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Growth and Volatility Analysis Using Wavelets

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Abstract

The magnitude and persistence of growth in gross domestic product are topics of intense scrutiny by economists. Although the existing techniques provide a range of tools to study the nature of growth and volatility time series, these usually come with shortcomings, including the need to arbitrarily define acceleration spells, and focus on a particular frequency at a time. This paper explores the application of "wavelet-based" techniques to study the time-varying nature of growth and volatility. These techniques lend themselves to a more robust analysis of short-term and long-term determinants of growth and volatility than the traditional decomposition techniques, as demonstrated on a small sample of countries. In addition to having desirable technical advantages, such as localization in time and frequency and the ability to work with non-stationary series, these techniques also make it possible to accurately decompose the association between growth trajectories of different countries over different time horizons. Such "co-movement" analysis can provide policy makers with important insights on regional integration, growth poles, and how short and long term developments in other countries affect their domestic economy.

Growth and Volatility Analysis Using Wavelets

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1 Introduction

An everlasting quest for policy makers is how to promote rapid and sustained growth. In practice, many economies have grown rapidly for short periods of time. However, sustaining the same performance in a longer time horizon is much less common. The differences between developed and developing economies are particularly striking in this regard. In many developed countries, a substantial part of the evolution of per capita GDP can be summarized by a single statistic: the average growth rate over time (Pritchett (2000)). This is mainly because the growth process is relatively stable in these countries, e.g. the variation around the long term trend is small. In comparison, growth exhibits significant volatility and instability in the majority of the developing countries. Frequent breaks in the long term trend as well as large variations around the trend are common. Therefore, the average growth rate will only explain a relatively small share of the information in these cases. We also need to investigate the pattern of fluctuations in order to understand the determinants of growth.

Starting from a similar observation, Hausmann *et al.* (2005) suggest that identifying the clear shifts in growth (breaks in the trend or volatility around the trend) can shed light on the relationship between growth and its fundamental determinants. Using a stylized definition of growth acceleration episodes, which is based on the magnitude and persistence of growth (e.g. an increase in per capita growth of 2 percentage points or more for 8 consecutive years), they find that the relationship between growth and its determinants varies on the basis of the time frame of the analysis. For instance, economic reform for openness, which is measured by a number of factors including structural (e.g. presence/absence of marketing boards) as well as macroeconomic (e.g. presence/absence of a large black market premium for foreign currencies) indicators, is found to be a significant determinant of growth accelerations that are sustained over the longer term, whereas externals shocks, defined as substantial improvements in the country's Terms of Trade, are found to generate growth accelerations that die out in the short term.

In this paper, we demonstrate a useful methodology to study the time-varying characteristics of growth in fine detail. Using a wavelet-based technique, we decompose the time series into high frequency (transitory) and low frequency (persistent) components. This, in turn, enables us to identify the growth acceleration and deceleration phases

without using arbitrary restrictions. Therefore, this technique lends itself to a robust analysis of the short-term and long-term determinants of growth. The same approach is extended to analyzing the volatility of the GDP series, where the focus is on changes in the growth rates as well as the levels of GDP. Figure 1 shows a decomposition of GDP per capita growth series in the United States by using this technique. Changes in the actual growth series between 1960 and 2010 (the top row) are decomposed into subcomponents due to variations at 2-4 years frequency (D1), 4-8 years frequency (D2), 8-16 years frequency (D3), and 16-32 years frequency (D4). Finally, the wavelet smooth (S4) denotes the trend term in the series.

Wavelet-based techniques have certain desirable characteristics that prove to be useful in growth and volatility analysis. First, wavelet decomposition provides an uncorrelated set of frequency scales, i.e. the sum of components is equal to the original series. When analyzing the growth fluctuations, this ensures that volatility due to different time scales are fully identified. This is not the case for common filtering techniques such as Hodrick-Prescott, where information "leaks" while filtering the series consecutively in order to separate the different frequencies. Second, wavelet decomposition is localized both in time and frequency, and the time domain and frequency domain information of the original series are preserved (the horizontal and vertical axes in Figure 1). Therefore, one-off events such as crises do not affect the decomposition at other points in time. In contrast, with traditional spectral analysis techniques, such as the Fourier transformation, the information is spread over the entire period of analysis. Therefore, one-off events have global impacts. Overall, these characteristics suggests that the wavelet techniques can be employed in several policy related studies including commodity price diagnostics and feasibility studies for economic unions. Table 1 shows a set of potential areas where wavelet techniques can be employed to enhance the existing analytical approaches.

This paper proceeds as follows. The next section discusses the fundamental characteristics of wavelet-based techniques with a comparison to other frequently used approaches in a non-technical manner. The third section introduces a basic description of wavelets for beginners. A more technical desription of wavelet transform with an em-

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There is a well established literature that investigates various aspects of volatility and growth relationship. For the impact of volatility on long term average growth rates, see Burnside and Tabova (2009), Hnatkovska and Loayza (2004), and Ramey and Ramey (1995); for the impact of openness on business cycle volatility and synchronization, see Calderon *et al.* (2007), Kose *et al.* (2003).
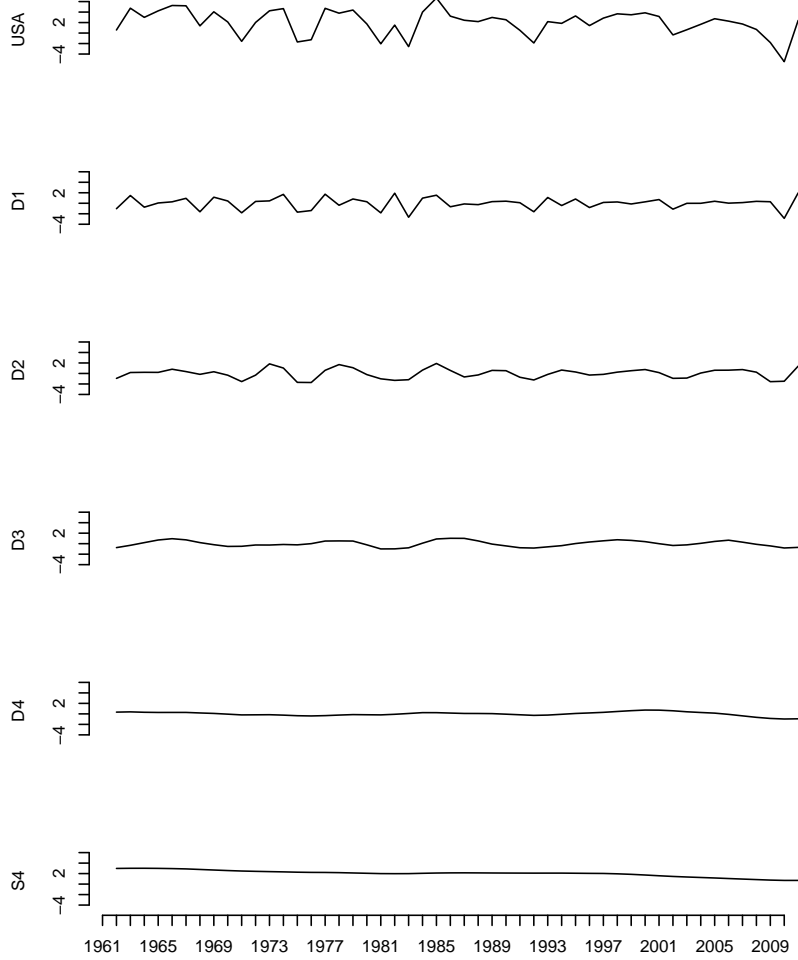
Figure 1: A wavelet-based multiresolution decomposition of income per capita growth series of USA.

Notes: This multiresolution decomposition is performed using Maximum Overlap Discrete Wavelet Transformation (MODWT) on first difference of the annual series from Penn World Tables 7.0. It is implemented using the pyramid algorithm shown in Figure 3. The top panel shows the actual series (growth rate of income per capita). Variations due to $2-4$ year frequency oscillations are shown in the second panel (D_1), others as follows: $4-8$ year frequencies in the third panel (D_2), $8-16$ year frequencies in the fourth panel (D_3)and $16-32$ year frequencies in the fifth panel (D_4). The last panel (S_4) shows the "smooth" component, e.g. all frequencies lower than 16 years. These components are approximately independent to each other and the original series can be recovered by aggregating the four sub-components.

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Table 1: Potential Applications of Wavelets

	Applications	Countries of Primary Interest
Growth Analytics	• Identify acceleration and deceleration phases (hills, plateaus, mountains, and plains)	All countries
	• Identify structural breaks	All countries
	• Investigate the country resilience by analyzing the persistence of impacts due to different types of shocks	All countries
Synchronization Analysis	• Analyze the co-movement of growth between two economies	All countries
	• Investigate the feasibility of economic union formation (monetary union, free trade areas, customs union) by analyzing the cyclical synchronization among a group of economies	Trade/monetary union members or candidates
Commodity Price Diagnostics	• Analyze the short-term and long-term behavior of key commodity prices	Commodity traders, resource-rich countries
	• Investigate the co-movement of commodity prices and desired macroeconomic aggregates	Commodity traders, resource-rich countries

phasis on Maximum Overlap Discrete Wavelet Transformation (MODWT) and wavelet variance analysis is presented in the appendix. The fourth section demonstrates an application of the wavelet scalogram using income, consumption, and investment series of a selected group of countries. The fifth section investigates the characteristics of variance and covariance of these series. The sixth section introduces an analysis of co-movement of growth across the countries. The last section concludes.

2   The Advantages of Using Wavelet Techniques

Economists have long been aware of the time varying characteristics of economic phenomena. Traditionally, these variations have been analyzed by using various spectral analysis methods, which enable decomposition of the time series into an independent set of frequency components. However, this is done under relatively strict assumptions in the spectral analysis. Fourier transformation is used to decompose a series into sinusoidal components when the series is stationary, and preservation of the information in time is not required (e.g. Granger (1966) and Nerlove (1964)). In the case of non-stationary data, however, the original series is filtered to be made stationary, which does not preserve all information from the series. Moreover, a single event in time, or an extension of the series by including new data points, can change the analysis at all frequencies; hence, the decomposition is not localized. This could be resolved by using a windowed Fourier analysis. However, it has weakness similar to the moving window averaging methods. It requires selection of a window where data are stationary and assumes that volatility range does not change over time. Wavelet-based techniques provide a robust alternative by allowing us to perform a volatility analysis in frequency domain with minimal specifications of analysis parameters.

In contrast to classical spectral analysis, wavelet-based techniques provide a decomposition that is localized both in time and frequency. By combining several combinations

Consumption smoothing over an economic agent's lifetime is one example. Permanent Income Hypothesis (PIH) suggests that agents consume out of permanent incomes and (dis)save out of transitory incomes, which implies that the marginal propensity to consume is expected to be greater for the former than the latter. Corbae *et al.* (1994) show that the marginal propensity to consume at high frequencies is lower than at low frequencies. Hence, decomposing the interaction into different time horizon components provides a better approximation of the true nature of the relationship.

of scaled and shifted versions of the mother wavelet (basis function), the wavelet transformation captures the localized information in time domain and presents the associated frequency information along with it. Therefore, standard time series measures such as correlation and covariance can be employed to analyze the association of the variables in the frequency scale of choice.

Another characteristic that makes the wavelet technique appealing in economic analysis is its ability to work with non-stationary data. In the case of trending data, detrending techniques like Hodrick-Prescott (HP) and band pass filters are used to derive the variations around the trend. These filters require selecting a window width for averaging on which the data are approximately stationary. Therefore, these processes depend on the assumptions regarding the underlying properties of the data. Unlike the HP filter, wavelet-based filtering does not require normality of the errors while extracting periodic components associated with multiple frequencies. Furthermore, these derived components are uncorrelated with each other. This enables the original series to be equal to the sum of the components, which is not the case for HP filter. In an attempt to analyze the medium term business cycles across countries, Comin and Gertler (2006) note that because the medium and high frequency variations in the data are not independent after the HP filtering, it is not feasible to compare the two components in isolation. Wavelet filtering provides a feasible filtering tool in similar conditions.

The class of non-stationarity that can be handled by the wavelet transform is broader than the existence of a mere unit root process (Ramsey and Lampart (1998). Time series models typically assume second order stationarity, i.e. the mean and the covariance of the process do not change over the period of analysis. Therefore, structural breaks require customized treatment depending on whether the break is considered to be in the mean or in the variance. Wavelet transform, on the other hand, provides a straightforward method to test and isolate the breaks. In the case of a sudden change in variance, the high frequency components in wavelet transform contain the shift and the low frequency components remain stationary. If the structural break is about the long term relationship, then all frequency scales in wavelet transform will reflect this (Gencay *et al.* (2001)). As discussed in Ramsey (1999) the ability of wavelets to represent complex structures without knowing the underlying functional form of the process is of great value in economic and financial research.
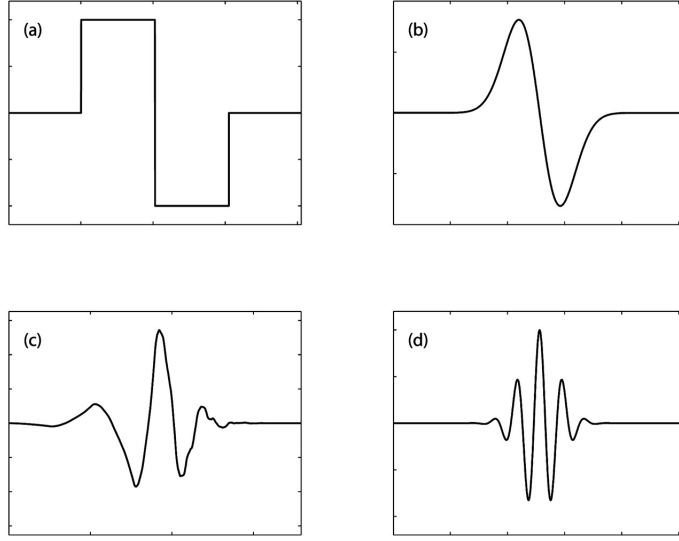
Figure 2: Examples of "mother wavelets": (a) Haar, (b) a wavelet related to the first derivative of the Gaussian PDF, (c) Daubechies, (d) Morlet (real component)

The next section provides a more formal introduction to the wavelet techniques.

3 An Introduction to the Wavelet Techniques

A wavelet (small wave) is a mathematical function with special characteristics, e.g. integration to zero and unit energy, that is used to transform a time series into components corresponding to different frequency ranges. This is done by filtering the original series via a selected algorithm, which uses the scaled and shifted versions (daughter wavelets) of the basis function (mother wavelet). Figure 2 demonstrates the commonly used basis functions. The simplest example of a wavelet filter is Haar mother wavelet, which is shown in panel (a) of Figure 2. The mathematical definition of this wavelet is the following;

$$\psi(t) = \begin{cases} -1/\sqrt{2} & -1 < t \leq 0 \\ 1/\sqrt{2} & 0 < t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Haar filter is preferred when the sample series is relatively short since it results in fewer artifacts around the end of series. In general, wavelet analysis can tell us how
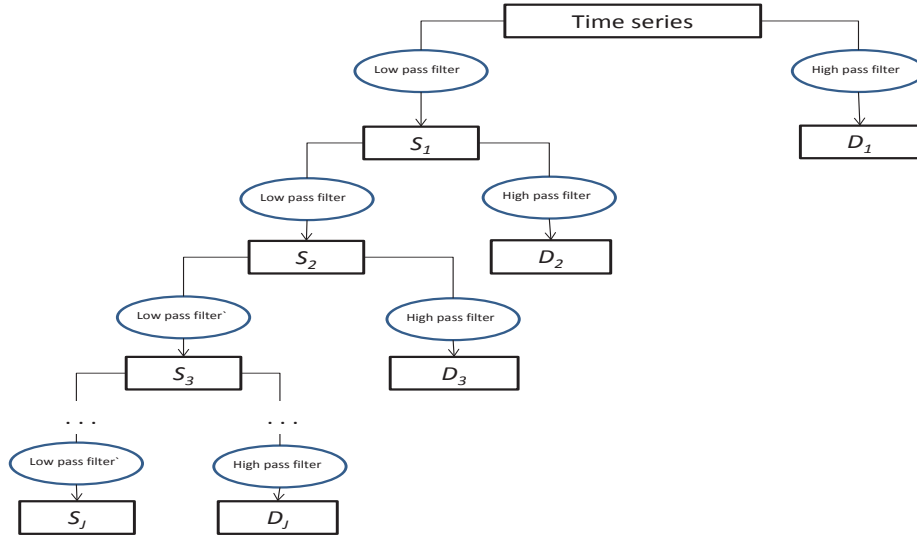
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Figure 3: Schematic representation of a wavelet multiresolution decomposition using Pyramid algorithm.

weighted averages of series vary from one averaging period to the next. The wavelet transform produces a series of wavelet coefficients that are associated with dyadic scales. These coefficients are proportional to the differences between the averages of the original data, and the scaling coefficients that are proportional to averages. The scaling coefficients capture the long term variations in the data. The discrete wavelet transform (DWT) effectively decorrelates even highly auto-correlated series, which can be perfectly reconstructed from the DWT coefficients making it an extremely useful technique for econometric analysis. The continuous wavelet transform is not considered in this paper due to small sample size of available time series.

4 Time and Frequency Decomposition

This section introduces a simple application of the MODWT that is defined in Section A.1. We first use a synthetic data example in order to introduce the idea of wavelet scalogram. Then, we use actual time series data for GDP in levels and first differences to demonstrate the application in different countries. The wavelet techniques discussed here do not assume stationarity of the series. Therefore, first differencing is performed only for interpretation

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purposes, not for detrending.

Synthetic data are simulated to illustrate the benefits and the potential of the wavelet-based analysis. Left panel of Figure 4a shows an example of the simulated GDP series that characterizes 10 years of stable growth in the beginning, followed by 5 years of no growth, and then 15 years of slower growth than the initial phase. Next, we introduce a sharp reduction in GDP that persists for a few years. This is followed by a 5 years of recovery period. After that there are 12 years of growth interrupted by a one year 5% drop that recovers immediately. Overall, this structure provides us the chance to elaborate on how the wavelet decomposition handles persistent and transitory shocks, growth slowdowns, and recoveries.

The right panel of Figure 4a shows the growth rate of the simulated series by using first differences, i.e. $X'_t = 100 \times (X_t/X_{t-1} - 1)$, where, X_t are the observations at time t, and X'_t are the resulting growth rate of the series at time t.

The resulting scalograms of the sythetic series in both levels and growth rates are given in Figure 4b. Panel (a) of Figure 4b can be interpreted as the changes in the synthetic GDP level series. One can clearly see that the growth phase during the first 10 years followed by a relatively short term period of no changes. Note that the "no growth" period of 5 years is captured by levels 1 and 2 that are associated with changes on scales from 2 to 8 years. In the long run, the series keeps growing (see levels 3 and 4 in green). The next 15 years correspond to the growth period which can be clearly seen in the scalogram by the green colors in all scales. One can notice that the growth rate is slightly lower than the beginning of the projection period, shown by a lighter shade of green. The simulated data contains two types of shocks: a short-term and a more persistent one. The difference between the two drops can be clearly seen in the wavelet scalogram. The first drop is more persistent and is captured by first 3 levels, scales from 2 to 16 years. The second drop is short term and is reflected by level 1.

The right panel of Figure 4b provides the wavelet transform of the growth series and can be interpreted as the change in the growth series, i.e. acceleration. The beginning of the series show periods of negative and positive acceleration that corresponds to the break points between the periods of growth and no growth. There is a 15 year period of zero acceleration that corresponds to the period of constant growth of the synthetic

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Figure 4: Decomposition of Synthetic GDP data (left panels), GDP growth (right panels).

(a) Synthetic GDP data (left panel), GDP growth (right panel).
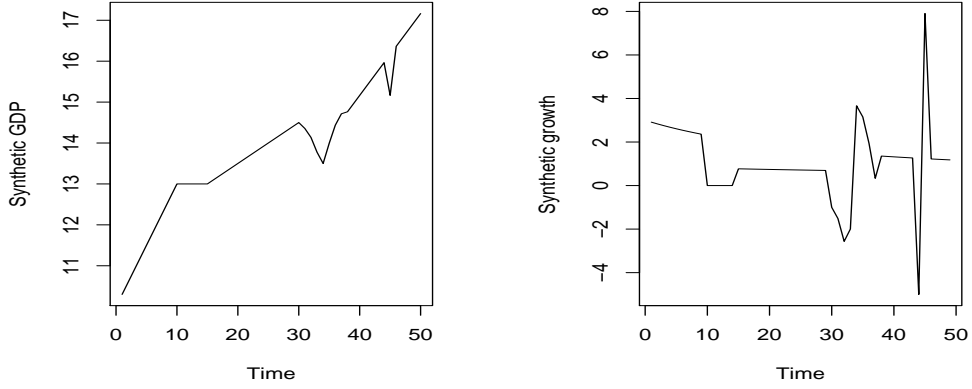
(b) MODWT scalogram of the synthetic GDP: (a) growth and (b) acceleration.
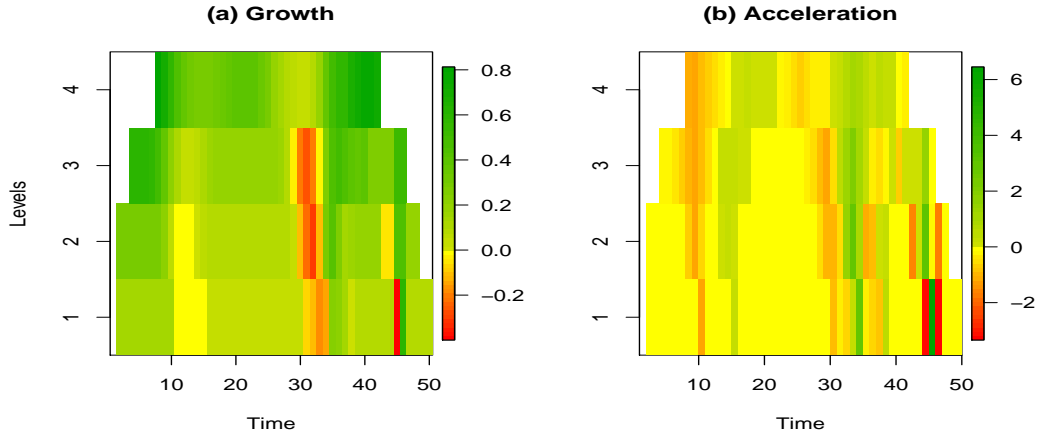
GDP. The acceleration scalogram reveals that for the long-term shock the decline was at a slower pace than the following recovery (see the red shade followed by a cluster of green around 30). The short-term shock results in sharp changes in the acceleration on a short-term scale (level 1), not visible in the long-term.

A potential limitation to the use of wavelet techniques for growth and volatility analysis in practice is limited data availability. The sample size of the analysis, i.e. the length of the time series, influences the power of the tests and estimation uncertainty as in the other statistical techniques. The methodology chosen here does not depend on the starting point of the time series. Figure 5 repeats the synthetic data examples for growth
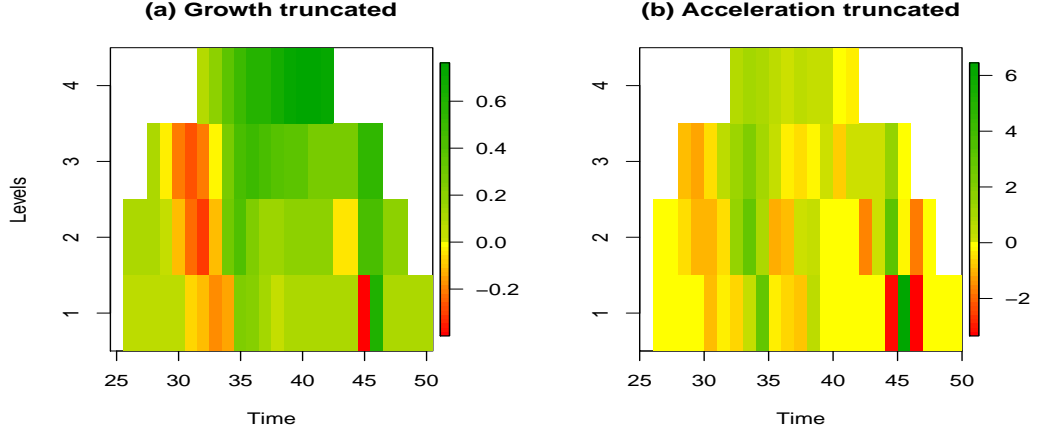
Figure 5: MODWT scalogram of the truncated synthetic GDP: (a) growth and (b) acceleration.

and acceleration series by using a truncated version of the data set introduced above. In particular, the new starting point is moved to 25th period of the synthetic data. The original synthetic series have 30 years of growth in the begining (with the exception of 5 years of stagnant GDP in the middle). Therefore, with the truncated data, there is only 5 years of growth in the beginning of the new projection horizon. The figures show that the decomposition by using the truncated data series does not differ from the one with longer series (compare to Figure 4b). This illustrates that the choice of starting period does not affect the decomposition at later periods.

Next, we consider actual data. We use annual series of income, consumption, and investment data from Penn World Tables 7.0, all in 2005 international dollars per capita terms. The income series are provided by the Purchasing Power Parity (PPP) converted GDP per capita (chain series) data, whereas consumption and investment series are computed by using the share of actual consumption and investment in the PPP converted GDP series at 2005 prices.

Decomposition for demonstration purposes are limited to the following five countries: Brazil (BRA), Fiji (FJI), Turkey (TUR), United States (USA), and Zimbabwe (ZWE). The selection of the countries roughly reflects a cross-section of the geographic spread, as well as per capita income levels as defined by the World Bank classification using 2011 Gross National Incomes.

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The available time series between 1961 and 2010 is relatively short in terms of number of observations ($N = 50$). A cross country analysis with a particular focus on low income countries is handicapped by data limitations. However, in the case of developed countries, longer time series with higher resolution, e.g. quarterly, or monthly, are often available and should be preferred for the gains in statistical significance.

First, we present the results of the wavelet analysis of the actual Output, Consumption, and Investment levels. Then, the same analysis is repeated for the growth rates of the latter components. In both cases a MODWT described in Section A.1 using the Haar filter is performed. Each level captured the changes in, for example Output, and Output growth at dyadic scale.

4.1 Growth Analysis

This section introduces a time and frequency decomposition of trending data by dyadic levels. We perform the wavelet analysis of the Output levels for a set of five countries. Figure 6 shows the MODWT based scalogram of the GDP per capita levels, which decomposes the changes in GDP per capita levels by time and frequency dimensions. The horizontal axis shows the time dimension, whereas the vertical axis denote the wavelet scales that identify the frequency categories: The first scale (1) shows the frequency intervals between 2 and 4 years,whereas the second scale (2) shows 4-8 years, the third scale (3) shows 8-16 years, and the fourth scale (4) shows 16-32 years. The shades of colors, on the other hand, correspond to changes in the GDP per capita at different frequencies: green shows an increase in GDP per capita, and red shows a decrease. The legends on the right hand sides of the figures map the different shades of green, yellow, and red with the corresponding magnitudes of change in each case. e.g. the greener the color is, the more positive is the change. The white areas around the edges are the omitted values that were affected by the wavelet circularity assumption (see Section A.1).

The figure for Brazil exhibits two economic downturns in the time frame of this analysis. A reduction in GDP per capita in the first half of the 1980s was followed by a short recovery, and then by a second decline in late 1980s and early 1990s. Our decomposition shows that the second downturn had stronger long-term impacts than the first one as shown in the decline of Output on the fourth scale (approximately 30

years frequency band). This can be interpreted as a result of more persistent shocks. In comparison, Fiji seems to have gone through several short-term changes in its GDP per capita levels, which can be seen from frequent appearances of shades of yellow and orange at the first scale (see level = 1 of panel (b) Figure 6). Beginning from the early 1980s, it experienced an economic downturn with long term implications, which lasted for about a decade in 1980s, and was followed by a long-term spell of growth. However, the short term variability in GDP growth persist. Turkey is demonstrating a stable long-term Output growth, with a moderate shock around 80's after which it quickly recovered. Another decrease in the Output on the 4 to 8 year scale was around the 2000. The impacts of the latter downturn were of significant magnitude, which can be seen as a dark red color in the graph. However, it did not have a long term impact. USA shows stable long-term growth with some moderate downturns through the past 50 years. However, the strongest decrease was in 2010, with the preceding signs of visible decrease in GDP per capita on 4 to 8 year scale. Finally, Zimbabwe exhibits frequent short-term and long-term economic downturns of significant magnitudes. There are multiple long-term declines in the Output during 1960 – 1990 that were followed by gradual and patchy recoveries. However, starting around the mid 1990s the economy spiraled into long-term downturn with a magnitude that exceeded the ones before.

This exercise briefly shows the benefits of using the wavelet methodology in decomposing the changes in income level in different economies. This approach allows us to distinguish between the transitory and persistent shocks while preserving the magnitude of impacts and localization in time. In this demonstration, where we used only five countries, the more developed countries in the small sample exhibit longer intervals of long-term Output growth (green colored intervals at Levels 3 and 4) than the developing countries. In addition to the cross country comparisons, we can also compare different shocks to a given economy in terms of the magnitude and duration of the impact.

Next, we consider the growth series.

4.2   Acceleration Analysis

This section introduces a time and frequency decomposition with first differenced series including consumption and investment. Therefore, the focus is on the changes in the
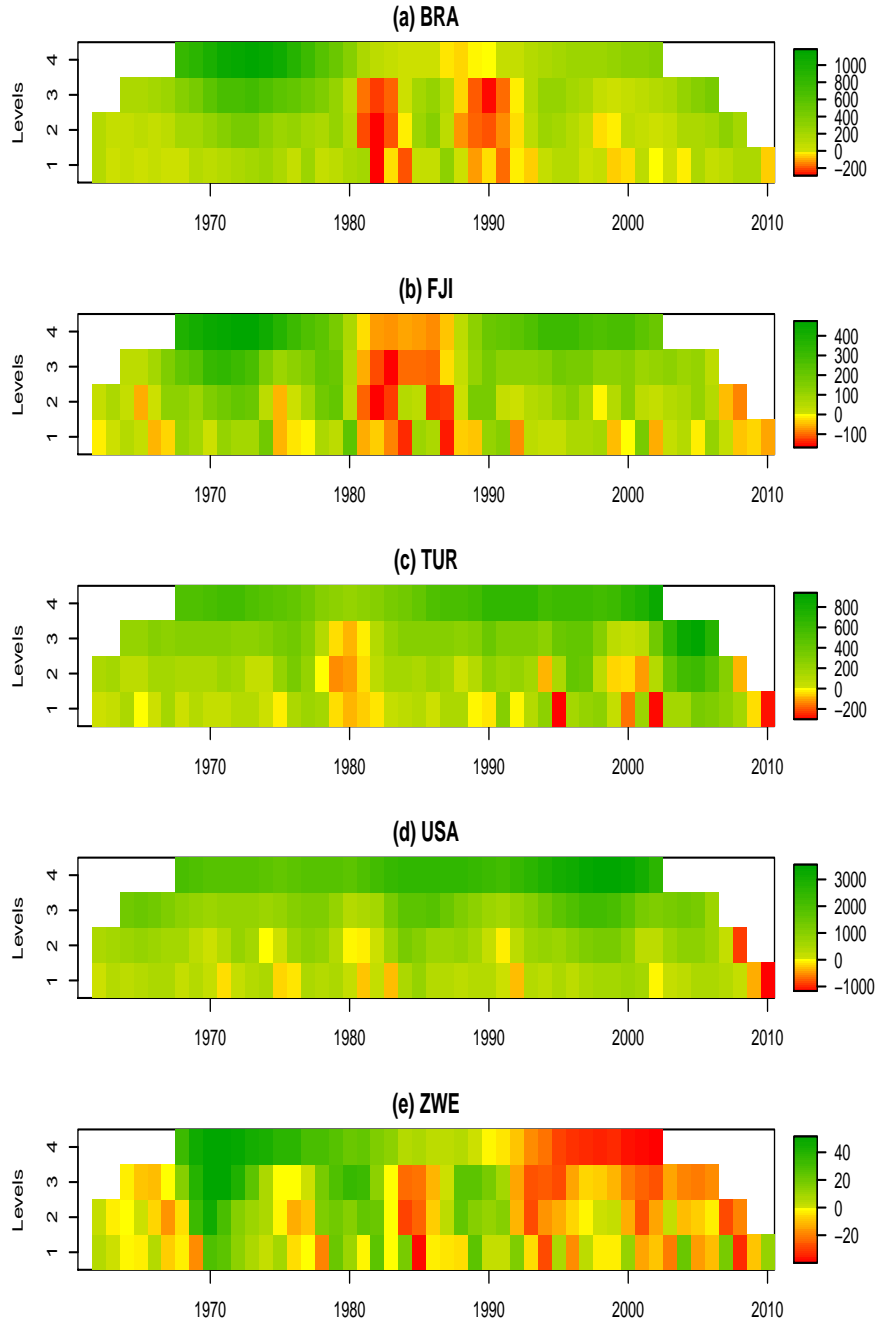
Figure 6: MODWT scalogram for Output of (a) BRA, (b) FJI, (c) TUR, (d) USA, and (e) ZWE.

growth rate of the original series. Figure 7 provides the scalograms of the MODWT wavelet coefficients using the first differences of the GDP level series for the five selected

countries. As opposed to the Figure 6 where the colors indicated the changes in the levels of GDP per capita, Figure 7 displays the changes in the growth rates of GDP per capita. Therefore, different shades of colors denote the *acceleration* or *deceleration* of the series. Note that the first four countries are plotted using the same scales for Output, Consumption and Investment growth despite the differences in vertical axis boundaries displayed here. Therefore, different shades and colors are comparable across these countries. Zimbabwe exhibits the largest variability and is plotted using its own scale. There are several short and medium term decelerations in the growth of per capita GDP in Zimbabwe in the beginning of the 1990s preceding the big decline. The lower middle income group representative in our sample, Fiji, also exhibits some variation in long term growth performance. One may observe a persistent decrease in the Output growth that started in mid 1970s. It was followed by a short-term recovery which did not last, the Output growth change slowed down in the 1980s, which resulted in a significant Output decline through the 1980s (see panel (b) of Figure 6).

Similar economic decline indicators can be observed in the changes in Output growth series of Brazil in 1970s, which resulted in a debt crisis in the 1980s. Based on the wavelet scalogram, there was a recovery phase, however did not translate into a long-term trend. Again, in the years before the South American crisis of 1990s, medium and long term scales indicate a reduction in the Output growth rate, signaling not a one-off deterioration, but a more persistent problem.

Turkey, on the other hand, went through a series of short- and mid-term changes in its growth rate. The most prominent short term reductions occurred in the early 1990s and 2000s, whereas the medium and long term reductions happened in the late 1990s and 2000s. In comparison, the changes in the growth rate in the United States are quite mild at all scales.

Note that, a green color in the scalogram of the first difference series could correspond to either a positive or negative growth rate. It shows whether the growth or decline rate is increasing. Wavelet volatility analysis used together with the growth analysis described in previous section can provide more details on the nature of the growth, i.e.

The growth literature generally uses the term "growth acceleration" for a positive change in the growth rate of output. We use the terminology in a similarly flexible way. However, technically speaking, acceleration denotes a second derivative of the original series, therefore a positive change in the growth rate is an "acceleration of output".

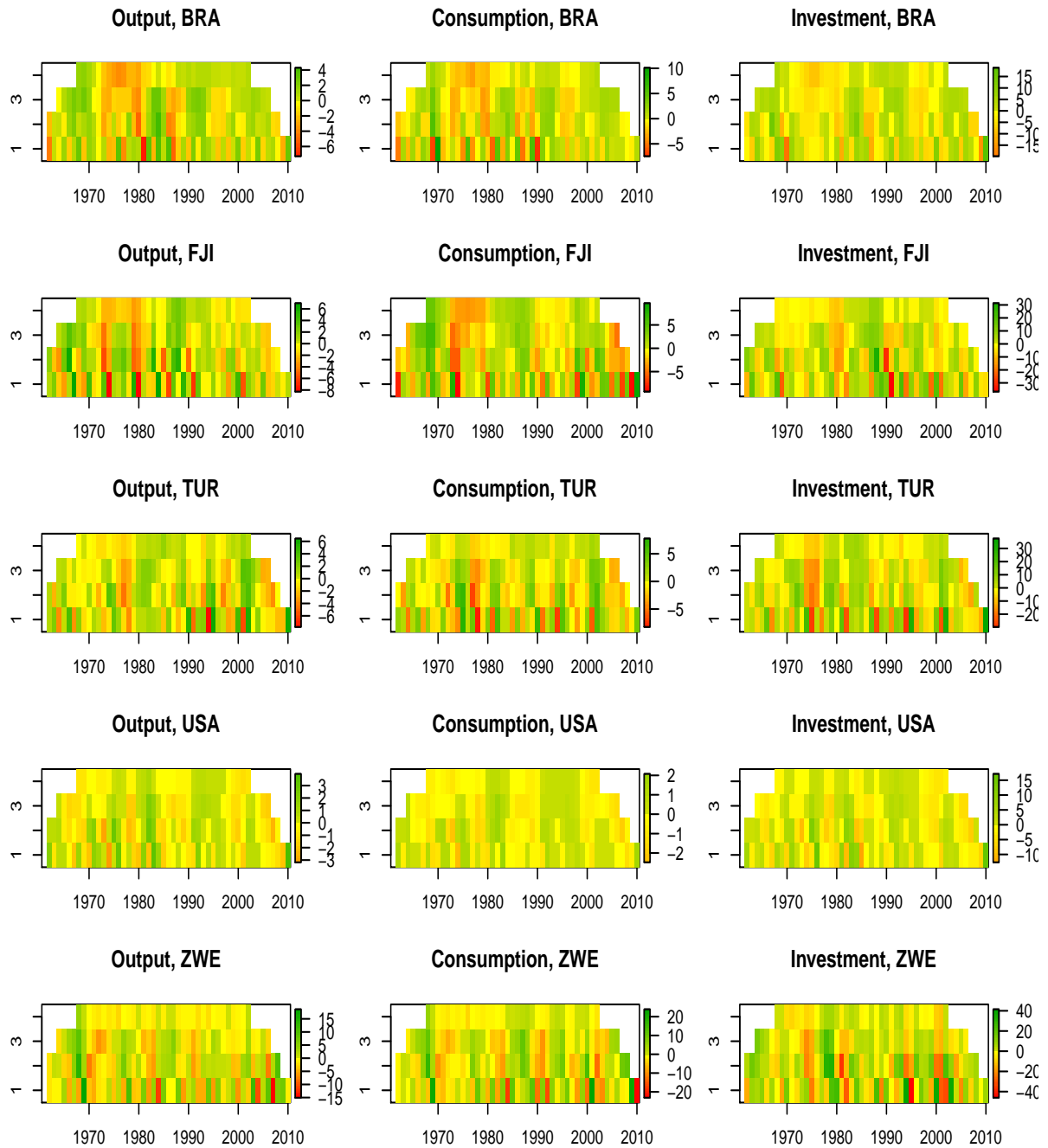Figure 7: MODWT scalogram for changes in Output, Consumption, and Investment growth for BRA, FJI, TUR, USA, and ZWE. The color scales are adjusted for the first four countries for Output, Consumption, and Investment growth.

acceleration of the economy. It picks up the indicators of upcoming economic decline, as well as shows how long it takes for the economy to start its way to recovery. For example,

output acceleration shows long-term decrease that lasted for about a decade prior to the Brazil crisis of 1980s (see top left of Figure 7).

An interesting observation here is that the Consumption series in the selected countries are as volatile as the output series in both the high frequency and low frequency scales, with the exception of United States. This implies an inability to smooth the consumption over time, which is a well-known phenomenon in the literature. Accordingly, several factors including credit constraints are considered to limit the consumption smoothing and risk sharing behavior in developing economies. Following sections provide a closer look at the volatility of output, consumption and investment series, and investigate the co-movement between these variables.

5 Wavelet Variance and Volatility Analysis

In this section, we demonstrate a comparison of wavelet variance estimates across the output series of five selected countries. Output variance decomposition at different frequencies are computed using a Haar filter and unbiased wavelet variance estimators as described by the equation (A.4). As Figure 8 shows, the output variability is the lowest for the USA in both high and low frequency scales. USA wavelet variance at levels 1 and 2 differs from other countries significantly. The low income country in our sample, Zimbabwe, presents high variability at all scales compared to other countries. However, despite having significantly greater variance, Zimbabwe's variance decomposition follows the same patters as other countries, i.e. the variance decreases as the scale increases. In general, low wavelet variance can be interpreted the same way as in the time domain analysis: lower variance in the first differences series means that Output follows a stable pattern. We can see this in more developed economies like high income USA, and upper middle income Turkey and Brazil. The higher the wavelet variance values the more volatile is the GDP growth series. Compared to Turkey Brazil presents lower average wavelet variance and spread on levels 1 and 2 (short term, up to 8 years changes) and higher values for levels 3 and 4 (8 to 32 years changes). The latter Output change behavior suggests that the economy of Brazil follows the patterns of high income countries closer than Turkey. However, there is quite a bit of overlap in the confidence intervals. Therefore, the differences are not statistically significant.
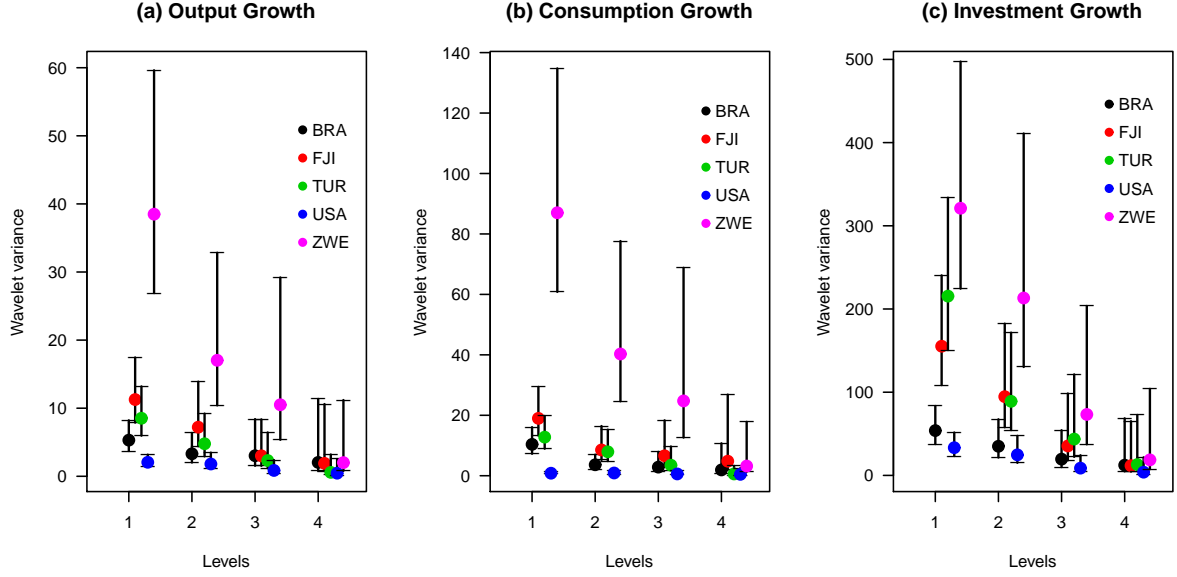
Figure 8: Wavelet variance and non-Gaussian confidence intervals of the (a) Output, (b) Consumption, (c) Investment growth for BRA, FJI, TUR, USA, and ZWE

Figure 8 also shows the wavelet variance estimates for consumption and investment growth series with associated confidence intervals. Changes in consumption growth rates show similar variability patterns (see panel (b) of Figure 8). Here, Zimbabwe, a country representing the low income group shows the most volatility for scales $j = 1, 2, 3$. The accuracy of confidence interval estimates at level $j = 4$ is rather low, due to small sample size and many coefficients missing because of wavelet circularity assumption. Panel (c) of Figure 8 shows that the changes in the Investment growth rates are much more volatile. It is worth mentioning that Brazil follows the same volatility pattern as the USA on high-frequency scales. Finally, the investment growth series for the other three countries (FJI, TUR, ZWE) show high short-term variability.

6   Co-movement Analysis

When analyzing the growth and volatility in a group of economies, one question that arises is to what extent they are synchronized. In this section, we demonstrate how the wavelet based techniques can help to explore this synchronization. In particular, as these

techniques generate a scale by scale decomposition of the time series data, we show that it is possible to decompose the covariance of two series into various scales. This multiscale analysis, in turn, enables us to understand how synchronization of the economic activities in different countries changes over different time horizons.

The first observation is that analyzing the co-movement of economic activities is more demanding on data than a simple decomposition. Therefore, in order to extend the number of observations in our sample, we use a monthly Industrial Production Index (IPI) data series. This series provides a sample size of $N = 237$ months. However, this selection also comes at a cost. As the IPI series does not include most of the low income countries, a modified sample of five countries was chosen for our analysis. These are Brazil (BRA), India (IND), United States (USA), Turkey (TUR), and Pakistan (PAK).

In the remaining part of this analysis, we use the industrial production of United States as a reference point. Therefore, the analysis shows to what extent industrial production in other countries in the sample are synchronized with the United States. Figure 9 shows the results for cross-correlation defined in (A.6) with lag $\lambda = 0$ and with scale τ ranging up to 32 months. Confidence intervals in all cases suggest that the correlations are insignificant for very high (0-1 month) and lower frequencies (16+ months). In addition, there is no statistically significant correlation between Pakistan IPI and the one of the United States at any frequency (Panel d). Similarly, the Indian series are not significantly correlated except at the 4 month scale. The correlations with Turkey and Brazil, on the other hand, follow a similar pattern with statistically significant associations at 2, 4, and 8 month scales. The peak of the correlation in Turkey is at 16 month scale, whereas the Brazil has its peak at 8 month scale. Overall, in our sample, industrial production in Turkey exhibits the highest correlation with the one in the United States at all scales.

The co-movement exercise has so far focused on contemporaneous synchronizations. Next, we study the lead and lag relationships between the industrial production in the United States and other countries in the sample. Note that the confidence intervals for levels higher than 4 (16 months) are too wide due to the boundary effect of the wavelet transform; hence, they are not included in this analysis.

Figure 10 shows the results for Turkey. The first scale (d1) denotes the correlation
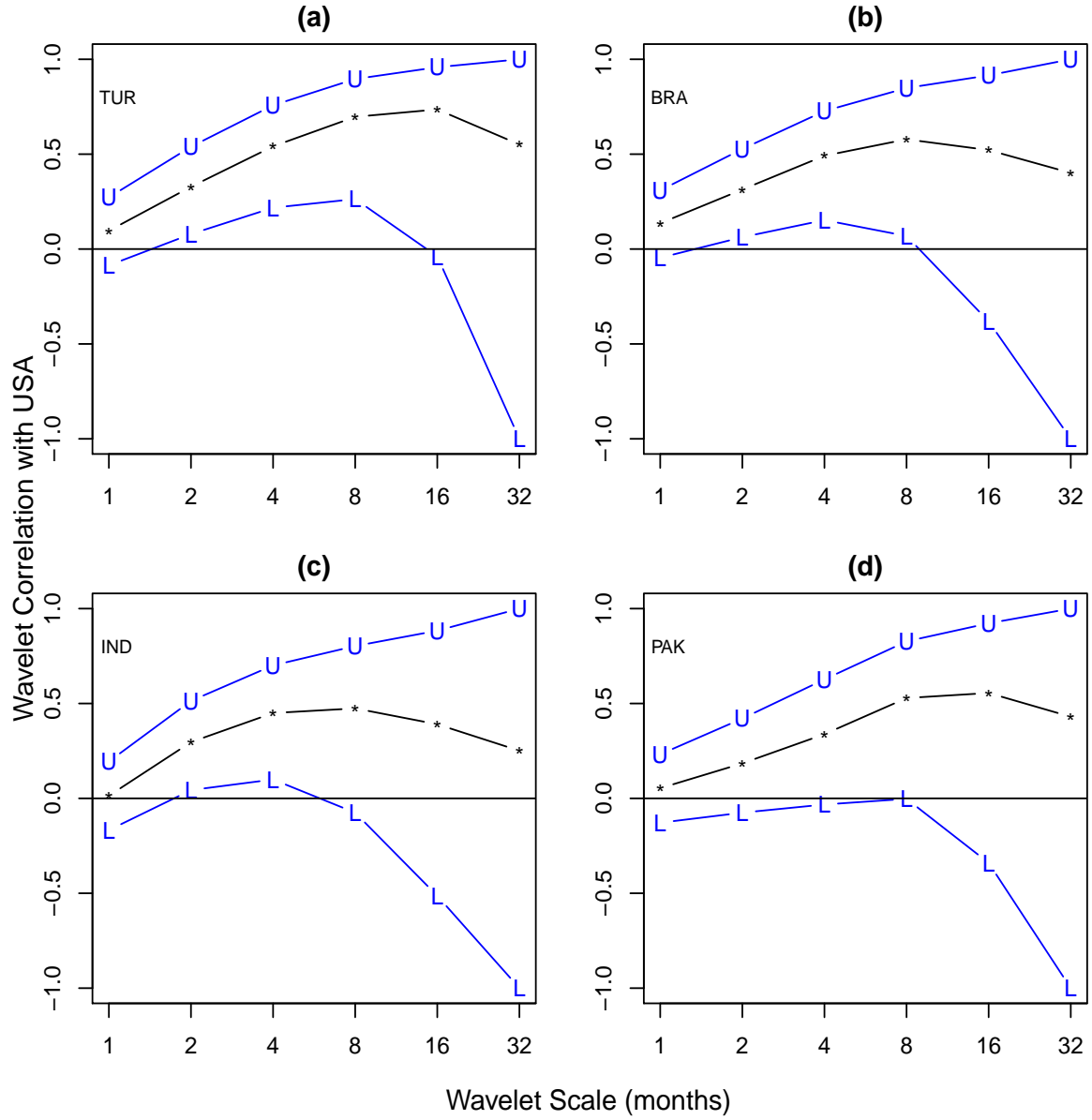
20

Figure 9: Wavelet cross-correlation between the monthly Industrial production index changes of USA (a) TUR, (b) BRA, (c) IND, and (d) PAK.

of industrial production series between Turkey and United States at 2 month frequency and by different lead and lag times. As displayed in the fourth panel, there is no significant association between the two series at this scale. In comparison, the changes in industrial production of the United States lead changes in the industrial production of Turkey on the scales between 4 and 16 months (d2 – d4) in a statistically significant way. These lead
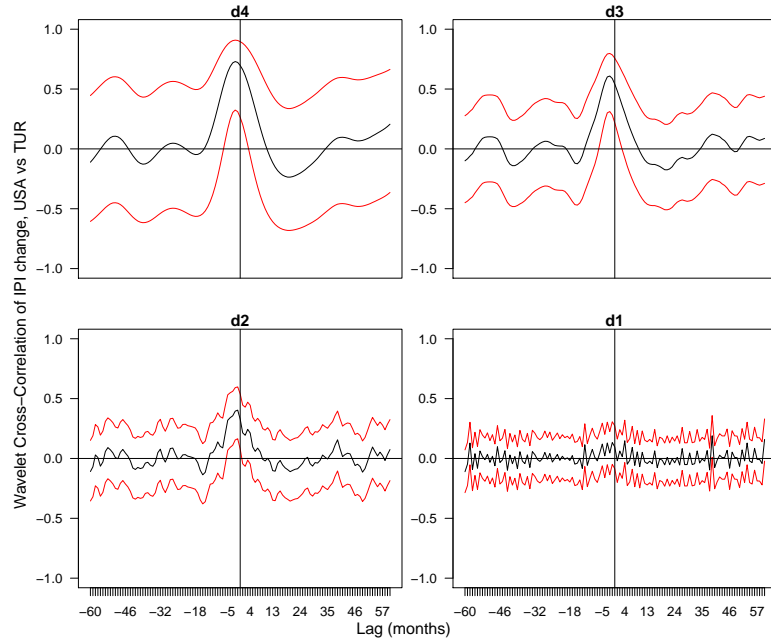
Figure 10: Wavelet cross-correlation between the monthly Industrial production index changes of USA and TUR for the first 6 scales. The range of the lags is ± 60 months.

times are found to be about 8 - 10 months. This result suggest that the medium term dynamics in the US industrial production is followed by Turkish industrial production with 8 – 10 months lag time. The analysis for Brazil is similar with the exception of lag times that are shorter than the ones for Turkey. Industrial production in Brazil at 4, 8, and 16 months scales follows the United States series with a 4-6 months lag time (see Figure 11). The co-movement of industrial productions in Brazil and United States are stronger at the longer time horizons (16 months), with no lead or lag time.

Figure 12 shows that the association between the industrial productions of India and the United States are weaker than in the previous cases. At 4 months scale (d2),

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We also investigate the cross-correlations between different scales. We found that the IPI changes in the United States at 2 months scale are positively associated with changes in IPI of Turkey at 4 and 8 months scale. The United States series at 2 months scale also leads the Turkish series at 16 months scale with a lead time of about 4 months. Responses at 4 months scale were similar. Longer-term changes in IPI of the United States, i.e. 8 and 16 months, provide changes in the Turkish IPI on scales 4 – 16 months, with about 2 months lag time.

As for cross-correlations, changes in the IPI of the United States at 2 months scale are positively associated with IPI changes of Brazil at 4 months scale with no lead or lag time. However, this turns to a slight lag between 4 and 10 months in the case of impact on the 8 and 16 months scales. Longer term dynamics in the United States industrial production (16 months scale), affect the Brazilian industrial production at scales between 2 and 8 months. In this case, there is an 8 month lag time.

Figure 11: Wavelet cross-correlation between the monthly Industrial production index changes of USA and BRA for the first 6 scales. The range of the lags is $\pm$ 60 months.

Figure 12: Wavelet cross-correlation between the monthly Industrial production index changes of USA and IND for the first 6 scales. The range of the lags is $\pm$ 60 months.

the IPI changes in India follow the ones in the United States with about 2 months lag
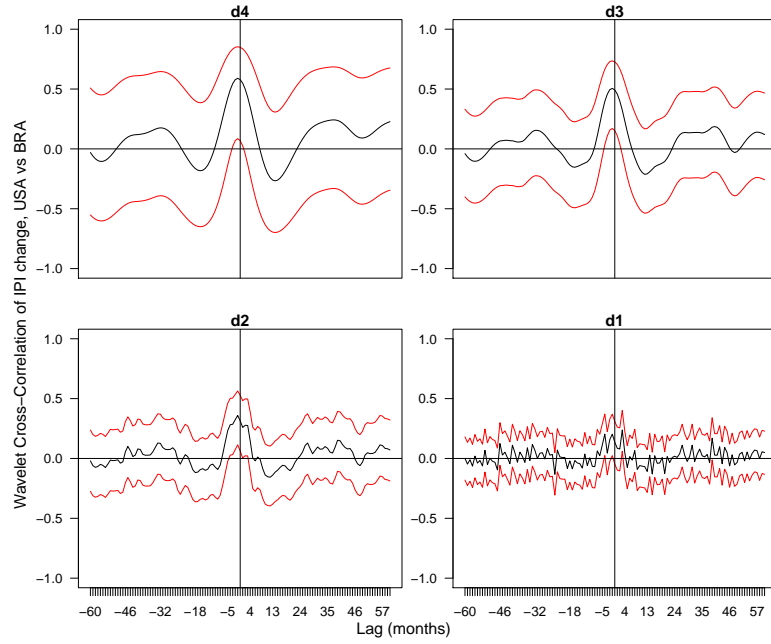
Figure 13: Wavelet cross-correlation between the monthly Industrial production index changes of USA and PAK for the first 6 scales. The range of the lags is ± 60 months.
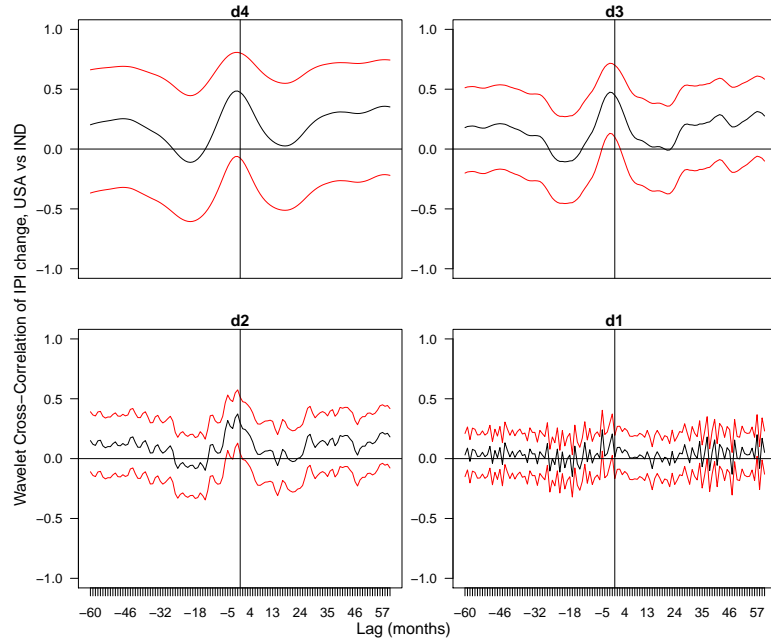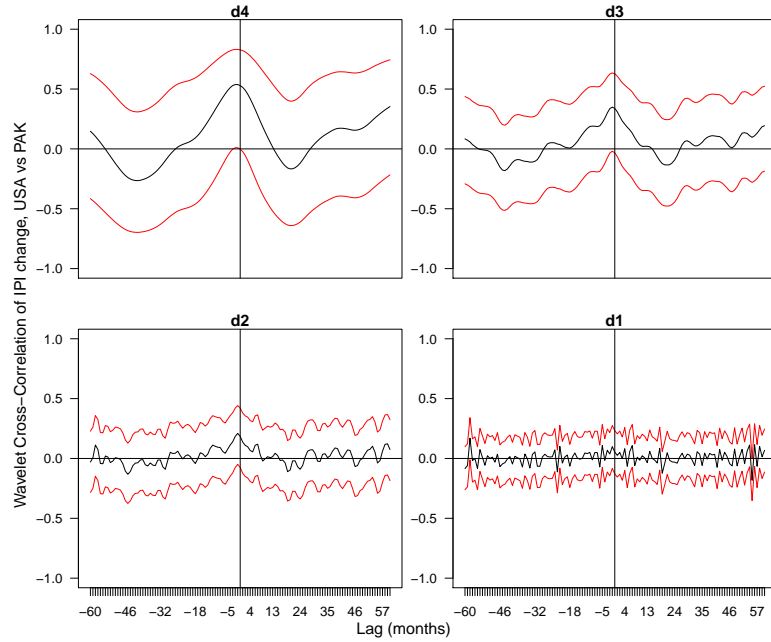
time. At 8 months scale (d3) this lead time increases to up to 6 months. There are no statistically significant associations on any other scales. The industrial production in Pakistan is uncorrelated with the one in United States at almost all time horizons except the 16 months scale (d4). As Figure 13 shows, there is a lag time of about 1-2 months in this case. However, this association is barely significant statistically. On the contrary to other countries, there are no significant effects on the IPI of Pakistan on 2 and 4 months scales.

Cross-correlation analysis between different scales reveals moderate to medium degree of negative correlations between the industrial productions in India and United States. The lag time varies from 18 to 24 months. Short term changes in Indian industrial production is not significantly correlated with any IPI changes in United States. However, the short-term changes in the IPI of United States have moderate negative effect on Indian IPI at long-term scales, i.e. 8 and 16 months. This implies that short term fluctuations in the industrial production of the United States may have have persistent effects on the Indian industrial production.

However, the changes in IPI of United States, both in short- and long-term, are positively associated with the long-term changes in IPI of Pakistan.

7   Conclusion

Recent contributions to the economic growth literature have shown that the persistence of growth is as important as its magnitude, if not more. Many developing countries exhibit sudden bursts of rapid growth that last a short period of time, whereas the developed economies tend to display more gradual but persistent growth trajectories. Therefore, determining the factors that lead to either beginning or end of the growth spells promises useful information regarding the fundamental determinants of growth. These, in turn, highlight the importance of a careful investigation for time varying characteristics of growth.

This study demonstrates an application of the wavelet based techniques for analyzing the growth and volatility over different time horizons. Overall, these techniques perform well in separating the short-term and long-term changes in the GDP levels and growth rates. Localization in time and frequency by using these techniques enables us to identify the persistence of the impacts due to one-off events in the data. These techniques also enable us to decompose the association between the growth trajectories of two different economies into different time horizons. Combined with a lead and lag relationships, this co-movement analysis can provide the policy makers with important insights on how short and long term developments in partner countries may affect the domestic economy.

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A   Technical Appendix: Wavelet Transform

Here we provide some technical details on the standard wavelet analysis techniques discussed in this paper.

A.1   Maximum Overlap Discrete Wavelet Transform

The MODWT is a non-orthogonal modification of the DWT, which produces a set of wavelet and scaling coefficients by linear filtering of the series. The MODWT is preferred over DWT for the following reasons. First, the MODWT retains the values that are downsampled and removed by DWT (redundancy). This preservation aligns the decomposed wavelet and scaling coefficients at each level with the original time series (see Percival and Walden (2000)). Therefore, the original series and its decomposition are easily comparable. The MODWT redundancy also slightly increases the effective degrees of freedom for each scale, improving the accuracy of certain wavelet-based estimates. Second, unlike the DWT that is limited to dyadic series, the MODWT is a well-defined procedure for all sample sizes N. Finally, MODWT is not affected by circular shifting of the time series, i.e. the resulting values do not depend on the starting point of the series like they do for DWT. Therefore, a shift-invariant transform like MODWT is considered in this paper.

Let X_t be a time series in discrete time $t = 0, \ldots, N-1$. As discrete time data series are finite, the MODWT requires an infinite series by definition. Therefore, in order to perform the MODWT, one needs to decide how to extend the time series to the unobserved values $X_0, X_{-1}, \ldots$, and $X_{N+1}, X_{N+2}, \ldots$. One way to solve this is to periodically extend the series, i.e. $X_0, X_1, \ldots, X_{N-1}, X_0, X_1, \ldots, X_{N-1}$. This extension, known as periodic boundary condition, might produce artifacts when there is a significant difference between the beginning and the end of the observed time series. It is the same edge effect seen in, for example, moving average smoothing methods and their variations. Several improvements have been proposed to deal with this. Some of them involve modifying the wavelets (see Cohen *et al.* (1993)), others focus on modifying the data (see Percival *et al.* (2011), Maslova *et al.* (2013)) Including the coefficients affected by the boundary condition can bias the statistical estimates, and therefore, the analysis that uses these estimates will be biased as well. Whereas, exclusion of these coefficients provides unbiased wavelet variance

estimates. However, since the sample size is reduced, the power of the corresponding tests is reduced as well. As this paper considers unbiased estimators, the choice of the boundary rule is not critical.

Decomposing the observed time series involves applying a high-pass wavelet filter from Daubechies compactly supported wavelet family $\{\tilde{h}_{j,l}\}$ (Daubechies (1992)). This yields a set of wavelet coefficients

$$\tilde{W}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{h}_{j,l} X_{t-l\,\mathtt{mod}N} \tag{A.1}$$

The application of a low-pass scaling filter, $\{\tilde{g}_{j,l}\}$, then, yields a set of scaling coefficients:

$$\tilde{V}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{g}_{j,l} X_{t-l\,\mathtt{mod}N} \tag{A.2}$$

where $t = 0, 1, ..., N-1$, "mod N" notation stands for circular convolution, $\tilde{h}_{j,l}$ and $\tilde{g}_{j,l}$ are the jth level MODWT wavelet and scaling filters, and L_j defines the length of the latter filters. There are $L_j - 1$ wavelet and scaling coefficients affected by the boundary rule. This number increases as scale j increases. These boundary coefficients do not have any meaningful interpretation, and result from the fact that the observed data are not an infinite sequence.

The jth level wavelet coefficients capture the changes of the time series associated with unitless scale $\tau_j = 2^{j-1}$. The MODWT uses approximate ideal band-pass filters, where the band is given on the interval of frequencies $[1/2^{j+1}, 1/2^j)$ and $1 \leq j \leq J$. In order to establish the association of the wavelet coefficients with the processes they are used for analyzing, we invert the frequency scale and produce equivalent periods of $[2^j, 2^{j+1}]\Delta t$ for the scales $1 \leq j \leq J$, where Δt are the time units.

A.2 Wavelet Variance

The MODWT decomposition of time series is energy conserving. Formally, this can be shown as the following:

$$\|\mathbf{X}\|^2 = \sum_{j=1}^{J_0} \|\tilde{\mathbf{W}}_j\|^2 + \|\mathbf{V}_{J_0}\|^2 \tag{A.3}$$

Intuitively, this equality shows that the variance of the original series is completely captured by the variance of the coefficients in the MODWT transform. One can consider two variance estimators based on the sample size. For large samples, we could only use non-boundary coefficients. In this case, the wavelet variance $\mathcal{V}_X^2$ is the expected value of $\tilde{W}_{j,t}^2$. The unbiased variance estimator is defined as follows:

$$\hat{\mathcal{V}}_X^2(\tau_j) = \frac{1}{M_j} \sum_{t=L_j-1}^{N-1} \tilde{W}_{j,t}^2 \tag{A.4}$$

where $\tau_j = 2^{j-1}$ is a unitless scale, $M_j = N - L_j + 1$ is the number of coefficients affected by the boundary condition at level j, and L_j is the filter width defined earlier.

In order to construct the confidence intervals for the wavelet variance, we use the fact that the unbiased variance estimator follows a χ^2 distribution with η_j equivalent degrees of freedom. We use one of the conservative methods to compute the degrees of freedom:

$$\eta_j = \max\{M_j/2^j, 1\}$$

as defined in Percival and Walden (2000). Then, the $(1 - 2\alpha) \times 100\%$ confidence interval can be approximated by

$$\left[\frac{\eta_j \hat{\mathcal{V}}_X^2(\tau_j)}{\chi_{\eta_j}^2(1-\alpha)}, \frac{\eta_j \hat{\mathcal{V}}_X^2(\tau_j)}{\chi_{\eta_j}^2(\alpha)} \right], \tag{A.5}$$

where $\chi_{\eta_j}^2(\alpha)$ is the $\alpha \times 100\%$ percentile of the χ^2 distribution with η_j effective degrees of freedom. In this paper, only the unbiased estimators and corresponding confidence intervals are computed (biased intervals result in practically the same confidence bounds), and the confidence level is set to 95% (see Figure 8).

A.3 Wavelet Cross-Correlation

The wavelet cross-correlation for scale τ_j at lag λ can be defined as:

$$\rho_{\lambda,XY}(\tau_j) = \frac{Cov\{\tilde{W}_{j,t}^X, \tilde{W}_{j,t}^Y\}}{(Var\{\tilde{W}_{j,t}^X\}Var\{\tilde{W}_{j,t}^Y\})^{1/2}} = \frac{\gamma_{\lambda,XY}(\tau_j)}{\mathcal{V}_X(\tau_j)\mathcal{V}_Y(\tau_j)}.$$

The wavelet cross-correlation coefficient is related to the bands of frequencies or scales. It is used to determine lead/lag relationship between two series on scale by scale basis.

The MODWT wavelet cross-correlation estimator is given as

$$\hat{\rho}_{\lambda,XY}(\tau_j) = \frac{\hat{\gamma}_{\lambda,XY}(\tau_j)}{\hat{\mathcal{V}}_X(\tau_j)\hat{\mathcal{V}}_Y(\tau_j)}, \tag{A.6}$$

where $\hat{\mathcal{V}}_X(\tau_j)$ and $\hat{\mathcal{V}}_Y(\tau_j)$ are the wavelet variance estimates defined in (A.4). Here $\hat{\gamma}_{\lambda,XY}(\tau_j)$ is the wavelet covariance estimator defined as follows

$$\hat{\gamma}_{\lambda,XY}(\tau_j) = \begin{cases} \tilde{N}_j^{-1}\sum_{l=L_j-1}^{N-\lambda-1} \tilde{W}_{j,l}^X \tilde{W}_{j,l+\lambda}^Y, & \lambda = 0,\ldots,\tilde{N}_j - 1, \\ \tilde{N}_j^{-1}\sum_{l=L_j-1-\lambda}^{N-\lambda-1} \tilde{W}_{j,l}^X \tilde{W}_{j,l+\lambda}^Y, & \lambda = -1,\ldots,-(\tilde{N}_j - 1), \\ 0, & \text{otherwise,} \end{cases}$$

where $\tilde{N}_j = N - L_j + 1$.