Onto Model-based Anomalous Link Pattern Mining on Feature-Rich Social Interaction Networks

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ABSTRACT

The detection of anomalies and exceptional patterns in social interaction networks is a prominent research direction in data mining and network science. For anomaly detection, typically two questions need to be addressed and defined: (1) What is an anomaly? (2) How do we identify an anomaly? This paper discusses model-based approaches and methods for addressing and formalizing these issues in the context of feature-rich social interaction networks. It provides a categorization of model-based approaches and provides perspectives and first promising directions for its implementation.

CCS CONCEPTS

• **Human-centered computing** → *Social networks*; *Social media*;
• **Information systems** → *World Wide Web*.

KEYWORDS

social media, social network analysis, social interaction networks, feature-rich networks, mining social networks

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1 INTRODUCTION

Social interactions of humans are mediated via social media in various forms and can be modeled using many diverse approaches, particularly using network theory. In the following, we adopt an intuitive definition of social media, regarding it as online systems and services in the ubiquitous web, which create and provide social data generated by human interaction and communication [2]. According to the idea of social interaction networks [2], we focus on interactions between humans, captured by social media. This also includes social relations implemented using specific resources, according to the principle of object-centric sociality [28].

Such data is typically multi-relational, heterogeneous, and usually includes several layers of interdependent temporal abstractions, e. g.,, corresponding to different time intervals of the captured interaction. Then, from a network perspective, these can be conveniently modeled using feature-rich social interaction networks, c. f., [27].

An important and challenging task in such contexts of ubiquitous and social interactions is the detection and analysis of anomalies, e. g., for fraud detection in online social networks, discovering events or unusual topics in heterogeneous network data, or identifying especially interesting or outstanding behavior such as given by influential or "central" actors. From an abstract point of view, an anomaly is defined as a pattern that does not conform to some notion of the expected, normal behavior. Therefore, a straightforward general anomaly detection approach defines a range covering the expected behavior. Then, it identifies any observation in the data that does not belong to this range as an anomaly. This kind of intuitive but relatively simplistic model mainly focuses on point anomalies, as discussed by [1, 43]. However, there is usually no clear formalization of the "normal behavior". Furthermore, current research mostly targets point anomalies, i. e., only relating to individual data points; this does not include anomalies with a more complex structure, e. g., those that encompass a group structure. Therefore, such complex (collective) anomaly patterns are often not detected if the individual contained points seem normal and only their interaction causes an anomaly. In addition, the complexity of anomaly detection is further enhanced by multi-relational and multi-dimensional data, e. g., given by a set of interconnected networks with spatial and temporal characteristics, for example, captured by mobility profiles, or by a set of online social networks and the captured set of activities and/or transactions. Thus, the notion of an anomaly includes other factors compared to a mere outlier which is typically defined by statistical criteria. The concept of an anomaly typically captures more complex criteria, including semantics, (user) expectations and complex data-driven structures.

Moving towards feature-rich social interaction networks, typically two questions then need to be addressed, defined, and formalized in the context of anomalous link patterns:

(1) What is an anomaly (pattern)?
(2) How do we identify an anomaly (pattern)?

One promising option in that context is given by model-based approaches, where an explicit model is used for detecting anomalies. In the following, we outline building blocks of such model-based approaches: We start by targeting the first question in order to address first approaches and promising directions for the second question, which we outline subsequently.

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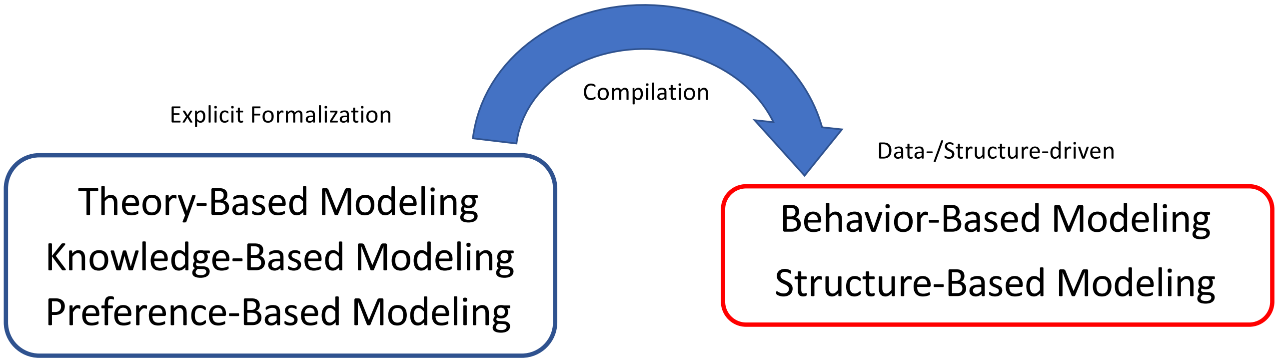

Figure 1: Categorization of Model-Based Approaches for Anomalous Link Pattern Mining

2 OUTLIERS VS. ANOMALIES

There are different definitions of an anomaly. According to the classical definition of [24], "an outlier is an observation that differs so much from other observations as to arouse suspicion that it was generated by a different mechanism". Adapted to anomalies in networks (represented by graphs), the general graph anomaly detection problem can be defined as follows: "Given a [...] graph database, find the graph objects [...] that are rare and that differ significantly from the majority of the reference objects in the graph" [1]. Considering networks (represented by graphs), we can focus on different types of graph objects. We can consider individual nodes [1, 46], links between nodes, or more complex substructures of nodes and/or links [6, 14], respectively. Currently, in the literature there are mainly approaches for handling individual point anomalies corresponding to detecting individual nodes, [c. f., 1, 46]. There exist a variety of techniques for anomaly or outlier detection [c. f., 38, 58], e. g., using subspace clustering [e. g., 29, 49], tensor factorization [e. g., 45], or community detection [e. g., 22]. In graph anomaly detection scenarios [c. f., 20, 39, 43] typically static plain graphs are considered.

However, in real-world networks the situation is typically more complex than only considering point anomalies, e. g., with respect to communities, their dynamics, and attribute information assigned to nodes and/or relations. Here, it is also quite difficult to capture the multi-relational nature of inter-connected heterogeneous networks. Therefore, we extend our focus from point anomalies to group anomalies, similar to [57] who define the general group anomaly detection problem as follows: "We are interested in finding the groups which exhibit a pattern that does not conform to the majority of other groups". Furthermore, we consider a range of complex networks, i. e., plain networks, attributed networks, as well as multi-layer networks.

Then, interesting methods for detecting and characterizing anomalous (or exceptional) link structures include exceptional (local) pattern mining and link prediction for anomalous link detection. We briefly discuss both approaches below.

3 MODEL CONSTRUCTION FOR MODEL-BASED ANOMALOUS LINK PATTERN MINING

For anomaly detection on ubiquitous and social interaction networks, we thus focus on the network structures and node properties. Therefore, this involves node topology, node features and attributes of node and/or edges, as formalized in feature-rich networks [27]. Since it is difficult to directly specify what an anomaly is (unless ground truth data is available), we aim for a model-based approach. Then, a specific model identifies a subgraph induced by a set of nodes and/or edges as an anomaly dependent on specific characteristics which are encoded in the respective model. We distinguish between the following options for constructing the models:

(1) Theory-based modeling.
(2) Knowledge-based modeling.
(3) Preference-based modeling.
(4) Behavior-based modeling.
(5) Structure-based modeling.

Regarding that categorization, we basically distinguish between approaches that require an *explicit formalization*, i. e., theory-based, knowledge-based and preference-based modeling, and those that rely on *data-driven* and *structure-driven* criteria, i. e., behavior-based and structure-based modeling. The former can often also be *compiled* into the latter, such that theories or (implicit) knowledge-structures are transformed into explicit structures to be used for anomaly detection (c. f., Figure 1).

3.1 Theory-Based Modeling

As one option for model-based detection of anomalies, we can consider social theories, e. g., homophily. In social interaction networks, homophily [35] has been identified as an important driving factor for establishing contacts, that is actors are more likely to engage with other actors if they are similar with respect to certain attributes, c. f., [10, 36, 37]. Other options include theories like the small world phenomenon [see 53], structural holes [see 18], or the strength of weak ties [see 23]. In [10], we have presented first results on detecting anomaly patterns using a homophilic model. Extensions consider, for example, more refined models on the local pattern formalization, e. g., [4].

3.2 Knowledge-Based Modeling

Knowledge-based approaches for model-based anomalous link pattern mining rely on a form of formalized domain or background knowledge. A powerful representation formalism is given by knowledge graphs, e. g., [16, 25] capturing the relations between concepts, their properties and further (inter-)relations. In such network structures, data is integrated into a comprehensive knowledge model capturing the relations between concepts and their properties in an explicit way, For instance, entities (concepts) are usually represented as nodes, there can be categories (labels) associated to node, and conceptual relations are given by directed edges between the nodes [42]. Following [41], from the point of construction, a knowledge network then mainly describes real world entities and their interrelations. Using such a structure for anomaly detection utilizes the contained relationships and dependencies, comparing expected and deviating paths and relationships on the graph with respect to the observed data. Example applications include, e. g., [8, 15] for anomaly detection in large knowledge graphs compared to complex interaction networks.

3.3 Preference-Based Modeling

Preference-based modeling incorporates a special form of knowledge, i. e., user preferences into building a model of the normal behavior. For example, in interaction networks in the context of social events, e. g., [7] or social interactions on student freshmen weeks [52] preferences can be expressed in order to determine the expected (normal) behavior. Then, e. g., simple approaches for anomaly detection consider correlation-based methods [52] or local pattern mining on the collected interaction data [7] in order to determine normal (expected) or anomalous (deviating) interactions.

3.4 Behavior-Based Modeling

Behavior-based modeling can be considered a data-driven approach, where we consider the respective observed data and compare it to a data-driven reference. This is given, for example, by typical quality function (interestingness functions) in local pattern mining, for example, relating to exceptional model mining and subgroup discovery [3]. Then, observed patterns are compared to the total population, or a null-model of the total population, respectively, as for community detection [6]. Furthermore, we can also analyze compositional structures in social interaction networks, e. g., using compositional subgroup discovery [4]. More complex behavioral modeling approaches consider, for example, Markov chains capturing transition structures in the observed data, in order to detect anomalies [8, 13].

3.5 Structure-Based Modeling

Structure-based modeling approaches take specific (graph-)structures in the network into account, in order to detect specific patterns that conform to those approaches, or to identify deviating groups. Thus, an important focus is the topological structure, for example, considering cliques, hub-authorities, or stars in a network. Then, for local patterns, also descriptive attributes are used for characterizing such structures. Exemplary methods include MinerLC [50] and MinerLSD [14] for exceptional local pattern mining.

4 MINING ANOMALOUS LINK PATTERNS

In the following, we target two directions: First, we consider local pattern mining for identifying exceptional subgroups indicating anomalous link patterns. Second, we focus on link prediction.

4.1 Mining Exceptional Local Patterns

The identification of interesting subgroups (often also called communities) is a prominent research direction in data mining and (social) network analysis, for detecting exceptional local patterns, e. g., [2, 3, 17, 21, 55], e. g., for description, characterization and introspection [5, 6, 9, 11, 12, 14]. Typically, a structural perspective is taken, such that specific (induced) subgraphs are investigated.

Attributed networks, where nodes and/or edges are labeled with additional information, allow further dimensions for detecting patterns that describe a specific subset of nodes of the graph representation of a (social) network As we have outlined above, local pattern mining is a prominent method for mining anomalous link patterns, for theory-based modeling [4, 10], knowledge-based modeling [8, 15], preference-based modeling [7], behavior-based modeling [4, 6] as well as structure-based modeling [14, 50].

4.2 Link Prediction for Anomalous Link Analysis

Link prediction [31, 32] considers the predictive modeling of links between network actors; in social interaction networks, it has a number of prominent applications, e. g., predicting missing links [31], improving collaborative filtering [26], or recommending new contacts [30, 40]. This also relates to mobility [54] and dynamic behavior [51, 56]. First experiments concerning feature-rich networks were presented in [19, 47]. First approaches for analyzing anomalous interrelations were described in [33, 34], also relating to previous work on assessing preferences and actual behavior, as described in [7]. Anomalous link discovery [44] can be implemented using link prediction, essentially focusing on the incorrectly predicted links. While there is no general method fit for all the different anomalous link modeling approaches discussed above, specific link prediction techniques can utilize specific implementations in order to detect anomalies, e. g., [48] for a preference-based approach.

5 CONCLUSION

This paper discussed model-based approaches for anomalous link pattern mining in the context of feature-rich social interaction networks. It categorized those, providing different perspectives, and outlined examples for its implementation. While we observe, that the outlined categories for model-based anomalous link pattern mining are not necessarily mutually exclusive, e. g., considering behavior-based and structure-based approaches, we nevertheless observe the capacity of the discussed approaches. In particular, adapting and extending those from descriptive methods, e. g., using local (exceptional) pattern mining, towards predictive methods for link analysis holds considerable potential for further research.

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