CALCULUS III

Vector:-

→ Magnitude

→ Direction

Scalar:-

→ Magnitude

Properties

i) Closed (+):-

For any two vectors $\vec{U}, \vec{V}$

$$\vec{U} + \vec{V} = \vec{W}$$

example:-

$$U = 2\hat{i} + 3\hat{j}$$
$$V = 4\hat{i} + 7\hat{j}$$

$$\vec{U} + \vec{V} = (2\hat{i} + 3\hat{j}) + (4\hat{i} + 7\hat{j})$$
$$= (2+4)\hat{i} + (3+7)\hat{j}$$
$$= 6\hat{i} + 10\hat{j}$$

ii) Associative Property:-

For any three vectors $\vec{U}, \vec{V}, \vec{W}$

$$\vec{U} + (\vec{V} + \vec{W}) = (\vec{U} + \vec{V}) + \vec{W}$$

For any three real numbers

$$a + (b + c) = (a + b) + c$$

CS CamScanner

Example:-

1) $9, 8, 7 \in \mathbb{R}$

$$9+(8+7) = (9+8)+7$$

$$9+15 = 17+7$$

$$24 = 24$$

2)
$$\vec{u} = 3\hat{i} + 7\hat{j} + 11\hat{k}$$
$$\vec{v} = 1\hat{i} - 2\hat{j} + 3\hat{k}$$
$$\vec{w} = 9\hat{i} - 11\hat{j} + 13\hat{k}$$

$$\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$$

$$L.H.S = \vec{u} + (\vec{v} + \vec{w})$$
$$= 3\hat{i} + 7\hat{j} + 11\hat{k} + \{(\hat{i} - 2\hat{j} + 3\hat{k} + 9\hat{i} - 11\hat{j} + 13\hat{k})$$
$$= (3\hat{i} + 7\hat{j} + 11\hat{k}) + (10\hat{i} - 13\hat{j} + 16\hat{k})$$
$$= 3\hat{i} + 7\hat{j} + 11\hat{k} + 10\hat{i} - 13\hat{j} + 16\hat{k}$$
$$= 13\hat{i} - 6\hat{j} + 27\hat{k}$$

$$R.H.S = (\vec{u} + \vec{v}) + \vec{w}$$
$$= (3\hat{i} + 7\hat{j} + 11\hat{k} + 1\hat{i} - 2\hat{j} + 3\hat{k}) + 9\hat{i} - 11\hat{j} + 13\hat{k}$$
$$= (4\hat{i} + 5\hat{j} + 14\hat{k}) + 9\hat{i} - 11\hat{j} + 13\hat{k}$$
$$= 4\hat{i} + 5\hat{j} + 14\hat{k} + 9\hat{i} - 11\hat{j} + 13\hat{k}$$
$$= 13\hat{i} - 6\hat{j} + 27\hat{k}$$

$$L.H.S = R.H.S$$

Day: M T W T F S

Date:

iii) **Identity:-**

For any vector $\vec{U}$ $\exists$ a vector $\vec{0}$ such that

$$\vec{U} + \vec{0} = \vec{U}$$

$\vec{0}$ is called a null vector.

Null vector: A vector having magnitude zero.

iv) **Inverse :-**

For any vector $\vec{U}$ $\exists$ a vector $\vec{U}'$ such that

$$\vec{U} + \vec{U}' = 0$$

$\vec{U}'$ is the inverse/negative ~~of~~ vector of $\vec{U}$.

Note:- Both $\vec{U}$ and $\vec{U}'$ having same magnitude but opposite direction.

v) **Commutative:-**

For any two vector $\vec{U}$ and $\vec{V}$

$$\vec{U} + \vec{V} = \vec{V} + \vec{U}$$

vi) **Distributive:-**

For any two vector $\vec{U}$ and $\vec{V}$ $\exists$ α scalar then

$$\alpha(\vec{U} + \vec{V}) = \alpha\vec{U} + \alpha\vec{V}$$

In General for any n-vectors $\vec{U}_1 + \vec{U}_2 + \cdots + \vec{U}_n$ $\exists \alpha$ Such that,

$$\alpha(\vec{U}_1 + \vec{U}_2 + \cdots + \vec{U}_n) = \alpha\vec{U}_1 + \alpha\vec{U}_2 + \cdots + \alpha\vec{U}_n.$$

Example

$$\vec{U} = (2, 3, 9) \quad , \quad \vec{V} = (7, 9, -11)$$

$$\alpha = 3$$

$$\alpha(\vec{U} + \vec{V}) = \alpha\vec{U} + \alpha\vec{V}$$

$$L.H.S = \alpha(\vec{U} + \vec{V})$$

$$= 3[(2, 3, 9) + (7, 9, -11)]$$

$$= 3[9, 12, -2]$$

$$= (27, 36, -6)$$

$$R.H.S = \alpha\vec{U} + \alpha\vec{V}$$

$$= 3(2, 3, 9) + 3(7, 9, -11)$$

$$= (6, 9, 27) + (21, 27, -33)$$

$$= (27, 36, -6)$$

$$L.H.S = R.H.S.$$

Example

$$\vec{U} = (\sin^2\theta, -5, 6, 1)$$

$$\vec{V} = (\cos^2\theta, 2, -4, \tan^2\theta)$$

$$\alpha = 5$$

$$\alpha(\vec{U} + \vec{V}) = \alpha\vec{U} + \alpha\vec{V}$$

$$L.H.S = \alpha(\vec{U} + \vec{V})$$

$$= 5[(\sin^2\theta - 5, 6, 1) + (\cos^2\theta, 2, -4, \tan^2\theta)]$$

$$= 5(\sin^2\theta + \cos^2\theta, -3, 2, 1 + \tan^2\theta)$$

$$= 5(\sin^2\theta + \cos^2\theta), -15, 10, 5(1 + \tan^2\theta)$$

$$= (5, -15, 10, 5(1 + \tan^2\theta))$$

$$= (5, -15, 10, 5\sec^2\theta$$

Date: Day: M T W T F S

$R.H.S = \alpha\vec{U} + \alpha\vec{V}$

$= 5(\sin^2\theta, -5, 6, 1) + 5(\cos^2\theta, 2, -4, \tan^2\theta)$

$= (5\sin^2\theta, -25, 30, 5) + (5\cos^2\theta, 10, -20, 5\tan^2\theta)$

$= (5\sin^2\theta + 5\cos^2\theta, -15, 10, 5 + 5\tan^2\theta)$

$= [5(\sin^2\theta + \cos^2\theta), -15, 10, 5(1+\tan^2\theta)]$

$= (5, -15, 10, 5\sec^2\theta)$

Associative (·x·)

$a\cdot(b\cdot c) = (a\cdot b)\cdot c$

For example $2, 5, 9 \in \mathbb{R}$

$2(5\cdot 9) = (2\cdot 5)\cdot 9$

$2\cdot(45) = (10)\cdot 9$

$90 = 90$

Let we have two scalars α, β and $\vec{U}$ is a vector.

$(\alpha\beta)\vec{U} = \alpha(\beta\vec{U})$

$U = a\hat{i} + b\hat{j} + c\hat{k}$

$1\times\vec{U} = \vec{U}$

$1\times\vec{U} = 1\times(a\hat{i} + b\hat{j} + c\hat{k})$

$= 1\times a\hat{i} + 1\times b\hat{j} + 1\times c\hat{k}$

$= a\hat{i} + b\hat{j} + c\hat{k}$

Dot Product:-

Let us consider we have two vectors

let $\vec{U} = (a_1, a_2, a_3, \text{------}, a_n)$

$\vec{V} = (b_1, b_2, b_3, \text{------}, b_n)$

Date:

then their dot product is defined as

$\vec{U}.\vec{V} = a_1b_1 + a_2b_2 + a_3b_3 + \text{------} + a_nb_n$

Example

$\vec{U} = (7, 8, 9, 8)$

$\vec{V} = (-2, 7, 11, 13)$

$\vec{w} = (6, 4, -1, 3)$

$\vec{x} = (2, 4, 6, 8)$

$(\vec{U} + \vec{V} + \vec{x})\,\vec{w} = \vec{U}.\vec{w} + \vec{V}.\vec{w} + \vec{x}.\vec{w}$

$L.H.S = (\vec{U} + \vec{V} + \vec{w})\,\vec{w}$

$= [(7, 8, 9, 8) + (-2, 7, 11, 13) + (6, 4, -1, 3)] \cdot (2, 4, 6, 8)$

$= (11, 19, 19, 24) \cdot (2, 4, 6, 8)$

$= 22 + 76 + 114 + \cancel{176}\ 192$

$= \cancel{388}\ 404$

$R.H.S = \vec{U}.\vec{w} + \vec{V}.\vec{w} + \vec{x}.\vec{w}$

$= (7, 8, 9, 8)(6, 4, -1, 3) + (-2, 7, 11, 13)(6, 4, -1, 3) + (2, 4, 6, 8)(6, 4, -$

$= (42 + 32 - 9 + 24) + (-12 + 28 - 11 + 39) + (12 + 16 - 6 + 24)$

$= 89 + 44 + 46$

$= 179$

example:-

$\vec{u} = (7, 8, 9, 8)$

$\vec{v} = (-2, 7, 11, 13)$

$\vec{w} = (6, 4, -1, 3)$

$\vec{x} = (2, 4, 6, 8)$

$$(\vec{u} + \vec{v} + \vec{x})\vec{w} = (\vec{u}\cdot\vec{w} + \vec{v}\cdot\vec{w} + \vec{x}\cdot\vec{w})$$

$$L.H.S = (\vec{u} + \vec{v} + \vec{x})\cdot\vec{w}$$

$$= [(7,8,9,8)+(-2,7,11,13)+(2,4,6,8)](6,4,-1,3)$$
$$= (7,19,26,29)(6,4,-1,3)$$
$$= 42+76-26+87$$
$$= 179$$

$$R.H.S = \vec{u}\cdot\vec{w} + \vec{v}\cdot\vec{w} + \vec{x}\cdot\vec{w}$$

$$= (7,8,9,8)\cdot(6,4,-1,3)+(-2,7,11,13)(6,4,-1,3)+(2,4,6,8)(6,4,-1,3)$$
$$= (42+32-9+24)+(-12+28-11+39)+(12+16-6+24)$$
$$= 89-44+46$$
$$= 179$$

$$L.H.S = R.H.S$$

Theorems:-

For any $\vec{u}, \vec{v}, \vec{w}$ in $\mathbb{R}^n$ and any scalar $k \in \mathbb{R}$

(i) $(\vec{u}+\vec{v})\cdot\vec{w} = \vec{u}\cdot\vec{w} + \vec{v}\cdot\vec{w}$

(ii) $\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$

(iii) $(k\vec{u})\cdot\vec{v} = k(\vec{u}\cdot\vec{v})$

(iv) $\vec{u}\cdot\vec{u} \geq 0$ if $\vec{u}\cdot\vec{u} = 0$ then $\vec{u} = \vec{0}$

Date:

Day: S M T W T F S

Proof:-

(i) $(\vec{u}+\vec{v}).\vec{w} = \vec{u}.\vec{w} + \vec{v}.\vec{w}$

Let $\vec{u} = (a_1, a_2, \ldots, a_n)$

$\vec{v} = (b_1, b_2, \ldots, b_n)$

$\vec{w} = (c_1, c_2, \ldots, c_n)$

L.H.S $= (\vec{u}+\vec{v}).\vec{w}$

$= [(a_1, a_2, \ldots, a_n) + (b_1, b_2, \ldots, b_n)].(c_1, c_2, \ldots, c_n)$

$= (a_1+b_1, a_2+b_2, \ldots, a_n+b_n)(c_1, c_2, \ldots, c_n)$

$= [(a_1+b_1)c_1 + (a_2+b_2)c_2 + \ldots + (a_n+b_n)c_n]$

$= (a_1c_1 + b_1c_1 + a_2c_2 + b_2c_2 + \ldots + a_nc_n + b_nc_n)$

$= (a_1c_1 + a_2c_2 + \ldots + a_nc_n) + (b_1c_1 + b_2c_2 + \ldots + b_nc_n)$

$= (a_1, a_2, \ldots a_n)(c_1, c_2, \ldots, c_n) + (b_1, b_2, \ldots, b_n)(c_1, c_2, \ldots, c_3)$

$= \vec{u}.\vec{w} + \vec{v}.\vec{w}$

= R.H.S (Proved)

ii) $\vec{u}.\vec{v} = \vec{v}.\vec{u}$

Let

$\vec{u} = (a_1, a_2, \ldots, a_n)$

$\vec{v} = (b_1, b_2, \ldots, b_n)$

L.H.S $= \vec{u}.\vec{v}$

$= (a_1, a_2, \ldots, a_n)(b_1, b_2, \ldots, b_n)$

$= (a_1b_1 + a_2b_2 + \ldots + a_nb_n)$

$= (b_1a_1 + b_2a_2 + \ldots + b_na_n)$

$= (b_1, b_2, \ldots, b_n)(a_1, a_2, \ldots, a_n)$

$= \vec{v}.\vec{u}$

= R.H.S (Proved)

Date: Day: M T W T F S

iii) $(k\vec{u}).\vec{v} = k(\vec{u}.\vec{v})$

$L.H.S = (k\vec{u}).\vec{v}$

let

$\vec{u} = (a_1, a_2, ----, a_n)$

$\vec{v} = (b_1, b_2, ----, b_n)$

$= [k(a_1, a_2, ----, a_n)].(b_1, b_2, ----, b_n)$

$= (ka_1, ka_2, ----, ka_n).(b_1, b_2, ----, b_n)$

$= (ka_1b_1 + ka_2b_2 + ---- + ka_nb_n)$

$= k(a_1b_1 + a_2b_2 + ---- + a_nb_n)$

$= k[(a_1, a_2, ----, a_n)(b_1, b_2, ----, b_n)]$

$= k(\vec{u}.\vec{v})$

$= R.H.S$

(Proved)

iv) $\vec{u}.\vec{u} \geq 0$ if $\vec{u}.\vec{u} = 0$ then $\vec{u} = \vec{0}$

let

$\vec{u} = (a_1, a_2, ----, a_n)$

$L.H.S = \vec{u}.\vec{u}$

$= (a_1, a_2, ---- a_n)(a_1, a_2, ----, a_n)$

$= (a_1^2 + a_2^2 + ---- a_n^2)$

$\because a_1^2 = 0 \Rightarrow a_1 = 0$

Similarly

$a_2^2 = 0$ & $a_n^2 = 0$

$= (0 + 0 + ---- + 0)$

$= 0$

$= R.H.S$ (Proved)

Orthogonal vectors:-

Let $\vec{u}$ and $\vec{v}$ any two vectors in $\mathbb{R}^n$ the $\vec{u}$ and $\vec{v}$ is said to be orthogonal if

$$\vec{u}\cdot\vec{v} = 0$$

$$(\vec{u} \perp \vec{v})$$

example:-

$$\vec{u} = (1,0,1,0,0)$$

$$\vec{v} = (0,1,0,0,0)$$

$$\vec{u}\cdot\vec{v} = (1,0,1,0,0)(0,1,0,0,0)$$

$$= (0+0+0+0+0)$$

$$= 0$$

$$\vec{u} \perp \vec{v}$$

For any two vectors $\vec{u}, \vec{v}$ in $\mathbb{R}^n$

$$\vec{u}\cdot\vec{v} = \|\vec{u}\| \, \|\vec{v}\| \cos\theta$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\| \, \|\vec{v}\|}$$

$$\|\vec{u}\|^2 = \vec{u}\cdot\vec{u}$$

$$\|\vec{u}\| = (\vec{u}\cdot\vec{u})^{1/2}$$

$$\|\vec{u}\| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}$$

CS CamScanner

Day: M T W T F S

Date:

Similarly:-

$$\|\vec{V}\| = \sqrt{b_1^2 + b_2^2 + \cdots + b_n^2}$$

$$\|\vec{w}\| = \sqrt{c_1^2 + c_2^2 + \cdots + c_n^2}$$

Unit Vector:-

let $\vec{v}$ be any vector

i.e:- $\vec{v} = \vec{v}(x_1, x_2, \ldots, x_n)$

$$\hat{v} = \frac{\vec{v}}{\|\vec{v}\|}$$

$$= \frac{(x_1, x_2, \ldots, x_n)}{\sqrt{x_1^2 + x_2^2 + \cdots, x_n^2}}$$

$$= \left(\frac{x_1}{\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}}, \frac{x_2}{\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}}, \ldots, \frac{x_n}{\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}} \right)$$

$$= \left(\frac{x_1}{\|\vec{v}\|}, \frac{x_2}{\|\vec{v}\|}, \ldots, \frac{x_n}{\|\vec{v}\|} \right)$$

example:-

$$\vec{v} = (1, -3, 2, 4)$$

$$\hat{v} = \frac{\vec{v}}{\|\vec{v}\|}$$

$$\|\vec{v}\| = \sqrt{(1)^2 + (-3)^2 + (2)^2 + (4)^2}$$

$$= \sqrt{1 + 9 + 4 + 16}$$

$$= \sqrt{30}$$

CamScanner

$$\hat{u} = \frac{(1, -3, 2, 4)}{\sqrt{30}}$$

$$= \left(\frac{1}{\sqrt{30}}, \frac{-3}{\sqrt{30}}, \frac{2}{\sqrt{30}}, \frac{4}{\sqrt{30}}\right)$$

example: 02:-

$$\vec{v} = (1, -3, 5, 6, \sin\theta, \cos\theta)$$

$$\|\vec{v}\| = \sqrt{(1)^2 + (-3)^2 + (5)^2 + (6)^2 + \sin^2\theta + \cos^2\theta}$$

$$= \sqrt{1 + 9 + 25 + 36 + 1}$$

$$= \sqrt{72}$$

$$\hat{v} = \frac{\vec{v}}{\|\vec{v}\|}$$

$$= \frac{(1, -3, 5, 6, \sin\theta, \cos\theta)}{\sqrt{72}}$$

$$= \left(\frac{1}{\sqrt{72}}, \frac{-3}{\sqrt{72}}, \frac{5}{\sqrt{72}}, \frac{\sin\theta}{\sqrt{72}}, \frac{\cos\theta}{\sqrt{72}}\right)$$

x———x———x———x

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \, \|\vec{v}\| \cos\theta$$

$$\because \vec{F} \cdot \vec{d} = Fd\cos\theta$$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \, \|\vec{v}\|}$$

Date: Day: M T W T F S

put

$$\theta = 90^\circ$$

$$\cos 90^\circ = \frac{\vec{U} \cdot \vec{V}}{\|\vec{U}\| \|\vec{V}\|}$$

$$0 = \frac{\vec{U} \cdot \vec{V}}{\|\vec{U}\| \|\vec{V}\|}$$

$$\boxed{\vec{U} \cdot \vec{V} = 0}$$

put $\theta = 0^\circ$

$$\cos 0^\circ = \frac{\vec{U} \cdot \vec{V}}{\|\vec{U}\| \|\vec{V}\|}$$

$$1 = \frac{\vec{U} \cdot \vec{V}}{\|\vec{U}\| \|\vec{V}\|}$$

$$\boxed{\vec{U} \cdot \vec{V} = \|\vec{U}\| \, \|\vec{V}\|}$$

Proof:-

$$\left| \frac{\vec{U} \cdot \vec{V}}{\|\vec{U}\| \|\vec{V}\|} \right| \leq |\cos \theta|$$

$$\frac{|\vec{U} \cdot \vec{V}|}{\|\vec{U}\| \, \|\vec{V}\|} \leq \cos 0^\circ$$

$$\frac{|\vec{U} \cdot \vec{V}|}{\|\vec{U}\| \, \|\vec{V}\|} \leq 1$$

$$|\vec{U} \cdot \vec{V}| \leq \|\vec{U}\| \, \|\vec{V}\|$$

schwaws inequality.

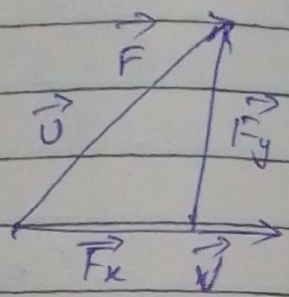
Projection:-
let $\vec{u}$ and $\vec{v}$ be two vectors then projection of $\vec{u}$ and $\vec{v}$ will be

$$(\vec{u} \text{ on } \vec{v}) = \frac{\vec{u}\cdot\vec{v}}{\|V\|^2}\cdot(\vec{v})$$

Resultant:-

$\vec{F} = \vec{F_x} + \vec{F_y}$

$\vec{F_x} = \vec{F}\cos\theta$



$$(\vec{u} \text{ on } \vec{v}) = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}\ \vec{u}$$

example:-

$\vec{u} = (1, -2, 3)$

$\vec{v} = (2, 4, 5)$

$$\vec{u} \text{ on } \vec{v} = \frac{\vec{u}\cdot\vec{v}}{\|V\|^2}(\vec{v})$$

$$\|V\| = \sqrt{(2)^2 + (4)^2 + (5)^2}$$

$$= \sqrt{4 + 16 + 25}$$

$$= \sqrt{45}$$

$$\|V\|^2 = (\sqrt{45})^2$$

Date: Day: M T W T F S

$$\begin{array}{r|l} 3 & 42 \\ 2 & 14 \\ 7 & 7 \\ & 1 \end{array} \qquad \begin{array}{r|l} 3 & 105 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$\|V\|^2 = 45$

$\vec{U}\cdot\vec{V} = (1,-2,3)(2,4,5)$

$= 2-8+15$

$= 9$

(Projection)

$\vec{V} \text{ on } \vec{V} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v}$

$= \frac{9}{45}(2,4,5)$

$= \left(\frac{2}{5}, \frac{4}{5}, \frac{5}{5}\right)$

$= \left(\frac{2}{5}, \frac{4}{5}, 1\right)$

Resultant

$\vec{U}\cdot\vec{V} = 9$

$\|V\|^2 = \sqrt{45}$

$\|U\| = \sqrt{(1)^2+(-2)^2+(3)^2}$

$= \sqrt{1+4+9}$

$= \sqrt{14}$

$\vec{U} \text{ on } \vec{V} = \frac{\vec{U}\cdot\vec{V}}{\|\vec{U}\|\,\|\vec{V}\|}\vec{U}$

$= \frac{9}{\sqrt{14}\cdot\sqrt{45}}(1,-2,3)$

$= \frac{9}{\sqrt{630}}(1,-2,3)$

$= \frac{9}{\ldots}(1,-2,3)$

$= \frac{3}{\sqrt{70}}(1, -2, 3)$

$= \left(\frac{3}{\sqrt{70}}, \frac{-6}{\sqrt{70}}, \frac{9}{\sqrt{70}}\right)$

Example:-

$F(t) = (\sin t, \cos t, t)$

$V(t) = (\cos t, -\sin t, 1)$

Find tangent vector on $\vec{V}(t)$

$\hat{v}(t) = ?$

$$\hat{v}(t) = \frac{\vec{V}(t)}{\|\vec{V}(t)\|}$$

$\|\vec{V}(t)\| = \sqrt{\cos^2 t + \sin^2 t + 1}$

$= \sqrt{1+1}$

$= \sqrt{2}$

$$\hat{v}(t) = \frac{\vec{V}(t)}{\|\vec{V}(t)\|}$$

$$= \frac{(\cos t, -\sin t, 1)}{\sqrt{2}}$$

$$= \left(\frac{\cos t}{\sqrt{2}}, \frac{-\sin t}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

CamScanner

Cross Product:-

Let $\vec{u}$ and $\vec{v}$ be two vectors

$\vec{u} = (a_1, b_1, c_1)$

$\vec{v} = (a_2, b_2, c_2)$

Then their cross product is

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \hat{i}(b_1c_2 - c_1b_2) - \hat{j}(a_1c_2 - c_1a_2) + \hat{k}(a_1b_2 - b_1a_2)$$

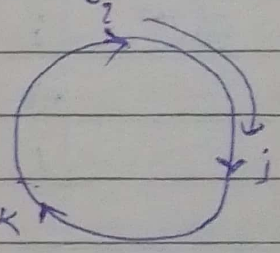
In general

$$\vec{u} \times \vec{v} = \|\vec{u}\| \|\vec{v}\| \sin\theta \, \hat{n}$$

is a vector quantity.

Cros Product:-

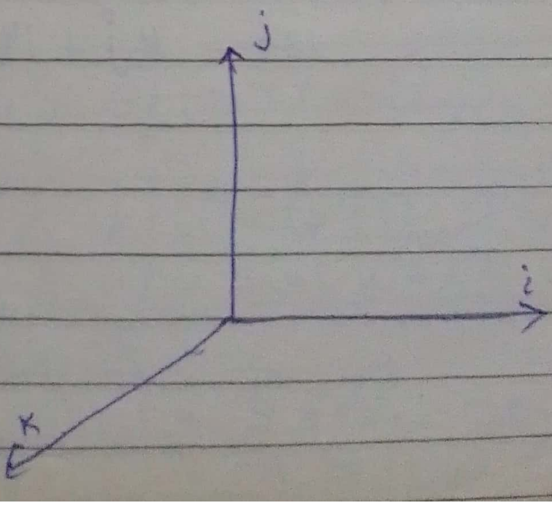
i) $\hat{i} \times \hat{j} = \hat{k}$	$\hat{j} \times \hat{i} = -\hat{k}$	$\hat{i} \times \hat{i} = 0$
ii) $\hat{j} \times \hat{k} = \hat{i}$	$\hat{k} \times \hat{j} = -\hat{i}$	$\hat{j} \times \hat{j} = 0$
iii) $\hat{k} \times \hat{i} = \hat{j}$	$\hat{i} \times \hat{k} = -\hat{j}$	$\hat{k} \times \hat{k} = 0$



$\because \sin\theta$ is zero

Dot Product:-

$i \cdot i = 1$	$i \cdot j = 0$
$j \cdot j = 1$	$j \cdot k = 0$
$k \cdot k = 1$	$k \cdot i = 0$



Date:

Problem: 1:-

let

$\vec{U} = (2\hat{i} - 3\hat{j} + 4\hat{k})$

$\vec{V} = 3\hat{i} + \hat{j} - 2\hat{k}$

$\vec{w} = \hat{i} + 5\hat{j} + 3\hat{k}$

evaluate:-

$\vec{U}\times\vec{V}$, $\vec{V}\times\vec{w}$, $\vec{U}\times\vec{w}$, $\vec{w}\times\vec{U}$, $\vec{V}\times\vec{w}$, $\vec{w}\times\vec{V}$

$$\vec{U}\times\vec{V} = \begin{vmatrix} i & j & k \\ 2 & -3 & 4 \\ 3 & 1 & -2 \end{vmatrix}$$

$= \hat{i}(6-4) - \hat{j}(-4 - 12) + \hat{k}(2+9)$

$= 2\hat{i} - 16\hat{j} - 11\hat{k}$

$$\vec{V}\times\vec{w} = \begin{vmatrix} i & j & k \\ 3 & 1 & -2 \\ 1 & 5 & 3 \end{vmatrix}$$

$= \hat{i}(3+10) - \hat{j}(9+2) + \hat{k}(15-1)$

$= 13\hat{i} - 11\hat{j} + 14\hat{k}$

Day: M T W T F S

Date:

Problem: 2:-

Find $\vec{u} \times \vec{v}$ and $\vec{v} \times \vec{u}$ if

(a) $\vec{u} = (1,2,3)$, $\vec{v} = (4,5,6)$

(b) $\vec{u} = (-4,7,3)$, $\vec{v} = (6,-5,2)$

Sol:

a) $\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$

$= \hat{i}(12-15) - \hat{j}(6-12) + \hat{k}(5-8)$

$= -3\hat{i} + 4\hat{j} - 3\hat{k}$

b) $\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 7 & 3 \\ 6 & -5 & 2 \end{vmatrix}$

$= \hat{i}(17+15) - \hat{j}(-8-18) + \hat{k}(20-42)$

$= 32\hat{i} + 26\hat{j} - 22\hat{k}$

Problem: 3:-

Find $\vec{v} \times \vec{w}$ also check $\vec{v} \times \vec{w} \cdot \vec{v} = 0$

and $\vec{v} \times \vec{w} \cdot \vec{w} = 0$

$v = (1,3,4)$, $w = (2,-6,-5)$

$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 4 \\ 2 & -6 & -5 \end{vmatrix} = \hat{i}(-15+24) - \hat{j}(-5-8) + \hat{k}(-6-6)$

$= 9\hat{i} + 13\hat{j} - 12\hat{k}$

$= (9, 13, -12)$

$\vec{v} \times \vec{w} \cdot \vec{v} = 0$

$L.H.S = \vec{v} \times \vec{w} \cdot \vec{v}$

$= (9, 13, -12)(1, 3, 4)$

$= 9 + 39 - 48$

$= 48 - 48$

$= 0$

$= R.H.S$

$\vec{v} \times \vec{w} \cdot \vec{w} = 0$

$L.H.S = \vec{v} \times \vec{w} \cdot \vec{w}$

$= (9, 13, -12)(2, -6, -5)$

$= 18 - 78 + 60$

$= 78 - 78$

$= 0$

$= R.H.S$

Problem 4:-

Find a unit vector $\perp$ to

$\vec{a} = (1, 2, 1)$ and $\vec{b}$ $(3, -4, 2)$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 3 & -4 & 2 \end{vmatrix}$$

$= \hat{i}(4+4) - \hat{j}(2-3) + \hat{k}(-4-6)$

$= 8\hat{i} + \hat{j} - 10\hat{k}$

$= (8, 1, -10)$

$$\|\vec{a}\times\vec{b}\| = \sqrt{(8)^2+(1)^2+(-10)^2}$$
$$= \sqrt{64+1+100}$$
$$= \sqrt{165}$$

Unit vector $\perp$ to $\vec{a}$

$$\frac{\vec{a}\times\vec{b}}{\|\vec{a}\times\vec{b}\|}\cdot a = \frac{1}{\sqrt{165}}(8,1,-10)(1,2,1)$$
$$= \frac{8+2-10}{\sqrt{165}}$$
$$= \frac{0}{\sqrt{165}}$$
$$= 0$$

Unit vector $\perp$ to $\vec{b}$

$$\frac{\vec{a}\times\vec{b}}{\|\vec{a}\times\vec{b}\|}\cdot b = \frac{1}{\sqrt{165}}(8,1,-10)(3,-4,2)$$
$$= \frac{1}{\sqrt{165}}(24-4-20)$$
$$= \frac{0}{\sqrt{165}}$$
$$= 0$$

Problem: 6:-

For any three vectors u, v, w

$$(\vec{u}\times\vec{v})\times\vec{w} \neq \vec{u}\times(\vec{v}\times\vec{w})$$

Sol:-

Let $\vec{u} = (2\hat{i} - 3\hat{j} + 4\hat{k})$

$\vec{v} =$

Date:

Day: M T W T F S

$= i(\cancel{42} + 44)$

Problem: 6:-

For any thre vectors u, v, w

$(\vec{u} \times \vec{v}) \times \vec{w} = \vec{u} \times (\vec{v} \times \vec{w})$

L.H.S $= (\vec{u} \times \vec{v}) \times \vec{w}$

let $\vec{u} = (a_1, a_2, a_3)$

$\vec{v} = (b_1, b_2, b_3)$

$\vec{w} = (c_1, c_2, c_3)$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= i(a_2 b_3 - a_3 b_2) - j(a_1 b_3 - a_3 b_1) + \hat{k}(a_1 b_2 - a_2 b_1)$$

$$(\vec{u} \times \vec{v}) \times \vec{w} = \begin{vmatrix} i & j & k \\ a_2 b_3 - a_3 b_2 & a_1 b_3 - a_3 b_1 & a_1 b_2 - a_2 b_1 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= i\left(c_3(a_1 b_3 - a_3 b_1) - c_2(a_1 b_2 - a_2 b_1)\right)$$

$$- j\left(c_3(a_2 b_3 - a_3 b_2) - c_1(a_1 b_2 - a_2 b_1)\right)$$

$$+ k\left(c_2(a_2 b_3 - a_3 b_2) - c_1(a_1 b_3 - a_3 b_1)\right)$$

$$= i(a_1 b_3 c_3 - a_3 b_1 c_3 - a_1 c_2 b_2 + a_2 b_1 c_2)$$

$$- j(a_2 b_3 c_3 - a_3 b_2 c_3 - a_1 b_2 c_1 + a_2 b_1 c_1)$$

$$+ k(a_2 b_3 c_2 - a_3 b_2 c_2 - a_1 b_3 c_1 + a_3 b_1 c_1)$$

$$R.H.S = \vec{u} \times (\vec{v} \times \vec{w})$$

$$\vec{v} \times \vec{w} = \begin{vmatrix} i & j & k \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= i(b_2 c_3 - c_2 b_3) - j(b_1 c_3 - c_2 b_3) + k(b_1 c_2 - b_2 c_1)$$

$$\vec{u} \times (\vec{v} \times \vec{w}) = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_2 c_3 - c_2 b_3 & b_1 c_3 - c_2 b_3 & b_1 c_2 - b_2 c_1 \end{vmatrix}$$

$$= i\left(a_2 (b_1 c_2 - b_2 c_1) - a_3 (b_1 c_3 - c_2 b_3)\right)$$

$$- j\left(a_1 (b_1 c_2 - b_2 c_1) - a_3 (b_2 c_3 - c_2 b_3)\right)$$

$$+ k\left(a_1 (b_1 c_3 - c_2 b_3) - a_2 (b_2 c_3 - c_2 b_3)\right)$$

$$= i(a_2 b_1 c_2 - a_2 b_2 c_1$$

Analytic Geometry

The branch of mathematics in which the position of any point can be observed / located / determined by pair this is also called Cartesian geometry.

Plane:-

The flat surface goes on infinitely in each direction.

Co-ordinates:-

Co-ordinates are the two ordered pair which defines the location of position on plane.

Types of coordinates system.

* Cartesian coordinates
* Polar coordinates
* Cylindrical coordinates
* spherical coordinates

Cartesian Coordinate system:-

In which position of any point represented by (x, y)

Polar coordinate system:-

In which the location (position) of a point is described by distance 'r' and angle 'θ'

$x = r\cos\theta$, $y = r\sin\theta$

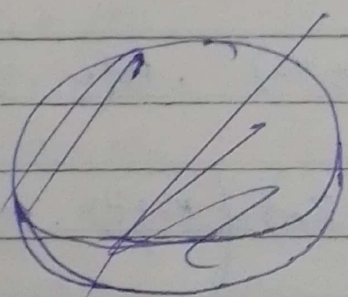
Cylindrical Coordinate system:-

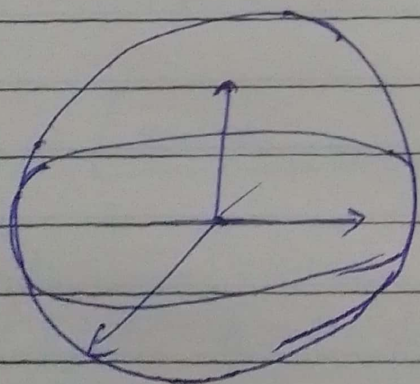
In which all the points are represented by height "h" distance "r" and angle "θ"

$P(h, r, \theta)$, $P(r, \theta, h)$

Spherical Coordinate system:-

In which the point in space is located by distar "r" and angle "θ" with ~~x~~xy-plance and another angle ϕ, naking wit z-axis





Three plane with the help of (x, y, z) coordinats

1) xy-plane
2) yz-plane
3) xz-plane.

Distance Formula:-

The distance between two Points $A(x_1, y_1)$ and $B(x_2, y_2)$ - The distance formula can be written as

$$d(A,B) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

In general

$A(x_1, x_2, x_3, \dots, x_n)$

$B(y_1, y_2, y_3, \dots, y_n)$

$$= \sqrt{(y_1-x_1)^2 + (y_2-x_2)^2 + (y_3-x_3)^2 + \dots + (y_n-x_n)^2}$$

Mid Point Formula:-

$A(x_1, x_2, \dots, x_n)$, $B(y_1, y_2, \dots, y_n)$

$$M(z_1, z_2, \dots, z_n) = \left(\frac{x_1+y_1}{2}, \frac{x_2+y_2}{2}, \dots, \frac{x_n+y_n}{2}\right)$$

Angle formula:-

$y = m_1 x + c$

$y = m_2 x + c$

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

if $\theta = 0°$ (Parallel)

CamScanner

Date: Day: M T W T F S

$$0 = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$m_1 - m_2 = 0$$

$$\boxed{m_1 = m_2}$$

if $\theta = 90^\circ$ $\perp$ (Perpendicular)

$$\tan 90^\circ = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\frac{\sin 90^\circ}{\cos 90^\circ} = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\frac{1}{0} = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\infty = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$1 + m_1 m_2 = \frac{m_1 - m_2}{\infty}$$

$$1 + m_1 m_2 = 0$$

$$\boxed{m_1 m_2 = -1}$$

example;

Find the distance and mid-point for the following.

i) $A(3, 2, 11)$, $B(6, 9, 17)$

ii) $C(5, 9, 11)$, $D(8, 17, 13)$

iii) $E(\cos\theta, 3\sin\theta)$, $F(\sin\theta, 3\cos\theta)$

CamScanner

Date:

i) $A(3,2,11)$, $B(6,9,17)$

Distance:-

$$d(A,B) = \sqrt{(6-3)^2+(9-2)^2+(17-11)^2}$$

$$= \sqrt{(3)^2+(7)^2+(6)^2}$$

$$= \sqrt{9+49+36}$$

$$= \sqrt{94}$$

Mid Point:-

$$M(z_1, z_2, z_3) = \left(\frac{3+6}{2}, \frac{2+9}{2}, \frac{11+17}{2}\right)$$

$$= \left(\frac{9}{2}, \frac{11}{2}, \frac{28}{2}\right)$$

$$= \left(\frac{9}{2}, \frac{11}{2}, 14\right)$$

Example:-

Find the slope between $(5,-3)$ and y-intercepts

$A(5,-3)$
$B(0,y)$

slope

$$m = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{y+3}{0-5}$$

CamScanner

Date: ____________ Day: M T W T F S

$$m = -\frac{(4+3)}{5}$$

Example:-

Q:1:- Find two unit vectors perpendicular to $(2,0,-3)$ and $(-1,4,2)$.

Sol:-

$\vec{u} = (2, 0, -3)$

$\vec{v} = (-1, 4, 2)$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 2 & 0 & -3 \\ -1 & 4 & 2 \end{vmatrix}$$

$$= i(0+12) - j(4-3) + k(8-0)$$

$$= 12i - j + 8k$$

$$\|\vec{u} \times \vec{v}\| = \sqrt{(12)^2 + (-1)^2 + (8)^2}$$

$$= \sqrt{144+1+64}$$

$$= \sqrt{209}$$

$$\frac{\vec{u} \times \vec{v}}{|\|\vec{u} \times \vec{v}\||} \cdot \vec{u} = \frac{(12, -1, 8)}{\sqrt{209}} \cdot (2, 0, -3)$$

$$= \frac{24 - 0 + 24}{\sqrt{209}} = 0$$

$$\frac{\vec{U}\times\vec{V}}{\|\vec{U}\times\vec{V}\|}\cdot\vec{V} = \frac{(12,-1,8)\cdot(-1,4,2)}{\sqrt{209}}$$

$$= \frac{-12-4+16}{\sqrt{209}}$$

$$= 0$$

$$\frac{\vec{V}\times\vec{U}}{\|\vec{V}\times\vec{U}\|}\cdot\vec{U} = 0$$

$$\frac{\vec{V}\times\vec{U}}{\|\vec{V}\times\vec{U}\|}\cdot\vec{V} = 0$$

Q:2:- If

$\vec{U}\cdot\vec{V} = 0$

then what will $\vec{U}=?$, $\vec{V}=?$

and if $\vec{U}\times\vec{V} = 0$

then what will $\vec{U}=?$, $\vec{V}=?$

Sol:-

let $\vec{U}\cdot\vec{V} = 0$

$\vec{U} = 5\hat{i}+6\hat{j}+-12\hat{k}$

$\vec{V} = 0\hat{i}+0\hat{j}+0\hat{k}$

(Hense proved)

$\vec{U}\times\vec{V} = 0$

$$L.H.S = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & -12 \\ 0 & 0 & 0 \end{vmatrix}$$

Date: Day: M T W T F S

$= i(0-0) - j(0-0) + k(0-0)$

$= 0$

~~Hense~~ proved.

$\vec{U}\cdot\vec{V} = 0 = a_1 a_2 + b_1 b_2 + c_1 c_2$

if $a_1 = a_2 = 0$

$b_1 b_2 = -c_1 c_2$

$= 0 - c_1 c_2 + c_1 c_2$

$= 0$

$\vec{U} = \vec{V} \Rightarrow a_1 = a_2,\ b_1 = b_2,\ c_1 = c_2$

(Exercise)

Q:6:- Using the identity

$a\times(b\times c) = (a\cdot c)b - (a\cdot b)c$

Calculate

$(i+j)\times[(i-j)\times(i+k)]$

$= [(i+j)\cdot(i+k)](i-j) - [(i+j)\cdot(i-j)](i+k)$

$= (i^2 + ij + ji + jk)(i-j) - (i^2 + ji - ji - j^2)(i+k)$

$= (1+0)(i-j) - (1-1)(i+k)$

$= i-j$ Ans

CALCULUS-III

VECTOR ANALYSIS:-

Differential Operator:-

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$= \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}, \frac{\partial}{\partial z}\hat{k}$$

In general for n variables

$$\vec{\nabla} = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}, \text{------}, \frac{\partial}{\partial x_n}\right)$$

$$f = f(x, y, z)$$

$$\vec{\nabla} f = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) f(x, y, z)$$

$$= \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

if $f = f(x_1, x_2, x_3, \text{-----}, x_n)$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \text{-----}, \frac{\partial}{\partial x_n}\right)$$

$$\vec{\nabla} f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \text{-----}, \frac{\partial f}{\partial x_n}\right)$$

$$\vec{\nabla} f = (f_{x_1}, f_{x_2}, ----, f_{x_n})$$

let $\vec{F}$ be a vector field function

$$\vec{F} = (f_x, f_y, f_z)$$

and

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$\vec{U} = (x, y)$

$\vec{V} = (a, b)$

$\vec{U} \cdot \vec{V} = xa + yb$

$$\vec{\nabla} \cdot \vec{F} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (f_x, f_y, f_z)$$

$$= \left(\frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial y}\right)$$

In general

$$\vec{\nabla} \cdot f = \left(\frac{\partial f_1}{\partial x_1}, \frac{\partial f_2}{\partial x_2}, \text{—————}, \frac{\partial f_n}{\partial x_n}\right)$$

if

$$\vec{f} = \vec{f}(f_1, f_2, f_3, \text{———} f_n)$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \partial \text{———}, \frac{\partial}{\partial x_n}\right)$$

Example:-

$$\vec{F} = (2+y, x-4)$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$$

$$\vec{\nabla}.\vec{F} = \frac{\partial}{\partial x}(2+y) + \frac{\partial}{\partial y}(x-4)$$

$$= 0 + 0$$

$$= 0$$

Example:-

Let

$$\vec{F} = (x^2+3y+z, \sin^2 y, \tan x + \sin y \cos z)$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\vec{\nabla}.\vec{F} = \frac{\partial}{\partial x}(x^2+3y+z) + \frac{\partial}{\partial y}(\sin^2 y) + \frac{\partial}{\partial z}(\tan x + \sin y \cos z)$$

$$= 2x + 2\sin y \cos y + \sin y(-\sin z)$$

$$= 2x + 2\sin y \cos y - \sin y \sin z$$

$$\boxed{\vec{\nabla}.\vec{F} = 2x + \sin 2y - \sin y \sin z}$$

Mon Tue Wed Thu Fri Sat

Date: / /20

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) - \hat{j}\left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right) + \hat{k}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2+3y+z & \sin^2 y & \tan x + \sin y \cos z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}(\tan x + \sin y \cos z) - \frac{\partial}{\partial z}\sin^2 y\right) - \hat{j}\left(\frac{\partial}{\partial x}(\tan x + \sin y \cos z) - \frac{\partial}{\partial z}(x^2+3y+z)\right)$$

$$+ \hat{k}\left(\frac{\partial}{\partial x}(\sin^2 y) - \frac{\partial}{\partial y}(x^2+3y+z)\right)$$

$$= \hat{i}\left(\cos y \cos z - \cancel{2\sin y \cos y}^{\,0}\right) - \hat{j}\left(\cot x - 1\right)$$

$$+ \hat{k}\left(0 - 3\right)$$

$$= \hat{i}(\cos y \cos z) + \hat{j}(\sec^2 x - 1) - 3\hat{k}$$

Date: __/__/20

$$\nabla^2 f = (\vec{\nabla}\cdot\vec{\nabla}) f$$

$$= \vec{\nabla}\cdot\vec{\nabla} f$$

$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

$$= \left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}\right)$$

$$= \nabla^2 f$$

Example:-

$$\vec{\nabla}(f+g) = \vec{\nabla}f + \vec{\nabla} g$$

$$L.H.S = \vec{\nabla}(f+g)$$

$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)(f+g)$$

$$= \left(\frac{\partial}{\partial x}(f+g), \frac{\partial}{\partial y}(f+g), \frac{\partial}{\partial z}(f+g)\right)$$

$$\left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial x}, \frac{\partial f}{\partial y} + \frac{\partial g}{\partial y}, \frac{\partial f}{\partial z} + \frac{\partial g}{\partial z}\right)$$

$$R.H.S = \vec{\nabla} f + \vec{\nabla} g$$

$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) f + \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) g$$

CamScanner

$$= \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) + \left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z}\right)$$

$$= \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial x}, \frac{\partial f}{\partial y} + \frac{\partial g}{\partial y}, \frac{\partial f}{\partial z} + \frac{\partial g}{\partial z}\right)$$

Hence proved

L.H.S = R.H.S

A measure that describes the spreadness and scatternes of individual data from their mean is called .

Absolute — unit wise

Relative — unit loss

$$M.D = \frac{\sum (X - \bar{X})}{n} \quad \text{ungrouped}$$

$$= \frac{\sum f(x - \bar{x})}{\sum f} \quad \text{grouped}$$

CamScanner

CALCULUS-III :-

Assignment # 01

2) let

$$\vec{\nabla} = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \text{———}, \frac{\partial}{\partial x_n}\right)$$

$$\vec{f} = \vec{f}(f_1, f_2, \text{———}, f_n)$$

Find

$$\vec{\nabla} \times \vec{f}$$

---

3) $\nabla(cf) = c\nabla f$

c any constant

$L.H.S = \nabla(cf)$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\vec{\nabla} f = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) f$$

$$\vec{\nabla} cf = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) cf$$

$$= \left(\frac{\partial(cf)}{\partial x}, \frac{\partial(cf)}{\partial y}, \frac{\partial(cf)}{\partial z}\right)$$

Date: __/__/20

$$= \left(c\frac{\partial f}{\partial x}, c\frac{\partial f}{\partial y}, c\frac{\partial f}{\partial z} \right)$$

$$= c\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$= c\nabla f$$

$= R.H.S$

③ $\nabla(fg) = g\nabla f + f\nabla g$

$L.H.S = \nabla(fg)$

$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)(fg)$$

$$= \left(\frac{\partial}{\partial x}(fg), \frac{\partial}{\partial y}(fg), \frac{\partial}{\partial z}(fg) \right)$$

$$= \left(f\frac{\partial g}{\partial x} + g\frac{\partial f}{\partial x}, f\frac{\partial g}{\partial y} + g\frac{\partial f}{\partial y}, f\frac{\partial g}{\partial z} + g\frac{\partial f}{\partial z} \right)$$

$$= \left(f\frac{\partial g}{\partial x}, f\frac{\partial g}{\partial y}, f\frac{\partial g}{\partial z} \right) + \left(g\frac{\partial f}{\partial x}, g\frac{\partial f}{\partial y}, g\frac{\partial f}{\partial z} \right)$$

$$= f\left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z} \right) + g\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$= f\nabla g + g\nabla f$$

$$= R.H.S$$

(4) $\nabla(f/g) = \dfrac{g\nabla f - f\nabla g}{g^2}$

$$L.H.S = \nabla(f/g)$$

$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) f/g$$

$$= \left(\frac{\partial}{\partial x} f/g, \frac{\partial}{\partial y} f/g, \frac{\partial}{\partial z} f/g\right)$$

(5) Let

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\vec{F} = (f_1, f_2, f_3)$$

$$\vec{G} = (g_1, g_2, g_3)$$

Prove that

$$\vec{\nabla}\cdot(\vec{F}+\vec{G}) = \vec{\nabla}.\vec{F} + \vec{\nabla}.\vec{G}$$

$$L.H.S = \vec{\nabla}(\vec{F}+\vec{G})$$

Mon Tue Wed Thu Fri Sat Date: / /20

$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\left(f_1+g_1, f_2+g_2, f_3+g_3\right)$$

$$= \frac{\partial}{\partial x}(f_1+g_1) + \frac{\partial}{\partial y}(f_2+g_2) + \frac{\partial}{\partial z}(f_3+g_3)$$

$$= \frac{\partial f_1}{\partial x} + \frac{\partial g_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial g_2}{\partial y} + \frac{\partial f_3}{\partial z} + \frac{\partial g_3}{\partial z}$$

$$= \left(\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial y}\right) + \left(\frac{\partial g_1}{\partial x} + \frac{\partial g_2}{\partial y} + \frac{\partial g_3}{\partial z}\right)$$

$$= \vec{\nabla}\cdot\vec{F} + \vec{\nabla}\cdot\vec{G}$$

$$= R.H.S$$

(6) $\vec{\nabla}\times(\vec{F}+\vec{G}) = \vec{\nabla}\times\vec{F} + \vec{\nabla}\times\vec{G}$

Proof

let $\vec{F} = (f_1, f_2, f_3)$, $\vec{G} = (g_1, g_2, g_3)$

$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

$$\vec{F}+\vec{G} = (f_1+g_1, f_2+g_2, f_3+g_3)$$

$$\vec{\nabla}\times(\vec{F}+\vec{G}) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_1+g_1 & f_2+g_2 & f_3+g_3 \end{vmatrix}$$

Mon Tue Wed Thu Fri Sat Date: / /20

$$= i\left[\frac{\partial}{\partial y}(f_3+g_3) - \frac{\partial}{\partial z}(f_2+g_2)\right] - j\left[\frac{\partial}{\partial x}(f_3+g_3) - \frac{\partial}{\partial z}(f_1+g_1)\right]$$

$$+ k\left[\frac{\partial}{\partial x}(f_2+g_2) - \frac{\partial}{\partial y}(f_1+g_1)\right]$$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_1 & f_2 & f_3 \end{vmatrix} + \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ g_1 & g_2 & g_3 \end{vmatrix}$$

$$= \vec{\nabla} \times \vec{F} + \vec{\nabla} \times \vec{G}$$

Date: / /20

CALCULUS - III

Curve

Curve is a continous and smooth following line without any sharp turn.

Continuity:-

* f is defined
* Limit should exist

$$\lim_{x \to a} f(x) = f(a)$$

* let $\lim_{\Delta x \to 0} f(x + \delta x) = f(x)$

$f(x)$ is continous if $|y - f(x)| < \varepsilon$ for

* A scalar function $\phi(u)$ is called continous at u if

$$\lim_{\Delta u \to 0} \phi(u + \Delta u) = \phi(u)$$

Differentiability:-

A vector function of 'u' is called Differentiable, of order 'n' if its n^{th} derivative exists.

Scalar function:-

A function whose range is one-dimension is called scalar function.

$$f(x,y,z) = xyz^2$$

Vector function:-

A vector expression of the form $(f(t), g(t), h(t))$ is called vector function: "A vector whose coordinates are itself are functions"

Partial derivatives:- of order 1:-

let

$$\vec{F} = \vec{F}(x,y,z)$$

$$\frac{\partial F}{\partial x} = \lim_{\delta x \to 0} \frac{F(x+\delta x, y, z) - F(x,y,z)}{\delta x}$$

similarly

$$\frac{\partial F}{\partial y} = \lim_{\delta y \to 0} \frac{F(x, y+\delta y, z) - F(x,y,z)}{\delta y}$$

$$\frac{\partial F}{\partial z} = \lim_{\delta z \to 0} \frac{F(x,y,z+\delta z) - F(x,y,z)}{\delta z}$$

Let $F = F(x_1, x_2, x_3, \ldots, x_n)$

$$\frac{\partial F}{\partial x_1} = \lim_{\delta x_1 \to 0} \frac{F(x_1+\delta_1, \ldots, x_n) - F(x_1, x_2, \ldots, x_n)}{\delta x_1}$$

Partial derivatives of order "2"

$$\frac{\partial^2 F}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial x}\right)$$

$$\frac{\partial^2 F}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial y}\right)$$

$$\frac{\partial^2 F}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial F}{\partial z}\right)$$

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial x}\right)$$

$$\lim_{\delta y \to 0} \frac{F_y(x, y+\delta y, z) - F_y(x, y, z)}{\delta y}$$

Example:-

$$\phi(x, y, z) = xy^2 z$$

$$\vec{A} = x\hat{i} + y\hat{j} + z\hat{k} = (x, y, z)$$

Find $\dfrac{\partial^3}{\partial x^2 \partial z}(\phi\vec{A})$ at $P(1, 2, 2)$
$P(2, 2, 3)$

Solution:-

$$\phi \vec{A} = xy^2 z (x, y, z)$$

$$= (x^2 y^2 z, x y^3 z, x y^2 z^2)$$

Mon Tue Wed Thu Fri Sat

Date: / /20

$$\frac{\partial}{\partial x}(\phi\vec{A}) = (2xy^2z, y^3z, y^2z^2)$$

$$\frac{\partial^2(\phi\vec{A})}{\partial x^2} = (2y^2z, 0, 0)$$

$$\frac{\partial^3}{\partial x^2\partial z}(\phi A) = (2y^2, 0, 0)$$

Let $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

Let $\phi = \phi(x, y, z)$

the gradiant is (Scalar function)

$\nabla\phi = \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}\right)$ and curl

of $\phi = \nabla \times \phi$ (Vector function)

if $\vec{\phi} = (f_1, f_2, f_3)$

and Divergence of $\phi = \vec{\nabla} \cdot \vec{\phi}$

↓

(Vector function)

example:-

$$\phi(x, y, z) = 3x^2y - y^2z^3$$

Find

$\nabla \phi$ at $P(1, -2, -1)$

example:-

Find $\nabla \phi$ if

$\phi = \ln|r|$, $\phi = \frac{1}{r}$

where $|r| = \sqrt{x^2 + y^2 + z^2}$

$S = \{ \quad \}$

* Close → semi group
* Associative → Grupoid
* Identity → Monoid
* Inverse → Group
* Commutative → Abelin Group

* identity (x)
* Associative → vector space
* → Distributive

~~Assignment~~ CALCULUS - III

$$\nabla\phi = \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}\right)$$

since ϕ is a scalar function

Here

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

Find

(i) $\nabla \times (\nabla\phi) = ?$

(ii) $\nabla \cdot (\nabla\phi) = ?$

(iii) $\nabla \times (\vec{\nabla} \times A) = ?$

where $\vec{A} = (A_1, A_2, A_3)$

(i) $\nabla \times (\nabla\phi) = ?$

$$\nabla \times (\nabla\phi) = \begin{vmatrix} i & j & k \\ \frac{\partial\phi}{\partial x} & \partial\phi/\partial y & \partial\phi/\partial z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \end{vmatrix}$$

$$= i\left(\frac{\partial\phi}{\partial y} \cdot \partial/\partial z - \frac{\partial\phi}{\partial z} \cdot \partial/\partial y\right) - j\left(\frac{\partial\phi}{\partial x} \cdot \partial/\partial z - \frac{\partial\phi}{\partial z} \cdot \frac{\partial\phi}{\partial x}\right)$$

$$+ k\left(\frac{\partial\phi}{\partial x} \cdot \partial/\partial y - \frac{\partial\phi}{\partial y} \cdot \frac{\partial}{\partial x}\right)$$

$$= i(0) - j(0) + k(0)$$

$$= \vec{0}$$

CamScanner

M Tu Wed Thu Fri Sat

Date: / /20

(ii) $\nabla \cdot (\nabla \phi) = ?$

$$\nabla \cdot (\nabla \phi) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}\right)$$

$$= \frac{\partial}{\partial x} \cdot \frac{\partial \phi}{\partial x} + \frac{\partial}{\partial y} \cdot \frac{\partial \phi}{\partial y} + \frac{\partial}{\partial z} \cdot \frac{\partial \phi}{\partial z}$$

$$= \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$$

iii) $\vec{\nabla} \times (\vec{\nabla} \times A) = ?$

$$\vec{\nabla} \times A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$$= i\left(\partial/\partial y\, A_3 - \partial/\partial z\, A_2\right) - j\left(\partial/\partial x\, A_3 - \partial/\partial z\, A_1\right)$$

$$+ k\left(\partial/\partial x\, A_2 - \partial/\partial y\, A_1\right)$$

$$\vec{\nabla} \times (\vec{\nabla} \times A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \partial/\partial y\, A_3 - \partial/\partial z\, A_2 & \partial/\partial x\, A_3 - \partial/\partial z\, A_1 & \partial/\partial x\, A_2 - \partial/\partial y\, A_1 \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}\left(\partial/\partial x\, A_2 - \frac{\partial}{\partial y} A_1\right) - \frac{\partial}{\partial z}\left(\frac{\partial}{\partial x} A_3 - \frac{\partial}{\partial z} A_1\right)\right]$$

$$-j\left(\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}A_2-\frac{\partial}{\partial y}A_1\right)-\frac{\partial}{\partial z}\left(\frac{\partial}{\partial y}A_3-\frac{\partial}{\partial z}A_2\right)\right)$$

$$+k\left(\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}A_3-\frac{\partial}{\partial z}A_1\right)-\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}A_3-\frac{\partial}{\partial z}A_2\right)\right)$$

$$= i\left(\frac{\partial^2}{\partial y\partial x}A_2-\frac{\partial^2}{\partial y^2}A_1-\frac{\partial^2}{\partial z\partial x}A_3+\frac{\partial^2}{\partial z^2}A_1\right)$$

$$-j\left(\frac{\partial^2}{\partial x^2}A_2-\frac{\partial^2}{\partial x\partial y}A_1-\frac{\partial^2}{\partial y\partial z}A_3+\frac{\partial^2}{\partial z^2}A_2\right)$$

$$+k\left(\frac{\partial^2}{\partial x^2}A_3-\frac{\partial^2}{\partial z\partial z}A_1-\frac{\partial^2}{\partial y^2}A_3+\frac{\partial^2}{\partial y\partial z}A_2\right)$$

X ———— X ———— X

if $\vec{A} = (A_1, A_2, A_3)$

1) $d\vec{A} = (dA_1, dA_2, dA_3)$, $d\vec{B} = (dB_1, dB_2, dB_3)$

2) $d(A\cdot B) = A\cdot dB + B\cdot dA$

$$= (A_1, A_2, A_3)(dB_1, dB_2, dB_3) + (B_1, B_2, B_3)(dA_1, dA_2, dA_3)$$

$$= A_1 dB_1 + A_2 dB_2 + A_3 dB_3 + B_1 dA_1 + B_2 dA_2 + B_3 dA_3$$

Let

$df = f(x, y, z)$

$A = A(x, y, z)$

$A_1 = A_1 A(x_1, y_1, z_1)$

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

$$dA_1 = \frac{\partial A_1}{\partial x}dx + \frac{\partial A_1}{\partial y}dy + \frac{\partial A_1}{\partial z}dz$$

let

$$f = f(x_1, x_2, x_3, \ldots x_n)$$

$$df = \frac{\partial f}{\partial x_1}dx_1 + \frac{\partial f}{\partial x_2}dx_2 + \ldots + \frac{\partial f}{\partial x_n}dx_n$$

Example:

$$\vec{A} = (x^2y^2z, xyz, x^2y^3z)$$

$$\vec{B} = (x^2, x^2-y^2, e^{xyz})$$

Find $d(\vec{A}\cdot\vec{B}) = ?$

$$dA_1 = \frac{\partial A_1}{\partial x}dx + \frac{\partial A_1}{\partial y}dy + \frac{\partial A_1}{\partial z}dz$$

$$= \frac{\partial}{\partial x}(x^2y^2z)dx + \frac{\partial}{\partial y}(x^2y^2z)dy + \frac{\partial}{\partial z}(x^2y^2z)dz$$

$$= 2xy^2z\,dx + 2x^2yz\,dy + x^2y^2dz$$

$$dA_2 = \frac{\partial}{\partial x}(xyz)dx + \frac{\partial}{\partial y}(xyz)dy + \frac{\partial}{\partial z}(xyz)dz$$

$$= yz\,dx + xz\,dy + xy\,dz$$

$$dA_3 = \frac{\partial}{\partial x}(x^2y^3z) + \frac{\partial}{\partial y}(x^2y^3z) + \frac{\partial}{\partial z}(x^2y^3z)$$

$$= 2xy^3z\,dx + 3x^2y^2z\,dy + x^2y^3\,dz$$

$$dB_1 = \frac{\partial}{\partial x}(x^2)\,dx + \frac{\partial}{\partial y}(x^2)\,dy + \frac{\partial}{\partial z}(x^2)\,dz$$

$$= 2x\,dx$$

$$dB_2 = \frac{\partial}{\partial x}(x^2-y^2)\,dx + \frac{\partial}{\partial y}(x^2-y^2)\,dy + \frac{\partial}{\partial z}(x^2-y^2)$$

$$= 2x\,dx - 2y\,dy$$

$$dB_3 = \frac{\partial}{\partial x}(e^{xyz})dx + \frac{\partial}{\partial y}(e^{xyz})dy + \frac{\partial}{\partial z}(e^{xyz})dz$$

$$= e^{xyz}(yz) + e^{xyz}.xz + e^{xyz}.xy$$

$$= yz\,e^{xyz}dx + xz\,e^{xyz}dy + xy\,e^{xyz}dz$$

$$= A_1 dB_1 + A_2 dB_2 + A_3 dB_3 + B_1 dA_1 + B_2 dA_2 + B_3 dA_3$$

$$= x^2y^2z\,(2x\,dx) + xyz\,(2x\,dx - 2y\,dy) + x^2y^3z\,(yz\,e^{xyz}dx + xz\,e^{xyz}dy + xy\,e^{xyz}dz)$$

$$+ x^2(2xy^2z\,dx - 2x^2yz\,dy + x^2y^2\,dz) + (x^2-y^2)(yz\,dx + xz\,dy + xy\,dz)$$

$$+ e^{xyz}(2xy^3z\,dx + 3x^2y^2z\,dy + x^2y^3\,dz)$$

$$= 2x^3y^2z\,dx + 2x^2yz\,dx - 2xy^2z\,dy + x^2y^3z^2e^{xyz}dx + x^3y^3z^2e^{xyz}dy + x^3y^4ze\text{[illegible]}$$

$$+ 2x^3y^2z\,dx - 2x^4yz\,dy + x^4y^2\,dz + x^2yz\,dx + x^3z\,dy + x^3y\,dz - y^3z\,dx - xy^2z\,dy - x\text{[illegible]}$$

$$+ 2xy^3z\,e^{xyz}dx + 3x^2y^2z\,e^{xyz}dy + x^2y^3e^{xyz}dz$$

$$= 4x^3y^2z\,dx + 3x^2yz\,dx - 3xy^2z\,dy - 2x^4yz\,dy + x^4y^2\,dz + x^3z\,dy + x^3y\,d\text{[illegible]}$$
$$- y^3z\,dx \; xy^3\,dz + (x^2y^3z^2dx + x^3y^3z^2dy + x^3y^4z\,dz + 2xy^3z\,dx + 3x^2y^2z\text{[illegible]}$$
$$+ x^2y^3dz)\,e^{xyz}$$

(CALCULUS-II)

Space curve:-

Consider a position vector $\vec{r}(t)$ joins origin O and any point (x, y, z) where $\{x = x(t), y = y(t), z = z(t)\}$ are parametric equations when t varies the r(t) make a space curve.

Here,

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

also can be written as

$$\vec{r}(t) = (x(t), y(t), z(t)) \quad \text{---} (1)$$

Velocity:-

The following vector is in the direction of $\delta\vec{r}$ if $t>0$, and is in the direction of $-\delta\vec{r}$ if $t<0$ where

$$\delta\vec{r} = \frac{\vec{r}(t+\delta t) - \vec{r}(t)}{\delta t}$$

if we apply $\lim\limits_{\delta t \to 0}$

Then

$$\lim_{\delta t \to 0} \frac{\delta\vec{r}}{\delta t} = \frac{d\vec{r}}{dt}$$

$$= \lim_{\delta t \to 0} \left(\frac{\vec{r}(t+\delta t) - \vec{r}(t)}{\delta t} \right)$$

from (1), we can write

$$\frac{d\vec{r}}{dt} = \lim_{\delta t \to 0} \frac{\delta \vec{r}}{\delta t} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right)$$

The above is the velocity of moving particle along the space curve and its direction is along tangent to any point on this curve.

Similarly the acceleration of the moving particle along the curve of parametric equations $(x = x(t), y = y(t), z = z(t))$ can be determined, by

$$\frac{d^2 r}{dt^2} = \vec{a} = \frac{d\vec{v}}{d\vec{t}} = \left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2} \right)$$

Example:-

Suppose the Parametric equation of the curve are

$$x = 40t^2 + 8t$$

$$y = 20 \cos 3t$$

$$z = 2 \sin 3t$$

Q:- 61, 62

Date: / /20

Find velocity and acceleration at
$t=0,\ t=1,\ t=10$

Differential Geometry:-

CamScanner