CALCULAS - I I I

PRESENTATION

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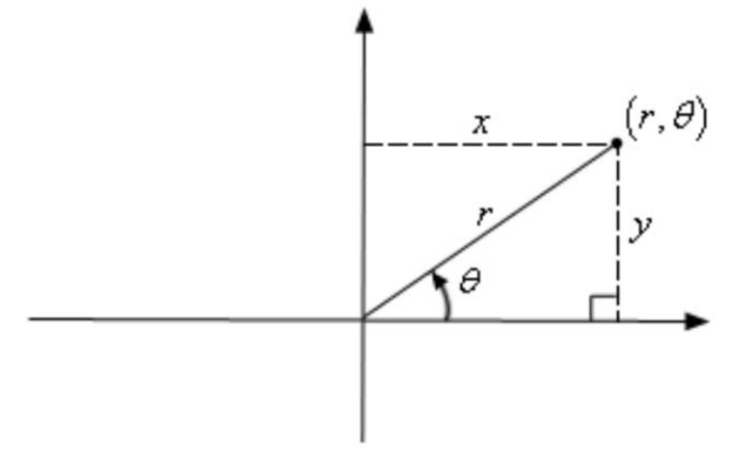
Polar Coordinates System
Gradient IN Polar Coordinates
Gradient in Spherical Coordinates

Polar Coordinates System

When each point on a plane of a two-dimensional coordinate system is decided by a distance from a reference point and an angle is taken from a reference direction, it is known as the polar coordinate system.

Explanation:

Pole = The reference point
Polar axis = the line segment ray from the pole in the reference direction
In the polar coordinate system, the origin is called a pole.



Polar Coordinates Formula:

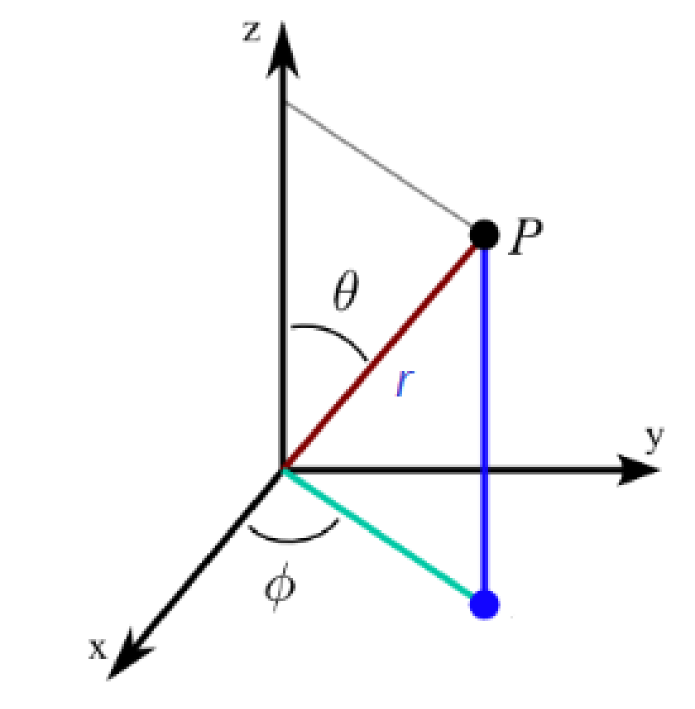
We can write an infinite number of polar coordinates for one coordinate point.

using the formula

- $(r, \theta+2\pi n)$ or $(-r, \theta+(2n+1)\pi)$, where n is an integer.
- The value of θ is positive if measured counterclockwise.
- The value of θ is negative if measured clockwise.
- The value of r is positive if laid off at the terminal side of θ.
- The value of r is negative if laid off at the prolongation through the origin from the terminal side of θ.

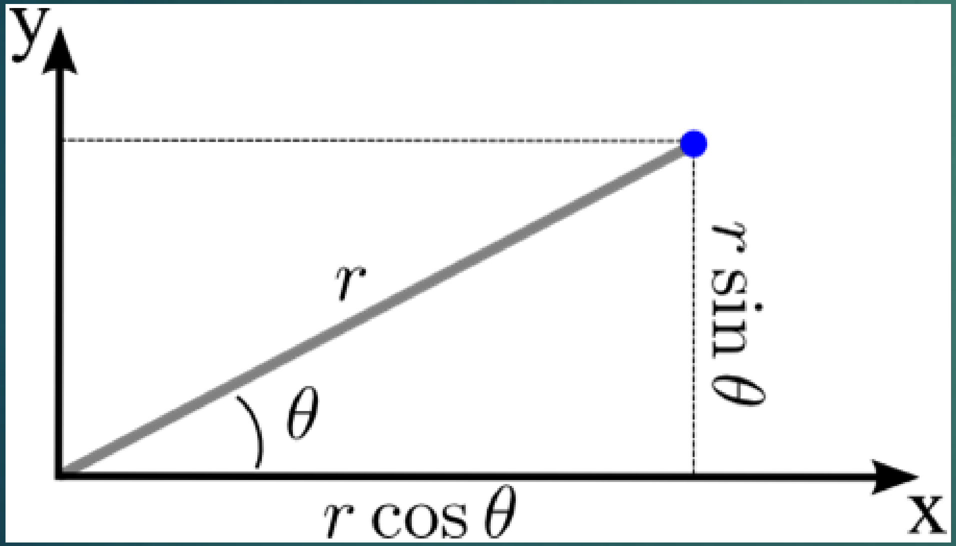
3D Polar Coordinates

3d polar coordinates or spherical coordinates will have three parameters: distance from the origin and two angles.
The 3d-polar coordinate can be written as (r, Φ, θ).
Here,
R = distance of from the origin
Φ = the reference angle from XY-plane (in a counter-clockwise direction from the x-axis)
θ = the reference angle from z-axis



Cartesian to Polar Coordinates

x = r cos θ
y = r sin θ



Finding r and θ using x and y:

$$x^2 + y^2 = (rcos\theta)^2 + (rsin\theta)^2$$

$$= r^2 \cos^2\theta + r^2 \sin^2\theta$$

$$= r^2(\sin^2\theta + \cos^2\theta)$$

$$= r^2(1)$$

$$= r^2$$

$$\Rightarrow r = \sqrt{x^2 + y^2}$$

$$\frac{y}{x} = \frac{rsin\theta}{rcos\theta}$$

$$\frac{y}{x} = tan\theta$$

$$\Rightarrow \theta = \tan^{-1}\frac{y}{x}$$

Example

Convert the rectangular or cartesian coordinates (2,2) to polar Coordinates.

Solution:-

Given.

$(x,y) = (2,2)$

$r = \sqrt{x^2+y^2}$

$r = \sqrt{(2)^2+(2)^2} \Longrightarrow r = \sqrt{4+4}$

$r = \sqrt{8} \Longrightarrow r = 2\sqrt{2}$

$y/x = 2/2 = 1$

$\theta = \tan^{-1} y/x$

$= \tan^{-1}(1)$

$\theta = \frac{\pi}{4}$

Hence, the polar Coordinates are $\left(2\sqrt{2}, \frac{\pi}{4}\right)$.

Convert the polar coordinate $(4, \pi/2)$ to a rectangular Point.

Solution :-

Given,

$$(r, \theta) = \left(4, \frac{\pi}{2}\right)$$

We know that,

$$x = r\cos\theta$$

$$x = 4\cos\frac{\pi}{2}$$

$$x = 4\cos 90^\circ$$

$$x = 4 \times 0 \Rightarrow x = 0$$

$$y = r\sin\theta$$

$$y = 4\sin\frac{\pi}{2} \Rightarrow y = 4\sin 90^\circ$$

$$y = 4 \times 1 \Rightarrow y = 4$$

Hence, the rectangular coordinates of the point is $(0, 4)$.

Gradient in polar coordinates

The change in the "y" coordinate with respect to the change in the "x" coordinate of that line.

If e_r and e_θ be the unit vectors parallel and perpendicular to the position vector r. Then gradient of a scalar of the function ϕ in the two dimensional Polar Coordinates is defined by:

$$\nabla\phi = \frac{\partial \theta}{\partial r} \cdot e_r + \frac{1}{r}\frac{\partial \phi}{\partial \theta} \cdot e_\theta$$

3-D Polar Coordinates:-

$$\nabla f = \frac{\partial f}{\partial r} e_r + \frac{1}{r}\frac{\partial f}{\partial \theta} e_\theta + \frac{1}{r\sin\theta}$$

Example

If $f = r\sin\theta + \tan\theta + \sin\theta$.
find In poler Coordinates.

Solution

$$\frac{\partial f}{\partial r} = \sin\theta$$

$$\frac{\partial f}{\partial \theta} = r\cos\theta + \sec^2\theta$$

$$\frac{\partial f}{\partial \phi} = \cos\phi$$

$$\therefore \nabla f = \frac{\partial f}{\partial r} \cdot e_r + \frac{1}{r} \cdot \frac{\partial f}{\partial \theta} \cdot e_\theta + \frac{1}{r\sin\theta} \cdot \frac{\partial f}{\partial \phi} e_\phi$$

$$= (\sin\theta) e_r + \frac{1}{r}(r\cos\theta + \sec^2\theta) e_\theta + \frac{1}{r\sin\theta} \cdot \cos\theta$$

Question:-

If $\phi = \sin\theta + r\cos\theta$
find $\nabla\phi$ in coordinates.

Solution:-

$$\frac{\partial\phi}{\partial r} = \cos\theta \quad , \quad \frac{\partial\phi}{\partial\theta} = \cos\theta - r\sin\theta$$

$$\because \nabla\phi = \frac{\partial\phi}{\partial r}\cdot e_r + \frac{\partial\phi}{\partial\theta}\cdot e_\theta$$

$$= (\cos\theta)\, e_r + \frac{1}{r}(\cos\theta - r\sin\theta)\, e_\theta$$

Gradient in Spherical Coordinates

Spherical coordinates of the system denoted as (r, θ, Φ) is the coordinate system mainly used in three dimensional systems. In three dimensional space, the spherical coordinate system is used for finding the surface area. These coordinates specify three numbers: radial distance, polar angles and azimuthal angle. These are also called spherical polar coordinates.

Gradient in Spherical Coordinates

In spherical coordinates, the gradient of a scalar function is a vector that points in the direction of the steepest increase of the function at a given point. The spherical coordinate system uses three coordinates to specify a point in space: radial distance (r), polar angle (θ), and azimuthal angle (φ).

The gradient in spherical coordinates is given by:

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

Gradient in Spherical Coordinates.

$x = r\cos\phi \sin\theta$
$y = r\sin\phi \sin\theta$
$z = r\cos\theta$

ϕ is between angle
x-axis & $\vec{r}$ vector θ z-axis
& $\vec{r}$ vector
The function x, y, z are
independent Variable w.r.t ϕ, r, θ
So Derative w.r.t to ϕ, r, θ.

$$\frac{\partial x}{\partial r} = \cos\phi \sin\theta$$

$$\frac{\partial x}{\partial \theta} = r \cos\phi \cos\theta$$

$$\frac{\partial x}{\partial \phi} = -r \sin\phi \sin\theta$$

$$\frac{\partial y}{\partial r} = \sin\phi \sin\theta$$

$$\frac{\partial y}{\partial \phi} = r \sin\phi \cos\theta$$

$$\frac{\partial y}{\partial \phi} = r \cos\phi \sin\theta$$

$$\frac{\partial z}{\partial r} = \cos\theta$$

$$\frac{\partial z}{\partial \theta} = -r \sin\theta$$

$$\frac{\partial z}{\partial \phi} = 0$$

$$\frac{\partial}{\partial r} = \frac{\partial x}{\partial r}\frac{\partial}{\partial x} + \frac{\partial y}{\partial r}\frac{\partial}{\partial y} + \frac{\partial z}{\partial r}\frac{\partial}{\partial z}$$

$$\frac{\partial}{\partial \phi} = \frac{\partial x}{\partial \phi}\frac{\partial}{\partial x} + \frac{\partial y}{\partial \phi}\frac{\partial}{\partial y} + \frac{\partial z}{\partial \phi}\frac{\partial}{\partial z}$$

$$\frac{\partial}{\partial \theta} = \frac{\partial x}{\partial \theta}\frac{\partial}{\partial x} + \frac{\partial y}{\partial \theta}\frac{\partial}{\partial y} + \frac{\partial z}{\partial \phi}\frac{\partial}{\partial z}$$

Subs method

$$\frac{\partial}{\partial r} = \cos\phi \sin\theta \frac{\partial}{\partial x} + \sin\theta \sin\phi \frac{\partial}{\partial y} + \cos\theta \frac{\partial}{\partial z}$$

$$\frac{\partial}{\partial \theta} = r\cos\phi\cos\theta \frac{\partial}{\partial x} + r\sin\phi \cos \frac{\partial}{\partial y} - r\sin\theta \frac{\partial}{\partial z}$$

$$\frac{\partial}{\partial \phi} = -r\sin\phi \frac{\partial}{\partial x} + r\cos\phi \sin\theta \frac{\partial}{\partial y}$$

$$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\hat{z}$$

$$\theta = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$$

$$\phi = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

$$\hat{x}\cdot\hat{r} = \sin\theta\cos\phi\,\hat{x}$$

$$\hat{x}\cdot\hat{\theta} = \cos\theta\cos\phi$$

$$\hat{x}\cdot\hat{\phi} = -\sin\phi$$

$$\hat{x} = \sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi - S$$

$$\hat{y}\cdot\hat{r} = \sin\theta\sin\phi$$

$$\hat{y}\cdot\hat{\theta} = \cos\theta\sin\phi$$

$$\hat{y}\cdot\hat{\phi} = \cos\theta$$

$$\hat{y} = \sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\theta\,\hat{\phi}$$

$$z\cdot\phi = -\sin\theta$$

$$\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

Defination according.

$$\bar{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$\hat{x}\frac{\partial}{\partial x} = \left[\sin\theta\cos\phi\,\hat{r} + \cos\phi\,\hat{\theta} - \sin\theta\,\hat{\phi}\right]$$

$$\left[\cos\phi\sin\theta\frac{\partial}{\partial\theta} + \frac{\sin\phi}{r\sin\theta}\frac{\partial}{\partial\phi}\right]$$

$$\hat{y} = \frac{\partial}{y}\left[\sin\theta\,\sin\phi\,\hat{r} + \cos\theta\sin\theta\,\hat{\theta}\right.$$

$$\left(\sin\theta\,\sin\phi\frac{\partial}{\partial r} + \frac{\sin\phi\cos\theta}{r}\frac{\partial}{\partial\theta} + \frac{\cos\phi}{r\sin\theta}\frac{\partial}{\partial\phi}\right]$$

$$\hat{z}\frac{\partial}{\partial z} = (\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})$$

$$= \left(\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\right)$$

$$\bar{\nabla} = \hat{r}\frac{\partial}{\partial r} + \frac{1}{r}\hat{\theta}\frac{\partial}{\partial \theta} + \frac{1}{r\sin\theta}\hat{\phi}\frac{\partial}{\partial \phi}$$

Example:

Find the function depending on the variable r, θ, ϕ which are co-ordinates in spherical system.

$$\bar{\nabla} V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$$

$$V = r^2\phi - 2\theta$$

Find gradient

$$\bar{\nabla} V = \frac{\partial}{\partial r}(r^2\phi - 2\theta)\hat{r} +$$

$$\frac{1}{r}\frac{\partial}{\partial \theta}(r^2\phi - 2\phi)\hat{\theta} +$$

$$\frac{1}{r\sin\theta}\frac{\partial}{\phi}(r^2\phi - 2\phi)\phi$$

$$\Rightarrow 2r\phi\,\hat{r} + \frac{1}{r}(-2)\hat{\theta} + \frac{1}{r\sin\theta}r^2\hat{\phi}$$

$$= 2r\phi\,\hat{r} - \frac{2}{r}\hat{\theta} + \frac{1}{\sin\theta}\hat{\phi}$$


Thank you!
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