Selling a Single Item with Negative Externalities

To Regulate Production or Payments?

Tithi Chattopadhyay
Princeton University
Princeton, New Jersey
tithic@princeton.edu

Nick Feamster
Princeton University
Princeton, New Jersey
feamster@princeton.edu

Matheus V. X. Ferreira
Princeton University
Princeton, New Jersey
mvxf@cs.princeton.edu

Danny Yuxing Huang
Princeton University
Princeton, New Jersey
yuxingh@cs.princeton.edu

S. Matthew Weinberg
Princeton University
Princeton, New Jersey
smweinberg@princeton.edu

ABSTRACT

We consider the problem of regulating products with negative externalities to a third party that is neither the buyer nor the seller, but where both the buyer and seller can take steps to mitigate the externality. The motivating example to have in mind is the sale of Internet-of-Things (IoT) devices, many of which have historically been compromised for DDoS attacks that disrupted Internet-wide services such as Twitter [5, 26]. Neither the buyer (i.e., consumers) nor seller (i.e., IoT manufacturers) was known to suffer from the attack, but both have the power to expend effort to secure their devices. We consider a regulator who regulates payments (via fines if the device is compromised, or market prices directly), or the product directly via mandatory security requirements.

Both regulations come at a cost—implementing security requirements increases production costs, and the existence of fines decreases consumers' values—thereby reducing the seller's profits. The focus of this paper is to understand the *efficiency* of various regulatory policies. That is, policy A is more efficient than policy B if A more successfully minimizes negatives externalities, while both A and B reduce seller's profits equally.

We develop a simple model to capture the impact of regulatory policies on a buyer's behavior. In this model, we show that for *homogeneous* markets—where the buyer's ability to follow security practices is always high or always low—the optimal (externality-minimizing for a given profit constraint) regulatory policy need regulate *only* payments *or* production. In arbitrary markets, by contrast, we show that while the optimal policy may require regulating both aspects, there is always an approximately optimal policy which regulates just one.

CCS CONCEPTS

• **Theory of computation → Theory and algorithms for application domains**; • **Security and privacy** → *Human and societal aspects of security and privacy.*

KEYWORDS

Mechanism Design and Approximation; Auction Design; Negative Externalities; Tragedy of the Commons; Policy and Regulation.

ACM Reference Format:
Tithi Chattopadhyay, Nick Feamster, Matheus V. X. Ferreira, Danny Yuxing Huang, and S. Matthew Weinberg. 2019. Selling a Single Item with Negative Externalities: To Regulate Production or Payments?. In *Proceedings of the 2019 World Wide Web Conference (WWW '19), May 13–17, 2019, San Francisco, CA, USA*. ACM, New York, NY, USA, 11 pages. https://doi.org/10.1145/3308558.3313692

1 INTRODUCTION

The Tragedy of the Commons is a well-documented phenomenon where agents act in their own personal interests, but their collective action brings detriments to the common good [13]. One motivating example that we will keep referencing in the paper is the sale of Internet-of-Things (IoT) devices, such as Internet-connected cameras, light bulbs, and refrigerators. Recent years have seen a proliferation of these "smart-home" devices, many of which are known to contain security vulnerabilities that have been exploited to launch high-profile attacks and disrupt Internet-wide services such as Twitter and Reddit [5, 26]. Both the owners and manufacturers of IoT devices have the ability to protect the common good (i.e., Internet-wide service for all users) from being attacked by securing their devices, but have little incentive to do so. For the manufacturers, implementing security features, such as using encryption or having no default passwords, introduces extra engineering cost [2]. Similarly, security practices, such as regularly updating the firmware or using complex and difficult-to-remember passwords, can be a costly endeavor for the consumers [9, 27]. The results of their actions cause a negative *externality*, where Internet service is disrupted for other users.

One way to reduce the negative externality is regulation. In the context of IoT sales, a regulator can, for instance, set minimum security standards for the manufacturers or impose fines on owners of hacked IoT devices that engage in attacks. Fines could come in a few forms: direct levies on the consumer, or indirect monetary incentives. For instance, ISPs could offer discounts to users whose networks have not displayed any signs of malicious activities. One might argue that such penalty-based policies could be too futuristic, but it is worth noting that similar practices are being adopted in other industries to mitigate negative externalities [3]. One example

WWW '19, May 13–17, 2019, San Francisco, CA, USA

ACM ISBN 978-1-4503-6674-8/19/05.
https://doi.org/10.1145/3308558.3313692

is the levying of fines on users (such as cars and factories) that cause pollution [11]. While there are a lot of practicalities that have to be kept in mind and the decision of when to implement consumer/user fines depends on various factors, this is certainly one of the various policy alternatives that is worthwhile to study. Such regulations, however, can potentially increase the cost of production, discourage consumers from purchasing IoT devices, and reduce the manufacturer's profit. Our focus is to compare the *efficiency* of various regulatory policies: for two policies which equally hurt the seller's profits, which one better mitigates externalities? We will also be interested in understanding the *optimal* policy: the minimum security standards for the manufactures and fines on owners that together best mitigate externalities subject to a minimum seller's profit.

We first develop a model that consists of a buyer (e.g., consumers interested in purchasing IoT devices) and a single product for sale (e.g., IoT device). The product may come with some (costly to increase) level of security, c, and a consumer purchasing the device may choose to spend additional effort h to further secure the device. We consider a mechanism for regulating the market through incentives, for example, by requiring that the product being sold implement security features that cost the seller c dollars, imposing a fine of y dollars on the buyer if the product is later compromised and used in attacks, or both. Which intervention is more appropriate depends on how efficient buyers are in securing the product. The goal of the regulator is to minimize the negative externalities subject to a cap on the negative impact on the seller's profits—the idea being that any policy which too negatively impacts the seller could be unimplementable due to industry backlash.

Understanding the effects of such regulations on the behavior of a *single* consumer is relatively straightforward. For example: as fines go up, consumers adjust (upwards) the optimal level of effort to expend, lowering their total value for the item. Yet, reasoning about how an entire market of consumers will respond to changes, and how these responses impact seller profits becomes more complex.

Our contributions are as follows: (i) We model the sale of a single item with negative externalities, using the sale of IoT devices as the motivating example (Section 3). (ii) We show in Sections 5 and 6 that when the population of consumers is *homogeneous* (i.e., all consumers are comparably effective at translating effort into security) that optimal policies need only to regulate *either* the product (via minimum security standards) *or* the payments (via fines). (iii) We provide an example of non-homogeneous markets where the optimal policy regulates both product and payments, but prove that in *all* markets, it is always approximately optimal to regulate only one (Sections 7.1 and 7.2). The technical sections additionally contain numerous examples witnessing the subtleties in reasoning about these problems, and that any assumptions made in our theorem statements are necessary.

2 RELATED WORK

Auction Design with Externalities. There is ample prior work studying auction design with network externalities in the following sense: if the item for sale is a phone, then one consumer's value for the phone increases when another consumer purchases a phone as well (which is a positive externality, because they can talk to more people). Similarly, the item could be advertising space, in which case one consumer's value for advertising space could decrease as other consumers receive space (which is a negative externality, as now each unit of space is less likely to grab attention) [4, 6, 12, 15, 17, 23]. Our work differs in that it is a third party, who is neither selling nor purchasing an item, who suffers the externalities.

Improving the Commons. There is also a large body of work studying the regulation of common goods (e.g., clean air, security, spectrum access) in the form of taxes or licenses. For example, a government agency can regulate the emission of pollution by auctioning licenses (perhaps towards minimizing the total social cost—regulation cost plus negative externalities) [10, 19, 22, 24, 28, 29]. Our work differs in that our regulations are constrained to guarantee minimum profit to the seller, rather than focusing exclusively on the social good.

Approximation in Auction Design. Owing to the inherent complexity of optimal auctions for most settings of interest, it is now commonplace in the Economics and Computation community to design simple but approximately auctions. Our work too follows this paradigm. We refer the reader to previous work [16] for an overview of this literature.

Mitigating Security Problems. Computer security is a particular example of the Tragedy of the Commons, where a software or hardware provider sells an insecure product, and where consumers may purchase the product without considering or taking actions to reduce the security risks. In addition, users might be unable to distinguish insecure products from insecure ones [1]. One mitigation strategy is to have the vendor release updates with security features, although this could be a costly process, as August observes [2]. However, identifying the existence of security vulnerabilities in the first place may take time for the vendors; for instance, a common software vulnerability known as buffer overflow remained in more than 800 open-source products for a median period of two years before the vendors fixed the problems, according to a study by Li [21].

An alternative to relying on a vendor to implement security features or releasing updates is to incentivize the users to follow security practices. Redmiles has found that users who adopt security practices, like using two-factor authentication, have a lower overall utility for themselves than if they adopt no security practices at all, as security practices may introduce inconvenience [27]. Even if users were notified of security problems that they were presumably unaware of, it took as long as two weeks for fewer than 40% of the users to take remedial actions, according to a study [20]. To introduce incentives, vendors could, for instance, offer discounts to users who adopt security behaviors [2]; regulators, on the other hand, could incur fines to users whose software or devices were hacked [18], which is a part of our model in this paper.

3 MODEL

In this section, we introduce our model, which consists of a population of rational buyers and a single item for sale. After introducing each of the concepts one-by-one, we include a table (Table 1) at the end of this section to remind the reader of each of the components.

Buyer properties. Buyers in our model have two parameters: $(v, k) \in \mathbb{R}^2_+$. v denotes the buyer's value for the item (i.e., how much value does the buyer derive from the IoT device in isolation, independent of fines, etc.). k denotes the buyer's *effectiveness* in translating effort into improved security. That is, a buyer with high k can spend little effort and greatly reduce the risk of being hacked (e.g. because they are well-versed in security measures). A buyer with low k requires significant effort for minimal security gains. We will often use $t := (v, k)$ to denote a buyer's *type*.

Security. A buyer who chooses to purchase an item will spend some level of effort $h \geq 0$ securing it, which causes disutility h to the buyer. The seller may also include some default security level c. If the buyer has effectiveness k, we then denote the combined effort by $\textsc{effort}(k, c, h) := c + kh$. The idea is that buyers with higher effectiveness are more effective at securing the device for the same disutility. Note that buyers with effectiveness $k > 1$ are more effective than the producer, and buyers with effectiveness $k < 1$ are less effective. Highly effective buyers should not necessarily be interpreted as "more skilled" than producers, but some security measures (e.g., password management) are simply more effective for consumers than producers to implement.

We model the probability that a device is compromised as a function $g(\cdot)$ of $\textsc{effort}$, with $g(x) := e^{-x}$. This modeling decision is clearly stylized, and meant as an approximation to practice which captures the following two important features: (a) as effort x approaches ∞, $g(x) \to 0$ (that is, it is possible to shrink the probability of being compromised arbitrarily small with sufficient effort), and (b) $g''(x) \geq 0$. That is, the initial units of effort are more effective (i.e. $g'(x)$ is larger in absolute value) than latter ones (when $g'(x)$ is smaller in absolute value). The idea is that consumers/producers will take the highest "bang-for-buck" steps first (e.g., setting a password). Note that our results do not qualitively change if, for instance, $g(x) := \lambda_1 e^{\lambda_2 x}$ for some constants $\lambda_1 \in (0, 1], \lambda_2 > 0$, but since the model is stylized anyway we set $\lambda_1 = \lambda_2 = 1$ for simplicity of notation.

Regulatory Policy. The regulator selects a policy/strategy $s = (y, c, p) \in \in\mathbb{R}^3_+$. Here, c denotes the security standards the producer must include which is equivalent to the production cost. y denotes the fine the consumer pays should their device be compromised. p denotes the price of the item. Conceptually, one should think of the regulator inducing the producer to set security standard c and price p via particular regulatory policies (e.g., requiring a minimum security level c', or mandating purchase of insurance). Mathematically, we will not belabor exactly how the regulator arrives at (y, c, p). We will also be interested in "simple" policies, which regulate either y or c.

Definition 3.1 (Simple Policy). For a policy $s = (y, c, p)$, we say s is a *fine policy* if $c = 0$, a *cost policy* if $y = 0$ and a *simple policy* if s is either a *fine policy* or a *cost policy.*

Utilities. Recall that so far our buyer has value v and efficiency k, and chooses to put in effort h. The regulator mandates security c (which is equivalent to the production cost) on the item (which has price p) and imposes fine y for compromised items. The probability that an item is compromised is $g(\textsc{effort}(k, c, h)) = e^{-c-kh}$. The buyer's utility is therefore: $v - p - h - y \cdot e^{-c-kh}$. Observe that the buyer is in control of h (but not v, p, y, c, k). So the buyer will optimize over $h \geq 0$ to minimize $h + y \cdot e^{-c-kh}$. By taking the derivative with respect to h, we get a closed form for the choice of effort $h^*(t, s)$ (recalling that we denote the buyer's type $t = (v, k)$ and the regulator's strategy $s = (y, c, p)$):

$$h^*(t, s) = \max\left(0, \frac{\ln(yk) - c}{k}\right) \tag{1}$$

We can now see that the probability that the buyer's item is compromised, conditioned on expending the optimally chosen effort is:

$$\textsc{risk}(t, s) := \min\left\{e^{-c}, \frac{1}{yk}\right\}. \tag{2}$$

We will additionally refer to the buyer's (security) *loss* as the expected fines they suffer plus the effort they spend. That is:

$$\ell(t, s) := y \cdot \textsc{risk}(t, s) + h^*(t, s) := \begin{cases} \frac{\ln(yk)-c+1}{k}, & yk \geq e^c \\ ye^{-c}, & yk < e^c \end{cases} \tag{3}$$

It then follows that the buyer's utility (value minus price minus expected fines) is:

$$u(t, s) := v - p - \ell(t, s) = \begin{cases} v - p - 1/k - \frac{\ln(yk)-c}{k}, & yk \geq e^c \\ v - p - ye^{-c}, & yk < e^c \end{cases} \tag{4}$$

Population of Buyers. We model the population of buyers as a distribution D over types t. Additionally, we make the now-typical assumption in the multi-dimensional mechanism design literature (e.g. [8, 14, and follow-up work]) that the parameters v and k are drawn independently, so that $D := D_v \times D_k$.[1] The seller's profits are then:

$$\textsc{Prof}_D(s) := (p - c) \cdot \Pr_{t \leftarrow D}[u(t, s) \geq 0] \tag{5}$$

Externalities. Finally, we define the externalities caused and the regulator's objective function. Each device sold has some probability of being compromised, and the regulator wishes to minimize the total fraction of compromised devices.[2] That is, we measure the externalities caused as:[3]

$$\textsc{Ext}_D(s) := \frac{\mathbb{E}_{t\leftarrow D}[\textsc{risk}(t, s) \cdot \mathbb{I}(u(t, s) \geq 0)]}{\Pr_{t\leftarrow D}[u(t, s) \geq 0]} \tag{6}$$

Optimization. The regulator's objective is to propose an $s = (y, c, p)$ that minimizes $\textsc{Ext}_D(s)$. Observe that, if left unconstrained, the regulator can simply propose $c \to \infty$, resulting in 0 externalities. Such a policy is completely unrealistic, as it would cause costs to approach ∞ and destroy the industry. Similarly, taking $y \to \infty$ would cause consumers to have negative utility even to get the item for free (again destroying the industry). We therefore impose a minimum profit constraint for a policy to be considered feasible. Indeed, this forces the regulator to trade off profits for externalities

[1] This assumption is even more justified in our setting than usual, as it is hard to imagine correlation between the *value* a consumer derives from using a smart refridgerator and their *ability* to secure IoT devices.
[2] It would be equally natural for the regulator to aim to minimize the total mass of compromised devices. Most of our results do not rely on optimizing one objective versus the other, but we stick with one in order to unify the presentation.
[3] Below, $\mathbb{I}(\cdot)$ denotes the indicator function, which takes value $\mathbb{I}(X) = 1$ whenever event X occurs, and 0 otherwise.

as effectively as possible. Therefore, our regulator is given some profit constraint R, and aims to find:

$$\arg\min_{s, \textsc{Prof}_D(s) \geq R} \{\textsc{Ext}_D(s)\}$$

We will only consider cases where there is *some* feasible s (that is, we will only consider R such that there exists a p with $R \leq \textsc{Prof}_D(0,0,p)$. If no such p exists, then the profit constraint exceeds the optimal achievable profit without regulation, and the problem is unsolvable).

Recap of model. Table 1 recaps the parameters of our model for future reference. Note also that many parameters (e.g. $\ell(\cdot,\cdot)$ are formally defined as a function of $t = (v,k)$ and $s = (y,c,p)$, but only depend on (e.g.) k, y, c. As such, it will often be clearer to overload notation and write $\ell(k,y,c)$, rather than defining a new $t = (v,k)$ with a meaningless parameter. Sometimes, though, it will be clearer to use the defined notation for a type t that was just defined. In the interest of clarity, we will overload notation for these variables, but it will be clear from context what they refer to.

Table 1: Model Variables

Variable	Text Definition	Formal Definition
$t = (v,k)$	(value, effectiveness)	N/A
$D := D_v \times D_k$	buyer population	N/A
$s = (y,c,p)$	(fine, security, price)	N/A
$h^*(t,s)$	buyer optimal effort	$\max\{0, \frac{\ln(yk)-c}{k}\}$,(1)
$\textsc{risk}(t,s)$	compromise prob.	$\min\{e^{-c}, \frac{1}{yk}\}$, (2)
$\ell(t,s)$	buyer security loss	Equation (3)
$u(t,s)$	buyer utility	$v - p - \ell(t,s)$, (4)
$\textsc{Prof}_D(s)$	seller profits	Equation (5)
$\textsc{Ext}_D(s)$	frac. compromised	Equation (6)

Final Thoughts on Model. We propose a stylized model to capture the following salient aspects of this market: (a) neither buyer nor seller suffer externalities when the item is compromised, (b) the regulator can regulate both the product (via c) and payments (via y, p), (c) there is a population of buyers, each with different value v and effectiveness k at translating effort into security, and (d) the regulator must effectively trade off externalities with profits by minimizing negative externalities, subject to a minimum profit constraint R. The goal of this model is not to capture every potentially relevant parameter, but to isolate the salient features above.

3.1 An Intuitive Example

In this section, we provide one example to help give intuition for the interaction between the fines y, default security c, and seller's profits $\textsc{Prof}_D(s)$. In particular, Figure 1 plots the maximum achievable $\textsc{Prof}_D(s)$ over all s with a fixed c (the x-axis) and y (the color of the plot). In all three examples, D_v is the uniform distribution on $[0, 20]$, and k is drawn from either the uniform distribution on $[0, 1]$, $[0, 3]$, or $[2, 3]$, respectively. Note that $k \geq 1$ is the threshold when a buyer is more efficient than the seller in mitigating externalities, so these examples cover two *homogeneous* populations, where all consumers are more (respectively, less) efficient than the producer, and one *heterogeneous* population, where some consumers are more efficient, and others are not.

For each possible (partial) regulation (y, c), the profit-maximizing choice of p is essentially a classic single-item problem (e.g. [25]), as the buyer's "modified value" v' is simply $v - \ell(t,s) - c$, and the seller's profit for setting price p is just $p \cdot \Pr_{t \leftarrow D}[v' \geq p]$. Therefore, for each partial regulation (y,c), we can construct the modified distribution and simply maximize $p \cdot \Pr_{t \leftarrow D}[v' \geq p]$ as above.

Observe in Figure 1, when $y = 0$, the seller gets greater profits with lower c. This should be intuitive, as neither the buyer nor seller suffer when the device is compromised. When $y > 0$, and $D_k = U([0,1])$, the seller's profits can increase with c. This should also be intuitive: now that the buyer suffers when the device is compromised, they prefer to buy a secure device.

On the other hand, when the market contains only efficient buyers ($k > 1$ always), the buyer prefers to provide her own security; any increased cost will always decrease the buyer's utility. Indeed, observe that $\frac{\partial \ell(t,s)}{\partial c}$ is either 0 (if $yk < e^c$) or $1 - 1/k$ (otherwise). If $k > 1$, then this is always positive, so higher c results in (weakly) higher loss for the consumer, and lower utility.

3.2 Preliminary Observations

We conclude with two observations which allow an easy comparison between the profits of certain policies. Intuitively, Observation 1 claims that any policy which makes *every single* consumer in the population have lower loss generates greater profits for the seller. We will make use of Observation 1 repeatedly throughout the technical sections to modify existing policies into ones which improve profits (ideally while also improving externalities, although that is not covered by Observation 1).

Observation 1. *Let $s = (y,c,p)$, $s' = (y',c',p')$ be such that $p' - c' = p - c \geq 0$ and for all $k \in$ support(D_k), $\ell(k,s) + c \leq \ell(k,s') + c'$. Then for all D_v, $\textsc{Prof}_{D_v \times D_k}(s) \geq \textsc{Prof}_{D_v \times D_k}(s')$.*

Proof. Observe that for both s and s', the seller's profit per sale is identical (as $p' - c' = p - c$). So we just wish to show that the probability of sale for s' is larger than that for s. Indeed, observe that for all t:

$$\begin{aligned} u(t,s) &= v - \ell(k,s) - p \\ &= v - (\ell(k,s) + c) + c - p \\ &\geq v - (\ell(k,s') + c') + c' - p' \\ &= v - \ell(k,s') - p' = u(t,s'). \end{aligned}$$

Therefore, any consumer (v,k) who chooses to purchase the item under policy s' will also choose to purchase under policy s, and therefore the probability of sale is at least as large for s as s'. □

Observation 2 below claims that the profit of any policy s is larger in populations D where every consumer is more effective than in D'.

Observation 2. *Let D_k stochastically dominate D'_k.[4] Then for all policies $s = (y,c,p)$ with $p \geq c$, and all D_v, $\textsc{Prof}_{D_v \times D_k}(s) \geq \textsc{Prof}_{D_v \times D'_k}(s)$.*

[4]That is, it is possible to couple draws (k, k') from (D_k, D'_k) so that $k \geq k'$ with probability 1. Equivalently: for all x, $\Pr[k \geq x, k \leftarrow D_k] \geq \Pr[k' \geq x, k' \leftarrow D'_k]$.

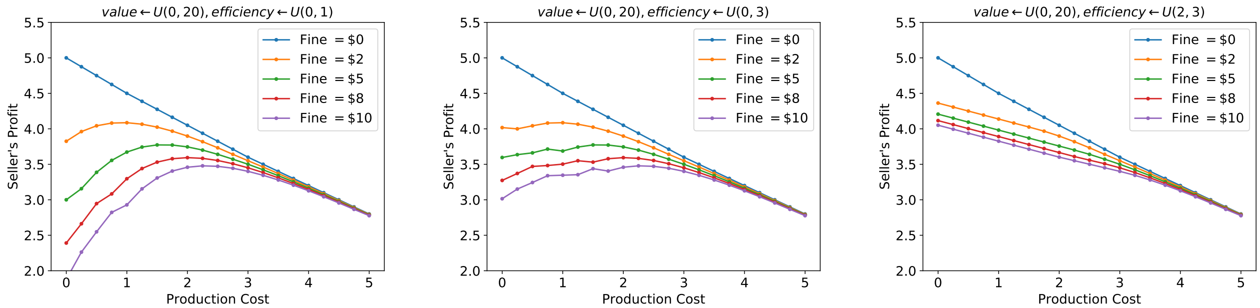

Figure 1: Seller's optimal profits under different distributions for efficiency, k. We plot the seller's profits on the vertical axis and the default security c on the horizontal axis. Each curve corresponds to different fines, y. Importantly, observe that when the fine is zero, the seller achieves greatest profits with lower default security. However, when the fine is non-zero, the seller may actually *increase* their profits with default security, but the benefits (to the seller) of default security decrease as the buyer population becomes more efficient.

Proof. As D_k stochastically dominates D'_k, it is possible to couple draws (t, t') from $(D_v \times D_k, D_v \times D'_k)$ such that $v = v'$ and $k \geq k'$. Observe simply that $u(t, s) \geq u(t', s)$ always. Therefore, $\Pr[u(t, s) \geq 0] \geq \Pr[u(t', s) \geq 0]$, and $\textsc{Prof}_{D_v \times D_k}(s) \geq \textsc{Prof}_{D \times D'_k}(s)$. □

Observe, however, that Observation 2, perhaps counterintuitively, does *not* hold if we replace profits with externalities. That is, for a fixed policy s, we might *increase* all consumers' effectiveness yet also *increase* the externalities caused. Intuitively, this might happen (for instance) in a fine policy which successfully only sells the item to extremely effective consumers who effectively secure their purchase. Ineffective consumers choose not to purchase the product to avoid fines. However, if these ineffective consumers are instead somewhat effective, they may now choose to purchase the item, thereby increasing externalities. Below is a concrete instantiation:

Example 3.2. Consider the population where D_v is a point-mass at e, and D_k takes on effectiveness 0 with probability 1/2 and $x > 1$ with probability 1/2. Consider the policy $s = (e, 0, e - 2.5)$. Then the $(e, 0)$ consumer chooses not to purchase: $\ell(e, 0, s) = e$, so their utility would be $e - e - (e - 2.5) < 0$. The (e, x) consumer chooses to purchase, as their loss is $\frac{2+\ln(x)}{x} < 2$ (as $x > 1$). So $\textsc{Ext}_{D_v \times D_k}(s) = \frac{1}{ex}$.

Consider now improving the effectiveness of the $k = 0$ consumers to $k = 1$ (so D'_k now takes on 1 with probability 1/2 and x with probability 1/2). The $(e, 1)$ consumer now chooses to purchase, as their loss is 2 (so their utility is $e - 2 - (e - 2.5) = 1/2$). So now $\textsc{Ext}_{D_v \times D'_k}(s) = (\frac{1}{e} + \frac{1}{ex})/2$. As $x > 1$, the externalities have gone up. If $x \geq 1$, the externalities may have gone up quite significantly.

In Example 3.2, of course "the right" thing to do is to also change the policy. Indeed, it is still the case that, for a *fixed consumer who purchases the item*, increasing effectiveness can only decrease externalities. But without fixing whether the consumer has purchased the item or not, the claim is false. Observation 3 captures what we can claim about risk, loss, etc. on a per-consumer basis. Proofs for the claims in Observation 3 all follow immediately from the definitions in Section 3.

Observation 3. *Let $k > k'$, then for all s:*

- $\textsc{risk}(k, s) \leq \textsc{risk}(k', s)$.
- $\ell(k, s) \leq \ell(k', s)$.
- $h^*(k, s) \geq h^*(k', s)$.
- $u(k, s) \geq u(k', s)$.

4 ROADMAP OF TECHNICAL SECTIONS

Now that we have the appropriate technical language, we provide a brief roadmap of the results to come.

- In Section 5, we provide a technical warmup to get the reader familiar with how to reason about our problem. The main result of this section is Theorem 5.1, which claims that the optimal policy when D_k is a point-mass is simple. The proof of this theorem helps illustrate one key aspect of our later arguments, and will also be used as a building block for later proofs.
- In Section 6, we prove our first main result (Theorem 6.1): as a function of R and D_v, there exists a cutoff T. If D_k is supported on $[0, T]$, then a cost policy is optimal. If D_k is supported on $[T, \infty)$, then a fine policy outperforms all *profits-maximizing* policies (we define this term in the relevant section — intuitively a policy is profits-maximizing if the price is the seller's best response to (y, c)). Section 6 also contains a surprising example witnessing that the additional profits-maximizing qualification is necessary.
- In Section 7, we consider general distributions. Unsurprisingly, simple policies are no longer optimal. Perhaps surprisingly, if one insists on exceeding the profits benchmark *exactly*, no simple policy can guarantee any bounded approximation to the optimal externalities (Corollary 7.5). However, we also show (Theorem 7.6) that it is possible to get a bicriterion approximation: if one is willing to approximately satisfy the profits constraint, it is possible to approximately minimize externalities with a simple policy. That is, for any s, D, there is a simple policy s' with $\textsc{Prof}_D(s') = \Omega(1) \cdot \textsc{Prof}_D(s)$ and $\textsc{Ext}_D(s') = O(1) \cdot \textsc{Ext}_D(s)$.

- We include complete proofs for most of our results on point-mass and homogeneous distributions, as these convey many of the key ideas. By Theorem 7.6, the proofs get quite technical so we provide a sketch of the main ideas. This and other omitted proofs can be found in [7].

5 WARM-UP: POINT-MASS EFFECTIVENESS

As a warm-up, we first study the case where D_k is a point mass (that is, all buyers in the population have the same effectiveness k). In this case, we show that a simple policy is optimal. The proof is fairly intuitive, with one catch. The intuitive part is that every consumer will put in the same effort, conditioned on buying the item. It therefore seems intuitive that if $k < 1$, it is better for all parties involved if any effort spent by the consumer is transferred to the producer instead (and this is true). It also seems intuitive that if $k > 1$, it is again better for all parties involved if any effort spent by the producer is "transferred" to the consumer instead (e.g. by raising fines so that the consumer chooses to spend the desired level of effort). This is not quite true: the catch is that the fine required to induce the desired buyer behavior may be too high to satisfy the profit constraint. But, the above argument does work for sufficiently large k. Importantly, there is some cutoff T such that for all $k \leq T$, the optimal policy is a cost policy ($y = 0$), while for all $k \geq T$, the optimal policy is a fine policy ($c = 0$). Below, when we write $D_v \times \{k\}$, we mean the distribution which draws v from D_v and outputs (v, k).

Theorem 5.1. *For all D_v, R, and k, the externality-minimizing policy for $D_v \times \{k\}$ is a simple policy. Moreover, for all R, D_v, there is a cutoff T such that if $k \leq T$, then the optimal policy is a cost policy. If $k \geq T$, then the optimal policy is a fine policy.*

Proof. Consider any policy $s = (y, c, p)$. Because all consumers have the same effectiveness k, s induces the same loss for all consumers. We first claim the following: □

Lemma 5.2. *Let $k \leq 1$. Then for all D_v and any policy $s = (y, c, p)$, there is an alternative policy $s' = (0, c', p)$ with $\text{Prof}_{D_v \times \{k\}}(s') \geq \text{Prof}_{D_v \times \{k\}}(s)$ and $\text{Ext}_{D_v \times \{k\}}(s') \leq \text{Ext}_{D_v \times \{k\}}(s)$.*

Proof. In policy s, all consumers have the same loss $\ell(k, s)$. This therefore is a good opportunity to try and make use of Observation 1. First, consider the possibility that $yk < e^c$. In this case, $h^*(k, s) = 0$, $\ell(k, s) = ye^{-c}$, and $\text{risk}(k, s) = e^{-c}$. This implies that $\text{Ext}_{D_v \times \{k\}}(s) = e^{-c}$. Consider instead the policy $s' = (0, c, p)$. Then $\ell(k, s') = 0$, but $\text{risk}(k, s) = e^{-c}$ and $\text{Ext}_{D_v \times \{k\}}(s) = e^{-c}$ like before. So the externalities are the same. An application of Observation 1 concludes that the profits have improved (indeed, (c, p) are the same in both policies, and the loss decreases as we switch from policy s to s').

Consider now the possibility that $yk \geq e^c$. In this case, $h^*(k, s) = \frac{\ln(yk)-c}{k}$, $\ell(k, s) = \frac{\ln(yk)-c+1}{k}$, and $\text{risk}(k, s) = \frac{1}{yk}$. Consider instead the policy $s' = (0, \ln(yk), p - c + \ln(yk))$. In this new policy, $\ell(k, s') = 0$ and $\text{risk}(k, s') = \frac{1}{yk}$. So indeed, the new policy has the same externalities. We just need to ensure that we can apply Observation 1. To this end, observe that:

$$\begin{aligned}\ell(k, s) + c - (\ell(k, s') + c') &= \frac{\ln(yk) - c + 1}{k} + c - \ln(yk) \\ &= (1/k - 1) \cdot (\ln(yk) - c) + 1/k \\ &\geq 0.\end{aligned}$$

The last line follows because $k \leq 1$ and $\ln(yk) \geq c$ (because $yk \geq e^c$). So the hypotheses of Observation 1 hold, and we can apply Observation 1 to conclude that the profits improve from s to s' as well. □

Lemma 5.2 covers the cases when $k \leq 1$: there is always an optimal cost policy. We now move to the case when $k > 1$. There are two cases to consider: one where the optimal policy will be a cost policy, and one where the optimal policy will be a fine policy. The distinguishing feature between these cases will be for a given c, how big of a fine is necessary to incentivize the consumer to put in effort c/k, and what the consumer's loss looks like for this choice of y. Below, c^* is defined to be the maximum c such that there exists a p such that $\text{Prof}_{D_v \times \{0\}}(0, c, p) \geq R$. Observe that c^* is also equal to the maximum ℓ such that there exists a p such that $\text{Prof}_{(D_v - \ell) \times \{0\}}(0, 0, p) \geq R$ (here, $D_v - \ell$ denotes the distribution which samples v from D_v and then subtracts ℓ, taking a maximum with 0 if desired). That is, c^* is the maximum loss that can be uniformly applied to all consumers (drawn from D_v) while still resulting in a distribution for which profit $\geq R$ is achievable.

Lemma 5.3. *Let c^* denote the maximum c such that there exists a p such that $\text{Prof}_{D_v \times \{0\}}(0, c, p) \geq R$. Then a cost policy is optimal for $D_v \times \{k\}$ if $k \in [1, 1 + 1/c^*]$.*

Proof. First, observe that the lemma hypothesis implies that *any* feasible policy must have $\ell(k, s) + c \leq c^*$ (if not, then an application of Observation 1 lets us contradict the lemma's hypothesis with a feasible $c' = \ell(k, s) + c > c^*$).

Consider now $k \in [1, 1 + 1/c^*]$, and start from some policy $s = (y, c, p)$. If this policy has $h^*(k, s) = 0$, then certainly we can just update $s' = (0, c, p)$ and get better profits with the same externalities (by Observation 1). If instead $h^*(k, s) > 0$, then $\ell(k, s) = \frac{\ln(yk)-c+1}{k}$, and $\text{risk}(k, s) = \frac{1}{yk}$. Consider instead $s^* = (0, c^*, p^*)$, for whichever p^* witnesses $\text{Prof}_D(s^*) \geq R$ (we know that such a p^* exists by the lemma's hypothesis). So now we just need to compare externalities. Assume for contradiction that $\text{risk}(k, s^*) > \text{risk}(k, s)$. Then we get:

$$\begin{aligned}\text{risk}(k, s^*) > \text{risk}(k, s) &\Rightarrow e^{-c^*} > \frac{1}{yk} \\ &\Rightarrow c^* < \ln(yk) \\ &\Rightarrow \frac{\ln(yk) - c + 1}{k} > \frac{c^* - c + 1}{k} \\ &\Rightarrow \ell(k, s) + c > \frac{c^* - c + 1}{k} + c \\ &\Rightarrow \ell(k, s) + c > \frac{c^* + 1}{k} \\ &\Rightarrow \ell(k, s) + c > c^* \Rightarrow\Leftarrow .\end{aligned}$$

The last implication uses the fact that $k \leq 1 + 1/c^*$. The line before this uses that $k \geq 1$. The contradiction arises because this would imply a scheme (s) with profit $\geq R$ with loss $> c^*$, contradicting the definition of c^* by the reasoning in the first paragraph of this proof. $\square$

Lemma 5.4. *Let c^* denote the maximum c such that there exists a p such that $\text{Prof}_{D_v \times \{0\}}(0, c, p) \geq R$. Then a fine policy is optimal for $D_v \times \{k\}$ if $k \geq 1 + 1/c^*$.*

Proof. Again start from some policy $s = (y, c, p)$, inducing some loss $\ell(k, s)$. First, maybe $h^*(k, s) > 0$. In this case, the risk is $\frac{1}{yk}$ and the loss plus cost is $\frac{\ln(yk) - c + 1}{k} + c$. In particular, observe that the partial derivative of the loss plus cost with respect to c is $1 - 1/k > 0$. So the policy $s' = (y, 0, p - c)$ has $\text{risk}(k, s') = \text{risk}(k, s)$ but also $\ell(k, s') + c' < \ell(k, s) + c$. So Observation 1 claims that this policy gets at least as much profits (and the risk is the same).

If instead, $h^*(k, s) = 0$, then the risk is e^{-c} and the loss is $y \cdot e^{-c}$. In this case, consider instead y^* such that $\frac{\ln(y^* k) + 1}{k} = c^*$ and using $s^* = (y^*, 0, p^*)$, for the p^* satisfying $\text{Prof}_D(s^*) \geq R$ (again, such a p^* must exist by definition of c^*, and the fact that $\ell(k, s^*) = c^*$, plus Observation 1). We just need to analyze the risk. Similar to the previous proof, assume for contradiction that $\text{risk}(k, s^*) > \text{risk}(k, s)$. Then:

$$\begin{aligned}
\text{risk}(k, s^*) > \text{risk}(k, s) &\Rightarrow e^{-c} < \frac{1}{y^* k} \\
&\rightarrow c > \ln(y^* k) \\
&\Rightarrow \frac{\ln(y^* k) + 1}{k} < \frac{c + 1}{k} \\
&\Rightarrow c^* < \frac{c + 1}{k} \\
&\Rightarrow c^* > c^* \cdot \frac{c + 1}{1 + c^*} \Rightarrow\Leftarrow .
\end{aligned}$$

The last inequality uses the fact that $k \geq 1 + 1/c^*$, and derives a contradiction as $c \leq c^*$ (if $c > c^*$, then certainly $\ell(k, s) + c > c^*$, contradicting the definition of c^*). $\square$

All three cases together prove Theorem 5.1. The T prescribed in the theorem statement is exactly $1 + 1/c^*$, where c^* is the maximum c such that there exists a p for which $\text{Prof}_{D_v \times \{0\}}(0, c, p) \geq R$.

We conclude with one last proposition regarding the behavior of the threshold with respect to the profits constraints R. Proposition 5.5 below states that as R increases, the threshold beyond which a fine policy is optimal increases as well.

Proposition 5.5. *Let $T(D_v, R)$ denote the threshold such that both a fine policy and cost policy are optimal for $D_v \times \{T(D_v, R)\}$ subject to profits constraints R. Then $T(D_v, R)$ is monotone increasing in R.*

Proof. To see this, let $c^*(D_v, R)$ denote the maximum c such that there exists a p such that $\text{Prof}_{D_v \times \{0\}}(0, c, p) \geq R$. Then $c^*(D_v, R)$ is *decreasing* in R (as the profits constraint goes up, we can't afford as much security). So $1 + 1/c^*(D_v, R)$ is *increasing* in R. This means that the threshold $T(D_v, R)$ beyond which a fine policy is optimal for $D_v \times \{T\}$ is increasing as a function of the profits constraint R (because $T = 1 + 1/c^*(D_v, R)$). $\square$

This concludes our treatment of the case where k is a point-mass. Theorem 5.1 should both be viewed as a warm-up to introduce some of our core techniques, and also as a building block towards our stronger theorems (in the following sections). The main technique we introduced is the ability to reduce risk and loss simultaneously to improve both profits and externalities. The idea was that if the buyer is less effective than the seller, everyone prefers that the seller put in effort ($y = 0, c > 0$). If the buyer is more effective than the seller, everyone prefers that the buyer put in effort. However, the regulator can not directly mandate that the buyer put in effort, and unfortunately the fines required to extract the desired buyer behavior may too negatively affect the profit. This is why the transition from cost to fine policies is $1 + 1/c^*$ instead of 1.

6 HOMOGENEOUS DISTRIBUTIONS

In this section, we show that for populations that are sufficiently homogeneous in effectiveness, the optimal policy remains simple. The second half of Theorem 6.1 requires a technical assumption. Specifically, we say that a policy (y, c, p) is *profits-maximizing* if, conditioned on y, c, p is set to maximize the seller's profits (that is, $\text{Prof}_D(y, c, p) \geq \text{Prof}_D(y, c, p')$ for all p').

Theorem 6.1. *For all D_v, R, there exists a cutoff T such that*

- *For all D_k supported on $[0, T]$, the externality-minimizing policy for $D_v \times D_k$ subject to profits R is a cost policy.*
- *For all D_k supported on $[T, \infty)$, the externality-minimizing policy for $D_v \times D_k$ subject to profits R is either a fine policy, or it is* not *profits-maximizing.*

The proof of Theorem 6.1 will follow from Lemmas 6.3 and 6.7, which handle the two claims in the theorem separately. Finally, we show in Section 6.3 that the profits-maximizing qualification in part two of Theorem 6.1 is necessary:

Proposition 6.2. *There exist distributions D_v, D_k, and profits constraint R such that:*

- *T is such that for all $k \geq T$, the externality-minimizing policy for $D_v \times \{k\}$ subject to profits constraints R is a fine policy.*
- *D_k is supported on $[T, \infty)$.*
- No *fine policy is externality-minimizing policy for $D_v \times D_k$ subject to profits constraints R.*
- *The externality-minimizing policy for $D_v \times D_k$ subject to profits constraints R is not simple, and not profits-maximizing (the latter is implied by the second bullet of Theorem 6.1).*

Proposition 6.2 is perhaps surprising: a fine policy is externality-minimizing for $D_v \times \{T\}$, and D_k stochastically dominates T, so the same fine policy has *even lower* externalities, and potentially greater profit for $D_v \times D_k$. Indeed, the optimal fine policy for $D_v \times D_k$ achieves lower externalities than that of $D_v \times \{T\}$. The catch is that an *even better* non-simple policy becomes viable, and achieves *still lower* externalities. Theorem 6.1 claims, however, that the optimal non-simple policy must not be profits-maximizing.

6.1 Extension Lemma for small k

The small k case follows roughly from the following intuition. For cost policies, neither the buyer's loss nor her risk depend on k. So whichever cost policy is optimal for $D_v \times \{T\}$ achieves the same profits and externalities as $D_v \times D_k$. Intuitively, going from $\{T\}$ to D_k supported on $[0, T]$ cannot possibly *increase* the profits of any scheme (formally: Observation 2), so the initial cost policy should remain optimal.

Lemma 6.3 (Extension of Cost Policy). *Let s be a cost policy that is optimal for $D_v \times \{T\}$ subject to profits R. Then for all D_k supported on $[0, T]$, s is optimal for $D_v \times D_k$ subject to profits R.*

Proof. First, we observe that $\text{Prof}_{D_v \times \{T\}}(s) = \text{Prof}_{D_v \times D_k}(s)$. This is simply because the loss of consumers is independent of k (as $y = 0$). Similarly, $\text{Ext}_{D_v \times \{T\}}(s) = \text{Ext}_{D_v \times D_k}(s)$. This is again because the risk of consumers is independent of k.

Now, assume for contradiction that there is some policy s' with profits $\text{Prof}_{D_v \times D_k}(s') \geq R$ and also $\text{Ext}_{D_v \times D_k}(s') < \text{Ext}_{D_v \times \{T\}}(s)$. Then we have the following inequality from Observation 2:

$$R \leq \text{Prof}_{D_v \times D_k}(s') \leq \text{Prof}_{D_v \times \{T\}}(s').$$

Therefore, as s is optimal for $D_v \times \{T\}$ subject to profits R, we must have:

$$\text{Ext}_{D_v \times \{T\}}(s') \geq \text{Ext}_{D_v \times \{T\}}(s).$$

This now lets us conclude the following chain of inequalities, where the first line is a corollary of Observation 3: the consumer in a population with D_k supported on $[0, T]$ whose device is least likely to be compromised is a consumer with $k = T$. The third line follows from the reasoning above (that s' achieves profits at least R on $D_v \times \{T\}$, and is therefore feasible). The final line follows because the externalities of a cost policy are independent of k.

$$\begin{aligned}
\text{Ext}_{D_v \times D_k}(s') &\geq \text{risk}(T, s') \\
&= \text{Ext}_{D_v \times \{T\}}(s') \\
&\geq \text{Ext}_{D_v \times \{T\}}(s) \\
&= \text{Ext}_{D_v \times D_k}(s).
\end{aligned}$$

□

Lemma 6.3 proves the first bullet of Theorem 6.1.

6.2 Extension Lemma for large k

In this section, we sketch the proof for the large k case of Theorem 6.1. Refer to Section 6 of [7] for omitted proofs. The proof will be a little more involved this time, since we can no longer claim that the externalities of a fine policy are independent of k (whereas this does hold for cost policies). The intuition for this case is the same though: if a fine policy is optimal for $D_v \times \{k\}$ for all $k \geq T$, and D_k is supported on $[T, \infty)$, fine policies should remain optimal for $D_v \times D_k$. Most of the proof does not make use of the technical assumption that the s we are competing with is a profits-maximizing policy: this assumption only arises at the very end.

The first step in our proof is the following concept, which captures the change in loss for a consumer (v, k) for regulation s versus s':

Definition 6.4 (Policy Comparison Function). For two policies s and s', we define the *policy comparison function* $g_{s,s'}(\cdot)$ so that $g_{s,s'}(k) = \ell(k, s) - \ell(k, s')$.

The policy comparison function takes as input an effectiveness k, and outputs the change in loss for a consumer under one policy versus another. Our first lemma argues that for certain pairs (s, s'), the policy comparison function is monotone in k. That is, consumers with more effectives have greater preference for one policy over another.

Lemma 6.5. *Let $s = (y, c, p)$ and $s' = (y', c', p')$ be such that $ye^{-c} \leq y'e^{-c'}$. Then $g_{s,s'}(\cdot)$ is monotone non-decreasing. Observe that the hypothesis holds if $y \leq y'$ and $c \geq c'$.*

We use Lemma 6.5 to claim the following corollary, which essentially states that if a policy change universally lowers loss *and* risk, then it is possible to adjust the price so that the profits go up and externalities go down.

Corollary 6.6. *Let $(y, c), (y', c')$ be such that (a) $ye^{-c} \leq y'e^{-c'}$ and (b) for all k in the support of D_k, $\ell(k, y, c) + c \geq \ell(k, y', c') + c'$ and $\text{risk}(k, y, c) \geq \text{risk}(k, y', c')$. Then for all p and all D_v, there exists a p' such that:*

$$\textit{Prof}_{D_v \times D_k}(y', c', p') \geq \textit{Prof}_{D_v \times D_k}(y, c, p),$$

$$\textit{Ext}_{D_v \times D_k}(y', c', p') \leq \textit{Ext}_{D_v \times D_k}(y, c, p).$$

Now we are ready to formally state the extension lemma for large k.

Lemma 6.7 (Extension of Fine Policy). *Let D_k be supported on $[T, \infty)$, where T is such that a fine policy is optimal for $D_v \times \{T\}$ subject to profits R. Then there is a fine policy s' with $\textit{Prof}_D(s') \geq R$ such that for all profits-maximizing s with $\textit{Prof}_D(s) \geq R$, $\textit{Ext}_D(s') \leq \textit{Ext}_D(s)$.*

Proof Sketch. Consider any proposed optimal policy $s = (y, c, p)$. We first consider the case where $\ell(T, s) + c \geq \ell(T, s^*)$ where s^* is the optimal fine policy on $D_v \times \{T\}$. If that is the case, then consider a fine policy $s' = (y', 0, p - c)$ where $\ell(T, s') = \ell(T, s) + c$. Observe we must have $y' \geq y$ since we can only obtain equality by increasing fines, then by Lemma 6.5, we have $\ell(k, s') \leq \ell(k, s) + c$ for all k in the support of D_k and the profit under s' can only be higher.

Now, we need to show that the risk is only lower for all $k \geq T$. First, as $\ell(T, s') \geq \ell(T, s^*)$ we conclude that $y' \geq y^*$. Next, we can argue (see [7] for the full proof) that we must have $\frac{1}{y^* T} \leq e^{-c}$. This allows us to conclude that $\frac{1}{y' T} \leq e^{-c}$, and we already have that $y' \geq y$ so $\frac{1}{y' k} \leq \frac{1}{yk}$. This gives us that $\text{risk}(k, s') \leq \text{risk}(k, s)$ for all $k \geq T$. Then Corollary 6.6 let's us claim that there exists a price p' for which both the profits and the externalities are better for $(y', 0, p')$ than s.

For the case where $\ell(T, s) + c < \ell(T, s^*)$, observe that $\ell(D_k, s) + c$ is strictly stochastic dominated by $\ell(D_k, s^*)$; therefore, there is a price $\hat{p}$ where the profit strictly higher than R. Lemma 6.9 bellow, allows us conclude that if s is optimal, then $\text{Prof}_{D_v \times D_k}(s) = R$; otherwise, we can compromise $\varepsilon > 0$ fraction of the profit to strictly improve externalities. This implies that if the policy s we start with is optimal, then it is not profits-maximizing. □

Definition 6.8 (Invariant Transformation). Given a policy $s = (y, c, p)$, define

$$\text{Inv}(s, \alpha) := \left(ye^{(p-c)(1-\alpha)}, \alpha c + (1 - \alpha)p, p\right)$$

where $\alpha \in [0, \frac{p}{p-c}]$.

Lemma 6.9 (Invariant Property). *Let $s' = \textsc{Inv}(s, \alpha)$, then for all $k \in \mathbb{R}^+$*

- $h^*(k, s') = h^*(k, s)$.
- $\ell(k, s') = \ell(k, s)$.
- $u(t, s) = u(t, s')$.

In addition,

$$\textsc{Prof}_D(s') = \alpha \textsc{Prof}_D(s)$$
$$\textsc{Ext}_D(s') = e^{-(1-\alpha)(p-c)} \textsc{Ext}_D(s, p)$$

Proof Sketch. By applying the definition of buyer's efficiency, we can see that for all k, $h^*(k, s') = h^*(k, s)$ which implies $\ell(k, s') = \ell(k, s)$, and $\textsc{risk}(k, s') = e^{-(1-\alpha)(p-c)} \textsc{risk}(k, s)$. □

This concludes the proof of bullet two of Theorem 6.1.

6.3 Example: The Profits-Maximizing Qualification is Necessary

In this section we provide the example promised in Proposition 6.2. Refer to Section 6 of [7] for omitted proofs. Consider the following distribution, and profits constraint $R := 0.5$:

$$D_v = \begin{cases} v_1 = 1 & \text{w. p. } \frac{1}{2} \\ v_2 = 16/15 & \text{w. p. } \frac{1}{2} \end{cases}$$

$$D_k = \begin{cases} k_1 = 3 & \text{w. p. } \frac{1}{2} \\ k_2 = x \to \infty & \text{w. p. } \frac{1}{2} \end{cases}$$

Above, x will be finite, but approaching ∞, and ε will be finite but approaching 0). The proposition will follow from the following sequence of claims. First, we will establish bullet one for $T := 3$.

Claim 4. *A fine policy is optimal for $D_v \times \{3\}$.*

Bullet two now immediately follows, as D_k is indeed supported on $[3, \infty)$. We now just need to find the optimal fine policy for $D_v \times D_k$, and establish a better policy that is not simple. We now search for the optimal fine policy. Such a policy might sell only to $(16/15, x)$, but then the profits is at most $4/5$, which is too little. Such a policy might sell only to $(16/15, x)$ and $(1, x)$. But since x is finite, such a policy certainly charges price < 1 (unless $y = 0$, in which case the policy sells to all four types), and sells with probability $\leq 1/2$, so the profits are also too small. Such a policy might sell to all four types, which we analyze below. Or it might sell to all types except $(1, 3)$, which we analyze after.

Claim 5. *The optimal fine policy s which sells to all four types has $\textsc{Ext}_{D_v \times D_k}(s) \geq \frac{1}{2\sqrt{e}}$.*

Claim 6. *The optimal fine policy s which sells to all types except $(1, 3)$ has $\textsc{Ext}_{D_v \times D_k}(s) \geq e^{-1/5}/3$.*

Corollary 6.10. *The optimal fine policy s has $\textsc{Ext}_{D_v \times D_k}(s) \geq e^{-1/5}/3$.*

Here's now some intuition for how we're going to design a better non-simple policy: given that we wish to sell to all types except $(1, 3)$, we can set y very close to 0 and have $\textsc{risk}(x, s) \approx 0$, because x is so large. The remaining question is then whether we wish to use y or c to make the risk of $(16/15, 3)$ as small as possible. Note that we must keep their loss under $2/5 < 1/2$ (as above). But for $k = 3$, a loss of $1/2$ is *exactly* the cutoff when it becomes more efficient to use a fine policy instead of a cost policy. So if we use c instead, we can get the risk lower for the same loss.

Claim 7. *Let ε be such that $\frac{\ln(x)+1}{x} \leq \varepsilon$. Then set $c = 1/3 - \varepsilon$, and $y = (2/5 - c)e^c$. Then $\textsc{Ext}_{D_v \times D_k}(y, c, 2/3 + c) = \frac{2}{3yx} + e^{-1/3+\varepsilon}/3$ and $\textsc{Prof}_{D_v \times D_k}(y, c, 2/3 + c) = 1/2$.*

Now, we just need to compare $e^{-1/5}/3$ and $e^{-1/3+\varepsilon}/3 + \frac{2}{3yx}$. Observe that as $x \to \infty$, $\varepsilon \to 0$ and $e^{-1/3+\varepsilon}/3$ approaches $e^{-1/3}/3$. So $\frac{2}{3yx} + e^{-1/3+\varepsilon}/3 \to 0 + e^{-1/3}/3 < e^{-1/5}/3$, and the externalities are indeed lower.

As a sanity check, we'll show that $((2/5-c)e^{1/3-\varepsilon}, 1/3-\varepsilon, 2/3+c)$ is not profits-maximizing (technically, Theorem 6.1 doesn't imply this, since we didn't prove that the scheme is optimal. But as this scheme is better than all fine policies, certainly the optimal policy is not simple, and therefore not profits-maximizing by Theorem 6.1. So the fourth bullet is already proven).

Claim 8. *$((2/5 - c)e^{1/3-\varepsilon}, 1/3 - \varepsilon, 2/3 + c)$ is not profits-maximizing.*

7 GENERAL DISTRIBUTIONS: AN APPROXIMATION

In this section, we consider general distributions. Clearly, one should not expect a simple policy to be optimal in general. Given that simple policies are optimal for homogeneous populations, one might reasonably expect that simple policies are approximately optimal for general distributions by simply ignoring half of the population and targeting the half that is responsible for most of the externalities. This idea works in one direction: if the "low k" region is responsible for most of the externalities in the optimum solution, then using a cost policy for the entire distribution is a good idea: the high k consumers may have significantly higher risk than previously, but this doesn't outweigh the original risk from the low k region.

This idea fails horribly, however, if the "high k" region is responsible for most of the externalities in the optimum solution. The problem is that while we can choose a policy to exclusively target this subpopulation, any low k (think: $k = 0$) consumers who choose to purchase anyway may have enormous risk in comparison to before (i.e. it could now be 1 when it was previously e^{-c} for large c). We first show that this intuition can indeed manifest in a concrete example by presenting a lower bound in Section 7.1. This rules out a *single-criterion* approximation that satisfies the profits constraint exactly, and approximates the externalities. In Section 7, we present a bicriterion approximation which approximately satisfies the profits constraint and also approximately minimizes externalities. This approximation is our most technical result. As such, we provide mainly proof sketches to overview the key steps.

7.1 Lower Bound on Heterogeneous Distributions

The key insight for our example is to make the profits constraint so binding that the only way to match it exactly is for the entire population to purchase the item. Part of the population will have $k = 0$, and part will have $k \to \infty$. With both c and y, it will be feasible to get the $k \to \infty$ consumers to have risk essentially 0, while the $k = 0$ consumers will have reasonably small risk. But with either $c = 0$ or $y = 0$, one of these will be lost, which causes significant risk increase.

Example 7.1. Let D_v be a point mass at $v_0 = 2e^{x/2} \cdot (x + e^{-x})$. Let D_k be a distribution with two point masses, one at $k = 0$ with probability $e^{-x/2}$, one at $e^{xe^{x/2}}$ with probability $1 - e^{-x/2}$. Let $R := v_0 - e^{-x} - x$.

Lemma 7.2. *The policy $(1, x, R+x)$ achieves profit R in Example 7.1, and has externalities $\leq e^{-x/2} \cdot e^{-x} + 1 \cdot e^{-xe^{x/2}}$.*

Proof. The utility of $(v_0, 0)$ is exactly $v_0 - e^{-x} - R - x = 0$, so they will choose to purchase. $(v_0, e^{xe^{x/2}})$ has only larger utility, so they will purchase as well. Therefore, the profit is indeed R.

The externalities are computed simply as the probability of having consumer $(v_0, 0)$ times their risk (e^{-x}) plus (upper bound on the) probability of consumer $(v_0, e^{xe^{x/2}})$ times their risk $(e^{-xe^{x/2}})$. ◻

Lemma 7.3. *Any cost policy that achieves profit R has externalities at least e^{-x+1}*

Proof. The maximum security we can set and still have profit R is $x + e^{-x}$. If we set this, then the risk of all consumers (which is now independent of k) is $e^{-x+e^{-x}} \geq e^{-x+1}$. ◻

Lemma 7.4. *Any fine policy that achieves profit R has externalities at least $e^{-x/2}$.*

Proof. To achieve profit R, the policy must sell to the entire population. The consumer with $k = 0$ will not put in any effort, and therefore their risk will be one, and the externalities will be at least $e^{-x/2}$. ◻

Corollary 7.5. *For all x, there exists a distribution $D_v \times D_k$ and profits constraint R such that the optimal policy is not simple, and any simple policy that satisfies profits constraints R has externalities at least a factor of x larger than the optimum.*

Corollary 7.5 is the main result of this section. Clearly the distribution witnessing Corollary 7.5 is highly contrived and unrealistic. And clearly, the way to get around this is to allow for a slight relaxation in the profits constraint so that we don't have to sell to the entire market (indeed, even allowing to relax the constraint by a $(1 - e^{-x/2})$ fraction in this case would suffice). So the subsequent section shows that by relaxing the profits constraint, an approximation guarantee is possible.

7.2 A Bicriterion approximation

Given the lower bound in Section 7.1, we show that simple policies guarantee a bicriterion approximation. As is traditional with worst-case approximation guarantees, our constants are not particularly close to 1, but are still relatively small. This is not meant to imply that the seller should be happy with (e.g.) a 1/8-fraction of the original profits, but more qualitatively to conclude that simple policies can reap many of the benefits of optimal ones (see [16] for further discussion about the role of approximation in mechanism design). As referenced previously, the proof of Theorem 7.6 is quite technical, so we sketch the key steps. The complete proof can be found on [7].

Theorem 7.6. *For all distributions D, and all policies s, there exists a simple policy s' such that*

$$\textit{Prof}_D(s') \geq \textit{Prof}_D(s)/8,$$

$$\textit{Ext}_D(s') \leq 40/3 \cdot \textit{Ext}_D(s).$$

Proof Sketch. Given an arbitrary policy $s = (y, c, p)$, consider the conditional distribution of buyers that purchase under s. If with constant probability a buyer has efficiency $k \leq 1$, then we output the cost policy $s_1 := (0, c + \ell(\sigma, s), p + \ell(\sigma, s))$ where σ is chosen such that a buyer continues to purchase with constant probability. We can show that $c + \ell(\sigma, s)$ is sufficiently large such that $\textsc{risk}(D_k, s') \leq \textsc{risk}(D_k, s)$ with constant probability.

For the case where with constant probability a buyer has efficiency $k > 1$, we define a blowup of the fines such that with constant probability a buyer continues to purchase but with the hope that inefficient buyers stop to purchase. The blowup can fail in two conditions: (1) D_k is not heavy tail, (2) D_v is heavy tail. For (1), we cannot derive a significant blowup if D_k is concentrated close to 1. For (2), we cannot drive inefficient buyers out of the market if they have high value. Either condition allow us to construct cost policies that give good externality guarantees. ◻

8 SUMMARY

We propose a stylized model to study regulation of single item sales with negative externalities, from which neither the buyer nor seller suffer. We first show that a simple policy is optimal in homogenous markets: That is, for all D_v, R, there exists a cutoff T such that when the effectiveness of consumers ranges in $[0, T]$, the optimal policy regulates only the product (and does not impose fines). Similarly, if all consumers have effectiveness in $[T, \infty)$, a policy which regulates only payments (via fines, and does not impose default security features) outperforms all profits-maximizing policies. Importantly, T is not necessarily the cutoff at which the consumers are more effective than the producer (which would be $T = 1$), but actually depends on the value distribution D_v and profit constraint R.

We then show in general markets that while a simple policy may not be optimal, one is always approximately optimal. In particular, we show that while no simple scheme can guarantee any finite approximation while satisfying the profit constraint exactly, a bicriterion approximation exist, which approximately satisfies the profit constraint and also approximately minimizes externalities. Going forward, we must better understand the effectiveness of consumers to decide which regulation strategy is more appropriate to approximately minimizes externalities.

While stylized, our model captures the key salient features of this problem. We chose to study the single seller/single item setting in order to isolate these features without bringing in additional complexities (and the numerous examples throughout our paper demonstrate that even the single seller/single item setting is quite rich). Now that our results develop this understanding, a good direction for future work is to consider competing sellers or multiple items.

REFERENCES

[1] George A Akerlof. 1978. The market for "lemons": Quality uncertainty and the market mechanism. In *Uncertainty in Economics*. Elsevier, 235–251.

[2] Terrence August, Duy Dao, and Kihoon Kim. 2016. Market Segmentation and Software Security: Pricing Patching Rights. *Workshop on the Economics of Information Security (WEIS)* (2016).

[3] William J. Baumol. 1972. On Taxation and the Control of Externalities. *The American Economic Review* 62, 3 (1972), 307–322. http://www.jstor.org/stable/1803378

[4] Sayan Bhattacharya, Janardhan Kulkarni, Kamesh Munagala, and Xiaoming Xu. 2011. On allocations with negative externalities. In *International Workshop on Internet and Network Economics*. Springer, 25–36.

[5] Brian Krebs. 2017. IoT Reality: Smart Devices, Dumb Defaults. *Krebs on Security Blog, https://krebsonsecurity.com/2016/02/iot-reality-smart-devices-dumb-defaults/* (2017).

[6] Ozan Candogan, Kostas Bimpikis, and Asuman Ozdaglar. 2012. Optimal pricing in networks with externalities. *Operations Research* 60, 4 (2012), 883–905.

[7] Tithi Chattopadhyay, Nick Feamster, Matheus V. X. Ferreira, Danny Yuxing Huang, and S. Matthew Weinberg. 2019. Selling a Single Item with Negative Externalities: To Regulate Production or Payments? *CoRR* (2019).

[8] Shuchi Chawla, Jason D. Hartline, and Robert D. Kleinberg. 2007. Algorithmic Pricing via Virtual Valuations. In *the 8th ACM Conference on Electronic Commerce (EC)*.

[9] Jay Pil Choi, Chaim Fershtman, and Neil Gandal. 2010. Network security: Vulnerabilities and disclosure policy. *The Journal of Industrial Economics* 58, 4 (2010), 868–894.

[10] Michal Feldman, David Kempe, Brendan Lucier, and Renato Paes Leme. 2013. Pricing public goods for private sale. In *Proceedings of the fourteenth ACM conference on Electronic commerce*. ACM, 417–434.

[11] Don Fullerton. 1997. Environmental Levies and Distortionary Taxation: Comment. *The American Economic Review* 87, 1 (1997), 245–251. http://www.jstor.org/stable/2950868

[12] Nima Haghpanah, Nicole Immorlica, Vahab Mirrokni, and Kamesh Munagala. 2013. Optimal auctions with positive network externalities. *ACM Transactions on Economics and Computation* 1, 2 (2013), 13.

[13] Garrett Hardin. 1968. The tragedy of the commons. *science* 162, 3859 (1968), 1243–1248.

[14] Sergiu Hart and Noam Nisan. 2017. Approximate revenue maximization with multiple items. *J. Economic Theory* 172 (2017), 313–347. https://doi.org/10.1016/j.jet.2017.09.001 Preliminary version in EC 2012.

[15] Jason Hartline, Vahab Mirrokni, and Mukund Sundararajan. 2008. Optimal marketing strategies over social networks. In *Proceedings of the 17th international conference on World Wide Web*. ACM, 189–198.

[16] Jason D Hartline. 2013. Mechanism design and approximation. *Book draft. October* 122 (2013).

[17] Philippe Jehiel, Benny Moldovanu, and Ennio Stacchetti. 1996. How (not) to sell nuclear weapons. *The American Economic Review* (1996), 814–829.

[18] Howard Kunreuther and Geoffrey Heal. 2003. Interdependent security. *Journal of risk and uncertainty* 26, 2-3 (2003), 231–249.

[19] William Lehr and Jon Crowcroft. 2005. Managing shared access to a spectrum commons. In *New Frontiers in Dynamic Spectrum Access Networks, 2005. DySPAN 2005. 2005 First IEEE International Symposium on*. IEEE, 420–444.

[20] Frank Li, Zakir Durumeric, Jakub Czyz, Mohammad Karami, Michael Bailey, Damon McCoy, Stefan Savage, and Vern Paxson. 2016. You've Got Vulnerability: Exploring Effective Vulnerability Notifications.. In *USENIX Security Symposium*. 1033–1050.

[21] Frank Li and Vern Paxson. 2017. A large-scale empirical study of security patches. In *Proceedings of the 2017 ACM SIGSAC Conference on Computer and Communications Security*. ACM, 2201–2215.

[22] David Martimort and Wilfried Sand-Zantman. 2016. A mechanism design approach to climate-change agreements. *Journal of the European Economic Association* 14, 3 (2016), 669–718.

[23] Vahab S Mirrokni, Sebastien Roch, and Mukund Sundararajan. 2012. On fixed-price marketing for goods with positive network externalities. In *International workshop on internet and network economics*. Springer, 532–538.

[24] Juan-Pablo Montero. 2008. A simple auction mechanism for the optimal allocation of the commons. *American Economic Review* 98, 1 (2008), 496–518.

[25] Roger B Myerson. 1981. Optimal auction design. *Mathematics of operations research* 6, 1 (1981), 58–73.

[26] Nicky Woolf. 2016. DDoS attack that disrupted internet was largest of its kind in history, experts say. *The Guardian, https://www.theguardian.com/technology/2016/oct/26/ddos-attack-dyn-mirai-botnet* (2016).

[27] Elissa M. Redmiles, Michelle L. Mazurek, and John P. Dickerson. 2018. Dancing Pigs or Externalities? Measuring the Rationality of Security Decisions. *To appear in ACM Conference on Economics and Computation* (2018).

[28] Paul Seabright. 1993. Managing local commons: theoretical issues in incentive design. *Journal of economic perspectives* 7, 4 (1993), 113–134.

[29] Martin L Weitzman. 1974. Prices vs. quantities. *The review of economic studies* 41, 4 (1974), 477–491.