Exerecise 4.6:-

1) $y'' + y = \sec x$

$m^2 + m = 0$

Taking squaring both side.

$\sqrt{m^2} = \sqrt{-1}$

$m = \pm i$

$y_c = c_1 \cos x + c_2 \sin(-x)$

$y_c = -c_1 \sin x + c_2 \cos x$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$= \cos x(\cos x) - \sin x(-\sin x)$

$= \cos^2 x + \sin^2 x$

$W = 1$

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$$W_1 = \begin{vmatrix} 0 & \sin x \\ \sec x & \cos x \end{vmatrix}$$

$$= 0 - \sin x(\sec x)$$

$$= -\sin x \sec x$$

$$= -\tan x$$

$$W_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \sec x \end{vmatrix}$$

$$= \cos x(\sec x) - 0$$

$$= \cos x \sec x$$

$$= 1$$

$$\cancel{u_1' = \sin x}$$

$$u_1' = \frac{-\tan x}{1}$$

$$= -\tan x$$

$$u_2' = \frac{1}{1} = 1$$

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$$\int u_1' = -\int \tan x \, dx$$
$$u_1 = \ln|\cos x|$$

$$\int u_2' = \int 1 \, dx$$
$$u_2 = x$$

$$y_p = \ln|\cos x|\cos x + x\sin x$$

So the general equation is

$$y = y_c + y_p$$
$$y = c_1\cos x + c_2\sin x + \ln|\cos x| \cos x + x\sin x.$$

2, $y'' + y = \tan x.$

$$m^2 + m = 0$$
$$\sqrt{m^2} = \sqrt{-1}$$
$$m = \pm i$$

$$y_c = c_1\cos x + c_2\sin x$$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$
$$= \cos^2 x + \sin^2 x$$
$$= 1$$

$$w_1 = \begin{vmatrix} 0 & \sin x \\ \tan x & \cos x \end{vmatrix}$$

$$w_1 = 0 - \sin x(\tan x)$$

$$w_1 = -\sin x \tan x$$

$$w_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \tan x \end{vmatrix}$$

$$w_2 = \cos x(\tan x) - 0$$

$$w_2 = \cos x \tan x = \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}}$$

$$w_2 = \sin x$$

$$u_1' = \frac{-\sin x \tan x}{1}$$

$$u_1' = -\sin x \cdot \frac{\sin x}{\cos x}$$

$$u_1' = -\frac{\sin^2 x}{\cos x}$$

$$u_1' = -\frac{(1-\cos^2 x)}{\cos x}$$

$$u_1' = \frac{\cos^2 x - 1}{\cos x}$$

3) $y'' + y = \sin x$

$m^2 + 1 = 0$

$\sqrt{m^2} = \sqrt{-1}$

$m = \pm i$

$y_c = c_1 \cos x + c_2 \sin x$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$W = \cos^2 x + \sin^2 x$

$W = 1$

$$W_1 = \begin{vmatrix} 0 & \sin x \\ \sin x & \cos x \end{vmatrix}$$

$W_1 = 0 - \sin^2 x$

$W_1 = -\sin^2 x$

$$W_2 = \begin{vmatrix} \cos x & 0 \\ \sin x & \sin x \end{vmatrix}$$

$W_2 = \cos x \sin x - 0$

$W_2 = \cos x \sin x$

$$u_1' = -\frac{\sin^2 x}{1}$$

$$\int u_1' = -\int \sin^2 x$$

$$u_1 = -\int \frac{1 - \cos 2x}{2}$$

$$= -\frac{1}{2} \int 1 - \cos 2x$$

$$= -\frac{1}{2}\left(x - \frac{\sin 2x}{2}\right)$$

$$u_1 = -\frac{x}{2} + \frac{\sin 2x}{2}$$

$$u_2' = \int \cos x \sin x \, dx$$

multiply and divided by 2.

$$\int u_2' = \frac{1}{2} \int 2 \cos x \sin x \, dx$$

$$u_2 = \frac{1}{2} \int \sin 2x \, dx$$

$$u_2 = \frac{1}{2} \cdot -\frac{\cos 2x}{2}$$

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$$u_2 = -\frac{1}{4}\cos 2x$$

$$y_p = \left(-\frac{x}{2} + \frac{1}{4}\sin 2x\right)\cos x + \left(-\frac{1}{4}\cos 2x\right)\sin x$$

$$y = y_c + y_p$$

$$y = c_1\cos x + c_2\sin x - \frac{x}{2} + \frac{1}{4}\sin 2x - \frac{1}{4}\cos 2x \sin x.$$

4) $y'' + y = \sec\theta\tan\theta$

$$m^2 + 1 = 0$$

$$\sqrt{m^2} = \sqrt{-1}$$

$$m = \pm i$$

$$y_c = c_1\cos x + c_2\sin x$$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$W = \cos^2 x + \sin^2 x$$

$$W = 1$$

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$$W_1 = \begin{vmatrix} \cos x & \sin x \\ \sec x \tan x & \cos x \end{vmatrix}$$

$$W_1 = 0 - \sin x(\sec x \tan x)$$

$$W_1 = -\sin x \sec x \tan x$$

$$W_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \sec x \tan x \end{vmatrix}$$

$$W_2 = \cos x(\sec x \tan x) + 0$$

$$W_2 = \cos x \sec x \tan x$$

$$\int u_1' = \int -\sin x \sec x \tan x$$

$$u_1 = -\int \sin x \cdot \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$u_1 = -\int \tan^2 x$$

$$u_1 = -\int (\sec^2 x - 1)$$

$$u_1 = -\tan x + x$$

$$\int u_2' = \int \cos x \sec x \tan x$$

$$u_2 = \int \cancel{\cos x} \cdot \frac{1}{\cancel{\cos x}} \tan x\,dx$$

$$u_2 = \int \tan x\,dx$$

$$u_2 = -\ln|\cos x|$$

$$y = y_c + y_p$$

$$y = c_1\cos x + c_2\sin x + x\cos x - \tan x\cos x - \sin x\ln|\cos x|$$

$$y = c_1\cos x + c_2\sin x + x\cos x - \overset{\sin}{\cancel{\tan x}} - \sin x\ln|\cos x|$$

$$y = c_1\cos x + c_2\sin x + x\cos x - \sin x\ln|\cos x|.$$

**Q5:** $y'' + y = \cos^2 x$

$$m^2 + 1 = 0$$

$$\sqrt{m^2} = \sqrt{-1}$$

$$m = \pm i$$

$$y_c = c_1\cos x + c_2\sin x$$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$W = \cos^2 x + \sin^2 x$

$W = 1$

$$W_1 = \begin{vmatrix} 0 & \sin x \\ \cos^2 x & \cos x \end{vmatrix}$$

$W_1 = 0 - \sin x \cos^2 x$

$W_1 = -\sin x \cos^2 x$

$$W_2 = \begin{vmatrix} \cos x & 0 \\ -\sin x & \cos^2 x \end{vmatrix}$$

$W_2 = \cos x(\cos^2 x) - 0$

$W_2 = \cos x \cos^2 x = \cos^3 x$

$\int u_1' = -\int \sin x \cos^2 x$

By Substituation.

We Assume

$u = \cos x$

$du = \sin x \, dx$

$u_1 = +\int u^2 du$

$u_1 = +\frac{u^3}{3}$

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$$u_1 = \frac{\cos^3 y}{3}$$

$$\int u_2 = \int \cos^3 x$$

$$= \int \cos x \cos^2 x$$

Integration by parts:-

$$= \cos^2 x \sin x - \int \sin x (2\cos x)(-\sin x)\,dx$$

$$= \cos^2 x \sin x + 2\int \sin^2 x \cos x\,dx$$

Using substitution

$$u = \sin x$$

$$du = \cos x$$

$$= \cos^2 x \sin x + 2\int u^2 dx$$

$$= \cos^2 x \sin x + 2 \cdot \frac{u^3}{3}$$

$$= \cos^2 x \sin x + 2 \cdot \frac{\sin^3 x}{3}$$

$$= \frac{\sin x}{3}\left(\cos^2 x + 2\sin^2 x\right)$$

$$= \frac{\sin x}{3}\left(\cos^2 x + 2\cos^2 x \sin^2 x\right)$$

$$= \frac{\sin x}{3}\left(\cos^2 x + 2(\cancel{1})\right)$$

$$y_p = \frac{\cos^3 x}{3}\cos x + \frac{\sin x}{3}(\cos^2 x + 2)\sin x$$

$$= \frac{\cos^4 x}{3} + \frac{\sin^2 x}{3}(\cos^2 x + 2)$$

$$y = y_c + y_p$$

$$= c_1\cos x + c_2\sin x + \frac{\cos^4 x}{3} + \frac{\sin^2 x}{3}(\cos^2 x + 2).$$

$$= \frac{1}{3}\left[\cos^4 x + \sin^2 x(\cos^2 x + 2)\right.$$

$$\left(\because (\cos^2 x)^2 = \left(\frac{1+\cos 2x}{2}\right)^2\right.$$

$$= \frac{1}{3}\left[\left(\frac{1+\cos 2x}{2}\right)^2 + \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) + 2\right]$$

$$= \frac{1}{3\cdot 2}\left[\left(\frac{1+\cos^2 2x}{2}\right)^2 + \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) + 2\right)$$

$$= \frac{1}{6}\left[\frac{1}{2} + \frac{\cos^2 2x}{2} + \frac{2\cos x}{2} + \frac{1}{2}(1-\cos^2 2x) + 2(1-\cos 2x)\right]$$

$$\frac{1}{6}\left[\frac{1}{2} + \frac{\cancel{\cos^2 2x}}{\cancel{2}} + \cos 2x + \frac{1}{2} - \frac{\cancel{\cos^2 2x}}{\cancel{2}} + 2 - 2\cos 2x\right]$$

$$= \frac{1}{6}\left[1 + 2 - \cancel{2}\cos 2x\right]$$

$$= \frac{1}{6}\left[3 - \cos 2x\right.$$

$$= \frac{1}{2} - \frac{1}{6}\cos 2x.$$