(Ex: 4.3)

8) $y'' + 4y' - y = 0$

Auxiliary Equation

$m^2 + 4m - 1 = 0$

By quadratic equation

$$m = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16+4}}{2}$$

$$= \frac{-4 \pm \sqrt{20}}{2}$$

$$= \frac{-4 \pm 2\sqrt{5}}{2}$$

$$= \frac{-4}{2} \pm \frac{2\sqrt{5}}{2}$$

$$= -2 \pm \sqrt{5}$$

$m_1 = -2 + \sqrt{5}$, $m_2 = -2 - \sqrt{5}$

Case - I :-

Distinct and Real roots

$$y = c_1 e^{(-2+\sqrt{5})x} + c_2 e^{(-2+\sqrt{5})x}$$

9) $y'' + 9y = 0$

Auxiliary Equation

$m^2 = -9$

$m = \pm \sqrt{3}\, i$

Case : III :-

Imaginary Root / conjugate

$y = e^{0x}(c_1 \cos \sqrt{3}x + c_2 \sin \sqrt{3}x)$

$y = c_1 \cos \sqrt{3}x + c_2 \sin \sqrt{3}x$

10) $3y'' + y = 0$

Axiliary Equation

$3m^2 + 1 = 0$

$3m^2 = -1$

$m^2 = -\frac{1}{3}$

$m = \pm \sqrt{\frac{1}{3}}\, i$

Case : III :-

Imaginary Root / conjugate.

$y = e^{0x}(c_1 \cos \sqrt{\frac{1}{3}}x + c_2 \sin \sqrt{\frac{1}{3}}x)$

11) $y'' - 4y' + 5y = 0$

Axiliary Equation

$m^2 - 4m + 5 = 0$

Day: M T W T F S S

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{4 \pm \sqrt{-4}}{2}$$

$$= \frac{4 \pm 2i}{2}$$

$$m = 2 \pm 1i$$

Case : III :-

Imaginary Root / conjugate

$$y = e^{2x}(c_1 \cos x + c_2 \sin x)$$

12) $2y'' + 2y' + y = 0$

Axiliary Equation

$$2m^2 + 2m + y = 0$$

$$2m^2 + = \frac{-(2) \pm \sqrt{(2)^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{-2 \pm \sqrt{4 - 8}}{4}$$

$$= \frac{-2 \pm 2i}{4}$$

$$m = -\frac{1}{2} \pm \frac{1}{2}i$$

Case: III:-

Imaginary Root / Conjugate

$$y = e^{-\frac{1}{2}x}\left(C_1 \cos \frac{1}{2}x + C_2 \sin \frac{1}{2}x\right)$$

13) $3y'' + 2y' + y = 0$

Auxiliary Equation

$$3m^2 + 2m + 1 = 0$$

$3m^2 + 2m$

$$m = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(1)}}{2(3)}$$

$$= \frac{-2 \pm \sqrt{4 - 12}}{6}$$

$$= \frac{-2 \pm \sqrt{-8}}{6}$$

$$= \frac{-2 \pm 2\sqrt{2}\,i}{6}$$

$$= -\frac{2}{6} \pm \frac{2\sqrt{2}}{6}i$$

$$m = -\frac{1}{3} \pm \frac{\sqrt{2}}{3}i$$

Day: M T W T F S S

Case : III :-

Imaginary Root / Conjugate

$y = e^{-\frac{1}{3}x}\left(C_1 \cos \frac{\sqrt{2}}{3}x + C_2 \sin \frac{\sqrt{2}}{3}x\right)$

B) $2y'' - 3y' + 4y = 0$

Axiliary Equation

$3m^2 - 3m + 4 = 0$

$m = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(4)}}{2(2)}$

$= \frac{3 \pm \sqrt{9 - 32}}{6}$

$= \frac{3 \pm \sqrt{-23}}{6}$

$= \frac{3 \pm \sqrt{23}\, i}{4}$

$m = \frac{3}{4} \pm \frac{\sqrt{23}}{4} i$

Case III :-

Imginary Root / Conjugate

$y = e^{\frac{3}{4}x}\left(C_1 \cos \frac{\sqrt{23}}{4}x + C_2 \sin \frac{\sqrt{23}}{4}x\right)$