CALCULUS -| | |

Presentation about Green's Theorem.

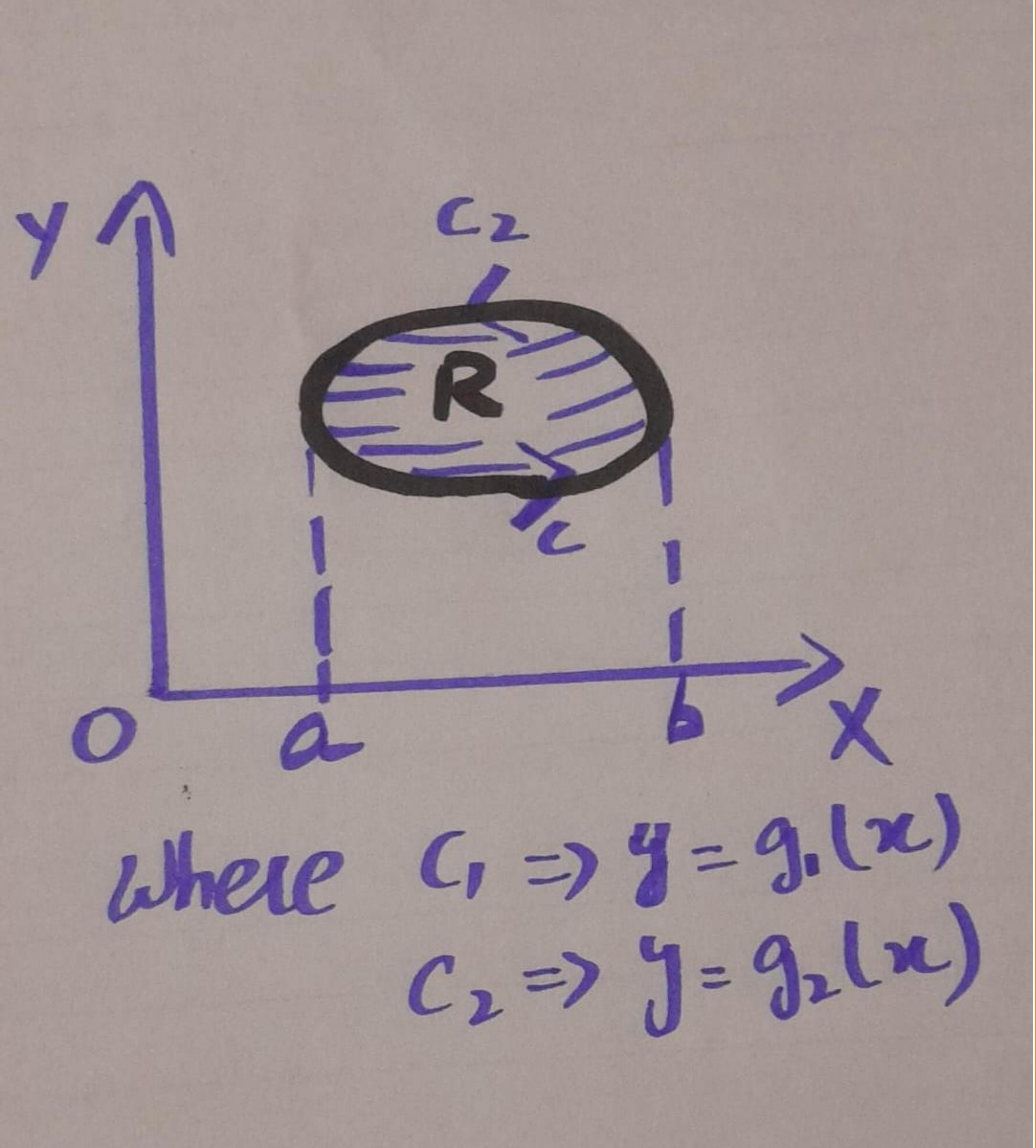
State green's theorem

- Green's theorem state that the line integral is equal to the double integral of this quantity over the enclosed region.

Statement.

- If $M, N, \partial M \backslash \partial Y$ and $\partial N \backslash \partial X$ are continuous function over a region R, bounded by closed curve C in xy plane .
- Then $\oint(Mdx+Ndy)= \int\int (\partial N\backslash\partial X- \partial M\backslash\partial Y)dxdy$.

DIAGRAM OF GREEN'S THEOREM.



In order to prove this theorem let us consider closed curve C divided into two curves C1 and C2 .And we represents C1 by y=g1(x) and C2 by y= g2(x).

Now, from R.H.S.

$$\iint_R \frac{\partial M}{\partial y}\, dx\, dy = \int_{x=a}^{b} \int_{y=g_1(x)}^{g_2(x)} \frac{\partial M}{\partial y}\, dx\, dy$$

$$= \int_{x=a}^{b} \Big[M(x, y) \Big]_{g_1(x)=y}^{g_2(x)=y}$$

$$= \int_{x=a}^{x=b} M\big(x, g_2(x) - x, g_1(x)\big)\, dx$$

$$= \int_{x=a}^{x=b} M(x, g_2)\, dx - \int_a^b M(x, g_1)\, dx$$

$$= -\int_{x=b}^{x=a} M(x, g_2)\, dx - \int_b^a M(x, g_1)\, dx$$

$$= -\left[\int_b^a M(x, g_2)\, dx + \int_b^a M(x, g_1)\, dx \right]$$

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$$= -\left[\int_{C_1} M(x, g_2)\,dx + \int_{C_2} M(x, g_2)\,dx \right]$$

$$= -\oint_C M\,dx \quad \text{or} \quad \oint_C M\,dx = \iint_R \frac{\partial M}{\partial y}\,dx\,dy \qquad \text{(1)}$$

~~Similarly~~ $\cancel{\oint_C N\,dy = -\iint_R \frac{\partial M}{\partial y}}$

$$\oint_C N\,dy = -\iint_R \frac{\partial N}{\partial x}\,dx\,dy$$

$$\Rightarrow -\oint_C N\,dy = \iint_R \frac{\partial N}{\partial x}\,dx\,dy \qquad \text{(2)}$$

Add eq (1) & (2) we get

$$\oint M\,dx - \oint N\,dy = -\iint_R \frac{\partial M}{\partial y}\,dx\,dy + \iint_R \frac{\partial N}{\partial x}\,dx\,dy$$

$$\oint M\,dx - \oint N\,dy = \iint_R \frac{\partial N}{\partial x}\,dx\,dy - \iint_R \frac{\partial M}{\partial y}\,dx\,dy$$

$$\oint (Mdx+Ndy)= \iint \left(\frac{\partial N}{\partial x}- \frac{\partial M}{\partial y}\right)dxdy$$

EXAMPLE.

Example:-

Solve $\oint_c y^3\,dx - x^3\,dy$ where c is a circle of radius 2 centered in origin.

Solution $\oint_c y^3\,dx - x^3\,dy$

Identify P & Q from the integral

Here $P = y^3$ & $Q = -x^3$

So, $\oint_c y^3\,dx - x^3\,dy = \iint_D -3x^2 - 3y^2\,dA$

$$= -3\int_0^{2\pi}\int_0^2 r^3\,dr\,d\theta$$

$$= -3\int_0^{2\pi} \frac{1}{4}\left[r^4\right]_0^2 d\theta$$

$$= -3\int_0^{2\pi} 4\,d\theta$$

$$= -3(4\theta)_0^{2\pi} \Rightarrow -24\pi$$

Therefore

$\oint_c y^3\,dx - x^3\,dy = -24\pi$

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