# DIVERGENCE AND CURL IN POLAR COORDINATES SPHERICAL COORDINATES:

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# introduction:

## Divergence:

Divergence is a vector operator that operates on a vector field, producing a scalar field giving the quantity of the vector field’s source at each point.

$$\text{Div } \vec{F} = \nabla \cdot \vec{F}$$

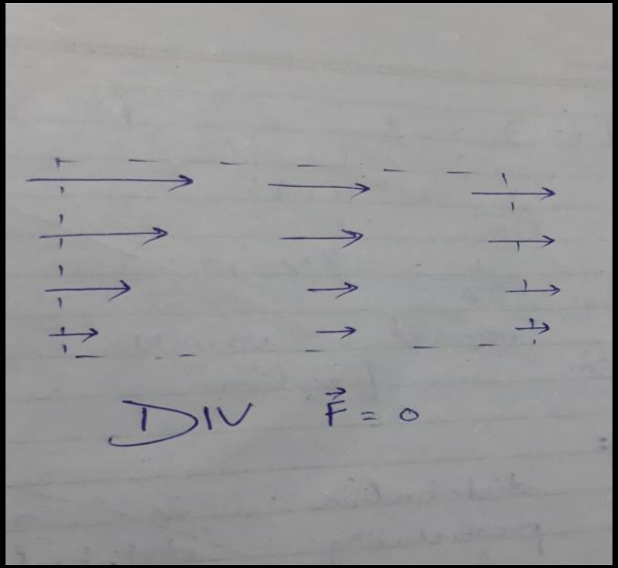
$$= \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$$

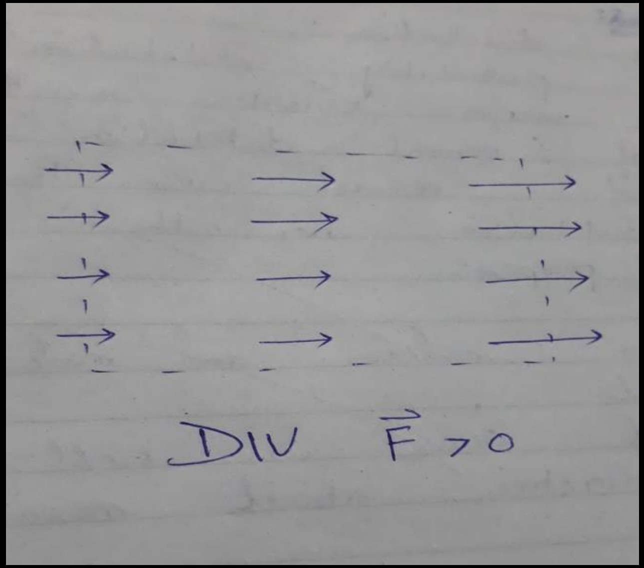
where

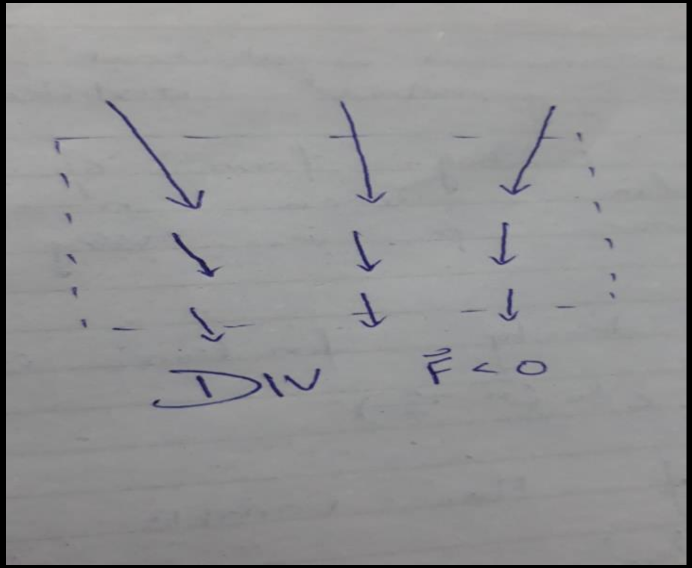
$$\vec{F} = M(x,y,z)\hat{i} + N(x,y,z)j + P(x,y,z)\hat{k}$$

THINK OF THE ARROWS REPRESENTING A VECTOR FIELD AS THE VELOCITY AND DIRECTION OF A MOVING FLUID

DIVERGENCE = THE NET GAIN (OR LOSS) OF FLUID ANYWHERE IN THE FEILD



# CURL:

In vector calculus, the curl, also known as rotor, is a vector operator that describes the infinitesimal circulation of a vector field in three-dimensional Enclidean space.

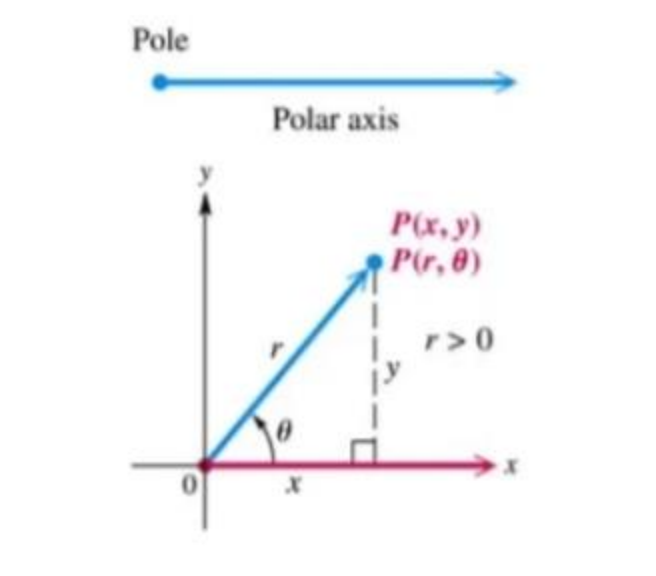
The curl at a point in the field is represented by a vector whose length and direction denote the magnitude and axis of the maximum circulation.

# Polar coordinates system:

- The polar coordinate system is bases on a point, called the pole, and a ray, called the polar axis.

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x},\ \text{if } x \neq 0$$



# DIVERGENCE AND CURL IN POLAR COORDINATE:

$$\vec{\nabla}\cdot\vec{V} = \frac{1}{h_i h_j h_k}\partial_i\left(h_j h_k V_i\right)$$

$$= \frac{1}{h_1 h_2 h_3}\partial_1\left(h_2 h_3 V_1\right) + \frac{1}{h_2 h_3 h_1}\partial_2\left(h_3 h_1 V_2\right) + \frac{1}{h_3 h_1 h_2}\partial_3\left(h_1 h_2 V_3\right)$$

$$= \frac{1}{r^2\sin\theta}\left\{\partial_r\left(r^2\sin\theta V_r\right) + \partial_\theta\left(r\sin\theta V_\theta\right) + \partial_\phi\left(r V_\phi\right)\right\}$$

$$\vec{\nabla}\times\vec{V} = \varepsilon_{ijk}\frac{\hat{e}_i}{h_j h_k}\partial_j\left(h_k V_k\right)$$

$$= \overset{+1}{\epsilon_{123}}\frac{\hat{e}_1}{h_2 h_3}\partial_2\left(h_3 V_3\right) + \overset{-1}{\epsilon_{132}}\frac{\hat{e}_1}{h_3 h_2}\partial_3\left(h_2 V_2\right)$$

$$+ \epsilon_{123}\frac{\hat{e}_2}{h_3 h_1}\partial_3\left(h_1 V_1\right) + \varepsilon_{213}\frac{\hat{e}_2}{h_1 h_3}\partial_1\left(h_2 V_3\right)$$

$$+ \epsilon_{312}\frac{\hat{e}_3}{h_1 h_2}\partial_1\left(h_2 V_2\right) + \epsilon_{321}\frac{\hat{e}_3}{h_2 h_1}\partial_2\left(h_1 V_1\right)$$

$$= \frac{\hat{e}_r}{r^2\sin\theta}\left\{\partial_\theta\left(r s\theta V_\phi\right) - \partial_\phi\left(r V_\theta\right)\right\} + \frac{\hat{e}_\theta}{r\sin\theta}\left\{\partial_\phi V_r - \partial_r\left(r\sin\theta V_\phi\right)\right\}$$

$$+ \frac{\hat{e}_\phi}{r}\left\{\partial_r\left(r V_\theta\right) - \partial_\theta V_r\right\}$$

LAME COEFFICIENTS

$h_1 = 1$ $\quad \partial_1 = \partial_r$

$h_2 = r$ $\quad \partial_2 = \partial_\theta$

$h_3 = r s\theta$ $\quad \partial_3 = \partial_\phi$

$$d\vec{s} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r s\theta\, d\phi\,\hat{\phi}$$

$\hat{e}_1 = \hat{r}$, $\hat{e}_2 = \hat{\theta}$, $\hat{e}_3 = \hat{\phi}$

$$\vec{V}(r,\theta,\phi) = V_r(r,\theta,\phi)\hat{r} + V_\theta(r,\theta,\phi)\hat{\theta} + V_\phi(r,\theta,\phi)\hat{\phi}$$

# SPHERICAL COORDINATES:

The **spherical coordinates** $(\rho, \theta, \phi)$ of a point $P$ in space are shown in Figure 6, where $\rho = |OP|$ is the distance from the origin to $P$, $\theta$ is the same angle as in cylindrical coordinates, and $\phi$ is the angle between the positive $z$-axis and the line segment $OP$.

Note that

$$\rho \geq 0 \qquad 0 \leq \phi \leq \pi$$

The spherical coordinate system is especially useful in problems where there is symmetry about a point, and the origin is placed at this point.

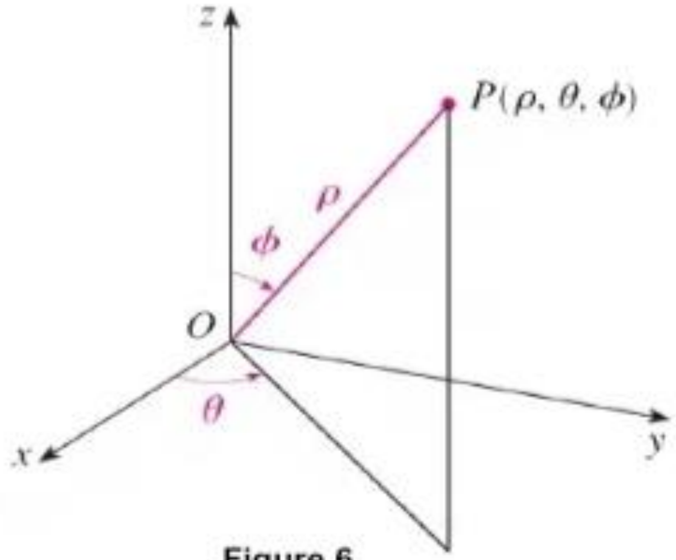



Figure 6

# Example:

For example, the sphere with center the origin and radius $c$ has the simple equation $\rho = c$ (see Figure 7); this is the reason for the name "spherical" coordinates.

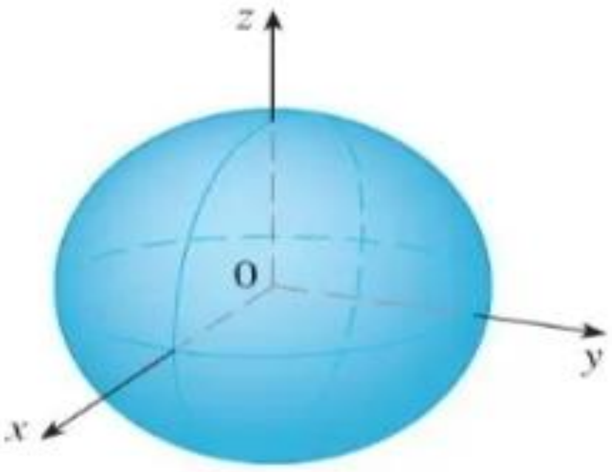



**Figure 7**

$\rho = c$, a sphere

# Graph of the equation:

The graph of the equation $\theta = c$ is a vertical half-plane (see Figure 8), and the equation $\phi = c$ represents a half-cone with the $z$-axis as its axis (see Figure 9).

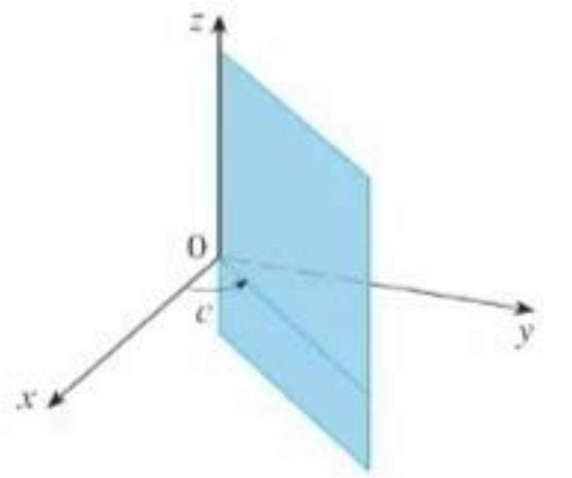



**Figure 8**

$\theta = c$, a half-plane

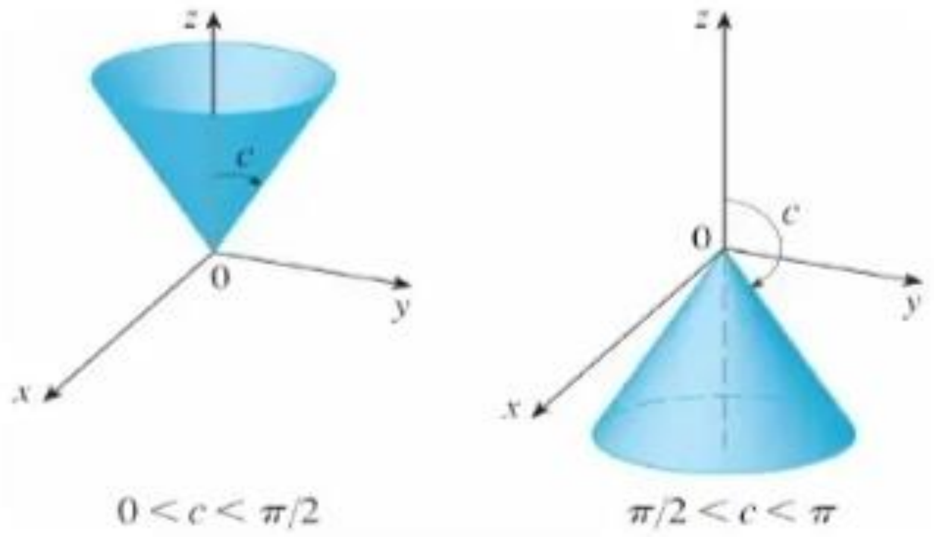



**Figure 9**

$\phi = c$, a half-cone

# Relation with rectangular coordinate:

The relationship between rectangular and spherical coordinates can be seen from Figure 10.

From triangles $OPQ$ and $OPP'$ we have

$$z = \rho \cos \phi \quad r = \rho \sin \phi$$

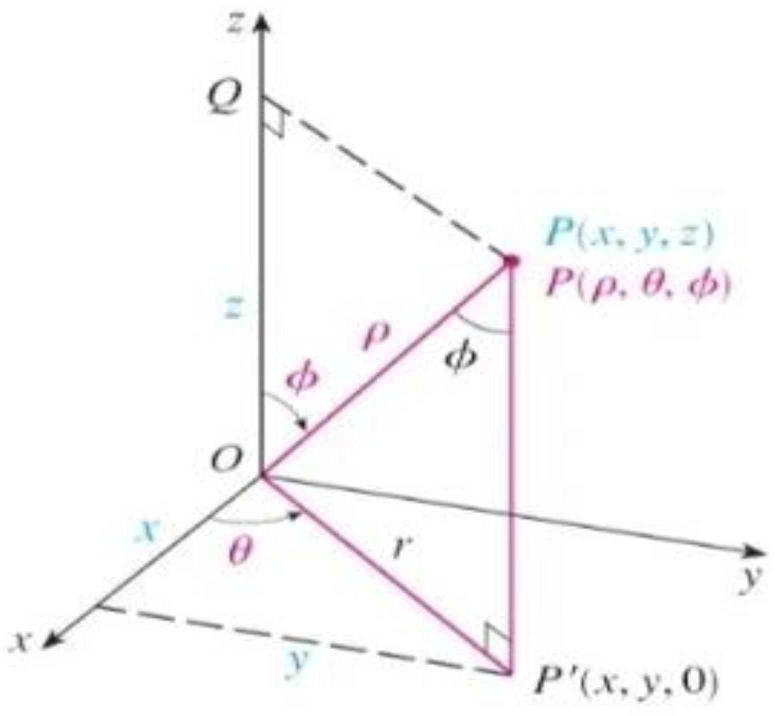



Figure 10

But $x = r\cos\theta$ and $y = r\sin\theta$, so to convert from spherical to rectangular coordinates, we use the equations

**3**
$$x = \rho \sin\phi \cos\theta \qquad y = \rho \sin\phi \sin\theta \qquad z = \rho\cos\phi$$

Also, the distance formula shows that

**4**
$$\rho^2 = x^2 + y^2 + z^2$$

We use this equation in converting from rectangular to spherical coordinates.

# Example: Converting from spherical to rectangular coordinates.

The point $(2, \pi/4, \pi/3)$ is given in spherical coordinates. Plot the point and find its rectangular coordinates.

Solution:

We plot the point in Figure 11.

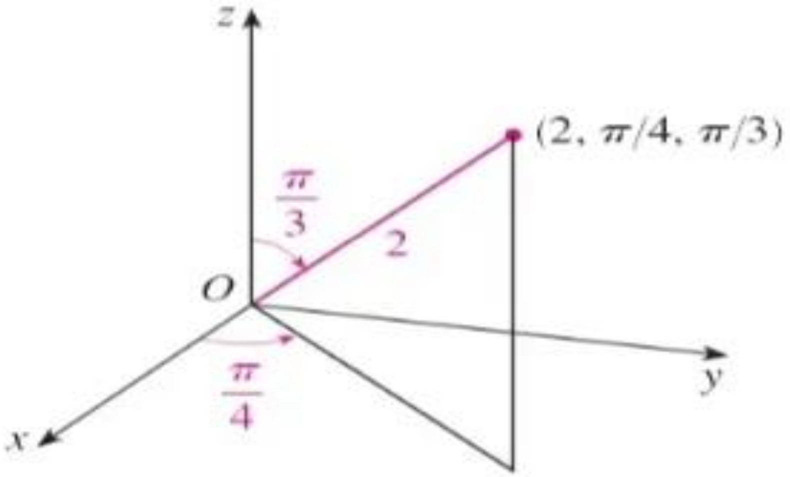



Figure 11

From Equations 3 we have

$$x = \rho \sin \phi \cos \theta \quad = 2 \sin \frac{\pi}{3} \cos \frac{\pi}{4} = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \sqrt{\frac{3}{2}}$$

$$y = \rho \sin \phi \sin \theta \quad = 2 \sin \frac{\pi}{3} \sin \frac{\pi}{4} = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \sqrt{\frac{3}{2}}$$

$$z = \rho \cos \phi \quad = 2 \cos \frac{\pi}{3} = 2\left(\tfrac{1}{2}\right) = 1$$

Thus the point $(2, \pi/4, \pi/3)$ is $(\sqrt{3/2}, \sqrt{3/2}, 1)$ectangular coordinates.