Incentive-Compatible Learning of Reserve Prices for Repeated Auctions

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ABSTRACT

Motivated by online advertising market, we consider a seller who repeatedly sells ex ante identical items via the second-price auction. Buyers' valuations for each item are drawn i.i.d. from a distribution F that is unknown to the seller. We find that if the seller attempts to dynamically update a common reserve price based on the bidding history, this creates an incentive for buyers to shade their bids, which can hurt revenue. When there is more than one buyer, incentive compatibility can be restored by using *personalized* reserve prices, where the personal reserve price for each buyer is set using the historical bids of *other* buyers. In addition, we use a *lazy* allocation rule, so that buyers do not benefit from raising the prices of their competitors. Such a mechanism asymptotically achieves the expected revenue obtained under the static Myerson optimal auction for F. Further, if valuation distributions differ across bidders, the loss relative to the Myerson benchmark is only quadratic in the size of such differences. We extend our results to a contextual setting where the valuations of the buyers depend on observed features of the items.

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Advertising is the main component of the monetization strategy of most Internet companies. A large fraction of online advertisements are sold via auctions where advertisers bid in real time for a chance to show their ads to users. Examples of such auction platforms, called advertisement exchanges (McAfee 2011, Muthukrishnan 2009), include Google's Doubleclick (AdX), Facebook, AppNexus, and OpenX.

The second-price auction is the dominant mechanism used by advertisement exchanges. Among the reasons for its prevalence are the simplicity of the second-price auction and the fact that it incentivizes advertisers to be truthful. The second-price auction can maximize the social welfare (i.e., the value created in the system) by allocating the item to the highest bidder.

To maximize the revenue earned in a second-price auction, the auctioneer can set a reserve price and not make any allocations when the bids are low. In fact, under symmetry and regularity assumptions, the second-price auction with an appropriately chosen reserve price is optimal and maximizes revenue among all selling mechanisms Myerson 1981, Riley and Samuelson 1981.

However, to set the reserve price effectively, the auctioneer requires information about the distribution of the valuations of the bidders. To understand the effects of changing reserve prices based on previous bids, we study a setting where the auctioneer sells impressions (advertisement space) via repeated second-price auctions. We demonstrate that the long-term incentives of advertisers play an important role in the performance of these repeated auctions.

We show that natural mechanisms that set a common reserve price using the history of the bids may create substantial incentives for the buyers to shade their bids. On the other hand, we propose an *incentive-compatible* mechanism that sets a personalized reserve price for each agent based on the bids of *other* agents.[1] Our mechanism allocates the item to the highest bidder if his bid exceeds his personal reserve price. If the item is allocated, the price is equal to the maximum of the second-highest bid and the personal reserve of the winner. This structure corresponds to mechanisms used in practice, as described in Paes Leme et al. 2016. By appropriately choosing the function that maps historical bids of others to a personal reserve price, we show that the expected revenue per round is asymptotically as large as that under the static Myerson optimal auction. Here we build on prior work that investigates how samples from a distribution can be used to set a near-optimal reserve price, cf., Dhangwatnotai et al. 2015. Importantly for incentives, our mechanism is "lazy" (see more on this in the section on related work below), in that it allocates the item only if the highest bidder clears her personal reserve; it is not enough for *some* bidder to clear her personal reserve. An "eager" variant would allocate the item to the highest bidder among those who clear their reserve; this would create an incentive for agents to overbid so as to increase the personal reserves of other agents in future, thereby increasing their likelihood of being eliminated.

A natural concern with such an approach is that if agents' valuation distributions differ from each other, it may lead to a lower personal reserve price for agents with a higher valuation distribution, and vice versa, thereby hurting revenue. We show that this issue is not significant when differences in valuation distributions are not too large. In particular, we show that the loss relative to the Myerson benchmark is only quadratic in the size of such differences, and supplement this theoretical result with numerical examples.

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[1] In case of unlimited supply the incentive compatibility directly follows if the price of each buyer only depends on the other buyers, cf., Balcan et al. 2018. With limited supply, obtaining incentive compatibility is more challenging because of "competition" among buyers.

We extend our results to a contextual setting with heterogeneous items that are represented by a feature vector of covariates. The valuations of the buyers are linear in the feature vectors (with a-priori unknown coefficients) plus an idiosyncratic private component. We present a learning algorithm that determines the reserve price for each buyer using an ordinary least squares estimator for the feature coefficients. We show that the loss of revenue is sub-linear in the number of samples (i.e., the number of previous auctions).

We also propose an essentially optimal mechanism that can extract (almost all of) the surplus of the agents. The basic idea is that using the bids of other agents, the seller can construct an estimate of the valuation distribution and hence of the expected utility per round of each agent when individual items are allocated using second-price auctions. Based on this estimate, the mechanism charges a surplus-extracting up-front payment at the beginning of each round. Since agents can influence the up-front payments of other agents, they may have an incentive to overbid to eliminate competing agents from future auctions. We propose a solution that asymptotically removes the incentive for agents to deviate from truthfulness: the mechanism simulates[2] agents who choose not to pay the entrance fee. We show that under our mechanism, truthfulness constitutes an approximate equilibrium.

In each of the mechanisms we propose, we overcome incentive issues using the same two key ideas: (i) we individually choose a pricing rule for each agent, based only on the bids of *other* agents, and (ii) we prevent an agent from benefitting if other agents are prevented from participating by raising the prices they face. In a setting where agents' valuation distributions are identical (or similar to each other), this approach enables the seller to obtain as much revenue as if he knew the valuation distribution F, while maintaining incentive compatibility. We believe that these design principles should be broadly applicable to overcome the lack of knowledge of F when there is competition between strategic agents/buyers.

Finally, we would like to highlight two of our technical contributions that would of broad interest to the mechanism design community: (i) our results on heterogeneous distribution of bidders extends to second price auction with common reserve and other related auctions. In this way, they complement the worst case guarantee (factor 2) of Hartline and Roughgarden 2009. More specifically, as a bi-product of the analysis leading to our main Theorem, we obtain that using a second price auction in the asymmetric valuations case, the seller can obtain expected revenue within $O(\delta^2)$ of the optimal, where δ is the maximum "distance" between valuation distributions. (ii) As part of our analysis on the revenue loss of our mechanism, we prove a lower bound of $\Omega(1/(\#\text{samples})^{2/3})$ up to logarithmic factors on the monopolist's revenue loss when i.i.d. samples are used to learn the optimal reserve price. To the extent of our knowledge, our work provides the first lower-bound for this problem, where the best known upper-bound is of $O(1/(\#\text{samples})^{1/2})$, cf., Dhangwatnotai et al. 2015.

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[2]We note that Gorokh et al. 2017, which is an independent and concurrent work to ours, present a similar simulation approach for mechanism design.