Lec1 M T W T F S Unit 1:- 5/9/2023 :تاریخ

Vector:-

-> Magnitude

-> Direction.

Scalar:-

-> Magnitude ✓

-> Direction. ✗

properties:

i) closed (+) property:- . clos

For any vectors $\vec{u}, \vec{v}$.

$$\vec{u} + \vec{v} = \vec{w}$$

Example:-

$$\vec{u} = 2\hat{i} + 3\hat{j}$$

$$\vec{v} = 4\hat{i} + 7\hat{j}$$

$$\vec{u} + \vec{v} = (2+4)\hat{i} + (3+7)\hat{j}$$

$$= 6\hat{i} + 10\hat{j}$$

ii) Associative property:-

For any three vector $\vec{u}, \vec{v}$ $\vec{w}$,

$$\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$$

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For any three Real number

a, b, c

$(\vec{a}+\vec{b})+\vec{c} = (b+c)+b.$

$\vec{u}+(\vec{v}+\vec{w}) = (\vec{u}+\vec{v})+\vec{w}$

$\vec{u} = 3\hat{i}+7\hat{j}+11\hat{k}$

$\vec{v} = \hat{i}-2\hat{j}+3\hat{k}$

$w = 9\hat{i}-11\hat{j}+13\hat{k}$

$= 3\hat{i}+7\hat{j}+11\hat{k}+(\hat{i}-2\hat{j}+3k+9\hat{i}-11\hat{j}+13\hat{k})$

$= 3\hat{i}+7\hat{j}+11\hat{k}+(10\hat{i}-13+16\hat{k})$

$= 13\hat{i}-6\hat{j}+27\hat{k}$

$(\vec{u}+\vec{v})+\vec{w}$

$= (3\hat{i}+7\hat{j}+11\hat{k}+\hat{i}-2\hat{j}+3\hat{k})+$

$9\hat{i}-11\hat{j}+13\hat{k}$

$= (4\hat{i}+5\hat{j}+14\hat{k})+(9\hat{i}-11\hat{j}+13\hat{k})$

$= 13\hat{i}-6\hat{j}+27\hat{k}$

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iii) identity:-

For Any vector $\vec{u}$ $\exists$ a vector $\vec{0}$ such that

$$\vec{u} + \vec{0} = \vec{u}$$

and 0 is called null vector.

Null vector:-

A vector having magnitude zero.

iv) Inverse:. property:-

For Any vector $\vec{u}$ $\exists$ a vector $\vec{u}'$ such that

$$\vec{u} + \vec{u}' = \vec{0}$$

and $\vec{u}'$ is called inverse negtive vector of u

Note:-
Both $\vec{u}$ and $\vec{u}'$ having same magnitude, but opposite direction.

Commutative

For Any two vector

$\vec{u}, v$

$$\vec{u} + \vec{v} = \vec{v} + \vec{u}$$

Distributive:- تقسیم

For Any two vector

$\vec{u}, \vec{v}$ $\exists$ α (scalar)

then $\alpha(\vec{u} + \vec{v}) = \alpha\vec{u} + \alpha\vec{v}$

In general:-

for Any n-vector

$\vec{u}_1 + \vec{u}_2 +, \ldots, + \vec{u}_n$

$\exists \alpha$

such that,

$$\alpha(\vec{u}_1 + \vec{u}_2 + \ldots + \vec{u}_n)$$

$$= \alpha\vec{u}_1 + \alpha\vec{u}_2 + \ldots + \alpha\vec{u}_n$$

$\vec{u} = (2, 3, 9)$

$\vec{v} = (7, 9, -11)$

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$\alpha = 3$

$R.H.S = \alpha\vec{u} + \alpha\vec{v}$

$= 3(2,3,9), + 3(7,9,-11)$

$= (6, 9, 27 + 21, 27, -33)$

$= (27, 36, -6)$ R.H.S

$\vec{u} = (\sin^2\theta, -5, 6, 1)$

$\vec{v} = (\cos^2\theta, 2, -4, \tan^2\theta)$

$\alpha = 2$

$= 5(\sin^2\theta, -5, 6, 1) + (\cos^2\theta, 2, -4, \tan^2\theta)$

$= 5(\sin^2\theta + \cos^2\theta), -3, 2, 1 + \tan^2\theta)$

$= 5(1), -3, 2, 1$

$= 5, 15, 10\ 5\sec^2\theta.$

$5(\sin^2\theta, -5, 6, 1) + 2(\cos^2\theta, 2, -4\ \tan^2$

$(5\sin^2\theta, -25, 30, 5) + 2\cos^2\theta, 4, -8$

$2\tan^2\theta)$

$= 5(\sin^2\theta + \cos^2 a), -15, 10$

$, 5(1 + \tan^2 a)$

$= 5, -15, 10, 5\sec^2 a$

Associative $(\cdot \times \cdot)$

$a \cdot (b \cdot c) = (a \cdot b) \cdot c$

For Example $2, 5, 9 \in IR$.

$2 \cdot (5 \cdot 9) = (2 \cdot 5) \cdot 9$

$2 \cdot (45) = (10) \cdot 9$

$90 = 90$

Let we have two scalars $\alpha ; \beta$ and $\vec{u}$ is a vector.

$(\alpha\beta)\vec{u} = \alpha(\beta\vec{u})$

$1 \times \vec{u} = \vec{u}$

$1 \times \vec{u} = 1 \times (a\hat{i} + b\hat{j} + c\hat{k})$

$= (1 \times a\hat{i} + 1 \times b\hat{j} + 1 \times c\hat{k})$

$\vec{u} = (a\hat{i} + b\hat{j} + c\hat{k})$.

$= \vec{u}$

Lecture: 2

Dot product:-

Let us consider we have two vectors.

let

$$\vec{u} = (a_1, a_2, a_3, \dots, a_n)$$

$$\vec{v} = (b_1, b_2, b_3, \dots, b_n)$$

then their dot product is defined as.

$$\vec{u}.\vec{v} = (a_1 b_1 + a_2 b_2 + a_3 . b_3 + \dots + a_n b_n)$$

Example:-

$$\vec{u} = (7, 8, 9, 8)$$

$$\vec{v} = (-2, 7, 11, 13)$$

$$\vec{w} = (6, 4, -1, 3)$$

$$\vec{x} = (2, 4, 6, 8)$$

find

$$(\vec{u} + \vec{v} + \vec{w}) . \vec{w} = \vec{u}.\vec{v} + \vec{u}.\vec{w} + x . \vec{w}$$

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$= (7,8,9,8)+(-2,7,11,13)+(2,4,6,8).$
$(6,4,-1,3)$

$= (7,19,26,29).(6,4,-1,3)$

$= (42,76,-26,87)$

$\vec{u}.\vec{w} + \vec{v}.\vec{w} + \vec{x}.\vec{w}$

$= (7,8,9,8).(6,4,-1,3)+$
$(-2,7,11,13).(6,4,-1,3)+$
$(2,4,6,8).(6,4,-1,3)$

$= (42,32,-9,24)+(-12,28,-11,3$
$+(12,16,-6,24)$

$= (42,76,-26,87).$

Therom:-

For Any $\vec{u}, \vec{v}, \vec{w}$ in $\mathbb{R}^n$ and any scalar $k \in \mathbb{R}$

(i) $(\vec{u}+\vec{v}).w = \vec{u}.\vec{w} + \vec{v}.\vec{w}$

(ii) $\vec{u}.\vec{v} = \vec{v}.\vec{u}$

(iii) $(k\vec{u}).\vec{v} = k(\vec{u}.\vec{v})$

(iv) $\vec{u}.\vec{u} \geq 0$ if $\vec{u}.\vec{u} = 0$ then $\vec{u} = \vec{0}$

proof:-

let

(i) $\vec{u} = (a_1 + a_2 + a_3,,, a_n)$

$\vec{v} = (b_1 + b_2 + b_3,,, b_n)$

$\vec{w} = (c_1 + c_2 +,,, c_n)$

$L.H.S = (\vec{u}+\vec{v}).w$

$= (a_1 + a_2 + a_3 +,,, a_n + b_1 + b_2 + b_3 +,,, b_n).(c_1 + c_2 + c_3,,, c_n)$

$= (a_1b_1 + a_2b_2 + a_3c_3).(c_1 + c_2 + c_3,, c_n)$

$= (a_1 b_1).c_1 + (a_2, b_2).c_2 +,,,,,, (a_n b_n).c_n.$

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$= (a_1 \times c_1 + b_1 \times c_1) + (a_2 \times c_2 + b_2 \times c_2)$
$+ , , , , , + (a_n \times c_n + b_n \times c_n)$

$= (a_1 \times c_1 + a_2 \times c_2 + , , , a_n c_n)$
$+ (b_1 \times c_1 + b_2 \times c_2 + , , , b_n c_n)$
$= \vec{u} . \vec{w} + \vec{v} . \vec{w}$

(ii) $\vec{u} = (a_1 , a_2 , , a_n)$
$\vec{v} = (b_1 , b_2 , , , b_n)$

$\vec{u} . \vec{v}$
$= (a_1 ; b_1 + a_2 b_2 \text{ to } a_n , b_n)$
$= (b_1 , a_1 + b_2 . a_2 . b_n . a_n)$

(iii) $(k\vec{u}) . \vec{v} = k (u . \vec{v})$

$= (ka_1 + ka_2 + ka_n) . (b_1 , b_2 , , b_n)$
$= (ka_1 . b_1 + ka_2 . b_2 + ka_n . b_n)$
$= k (a_1 . b_1 + (a_2 . b_2) + (a_n . b_n)$

(iv) $\vec{u}\cdot\vec{u} \geq 0$ if $\vec{u}\cdot\vec{u} = 0$ then $\vec{u} = 0$

$= (a_1, a_2, a_n)\cdot(a_1, a_2, a_n) = 0$

$= (a_1^2 + a_2^2 + a_n^2) = 0$

$= a_1^2 = 0, \quad a_2^2 = 0$

$a_n^2 = 0$

$= \vec{u} = 0$

Othogonal Vectors:.

let $\vec{u}$ and $\vec{v}$ be any two vector in $\mathbb{R}^n$ then $\vec{u}$ and $\vec{v}$ are said to be orthogonal if $\vec{u}\cdot\vec{v} = 0$ $(\vec{u} \perp \vec{v})$.

Example:.

$\vec{u} = (1, 0, 1, 0, 0)$

$\vec{v} = (0, 1, 0, 0, 0)$

$\vec{u}\cdot\vec{v} = 0 + 0 + 0 + 0 + 0$

$\vec{u}\cdot\vec{v} = 0$

$\vec{u} \perp \vec{v}$

For Any two vector $\vec{u}, \vec{v}$, in $\mathbb{R}^n$

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|u\| \|v\|}$$

$$\|u\|^2 = \vec{u} \cdot \vec{u}$$

$$\|\vec{u}\| = (\vec{u} \cdot \vec{u})^{1/2}$$

$$= (a_1^2 + a_2^2 + a^3, , , , a_n^2)$$

$$\|u\| = \sqrt{a_1^2 + a^2 + a^2}$$

Similary

$$\|v\| = \sqrt{b_1^2 + b_2^2 + , b_n^2}$$

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Unit Vector:-

let $\vec{u}$ be any vector.

$$\vec{u} = \vec{u}(x_1, x_2, x_3, \ldots, x_n)$$

Then $\hat{u} = \dfrac{\vec{u}}{\|\vec{u}\|}$

$$= \frac{|x_1, x_2, x_3, \ldots, x_n|}{\sqrt{x_1^2 + x_2^2 + \ldots + x_n^2}}$$

$$= \left(\frac{x_1}{\sqrt{x_1^2 + x_2^2 + \ldots x_n^2}}, \frac{x_2}{\sqrt{x_1^2 + x_2^2 + \ldots x_n^2}}, \ldots, \right.$$

$$\left. \frac{x_n}{\sqrt{x_1^2 + x_2^2 + \ldots x_n^2}} \right)$$

$$= \left(\frac{x_1}{\|u\|}, \frac{x_2}{\|u\|}, \ldots, \frac{x_n}{\|u\|} \right).$$

Example:-

$\vec{u} = (1, -3, 2, 4)$

$$||u|| = \sqrt{(1)^2 + (-3)^2 + (2)^2 + (4)^2}$$

$$||u|| = \sqrt{1 + 9 + 4 + 16}$$

$$||u|| = \sqrt{30}$$

$$\vec{u} = \frac{\vec{u}}{||u||}$$

$$\vec{u} = \frac{(1, -3, 2, 4)}{\sqrt{30}}$$

$$= \frac{1}{\sqrt{30}}, \frac{-3}{\sqrt{30}}, \frac{2}{\sqrt{30}}, \frac{4}{\sqrt{30}}$$

Example:-

$$\vec{v} = (1, -3, 5, 6, \sin\theta, \cos\theta)$$

$$\|v\| = \sqrt{(1)^2 + (-3)^2 + (5)^2 + (6)^2 + (\sin\theta)^2 + (\cos\theta)^2}$$

$$\|v\| = \sqrt{1 + 9 + 25 + 36 + \sin^2\theta + \cos^2\theta}$$

$$\|v\| = \sqrt{71 + (\sin^2\theta + \cos^2\theta)}$$

$$\|v\| = \sqrt{72}$$

$$\vec{v} = \frac{(1, -3, 5, 6, \sin\theta, \cos\theta)}{\sqrt{72}}$$

$$\vec{v} = \frac{1}{\sqrt{72}}, \frac{-3}{\sqrt{72}}, \frac{5}{\sqrt{72}}, \frac{6}{\sqrt{72}}$$

$$, \frac{\sin\theta}{\sqrt{72}}, \frac{\cos\theta}{\sqrt{72}}.$$

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Proof:- $|\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \|\vec{v}\|$

$\vec{u} \cdot \vec{v} = \|u\| \|v\| \cos\theta$

$\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$

$\theta = 90^\circ$

$\cos 90^\circ = 0$

$\cancel{\theta = 0^\circ}$

$\cancel{\cos 0^\circ = 1}$

$\vec{u} \cdot \vec{v} = 0$

$\theta = 0^\circ$

$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\|$

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$$\left| \frac{\vec{u}\cdot\vec{v}}{\|u\|\,\|v\|} \right| = |\cos\theta|$$

$$\frac{|\vec{u}\cdot\vec{v}|}{|\,\|u\|\,\|v\|\,|} \leq 1$$

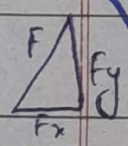
$$|\vec{u}\cdot\vec{v}| \leq \|\vec{u}\|\,\|\vec{v}\|.$$

Projection:-

Let $\vec{u}$ and $\vec{v}$ be two vector then projection of $\vec{u}$ and $\vec{v}$ will be

$$\Rightarrow (\vec{u} \text{ on } \vec{v}) = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\, v$$

$$\Rightarrow (\vec{u} \text{ on } \vec{v}) = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}\, \vec{u}$$



$$\vec{F} = \vec{F_x} + \vec{F_y}$$

$$F_x = F\cos\theta$$

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Example:

$\vec{u} = (1,-2,3)$

$\vec{v} = (2,4,5)$

1 formula

$\vec{u}\cdot\vec{v} = (1,-2,3)(2,4,5)$

$\vec{u}\cdot\vec{v} = (2-8+15)$

$= 9$

$\vec{u}$ on $\vec{v}$

$\|v\| = \sqrt{(2)^2+(4)^2+(5)^2}$

$= \sqrt{4+16+25}$

$\|v\| = (\sqrt{45})^2$

$\|v\| = 45$

$\vec{u}$ on $\vec{v} = \frac{9}{45}(2,4,5)$

$= (\frac{2}{5}, \frac{4}{5}, 1)$

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Example:

$\vec{u} = (1, 2, 3)$

$\vec{v} = (2, 4, 5)$

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Example:

$F(t) = (\sin t, \cos t, t)$

$\vec{v}(t) = (\cos t, -\sin t, 1)$

find tangent on $\vec{v}(t)$?

$\hat{v}(t) = ?$

Solution:-

$\|\vec{v}\| = \sqrt{(\cos t)^2 + (-\sin t)^2 + (1)}$

$= \sqrt{\cos^2 t + \sin t + 1}$

$= \sqrt{1+1}$

$\|\vec{v}\| = \sqrt{2}$

$\hat{v}(t) = \dfrac{\vec{v}(t)}{\|\vec{v}\|}$

$\hat{v}(t) = \dfrac{(\cos t, -\sin t, 1)}{\sqrt{2}}$

$$= \frac{\cos t}{\sqrt{2}}, -\frac{\sin t}{\sqrt{2}}, \frac{1}{\sqrt{2}}$$

Cross Product:-

let $\vec{u}$ and $\vec{v}$ be two vector

$$\vec{u} = (a_1, b_1, c_1)$$

$$\vec{v} = (a_2, b_2, c_2)$$

Then their cross product is

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \hat{i}(b_1c_2 - c_1b_2) - \hat{j}(a_1c_2 - c_1a_2) + \hat{k}(a_1b_2 - b_1a_2)$$

$$= \hat{i}(b_1c_2 - b_2c_1) + \hat{j}(a_2c_2 + a_1c_2) + \hat{k}(a_1b_2 - a_2b_1)$$

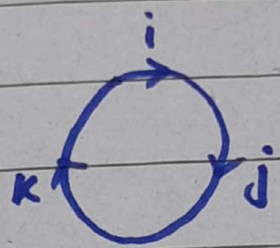
⇒ General form:-

⇒ cross product:-

$$\vec{u} \times \vec{v} = \|\vec{u}\| \|\vec{v}\| \sin\theta \, \hat{n}$$

$\vec{u} \times \vec{v}$ ↓ is a vector quantity.

⇒ properties:-



(i)

- $\hat{i} \times \hat{j} = \hat{k}$
- $\hat{j} \times \hat{k} = \hat{i}$
- $\hat{k} \times \hat{j} = \hat{k}$

(ii)

$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{j} \times \hat{k} = -\hat{j}$

(iii)

$\hat{i} \times \hat{i} = 0$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$

Calculate the vector product of a and b given that

$a = 2i + j + k$

$b = i - j - k$

$a.b = (2i + j + k).(i - j - k)$

$= 2 - 1 - 1$

$a.b = 0$

Lecture no 4:-

$\vec{u} = 2i - 3j + 4k$

$\vec{v} = 3i + j - 2k$

$\vec{w} = i + 5j + 3k.$

Evaluate:-

$\vec{u} \times \vec{v}$, $\vec{v} \times \vec{u}$, $\vec{u} \times \vec{w}$

$\vec{w} \times \vec{u}$, $\vec{v} \times \vec{w}$ $\vec{w} \times \vec{v}$?

Sol:-

(i) $\vec{u} \times \vec{v} = (2\hat{i} - 3\hat{j} + 4\hat{k}) \times (3\hat{i} + \hat{j} - 2\hat{k})$

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(iv)

$$\vec{u} \times \vec{v} \quad \begin{vmatrix} i & j & k \\ 2 & -3 & 4 \\ 3 & 1 & -2 \end{vmatrix}$$

$= i(6-4) - j(-4-12) + k(2+9)$

$= \quad 2i + 16j + 11k$

(ii) $\vec{v} \times \vec{u} = \begin{vmatrix} i & j & k \\ 3 & 1 & -2 \\ 2 & -3 & 4 \end{vmatrix}$

$= i(4-6) - j(12+4) + k(-9-2)$

$= \quad -2i - 16j - 11k$

(iii) $\vec{u} \times \vec{w} \quad \begin{vmatrix} i & j & k \\ 2 & -3 & 4 \\ 1 & 5 & 3 \end{vmatrix}$

$= i(-9-20) - j(6-4) + k(10+3)$

$= \quad -29i - 2j + 13k.$

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(iv) $\vec{w} \times \vec{u}$ $\begin{vmatrix} i & j & k \\ 1 & 5 & 3 \\ 2 & -3 & 4 \end{vmatrix}$

$i(20+9) - j(4-6) + k(-3-10)$

$29i + 2j - 13k.$

(v) $\vec{v} \times \vec{w}$ $\begin{vmatrix} i & j & k \\ 3 & 1 & -2 \\ 1 & 5 & 3 \end{vmatrix}$

$= i(3+10) - j(9+2) + k(15-1)$

$= 13i - 11j + 14k.$

(vi) $\vec{w} \times \vec{v}$ $\begin{vmatrix} i & j & k \\ 1 & 5 & 3 \\ 3 & 1 & -2 \end{vmatrix}$

$= i(-10-3) - j(-2-9) + k(1-15)$

$= -13i + 11j - 14k$

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find $\vec{u}\times\vec{v}$ and $\vec{v}\times\vec{u}$.

a) $u=(1,2,3)$ $v=(4,5,6)$

$$\vec{u}\times\vec{v}\begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

$= \hat{i}(12-15)-\hat{j}(6-12)+\hat{k}(12-15)$

$= -3i+6j-3k.$

$$\vec{v}\times\vec{u} = \begin{vmatrix} i & j & k \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix}$$

$= i(15-12)-j(12-6)+k(8-5)$

$= +3i-6j+3k.$

b) $u=(-4,7,3)$ $v=(6,-5,2)$

$$\vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ -4 & 7 & 3 \\ 6 & -5 & 2 \end{vmatrix}$$

$= i(14+15)-j(-8-18)+k(20-42)$

$= 29i - 26j - 18k.$

$$\vec{v} \times \vec{u} = \begin{vmatrix} i & j & k \\ 6 & -5 & 2 \\ -4 & 7 & 3 \end{vmatrix}$$

$= i(-15-14) - j(18+8) + k(42-20)$

$= -29i - 26j + 18k.$

$\vec{v} \times \vec{w}$ find ?

$v = \langle 1, 3, 4 \rangle$

$w = (2, -6, -5)$

$$\text{(i) } \vec{v} \times \vec{w} = \begin{vmatrix} i & j & k \\ 1 & 3 & 4 \\ 2 & -6 & -5 \end{vmatrix}$$

$= i(-15+24) - j(-5-8) + k(-6-6)$

$= 9i - 13j - 12k$

(iii) $\vec{v} \times \vec{w} \cdot \vec{w} = 0$

$(9, -13, -12)(1, 3, 4)$

$(9, -39 - 48) = 0$

$-39 + 39 = 0$

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Q1 Find two unit vector prependicalar two (2, 0, −3) and (−1, 4, 2).
if $\vec{u} \times \vec{v} = 0$ Then $\vec{u} = ?$ $\vec{v} = ?$

Sol

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 2 & 0 & -3 \\ -1 & 4 & 2 \end{vmatrix}$$

$$= i \begin{vmatrix} 0 & -3 \\ 4 & 2 \end{vmatrix} - j \begin{vmatrix} 2 & -3 \\ -1 & 2 \end{vmatrix} + k \begin{vmatrix} 2 & 0 \\ -1 & 4 \end{vmatrix}$$

$$= i(0+12) - j(4-3) + k(8-0)$$

$$= 12i - j + 8k$$

$$\vec{v} \times \vec{u} = \begin{vmatrix} i & j & k \\ -1 & 4 & 2 \\ 2 & 0 & -3 \end{vmatrix}$$

$$= i(-12-0) - j(3-4) + k(0-8)$$

$$= -12i + j - 8k$$

$\vec{u}\cdot\vec{w} = 0$

$= (2, 0, -3)(12, -1, 8)$

$= 24 + 0, -24$

$= 0$

$\vec{v}\cdot\vec{w} = (-1, 4, 2)\cdot(12, -1, 8)$

$= -12 - 4 + 16$

$= 0$

$\bar{u}\cdot\bar{z} = (2, 0, -3)(-12, 1, -8)$

$= -24 + 0 + 24$

$= 0$

$\vec{v}\cdot\vec{z} = (-1, 4, 2)(-12, 1, -8)$

$= 12 + 4 - 16$

$= 16 - 16$

$= 0$

$$\vec{u} \times \vec{v} = 12\hat{i} - \hat{j} + 8\hat{k}$$

$$\|\vec{u} \times \vec{v}\| = \sqrt{144 + 1 + 64}$$

$$= \sqrt{209}$$

$$\frac{\vec{u} \times \vec{v}}{\|\vec{u} \times \vec{u}\|} = \frac{12}{\sqrt{209}} \hat{i} - \frac{1}{\sqrt{209}} \hat{j} + \frac{8}{\sqrt{209}} \hat{k}$$

$$\frac{\vec{v} \times \vec{w}}{\|\vec{v} \times \vec{w}\|} = \frac{-12}{\sqrt{209}} \hat{i} + \frac{1}{\sqrt{209}} \hat{j} - \frac{8}{\sqrt{209}} \hat{k}$$

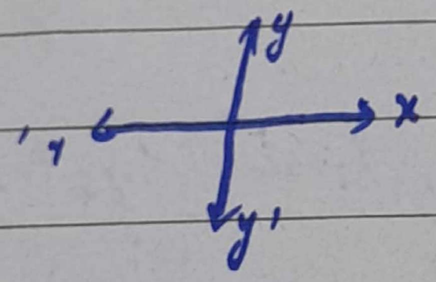
Lec # 06

Analytic Geometry:-

The branch of mathematics in which the position of any point can be (observed/ located/ determined) by ordered pair. This also called Cartesin geometry.

Plane:-

A flate Surface Goes on infinitely in each of the direction.



Cordinates:-

are two ordered pair which defines the location of point:-

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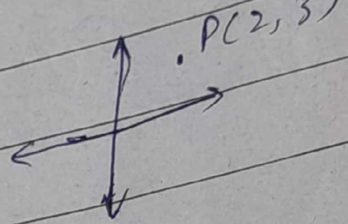
On plane:-

Type of coordinate System:-

=> cartesin coordinate.
=> polar coordinates
=> cylindrical coordinates
=> spherical coordinates

cartesin coordinates:-

In whicih the position of any point represented by (x, y).



Polar coordinate System:-

In whicih the location / position of a point is described by distance 'r' angle 'θ'

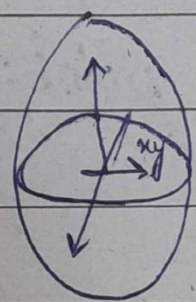
Cylindrical coordinates:-

In whicih all the points are represented by height "h" distance r and angle θ.

$P(h, r, \theta)$, $P(r, \theta, h)$

Spherical coordinate system:-

In whicih the point in space located by distance r and angle θ with xy-plane and another angle ϕ. making with z-axis



Three planes with the help of (x, y, z).

1, xy-plane.
2, yz-plane
3, xz plane.

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Distance formula:-

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$.

- The distance formula can be written as.

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In general

$A(x_1, x_2, x_3, \dots, x_n)$

$B(y_1, y_2, y_3, \dots, y_n)$

$$d = \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2 + \dots + (y_n - x_n)^2}$$

Mid point:-

$A(x_1, x_2, x_3, \dots, x_n)$ B

$(y_1, y_2, \dots, y_n)$

$M(z_1, z_2, \dots, z_n)$

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$$\left(\frac{x_1+y_1}{2}, \frac{x_2+y_2}{2}, \ldots, \frac{x_n+y_n}{2}\right)$$

Angle formula:-

$$y = m_1x + c$$
$$y = m_2x + c$$

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1m_2}$$

if $\theta = 0^\circ$

$$0 = \frac{m_1 - m_2}{1 + m_1m_2} \text{ (parallel)}$$

$$m_1 - m_2 = 0$$
$$m_1 = m_2$$

if $\theta = 90^\circ$ $(\perp)$

$$\tan 90^\circ = \frac{m_1 - m_2}{1 + m_1m_2}$$

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$$\frac{\sin 90^\circ}{\cos 90^\circ} = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\frac{1}{0} = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\infty = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$1 + m_1 m_2 = \frac{m_1 - m_2}{\infty}$$

$$1 + m_1 m_2 = 0$$

$$m_1 m_2 = -1$$

Example:-

~~Example~~:-

Q Find the distance for the folling points

=> i) $A(3, 2, 11)$, $B(6, 9, 17)$

=> ii) $C(5, 9, 11)$ $B(8, 17, 13)$

=> iii) $E(\cos\theta, 3\sin\theta)$. $F(\sin\theta, 3\cos\theta)$

=> Also find their mid points.

Example:-

Q Find the slope between (5, -3) and intercept.

A (5, -3)
B (0, y)

Solution:-

Q no1 :- Distance.

A (3, 2, 11)
B (6, 9, 17)

$$|AB| = \sqrt{(6-3)^2 + (9-2)^2 + (17-11)^2}$$

$$= \sqrt{(3)^2 + (7)^2 + (6)^2}$$

$$= \sqrt{9 + 49 + 36}$$

$$= \sqrt{94}$$

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Q no 2:-

C(5, 9, 11)

D(8, 17, 13)

$$|CD| = \sqrt{(8-5)^2+(17-9)^2+(13-11)^2}$$

$$|CD| = \sqrt{(3)^2+(8)^2+(2)^2}$$

$$= \sqrt{9+64+4}$$

$$|CD| = \sqrt{77}$$

Q no 3:-

$E(\cos\theta, 3\sin\theta)$

$F(\sin\theta, 3\cos\theta)$

$$|EF| = \sqrt{(\sin\theta-\cos\theta)^2+(3\cos\theta-3\sin\theta)^2}$$

$$|EF| =$$

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Qno1 mid points:-

A (3, 2, 11)

B (6, 9, 17)

$$= \left(\frac{3+6}{2}, \frac{2+9}{2}, \frac{11+17}{2} \right)$$

$$= \left(\frac{9}{2}, \frac{11}{2}, \frac{28}{2} \right)$$

Qno2:-

C (5, 9, 11)

D (8, 17, 13)

$$= \left(\frac{5+8}{2}, \frac{9+17}{2}, \frac{11+13}{2} \right)$$

$$= \left(\frac{13}{2}, \frac{26}{2}, \frac{24}{2} \right)$$

$$= \left(\frac{13}{2}, \frac{13}{2}, \frac{12}{2} \right)$$

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Qno 3 :-

$E = (\cos\theta, 3\sin\theta)$

$F = (\sin\theta, 3\cos\theta)$

$EF = \frac{\cos\theta + \sin\theta}{2}, \frac{(3\sin\theta + 3\cos\theta)}{2}$

$= \left(\frac{1}{2}, \frac{3}{2}\right)$

Q no 3

Find the slop

A (5, -3)

B (0, 9)

Vector identity:-
differential operator.

$$\bar{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$= \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial}\hat{y} + \frac{\partial}{\partial z}\hat{k}$$

In general for n variable.

$$\nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \ldots, \frac{\partial}{\partial x_n}\right)$$

$$f = (x, y, z)$$

$$\vec{\nabla} f = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot f(x, y, z)$$

$$= \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$$

if $f = f(x_1, x_2, \ldots, x_n)$

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$$\nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \ldots, \frac{\partial}{\partial x_n}\right)$$

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n}\right)$$

$$\vec{\nabla} f = (f_{x_1}, f_{x_2}, \ldots, f_{x_n})$$

Let $\bar{F}$ be vector field function.

$$\vec{F} = (f_x, f_y, f_z)$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\vec{\nabla} \cdot \vec{F} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (f_x, f_y, f_z)$$

$$= \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$

$$\overline{\nabla}\cdot\overline{f} = \left(\frac{\partial f_1}{\partial x_1}, \frac{\partial f_2}{\partial x_2}, \dots, \frac{\partial f_n}{\partial x_n}\right)$$

if $\overrightarrow{f} = \overline{f}(f_1, f_2, \dots, f_n)$

$$\nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \dots, \frac{\partial}{\partial x_n}\right)$$

=> **Example:-**

$F = (x^2 + 3y + z, \sin^2 y, \tan x^t \sin y \cos z)$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\nabla \cdot F = \frac{\partial}{\partial x}(x^2 + 3y + z) + \frac{\partial}{\partial z}\sin^2 y$$

$$\frac{\partial}{\partial z}(\tan x^t \sin y \cos x).$$

$= 2x + 2\sin y \cos y + \sin y(-\sin z)$

$\overline{\nabla} \cdot \overline{F} = 2x + \sin 2y - \sin y \sin z.$

$$\overline{\nabla} \times \overline{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \hat{i}\left(\partial/\partial y F_z - \frac{\partial}{\partial z} F_y\right) -$$

$$\hat{j}\left(\partial/\partial s F_z - \partial/\partial z F_x\right) + \hat{k}\left(\partial \partial x F_y - \partial f y F_x\right)$$

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⇒ **Example :-**

let

$F = (x^2 + 3y + 2, \sin^2 y, \tan x + \sin y \cos z)$

$$\vec{\nabla} \cdot \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2+3y+2 & \sin^2 y & \tan \end{vmatrix}$$

grad

$$\hat{i}\left(\frac{\partial}{\partial y} \tan x + \sin y \cos z - \frac{\partial}{\partial z}(\sin^2 y)\right)$$

$$- \hat{j}\left(\frac{\partial}{\partial x} \tan x + \sin y \cos z - \frac{\partial}{\partial z}(x^2 + 3y + 2)\right)$$

$$+ \hat{k}\left(\frac{\partial}{\partial x} \sin^2 y - \frac{\partial}{\partial y}(x^2 + 3y + z)\right).$$

$$= \hat{i}(\cos y \cos z - \cancel{2 \sin y}^{\,0}) - \hat{j}(\sec^2 x + 0)$$
$$- k(0 - 1) + \hat{k}(0 - 3)$$

$$= \hat{i}(\cos z \cos y) - \hat{j}(\sec^2 x) + \hat{k}(3)$$

(1) $\nabla^2 f = (\vec{\nabla}\cdot\vec{\nabla})f$

$$\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\cdot\left(\frac{\partial f}{\partial x}\cdot\frac{\partial f}{\partial y}\cdot\frac{\partial f}{\partial z}\right)$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z}$$

(2) $\vec{\nabla}(f+g) = \vec{\nabla}f + \vec{\nabla}g$

$= L.H.S \quad \vec{\nabla}(\vec{f}+\vec{g})$

$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)(f+g)$$

$$= \left(\frac{\partial}{\partial x}(f+g) + \frac{\partial}{\partial y}(f+g) + \frac{\partial}{\partial z}(f+g)\right)$$

$$= \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial x}, \frac{\partial f}{\partial y} + \frac{\partial g}{\partial y}, \frac{\partial f}{\partial z} + \frac{\partial g}{\partial z}\right).$$

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$$= \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) + \left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z}\right)$$

$$= \vec{\nabla} f + \vec{\nabla} g$$

(3) $\vec{\nabla}(cf) = c\nabla f$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\nabla f = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) f$$

Similarly:-

$$\left(\frac{\partial}{\partial x}(cf), \frac{\partial}{\partial y}(cf), \frac{\partial}{\partial z}(cf)\right)$$

$$= c\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

$$= c\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

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4 $\nabla(fg) = g\nabla f + f\nabla g$

$\nabla(fg) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

$= \left(\frac{\partial}{\partial x}(fg), \frac{\partial}{\partial y}(fg), \frac{\partial}{\partial z}(fg)\right)$

$\left(f\frac{\partial g}{\partial x} + g\frac{\partial f}{\partial x}, f\frac{\partial g}{\partial y} + g\frac{\partial f}{\partial y}, f\frac{\partial g}{\partial z} + g\frac{\partial f}{\partial z}\right)$

$= \left(f\frac{\partial g}{\partial x}, f\frac{\partial g}{\partial y}, f\frac{\partial g}{\partial z}\right) + \left(g\frac{\partial f}{\partial x}, g\frac{\partial f}{\partial y}, g\frac{\partial f}{\partial z}\right)$

$= f\left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z}\right) + g\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

$= f\nabla g + g\nabla f$ Hence proved.

proof

∇

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proof # 4:.

$$\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}$$

$$= \nabla\left(\frac{f}{g}\right)$$

$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\left(\frac{f}{g}\right)$$

$$= \left(\frac{\partial}{\partial x}\left(\frac{f}{g}\right), \frac{\partial}{\partial y}\left(\frac{f}{g}\right), \frac{\partial}{\partial z}\left(\frac{f}{g}\right)\right)$$

$$= \left(\frac{g\frac{\partial f}{\partial x} - f\frac{\partial g}{\partial x}}{g^2}, \frac{g\frac{\partial f}{\partial y} - f\frac{\partial g}{\partial y}}{g^2}, \frac{g\frac{\partial f}{\partial z} - f\frac{\partial f}{\partial z}}{g^2}\right.$$

$$= \frac{1}{g^2}\left(g\frac{\partial f}{\partial x} - f\frac{\partial g}{\partial x}, g\frac{\partial f}{\partial y} - f\frac{\partial f}{\partial y}, g\frac{\partial f}{\partial z} - \frac{\partial g}{\partial z}\right)$$

$$= \frac{1}{g^2}\left(g\frac{\partial f}{\partial x}, g\frac{\partial f}{\partial y}, g\frac{\partial f}{\partial z}\right) - \left(f\frac{\partial g}{\partial x}, f\frac{\partial g}{\partial y}, f\frac{\partial g}{\partial z}\right).$$

(5)

prove that $\vec{\nabla}\cdot(\vec{F}+\vec{G}) = \vec{\nabla}\cdot\vec{F} + \vec{\nabla}\cdot\vec{G}$.

$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\cdot(f_1+g_1, f_2+g_2, f_3+g_3)$$

$$= \frac{\partial}{\partial x}(f_1+g_1) + \frac{\partial}{\partial y}(f_2+g_2) + \frac{\partial}{\partial z}(f_3+g_3).$$

$$\left(\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}\right) + \left(\frac{\partial g_1}{\partial x} + \frac{\partial g_2}{\partial y} + \frac{\partial g_3}{\partial z}\right)$$

$$= \frac{\partial f_1}{\partial x} + \frac{\partial g_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial g_2}{\partial y} + \frac{\partial f_3}{\partial z} + \frac{\partial g_3}{\partial z}$$

$$= \vec{\nabla}\cdot\vec{F} + \vec{\nabla}\cdot\vec{G}$$

$$\frac{1}{g^2}\left[g\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) - f\left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z}\right)\right]$$

$$= \frac{1}{g^2}\left[g\vec{\nabla}f - f\vec{\nabla}g\right]$$

$$= \frac{g\vec{\nabla}f - f\vec{\nabla}g}{g^2}$$

(6)

proof #6 :-

$$\vec{\nabla} \times (\vec{F} \times \vec{G}) = \vec{\nabla} \times \vec{F} + \vec{\nabla} \times \vec{G}$$

$$\vec{\nabla} \times (\vec{F} \times \vec{G})$$

$$\vec{F} = (f_1, f_2, f_3)$$

$$\vec{G} = (g_1, g_2, g_3)$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

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$$\vec{F} + \vec{G} = (f_1+g_1, f_2+g_2, f_3+g_3)$$

$$\nabla \times (\vec{F} \times \vec{G}) =$$

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_1+g_1 & f_2+g_2 & f_3+g_3 \end{vmatrix}$$

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_1 & f_2 & f_3 \end{vmatrix} + \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ g_1 & g_2 & g_3 \end{vmatrix}$$

$$= \vec{\nabla} \times \vec{F} + \vec{\nabla} \times \vec{G}.$$

Remaing Q# of cross product:-

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$\vec{v} \times \vec{w} \cdot \vec{w}$

$(9, -13, -12)(2, -6, -5)$

$= 18 - 78 + 60$

$= -60 + 60$

$= 0.$

⟹ Qno4:-

$\vec{a} \times \vec{b}$

$a = (1, 2, 1)$

$b = (3, 4, 2)$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 3 & -4 & 2 \end{vmatrix}$$

$= \hat{i}(4+4) - \hat{j}(2-3) + \hat{k}(-4-6)$

$= 8\hat{i} + \hat{j} - 10\hat{k}$

$= \sqrt{(8)^2 + (1)^2 + (-10)^2}$

$= \sqrt{64 + 1 + 100}$

$= \sqrt{165}$

$\frac{8}{\sqrt{165}} i, \frac{1}{\sqrt{165}}, \frac{10}{\sqrt{165}} k,$

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⇒ **Qno5:-**

$[(i+j)\times(i-j)]\cdot(k-i)$

$= (i^2 - ji + ji - jk^2)\cdot(k-i)$

$= (i^2 - j^2)\cdot(k-i).$

⇒ **Q#6:-**

$[(i-j)\times(k-i)]\cdot(i+j)$

$= (i\times k + i\times i - j\times k + j\times i]\cdot(i+j)$

$= (j + 0 - i + k).(i+j)$

$= (j - i + k)\cdot(i+j)$

$= i\times j - i\times i + k\times j$

$= k - 0 + i$

$= k + i$

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Curve:-

Curve is a collection and Smoth following line without any sharp turn.

Countinuity:-

f is defined

limit should exist.

$$\lim_{x \to a} f(x) = f(a)$$

$$\lim_{\Delta x \to 0} f(x + \delta x) = f(x)$$

$f(x)$ is countinous if $|y - f(u)| < \varepsilon$ for. $|\delta x| < \delta'$

A Scalar formoula $\phi(u)$ is called continous at u if

$$\lim_{\Delta u \to 0} \phi(u + \Delta u) = \phi(u)$$

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Differentiability:-

A vector function of ᵛ called differentiable of order if its n^{th} of the derivates exists.

Scalar Function:-

A function whose Range is one-dimension is called Scalar function.

$$f(x, y, z) = xyz^2$$

Vector Function:

A vector expression of the form $(f(t), g(t), h(t))$ is called vector function A vector whose cordinates are itself are function.

partial derivates of order 1:.

let

$$\bar{F} = \bar{F}(x, y, z)$$

$$\frac{\partial F}{\partial x} = \lim_{\delta x \to 0} \frac{F(x+\delta x, y, z) + F(x, y, z)}{\delta x}$$

Similarly

$$\frac{\partial F}{\partial y} = \lim_{\delta y \to 0} \frac{F(x, y+\delta y, z) - F(x, y, z)}{\delta y}$$

$$\frac{\partial F}{\partial z} = \lim_{\delta z \to 0} \frac{F(x, y, +z+\delta z) - F(x, y, z)}{\delta z}$$

let $F = F(x_1, x_2, x_3, \ldots x_n)$

$$\frac{\partial f}{\partial x_1} = \lim_{\delta x_1 \to 0} \frac{F(x_1 + \delta_1, \ldots, x_n) - F(x_1, x_2, \ldots, x_n)}{\delta x_1}$$

partial derivative of order "2"

$$\frac{\partial^2 F}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$$

$$\frac{\partial^2 F}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial y}\right)$$

$$\frac{\partial^2 F}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial F}{\partial z}\right)$$

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial x}\right)$$

$$\lim_{\delta y \to 0} \frac{F_y(x, y+\delta y, z) - F_y(x, y, z)}{\delta y}$$

Example:-

$$\phi(x,y,z) = xy^2z$$

$$\vec{A} = x\hat{i} + y\hat{j} + z\hat{k} = (x,y,z)$$

Find $\frac{\partial^3}{\partial x^2 \partial z}(\phi\vec{A})$ at $P(1,2,2)$
$P(2,2,3)$

Solution:-

$$\phi\vec{A} = xy^2z(x,y,z)$$

$$= (x^2y^2z, xy^3z, xy^2z^3)$$

$$\frac{\partial}{\partial x}(\phi\vec{A}) = (2xy^2z, y^3z, y^2z^3)$$

$$\frac{\partial^2}{\partial x^2}(\phi\vec{A}) = (2xy^2z, 0, 0)$$

$$\frac{\partial^3}{\partial x^3}(\phi\vec{A}) = (2y^2, 0, 0).$$

Therom:-

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

Let $\phi = \phi(x, y, z)$ the gradient is (scalar ~~Theorm~~ function)

$$\nabla\phi = \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}\right)$$ and

curl of $\phi = \nabla \times \phi$ (Vector function

if $\vec{\phi} = (f_1, f_2, f_3)$ and

divergence of $\phi = \vec{\nabla} \cdot \vec{\phi}$

Example:-

$\Rightarrow$ $\phi(x, y, z) = 3x^2y - y^2z^3$

Find $\nabla\phi =$ at $P(1, -2, 1)$.

$\Rightarrow$ Find $\nabla\phi$ if $\phi = \ln|r|$, $\phi = \frac{1}{r}$

where $|r| = \sqrt{x^2 + y^2 + z^2}$.

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problem:- Assignment 02:-

$$\nabla\phi = \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}\right)$$

Here

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

Since ϕ is scalar formula.

Find

i) $\nabla \times (\nabla\phi) = ?$

ii) $\nabla \cdot (\nabla\phi) = ?$

iii) $\nabla \times (\nabla \times A) = ?$

i) $\nabla \times \nabla\phi$:-

$$\nabla\phi = \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}\right)$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

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$$\nabla \times (\nabla \phi) \begin{bmatrix} i & j & k \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix}$$

$$i\left[\frac{\partial \phi}{\partial y} \cdot \frac{\partial}{\partial z} - j\,\frac{\partial \phi}{\partial z} \cdot \frac{\partial}{\partial y}\right) - j$$

$$\left[\frac{\partial \phi}{\partial x} \cdot \frac{\partial}{\partial z} - \frac{\partial \phi}{\partial z} \cdot \frac{\partial}{\partial x}\right) + k$$

$$\left[\frac{\partial \phi}{\partial x} \cdot \frac{\partial}{\partial y} - \frac{\partial \phi}{\partial y} \cdot \frac{\partial}{\partial x}\right).$$

ii) $\nabla \cdot (\nabla \phi)$:-

$$\nabla \phi = \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}\right)$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

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$$\nabla\cdot(\nabla\phi) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\cdot\left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}\right)$$

$$= \left[\frac{\partial}{\partial x}\cdot\frac{\partial\phi}{\partial x} + \frac{\partial}{\partial y}\cdot\frac{\partial\phi}{\partial y} + \frac{\partial}{\partial z}\cdot\frac{\partial\phi}{\partial z}\right]$$

$=$

iii) $\nabla\times(\nabla\times A)$:- $(\because$ where $\vec{A}\,(A_1, A_2, A_3))$

$$(\vec{\nabla}\times A) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\times(A_1, \cancel{\times} A_2, A_3)$$

$$\vec{\nabla}\times A \quad \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_1 & A_2 & A_3 \end{bmatrix}$$

$$\hat{i}\left[\frac{\partial}{\partial y}\cdot A_3 - \frac{\partial}{\partial z}\cdot A_2\right] - \hat{j}\left[\frac{\partial x}{\partial x} A_3 - \frac{\partial}{\partial z} A_1\right] + k$$

$$\left[\frac{\partial}{\partial x}\cdot A_3 - \frac{\partial}{\partial y}\cdot A_1\right]$$

$$\vec{\nabla}\times(\vec{\nabla}\times A) = \begin{array}{ccc} i & j & k \end{array}\left[\begin{array}{ccc} \frac{\partial A_3}{\partial y}-\frac{\partial A_2}{\partial z} & \frac{\partial A_3}{\partial x}-\frac{\partial A_1}{\partial z} & \frac{\partial A_3}{\partial x}-\frac{\partial A_1}{\partial y} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \end{array}\right]$$

$$i\left[\frac{\partial A_3}{\partial x}-\frac{\partial A_1}{\partial z}\cdot\frac{\partial}{\partial z} - \frac{\partial A_3}{\partial x}-\frac{\partial A_1}{\partial y}\cdot\frac{\partial}{\partial y}\right]$$

$$-j\left[\frac{\partial A_3}{\partial y}-\frac{\partial A_2}{\partial z}\cdot\frac{\partial}{\partial z} - \frac{\partial A_3}{\partial x}-\frac{\partial A_1}{\partial y}\cdot\frac{\partial}{\partial x}\right]$$

$$+k\left[\frac{\partial A_3}{\partial y}-\frac{\partial A_2}{\partial z}\cdot\frac{\partial}{\partial y} - \frac{\partial A_3}{\partial x}-\frac{\partial A_1}{\partial z}\cdot\frac{\partial}{\partial x}\right]$$

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if $\vec{A} = (A_1, A_2, A_3)$

$\Rightarrow d\vec{A} = (dA_1, dA_2, dA_3)$

, $d\vec{B} = (dB_1, dB_2, dB_3)$

$\Rightarrow d(A \cdot B) = A dB + B dA$

$= (A_1, A_2, A_3) \cdot (dB_1, dB_2, dB_3)$

$+ (B_1, B_2, B_3)(dB_1, dA_2, dA_3)$

$= A_1 dB_1 + A_2 dB_2 + A_3 dB_3 +$

$B_1 dA_1 + B_2 dA_2, B_3 dA_3.$

Q #01:-

Let $df = f(x, y, z)$

$A = A(x_1, x_2, x_3)$

$df = \left(\frac{\partial f}{\partial x} dx, \frac{\partial f}{\partial y} dy, \frac{\partial f}{\partial z} dz\right)$

Example:-

$\vec{A} = (x^2 y^2 z, x, y, z, x^2 y^2 z)$

$\vec{B} = (x^2, x^2 - y^2, e^{xyz})$

Find $d(\vec{A} \cdot \vec{B})$

$d(\vec{A} \cdot \vec{B}) = A \cdot dB + B \cdot dA$

$= (A_1, A_2, A_3) \cdot (dB_1, dB_2, dB_3) +$

$(B_1, B_2, B_3)(dA_1, dA_2, dA_3).$

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$A_1 dB_1 + A_2 dB_2 + A_3 dB_3 + B_1 dA_1$
$+ B_2 dB_2 + B_3 dA_3$. —— ①

$$dA_1 = \frac{\partial}{\partial x}(x^2y^2z)dx + \frac{\partial}{\partial y}(x^2y^2z)dy$$

$$+ \frac{\partial}{\partial x}(x^2y^2z)dz$$

$$= 2xy^2z\,dx + 2x^2yz + x^2y^2dz$$

$$dA_2 = yz\,dx + xz\,dy + xy\,dz$$

$$dA_3 = 2xy^3z\,dx + 3x^2y^2z\,dy + x^2y^3dz$$

$$dB_1 = 2x\,dx$$

$$dB_2 = 2x\,dx - 2y\,dy$$

$$dB_3 = yze^{xyz}dx + xze^{xyz}dy + xye^{xyz}dz$$

put in eq ①

$$= (2x\,dx)(x^2y^2z) + xyz(2x\,dx - 2y\,dy) + x^2y^3z^2 + (yze^{xyz}dx + xze^{xyz}dy + xye^{xyz}dz)$$

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$x^2(2xy^2dx + 2x^2yz\,dy + x^2y^2dz)$
$+ (x^2 - y^2)(yz\,dx + xz\,dy +$
$xy\,dz) + e^{xyz}(2xy^3z\,dx$
$+ 3x^2y^2z\,dy + x^3y^3\,dz)$.

Space curve:-

Consider a position vector $\vec{r}(t)$ joins orgin O and any point (x, y, z) where $\{x(t), y = y(t), z = z(t)\}$ when t varies the $r(t)$ make a space curve.

Here

$\vec{r}(t) = x(t)i$