

Exercise 4.3

Q # 01:

$$4y'' + y' = 0$$

Auxiliary Equation

$$4m^2 + m = 0$$

$$m(4m + 1) = 0$$

$$m = 0 \quad ; \quad 4m + 1 = 0$$

$$4m = -1$$

$$m = -\frac{1}{4}$$

$$y = c_1 e^0 + c_2 e^{-1/4x}$$

$$y = c_1 + c_2 e^{-1/4x}$$

Q #02:-

$$y'' - 36y = 0$$

Auxiliary equation

$$m^2 = 36$$

$$m = \pm 6$$

$$y = c_1 e^{6x} + c_2 e^{-6x}$$

Q #03

$$y'' - y' - 6y = 0$$

Auxiliary equation

$$m^2 - m - 6m = 0$$

$$a = 1$$

$$b = -1$$

$$c = -6$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1+24}}{2}$$

$$= \frac{1 \pm \sqrt{25}}{2}$$

$$= \frac{1 \pm 5}{2}$$

$$m_1 = \frac{1+5}{2} ; m_2 = \frac{1-5}{2}$$

$$m_1 = \frac{6}{2} ; m_2 = -\frac{4}{2}$$

$$m_1 = 3 ; m_2 = -2$$

$$y = c_1 e^{3x} + c_2 e^{-2x}$$

Q# 04:-

$$y'' - 3y' + 2y = 0$$

Auxiliary equation

$$m^2 - 3m + 2m = 0$$

$$a = 1$$

$$b = -3$$

$$c = 2$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9-8}}{2}$$

$$= \frac{3 \pm \sqrt{1}}{2}$$

$$m = \frac{3 \pm 1}{2}$$

$$m_1 = \frac{3+1}{2} ; \quad m_2 = \frac{3-1}{2}$$

$$m_1 = \frac{4}{2} ; \quad m_2 = \frac{2}{2}$$

$$m_1 = 2 ; \quad m_2 = 1$$

$$y = C_1 e^{2x} + C_2 e^x$$

Q#05:-

$$y'' + 8y' + 16y = 0$$

Auxiliary equation

$$m^2 + 8m + 16m = 0$$

$$a = 1$$

$$b = 8$$

$$c = 16$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-8 \pm \sqrt{(8)^2 - 4(1)(16)}}{2(1)}$$

$$m = \frac{-8 \pm \sqrt{64 - 64}}{2}$$

$$m = \frac{-8 \pm 0}{2}$$

$$m_1 = \frac{-8 + 0}{2} ; m_2 = \frac{-8 - 0}{2}$$

$$m_1 = \frac{-8}{2} ; m_2 = \frac{-8}{2}$$

$$m_1 = -4 ; m_2 = -4$$

$$y = c_1 e^{-4x} + c_2 e^{-4x}$$

Q#6:

$$y'' - 10y' + 25y = 0$$

Auxiliary equation

$$m^2 - 10m + 25m = 0$$

$$a = 1$$

$$b = -10$$

$$c = 25$$

$$m = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(25)}}{2(1)}$$

$$m = \frac{10 \pm \sqrt{100 - 100}}{2}$$

$$m = \frac{10 \pm 0}{2}$$

$$m_1 = \frac{10+0}{2} ; m^2 = \frac{10-0}{2}$$

$$m_1 = \frac{10}{2} ; m^2 = \frac{10}{2}$$

$$m_1 = 5 ; m^2 = 5$$

$$y = c_1 e^{5x} + c_2 e^{5x}$$

Q#07:-

$$12y'' - 5y' - 2y = 0$$

Auxiliary equation

$$12m^2 - 5m - 2m = 0$$

$$a = 12$$

$$b = -5$$

$$c = -2$$

$$m = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(12)(-2)}}{2(12)}$$

$$m = \frac{5 \pm \sqrt{25 + 96}}{24}$$

$$m = \frac{5 \pm \sqrt{121}}{24}$$

$$m = \frac{5 \pm 11}{24}$$

$$m_1 = \frac{5+11}{24} ; m_2 = \frac{5-11}{24}$$

$$m_1 = \frac{16}{24}$$

$$m_2 = -\frac{6}{24}$$

$$m_1 = \frac{2}{3}$$

$$m_2 = -\frac{1}{4}$$

$$y = c_1 e^{2/3 x} + c_2 e^{-1/4 x}$$

