WAVELET-BASED ANALYSIS OF TIME SERIES: AN EXPORT FROM ENGINEERING TO FINANCE

John R. Norsworthy, Ding Li, Rifat Gorener
Center for Financial Technology
Lally School of Management and Technology
Rensselaer Polytechnic Institute
Troy NY 12180
2000 IEEE International Engineering Management Society Conference
August 10-13, 2000
Albuquerque, New Mexico

Abstract

Wavelet analysis is an adaptation of a class signal processing techniques that has been widely applied in the past 10 years to such other fields as data compression and image processing. It operates in a manner similar to Fourier analysis, in that it permits the transformation of information from a time series domain to the frequency domain. Applications are just beginning in finance. In this paper we show how basic wavelet techniques can shed light on a central problem of finance, the modeling of asset prices. Wavelet analysis identifies the relationship between asset returns and returns on a market portfolio as primarily located in higher frequencies. This simple conclusion suggests an important agenda for future wavelet-based research into asset pricing.

Introduction: Wavelet Analysis

Wavelet analysis is a new development in applied mathematics about ten years in duration. Brief historical reviews of wavelets can be found in Meyer (1993) and Graps (1995). Y.Meyer, I.Daubechies, S.Mallat and others developed the theory of wavelets in the late 1980's. Wavelet analysis has been applied to many situations with favorable results. Wavelet analysis has remarkable impact mainly on three fields, signal processing (Donoho and Johnstone 1998), image analysis, and data compression (Bradley, Brislawn et al. 1993). Wavelets also have special statistic properties (Donoho and Johnstone 1995), which can be used in adaptive filtering and smoothing. Many researchers are expanding its application on other fields, such as financial engineering (Ramsey and Lampart 1998). Following is a brief review of wavelet methods and its applications. Our empirical study is developed on S-PLUS (Bruce and Gao 1996) wavelet module platform.

Generally speaking, wavelet analysis is a refinement of Fourier analysis. The Fourier Transform (FT) processes the raw signal (a time series) by using a mathematical transformation, which transforms the signal from time domain into frequency spectrum. The processed signal tells us how many frequencies and how much (energy) of each frequency exists in the raw signal, but it does not give us the time information (where a particular frequency appears in the time domain). If the signal is stationary, we don't need the "location" information, but in the real world most of our data sets are non-stationary.

The Windowed Fourier Transform (WFT) can locate the window of the data that are transformed in time.[1] The WFT only transforms part of a signal and that segment of signal is small enough that we can assume that portion of signal is stationary. By using a particular window function and shifting the window along the time dimension of the signal we can localize the frequency in the signal, and we obtain a time-frequency representation of the signal. The transformation coefficients are the amplitudes of different frequencies at different times. But WFT has a problem, known as the resolution problem. The Heisenberg uncertainty principle states that we cannot know the exact time-frequency representation of a signal. We can know however the bands of frequencies associated with the time intervals in the signal. Here we have to have a requirement on the width of the window function. The wavelet transformation is a solution to the problem.

The Continuous Wavelet Transform (CWT) uses a particular wavelet waveform, which has some required or desired properties, as does the window function (which applies the same logic as WFT). There are two main differences between WT and WFT. First, in CWT we use a wavelet to replace the cosine in WFT, which will give us many spikes in the decomposed signal. Second, the most significant characteristic of a particular CWT is the width of the window, which is changed for different frequencies.

[1] The WFT is attributed to Denis Gabor (1946).

As we noted for CWT, the windowed signal multiplied with window function is then continuously integrable across time. That is not a discrete transformation and contains highly redundant information.

The Discrete Wavelet Transform (DWT) reduces the signal sample by a factor of two each time according to Nyquist's rule, and then decomposes (resolves) the signal at different frequency bands with different resolution for each frequency band. Each frequency is localized in a particular place in the time domain, according to that band's resolution.

The orthogonal wavelet series approximation to a time series *f(t)* is given by

$$f(t) \approx \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t) \qquad (1)$$

J is the number of levels (scales or frequencies), and k ranges from 1 to the number of coefficients in the specified component. $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$ are the approximating orthogonal wavelet functions given by

$$\phi_{j,k}(t) = 2^{-j/2}\phi\left(\frac{t-2^j k}{2^j}\right) \qquad j = 1 \; to \; J \qquad (2)$$

$$\psi_{j,k}(t) = 2^{-j/2}\psi\left(\frac{t-2^j k}{2^j}\right) \qquad j = 1 \; to \; J \qquad (3)$$

The wavelet coefficients are given by

$$s_{J,k} \approx \int \phi_{J,k}(t) f(t) dt \qquad (4)$$

$$d_{j,k} \approx \int \psi_{j,k}(t) f(t) dt \qquad (5)$$

The most common application of wavelets is in signal processing.[2] In this sense, a signal is a sequence of numerical measurements. In signal processing, the main or basic tasks are coding the signal (possibly compressing or encrypting it) and transmitting the signal. The objective of this process is to optimize the efficiencies of signal coding, transmitting, and reconstruction. Another emphasis of the wavelets literature of signal processing is denoising, i.e., extracting the true signal from the observations spoiled by noise. Wavelet analysis performed well in those applications. Data compression is another field in which wavelet analysis has impressive results. The purpose of data compression is to transform large data set into smaller data set, which contains the most important elements and can be reconstructed later with only a minimum of loss. FBI fingerprints storage is a success application of wavelets.

In statistical applications, wavelet analysis has both linear smoothing and nonlinear smoothing properties. Donoho (1993) developed the thresholding technique, a procedure of estimating an unknown signal from observed data by using the smoothing statistic properties of wavelets.

The Finance Question: Are the Relationships between Asset and Market Returns Chiefly at High Frequencies?

Some of the uses of wavelet analysis in finance seem to be primarily the deployment of wavelet technique to financial data; that is, bringing financial data into the world of wavelets rather than bringing wavelets to address the traditional issues in finance. This paper attempts the latter. We apply wavelets to analyze the relationship between the return on an asset and the return on the market portfolio, or investment alternative. This relationship lies at the core of asset pricing research. Our findings are interesting though not surprising: movements in the asset return are associated strongly with high frequency movements in the market return, while low frequency movements are much more weakly linked.

The classic asset pricing model is the Capital Asset Pricing Model (Sharpe 1964, Lintner 1965). A more general model with the same mathematical formulation, usually called the market model[3], is written

$$r_i = \alpha + \beta r_M + e_i \qquad (6)$$

where r_i is the return on asset *i*, r_M is the return on the alternative investment in the market. Returns are calculated as

$$R_{it} = (P_{it} - P_{it-1} + d_{it}) / P_{it-1} \qquad (7)$$

[2] S-PLUS, which has good support for wavelet analysis as does Matlab, uses a fast pyramid algorithm to calculate the coefficients of the wavelet series approximation for the discrete signal $f_1, \ldots, f_n$ of finite length.

[3] See e.g. Copeland and Weston (1992).

and similarly for r_M. The log transformation accommodates negative returns while providing instantaneous growth rates:

$$r_{it} = \ln(1 + R_{it}) \qquad (8)$$

The β coefficient in (6) is THE β (beta) that is often discussed in the business as well as financial press. β measures the responsiveness of the asset return to changes in the market return, and is used in classical portfolio selection. Value Line and other financial information companies regularly publish beta coefficients calculated for individual stocks.

Our objective for an initial foray into wavelet analysis is to parse the r_i to r_M relationship into relationships at different frequencies, and to observe whether there are different quantitative links at different frequencies. Another way to state the issue is: does β change for different frequencies? While it is simple, the question has at least two important implications. If β for individual assets changes across different frequencies, and the market influence is greatest (β is largest) for high frequencies, then

- Movements that are specific to asset *i* will be captured primarily in lower frequency movements.
- Filtering techniques that screen out high frequency movements in r_i will make it easier to detect and measure the effects of unusual events, such as acquisitions of a company by another or changes in dividend policy.

Study Design

To address this question, we estimated the β coefficients for five different frequencies – two, four, eight, 16, and 32 day frequencies[4] – for 99 stocks with the Standard and Poor's (S&P) 500 Index representing the market. The estimates were based on daily returns for the year 1998 taken from the CRSP database[5] for 99 stocks, 29 from the Dow-Jones 30 industrials, 30 drawn randomly from the S&P 400 middle capitalization (mid cap) index list of companies, and 40 drawn randomly from the S&P 400 small capitalization (small cap) index list.[6] Ordinary least squares is the estimation method applied to estimate the market model.[7]

Wavelet analysis transforms a series from the time domain to the frequency (or scale) domain. For a discrete variable r_{it} with $t = 1 \ldots n$ observations, coefficients in the frequency domain are extracted for "crystals" associated with powers of 2: *n/2, n/4, n/8* … etc. In fact, there are (up to) $k = n/2$ (non-zero) coefficients in crystal 1, denoted d_1, $k = n/4$ coefficients in crystal 2, denoted d_2, and in general, $k = n/2^j$ in crystal d_j where j is the crystal index in equations (1) through (5) above.

At each stage of decomposition into J crystals d_j, $j = 1 \ldots J$ and the residue s_J the entire time series (signal) is replicated in

$$\hat{f}^J = D_1 + \ldots + D_J + S_J \qquad (9)$$

where D_j is the recomposed series in the time domain from the crystal d_j and S_J is the recomposition of the residue s_J. Now let $S_{j,t}$ and $D_{j,t}$ represent the t[th] elements in S_j and D_j respectively and define the contribution of frequency $J + 1$ by the (Euclidean) distance function

$$\begin{aligned}\delta(J-1, J) &= \left[\sum_{t=1}^{n} (D_{J,t})^2\right]^{\frac{1}{2}} \\ &= \left[\sum_{t=1}^{n} \left(S_{J,t} - \sum_{j=1}^{J-1} D_{j,t}\right)^2\right]^{\frac{1}{2}}\end{aligned} \qquad (10)$$

In this study, five crystals were extracted for each of the 99 stock returns series and for the S&P 500 Index returns. To see the contribution of crystal d_j for series r_i, the series is first transformed (decomposed) from the time to the frequency domain, and then d_j is reconstituted (recomposed) into the time domain. The resulting series in the time domain is the contribution of frequency j to the original series or the component of r_i that has frequency j, a contribution in the literal sense of equation (9).

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[4] A few extra days were added from the end of 1997 to bring the number of trading days to 256, an integral power of two. Typical wavelets software maps time series into sampling frequencies defined on powers of two.

[5] CRSP, the Center for Research in Securities Prices at the University of Chicago, makes available a large menu of daily and monthly stock prices, returns, volumes, etc. for the stocks listed on the NYSE, AMEX and NASDAQ markets. CRSP is the definitive source for such data.

[6] There are only 29 companies from the Dow-Jones list because one company – Chrysler – was removed from the list in November 1998 because it was judged to be no longer an American company (after its acquisition by Daimler-Benz).

[7] We cannot report here the individual results of the 495 estimations of the market model. The results are available on request.

After obtaining the recomposed time series for each frequency, the procedure was then to estimate by OLS the equations for each stock i and each (recomposed) crystal j

$$r_i^j = \alpha_i^j + \beta_i^j r_M^j + e_i^j = \alpha_i^j + \beta_i^j D_M^j + e_i^j \qquad j = 1 \text{ to } 6 \qquad (11)$$

which is quite similar in spirit to the market model.

Our expected results are as follows:

1. The effect of the market return on an individual asset's return will be greater in the higher frequencies than in the lower. One straightforward way to address this issue is in terms of the distance function in equation 10 above. The mean contributions of D_i^j should generally decline as j rises toward J.
2. Also addressing the frequency focus of the asset-market relationship: the effect of successively lower frequencies in the market returns on the corresponding frequencies in the asset returns, as measured by the explanatory power (adjusted R-squared) of β -type regressions in equation 11, will generally decline as the frequency **de**creases.

Study Results

Table 1 shows the results of estimating equation 11 for the 99 companies. The mean, variance, skew and kurtosis are shown for the total of 99 values of the β_i^j coefficients, which measure the effect of frequency j movements in the market return on the return for asset i. These β_i^j coefficients may be thought of as frequency-specific β s for each asset. Table 1 demonstrates the relationship that was expected: on average, across 99 stocks, the betas decline as the frequency declines. This evidence supports the proposition that the major part of the market's influence on individual asset prices is at higher frequencies. Once again, there is a clear tendency for

Table 1. Wavelet: "haar", Market Model Regressions by Recomposed Crystals

Total			Beta					R-sqr				
Firm	**Beta**	**R-sqr**	**D1**	**D2**	**D3**	**D4**	**D5**	**D1**	**D2**	**D3**	**D4**	**D5**
Mean	0.7785	0.1811	0.7324	0.7824	0.7996	1.0701	0.7175	0.0897	0.0504	0.0325	0.0178	0.0035
Stdev	0.3449	0.1277	0.3416	0.4734	0.4463	0.7896	1.2458	0.0696	0.0467	0.0258	0.0179	0.0047
Skew	0.3285	1.1577	0.1843	0.1884	1.0343	0.9467	0.5154	1.2868	1.3045	0.6682	1.5607	1.5226
Kurt	-0.0475	1.6217	0.2950	-0.2460	1.9136	1.5176	0.3677	1.3774	1.4033	-0.5665	3.1118	1.3300
Large			**Beta**					**R-sqr**				
Firm	**Beta**	**R-sqr**	**D1**	**D2**	**D3**	**D4**	**D5**	**D1**	**D2**	**D3**	**D4**	**D5**
Mean	0.9098	0.3084	0.8879	0.9852	0.8976	0.8158	0.6951	0.1576	0.0902	0.0497	0.0157	0.0038
Stdev	0.2392	0.1282	0.2344	0.3622	0.2750	0.6241	1.1200	0.0705	0.0506	0.0245	0.0138	0.0051
Skew	0.4425	0.8739	0.2766	0.4228	-0.6127	-0.2386	0.0795	0.5851	0.3616	0.0872	0.8355	1.5436
Kurt	0.7285	1.0095	-0.7172	-0.1038	1.0020	-0.5134	0.1031	-0.3182	-0.8501	-0.7704	-0.0837	1.6748
Middle			**Beta**					**R-sqr**				
Firm	**Beta**	**R-sqr**	**D1**	**D2**	**D3**	**D4**	**D5**	**D1**	**D2**	**D3**	**D4**	**D5**
Mean	0.8069	0.1607	0.7649	0.7554	0.9264	1.0299	0.8729	0.0794	0.0392	0.0373	0.0140	0.0040
Stdev	0.4038	0.0876	0.3923	0.4673	0.5779	0.7602	1.5057	0.0528	0.0393	0.0259	0.0122	0.0051
Skew	0.3090	0.7517	0.6829	-0.0234	0.9845	1.3885	0.6793	2.3165	2.5512	0.3026	0.5941	1.3242
Kurt	-0.6433	0.2210	-0.3574	-0.4356	0.6867	2.6408	-0.1811	6.4881	9.4437	-0.8058	-1.0154	0.3429
Small			**Beta**					**R-sqr**				
Firm	**Beta**	**R-sqr**	**D1**	**D2**	**D3**	**D4**	**D5**	**D1**	**D2**	**D3**	**D4**	**D5**
Mean	0.6621	0.1040	0.5953	0.6555	0.6335	1.2846	0.6171	0.0482	0.0298	0.0164	0.0222	0.0028
Stdev	0.3305	0.0708	0.3181	0.5088	0.3873	0.8722	1.1332	0.0353	0.0282	0.0155	0.0230	0.0040
Skew	0.7756	0.8658	-0.0336	0.7006	1.2637	0.8577	0.3100	0.6778	0.9319	1.7746	1.3151	1.7114
Kurt	1.1912	0.1776	1.0069	0.3220	2.6423	0.8208	0.7203	-0.3998	0.0617	3.8915	1.3852	2.3708

Table 2. Wavelet: "haar", Market Model Regressions by Recomposed Residues												
Total			Beta					R-sqr				
Firm	Beta	R-sqr	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5
Mean	0.7785	0.1811	0.8301	0.8798	1.0516	1.0216	1.1214	0.0970	0.0522	0.0255	0.0101	0.0094
Stdev	0.3449	0.1277	0.4013	0.4329	0.6971	0.7956	0.9117	0.0723	0.0351	0.0241	0.0115	0.0103
Skew	0.3285	1.1577	0.4370	0.8613	0.9231	1.0874	0.8152	1.0926	0.6967	1.6124	2.0528	1.6533
Kurt	-0.0475	1.6217	-0.0736	1.1847	1.4818	2.2460	1.0259	1.1808	-0.1234	3.7636	5.8691	3.4340
Large			Beta					R-sqr				
Firm	Beta	R-sqr	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5
Mean	0.9098	0.3084	0.9343	0.8812	0.8460	0.8952	0.9609	0.1557	0.0690	0.0243	0.0104	0.0094
Stdev	0.2392	0.1282	0.2984	0.2950	0.5452	0.5674	0.6403	0.0750	0.0347	0.0211	0.0107	0.0084
Skew	0.4425	0.8739	0.2043	-0.2196	0.0035	0.4250	-0.4074	0.4290	0.2511	0.5985	1.2591	0.9210
Kurt	0.7285	1.0095	0.6799	0.3281	-0.7065	-0.6936	-1.0820	0.2478	0.1378	-0.8087	0.8750	0.6741
Middle			Beta					R-sqr				
Firm	Beta	R-sqr	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5
Mean	0.8069	0.1607	0.8537	0.9561	1.0198	1.0034	1.0463	0.0874	0.0547	0.0218	0.0105	0.0085
Stdev	0.4038	0.0876	0.4638	0.5427	0.7514	0.9871	1.0262	0.0622	0.0336	0.0197	0.0144	0.0115
Skew	0.3090	0.7517	0.2526	0.7351	0.8794	1.3043	1.2596	1.6449	0.4911	1.2643	2.5645	2.5591
Kurt	-0.6433	0.2210	-0.1560	0.5266	0.3502	2.8514	2.5484	5.0942	-0.1135	1.7609	8.1743	8.5142
Small			Beta					R-sqr				
Firm	Beta	R-sqr	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5
Mean	0.6621	0.1040	0.7368	0.8214	1.2245	1.1269	1.2942	0.0617	0.0381	0.0292	0.0097	0.0102
Stdev	0.3305	0.0708	0.4042	0.4260	0.7246	0.7818	0.9784	0.0480	0.0311	0.0287	0.0099	0.0109
Skew	0.7756	0.8658	1.0007	1.0270	1.1376	0.7928	0.5262	1.2530	1.5504	1.7935	1.4776	1.1989
Kurt	1.1912	0.1776	0.4488	0.9251	2.2732	0.5585	-0.4046	1.4967	2.1801	3.8507	1.7422	0.5189

the explanatory power to decline as the frequency declines. by the adjusted R^2 coefficient averages and other distribution statistics for the 99 company returns.

The second panel of Table 1 shows the distribution of coefficients for the 29 Dow-Jones stocks, the third panel shows the 30 mid-cap stocks, and the last panel the 40 small cap stocks. There seems to be a size effect: the market model beta coefficients are on average higher for the large cap Dow-Jones stocks compared to the mid- and small cap stocks. On this basis, we would expect to observe weaker links for mid- and small cap firms to the market. In fact for each frequency, the R^2 declines generally as the size of firm declines. The exceptions are only in the lowest frequencies.

Table 2 displays another dimension of the same issue. This table shows the explanatory power of the residues of various frequencies, also shown to decline as the frequency declines. This is as we would expect: here we expect to see the explanatory power of the residuals decline as the frequency increases, and that is what we observe. Panel A shows the distribution of R^2 coefficients for the 29 Dow-Jones stocks, Panel B shows the 30 mid-cap stocks, and Panel C the 40 small cap stocks. There is some evidence of a size effect: the R^2 coefficients are on average higher and more tightly clustered for the large cap Dow-Jones stocks. Moreover, for the total panel, and large cap and mid-cap sets, the average R^2

declines monotonically with frequency, and nearly so for the small cap panel, where the explanatory power of all frequencies is quite low. This pattern in the explanatory power of the residues (statistically, the residuals) once again reinforces the finding that the higher frequencies are the primary link between asset returns and market returns.

Full detail for the β and psuedo- β estimates is shown in Table 3 for a sample of six stocks. These companies, whose ticker symbols are shown in the first column of Table 3, were chosen at random from among the 99 companies. These company models show the same characteristics as are observed in the panel averages: declining links between the asset and market returns as the frequency declines. The P-values associated with the beta coefficients correspondingly rise as the frequency declines.

Conclusions and Future Wavelet Research

Wavelet analysis demonstrates rather strong evidence that the market's influence on asset returns is principally in the high frequency movements of the asset returns. This suggests that filtering out the market "noise" will permit the investigator to focus on longer-term movements specific to the company and its industry. It also confirms – as has been apparent

Table 3. Complete Regression Results for 6 Companies							
Firm Ticker	Intercept			Regression Coefficient			R-sqr
	Intercept	S.E.	P-value	Coefficient	S.E.	P-value	
	Market Model			Market Model			
XON	0.0004	0.0009	0.7106	0.4686	0.0741	0.0000	0.1361
IP	-0.0006	0.0011	0.5604	0.8605	0.0870	0.0000	0.2782
FRX	0.0023	0.0017	0.1712	1.0025	0.1297	0.0000	0.1905
SMBL	0.0027	0.0016	0.1013	0.9803	0.1285	0.0000	0.1865
JEF	-0.0007	0.0019	0.7094	1.3812	0.1466	0.0000	0.2588
TRST	0.0004	0.0012	0.7605	0.6605	0.0927	0.0000	0.1665
	D1	**D1**	**D1**	**D1**	**D1**	**D1**	**D1**
XON	0.0009	0.0010	0.3781	0.5177	0.1048	0.0000	0.0877
IP	0.0003	0.0012	0.8234	0.6791	0.1343	0.0000	0.0914
FRX	0.0034	0.0018	0.0567	0.9450	0.1893	0.0000	0.0893
SMBL	0.0038	0.0017	0.0321	0.8686	0.1883	0.0000	0.0773
JEF	0.0008	0.0021	0.7042	1.1219	0.2237	0.0000	0.0901
TRST	0.0011	0.0012	0.3782	0.7666	0.1313	0.0000	0.1183
	D2	**D2**	**D2**	**D2**	**D2**	**D2**	**D2**
XON	0.0009	0.0010	0.3960	0.3288	0.1610	0.0422	0.0161
IP	0.0003	0.0012	0.8181	1.2644	0.1929	0.0000	0.1447
FRX	0.0034	0.0018	0.0648	0.8134	0.2892	0.0053	0.0302
SMBL	0.0038	0.0018	0.0332	1.1875	0.2804	0.0000	0.0659
JEF	0.0008	0.0021	0.7067	1.4978	0.3341	0.0000	0.0733
TRST	0.0011	0.0013	0.3971	0.7010	0.2022	0.0006	0.0452
	D3	**D3**	**D3**	**D3**	**D3**	**D3**	**D3**
XON	0.0009	0.0010	0.3923	0.5683	0.1974	0.0043	0.0316
IP	0.0003	0.0013	0.8257	1.0634	0.2489	0.0000	0.0671
FRX	0.0034	0.0018	0.0586	1.5917	0.3487	0.0000	0.0758
SMBL	0.0038	0.0018	0.0340	1.3564	0.3482	0.0001	0.0564
JEF	0.0008	0.0022	0.7143	0.9966	0.4242	0.0196	0.0213
TRST	0.0011	0.0013	0.4064	0.3267	0.2548	0.2011	0.0064
	D4	**D4**	**D4**	**D4**	**D4**	**D4**	**D4**
XON	0.0009	0.0010	0.3990	0.3561	0.3726	0.3402	0.0036
IP	0.0003	0.0013	0.8313	0.3974	0.4789	0.4073	0.0027
FRX	0.0034	0.0018	0.0688	0.3582	0.6747	0.5960	0.0011
SMBL	0.0038	0.0018	0.0391	0.6775	0.6658	0.3098	0.0041
JEF	0.0008	0.0021	0.7033	3.9300	0.7589	0.0000	0.0955
TRST	0.0011	0.0013	0.4076	0.3026	0.4754	0.5251	0.0016
	D5	**D5**	**D5**	**D5**	**D5**	**D5**	**D5**
XON	0.0009	0.0010	0.3996	0.5382	0.9570	0.5743	0.0012
IP	0.0003	0.0013	0.8311	1.4574	1.2266	0.2359	0.0055
FRX	0.0034	0.0018	0.0689	0.6614	1.7312	0.7028	0.0006
SMBL	0.0038	0.0018	0.0393	-0.9747	1.7102	0.5692	0.0013
JEF	0.0008	0.0022	0.7154	3.7418	2.0333	0.0669	0.0132
TRST	0.0011	0.0013	0.4059	1.7962	1.2153	0.1406	0.0085

from other studies of asset pricing models – that an asset pricing approach to identification of unusual events should be fruitful when the market effect is removed.

Wavelet analysis may very likely enable a more informative picture of two other widely applied approaches to asset pricing, the Arbitrage Pricing Theory (APT) and Multifactor Model (MFM). In different ways, each of these models adds explanatory variables to the CAPM and Market Models. The added depth to the analysis derives from the ability through wavelet analysis to associate lower frequency spectra of macroeconomic time series known to influence asset returns (e.g. GDP growth, reports of money supply growth, decisions of the Federal Open Market Committee, movements in consumer prices, unemployment, productivity, etc.) with the asset returns. The same is true regarding company-specific information reported in quarterly and annual information cycles to stockholders and the U.S. Securities and Exchange Commission.

Wavelet analysis also suggests non-parametric methods of removing market noise from asset returns, based on de-noising wavelet techniques applied in signal processing. This offers the possibility of more sensitive event studies, where the influence of some unusual event **is measured**, such as a dividend increase, or unexpectedly high or low quarterly earnings.

References

Bradley, J. N., C. M. Brislawn, et al. (1993). "The FBI Wavelet/Scale Quantization Standard For Grey-Scale Fingerprint Image Compression." SPIE Proceedings, Visual Information Process **2**.

Bruce, A. and H.-Y. Gao (1996). Applied Wavelet Analysis with S-PLUS, Springer.

Bruce, A., D. Donoho, et al. (1996). "Wavelet Analysis." IEEE Spectrum **33**(10): 26.

Copeland, T.E. and J.F. Weston (1992) Financial Theory and Corporate Policy, 3rd ed. pp. 361-362.

Criminal Justice Information Services, (1993). WSQ Gray-Scale Fingerprint Image Compression Specification, Federal Bureau of Investigation

Davidson, R., W. C. Labys, et al. (1998). "Wavelet Analysis of Commodity Price Behavior." Computational Economics **11**: 103-128.

Donoho, D. L. (1993). "Nonlinear wavelet methods for recovery of signals, densities, and spectra from indirect and noisy data." American Mathematical Society **47**: 173-205.

Donoho, D. L. and I. M. Johnstone (1995). "Wavelet Shrinkage: Asymptopia." Journal of the Royal Statistical Society **57**(2): 301-369.

Donoho, D. L. and I. M. Johnstone (1998). "Minimax Estimation Via Wavelet Shrinkage." The Annals of Statistics **26**(3): 879-921.

Gabor, D. (1946) "Theory of Communication", Journal of the Institute of Electrical Engineering, **93**(3): 429-457.

Kim, M. and J. R. Ritter (1999). "Valuing IPOs." Journal of Financial Economics **53**(3): 409-437.

Graps, A. (1995). "An Introduction to Wavelets." IEEE Computational Science and Engineering **2**(2).

Lintner, J. (1965) "Security Prices and Maximal Gains from Diversification." Journal of Finance, December: 587-616.

Meyer, Y. (1993). Wavelets: Algorithms and Applications. Philadelphia, Society for Industrial and Applied Mathematics.

Ramsey, J. B. and C. Lampart (1998a). "Decomposition of Economic Relationships by Timescale Using Wavelets." Macroeconomic Dynamics **2**: 49-71.

Ramsey, J.B. and C. Lampart (1998b). "The Decomposition of Economic Relationships by Time Scale Using Wavelets: Expenditure and Income." Studies in Nonlinear Dynamics and Econometrics, **3**(4): 23-42.

Ramsey, J.B. and Z. Zhang (1997). "The Analysis of Foreign Exchange Data Using Waveform Dictionaries." Journal of Empirical Finance **4**: 341-372.

Sharpe, W.F. (1964) "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." Journal of Finance, September: 425-442.