Ex # 4.7

Q # 11 :-

$$x^2y'' + 5xy' + 4y = 0 \quad \text{— (1)}$$

Sol:-

$$y = x^m \quad \text{— (2)}$$

$$\frac{dy}{dx} = mx^{m-1} \quad \text{— (3)}$$

$$\frac{d^2y}{d^2x} = m(m-1)x^{m-2} \quad \text{— (4)}$$

Substitude eq (1), (2), (3), (4)

$$x^2m(m-1)x^{m-2} + 5xmx^{m-1} + 4x^m = 0$$

$$m(m-1)x^m + 5mx^m + 4x^m = 0$$

$$x^m(m(m-1) + 5m + 4) = 0$$

$$x^m \neq 0$$

$$m^2 - m + 5m + 4 = 0$$

$$m^2 + 4m + 4 = 0$$

$$m^2 + 2m + 2m + 4 = 0$$

$$m(m+2) + 2(m+2) = 0$$

$$(m+2)(m+2) = 0$$

$m = -2, -2$

$y = c_1 x^{-2} + c_2 x^{-2}(\ln x) = 0$

Q #12:-

$x^2y'' + 8xy' + 6y = 0$

$y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

Substitute eq (1), (2) and (3)

$x^2 m(m-1)x^{m-2} + 8x\,mx^{m-1} + 6x^m = 0$

$m(m-1)x^m + 8xmx^m + 6x^m = 0$

$x^m(m(m-1) + 8m + 6) = 0$

$x^m \neq 0$

$m^2 - m + 8m + 6 = 0$

$m^2 + 7m + 6 = 0$

$a = 1, \quad b = 7$

$c = 6$

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$$\frac{-7 \pm \sqrt{49 - 24}}{2}$$

$$= \frac{-7 \pm \sqrt{25}}{2}$$

$$= \frac{-7 \pm 5}{2}$$

$$= \frac{-7+5}{2} \; ; \; \frac{-7-5}{2}$$

$$= \frac{-2}{2} = \frac{-12}{2}$$

$$m = -1 = -6$$

$$y = c_1 x^{-1} + c_2 x^{-6}$$

Q # 13

$$3x^2 y'' + 6xy' + y = 0$$

$$y = x^m$$

$$y' = m x^{m-1}$$

$$y'' = m(m-1) x^{m-2}$$

Substitute.

$$3x^2 m(m-1)x^{m-2} + 6xmx^{m-1} + x^m = 0$$

$$3m(m-1)x^m + 6mx^m + x^m = 0$$

$$x^m(3m(m-1) + 6m + 1) = 0$$

$$x^m \neq 0$$

$$3m^2 - 3m + 6m + 1 = 0$$

$$3m^2 + 3m + 1 = 0$$

$$a = 3 \quad b = 3 \quad c = 1$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-3 \pm \sqrt{9 - 12}}{6}$$

$$m = \frac{-3 \pm \sqrt{-3}}{6}$$

$$m = \frac{-3 \pm \sqrt{3}i}{6}$$

$$m = -\frac{3}{6} \pm \frac{\sqrt{3}}{6}i$$

$$m = -\frac{1}{2} \pm \frac{\sqrt{3}}{6}i$$

$$y = x^{-1/2}\left[c_1\left(\cos\frac{\sqrt{3}}{6}\ln x\right) + c_2 \sin\left(\frac{\sqrt{3}}{6}\ln x\right)\right]$$

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Q # 14 :-

$$x^2y'' - 7xy' + 41y = 0$$

$$y = x^m$$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$x^2 m(m-1)x^{m-2} - 7x\,mx^{m-1} + 41x^m = 0$$

$$m(m-1)x^m - 7mx^m + 41x^m = 0$$

$$x^m(m(m-1) - 7m + 41) = 0$$

$$x^m \neq 0$$

$$m^2 - m - 7m + 41 = 0$$

$$m^2 - 8m + 41 = 0$$

$$a = 1 \qquad b = -8 \qquad c = 41$$

$$= \frac{8 \pm \sqrt{64 - 164}}{2}$$

$$= \frac{8 \pm \sqrt{-100}}{2}$$

$$= \frac{8 \pm \sqrt{100}\,i}{2}$$

$$= \frac{8}{2} \pm \frac{\sqrt{100}}{2}\,i$$

$$\frac{4 \pm \sqrt{50 \times 2i^2}}{2}$$

$$\frac{4 \pm \sqrt{25 \times 2 \times 2i}}{2}$$

$$4 \pm 5^2$$

Q15 $x^3y''' - 6y = 0$

$y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$y''' = m(m-1)(m-2)x^{m-3}$

$x^3 m(m-1)(m-2)x^{m-3} - 6x^m = 0$

$m(m-1)(m-2)x^m - 6x^m = 0$

$x^m(m(m-1)(m-2) - 6) = 0$

$x^m \neq 0$

$m^3 - 3m^2 + 2m - 6 = 0$

$m^2(m-3) + 2(m-3) = 0$

$(m^2 + 2)(m-3) = 0$

$\sqrt{m^2} = \sqrt{-2}$

$m = \pm \sqrt{2}i$

$m = 3$

$y = c_1x^3 + c_2 \cos(\sqrt{2}\ln x) + c_3 \sin(\sqrt{2}\ln x)$

Q16:- $x^3y''' + xy' - y = 0$

$y = x^m$

$y' = mx^{m-1}$

$y'' = m(m-1)x^{m-2}$

$y''' = m(m-1)(m-2)x^{m-3}$

$x^3 m(m-1)(m-2)x^{m-3} + xmx^{m-1} - y = 0$

$m(m-1)(m-2)x^m + mx^m - x^m = 0$

$m(m^2 - 2m - m + 2) + m - x^m = 0$

$m(m^2 - 3m + 2) + m - 1) = 0$

$m^3 - 3m^2 + 2m + m - 1 = 0$

$m^3 - 3m^2 + 3m - 1 = 0$

$(m-1)^3 = 0$

$(m-1)(m-1)(m-1) = 0$

$y = c_1 x^1 + c_2 x^1 \ln x + c_3 x^1 (\ln x)^2$

Q17:- $xy^4 + 6y''' = 0$

$y''' = m(m-1)(m-2)x^{m-3}$

$y^{iv} = m(m-1)(m-2)(m-3)x^{m-4}$

$x\,m(m-1)(m-2)(m-3)x^{m-4} + 6m(m-1)$

$(m-2)x^{m-3}$

$x \cdot m(m-1)(m-2)(m-3)x^{m-4} + 6$

$m(m-1)(m-2)x^{m-3}$

$x^{m-3} \cdot m(m-1)(m-2)(m-3) +$

$6 x^{m-3}(m-1)(m-2)$

$x^{m-3} \; m(m-1)(m-2)(m-3+6) = 0$

$m(m-1)(m-2)(m+3) = 0$

$m_1 = 0$, $m_2 - 1 = 0$

$m_3 - 2 = 0$ $m_1 + 3 = 0$

$m_1 = 0$ $m_2 = +1$

$m_3 = +2$

$m_4 = -3$

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$$y = c_1 0^4 + c_2 x^1 + c_3 x^2 + c_4 x - 3$$

$$y = c + c_2 x + c_3 x^2 + c_4 x - 3$$

Qno 18:-

$$x^4 y^{(4)} + 6x^3 y''' + 9x^2 y'' + 3x y' + y = 0$$

$$x^4 \cdot m(m-1)(m-2)(m-3)x^{m-4} + 6x^3 \cdot m(m-1)(m-2)x^{m-3} + 9x^2 \cdot m(m-1)x^{m-2} + 3mx^{m-1} + x^m = 0$$

$$\{(m^2-m)(m^2-5m+6) + 6m(m^2-3m+2) + 9m(m-1) + 7m + 1\} x^m = 0$$

$$m^4 - 5m^3 + 6m^2 - 6m^3 + 5m^2 - 6m + 6m^3 - 18m^2 + 12m + 9m^2 - 9m + 3m + 1 = 0$$

$$m^4 + 2m^2 + 1 = 0$$

$$(m^2+1)^2 = 0$$

$$m^2 + 1 = 0$$

$$m^2 = -1$$

$$m = \pm i, \pm i$$

$$y_c(x) = c_1 \cos(\ln x) + c_2 \sin(\ln x) + \ln x \{c_3 \cos(\ln x) + c_4 \sin(\ln x)\}.$$

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Q no 19

$$xy'' - 4y' = x^4$$

$$y = x^m$$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$x^2(m(m-1)x^{m-2}) - 4xmx^{m-1} = 0$$

$$(m^2 - m - 4m)x^m = 0$$

$$m^2 - 5m = 0$$

$$m(m-5) = 0$$

$$m = 0, 5$$

$$y_c(x) = c_1 0^x + c_2 x^5$$

$$= c_1 + c_2 x^5$$

$$y_1 = 1 \quad ; \quad y_2 = x^5$$

$$w(1, x^5) = \begin{vmatrix} 1 & x^5 \\ 0 & 5x^4 \end{vmatrix}$$

$$= 5x^4 - 0$$

$$= 5x^4$$

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$x^2y'' - 4xy' = x^5$

Dividind by x^2

$y'' - \frac{4}{x}y' = x^3$

$f(x) = x^3$

$$w_1 = \begin{vmatrix} 0 & x^5 \\ x^3 & 5x^4 \end{vmatrix}$$

$w_1 = 0 - x^5 x^3$

$w_1 = -x^5 x^3$

~~$w_2 = \begin{vmatrix} 1 & 0 \\ 5x^4 & x^3 \end{vmatrix}$~~

$$w_2 = \begin{vmatrix} 1 & 0 \\ x & 3x^3 \end{vmatrix}$$

$= x^3 - 0$

$= x^3$

$$u_1' = -\frac{x^5 x^3}{5x^4}$$

$$u_2' = \frac{x^3}{5x^4}$$

Integrate w.r.t 'x'

$$\int u_1' = -\int \frac{x^5 x^3}{5x^4} dx$$

$$= -\frac{1}{5}\int \frac{x^3}{x^4} dx$$

$$= -\frac{1}{5}\int x^4 dx$$

$$= -\frac{x^5}{25}$$

$$\int u_2' = \int \frac{x^3}{5x^4}$$

$$= \frac{1}{5}\int \frac{1}{x}\,dx$$

$$= \frac{1}{5}\ln x$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$= -\frac{x^5}{25}\cdot 1 + \frac{1}{5}\ln x \cdot x^5$$

$$y(x) = y_c + y_p$$

$$= c_1 + c_2 x^5 - \frac{x^5}{25} + \frac{x^5}{5}$$