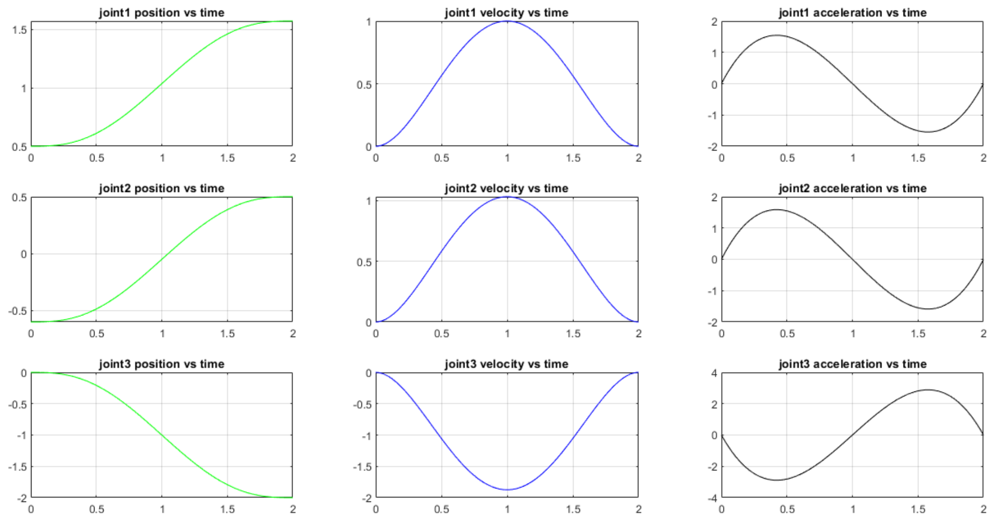

Figure 2: Joint Trajectory Polynomial

3 Joint Trajectory - Trapezoidal

The first step in solving this task is calculating t_a and t_f taking into consideration all the given constraints. Next step is adapting all the joints to the slowest joint in the system so they all start and finish at the same exact time. such adaptation will require calculating new velocities and accelerations of the other two joints. After that the trajectory is divided into three intervals each of specific degree.

$$q(t) = \begin{cases} a_{10} + a_{11}t + a_{12}t^2 & t \leq t_a \\ a_{20} + a_{21}t & t_a < t \leq t_f - t_a \\ a_{30} + a_{31}t + a_{32}t^2 & t_f - t_a < t \leq t_f \end{cases}$$

Coefficients are computed and plots can be obtained and visualized as follows.

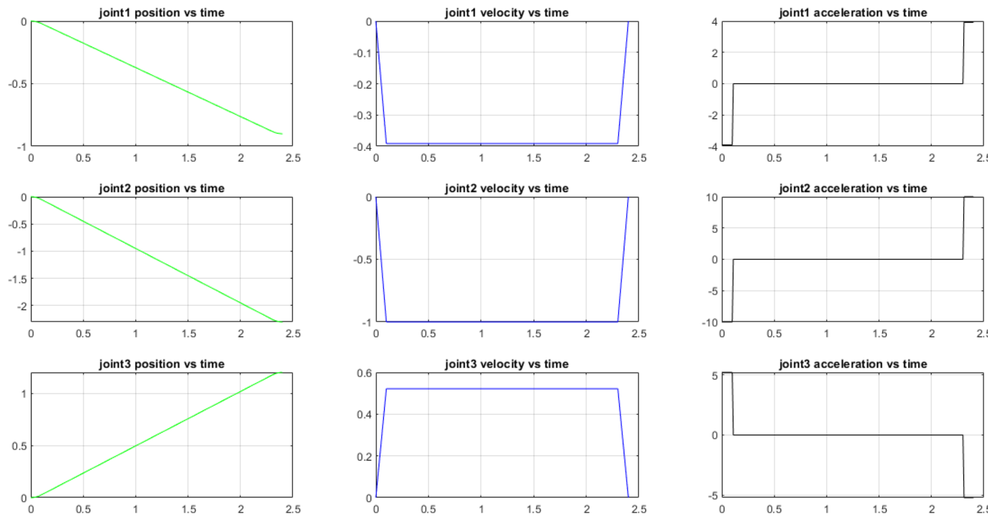

Figure 3: Joint Trajectory Trapezoidal

4 Joint Trajectory - line

In this task we are given two points in the task space and we need to follow a straight line in order to move between them.
For this task we were able to rebuild the path in the task space for each axis by finding all the needed coefficients which satisfy the constraints and adapting to the slowest joint in the system. So we were able to obtain the positions, velocities and accelerations on all the axis as shown in the following graphs.

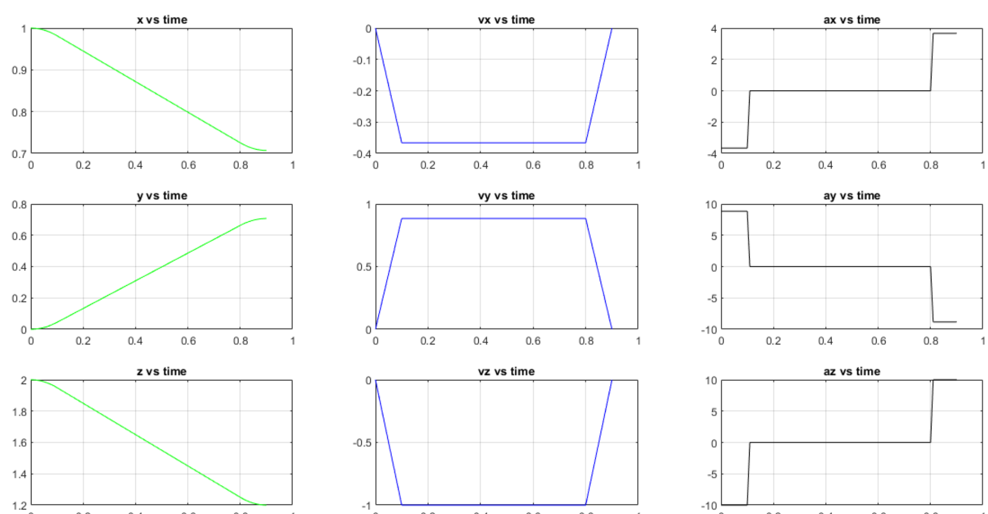

Figure 4: Motion on the 3 axis

The following step is feeding all these points to the IK function in order to find the matching configurations at each time step. Next, joint velocities are calculated using the inverse of the jacobian and velocities on the axis.

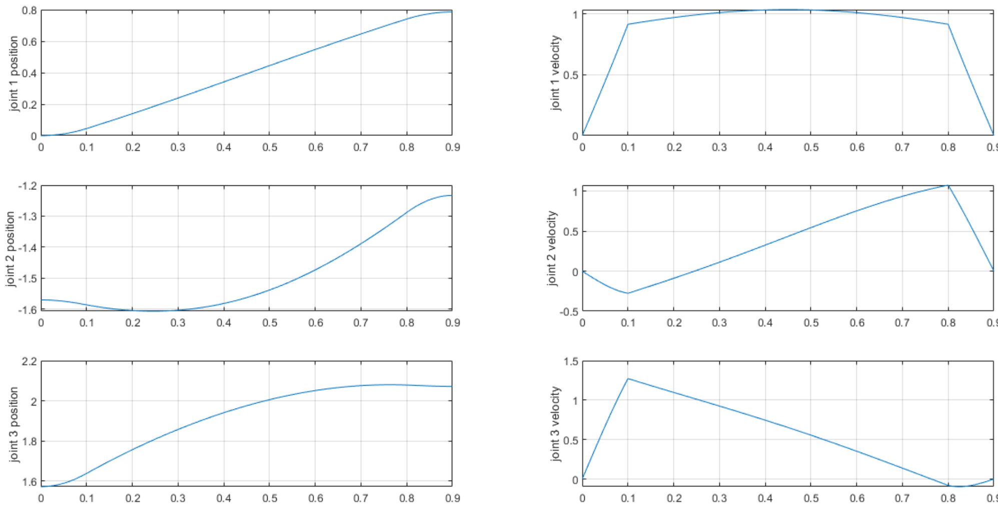

Figure 5: Joint Trajectory

Final step is feeding the obtained configurations to the FK function to find the actual path and plot the results against the desired points on each one of the axis.

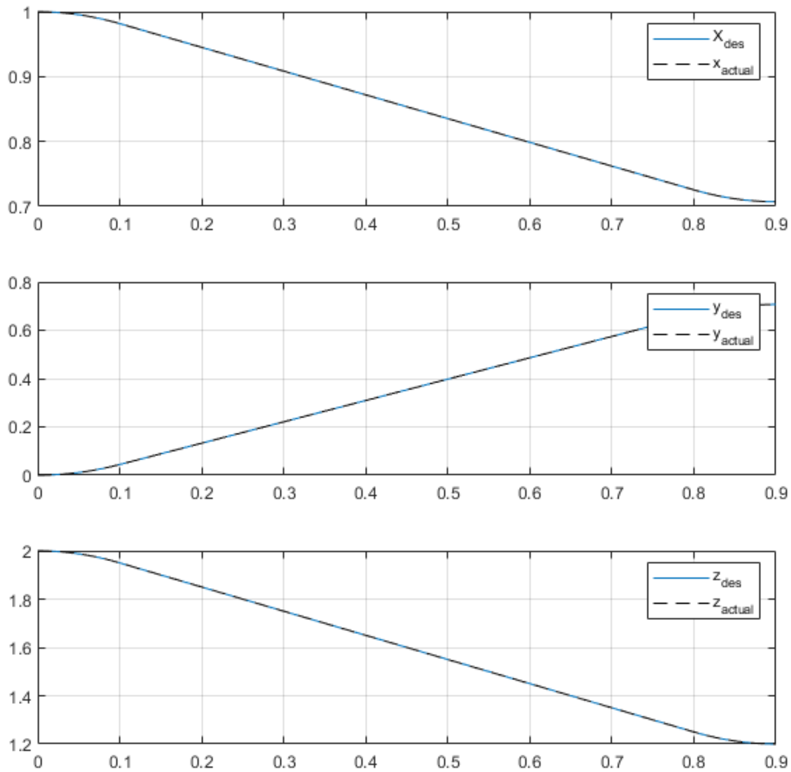

Figure 6: Actual vs desired paths

5 GitHub

All files can be found here