Ontology Adaptation upon Updates

Alessandro Solimando, Giovanna Guerrini

Dipartimento di Informatica, Bioingegneria, Robotica e Ingegneria dei Sistemi Università di Genova, Italy name.surname@unige.it

Abstract. Ontologies, like any other model, change over time due to modifications in the modeled domain, deeper understanding of the domain by the modeler, error corrections, simple refactoring or shift of modeling granularity level. Local changes usually impact the remainder of the ontology as well as any other data and metadata defined over it. The massive size of ontologies and their possible fast update rate requires automatic adaptation methods for relieving ontology engineers from a manual intervention, in order to allow them to focus mainly on high-level inspection. This paper, in spirit of the *Principle of minimal change*, proposes a fully automatic ontology adaptation approach that reacts to ontology updates and computes sound reformulations of ontological axioms triggered by the presence of certain preconditions. The rule-based adaptation algorithm covers up to \mathcal{SROIQ} DL.

1 Introduction and Motivations

Ontologies, like any other model, change over time and a revalidation of all data and metadata defined on top of the modified ontology is needed upon updates. Massive ontology size and fast update rate¹ call for automated support and adaptation algorithms. Despite the great attention devoted in the last ten years to ontology evolution [1, 7], to the best of our knowledge there are no proposals in the literature coping with ontology adaptation upon updates. With similar motivations, an adaptation algorithm for a subset of SPARQL queries (with expressivity equivalent to union of Conjunctive Queries) in response to ontology updates is proposed in [6]. Protégé², one of the most complete ontology frameworks, does not support any kind of adaptation w.r.t. ontology updates: when a concept or a role is deleted, all the axioms referring it are removed as well. Even if there are cases in which this behavior is acceptable (e.q., error corrections), there are others for which it is detrimental, for instance a modification of the modeling granularity of the ontology. In this scenario, a sound reformulation of axioms by means of super/sub concepts or roles is not only desirable but usually manually performed by the modeler. Additionally, in Artificial Intelligence (Belief Revision), knowledge deletion usually follows the Principle of Minimal

¹ An example is the *Gene Ontology* (http://www.geneontology.org/), with $\sim 416K$ axioms and $\sim 40K$ entities, daily updated (statistics for data-version 2013-02-22).

² Available here: http://protege.stanford.edu/

Change [5], which suggests that the amount of lost information should be as minimal as possible. Given that ontologies do not necessarily (explicitly) include all their logical consequences, also the implicit knowledge should be taken into account, as well as explicit one (that is, ontology axioms).

While a set of basic ontology changes can be easily defined, it is impossible to identify a set of complex changes without fixing the granularity level, *i.e.*, updates expressed as arbitrarily complex graph patterns (see [10], Section 3.2.1). In this proposal we consider the basic updates proposed by [3]: addition, deletion and update of entities (concepts and roles). Given that adding or updating entities do not reduce knowledge, and that ontology consistency can be tested using ontology reasoners, our adaptation algorithm focuses only on entity deletions.

In this paper, we propose an algorithm that, given an ontology and an entity (concept or role) to delete, scans for an equivalent, a super and a sub-entity and tries to reformulate the axioms involving the entity in question, with a rule-based approach. Our reformulated axioms are a fraction of the *implicit knowledge* of the ontology under update that would be lost by deleting all of the axioms involving the removed entity. An alternative would be to compute the closure (that is, complete inference of implicit knowledge) for the ontology prior to entity deletion. Due to its high computational cost and possible non-finiteness of the result, a suboptimal but less expensive approach is preferable for our target scenario, that is interactive modeling.

Even if the adaptation algorithm is completely automatic, it may not always be aligned with the modeler's intention. For this reason, the present proposal has to be intended as an optional feature. When activated, it provides a preview of the changes to show the automatic adaptation effects. On this basis, the modeler can accept or ignore the proposed changes. In addition, a straightforward extension could be the possibility, for the modeler, to select the equivalent (resp. sub/super) entity for the reformulation, when different alternatives are available.

The contribution of the present paper can be summarized as follows: an automatic adaptation algorithm supporting up to \mathcal{SROIQ} expressivity, its correctness proof, and temporal complexity analysis (Section 3), an experimental evaluation of the percentage of adaptable entities and axioms on a dataset of real ontologies (Section 4). First, DL basics are introduced (Section 2), and the paper concludes discussing future work (Section 5).

2 Preliminaries

Our proposal covers up to \mathcal{SROIQ} Description Logic (DL), on top of which the Ontology Web Language (OWL2) [11] is defined. The notations and definitions used in this section are borrowed from [4]. An ontology is defined by a set of axioms and a set of entity names (signature), composed by three disjoint subsets: $N_{\mathcal{R}}$ for role names, $N_{\mathcal{I}}$ for individual names, $N_{\mathcal{C}}$ for concept names. These entities are defined by means of expressions. We have Role expressions $\mathbf{R} ::= U \mid N_{\mathcal{R}} \mid N_{\mathcal{R}}^-$, and Concept expressions $\mathbf{C} ::= N_{\mathcal{C}} \mid (\mathbf{C} \sqcup \mathbf{C}) \mid (\mathbf{C} \sqcap \mathbf{C}) \mid \neg \mathbf{C} \mid \mathsf{T} \mid \bot \mid \exists \mathbf{R}.\mathbf{C} \mid \forall \mathbf{R}.\mathbf{C} \mid \geq_n \mathbf{R}.\mathbf{C} \mid \leq_n \mathbf{R}.\mathbf{C} \mid \exists \mathbf{R}.Self \mid \{N_{\mathcal{I}}\}$, with $n \geq 0$. For the

	Precondition	\mathbf{Rule}
a.1	$C \equiv C'$,	axiom o axiom[C/C']
	$C \in signature(axiom)$	
a.2	$C \sqsubseteq D$	$E \equiv \exists R.C \to E \sqsubseteq \exists R.D$
a.3	$C \sqsubseteq D$	$E \equiv \geq_n R.C \to E \sqsubseteq \geq_n R.D$
a.4	$C \sqsubseteq D$	$E \equiv C \sqcup F \to E \sqsubseteq D \sqcup F$
a.5	$C \sqsubseteq D$	$E \equiv C \sqcap F \to E \sqsubseteq D \sqcap F$
a.6	$C \sqsubseteq D$	$E \equiv \neg C \to \neg D \sqsubseteq E$
a.7	$C \sqsubseteq D$	$C(a) \to D(a)$
a.8	$C \sqsubseteq D$	$E \equiv \forall R.C \rightarrow E \sqsubseteq \forall R.D$
a.9	$B \sqsubseteq C$	$E \equiv \leq_n R.C \to E \sqsubseteq \leq_n R.B$
a.10	$B \sqsubseteq C$	$E \equiv C \sqcup F \to B \sqcup F \sqsubseteq E$
a.11	$B \sqsubseteq C$	$E \equiv C \sqcap F \to B \sqcap F \sqsubseteq E$
a.12	$B \sqsubseteq C$	$E \equiv \neg C \to E \sqsubseteq \neg B$
a.13	$B \sqsubseteq C$	$C \sqsubseteq E \to B \sqsubseteq E$

Table 1. Adaptation rules for concept deletion DEL(C), where $B, C, C', D, E, F \in N_{\mathcal{C}}$, $R \in N_{\mathcal{R}}$ and $a \in N_{\mathcal{I}}$.

semantics associated with nominals, role and concept expressions the reader may refer to [4]. The set of axioms of an ontology, denoted with Axioms, is defined as $Axiom := ABox \cup RBox \cup TBox$. The reader may refer to [4] also for a detailed description of the different available axioms for SROIQ DL, and to [9] for the definitions of ontology interpretation and ontology satisfiability. W.l.o.g. in the paper we will consider normalized ontologies in Negation Normal Form (NNF), with an application of Structural Reduction (SR), as shown in [9] (Subsection 5.3). SR introduces fresh concept names for (complex) concept expressions, thus letting us to easily refer to each concept expression by means of its associated concept name. Neither the SR nor the NNF are required for the application of our method. NNF, however, may increase the ratio of adapted axioms.

3 Algorithm

This section introduces the adaptation rules (Section 3.1), the rule-based adaptation algorithm (Section 3.2), the correctness proof for the given rules (Section 3.3), and the temporal complexity of the algorithm (Section 3.4).

3.1 Adaptation Rules

The adaptation rules are presented in Table 1 (rules for concepts) and Table 2 (rules for roles). We denote by axiom[A/B] the alpha renaming of an axiom of entity A by entity B. A rule r is composed by a left hand side, LHS(r), a right hand side, RHS(r), and a precondition prec(r). A rule is defined **applicable** iff prec(r) is satisfied by at least one concept (resp. role). Given an ontology o and an entity o to delete, the o a rule o is said to be **matching** iff an axiom in o exists that is equal to o o modulo alpha renaming of o (resp. o)

	Precondition	Rule
b.1	$R \equiv R',$	$axiom \rightarrow axiom[R/R']$
	$R \in signature(axiom)$	
b.2	$Q \sqsubseteq R$	$T_0 \circ \ldots \circ T_m \circ R \circ T'_0 \circ \ldots \circ T'_p \sqsubseteq T$
		$\rightarrow T_0 \circ \ldots \circ T_m \circ Q \circ T'_0 \circ \ldots \circ T'_p \sqsubseteq T$
b.3	$Q \sqsubseteq R$	$E \equiv \forall R.C \to E \sqsubseteq \forall Q.C$
b.4	$Q \sqsubseteq R$	$E \equiv \leq_n R.C \to E \sqsubseteq \leq_n Q.C$
b.5	$Q \sqsubseteq R$	$T \equiv R^- \to Q^- \sqsubseteq T$
b.6	$Q \sqsubseteq R$	$Disjoint(R,T) \rightarrow Disjoint(Q,T)$
b.7	$R \sqsubseteq S$	$R(a,b) \to S(a,b)$
b.8	$R \sqsubseteq S$	$E \equiv \exists R.C \to E \sqsubseteq \exists S.C$
b.9	$R \sqsubseteq S$	$E \equiv \exists R.Self \rightarrow E \sqsubseteq \exists S.Self$
b.10	$R \sqsubseteq S$	$E \equiv \geq_n R.C \to E \sqsubseteq \geq_n S.C$
b.11	$R \sqsubseteq S$	$T \equiv R^- \to T \sqsubseteq S^-$
b.12	$R \sqsubseteq S$	$T_0 \circ \ldots \circ T_q \sqsubseteq R \to T_0 \circ \ldots \circ T_q \sqsubseteq S$

Table 2. Adaptation rules for role deletion DEL(R), where $E, C \in N_C$, $Q, R, R', S, T, T_i, T'_j \in N_R$, with $m, n, p, q \ge 0$, and $a, b \in N_T$.

with e, denoted with LHS(r)[e]. The **application** of an applicable rule r w.r.t. o and e rewrites any axiom of o matching LHS(r)[e] into RHS(r)[e'], where e' is the selected entity for reformulation. It is worth noting that if a DL less expressive than \mathcal{SROIQ} is adapted, only a subset of the rules will be applicable, depending on the axioms and constructors available. For instance, for basic \mathcal{ALC} with $General\ Concept\ Inclusion\ (i.e., <math>C \sqsubseteq D$), rules a.3, a.9, b.2, b.4, b.5, b.9, b.10, b.11, b.12 are not applicable.

3.2 Adaptation Algorithm

Algorithm 1 presents the adaptation algorithm for ontology updates. It takes as input the entity e to be deleted and the ontology o it belongs to. By means of function computePrec, the set of axioms related to e is computed, as well as a triple p consisting of a (nondeterministically choosen) equivalent, a sub and a super entity, if any (line 3). For each axiom a having e in its signature (line 4), it tests if the axiom matches the left hand side of the rule (line 5). At this point, function satisfies (line 6) checks if the current axiom is compatible with rule r and if the required element in p is not null. The reformulated axiom is inserted in o (line 7). Finally, all the axioms involving entity e are removed from o (line 8). Even if a preliminar classification phase is not required, it may increase the algorithm effectiveness. In what follows we give a toy example of ontology update, comparing the result of adaptation to classical deletion approach.

Example 1. Consider an ontology o consisting of these axioms and the obvious associated signature: $Human \equiv \exists eats.Food$, Food(cheese), $Eater \equiv \forall eats.Food$, $\bot \equiv Plastic \sqcap Food$, $Uneatable \equiv \neg Eatable$, $Pizza \sqsubseteq Food$, $Food \sqsubseteq Eatable$. Deleting Food concept from o with adaptation we obtain: $Human \sqsubseteq \exists eats.Eatable$, Eatable(cheese), $Eater \sqsubseteq \forall eats.Eatable$, $Plastic \sqcap$

Algorithm 1 Ontology Update Adaptation

```
1: function OntoUpdateAdapt(Entity e, Ontology o) 2: axioms = \emptyset
3:
4:
5:
6:
7:
8:
9:
          p := \langle eq, sub, sup \rangle \leftarrow computePrec(e, axioms, o)
          for a \in axioms do
               for r \in Rules . a = LHS(r)[e] do
                    \textbf{if} \ \ satisfies(\langle a,e,e'\rangle, \overrightarrow{prec}(r)), \ e' \in \{eq,sub,sup\} \ \ \textbf{then}
                         Axioms(o) \leftarrow Axioms(o) \cup \{RHS(r)[e']\}
                    end if
               end for
10:
           end for
11:
           Axioms(o) \leftarrow Axioms(o) \setminus axioms
12: end function
13: function ComputePrec(Entity e, Set axioms, Ontology o)
14:
           eq, sub, sup \leftarrow \epsilon
           for a \in Axioms(o) . e \in signature(a) do
15:
16:
                axioms \leftarrow axioms \cup \{a\}
17:
                if eq, sub, sup \neq \epsilon then
18:
19:
20:
21:
                    break
                end if
               if a = e \equiv e' or a = e' \equiv e then
22:
23:
24:
25:
26:
27:
28:
                else if a = e \sqsubseteq e' then
                else if a = e \supseteq e' then
                    sub \leftarrow e'
                end if
           end for
           return \langle eq, sub, sup \rangle
\widetilde{29}: end function
```

 $Pizza \sqsubseteq \bot$, $Pizza \sqsubseteq Eatable$, $Uneatable \equiv \neg Eatable$ (using rule a.2, a.7, a.8, a.11 and a.13, respectively). Without adaptation, instead, only the last axiom would be present in o after concept deletion.

3.3 Rules Correctness Proof

Before stating the proposition about the correctness of the adaptation rules we introduce some definitions and lemmata. For sake of brevity we will interchangeably refer to the axioms and their semantics, according to [4].

Definition 1. An axiom A_1 entails an axiom A_2 iff, for any interpretation I, $I \models A_2 \implies I \models A_1$, that is $A_2^I \subseteq A_1^I$.

Definition 2. An adaptation rule r is **sound** iff $\{LHS(r), prec(r)\}$ entails RHS(r).

Lemma 1. $\forall C, D, F \in N_{\mathcal{C}}$. $C \sqsubseteq D \implies C \sqcup F \sqsubseteq D \sqcup F$.

Proof. By considering the associated semantics the Lemma can be restated as $C^I \subseteq D^I \implies \underbrace{C^I \cup F^I}_{\alpha} \subseteq \underbrace{D^I \cup F^I}_{\beta}$. Assume that the preceding formula does

not hold, that is $\alpha \not\subseteq \beta$. This means $\exists x \in \beta$. $x \not\in \alpha$, and requires that at least one of the following conditions holds:

- $-x \in F^I$, but this implies $x \in \alpha$, resulting in a contradiction, $-x \in C^I$, and thus this implies $C^I \subseteq D^I \implies x \in \alpha$, contradicting the hypothesis. \square

Lemma 2. $\forall C, D, F \in N_{\mathcal{C}} : C \sqsubseteq D \implies C \sqcap F \sqsubseteq D \sqcap F$.

Proof. By considering the associated semantics the Lemma can be restated as $C^I \subseteq D^I \Longrightarrow \underbrace{C^I \cap F^I}_{\alpha} \subseteq \underbrace{D^I \cap F^I}_{\beta}$. Assume that the preceding formula does not

hold, that is $\alpha \not\subseteq \beta$. This means $\exists x \in \beta \ . \ x \not\in \alpha$. Note that $x \in \beta$ is equivalent to requiring that $x \in F^I \land x \in D^I$ holds. However, $x \in F^I \land x \notin \alpha \implies x \notin C^I$. Given that $x \in D^I$ holds, this contradicts the premise $C \sqsubseteq D$. \square

Lemma 3. $\forall C, D \in N_C$. $C \sqsubseteq D \implies \exists R.C \sqsubseteq \exists R.D$.

Proof. Assume that $\{x \mid \exists y \in C^I : \langle x,y \rangle \in R^I\} \not\subseteq \{x \mid \exists y \in D^I : \langle x,y \rangle \in R^I\}$ holds, that is, $\exists R.C \not\sqsubseteq \exists R.D$. This requires that the following condition holds: $\exists \langle x,y \rangle \in R^I : y \in C^I \land y \notin D^I$. But, if such condition holds, then $C \not\sqsubseteq D$, contradicting the premise. \square

Lemma 4. $\forall C, D \in N_C$. $C \sqsubseteq D \implies \forall R.C \sqsubseteq \forall R.D$.

Proof. Assume that $\{x \mid \forall \langle x,y \rangle : \langle x,y \rangle \in \mathbb{R}^I \implies y \in \mathbb{C}^I\} \not\subseteq \{x \mid x \mid y \in \mathbb{C}^I\}$ $\forall \langle x,y \rangle : \langle x,y \rangle \in R^I \implies y \in D^I \}$ holds, that is, $\forall R.C \not\sqsubseteq \forall R.D.$ This requires that the following condition holds: $(\exists x . \forall \langle x, y \rangle . \langle x, y \rangle \in R^I \implies y \in C^I) \land (\exists \bar{y})$. $\langle x, \bar{y} \rangle \in R^I \wedge \bar{y} \notin D^I$). But, if this condition holds, then an \bar{y} exists and R^I is not empty. Therefore, since the left operand of the implication holds, then right operand also does. From this, we obtain $C^I \not\subseteq D^I$, contradicting the premise. \square

Proposition 1. Adaptation rules application preserves ontology satisfiability.

Proof. Ontology satisfiability is preserved because every adaptation rule is sound. We prove this for each rule separately:

- a.1 The proof directly follows from Concept Equivalence axiom definition.
- a.2 $E \equiv \exists R.C \rightarrow E \sqsubseteq \exists R.D. \exists R.C \sqsubseteq \exists R.D$ must hold: thanks to the rule precondition, $C \sqsubseteq D$, we can apply Lemma 3.
- a.3 $E \equiv \geq_n R.C \to E \sqsubseteq \geq_n R.D. \geq_n R.C \sqsubseteq \geq_n R.D$ must hold, but it is sufficient that $\{x \mid \exists y \in C^I : \langle x,y \rangle \in R^I\} \subseteq \{x \mid \exists y \in D^I : \langle x,y \rangle \in R^I\}$ holds. Thanks to the rule precondition, $C \sqsubseteq D$, we can apply Lemma 3.
- a.4 $E \equiv C \sqcup F \to E \sqsubseteq D \sqcup F$. $C \sqcup F \sqsubseteq D \sqcup F$ holds for Lemma 1 because $C \sqsubseteq D$ holds.
- a.5 $E \equiv C \sqcap F \to E \sqsubseteq D \sqcap F$. $C \sqcap F \sqsubseteq D \sqcap F$ holds for Lemma 2 because $C \sqsubseteq D$ holds.
- a.6 $E \equiv \neg C \rightarrow \neg D \sqsubseteq E$. $\neg D \sqsubseteq \neg C$ must hold: the semantics is $\Delta^I \setminus D^I \subseteq$ $\Delta^I \setminus C^I$, but this contradicts $C \sqsubseteq D$.
- a.7 $C(a) \to D(a)$. $C(a) \Longrightarrow D(a)$ is guaranteed by the rule precondition.
- a.8 $E \equiv \forall R.C \rightarrow E \sqsubseteq \forall R.D.$ $E \equiv \forall R.C \implies E \sqsubseteq \forall R.D$ holds for Lemma 4 because $C \sqsubseteq D$ holds.

- a.9 $E \equiv \leq_n R.C \rightarrow E \subseteq \leq_n R.B. \leq_n R.B \subseteq \leq_n R.C$, but it is sufficient that $\{x \mid \exists y \in B^I : \langle x, y \rangle \in R^I\} \subseteq \{x \mid \exists y \in C^I : \langle x, y \rangle \in R^I\}$. Thanks to the rule precondition, $B \sqsubseteq C$, we can apply Lemma 3.
- a.10 $E \equiv C \sqcup F \to B \sqcup F \subseteq E$. The proof for $B \sqcup F \subseteq C \sqcup F$ is the dual of the one given in item (a.4).
- a.11 $E \equiv C \sqcap F \to B \sqcap F \sqsubseteq E$. The proof for $B \sqcap F \sqsubseteq C \sqcap F$ is the dual of the one given in item (a.5).
- a.12 $E \equiv \neg C \rightarrow E \sqsubseteq \neg B$. the proof for $\neg C \sqsubseteq \neg B$ is the dual of the one given in item (a.6).
- a.13 $C \subseteq E \to B \subseteq E$. The rule precondition, $B \subseteq C$. By transitivity, this implies $B \sqsubseteq E$.
- b.1 The proof directly follows from *Role Equivalence* axiom definition.
- b.2 $\underbrace{T_0 \circ \ldots \circ T_m \circ R \circ T'_0 \circ \ldots \circ T'_p}_{\alpha} \sqsubseteq T \to \underbrace{T_0 \circ \ldots \circ T_m \circ Q \circ T'_0 \circ \ldots \circ T'_p}_{\beta} \sqsubseteq$
 - T. assume that $\beta^I \not\subseteq \alpha^I$ holds. This requires that $\exists x_0, \ \dots, \ x_{m+p+3}$. $\langle x_0, x_1 \rangle \in T_0^I \wedge \ldots \wedge \langle x_{m+1}, x_{m+2} \rangle \in Q^I \wedge \langle x_{m+p+2}, x_{m+p+3} \rangle \in T_p^I \wedge$ $\langle x_{m+1}, x_{m+2} \rangle \notin R^I$. This contradicts $Q \sqsubseteq R$.
- b.3 $E \equiv \underbrace{\forall R.C}_{\alpha} \rightarrow E \sqsubseteq \underbrace{\forall Q.C}_{\beta}$. Assume that $\alpha^I \not\subseteq \beta^I$ holds. This requires that
 - $\exists x \;.\; x \in \alpha^I \land x \not\in \beta^I \text{, that is, } \exists x. ((\forall y \;.\; \langle x,y \rangle \in R^I \implies y \in C^I) \land (\exists y') \land (\exists x') \land (\exists x')$ $(x,y') \in Q^I \wedge y' \notin C^I$. Given that $Q \sqsubseteq R$, if such y' exists, α cannot hold, leading to a contradiction.
- b.4 $T \equiv \underbrace{\leq_n R.C}_{\alpha} \to T \sqsubseteq \underbrace{\leq_n Q.C}_{\beta}$. Assume that $\alpha^I \not\subseteq \beta^I$. This requires that
 - $\begin{array}{l} \exists x \;.\; |\{y \mid y \in C^I \wedge \langle x,y \rangle \overset{\smile}{\in} R^I\}| \leq n \wedge |\{y \mid y \in C^I \wedge \langle x,y \rangle \in Q^I\}| > n. \\ \text{This implies}\; |Q^I| > |R^I|, \; \text{contradicting}\; Q \sqsubseteq R. \end{array}$
- b.5 $T \equiv R^- \to Q^- \sqsubseteq T$. Assume that $Q^{-\bar{I}} \not\subseteq R^{-\bar{I}}$. This requires that $\exists \langle x, y \rangle$. $\langle y, x \rangle \in Q^I \wedge \langle y, x \rangle \notin R^I$. This contradicts $Q^I \subseteq R^I$.
- b.6 $Disjoint(R,T) \to Disjoint(Q,T)$. Assume that $R^I \cap T^I = \emptyset \implies Q^I \cap Q$ $T^I=\emptyset$ does not hold. This requires that $\exists \langle x,y\rangle \in Q^I \land \langle x,y\rangle \in T^I \land \langle x,y\rangle \not\in R^I$ holds, but $\langle x,y\rangle \in Q^I \land \langle x,y\rangle \not\in R^I$ contradicts $Q\sqsubseteq R$.
- b.7 $R(a,b) \to S(a,b)$. From $R \subseteq S$ we have that $\forall \langle x,y \rangle : \langle x,y \rangle \in R \implies$ $\langle x, y \rangle \in S.$
- b.8 $E \equiv \underbrace{\exists R.C}_{\alpha} \to E \sqsubseteq \underbrace{\exists S.C}_{\beta}$. Assume that $\alpha^I \not\subseteq \beta^I$. This requires that $\exists x$.
 - $\exists y \in C^I \ . \ \langle x,y \rangle \in S^I \land \langle x,y \rangle \not \in R^I \ \text{holds. This contradicts} \ R \sqsubseteq S.$
- b.9 $E \equiv \underbrace{\exists R.Self}_{\alpha} \rightarrow E \sqsubseteq \underbrace{\exists S.Self}_{\beta}$. Assume that $\alpha^{I} \not\subseteq \beta^{I}$. This requires that $\exists x : \langle x, x \rangle \in S^{I} \land \langle x, x \rangle \not\in R^{I}$ holds. This contradicts $R \sqsubseteq S$.
- b.10 $E \equiv \underbrace{\geq_n R.C}_{\alpha} \to E \sqsubseteq \underbrace{\geq_n S.C}_{\beta}$. Assume that $\alpha^I \not\subseteq \beta^I$. This requires that
 - $\exists x \;.\; |\{y \mid y \in C^I \land \langle x,y \rangle \overset{\scriptscriptstyle \rho}{\in} R^I\}| \geq n \land |\{y \mid y \in C^I \land \langle x,y \rangle \in S^I\}| < n.$ This implies $|R^I| > |S^I|$, contradicting $R \sqsubseteq S$.

b.11 $T \equiv R^- \to T \sqsubseteq S^-$. Assume that $R^{-I} \not\subseteq S^{-I}$. This requires that $\exists \langle x, y \rangle$. $\langle y, x \rangle \in R^I \land \langle y, x \rangle \notin S^I$, thus contradicting $R \sqsubseteq S$.

b.12 $T_0 \circ \ldots \circ T_q \sqsubseteq R \to T_0 \circ \ldots \circ T_q \sqsubseteq S$. This immediately follows, by transitivity, from $R \sqsubseteq S$. \square

3.4 Temporal Complexity

Proposition 2. The time complexity of the algorithm is in $\mathcal{O}(n)$, where n is the number of axioms of the input ontology o.

Proof. computePrecond scans all the axioms of ontology o. For each of them it performs some comparison having a total cost of c_1 , so it has a cost of $n \cdot c_1$. The for statement of line 4 in Algorithm 1 is executed n times in the worst case (each axiom of the ontology refers to the entity in question). The for statement of line 5 is executed $c_2 = |Rules|$ times, where Rules is the set of adaptation rules. satisfies test requires a constant (c_3) time for checking the required conditions. Axiom rewriting and its insertion requires constant (c_4) time. The removal of old axioms requires constant time (c_5) too. The overall complexity is therefore equal to $n \cdot c_1 + c_2 \cdot c_3 \cdot c_4 \cdot n + c_5$, that belongs to $\mathcal{O}(n)$. \square

4 Experiments

In order to evaluate the practical applicability of our proposal we implemented a Java prototype based on the OWL API library³. In OWL API the axioms are immutable objects, and it supports only axiom addition and removal. Whenever possible, the rule application has been simulated with a pair of add and delete changes. In the other cases we employed Java Reflection for directly modifying the involved axiom. In addition to correctness, we also experimentally evaluated the coverage of OWL2 axioms and constructors of our set of rules. The dataset is presented in Table 3 (manual selection on the Web based on ontology size and DL expressivity).

Correctness The developed proof-of-concept prototype has been used for testing correctness of our adaptation rules, the experimental counterpart of the proofs given in Section 3.3. More precisely, the test consists in taking as input a satisfiable ontology composed by the precondition and an axiom corresponding to the LHS of a rule r (modulo alpha renaming of the entity to delete). At this point, using Hermit reasoner $(v1.3.7)^4$, we check the entailment of RHS(r)[e'].

Evaluation An entity e is **adaptable** iff it satisfies at least one rule precondition, while an axiom a is **adaptable** iff it at least one rule r s.t. LHS(r)[e] = a exists, in case prec(r) holds w.r.t. e, the axiom is said **fully adaptable**. As an estimation of the practical effectiveness of our algorithm, we consider, for

³ Available here: http://owlapi.sourceforge.net/

⁴ Hermit and related information are available at http://hermit-reasoner.com/

(C.4)*	N/A	76.17	45.16	96.99	N/A	N/A	93.55	100.00	0.00	N/A	67.73	N/A	100.00	N/A	N/A	81.25	100.00	100.00	100.00	N/A	N/A	100.00	11.67	100.00	N/A	81.25	92.47	N / N
(C.2)*	49.49	17.95	37.10	88.29	50.00	46.91	48.43	22.15	42.86	50.47	36.92	50.00	100.00	49.84	38.93	71.18	39.62	100.00	100.00	57.18	46.73	50.00	51.69	42.86	50.00	40.91	42.82	000
(C.4)	N/A		45.16	96.99	N/A	N/A		100.00		N/A		N/A		N/A			100.00	100.00	100.00	N/A			11.67		N/A	81.25	92.23	NI / A
(C.3)	0.00	97.22		84.62	N/A		100.00	75.00	33.33					0.00		71.43	18.18	0.001 (75.00	0.00		33.33	95.63	22.22		80.00		000
(C.2)	00.00 49.49	0 17.92	7 35.11	67.88	0 50.00) 45.50	048.07	0 22.14	0 8.20	2 38.53	0 34.67	0.50.00	.00.00 100.00	2 49.84	36.99	069.94	39.62	0 100.00	1 77.14	057.03	046.73	049.61	051.69	3 42.86	050.00	3 40.91	0 42.70	00000
1S (C.1	100.0	100.00	97.67	94.81	100.00	90.80	100.00	100.00	100.00	96.22	100.00	100.00	100.0	99.82	67.02	100.00	99.75	100.00	94.44	100.00	100.00	100.00	100.00	84.48	100.00	63.16	100.00	100 00
Axioms Logical Axioms (C.1) (C.2) (C.3) (C.4) (C.2)* (C.4)*	100	2712	93	657	84	215	1747	712	475	8493	1025	350	22834	11545	4838	108	2930	204	38	216	398	93	2444	81	290	63	647	n C
Axioms I	186	2492	243	747	576	353	5043	939	503	20027	3499	5649	23527	30364	11043	396	19849	874	09	404	3508	145	4428	165	1096	103	2044	00
URI	http://omv.ontoware.org/2009/09/OWLChanges	http://ccdb.ucsd.edu/SAO/1.2	http://swat.cse.lehigh.edu/onto/univ-bench.owl	http://www.w3.org/TR/2003/CR-owl-guide-20030818/wine	http://purl.obolibrary.org/obo/ogms.owl	http://owlodm.ontoware.org/OWL2	http://semanticscience.org/ontology/sio-core.owl	http://www.co-ode.org/ontologies/pizza/pizza.owl	http://sweet.jpl.nasa.gov/2.1/reprSciUnits.owl	http://purl.obolibrary.org/obo/hao/2011-11-03/hao.owl	http://purl.obolibrary.org/obo/ido.owl	http://ontology.neuinfo.org/NIF/Dysfunction/NIF-Dysfunction.owl	http://aims.fao.org/aos/geopolitical.owl	http://human.owl	http://mouse.owl	http://owl.man.ac.uk/2005/07/sssw/people.owl	http://ontology.neuinfo.org/NIF/BiomaterialEntities/NIF-GrossAnatomy.owl	http://purl.obolibrary.org/obo/flu/dev/flu.owl	http://www.owl-ontologies.com/generations.owl	http://protege.stanford.edu/plugins/owl/owl-library/ka.owl	http://ontology.neuinfo.org/NIF/BiomaterialEntities/NIF-Cell.owl	http://www.owl-ontologies.com/travel.owl	http://www.hozo.jp/owl/YAMATO20120714.owl	http://purl.org/net/ontology/beer.owl	http://msi-ontology.sourceforge.net/ontology/NMR.owl	http://www.nada.kth.se/~mehrana/Delegation.owl	http://www.biomodels.net/kisao/KISAO	http://min opolibnom: one/owness our
DI	ALUQ(D)	SHIN(D)	TEHI+(D)	SHOIN(D)	ALCO	ALQ(D)	SRIQ(D)	SHOIN	SHOIN(D)	$_{ m SR}$	SROIF	SROIF	SHIN(D)	w	ALE	ALCHOIN	SROIF	SROIN(D)	ALCOIF	AL(D)	SROIF	SOIN(D)	ALUHN	ALHIF(D)	SH(D)	SHIF(D)	ALCRIQ(D)	VI CO
П	1.	5.	3. A			9.								14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27. A	30

Table 3. Dataset and coverage results presented in Section 4. Ontologies 14 and 15 are part of the dataset used by Ontology Alignment Evaluation Initiative, and are available at http://oaei.ontologymatching.org/. Coverage results are unaggregated and based on the data of Table 4. N/A means the ontology contains no roles/adaptable roles.

each ontology in our dataset, the following scenario: we simulate the deletion of each single entity, in isolation, and we take into account the percentage of adaptable ones (i.e., such that another entity suitable for reformulation exists). For each of these adaptable entities, we also inspect how many axioms involving them would be adapted instead of simply deleted. For this reference scenario we defined *Coverage* measure as: (C.1) the percentage of adaptable concepts (resp. roles (C.3)) out of the total number of concepts (resp. roles), and (C.2) the percentage of adaptable axioms w.r.t. the deleted concept (resp. role, (C.4)) out of the number of axioms to be deleted (that is, presenting the deleted entity in their signature). The (C._)* variants count the fully adaptable axioms, and evaluate the completeness of our adaptation rules (the complement of the fully adaptable axioms is not supported by our rules).

In Table 3 the coverage for each ontology in isolation is reported (computed from the raw data of Table 4), while the result considering the dataset as a whole ontology is the following: (C.1) 93.247%, (C.2) 41.757%, (C.2*) 44.185%, (C.3) 73.647%, (C.4) 79.63%, (C.4*) 80.847%. Table 3 shows that 10 out of 12 of the worst performing ontologies w.r.t. role coverage ((C.3), (C.4)) and (C.4)are expressed in a DL missing role hierarchy constructs (identified by letter H in the DL name). Without role hierarchy constructs only role equality can be used for adaptation, thus reducing the number of adaptable roles. Concept coverage (C.1) presents, instead, high values (above 60%) for all the considered ontologies, independently from the DL they are expressed with. This is not surprising because concept hierarchy constructs are available for DLs at least as expressive as \mathcal{AL} . On the contrary, coverage results for concept rules w.r.t. OWL2 axioms and constructors seem to be unrelated to either the underpinning DL or the ontology size (in terms of number of axioms and/or entities). For instance, the ontologies with worst values for $(C.2)^*$ are 2. (SHIN(D)), 8. (SHOIN(D))11. (\mathcal{SROIF}) 3. $(\mathcal{ALEHI} + (\mathcal{D}))$ and 15. (\mathcal{ALE}) , with very different number of concepts and axioms (Table 4). Similarly, among the best results for (C.2)* the expressivity ranges from $\mathcal{AL}(\mathcal{D})$ to $\mathcal{SROIN}(\mathcal{D})$, again with varying number of axioms and concepts. Ideally the proposal should adapt all the axioms: (C.2)*, in particular, is far from this result, but it is well known that OWL2, despite being based on SROIQ, adds new constructors and axioms, that are derivable from \mathcal{SROIQ} ones (they do not add expressive power). For example, Concept Disjointness axiom (i.e., Disjoint(C, D), with $C, D \in \mathcal{N}_C$) is only a shortcut for $C \sqcap D \sqsubseteq \perp^5$. Our prototype strictly applies the rules of Table 1 and Table 2, so it cannot directly process the axioms and constructors not available in \mathcal{SROIQ} DL, thus diminishing the number of adaptable axioms.

5 Future Work

The paper represents, to the best of our knowledge, the first proposal for ontology adaptation upon updates. In addition, the algorithm is totally automatic and supports ontology expressivity up to \mathcal{SROIQ} , on top of which OWL2 is defined.

⁵ Refer to [9], Chapter 9, for further examples and details.

The present paper could be extended in several directions. The set of adaptation rules is a preliminary proposal, we plan to further enrich it in order to increase the coverage rate reported in Section 4 and to consider reasonable alternatives for each single rule (e.g., sound alternatives for a.8 could be $C \subseteq D, E \equiv \forall R.C \to \forall R.D \subseteq E$ or $B_0 \dots B_n \subseteq C, E \equiv \forall R.C \to E \subseteq \forall R. \bigsqcup_{i=0}^n B$. We also plan to consider the integration of anonymous entities (e.g., using \top as superclass). Another possible extension is the integration of a complex update (e.g., concept merge and split) proposals, such as [2]. The relationship between DL updates and Belief Revision has been investigated [8], we plan to further investigate it w.r.t. our proposal. We also intend to improve our prototype up to a full support of OWL2. Our final goal will be a Protégé plugin, from which we hope to receive feedbacks from the community of ontology engineers and practitioners. The experimental evaluation will also be strengthened with an extended ontology dataset and temporal profiling of the prototype.

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$\begin{array}{c} 0 \\ 8 \\ 7 \\ 7 \\ 95 \\ 111 \\ 11 \\ 148 \\ 2321 \\ 133 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\$	ID Concepts	Adaptable Concepts	Concept Axioms	Adaptable Concept Axioms	\mathbf{Roles}	Adaptable Roles	Role Axioms	Adaptable Role Axioms	Unsatisfiable Concept Axioms	Unsatisfiable Role Axioms
736 736 5129 919 36 35 256 195 8 43 42 131 279 13 14 7 77 73 411 279 13 11 345 231 14 7 92 92 168 84 0 0 0 0 0 0 87 79 367 167 184 184 96 0 0 0 1021 1021 283 1357 184 184 96 885 21 1021 123 183 15 6 2 137 0 0 148 130 1857 9809 3759 30 26 285 191 133 130 123 460 460 6 2 285 191 133 150 123 123 2 0 0 0 0 2		100	198	98	2	0	0	0	0	0
43 42 131 46 25 9 31 14 7 77 73 411 279 118 84 0 0 0 0 95 92 92 168 84 0 0 0 0 0 95 87 79 367 167 44 0 0 0 11 1021 122 183 1357 184 184 96 885 21 1021 120 1477 327 8 6 180 180 11 112 12 183 15 6 2 137 0 148 1930 1857 9809 3759 30 26 285 191 133 1930 1857 9809 3779 4 0 0 0 2321 1930 188 189 189 488 4888 0 <td< td=""><td></td><td>736</td><td>5129</td><td>919</td><td>36</td><td>35</td><td>256</td><td>195</td><td>∞</td><td>0</td></td<>		736	5129	919	36	35	256	195	∞	0
77 73 411 279 13 11 345 231 95 92 92 168 84 0 0 0 0 10 87 79 367 167 44 0 0 0 10 1021 1021 2823 1357 184 184 946 885 21 100 100 1477 327 8 6 180 180 1 110 100 1477 327 8 6 180 180 1 110 100 1477 327 8 6 180 180 1 121 143 15 6 285 191 133 150 162 460 460 6 4888 4888 0 162 183 1624 6116 2423 11 12 486 86 0 188 17		42	131	46	25	9	31	14	7	0
92 92 168 84 0 0 0 0 0 87 79 367 184 0 0 0 0 11 1021 1021 2823 1357 184 184 946 885 21 100 100 1477 327 8 6 180 180 1 100 100 1477 327 8 6 180 180 1 110 12 183 15 6 2 137 0 148 190 1857 9809 759 30 26 285 191 133 150 509 2189 759 30 26 285 191 133 352 352 700 460 6 4888 4888 0 121 143 3 0 0 0 0 0 162 163 <		73	411	279	13	11	345	231	95	0
87 79 367 167 44 0 0 11 1021 1021 2823 1357 184 184 946 885 21 102 100 1477 327 8 6 180 180 1 12 12 183 15 6 2 137 0 180 1 1930 1857 9809 3779 4 0 0 0 2321 509 2189 759 30 26 285 191 133 509 2189 759 30 26 285 191 133 509 352 700 460 6 488 4888 0 12 12 460 460 6 4888 4888 0 127 14 83 0 0 0 0 312 60 163 161 47 19		92	168	84	0	0	0	0	0	0
1021 1021 2823 1357 184 184 946 885 21 100 100 1477 327 8 6 180 180 1 12 12 183 15 6 2 137 0 1 1930 1857 9809 3779 4 0 0 0 2321 509 509 2189 759 30 26 285 191 133 352 700 350 0 0 0 0 0 0 0 122 12 460 460 6 488 488 0		79	367	167	44	0	0	0	11	0
100 100 1477 327 8 6 180 180 1 12 12 183 15 6 2 137 0 148 1930 1857 9809 3779 4 0 0 0 148 509 509 2189 759 30 26 285 191 133 509 509 2189 759 30 26 285 191 133 509 509 2189 759 30 26 285 191 133 352 700 350 0 0 0 0 0 12 12 460 460 6 6 4888 4888 0 2744 1839 6256 2314 3 0 0 0 0 0 1628 1624 6116 2423 11 2 86 86 0 0		1021	2823	1357	184	184	946	885	21	0
12 12 183 15 6 2 137 0 148 1930 1857 9809 3779 4 0 0 2321 509 509 2189 759 30 26 285 191 133 352 352 700 350 0 0 0 0 2321 12 12 460 460 6 4888 4888 0 2744 1839 6256 2314 3 0 0 0 0 2744 1839 6256 2314 3 0 0 0 312 1628 1624 6116 2423 11 2 86 86 0 213 213 447 447 19 19 83 83 0 18 17 35 27 4 3 20 20 8 373 794		100	1477	327	œ	6	180	180	<u></u>	0
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509 509 2189 759 30 26 285 191 133 352 352 700 350 312 0 0 0 312 0 0 0 312 0 0 0 312 0 0 0 312 0 0 0 312 0 0 0 312 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0		1857	9809	3779	4	0	0	0	2321	0
352 352 700 350 0		509	2189	759	30	26	285	191	133	ω
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3304 3298 10880 5423 2 0 0 0 0 2744 1839 6256 2314 3 0 0 0 312 2744 1839 6256 2314 3 0 0 0 312 260 60 173 121 14 10 48 39 3 1628 1624 6116 2423 11 2 86 86 0 213 213 447 447 19 19 83 83 0 18 17 35 27 4 3 20 20 8 18 17 35 27 4 3 20 20 1 373 373 794 371 2 0 0 0 1 925 925 4589 2372 183 175 1542 180 0 19 <td></td> <td>12</td> <td>460</td> <td>460</td> <td>6</td> <td>6</td> <td>4888</td> <td>4888</td> <td>0</td> <td>0</td>		12	460	460	6	6	4888	4888	0	0
2744 1839 6256 2314 3 0 0 0 60 173 121 14 10 48 39 1628 1624 6116 2423 11 2 86 86 213 213 447 447 19 19 83 83 18 17 35 27 4 3 20 20 96 377 215 60 0 0 0 0 373 373 794 371 2 0 0 0 35 35 129 64 6 2 11 11 925 925 4589 2372 183 175 1542 180 58 49 105 45 9 2 6 6 19 12 2 9 0 0 0 0 19 12 2 9		3298	10880	5423	2	0	0	0	0	0
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1628 1624 6116 2423 11 2 86 86 213 213 447 447 19 19 83 83 18 17 35 27 4 3 20 0 96 96 377 215 60 0 0 0 373 373 794 371 2 0 0 0 35 129 64 6 2 11 11 1 925 925 4589 2372 183 175 1542 180 58 49 105 45 9 2 6 6 50 301 580 290 0 0 0 0 19 12 22 9 20 16 48 39 202 20 1335 570 9 9 386 356 27 27 <td< td=""><td></td><td>60</td><td>173</td><td>121</td><td>14</td><td>10</td><td>48</td><td>39</td><td>ယ</td><td>0</td></td<>		60	173	121	14	10	48	39	ယ	0
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18 17 35 27 4 3 20 20 96 377 215 60 0 0 0 373 373 794 371 2 0 0 0 35 35 129 64 6 2 11 11 925 925 4589 2372 183 175 1542 180 58 49 105 45 9 2 6 6 301 301 580 290 0 0 0 0 19 12 2 9 20 16 48 39 202 202 1335 570 9 9 386 356 27 27 50 25 2 0 0 0		213	447	447	19	19	83	83	0	0
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373 373 794 371 2 0 0 0 35 35 129 64 6 2 11 11 925 925 4589 2372 183 175 1542 180 58 49 105 45 9 2 6 6 301 301 580 290 0 0 0 0 19 12 22 9 20 16 48 39 202 202 1335 570 9 9 386 356 27 27 50 25 2 0 0 0		96	377	215	60	0	0	0	1	0
35 35 129 64 6 2 11 11 925 925 4589 2372 183 175 1542 180 58 49 105 45 9 2 6 6 301 301 580 290 0 0 0 0 19 12 22 9 20 16 48 39 202 202 1335 570 9 9 386 356 27 27 50 25 2 0 0 0		373	794	371	2	0	0	0	0	0
925 925 4589 2372 183 175 1542 58 49 105 45 9 2 6 301 301 580 29 0 0 0 19 12 22 9 20 16 48 202 202 1335 570 9 9 386 27 27 50 25 2 0 0		35	129	64	6	2	11	11	1	0
58 49 105 45 9 2 6 301 301 580 290 0 0 0 19 12 22 9 20 16 48 202 202 1335 570 9 9 386 27 27 50 25 2 0 0		925	4589	2372	183	175	1542	180	0	0
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19 12 22 9 20 16 48 202 202 1335 570 9 9 386 27 27 50 25 2 0 0		301	580	290	0	0	0	0	0	0
202 202 1335 570 9 9 386 27 27 50 25 2 0 0		12	22	9	20	16	48	39	0	0
27 27 50 25 2 0 0		202	1335	570	9	9	386	356	4	1
		27	50	25	2	0	0	0	0	0

Table 4. Raw data used for coverage analysis of Section 4. Unsatisfied axioms stands for axioms matching the LHS of a rule having a precondition not satisfied by the entity under deletion.