Package ‘biwavelet’

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Imports fields

Description This is a port of the WTC MATLAB package written by Aslak Grinsted and the wavelet program written by Christopher Torrence and Gibert P. Compo. This package can be used to perform univariate and bivariate (cross-wavelet, wavelet coherence, wavelet clustering) analyses.

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`biwavelet-package` *Conduct Univariate and Bivariate Wavelet Analyses*

Description

This is a port of the WTC MATLAB package written by Aslak Grinsted and the wavelet program written by Christopher Torrence and Gibert P. Compo. This package can be used to perform univariate and bivariate (cross-wavelet, wavelet coherence, wavelet clustering) wavelet analyses.

Details

Package:	biwavelet
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Version:	0.17.10
Date:	2015-04-29
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LazyLoad:	yes

As of biwavelet version 0.14, the bias-corrected wavelet and cross-wavelet spectra are automatically computed and plotted by default using the methods described by Liu et al. (2007) and Veleda et al. (2012). This correction is needed because the traditional approach for computing the power spectrum (e.g., Torrence and Compo 1998) leads to an artificial and systematic reduction in power at lower periods.

Author(s)

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Code based on WTC MATLAB package written by Aslak Grinsted and the wavelet MATLAB program written by Christopher Torrence and Gibert P. Compo.

References

Cazelles, B., M. Chavez, D. Berteaux, F. Menard, J. O. Vik, S. Jenouvrier, and N. C. Stenseth. 2008. Wavelet analysis of ecological time series. *Oecologia* 156:287-304.

Grinsted, A., J. C. Moore, and S. Jevrejeva. 2004. Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics* 11:561-566.

Liu, Y., X. San Liang, and R. H. Weisberg. 2007. Rectification of the Bias in the Wavelet Power Spectrum. *Journal of Atmospheric and Oceanic Technology* 24:2093-2102.

Rouyer, T., J. M. Fromentin, F. Menard, B. Cazelles, K. Briand, R. Pianet, B. Planque, and N. C. Stenseth. 2008. Complex interplays among population dynamics, environmental forcing, and exploitation in fisheries. *Proceedings of the National Academy of Sciences* 105:5420-5425.

Rouyer, T., J. M. Fromentin, N. C. Stenseth, and B. Cazelles. 2008. Analysing multiple time series and extending significance testing in wavelet analysis. *Marine Ecology Progress Series* 359:11-23.

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

Torrence, C., and P. J. Webster. 1998. The annual cycle of persistence in the El Nino/Southern Oscillation. *Quarterly Journal of the Royal Meteorological Society* 124:1985-2004.

Veleda, D., R. Montagne, and M. Araujo. 2012. Cross-Wavelet Bias Corrected by Normalizing Scales. *Journal of Atmospheric and Oceanic Technology* 29:1401-1408.

Examples

```
# As of biwavelet version 0.14, the bias-corrected wavelet and cross-wavelet spectra
# are automatically computed and plotted by default using the methods
# described by Liu et al. (2007) and Veleda et al. (2012). This correction
# is needed because the traditional approach for computing the power spectrum
# (e.g., Torrence and Compo 1998) leads to an artificial and systematic reduction
# in power at low periods.

# EXAMPLE OF BIAS CORRECTION:
require(biwavelet)
# Generate a synthetic time series 's' with the same power at three distinct periods
t1=sin(seq(from=0, to=2*5*pi, length=1000))
t2=sin(seq(from=0, to=2*15*pi, length=1000))
t3=sin(seq(from=0, to=2*40*pi, length=1000))
s=t1+t2+t3

# Compare non-corrected vs. corrected wavelet spectrum
wt1=wt(cbind(1:1000, s))
par(mfrow=c(1,2))
plot(wt1, type="power.corr.norm", main="Bias-corrected")
plot(wt1, type="power.norm", main="Not-corrected")

# ADDITIONAL EXAMPLES
t1=cbind(1:100, rnorm(100))
t2=cbind(1:100, rnorm(100))
# Continuous wavelet transform
wt.t1=wt(t1)
# Plot power
# Make room to the right for the color bar
par(oma=c(0, 0, 0, 1), mar=c(5, 4, 4, 5) + 0.1)
plot(wt.t1, plot.cb=TRUE, plot.phase=FALSE)
```

```
# Cross-wavelet
xwt.t1t2=xwt(t1, t2)
# Plot cross wavelet power and phase difference (arrows)
plot(xwt.t1t2, plot.cb=TRUE)

# Wavelet coherence; nrands should be large (>= 1000)
wtc.t1t2=wtc(t1, t2, nrands=10)
# Plot wavelet coherence and phase difference (arrows)
# Make room to the right for the color bar
par(oma=c(0, 0, 0, 1), mar=c(5, 4, 4, 5) + 0.1)
plot(wtc.t1t2, plot.cb=TRUE)

# Perform wavelet clustering of three time series
t1=cbind(1:100, sin(seq(from=0, to=10*2*pi, length.out=100)))
t2=cbind(1:100, sin(seq(from=0, to=10*2*pi, length.out=100)+0.1*pi))
t3=cbind(1:100, rnorm(100))
# Compute wavelet spectra
wt.t1=wt(t1)
wt.t2=wt(t2)
wt.t3=wt(t3)
# Store all wavelet spectra into array
w.arr=array(NA, dim=c(3, NROW(wt.t1$wave), NCOL(wt.t1$wave)))
w.arr[1, , ]=wt.t1$wave
w.arr[2, , ]=wt.t2$wave
w.arr[3, , ]=wt.t3$wave

# Compute dissimilarity and distance matrices
w.arr.dis=wclust(w.arr)
plot(hclust(w.arr.dis$dist.mat, method="ward"), sub="", main="",
        ylab="Dissimilarity", hang=-1)
```

`ar1.spectrum` *Power spectrum of a random red noise process*

Description

Generate the power spectrum of a random time series with a specific AR(1) coefficient

Usage

```
ar1.spectrum (ar1, periods)
```

Arguments

`ar1`	first order coefficient desired.
`periods`	periods of the time series at which the spectrum should be computed.

Value

Returns the power spectrum.

Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Code based on WTC MATLAB package written by Aslak Grinsted.

References

Cazelles, B., M. Chavez, D. Berteaux, F. Menard, J. O. Vik, S. Jenouvrier, and N. C. Stenseth. 2008. Wavelet analysis of ecological time series. *Oecologia* 156:287-304.

Grinsted, A., J. C. Moore, and S. Jevrejeva. 2004. Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics* 11:561-566.

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

Examples

```
p=ar1.spectrum(0.5, 1:25)
```

---

check.data — *Check the format of time series*

Description

Check the format of time series

Usage

```
check.data (y, x1 = NULL, x2 = NULL)
```

Arguments

`y`	time series y in matrix format (`n` rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
`x1`	time series x1 in matrix format (`n` rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
`x2`	time series x2 in matrix format (`n` rows x 2 columns). The first column should contain the time steps and the second column should contain the values.

Value

Returns a named list containing:

`t`	time steps
`dt`	size of a time step
`n.obs`	number of observations

Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

References

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

Examples

```
# Not run:
# t1=cbind(1:100, rnorm(100))
# check.data(y=t1)
```

`convolve2D` *Fast column-wise convolution of a matrix*

Description

Use the Fast Fourier Transform to perform convolutions between a sequence and each column of a matrix.

Usage

```
convolve2D (x, y, conj = TRUE, type = c("circular", "open"))
```

Arguments

`x`	M x n matrix.
`y`	numeric sequence of length N.
`conj`	logical; if `TRUE`, take the complex conjugate before back-transforming. Default is `TRUE` and used for usual convolution.
`type`	character; one of `circular`, `open` (beginning of word is ok). For `circular`, the two sequences are treated as circular, i.e., periodic. For `open` and `filter`, the sequences are padded with zeros (from left and right) first; `filter` returns the middle sub-vector of open, namely, the result of running a weighted mean of `x` with weights `y`.

Details

This is a corrupted version of convolve made by replacing fft with mvfft in a few places. It would be nice to submit this to the R Developers for inclusion.

Note

This function was copied from `waveslim` to limit package dependencies.

Author(s)

Brandon Whitcher

`enviro.data` *Multivariate ENSO (MEI), NPGO, and PDO indices*

Description

Monthly indices of ENSO, NPGO, and PDO from 1950 to 2009

Usage

```
data (enviro.data)
```

Format

A data frame with 720 observations on the following 6 variables.

`month` a numeric vector containing the month

`year` a numeric vector containing the year

`date` a numeric vecor containing the date

`mei` a numeric vector containing the MEI index

`npgo` a numeric vector containing the NPGO index

`pdo` a numeric vector containing the PDO index

Source

MEI: http://www.esrl.noaa.gov/psd/enso/mei

NPGO: http://www.o3d.org/npgo

PDO: http://jisao.washington.edu/pdo

References

Di Lorenzo, E., N. Schneider, K. M. Cobb, P. J. S. Franks, K. Chhak, A. J. Miller, J. C. McWilliams, S. J. Bograd, H. Arango, E. Curchitser, T. M. Powell, and P. Riviere. 2008. North Pacific Gyre Oscillation links ocean climate and ecosystem change. *Geophys. Res. Lett.* 35:L08607.

Mantua, N. J., and S. R. Hare. 2002. The Pacific decadal oscillation. *Journal of Oceanography* 58:35-44.

Zhang, Y., J. M. Wallace, and D. S. Battisti. 1997. ENSO-like interdecadal variability: 1900-93. *Journal of Climate* 10:1004-1020.

Examples

```
data(enviro.data)
head(enviro.data)
```

`meshgrid` *Rectangular grid in two dimensions*

Description

Replicates the grid vectors `xv` and `yv` to generate a full grid

Usage

```
meshgrid (xv, yv)
```

Arguments

`xv`	vector of numeric values
`yv`	vector of numeric values

Value

Returns a list containing the full grid components `xv` and `yv`:

`x`	replicated values of xv
`y`	replicated values of yv

Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Examples

```
xv=runif(10)
yv=runif(5)
g=meshgrid(xv, yv)
```

`phase.plot` *Plot phases*

Description

Plot phases with arrows

Usage

```
phase.plot (x, y, phases, arrow.len=min(par()$pin[2]/30,par()$pin[1]/40),
            arrow.col="black", arrow.lwd=arrow.len*0.3)
```

Arguments

`x`	x-coordinates
`y`	y-coordinates
`phases`	phases
`arrow.len`	size of the arrows. Default is based on plotting region (min(par()$pin[2]/30,par()$pin[1]/40).
`arrow.lwd`	width/thickness of arrows. Default is arrow.len*0.3.
`arrow.col`	arrow line color. Default is `black`.

Note

Arrows pointing to the right mean that `x` and `y` are in phase.

Arrows pointing to the left mean that `x` and `y` are in anti-phase.

Arrows pointing up mean that `y` leads `x` by $\pi/2$.

Arrows pointing down mean that `x` leads `y` by $\pi/2$.

Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Huidong Tian provided a much better implementation of the phase.plot function that allows for more accurate phase arrows.

Original code based on WTC MATLAB package written by Aslak Grinsted.

Examples

```
## Not run: phase.plot(x, y, phases)
```

`plot.biwavelet` *Plot* `biwavelet` *objects*

Description

Plot `biwavelet` objects such as the cwt, cross-wavelet and wavelet coherence

Usage

```
## S3 method for class 'biwavelet'
 plot(x, ncol = 64, fill.cols = NULL, xlab = "Time", ylab = "Period",
              tol = 1, plot.cb = FALSE, plot.phase = FALSE,
              type=c("power.corr.norm", "power.corr", "power.norm", "power",
                     "wavelet", "phase"),
              plot.coi = TRUE, lwd.coi = 1, col.coi = "white",
              lty.coi = 1, alpha.coi = 0.5,
              plot.sig = TRUE, lwd.sig = 4, col.sig = "black", lty.sig = 1,
              bw = FALSE, legend.loc = NULL, legend.horiz = FALSE,
```

```
arrow.len = min(par()$pin[2]/30, par()$pin[1]/40),
arrow.lwd = arrow.len*0.3,
arrow.cutoff = 0.9, arrow.col = "black",
xlim = NULL, ylim = NULL, zlim = NULL,
xaxt = "s", yaxt = "s",
form = '%Y', ...)
```

Arguments

`x`	`biwavelet` object generated by `wt`, `xwt`, or `wtc`.
`ncol`	number of colors to use. Default is 64.
`fill.cols`	Vector of fill colors to be used. Users can specify color vectors using `colorRampPalette` or `brewer.pal` from package `RColorBrewer`. Default is `NULL` and will generate MATLAB's jet color palette.
`xlab`	xlabel of the figure. Default is "Time"
`ylab`	ylabel of the figure. Default is "Period"
`tol`	tolerance level for significance contours. Sigificance contours will be drawn around all regions of the spectrum where `spectrum/percentile >= tol`. Default is `1`. If strict `i^{th}` percentile regions are desired, then must be set to `tol` must be set to `1`.
`plot.cb`	plot color bar if TRUE. Default is FALSE.
`plot.phase`	Plot phases with black arrows. Default is FALSE.
`type`	type of plot to create. Can be `power` to plot the power, `power.corr` to plot the bias-corrected power, `power.norm` to plot the power normalized by the variance, `power.corr.norm` to plot the bias-corrected power normalized by the variance, `wavelet` to plot thewavelet coefficients, or `phase` to plot the phase. Default is `power.corr.norm`.
`plot.coi`	plot cone of influence (COI) as a semi-transparent polygon if TRUE. Default is TRUE. Areas that fall within the polygon can be affected by edge effects.
`lwd.coi`	Line width of COI. Default is 1.
`col.coi`	Color of COI. Default is `white`.
`lty.coi`	Line type of COI. Default is 1 for solide lines.
`alpha.coi`	Transparency of COI. Range is 0 (full transparency) to 1 (no transparency). Default is 0.5.
`plot.sig`	plot contours for significance if TRUE. Default is TRUE.
`lwd.sig`	Line width of significance contours. Default is 4.
`col.sig`	Color of significance contours. Default is `black`.
`lty.sig`	Line type of significance contours. Default is 1.
`bw`	plot in black and white if TRUE. Default is FALSE.
`legend.loc`	legend location coordinates as defined by `image.plot`. Default is `NULL`.
`legend.horiz`	plot a horizontal legend if TRUE. Default is FALSE.
`arrow.len`	size of the arrows. Default is based on plotting region (min(par()$pin[2]/30,par()$pin[1]/40).

`arrow.lwd`	width/thickness of arrows. Default is arrow.len*0.3.
`arrow.cutoff`	cutoff value for plotting arrows. z-values that fall below the `arrow.cutoff` quantile will not be plotted. Default is 0.9.
`arrow.col`	Color of arrows. Default is `black`.
`xlim`	the x limits. The default is `NULL`.
`ylim`	the y limits. The default is `NULL`.
`zlim`	the z limits. The default is `NULL`.
`xaxt`	Add x-axis? The default is `s`; use `n` for none.
`yaxt`	Add y-axis? The default is `s`; use `n` for none.
`form`	format to use to display dates on the x-axis. Default is '%Y' for 4-digit year. See `?Date` for other valid formats.
`...`	other parameters.

Details

Arrows pointing to the right mean that `x` and `y` are in phase.

Arrows pointing to the left mean that `x` and `y` are in anti-phase.

Arrows pointing up mean that `y` leads `x` by $\pi/2$.

Arrows pointing down mean that `x` leads `y` by $\pi/2$.

Author(s)

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Code based on WTC MATLAB package written by Aslak Grinsted.

References

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Liu, Y., X. San Liang, and R. H. Weisberg. 2007. Rectification of the Bias in the Wavelet Power Spectrum. *Journal of Atmospheric and Oceanic Technology* 24:2093-2102.

See Also

`image.plot`

Examples

```
t1=cbind(1:100, rnorm(100))
t2=cbind(1:100, rnorm(100))
## Continuous wavelet transform
wt.t1=wt(t1)
## Plot power
## Make room to the right for the color bar
par(oma=c(0, 0, 0, 1), mar=c(5, 4, 4, 5) + 0.1)
plot(wt.t1, plot.cb=TRUE, plot.phase=FALSE)

## Cross-wavelet transform
xwt.t1t2=xwt(t1, t2)
## Plot cross-wavelet
par(oma=c(0, 0, 0, 1), mar=c(5, 4, 4, 5) + 0.1)
plot(xwt.t1t2, plot.cb=TRUE)

## Example of bias-correction
t1=sin(seq(from=0, to=2*5*pi, length=1000))
t2=sin(seq(from=0, to=2*15*pi, length=1000))
t3=sin(seq(from=0, to=2*40*pi, length=1000))
## This aggregate time series should have the same power at three distinct periods
s=t1+t2+t3
## Compare plots to see bias-effect on CWT: biased power spectrum artificially
## reduces the power of higher-frequency fluctuations.
wt1=wt(cbind(1:1000, s))
par(mfrow=c(1,2))
plot(wt1, type="power.corr.norm", main="Bias-corrected")
plot(wt1, type="power.norm", main="Biased")

## Compare plots to see bias-effect on XWT: biased power spectrum artificially
## reduces the power of higher-frequency fluctuations.
x1=xwt(cbind(1:1000, s), cbind(1:1000, s))
par(mfrow=c(1,2))
plot(x1, type="power.corr.norm", main="Bias-corrected")
plot(x1, type="power.norm", main="Biased")
```

---

pwtc *Compute partial wavelet coherence*

Description

Compute partial wavelet coherence between `y` and `x1` by partialling out the effect of `x2`

Usage

```
pwtc (y, x1, x2, pad = TRUE, dj = 1/12, s0 = 2 * dt, J1 = NULL,
     max.scale = NULL, mother = c("morlet", "paul", "dog"), param = -1,
     lag1 = NULL, sig.level = 0.95, sig.test = 0, nrands = 300, quiet = FALSE)
```

Arguments

y	time series y in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
x1	time series x1 in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
x2	time series x2 whose effects should be partialled out in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
pad	pad the values will with zeros to increase the speed of the transform. Default is TRUE.
dj	spacing between successive scales. Default is 1/12.
s0	smallest scale of the wavelet. Default is 2*dt.
J1	number of scales - 1.
max.scale	maximum scale. Computed automatically if left unspecified.
mother	type of mother wavelet function to use. Can be set to morlet, dog, or paul. Default is morlet. Significance testing is only available for morlet wavelet.
param	nondimensional parameter specific to the wavelet function.
lag1	vector containing the AR(1) coefficient of each time series.
sig.level	significance level. Default is 0.95.
sig.test	type of significance test. If set to 0, use a regular χ^2 test. If set to 1, then perform a time-average test. If set to 2, then do a scale-average test.
nrands	number of Monte Carlo randomizations. Default is 300.
quiet	Do not display progress bar. Default is FALSE

Value

Return a biwavelet object containing:

coi	matrix containg cone of influence
wave	matrix containing the cross-wavelet transform of y and x1
rsq	matrix of partial wavelet coherence between y and x1 (with x2 partialled out)
phase	matrix of phases between y and x1
period	vector of periods
scale	vector of scales
dt	length of a time step
t	vector of times
xaxis	vector of values used to plot xaxis
s0	smallest scale of the wavelet
dj	spacing between successive scales
y.sigma	standard deviation of y

`x1.sigma`	standard deviation of `x1`
`mother`	mother wavelet used
`type`	type of `biwavelet` object created (`pwtc`)
`signif`	matrix containg `sig.level` percentiles of wavelet coherence based on the Monte Carlo AR(1) time series

Note

The Monte Carlo randomizations can be extremely slow for large datasets. For instance, 1000 randomizations of a dataset consisting of 1000 samples will take ~30 minutes on a 2.66 GHz dual-core Xeon processor.

Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Code based on WTC MATLAB package written by Aslak Grinsted.

References

Aguiar-Conraria, L., and M. J. Soares. 2013. The Continuous Wavelet Transform: moving beyond uni- and bivariate analysis. *Journal of Economic Surveys* In press.

Cazelles, B., M. Chavez, D. Berteaux, F. Menard, J. O. Vik, S. Jenouvrier, and N. C. Stenseth. 2008. Wavelet analysis of ecological time series. *Oecologia* 156:287-304.

Grinsted, A., J. C. Moore, and S. Jevrejeva. 2004. Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics* 11:561-566.

Ng, E. K. W., and J. C. L. Chan. 2012. Geophysical applications of partial wavelet coherence and multiple wavelet coherence. *Journal of Atmospheric and Oceanic Technology* 29:1845-1853.

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

Torrence, C., and P. J. Webster. 1998. The annual cycle of persistence in the El Nino/Southern Oscillation. *Quarterly Journal of the Royal Meteorological Society* 124:1985-2004.

Examples

```
y=cbind(1:100, rnorm(100))
x1=cbind(1:100, rnorm(100))
x2=cbind(1:100, rnorm(100))
## Partial wavelet coherence of y and x1
pwtc.yx1=pwtc(y, x1, x2, nrands=0)
## Partial wavelet coherence of y and x2
pwtc.yx2=pwtc(y, x2, x1, nrands=0)
## Plot partial wavelet coherence and phase difference (arrows)
## Make room to the right for the color bar
par(mfrow=c(2,1), oma=c(4, 0, 0, 1), mar=c(1, 4, 4, 5), mgp = c(1.5, 0.5, 0))
plot(pwtc.yx1, xlab="", plot.cb=TRUE, main="Partial wavelet coherence of y and x1 | x2")
plot(pwtc.yx2, plot.cb=TRUE, main="Partial wavelet coherence of y and x2 | x1")
```

`smooth.wavelet` *Smooth wavelet in both the time and scale domains*

Description

The time smoothing uses a filter given by the absolute value of the wavelet function at each scale, normalized to have a total weight of unity, which is a Gaussian function for the Morlet wavelet. The scale smoothing is done with a boxcar function of width 0.6, which corresponds to the decorrelation scale of the Morlet wavelet.

Usage

```
smooth.wavelet (wave, dt, dj, scale)
```

Arguments

`wave`	wavelet coefficients
`dt`	size of time steps
`dj`	number of octaves per scale
`scale`	wavelet scales

Value

Returns the smoothed wavelet.

Note

This function is used internally for computing wavelet coherence. It is only appropriate for the morlet wavelet.

Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Code based on WTC MATLAB package written by Aslak Grinsted.

References

Torrence, C., and P. J. Webster. 1998. The annual cycle of persistence in the El Nino/Southern Oscillation. *Quarterly Journal of the Royal Meteorological Society* 124:1985-2004.

Examples

```
## Not run: smooth.wt1=smooth.wavelet(wave, dt, dj, scale)
```

wclust — *Compute dissimilarity between multiple wavelet spectra*

Description

Compute dissimilarity between multiple wavelet spectra

Usage

```
wclust (w.arr)
```

Arguments

`w.arr`	`N x p x t` array of wavelet spectra where `N` is the number of wavelet spectra to be compared, `p` is the number of periods in each wavelet spectrum and `t` is the number of time steps in each wavelet spectrum.

Value

Returns a list containing:

`diss.mat`	square dissimilarity matrix
`dist.mat`	(lower triangular) distance matrix

Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

References

Rouyer, T., J. M. Fromentin, F. Menard, B. Cazelles, K. Briand, R. Pianet, B. Planque, and N. C. Stenseth. 2008. Complex interplays among population dynamics, environmental forcing, and exploitation in fisheries. *Proceedings of the National Academy of Sciences* 105:5420-5425.

Rouyer, T., J. M. Fromentin, N. C. Stenseth, and B. Cazelles. 2008. Analysing multiple time series and extending significance testing in wavelet analysis. *Marine Ecology Progress Series* 359:11-23.

Examples

```
t1=cbind(1:100, sin(seq(from=0, to=10*2*pi, length.out=100)))
t2=cbind(1:100, sin(seq(from=0, to=10*2*pi, length.out=100)+0.1*pi))
t3=cbind(1:100, rnorm(100))

## Compute wavelet spectra
wt.t1=wt(t1)
wt.t2=wt(t2)
wt.t3=wt(t3)

## Store all wavelet spectra into array
```

```
w.arr=array(NA, dim=c(3, NROW(wt.t1$wave), NCOL(wt.t1$wave)))
w.arr[1, , ]=wt.t1$wave
w.arr[2, , ]=wt.t2$wave
w.arr[3, , ]=wt.t3$wave

## Compute dissimilarity and distance matrices
w.arr.dis=wclust(w.arr)
plot(hclust(w.arr.dis$dist.mat, method="ward"), sub="", main="",
        ylab="Dissimilarity", hang=-1)
```

`wdist` *Compute dissimilarity between two wavelet spectra*

Description

Compute dissimilarity between two wavelet spectra

Usage

```
wdist (wt1, wt2, cutoff = 0.99)
```

Arguments

`wt1`	`power`, `wave` or `rsq` matrix from `biwavelet` object generated by `wt`, `xwt`, or `wtc`.
`wt2`	`power`, `wave` or `rsq` matrix from `biwavelet` object generated by `wt`, `xwt`, or `wtc`.
`cutoff`	cutoff value used to compute dissimilarity. Only orthogonal axes that contribute more than `1-cutoff` to the total covariance between the two wavelet spectra will be used to compute their dissimilarity. Default is `0.99`.

Value

Returns wavelet dissimilarity.

Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

References

Rouyer, T., J. M. Fromentin, F. Menard, B. Cazelles, K. Briand, R. Pianet, B. Planque, and N. C. Stenseth. 2008. Complex interplays among population dynamics, environmental forcing, and exploitation in fisheries. *Proceedings of the National Academy of Sciences* 105:5420-5425.

Rouyer, T., J. M. Fromentin, N. C. Stenseth, and B. Cazelles. 2008. Analysing multiple time series and extending significance testing in wavelet analysis. *Marine Ecology Progress Series* 359:11-23.

Examples

```
t1=cbind(1:100, sin(seq(from=0, to=10*2*pi, length.out=100)))
t2=cbind(1:100, sin(seq(from=0, to=10*2*pi, length.out=100)+0.1*pi))
## Compute wavelet spectra
wt.t1=wt(t1)
wt.t2=wt(t2)
## Compute dissimilarity
wdist(wt.t1$wave, wt.t2$wave)
```

---

wt — *Compute wavelet transform*

Description

Continuous wavelet transform

Usage

```
wt (d, pad = TRUE, dt = NULL, dj = 1/12, s0 = 2 * dt, J1 = NULL,
    max.scale = NULL, mother = c("morlet", "paul", "dog"),
    param = -1, lag1 = NULL, sig.level = 0.95, sig.test = 0, do.sig=TRUE)
```

Arguments

`d`	time series in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
`pad`	pad the values will with zeros to increase the speed of the transform. Default is TRUE.
`dt`	length of a time step.
`dj`	spacing between successive scales. Default is 1/12.
`s0`	smallest scale of the wavelet. Default is `2*dt`
`J1`	number of scales - 1.
`max.scale`	maximum scale. Computed automatically if left unspecified.
`mother`	type of mother wavelet function to use. Can be set to `morlet`, `dog`, or `paul`. Default is `morlet`.
`param`	nondimensional parameter specific to the wavelet function.
`lag1`	AR(1) coefficient of time series used to test for significant patterns.
`sig.level`	significance level. Default is 0.95.
`sig.test`	type of significance test. If set to 0, use a regular χ^2 test. If set to 1, then perform a time-average test. If set to 2, then do a scale-average test.
`do.sig`	perform significance testing if `TRUE`. Default is `TRUE`.

Value

Returns a `biwavelet` object containing:

`coi`	matrix containg cone of influence
`wave`	matrix containing the wavelet transform
`power`	matrix of power
`power.corr`	matrix of bias-corrected power using the method described by `Liu et al. (2007)`
`phase`	matrix of phases
`period`	vector of periods
`scale`	vector of scales
`dt`	length of a time step
`t`	vector of times
`xaxis`	vector of values used to plot xaxis
`s0`	smallest scale of the wavelet
`dj`	spacing between successive scales
`sigma2`	variance of time series
`mother`	mother wavelet used
`type`	type of `biwavelet` object created (`wt`)
`signif`	matrix containg significance levels

Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Code based on wavelet MATLAB program written by Christopher Torrence and Gibert P. Compo.

References

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

Liu, Y., X. San Liang, and R. H. Weisberg. 2007. Rectification of the Bias in the Wavelet Power Spectrum. *Journal of Atmospheric and Oceanic Technology* 24:2093-2102.

Examples

```
t1=cbind(1:100, rnorm(100))
## Continuous wavelet transform
wt.t1=wt(t1)
## Plot power
## Make room to the right for the color bar
par(oma=c(0, 0, 0, 1), mar=c(5, 4, 4, 5) + 0.1)
plot(wt.t1, plot.cb=TRUE, plot.phase=FALSE)
```

`wt.bases` *Compute wavelet*

Description

Computes the wavelet as a function of Fourier frequency.

Usage

```
wt.bases (mother = c("morlet", "paul", "dog"), k, scale, param = -1)
```

Arguments

`mother`	type of mother wavelet function to use. Can be set to `morlet`, `dog`, or `paul`. Default is `morlet`.
`k`	vector of frequencies at which to calculate the wavelet.
`scale`	the wavelet scale.
`param`	nondimensional parameter specific to the wavelet function.

Value

Returns a list containing:

`daughter`	wavelet function
`fourier.factor`	ratio of fourier period to scale
`coi`	cone of influence
`dof`	degrees of freedom for each point in wavelet power

Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Code based on wavelet MATLAB program written by Christopher Torrence and Gibert P. Compo.

References

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

Examples

```
## Not run: wb=wt.bases(mother, k, scale[a1], param)
```

wt.sig *Determine significance of wavelet transform*

Description

Determine significance of wavelet transform

Usage

```
wt.sig (d, dt, scale, sig.test = 0, sig.level = 0.95, dof = 2,
        lag1 = NULL, mother = c("morlet", "paul", "dog"),
        param = -1, sigma2 = NULL)
```

Arguments

d	time series in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
dt	length of a time step.
scale	the wavelet scale.
sig.test	type of significance test. If set to 0, use a regular χ^2 test. If set to 1, then perform a time-average test. If set to 2, then do a scale-average test.
sig.level	significance level. Default is 0.95.
dof	degrees of freedom for each point in wavelet power.
lag1	AR(1) coefficient of time series used to test for significant patterns.
mother	type of mother wavelet function to use. Can be set to `morlet`, `dog`, or `paul`. Default is `morlet`.
param	nondimensional parameter specific to the wavelet function.
sigma2	variance of time series

Value

Returns a list containing:

signif	vector containing significance level for each scale
signif	vector of red-noise spectrum for each period

Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Code based on wavelet MATLAB program written by Christopher Torrence and Gibert P. Compo.

References

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

Examples

```
## Not run: wt.sig (d, dt, scale, sig.test, sig.level, lag1, dof=-1,
##  mother=morlet, sigma2=1)
```

`wtc` *Compute wavelet coherence*

Description

Compute wavelet coherence

Usage

```
wtc (d1, d2, pad = TRUE, dj = 1/12, s0 = 2 * dt, J1 = NULL,
     max.scale = NULL, mother = c("morlet", "paul", "dog"), param = -1,
     lag1 = NULL, sig.level = 0.95, sig.test = 0, nrands = 300, quiet = FALSE)
```

Arguments

`d1`	time series 1 in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
`d2`	time series 2 in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
`pad`	pad the values will with zeros to increase the speed of the transform. Default is TRUE.
`dj`	spacing between successive scales. Default is 1/12.
`s0`	smallest scale of the wavelet. Default is `2*dt`.
`J1`	number of scales - 1.
`max.scale`	maximum scale. Computed automatically if left unspecified.
`mother`	type of mother wavelet function to use. Can be set to `morlet`, `dog`, or `paul`. Default is `morlet`. Significance testing is only available for `morlet` wavelet.
`param`	nondimensional parameter specific to the wavelet function.
`lag1`	vector containing the AR(1) coefficient of each time series.
`sig.level`	significance level. Default is `0.95`.
`sig.test`	type of significance test. If set to 0, use a regular χ^2 test. If set to 1, then perform a time-average test. If set to 2, then do a scale-average test.
`nrands`	number of Monte Carlo randomizations. Default is 300.
`quiet`	Do not display progress bar. Default is `FALSE`

Value

Return a `biwavelet` object containing:

- `coi` matrix containg cone of influence
- `wave` matrix containing the cross-wavelet transform
- `wave.corr` matrix containing the bias-corrected cross-wavelet transform using the method described by `Veleda et al. (2012)`
- `power` matrix of power
- `power.corr` matrix of bias-corrected cross-wavelet power using the method described by `Veleda et al. (2012)`
- `rsq` matrix of wavelet coherence
- `phase` matrix of phases
- `period` vector of periods
- `scale` vector of scales
- `dt` length of a time step
- `t` vector of times
- `xaxis` vector of values used to plot xaxis
- `s0` smallest scale of the wavelet
- `dj` spacing between successive scales
- `d1.sigma` standard deviation of time series 1
- `d2.sigma` standard deviation of time series 2
- `mother` mother wavelet used
- `type` type of `biwavelet` object created (`wtc`)
- `signif` matrix containg `sig.level` percentiles of wavelet coherence based on the Monte Carlo AR(1) time series

Note

The Monte Carlo randomizations can be extremely slow for large datasets. For instance, 1000 randomizations of a dataset consisting of 1000 samples will take ~30 minutes on a 2.66 GHz dual-core Xeon processor.

Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Code based on WTC MATLAB package written by Aslak Grinsted.

References

Cazelles, B., M. Chavez, D. Berteaux, F. Menard, J. O. Vik, S. Jenouvrier, and N. C. Stenseth. 2008. Wavelet analysis of ecological time series. *Oecologia* 156:287-304.

Grinsted, A., J. C. Moore, and S. Jevrejeva. 2004. Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics* 11:561-566.

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

Torrence, C., and P. J. Webster. 1998. The annual cycle of persistence in the El Nino/Southern Oscillation. *Quarterly Journal of the Royal Meteorological Society* 124:1985-2004.

Veleda, D., R. Montagne, and M. Araujo. 2012. Cross-Wavelet Bias Corrected by Normalizing Scales. *Journal of Atmospheric and Oceanic Technology* 29:1401-1408.

Examples

```
t1=cbind(1:100, rnorm(100))
t2=cbind(1:100, rnorm(100))
## Wavelet coherence
wtc.t1t2=wtc(t1, t2, nrands=10)
## Plot wavelet coherence and phase difference (arrows)
## Make room to the right for the color bar
par(oma=c(0, 0, 0, 1), mar=c(5, 4, 4, 5) + 0.1)
plot(wtc.t1t2, plot.cb=TRUE)
```

---

wtc.sig *Determine significance of wavelet coherence*

Description

Determine significance of wavelet coherence

Usage

```
wtc.sig (nrands = 300, lag1, dt, ntimesteps, pad = TRUE, dj = 1/12, s0, J1,
         max.scale=NULL, mother = c("morlet", "paul", "dog"), sig.level = 0.95,
         quiet = FALSE)
```

Arguments

`nrands`	number of Monte Carlo randomizations. Default is 300.
`lag1`	vector containing the AR(1) coefficient of each time series.
`dt`	length of a time step.
`ntimesteps`	number of time steps in time series.
`pad`	pad the values will with zeros to increase the speed of the transform. Default is `TRUE`.
`dj`	spacing between successive scales. Default is 1/12.

`s0`	smallest scale of the wavelet. Default is `2*dt`
`J1`	number of scales - 1.
`max.scale`	maximum scale
`mother`	type of mother wavelet function to use. Can be set to `morlet`, `dog`, or `paul`. Default is `morlet`. Significance testing is only available for `morlet` wavelet.
`sig.level`	significance level to compute. Default is `0.95`
`quiet`	Do not display progress bar. Default is `FALSE`

Value

Returns significance matrix containing the `sig.level` percentile of wavelet coherence at each time step and scale.

Note

The Monte Carlo randomizations can be extremely slow for large datasets. For instance, 1000 randomizations of a dataset consisting of 1000 samples will take ~30 minutes on a 2.66 GHz dual-core Xeon processor.

Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Code based on WTC MATLAB package written by Aslak Grinsted.

References

Cazelles, B., M. Chavez, D. Berteaux, F. Menard, J. O. Vik, S. Jenouvrier, and N. C. Stenseth. 2008. Wavelet analysis of ecological time series. *Oecologia* 156:287-304.

Grinsted, A., J. C. Moore, and S. Jevrejeva. 2004. Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics* 11:561-566.

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

Torrence, C., and P. J. Webster. 1998. The annual cycle of persistence in the El Nino/Southern Oscillation. *Quarterly Journal of the Royal Meteorological Society* 124:1985-2004.

Examples

```
## Not run: wtcsig=wtc.sig(nrands, lag1 = c(d1.ar1, d2.ar1), dt,
##                          pad, dj, J1, s0, mother = morlet)
```

xwt Compute cross-wavelet

Description

Compute cross-wavelet

Usage

```
xwt (d1, d2, pad = TRUE, dj = 1/12, s0 = 2 * dt, J1 = NULL,
     max.scale = NULL, mother = c("morlet", "paul", "dog"),
     param = -1, lag1 = NULL, sig.level = 0.95, sig.test = 0)
```

Arguments

d1	time series 1 in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
d2	time series 2 in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
pad	pad the values will with zeros to increase the speed of the transform. Default is TRUE.
dj	spacing between successive scales. Default is 1/12.
s0	smallest scale of the wavelet. Default is 2*dt
J1	number of scales - 1.
max.scale	maximum scale. Computed automatically if left unspecified.
mother	type of mother wavelet function to use. Can be set to morlet, dog, or paul. Default is morlet. Significance testing is only available for morlet wavelet.
param	nondimensional parameter specific to the wavelet function.
lag1	vector containing the AR(1) coefficient of each time series.
sig.level	significance level. Default is 0.95.
sig.test	type of significance test. If set to 0, use a regular χ^2 test. If set to 1, then perform a time-average test. If set to 2, then do a scale-average test.

Value

Returns a biwavelet object containing:

coi	matrix containg cone of influence
wave	matrix containing the cross-wavelet transform
wave.corr	matrix containing the bias-corrected cross-wavelet transform using the method described by Veleda et al. (2012)
power	matrix of power

`power.corr`	matrix of bias-corrected cross-wavelet power using the method described by `Veleda et al. (2012)`
`phase`	matrix of phases
`period`	vector of periods
`scale`	vector of scales
`dt`	length of a time step
`t`	vector of times
`xaxis`	vector of values used to plot xaxis
`s0`	smallest scale of the wavelet
`dj`	spacing between successive scales
`d1.sigma`	standard deviation of time series 1
`d2.sigma`	standard deviation of time series 2
`mother`	mother wavelet used
`type`	type of `biwavelet` object created (`xwt`)
`signif`	matrix containg significance levels

Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Code based on WTC MATLAB package written by Aslak Grinsted.

References

Cazelles, B., M. Chavez, D. Berteaux, F. Menard, J. O. Vik, S. Jenouvrier, and N. C. Stenseth. 2008. Wavelet analysis of ecological time series. *Oecologia* 156:287-304.

Grinsted, A., J. C. Moore, and S. Jevrejeva. 2004. Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics* 11:561-566.

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

Torrence, C., and P. J. Webster. 1998. The annual cycle of persistence in the El Nino/Southern Oscillation. *Quarterly Journal of the Royal Meteorological Society* 124:1985-2004.

Veleda, D., R. Montagne, and M. Araujo. 2012. Cross-Wavelet Bias Corrected by Normalizing Scales. *Journal of Atmospheric and Oceanic Technology* 29:1401-1408.

Examples

```
t1=cbind(1:100, rnorm(100))
t2=cbind(1:100, rnorm(100))
## Cross-wavelet
xwt.t1t2=xwt(t1, t2)
## Plot cross-wavelet and phase difference (arrows)
plot(xwt.t1t2, plot.cb=TRUE, plot.phase=TRUE)
## Real data
data(enviro.data)
```

```
## Cross-wavelet of MEI and NPGO
xwt.mei.npgo=xwt(subset(enviro.data, select=c("date", "mei")),
        subset(enviro.data, select=c("date", "npgo")))
## Make room to the right for the color bar
par(oma=c(0, 0, 0, 1), mar=c(5, 4, 4, 5) + 0.1)
plot(xwt.mei.npgo, plot.cb=TRUE, plot.phase=TRUE)
```

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