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Systematic risk and time scales: New evidence from an application of wavelet approach to the emerging Gulf stock markets

Mansur Masih *, Mohammed Alzahrani, Omar Al-Titi

Department of Finance and Economics, King Fahd University of Petroleum and Minerals, P.O. Box: 1764, Dhahran 31261, Saudi Arabia

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ABSTRACT

The paper is the first attempt to estimate systematic risk 'beta' at different time scales in the context of the emerging Gulf Cooperation Council (GCC) equity markets by applying a relatively new approach in finance known as wavelet analysis. Our results indicate that on average beta coefficients in all GCC countries show a multiscale tendency. This is consistent with our theoretical expectation that stock market investors have different time horizons due to different trading strategies and that is also reflective of the characteristics of the GCC markets in particular in that they are less developed, less liquid, involve more transaction costs, highly dependent on individual investors, and prone to infrequent trading. Further, we analyze the impact of different time scales on Value at Risk (VaR) and find that VaR measured at different time scales suggests that risk tends to be concentrated more at the higher frequencies (lower time scales) of the data. The results are plausible and intuitive and have strong policy implications.

1. Introduction

The systematic risk measure "beta" is a key concept in modern finance theory. Beta measures the covariation between a security return with the market return scaled by market variability. Estimating beta is an essential step towards testing the capital asset pricing model of Sharpe (1964), Lintner (1965), and Mossin (1966). Black, Jensen, and Scholes (1972), and Fama and MacBeth (1973) are the first studies that test the CAPM by estimating beta first and then regress the portfolio return against the estimated beta. CAPM is supported if the relation between expected return and beta is significant and equal to the market risk premium. The findings of Black et al. (1972) and Fama and MacBeth (1973) are generally supportive of CAPM, however, the slope of the regression is smaller than the market risk premium.

Many studies after that tried to discuss the difficulties in beta estimation that made CAPM testing a very difficult if not an impossible job.[1] For example, CAPM assumes that investors are concerned with return measured over one period and have the same investment horizon. In practice, however, a security market consists of thousands of traders and investors with different time horizons in their minds regarding their investment decisions and owing to the different decision-making time horizons among investors, the true dynamics of the relationship between stock returns and risk factors is likely to vary depending on the time horizon of the investors. In addition, CAPM assumes that investors are mean-variance optimizers with homogenous expectations. i.e. investors perceive the same means, variances, and covariances for returns. Therefore, in theory, if we can observe the market portfolio, all investors should agree on the calculated beta of any portfolio or security. In reality, however, this is difficult to obtain. Even if investors agree on a well diversified portfolio to be the market portfolio proxy, they will not necessarily adhere to the same investment horizon. Therefore, their perception and measurement of risk will not be the same. Financial analysts have long recognized the need to incorporate different time scales in regard to the investors' decision making in the financial markets but mainly due to the lack of appropriate analytical tool to decompose data into more than two time scales, the analysis was restricted until recently to two time scales — short- and long-run only (In & Kim, 2006). A relatively new approach known as wavelet analysis that takes care of different time scales or horizons in decision making would hopefully address that gap.

In this study we show evidence that risk has a multihorizon nature embedded in it using daily return of individual stock and market indices in 7 Gulf Cooperation Council (GCC hereafter) markets between 2007 and 2008. We use Haar wavelet transform technique to decompose the daily returns into orthogonal components with different timescales. We then estimate beta using returns measured at

* Corresponding author. Tel.: +966 3 860 5257; fax: +966 3 860 2585.
E-mail address: masih@kfupm.edu.sa (M. Masih).

[1] See Cochrane(1999) and Campbell (2000) for detailed reviews of several issues related to CAPM testing.

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those different time scales. The results show that beta changes with the different time scale used. We also provide further evidence that risk is concentrated in high frequency data by measuring the Value at Risk (VaR) at different time scales and show that VaR is higher in shorter horizons with less contribution to the VaR measure when we consider longer horizons.

The rest of the paper is organized as follows: in Section 2, we review the literature on beta estimation and the wavelet applications. In Section 3, we provide a brief introduction on the stock market considered and their unique aspects. The wavelet technique is explained in Section 4. Empirical results are discussed in Section 5. We conclude in Section 6.

2. Literature review

Several studies raised this particular issue and show that estimated beta differs depending on the interval used in calculating return,[2] see for example, Fama (1970), Pogue and Solnic (1974), Levhari and Levy (1977), Smith (1978), and Hawawini (1980). More recently, Frankfurter, Leung, and Brockman (1994) use US data to show that the mean and variance of beta increase as return interval increases from daily to yearly.

Different explanation for the interval bias in beta can be found in literature. The most popular is that multiscale beta can be caused by infrequent trading and delays in price adjustment to new information that lead to cross-correlations among the security returns which in turn lead to autocorrelation in market returns and bias estimated beta. See for example Fisher (1966) and Cohen, Hawawini, Maier, Schwartz, and Whitcomb (1980).

Bjornson, Kim, and Lee (1999) suggest that beta reflects macroeconomic risks that may have different frequencies (high or low) and thus the sensitivity of beta to different time intervals may reflect the impact of those risks.

Handa, Kothari, and Wasley (1989) suggest that the sensitivity of beta estimation to the return interval is caused by the fact that the covariance between the security and the market and the variance of the market index do not change proportionately as the return interval changes. They found that betas of high-risk securities increased with the return interval, whereas betas of low-risk securities decreased with the return interval. Handa et al. (1989) use US data to show that estimated beta changes with return interval while the direction of the change is inversely related to the firm size. Specifically, they report larger estimated beta as we move towards larger return interval for smaller firms and smaller estimated beta as we move towards larger return interval for large firms. Brailsford and Josev (1997) reported similar results using Australian data. Corhay (1992) show return interval bias using Belgium data and Diacogiannis and Makri (2008) using Greek data, Fung, Schwartz and Whitcomb (1985) using French data, and Fowler, Rorke, and Jog (1980) using Canadian data.

An alternative explanation for the sensitivity of beta estimation to return interval is that standard error of the beta estimates increases as the return interval is lengthened. Daves, Ehrhardt, and Kunkel (2000) using US data report smaller standard error of estimated beta with smaller return interval than larger intervals. The larger standard error of beta in longer intervals makes it less able to explain return variation. Handa et al. (1989) regress returns on monthly and annual betas to test this explanation, however, their evidence based on both OLS and GLS estimation does not support this explanation.

The findings of Daves et al. (2000) suggest that testing CAPM may depend on the interval chosen. In fact, Handa, Kothari, and Wasley (1993) show that CAPM is rejected when using monthly return interval while it cannot be rejected when using yearly return interval. Brailsford and Faff (1997) failed to reject the CAPM using weekly returns while they rejected the CAPM using daily return. Kothari and Shanken (1998) provide evidence that monthly returns can lead to rejecting CAPM while using yearly return interval provide better results as it avoids non-synchronous trading and seasonality effects on returns.

Other studies suggest ways to reduce this bias in estimating beta. For example, Scholes and Williams (1977), Dimson (1979) suggest use of lead and lag returns to calculate a less biased beta. Vasicek (1973) suggests a Bayesian correction for the bias. Cohen, Hawawini, Maier, Schwartz, and Whitcomb (1983) present an analytical model to adjust estimated beta by estimating the cross-sectional interval–thinness relationship. There is no agreement in the literature on which adjustment method is superior over the rest. Diacogiannis and Makri (2008) tested the efficiency of the Scholes and Williams (1977) and Cohen et al. (1983) methods using Greek data. They found that neither method provides any incremental benefit over standard OLS estimation. Bartholdy and Riding (1992) make the same conclusion about Scholes and Williams (1977) and Dimson (1979) using New Zealand market data. Fung et al. (1985) provide evidence that support Cohen et al. (1983) method. McInish and Wood (1986) use a linear programming model to test the efficiency of the different adjustment methods and find evidence supporting Dimson (1979) model.

So far, empirical observations reveal that beta changes as we change investment horizon or time scale at which return is measured. Therefore, the traditional ways of estimating beta through picking an arbitrary time scale to measure return and simply run OLS regression to estimate beta will not be statistically appropriate and we will lose a lot of information about beta dynamics across different intervals. To accommodate for the multihorizon nature of beta we can use the help of wavelet analysis.[3]

Wavelets are localized in both time and scale and can be used to cut (filter) data up into different frequency components. We can use wavelet filters to decompose returns into different time scales and analyze the dynamics of beta across the different horizons without losing any information and without having to assume stationary process for beta. Wavelet analysis can also accommodate structural changes, discontinuity, and regime shifts.

Financial and economics application of wavelet analysis is introduced by Ramsey and Lampart (1998a,b) in which the relationship between money-income and money-expenditure are analyzed. Almasri and Shukur (2003) examined the causality relationship between public expenditures and incomes across different time scales. Dalkir (2004) analyzed the multiscale causality relationship between money supply and income. Gencay, Selcuk, Whitcher (2001a) introduced a method to extract intraday seasonality using wavelet analysis with no need for model selection parameters. Ramsey and Zhang (1997) use wavelet analysis to depict the non-stationary nature of foreign exchange rate series. Gencay, Selcuk, and Whitcher (2001b) also used wavelet analysis to show that foreign exchange volatility differs across different time horizons. Karuppiaha and Los (2005) use wavelet analysis to describe the multiresolution nature of foreign exchange in Asia.

Other examples on wavelet applications in finance include, Kim and In (2005a) in testing the relation between stock return and inflation, Lin and Stevenson (2001) and In and Kim (2006) in

[2] Other studies discussed other aspects of unstable beta estimation. Blume (1971) and Levy (1971) show that estimated beta tends to regress towards the over all mean of the market beta which is one. Blume (1971) and Blume (1975) introduced a cross-sectional first-order autoregressive model to remedy this bias. Fabozzi and Francis (1978) suggest that beta is not stable and moves randomly over time. Brailsford and Faff (1997) and Ghosh (1992) reach similar conclusion using GARCH process. Harvey (1989) discuss instability of beta overtime. Chan and Lakonishok (1993) analyzed beta estimated in different periods in over 60 years to show that the estimated betas are sensitive to the time period examined. Also, Garcia and Ghysels (1998) discussed the impact of structural change and regime switches on beta estimation.

[3] See Ramsey (1999) and Ramsey (2002) for a comprehensive review if the wavelet applications in finance and economics. Gençay, Selcuk, and Whitcher (2002) also provide a number of financial and economic application of wavelet analysis.

examining the relation between the future market and spot market, Kim and In (2003) and Gallegati (2005a) in testing the causality test between financial market and economic activity, Kim and In (2005b) and In, Kim, Marisetty, and Faff (2008) in calculation of multiscale Sharp ratio, and Zhang and Farley (2004) and Lee (2004) in analyzing causality of the international stock market, Kim and In (2007) in analyzing the mutiscale relation between stock prices and bond returns.

As far as the multiscale nature of beta, Gallegati (2005b) analyzed MENA markets and shows that variance and covariance of stock returns change according to time scale. Norsworthy, Li and Gorener (2000) analyzed stocks from the US market and find that beta coefficients generally decrease as we move into higher scales (longer interval). They also reported a decline in ability of the market to explain stock returns as we move into higher scales. Gençay, Selcuk, and Whitcher (2003, 2005) propose a new method to estimate beta using multiscale decomposition through wavelet filters. Their findings in three markets, US, UK, and Germany support the multihorizon nature of beta; and, contrary to the findings of Norsworthy et al. (2000), they report a stronger relation between return and risk as we move into higher time interval. Fernandez (2005) uses wavelet analysis to test multiscale CAPM using portfolio from emerging markets and find that beta changes with different time scales. Fernandez (2006) finds similar results using Chilean data. Despite all these works, ours is the first attempt at estimating multiscale beta in the context of the emerging Gulf stock markets with the help of wavelet analysis.

3. GCC market data

We use daily stock returns data for all 7 stock markets in Gulf Cooperation Council (GCC) countries. The GCC stock markets are relatively young. The oldest stock market in the GCC is the Kuwaiti stock market which was established in 1977. Stock markets in Muscat, Bahrain, and Saudi Arabia started around 1989. Doha stock market in Qatar was established in 1997. The youngest markets in the GCC are Dubai and Abu Dhabi securities markets which were established in 2000.

Market capitalization and trading volume in the GCC markets is significantly smaller than other markets in developed countries. In addition, GCC markets lack the significant role of institutional and foreign investors. GCC markets put restrictions on foreign capital and institutional investors invest in these markets with cold feet especially after the burst of the bubble in those markets in 2006. The authorities in those markets are developing new rules and regulations to reform those markets, however, the process is gradual and may be slow. Abdmoulah (forthcoming) provides evidence that the regulatory efforts in the GCC markets were not effective enough to enhance efficiency of those markets.

A number of studies try to test the efficiency of the GCC markets in the weak-form. Dahel and Laabas (1999) test the efficiency of four GCC stock markets in Bahrain, Kuwait, Oman and Saudi Arabia and show that only Kuwaiti stock market is efficient in the weak-form while other markets failed the random walk tests including unit Root, variance ratio, and auto correlation tests. Rao and Shankaraiah (2003) provide evidence that GCC stock markets are not informationally efficient indicating a delay in price adjustment to new and relevant information. Sharma (2005) provides evidence that daily stock returns in the Saudi, Qatari, Kuwaiti, and Omani stock markets deviate significantly from normal distribution. Elango and Hussein (2007) provide evidence against GCC market efficiency using runs test and also show that stock returns in these markets are not normally distributed. Simpson (2004) analyzes the GIC index that comprises all GCC markets and shows that GCC stock return to have auto-correlations up to 36 lags, non-normal distribution, non-stationary series, and structural breaks. Abdmoulah (forthcoming) examined the efficiency of Arab markets including all GCC markets over time. He shows that GCC markets are not efficient in the weak-form and that has not improved despite the regulatory reforms in those markets except for the Saudi stock market. Al Janabi, Hatemi-J and Irandoust (forthcoming) find evidence that support efficiency of the GCC markets in the weak and semi-strong form using bootstrapping that account for non-normality and varying volatility.

The general observation about the GCC markets is the existence of thin and infrequent trading. Abraham, Sayyed, and Alsakran (2002) provide evidence that weak-form efficiency test is improved in the GCC markets when we adjust returns for infrequent trading. Al-Khazali, Ding, and Soo Pyun (2007) test the random walk hypothesis using nonparametric variance-ratio test in eight MENA countries including four GCC countries and find evidence supporting market efficiency in those markets only after correcting for infrequent and thin trading.

These empirical findings combined with characteristics of the GCC markets make the GCC markets good candidates for applying wavelet to estimate beta. The GCC markets suffer from thin and infrequent trading with high variability in return. In addition, the active investors in these markets are individual investors with different investment horizons and thus they perceive risk from different time perspectives.

The Saudi stock market data set includes the daily price series of 88 companies and the Saudi market index (TASI), from 12 February 2007 to 2 March 2008. Muscat securities market data set includes the daily price series of 114 companies and the Muscat market index (MSM), from 28 March 2007 to 1 April 2008. Kuwait stock exchange data set includes the daily price series of 189 companies and the Kuwait market index (KSE), from 28 March 2007 to 1 April 2008. Bahrain stock exchange data set includes the daily price series of 43 companies and the Bahrain market index (BSE), from 28 March 2007 to 1 April 2008. Doha securities market data set includes the daily price series of 38 companies and the Doha market index (DSM), from 28 March 2007 to 1 April 2008. Abu Dhabi securities market data set includes the daily price series of 61 companies and Abu Dhabi market index (ADSM), from 28 March 2007 to 1 April 2008. Dubai financial market data set includes the daily price series of 46 companies and Dubai market index (DFM), from 28 March 2007 to 1 April 2008.[4]

4. Methodology

For each stock market, we collect daily return series (256 observations) for each stock in the sample as well as for the market index. Daily stock returns are calculated from stock price (P) as follows,

$$r_{it} = \ln\left(\frac{P_{it}}{P_{it-1}}\right) \text{ for stock } i \text{ at day } t \tag{1}$$

While the daily return on the market index is calculated from the index value (X) as follows:

$$r_{mt} = \ln\left(\frac{X_t}{X_{t-1}}\right) \text{ at day } t \tag{2}$$

After calculating the return series for every stock and for the market, we use wavelet analysis to be able to separate out each return series into its constituent multiresolution (multihorizon) components. To do that we apply discrete wavelet transformation (DWT) on daily return series by sampling the return series at evenly-spaced points in time. By doing

[4] The number of stocks in each markets is larger than those in our sample because we exclude some stocks that does not have complete return series 256 trading days) during our sample period. As a result we exclude18 stocks listed in Saudi stock market, 11 listed in Muscat securities market, 9 listed in Kuwait stock exchange. 4 listed in Bahrain stock exchange, 4 listed in Doha securities market, 2 listed in Abu Dhabi securities market, and 13 listed in Dubai financial market.

the transformation we basically transform the return series from time domain into scale (interval) domain in order to understand the frequency at which the activity in the time series occurs. In our study, we sample the daily return series at different scale crystals (j) as follows: d1 (2–4 days), d2 (4–8 days) days, d3 (8–16 days), d4 (16–32 days), and d5 (32–64 days).[5]

We use orthogonal Haar wavelet transformation to obtain a multi-scale decomposition of the return series.[6] The transformed return series $r(t)$ is represented as a linear combination of wavelet functions as follows:

$$r(t) \approx \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \ldots \sum_k d_{1,k}\psi_{1,k}(t) \quad (3)$$

where:

j is the number of scale crystals (intervals or frequencies)
k is the number of coefficients in the specified component

$\phi_{j,k}(t)$ and $\psi_{j,k}(t)$ are the father and mother orthogonal wavelet pair that are given respectively by

$$\phi_{j,k}(t) = 2^{-j/2}\phi\left(\frac{t-2^jk}{2^j}\right) \text{ for } j = 1 \text{ to } J \quad (4)$$

$$\psi_{j,k}(t) = 2^{-j/2}\psi\left(\frac{t-2^jk}{2^j}\right) \text{ for } j = 1 \text{ to } J \quad (5)$$

Father wavelets represent the low-frequency (smooth) parts of the series, whereas mother wavelets represent the high-frequency (detailed) parts of the series.

$s_{j,k}$ and $d_{j,k}$ are wavelet coefficients that are approximated by the following integrals:

$$s_{J,k} \approx \int \phi_{J,k}(t) f(t) dt \quad (6)$$

$$d_{j,k} \approx \int \psi_{j,k}(t) f(t) dt \quad (7)$$

$s_{J,k}$ are called the 'smooth' coefficients that represent the underlying smooth behavior of the series, while $d_{j,k}$ are called the 'detail' coefficients that represent the scale deviations from the smooth process. These coefficients are measures of the contribution of the corresponding wavelet function to the total series.

After we decompose the return series into j crystals, the crystals d_j are recomposed into a time domain. The entire return series is replicated in multiresolution decomposition as follows:

$$\hat{r}^J = D_1 + \ldots\ldots + D_J + S_J \quad (8)$$

where Dj is the recomposed series in the time domain from the crystal d_j and S_J is the recomposition of the residue.

The reconstituted return series $\hat{r}^J$ contain the separate components of the original series at each frequency j. D_j represent the contribution of frequency j to the original series.[7]

[5] The number of j crystals that can be produced has to satisfy the following relation: $N \geq 2^j$ where N is the number of observations. In our study we have 256 observations. Thus, the maximum number of crystals that can be produced is 8. However, the recommended number if crystals is less than that (5 in our case) as higher crystals can only just be properly resolved.

[6] For more details and applications on orthogonal wavelet, see Mallat (1998) and Gencay et al. (2002).

[7] Wavelet filter has unit energy. This property ensures that the coefficients from the wavelet transform will have the same energy as the data. Therefore, no information are excluded thorough the wavelet transform and also no extra information re added in the process.

After obtaining the recomposed series for each frequency, we run an OLS regression of each stock on each recomposed crystal of the market portfolio R_m^j:

$$R_i = \alpha_i^j + \beta_i^j R_m^j + \varepsilon_i^j = \alpha_i^j + \beta_i^j D_m^j + \varepsilon_i^j \quad \text{for } j = 1 \text{ to } 5 \quad (9)$$

The coefficient β_i^j is the *key* variable we are trying to examine. If β_i^j is essentially similar across scales j, then beta does not have any multiscale nature embedded in it, i.e., there is no reason to believe that beta calculated using daily return to be different from beta calculated using weekly or monthly returns. However, if β_i^j changes depending on the scale j, then beta is multiscale and thus, return interval cannot be arbitrarily chosen. In addition, return interval would have an effect on the risk measure as well as on the CAPM testing results.

4.1. Computation of wavelet variance, covariance and value at risk

Wavelet variance analysis requires partitioning the variance of a time series into pieces that are associated to different time scales. It tells us which scales are important contributors to the overall variability of a series (Percival & Walden, 2000).

In particular, let $x1, x2, \ldots, xn$ be a time series of interest, which is assumed to be a realization of a stationary process with variance σ_X^2. If $v_X^2(\tau_j)$ denotes the wavelet variance for scale $\tau_j = 2^{j-1}$, then the following relationship holds:

$$\sigma_X^2 = \sum_{j=1}^{\infty} v_X^2(\tau_j) \quad (10)$$

This relationship is analogous to that between the variance of a stationary process and its spectral density function (SDF):

$$\sigma_X^2 = \int_{-1/2}^{1/2} S_X(f) df \quad (11)$$

where $S_X(f)$ is the SDF at the frequency f $[-1/2, 1/2]$.

An unbiased estimator of the wavelet variance is defined as

$$\hat{v}_X^2(\tau_j) = \frac{1}{(n_j' - L_j')2^j} \sum_{t=L_j'-1}^{n_j'-1} d_{j,t}^2 \quad (12)$$

where $n_j' = (n/2^j)$ be the number of discrete wavelet transform (DWT) coefficients at level j, where n is the sample size, and $L_j' = (L-2)\left(1-\frac{1}{2^j}\right)$ is the number of DWT boundary coefficients at level j (provided that $n'_j \succ L_j'$), where L is the width of the wavelet filter.

Similarly, the unbiased wavelet covariance between the time series X and Y, at scale j, can be defined as

$$\hat{v}_{XY}^2(\tau_j) = \frac{1}{(n_j' - L_j')2^j} \sum_{t=L_j'}^{n_j'-1} d_{j,t}^{(X)} d_{j,t}^{(Y)} \quad (13)$$

Provided that $n_j' \succ L_j'$.

Under CAPM, the wavelet beta estimator for asset i, at scale j, is defined as

$$\hat{\beta}_i(\tau_j) = \frac{\hat{v}_{R_iR_m}^2(\tau_j)}{\hat{v}_{R_m}^2(\tau_j)} \quad (14)$$

where $\hat{v}_{R_iRm}^2(\tau_i)$ is the wavelet covariance of asset i and the market portfolio at scale j, and $\hat{v}_{R_m}^2(\tau_i)$ is the wavelet variance of the market portfolio at scale j

Also, the wavelet R^2estimator for asset i, at scale j, is defined as

$$R_i^2\left(\tau_j\right) = \hat{\beta}_i^2\left(\tau_j\right)\frac{\hat{v}_{R_m}^2(\tau_j)}{\hat{v}_{R_i}^2(\tau_j)} \tag{15}$$

(See Gençay et al., 2003).

We turn next to measure Value at Risk (VaR). VaR is a popular measure of market risk (see, for example, Jorion, 2001, chapter 1), whose origin dates back to the late 1980.s at J.P. Morgan. In particular, VaR answers the question as to how much we can lose, with a given probability, over a certain time horizon.

To obtain the contribution of scale j to total value at risk, we follow Fernandez (2006) and use the measure:

$$\frac{\sigma_m^2\left(\tau_j\right)\left(\sum_{i=1}^{k}\beta_i(\tau_j)/k\right)^2 + \frac{1}{k^2}\sum_{i=1}^{k}\sigma_{\varepsilon_i}^2(\tau_j)}{\sigma_m^2\left(\sum_{i=1}^{k}\beta_i/k\right)^2 + \frac{1}{k^2}\sum_{i=1}^{k}\sigma_{\varepsilon_i}^2} \tag{16}$$

In order to obtain $\sigma_{\varepsilon_i}^2(\tau_j)$, we use the relation $\sigma_i^2(\tau_j) = \beta_i^2(\tau_j)\sigma_m^2(\tau_j) + \sigma_\varepsilon^2(\tau_j)$.That is,

$$\sigma_\varepsilon^2(\tau_j) = \sigma_i^2(\tau_j) - \beta_i^2(\tau_j)\sigma_m^2(\tau_j) \tag{17}$$

The variance of stock i at scale j, $\sigma_i^2(\tau_j)$, the beta of stock i return at scale j, $\beta_i(\tau_j)$, and the variance of the market portfolio at scale j, $\sigma_m^2(\tau_j)$, can be computed using Eqs. (12) and (14).

5. Empirical results and discussions

Table 1 shows the results of estimating beta coefficients β_i^j and $R^2$for each GCC market by regressing individual stock returns on each recomposed scale crystal j of the market index D_m^j of the specific GCC market. The mean, variance, skewness and kurtosis are shown for distribution of all estimated betas at each time scale. The coefficients β_i^j measure the contribution of scale j movements in the market return to the stock return. These coefficients represent scale-specific betas for each asset. As we can see in Table 1, beta coefficients in all GCC markets show a multiscale tendency. Beta changes non-monotonically with the time scale. In general, beta seems to slightly increase between lowest and highest scale. This can be understood in the context of the relation between risk and trading interval as long-term investors are more exposed to systematic risk than short-term investors. However, we can also notice that the variability of beta (measured based on the distribution of betas of all stocks) increases as we move from low to high scale. This result suggests that the high frequency component of systematic risk (low scale beta) is more stable at the lower scale component of the market return.

Table 1
Market model regression by recomposed crystals.

	Total		Beta					R^2				
	Beta	R^2	D1	D2	D3	D4	D5	D1	D2	D3	D4	D5
Panel A: Saudi Stock Market												
Mean	1.06	0.37	1.14	0.98	1.18	0.97	1.20	0.18	0.10	0.04	0.03	0.02
SD	0.22	0.11	0.25	0.29	0.39	0.32	0.48	0.05	0.05	0.02	0.02	0.01
Skew.	−0.61	1.21	−0.34	−0.06	0.14	−0.03	0.91	−0.06	1.63	0.37	1.06	1.44
Kurtosis	−0.51	3.08	−0.65	−0.29	−0.66	−0.41	1.10	−0.28	4.70	−0.40	0.65	2.04
Panel B: Muscat Securities Market												
Mean	0.42	0.05	0.43	0.34	0.51	0.60	0.27	0.02	0.02	0.01	0.01	0.01
SD	0.62	0.08	0.87	0.64	0.92	1.40	1.02	0.04	0.03	0.01	0.01	0.01
Skew.	2.45	2.45	4.62	1.11	1.49	3.66	−2.10	2.79	2.46	1.52	3.81	2.76
Kurtosis	9.72	7.96	26.56	8.63	10.26	18.62	10.45	8.33	6.24	1.78	18.82	10.52
Panel C: Kuwait Stock Exchange												
Mean	0.91	0.08	0.91	0.91	0.89	0.92	1.05	0.03	0.02	0.02	0.01	0.01
SD	0.60	0.07	0.69	0.65	0.73	1.44	1.24	0.03	0.02	0.02	0.01	0.01
Skew.	0.38	0.96	0.53	0.04	0.34	−0.03	1.17	1.51	1.39	1.27	2.53	2.71
Kurtosis	−0.07	0.43	0.57	−0.48	−0.08	0.48	4.10	2.75	2.09	1.15	9.17	8.79
Panel D: Bahrain Stock Exchange												
Mean	0.33	0.03	0.31	0.36	0.27	0.43	0.38	0.02	0.01	0.01	0.00	0.01
SD	0.54	0.07	0.58	0.70	0.84	0.77	1.07	0.05	0.02	0.01	0.01	0.01
Skew.	3.44	5.66	3.67	1.71	0.70	0.74	−0.43	5.79	4.16	2.07	2.50	2.17
Kurtosis	16.06	34.75	17.64	5.69	0.70	0.74	1.13	35.95	20.06	4.24	7.27	5.64
Panel E: Doha Securities Market												
Mean	0.67	0.18	0.49	0.80	0.65	0.69	0.81	0.04	0.10	0.02	0.03	0.01
SD	0.25	0.13	0.29	0.31	0.57	0.39	0.53	0.03	0.07	0.02	0.02	0.01
Skew.	−0.18	0.92	−0.66	−0.81	−0.37	−0.16	−0.90	0.49	1.36	1.46	1.00	0.66
Kurtosis	−0.09	0.88	0.39	1.69	0.01	−0.46	1.46	−0.83	3.05	2.18	0.65	−0.60
Panel F: Dubai Financial Market												
Mean	0.49	0.20	0.51	0.43	0.48	0.52	0.54	0.12	0.05	0.02	0.01	0.01
SD	0.57	0.28	0.61	0.58	0.59	0.70	0.67	0.17	0.06	0.03	0.01	0.01
Skew.	0.68	0.92	0.60	0.60	0.65	1.16	0.32	1.02	1.06	1.77	2.84	1.00
Kurtosis	−1.16	−0.92	−1.24	−1.08	−0.50	2.38	−0.98	−0.55	−0.30	2.52	9.20	−0.12
Panel G: Abu Dhabi Securities Market												
Mean	0.53	0.09	0.48	0.59	0.57	0.69	0.70	0.05	0.02	0.01	0.01	0.01
SD	0.47	0.12	0.50	0.61	0.59	1.02	0.81	0.07	0.03	0.02	0.01	0.01
Skew	0.24	1.19	−0.03	−0.04	0.17	3.01	0.47	1.10	1.35	1.67	2.14	1.99
Kurtosis	−0.95	−0.06	−0.42	−0.42	−0.21	16.69	−0.25	−0.24	0.67	1.98	5.01	3.70

A more interesting result appears from observing the behavior of R^2as it decreases monotonically when we move to higher scales (longer interval). This result means that the market return is more able to explain individual stock return at lower scales (shorter intervals) than higher scales. In other words, systematic risk of individual stocks is better captured in the high frequency (short horizon) part of the market return rather than the low frequency (long horizon). The results also suggest that CAPM will be supported more if we use returns that are measured over shorter periods.

The results are in line with the findings of Norsworthy et al. (2000) and Fernandez (2006) but opposite to those of Gençay et al. (2005) on developed markets. This result leads us to question whether developed markets are different from emerging markets in terms of the appropriate investment horizon used by the marginal investor when determining his investment objectives and taking his investment decisions. The difference between developed and emerging markets may also stem from the identity of the marginal investors in those markets as individual investor may have different horizon length than institutional investors.

Our findings can be easily understood within the GCC markets context as individual investors represent the majority of investors and their trades on average are believed to be driven by shorttermism.

In Table 2, we use the recomposed residues from the market recomposed series to explain individual stock returns using the following regression:

$$R_i^j = \alpha_i^j + \beta_i^j S_m^j + \varepsilon_i^j \tag{17}$$

where

$$S_m^j = R_m - \sum_{i=1}^{j} D_m^i \text{ for } j = 1 \text{ to } 5$$

The recomposed residue is the difference between the raw signal and the accumulated wavelet components that represent the signal at each scale j. Therefore, as we move to higher scales, there will be less left in the variability of the market return and as a result less ability to explain the return variation in the individual stock. Table 2 shows less contribution of the recomposed residues in explaining stock returns as we move into higher scales. The beta coefficients are lower at higher scales in all GCC markets except for Muscat market. Also, by looking at the R^2s of those regressions we can see that they decline as we move into higher scales suggesting that most of the useful variations in the market in explaining returns are at the high

Table 2
Market model regression by recomposed residues.

	Total		Beta					R^2				
	Beta	R^2	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5
Panel A: Saudi Stock Market												
Mean	1.06	0.37	1.01	1.04	0.96	0.95	0.71	0.19	0.09	0.05	0.03	0.01
SD	0.22	0.11	0.24	0.23	0.23	0.29	0.41	0.08	0.04	0.03	0.02	0.01
Skew.	−0.61	1.21	−0.16	−0.22	−0.32	0.30	0.18	1.51	0.74	0.98	1.35	1.30
Kurtosis	−0.51	3.08	−0.33	−0.28	−0.43	0.06	−0.19	3.92	0.17	1.22	2.08	1.00
Panel B: Muscat Securities Market												
Mean	0.42	0.05	0.41	0.51	0.51	0.42	0.75	0.03	0.02	0.01	0.01	0.01
SD	0.62	0.08	0.62	0.72	0.92	1.01	1.71	0.05	0.02	0.02	0.01	0.01
Skew.	2.45	2.45	2.32	1.91	0.99	−0.78	1.79	2.65	2.71	3.08	3.20	2.19
Kurtosis	9.72	7.96	12.92	13.22	5.47	5.66	6.45	8.88	10.81	12.52	13.98	4.70
Panel C: Kuwait Stock Exchange												
Mean	0.91	0.08	0.91	0.90	0.92	0.92	0.80	0.05	0.03	0.01	0.01	0.01
SD	0.60	0.07	0.61	0.67	0.79	0.78	0.90	0.05	0.03	0.01	0.01	0.01
Skew.	0.38	0.96	0.26	0.28	0.31	0.50	0.81	1.37	1.42	1.82	2.23	3.15
Kurtosis	−0.07	0.43	−0.43	−0.26	0.44	0.37	3.00	2.38	2.16	3.59	6.87	13.91
Panel D: Bahrain Stock Exchange												
Mean	0.33	0.03	0.35	0.35	0.39	0.36	0.35	0.02	0.01	0.01	0.01	0.01
SD	0.54	0.07	0.55	0.62	0.76	0.94	1.04	0.03	0.01	0.01	0.01	0.01
Skew.	3.44	5.66	2.49	0.07	−0.55	−0.68	−0.51	4.31	2.96	1.85	1.64	1.43
Kurtosis	16.06	34.75	9.03	4.42	4.17	3.45	2.66	21.62	9.45	3.22	2.22	1.37
Panel E: Doha Securities Market												
Mean	0.67	0.18	0.75	0.68	0.69	0.69	0.55	0.15	0.05	0.04	0.02	0.01
SD	0.25	0.13	0.28	0.33	0.36	0.45	0.61	0.10	0.05	0.04	0.02	0.01
Skew	−0.18	0.92	−0.47	−0.16	−0.31	−0.87	−0.44	1.19	1.17	1.12	1.43	1.40
Kurtosis	−0.09	0.88	0.65	−0.60	0.23	2.28	−0.15	2.10	1.68	0.72	1.97	1.25
Panel F: Dubai Financial Market												
Mean	0.49	0.20	0.46	0.49	0.50	0.50	0.47	0.09	0.04	0.03	0.02	0.01
SD	0.57	0.28	0.53	0.52	0.53	0.59	0.69	0.12	0.06	0.03	0.02	0.02
Skew.	0.68	0.92	0.76	0.70	0.61	0.11	0.05	1.14	1.44	1.43	1.00	1.50
Kurtosis	−1.16	−0.92	−0.83	−0.34	−0.22	−0.72	0.20	−0.08	1.27	1.32	−0.41	2.04
Panel G: Abu Dhabi Securities Market												
Mean	0.53	0.09	0.60	0.61	0.65	0.62	0.56	0.04	0.02	0.02	0.01	0.01
SD	0.47	0.12	0.49	0.51	0.62	0.58	0.67	0.06	0.03	0.02	0.01	0.01
Skew	0.24	1.19	0.63	0.47	1.12	0.21	0.32	1.45	1.52	1.30	1.58	1.67
Kurtosis	−0.95	−0.06	−0.13	−0.10	4.13	−0.79	−0.25	0.69	1.08	0.58	1.84	2.56

Table 3
Beta and R^2 computed from recomposed crystals of individual stocks and the market portfolio.

	Total		Beta					R^2				
	Beta	R^2	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5
Panel A: Saudi Stock Market												
Mean	1.06	0.37	1.14	0.98	1.18	0.97	1.20	0.36	0.38	0.36	0.47	0.59
SD	0.22	0.11	0.25	0.29	0.39	0.32	0.48	0.10	0.14	0.17	0.18	0.21
Skew.	−0.61	1.21	−0.34	−0.06	0.14	−0.03	0.91	0.80	0.77	0.34	−0.12	−0.48
Kurtosis	−0.51	3.08	−0.65	−0.29	−0.66	−0.41	1.10	1.86	2.28	−0.32	−0.26	−0.30
Panel B: Muscat Securities Market												
Mean	0.42	0.05	0.43	0.34	0.51	0.60	0.27	0.04	0.07	0.10	0.13	0.17
SD	0.62	0.08	0.88	0.65	0.93	1.40	1.03	0.07	0.12	0.12	0.16	0.17
Skew.	2.45	2.45	4.62	1.11	1.48	3.66	−2.10	2.53	2.15	1.79	1.97	1.03
Kurtosis	9.72	7.96	26.56	8.63	10.26	18.62	10.45	7.05	4.65	2.78	4.26	0.25
Panel C: Kuwait Stock Exchange												
Mean	0.91	0.08	0.91	0.92	0.90	0.93	1.06	0.07	0.09	0.13	0.12	0.22
SD	0.60	0.07	0.70	0.65	0.73	1.45	1.24	0.07	0.08	0.13	0.13	0.22
Skew.	0.38	0.96	0.53	0.04	0.34	−0.03	1.17	1.31	1.26	1.07	1.38	0.88
Kurtosis	−0.07	0.43	0.57	−0.48	−0.07	0.48	4.10	1.23	1.34	0.78	1.81	−0.17
Panel D: Bahrain Stock Exchange												
Mean	0.33	0.03	0.31	0.36	0.27	0.43	0.38	0.03	0.04	0.06	0.07	0.17
SD	0.54	0.07	0.58	0.70	0.84	0.77	1.07	0.08	0.07	0.08	0.08	0.16
Skew.	3.44	5.66	3.67	1.71	0.70	0.74	−0.43	5.53	4.22	1.45	1.44	0.94
Kurtosis	16.06	34.75	17.64	5.68	0.70	0.74	1.13	33.51	21.12	0.86	1.40	−0.18
Panel E: Doha Securities Market												
Mean	0.67	0.18	0.49	0.81	0.65	0.69	0.81	0.08	0.31	0.16	0.38	0.43
SD	0.25	0.13	0.29	0.31	0.57	0.39	0.53	0.07	0.17	0.15	0.26	0.27
Skew.	−0.18	0.92	−0.66	−0.81	−0.37	−0.16	−0.89	0.84	0.25	0.86	0.11	−0.02
Kurtosis	−0.09	0.88	0.39	1.69	0.01	−0.46	1.46	0.05	0.31	−0.42	−1.12	−1.23
Panel F: Dubai Financial Market												
Mean	0.49	0.20	0.51	0.43	0.48	0.52	0.54	0.22	0.20	0.20	0.18	0.32
SD	0.57	0.28	0.61	0.58	0.59	0.70	0.67	0.30	0.27	0.25	0.21	0.32
Skew.	0.68	0.92	0.60	0.60	0.65	1.16	0.32	0.87	0.95	1.26	1.62	0.64
Kurtosis	−1.16	−0.92	−1.24	−1.08	−0.50	2.38	−0.98	−1.09	−0.71	0.44	2.11	−1.03
Panel G: Abu Dhabi Securities Market												
Mean	0.53	0.09	0.49	0.59	0.57	0.69	0.70	0.10	0.10	0.11	0.16	0.23
SD	0.47	0.12	0.51	0.61	0.59	1.02	0.82	0.12	0.12	0.14	0.16	0.22
Skew	0.24	1.19	−0.03	−0.04	0.17	3.01	0.47	1.06	1.42	1.48	1.09	0.82
Kurtosis	−0.95	−0.06	−0.42	−0.42	−0.21	16.69	−0.25	−0.41	0.85	1.04	0.27	−0.29

frequency parts. Similar findings are also reported in Norsworthy et al. (2000) and by Fernandez (2006). The results are consistent with our theoretical expectation that stock market investors have different time horizons due to different trading strategies.

Next, β_i^j in Eq. (9) is not a true beta because the individual stock and the market return are measured at different time scales (Fernandez, 2006). Therefore, a more accurate measure of beta, at scale j, is given by Eq. (14). We also report R^2 for each scale, which is computed by Eq. (15).

Table 3 shows the results of estimating beta coefficients β_i^j and R^2 for each GCC market by regressing recomposed scale crystal j of individual stock returns D_i^j on each recomposed scale crystal j of the market index D_m^j of the specific GCC market.

Unlike the results reported in Table 1 and 2, the linear relationship between an individual stock and the market portfolio becomes generally stronger at higher scales of the two variables. Similar conclusions are drawn by Gencay et al. (2005). This evidence implies that the fraction of systematic risk contained in an individual stock at lower frequencies has a higher association with lower frequencies of the market portfolio.

This result indicates the effect of mismatching the frequency between individual stock returns and the market returns. The mismatching problem is more serious when stocks suffer from infrequent trading. Scholes and Williams (1977), Dimson (1979), Fowler et al. (1980), and Cohen et al. (1983) provide evidence that betas of stocks that trade more (less) frequently than their market index are biased upward (downward). This is exactly what we find in our GCC market data which suggest that GCC markets may suffer from thin and infrequent trading (Abraham et al., 2002 and Al-Khazali et al., 2007). Recent studies that describe the GCC markets usually refer to them as less developed, less liquid, and highly dependent on individual investors, e.g. Al Janabi et al. (forthcoming) and Abdmoulah (forthcoming).

To support our findings more, we calculated the Value at Risk (VaR) for an equally weighted portfolio of all stocks in each GCC markets separately (Table 4) as well as for the GCC market indices (Table 5) at different time scale. The VaR represents the potential loss per 1000 units of the specific GCC market's currency invested in 1-day horizon at the 95% confidence level.

As we can see from Tables 4 and 5, VaR declines monotonically as we move into higher scales. In addition it declines at a faster rate, i.e. the contribution to the VaR from each higher scale is decreasing. The results tend to suggest that risk is concentrated at the lower scale (higher frequencies) of the data.

6. Conclusion

We made an initial attempt to analyze the 7 GCC stock markets between 2007 and 2008 to examine the multihorizon nature of systematic risk beta. We applied the wavelet analysis to decompose the daily returns into multiscale orthogonal components. We run a

Table 4
Value at risk (VaR) at different time scales for equally weighted portfolio.

	Saudi Stock Market		Muscat Securities Market		Kuwait Stock Exchange		Bahrain Stock Exchange	
	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)
Scale 1	0.0451	48.04	0.0061	48.14	0.0027	40.94	0.0013	45.99
Scale 2	0.0260	27.64	0.0029	22.91	0.0017	26.15	0.0007	25.87
Scale 3	0.0106	11.27	0.0021	16.44	0.0013	18.91	0.0003	11.70
Scale 4	0.0063	6.66	0.0009	7.11	0.0003	4.84	0.0002	6.43
Scale 5	0.0040	4.27	0.0002	1.62	0.0003	4.80	0.0001	4.78
Recomposed data	0.092		0.0121		0.0064		0.0027	
Raw data	0.094		0.0126		0.0067		0.0029	

	Doha Securities Market		Dubai Financial Market		Abu Dhabi Securities Market	
	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)
Scale 1	0.0092	36.18	0.011	56.08	0.0111	50.28
Scale 2	0.0103	40.42	0.00403	20.64	0.0049	22.29
Scale 3	0.0021	8.31	0.00204	10.42	0.0031	13.96
Scale 4	0.0022	8.54	0.00091	4.65	0.0012	5.43
Scale 5	0.0009	3.69	0.000751	3.84	0.0010	4.50
Recomposed data	0.0247		0.0187		0.0214	
Raw data	0.0254		0.0195		0.0222	

Table 5
Value at risk (VaR) at different time scales for the market indices.

	Saudi Stock Market		Muscat Securities Market		Kuwait Stock Exchange		Bahrain Stock Exchange		Doha Securities Market		Dubai Financial Market		Abu Dhabi Securities Market	
	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)
Scale 1	0.0331	48.55	0.0030	48.08	0.0024	40.31	0.0009	48.23	0.0037	22.75	0.0089	56.29	0.0063	49.93
Scale 2	0.0189	27.66	0.0014	22.02	0.0016	26.11	0.0005	26.77	0.0086	52.73	0.0032	20.30	0.0030	23.84
Scale 3	0.0073	10.77	0.0009	14.72	0.0012	19.96	0.0002	9.14	0.0013	7.86	0.0017	10.86	0.0013	10.68
Scale 4	0.0050	7.28	0.0007	10.49	0.0003	4.69	0.0001	6.81	0.0020	12.20	0.0007	4.48	0.0009	7.37
Scale 5	0.0033	4.89	0.0001	1.92	0.0003	5.20	0.0001	4.18	0.0009	5.32	0.0007	4.20	0.0006	4.84
Recomposed data	0.0676		0.0061		0.0057		0.0017		0.0165		0.0152		0.0121	
Raw data	0.0681		0.0063		0.0059		0.0018		0.0163		0.0158		0.0126	

market model to estimate beta at different time scales. We further support our results by analyzing the impact of different time scales on Value at Risk (VaR). We find that VaR measured at different time scales suggests that risk tends to be concentrated more at the higher frequencies (lower time scales) of the data. The results are plausible and intuitive and have strong policy implications.

Our results demonstrate the multiscale tendency of the average beta coefficients in all GCC countries. Our findings are in line with the existence of multi investment horizons due to multi trading strategies pursued by investors. This multi horizon nature and its impact on the dynamics of beta coefficient in the GCC equity markets are motivated by the characteristics of those markets, such as under development, illiquidity, high transaction costs, high dependence on individual investors, and infrequent trading.

Our results indicate the importance of improving the GCC markets with regard to aforementioned characteristics. Abdmoulah (forthcoming) examines the evolving efficiency of the GCC markets in relation to the regulatory efforts to develop those markets. Abdmoulah finds insignificant improvement in the efficiency of those markets despite the efforts by regulators. Although the GCC markets implemented important regulations with regard to disclosure and transparency enhancement, the GCC markets are lacking important steps to fix other important aspects that affect market efficiency. For example, the GCC markets are still dependent on individual investors, who are more likely to be less informed, subject to high transaction cost, providers of limited liquidity, and subject to behavioral biases.

To this end, it is imperative to the GCC markets to improve their depth by encouraging the participation of smart investors or institutional investors. Several steps can encourage smart investors to participate. First: integrating the GCC markets and facilitating cross listing can enhance liquidity, mitigate thin trading, and provide more depth to the market. Second: restructuring the trading mechanism and introducing the market maker role in the GCC markets can reduce transaction cost, reduce excessive volatility, and keep prices closer to their true value. Third: expanding equity markets by encouraging public offering by family and public firms to attract more local and foreign savings and to promote the GCC markets as diversification haven for international investors. This last point is especially important in light of the recent financial crisis. Relatively, the GCC markets were less impacted by the crisis; and that presents an opportunity for those markets to attract international investments that are concerned with international diversification. However, this also presents a challenge for the GCC markets to improve their investment climate through effective financial reforms in order to attract the international funds. This study along with other few studies on the GCC markets is trying to shed more light on the characteristics of the GCC markets and study the impact of those characteristics on different financial analysis issues.

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