Comovement of Central European stock markets using wavelet coherence: Evidence from high-frequency data

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Abstract. In this paper, we contribute to the literature on international stock market comovement and contagion. The novelty of our approach lies in usage of wavelet tools to high-frequency financial market data, which allows us to understand the relationship between stock market returns in completely different way. Major part of economic time series analysis is done in time or frequency domain separately. Wavelet analysis can combine these two fundamental approaches, so we can work in time-frequency domain. Using wavelet coherence, we have found very interesting dynamics of cross-correlations between Central European and Western European stock markets. We analyze the high-frequency (5 minute) and low-frequency (daily) data of Czech (PX), Hungarian (BUX) and Polish (WIG) stock indices with a benchmark of German stock index (DAX) on the period of 2008-2009. Our findings provide possibility of a new approach to financial risk modeling.

Keywords: comovement, contagion, wavelet analysis, wavelet coherence

JEL classification: C22, C40, E32, F30, G15
AMS classification: 62P05, 60-08, 60G35

1 Introduction

During last decades and mainly during last several years, the interconnection between all stock markets has grown significantly. As the stock markets of Latin America, Central and Eastern Europe and South and Southeast Asia are becoming more open to foreign investors, the traded volumes have increased considerably. Such increase in liquidity and availability of stocks enlarges the possibilities of an international portfolio diversification. The recent events on financial markets between years 2007 and 2009 raised some serious questions about a potential of the diversification during critical events.

The issue of diversification is closely connected with correlations, comovements, spillovers and contagion. Whereas correlations and comovements are well defined through linkages based on fundamentals, definition of contagion varies across literature. [2] defines contagion as "correlation over and above what one would expect from economic fundamentals" whereas [3] defines it as a situation when "international propagation mechanisms are discontinuous". In globalized financial markets with growing trading volumes and liquidity, the integration and comovements are becoming stronger in time so that the use of diversification has been becoming more limited. Therefore, examination and research on different types of comovements and correlations in time is of a great importance. In addition to the time dimension of the market dynamics, there are different types of investors who influence such dynamics. Starting with noise traders with an investment horizon of several minutes or hours, the spectrum of investors ranges through technicians with the horizon of several days to fundamentalists with the horizon of several weeks or months to pension funds with the investment horizon of several years. Thus, apart from the time domain, there is a frequency domain, which represents various investment horizons.

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As both frequency and time domains are equally important and valid for a deeper research of the dynamics of the financial markets, it is needed to contain them in the analysis. However, majority of used models focus on one of the domains solely. In the time domain, [6] finds that comovements increase during volatile periods but remain strong afterwards, which is explained by Bayesian learning theory. [7] applies a time-dependent adjusted correlation analysis and argue that there are strong linkages between stock markets which become stronger during volatile periods. [8] uses cross-country time-varying correlation coefficients and apply it on Asian crisis of 1997 and shows that even after controlling for economic fundamentals, strong herding contagion remains. Similarly, [9] uses time-varying correlations on different East-Asian stock indices and finds increasing integration in time with the strongest correlations during the crisis of 1997. [11] examines four developed markets (the USA, the UK, Germany and Japan) and find that the comovements in the markets are increasing in time for prices, returns, volatility and correlations. [14] researches on dynamic conditional correlations between the US, English, German and French stock markets and show that correlations increased after Euro adoption whereas the increase was the most profound for Germany and France comovements. The literature is less numerous for the frequency domain. One of a few examples is [4] who examines relationship between the US and German stock markets and argue that there is only a short-run and no long-run interdependence. Moreover, the authors show that contagion exists only between abnormal fluctuations and specifically in a non-linear and a regime-switching manner.

In our research, we combine both time and frequency domain and we apply wavelet coherence on high-frequency (5 minute) and low-frequency (daily) data of Czech (PX), Hungarian (BUX) and Polish (WIG) stock indices with a benchmark of German stock index (DAX). By analyzing wavelet coherence, which was recently used in financial analysis by [1, 13, 5], we show how cross-correlations are changing in time and across frequencies, continuously. The novelty of our approach lies in the usage of these wavelet tools to high-frequency financial market data, which allows us to understand the relationship between Central European stock market returns in completely different way. The paper is structured as follows. We present very brief introduction to the methodology of the wavelet coherence. After the methodology is set, we employ high-frequency (5 minute) and low-frequency (daily) data of Czech (PX), Hungarian (BUX), Polish (WIG) and German (DAX) stock indices and study their interdependence in both time and frequency domain.

2 Wavelet analysis

In our paper, we use the wavelet coherence which measures local correlation of two time series in time-frequency domain. First, we briefly define the continuous wavelet transform, followed by the wavelet coherence. The continuous wavelet transform $W_x(u,s)$ is obtained by projecting a wavelet $\psi(.)$ onto the examined time series $x(t) \in L^2(\mathbb{R})$: $W_x(u,s) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{s}} \psi\left(\frac{t-u}{s}\right) dt$, where $1/\sqrt{s}$ denotes a normalization, u is a location parameter and s is a scale parameter [12]. We use the Morlet wavelet ($\omega_0 = 6$) that is a complex wavelet with a good time-frequency localization [1].

Following approach of [15], we define the wavelet coherence as the squared absolute value of the smoothed cross wavelet spectra, $W_{xy}(u,s)$[1], normalized by the product of the smoothed individual wavelet power spectra of each series i.e.,

$$R^2(u,s) = \frac{|S(s^{-1}W_{xy}(u,s))|^2}{S(s^{-1}|W_x(u,s)|^2)S(s^{-1}|W_y(u,s)|^2)}, \tag{1}$$

where S is a smoothing operator. The squared wavelet coherency coefficient is in the range $0 \leq R^2(u,s) \leq 1$, values close to zero indicates weak correlation, while values close to one are evidence of strong correlation. Thus it provides useful tool for analysis of comovement across the stock markets.

The phase difference, indicated by arrows, gives us details about delays of oscillation of the two examined time series. Arrows pointing to the right (left) when the time series are in-phase (anti-phase) or are positively (negatively) correlated. Arrow pointing up means that the first time series leads the second one, arrow pointing down indicates that the second time series leads the first one.

[1] $W_{xy}(u,s) = W_x(u,s)W_y^*(u,s)$

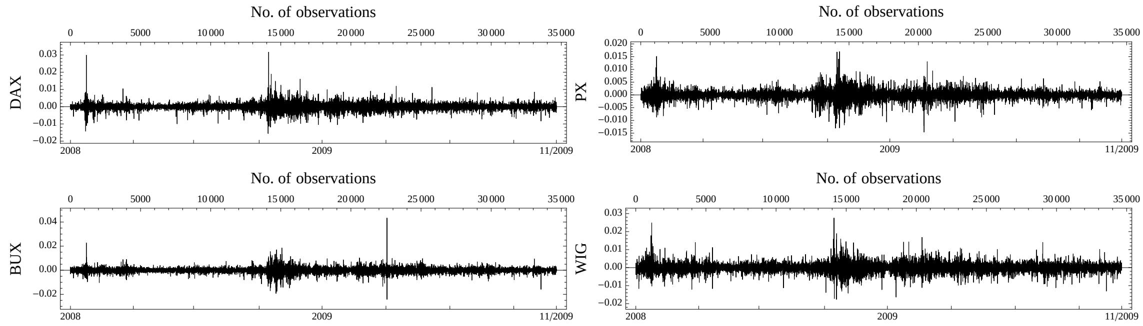

Figure 1: Plots of 5-min logarithmic returns for DAX, PX, BUX and WIG indices.

3 High-frequency data analysis

3.1 Data description

In our analysis, we use 5-minute high-frequency data of Czech (PX), Hungarian (BUX) and Polish (WIG) stock indices with a benchmark of German stock index (DAX). Central European stock markets data were collected over a period of 2 years beginning with January 2, 2008 and ending by November 30, 2009. The data were obtained from TICK data. When looking at the data, one quickly observes that number of observations for each trading day differs among the indices. This problem arises due to different stock market opening hours. Prague Stock Exchange, as well as Warsaw Stock Exchange, is open from 9:30 to 16:00 Central European Time (CET). Budapest Stock Exchange is open from 9:00 to 16:30 CET. Finally, Frankfurt Stock Exchange is open from 9:00 to 17:30 CET. Thus we need to adjust the dataset by including only the periods of day where the data is available for all analyzed stock indices. We compute logarithmic returns for the period from 9:30 to 16:00 CET for each day separately in order to avoid overnight returns. Finally, we are left with 77 return observations for each stock market for each day of the analyzed period. By discarding major public holidays, the final sample includes 450 trading days.

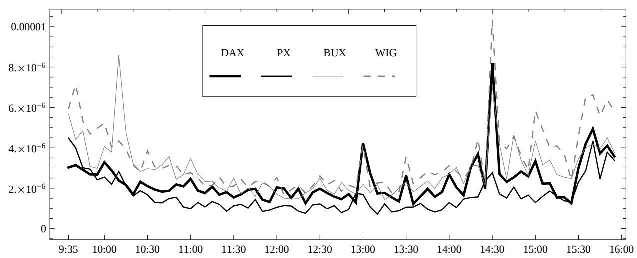

Figure 2: Intraday pattern of average squared returns (volatility).

Another issue, which may bias the results of our analysis, is a known effect of news arrival during the first hours of the trading day as well as higher activity during the end of the trading day [10]. Variance at the open is found to be more than three times the midday variance and variance at the close is about 1.5 times the midday variance. Figure 2 shows average variances through the trading day for all analyzed stock market series. It confirms the U-shape of volatility during the trading day. It is also interesting to observe quite strong activity of DAX, BUX and WIG at 14:30 CET. This activity may be contributed by the usual U.S. macroeconomic announcements during 14:30 CET. Our analyzed period includes the mortgage crisis thus the stock market volatility is generally higher indicating higher risk and also nervousness of investors. This clearly corresponds to the sudden increase in the activity at 14:30 CET for all markets. It is interesting to note that Prague Stock Exchange does not react to the U.S. announcements at 14:30 CET so strongly. In order to prevent the bias from this effect, we tried also to estimate different periods which do not include the U.S. announcements time and do not include the beginning and end of the trading so the U-shape volatility effect is eliminated. The results have not been affected, thus we finally utilize all available data consisting of 34 650 observations for each analyzed index. Figure 1 shows the plots for our final sample of 5-minute high-frequency returns.

3.2 Results

In our analysis, we utilize the wavelet transform to analyze the time-scale properties of stock returns. Moreover, the main analysis of comovement of the studied stock markets is the wavelet coherence[2] as it allows to quantify the relation between two time series in the time-frequency domain. To asses the

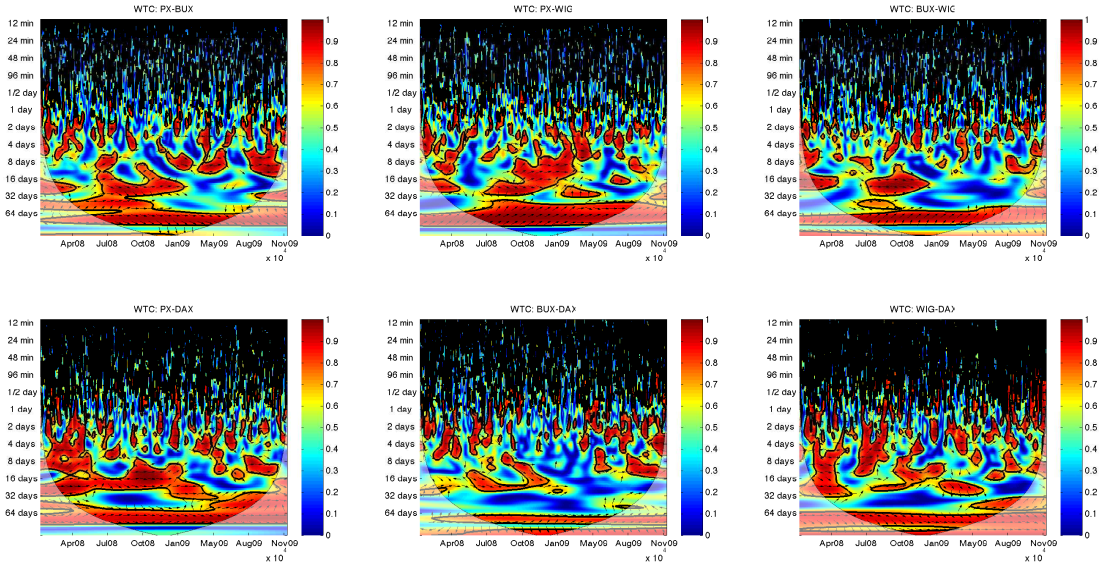

Figure 3: Wavelet coherence of PX, BUX, WIG and DAX indices pairs on the 5 minutes high-frequency returns. Horizontal axis shows time, while vertical axis shows period in minutes/days. The warmer the color of region, the higher the degree of dependence between the pair.

statistical significance of the wavelet coherence, we use the Monte Carlo simulations. Figure 3 shows the estimated wavelet coherency and the phase difference for all examined pairs of indices. Time is on the horizontal axis while vertical axis refers to frequency (the lower the frequency, the higher the scale, or period). The wavelet coherence finds the regions in time frequency space where the two time series co-vary (but do not necessarily have high power). Regions inside the black lines plotted in warmer colors represent regions where significant dependence has been found. The colder the color is the less dependent the series are. Blue regions represent periods and frequencies with no dependence in the indices. Thus the plot clearly identifies both frequency bands and time intervals where the series move together. Continuous wavelet transform at a given point uses information of neighbor data points, thus areas at the beginning and at the end of the time interval should be interpreted with caution. Especially the time-frequency blocks inside the cone of influence on lower frequencies where the transform does not have sufficient number of data.

From the analysis of the wavelet coherence, we can observe very interesting results. First of all, there are quite large significant comovement periods among all tested stock markets through several frequencies. As for the high-frequency patterns, it is quite hard to see from the pictures as the black regions consist of many small periods of significant comovement at various frequencies (5min, 10min, etc.). Each of the pairs also shows strong comovement periods on several daily frequencies up to two to three weeks as well as periods where pairs comove on the several months scales.

When we look at the comovement of PX, BUX and WIG (Figure 3), we can observe that PX is positively correlated with WIG on lower frequencies up to several months. PX-WIG pair also shows very interesting development of changing cross-correlation from the second half of year 2008 until the end of the first half of 2009. Correlations are strongly significant through this time period but they change from the month period (lower frequency) to the shorter period of one week (higher frequency). This dynamics of interdependence visible from cross-wavelet transform of high-frequency data is unique and allows to understand the relationship between the analyzed stock markets in completely different way. Morover, phases represented by arrows reveal that WIG is positively influenced by PX; these markets also have the largest period of comovement through time and scales. PX is also positively correlated with BUX at

[2]For estimation, we use the MATLAB wavelet coherence package developed by A.Grinsted, J.C.Moore and S.Jevrejeva.

several large time and scale periods but the phases do not point to any directional influence. As to the dependence of these markets on DAX, pair PX-DAX shows the largest periods of comovement. WIG is dependent on DAX while BUX again shows the weakest dependence through different time and scales periods.

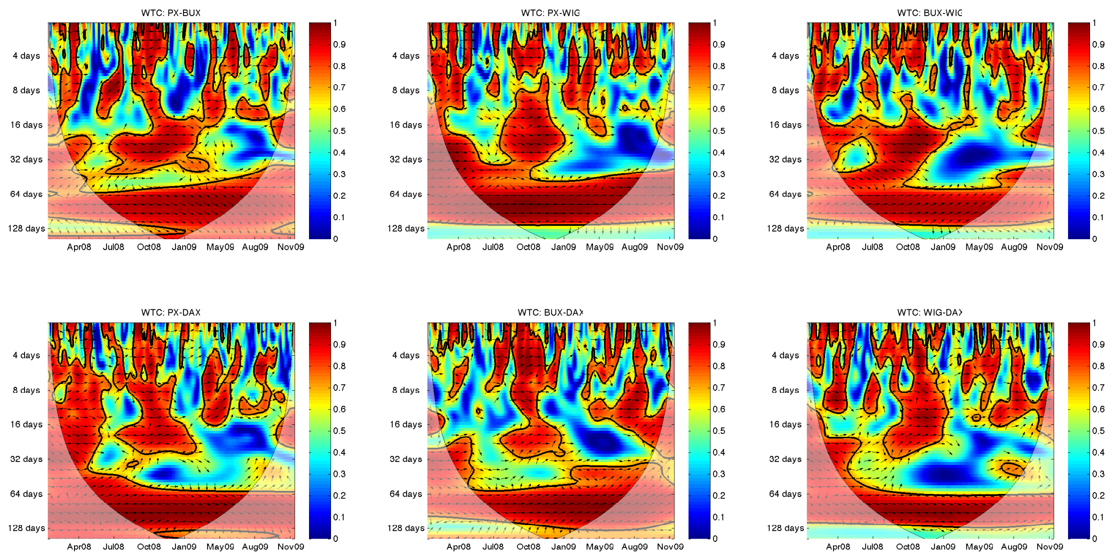

Figure 4: Wavelet coherence of PX, BUX, WIG and DAX indices pairs on the daily returns. Horizontal axis shows time, while vertical axis shows period in minutes/days. The warmer the color of region, the higher the degree of dependence between the pair.

We also perform the same analysis on the daily data including the same period. The cross wavelet coherence is plotted in Figure 4. Daily data show large time-scale periods of comovement on lower frequencies around 3-6 months. During the second half of the year 2008, all the indices show strong comovement at the scale of 2-4 weeks, while this dependence disappears in the year 2009. Comovement periods on the daily scales up to one week are also visible for all pairs.

4 Conclusion

In this paper, we contribute to the literature on the international stock market comovement and contagion by researching the interconnections between Central European stock markets in time-frequency space. The novelty of our approach lies in the usage of the wavelet tools to high-frequency financial market data, which allows to understand the relationship between stock market returns in completely different way. In our research, we combine both time and frequency domain and we apply the cross-wavelet analysis as a main tool of studying the comovements. Using the wavelet coherence, we show how cross-correlations are changing in time and across frequencies, continuously. We employ the high-frequency (5 minute) and low-frequency (daily) data of Czech (PX), Hungarian (BUX) and Polish (WIG) stock indices with a benchmark of German stock index (DAX) in the period of 2008-2009.

The main result of our analysis is that we find that interconnection between all stock markets changes significantly in time and varies across frequencies. Using the 5 minutes high-frequency data, we find the strongest interdependencies among Czech (PX) and Polish (WIG) stock markets. Cross-correlations were significant through various frequencies starting at intraday period and ending at periods up to three months. PX-WIG pair also shows very interesting development of changing cross-correlation from the second half of year 2008 until the end of the first half of 2009. Correlations are strongly significant through this time period but they change from the one month period (lower frequency) to the shorter period of one week (higher frequency). This dynamics of interdependence visible from the wavelet coherence of high-frequency data is unique and allows us to understand the relationship between analyzed stock markets in a new way. To conclude, we have shown very interesting dynamics of cross-correlations between Central European and Western European stock markets using a novel approach. Our findings are model-free and provide a possibility of the new approach to financial risk modeling. Thus, they have strong implications to portfolio management.

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