Growing Attributed Networks through Local Processes

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ABSTRACT

This paper proposes an attributed network growth model. Despite the knowledge that individuals use limited resources to form connections to similar others, we lack an understanding of how local and resource-constrained mechanisms explain the emergence of structural properties found in real-world networks. We make three contributions. First, we propose a simple and accurate model of attributed network growth that jointly explains the emergence of in-degree, local clustering, clustering-degree relationship and attribute mixing patterns. Second, we make use of biased random walks to develop a model that forms edges locally, without recourse to global information. Third, we account for multiple sociological phenomena: bounded rationality; structural constraints; triadic closure; attribute homophily; preferential attachment. Our experiments show that the proposed Attributed Random Walk (ARW) model accurately preserves network structure and attribute mixing patterns of real-world networks; it improves upon the performance of eight well-known models by a significant margin of 2.5–10×.

CCS CONCEPTS

• **Information systems** → **Web applications**; *Data mining*; *Web mining*; • **Applied computing** → *Sociology*.

KEYWORDS

Network growth; Network Structure; Attributed networks

1 INTRODUCTION

We present a network growth model that explains how distinct structural properties of attributed networks can emerge from a local edge formation process. In real-world networks, individuals form edges with limited information and partial network access. Moreover, phenomena such as triadic closure and homophily simultaneously influence individuals' decisions to form connections. Over time, these decisions cumulatively shape real-world networks to exhibit rich structural properties: heavy-tailed in-degree distribution, skewed local clustering and diverse attribute mixing patterns. However, we lack an understanding of local, resource-constrained

American Physical Society (APS) Dataset   Attributed Random Walk (ARW) Model   Holme-Kim (HK) Model

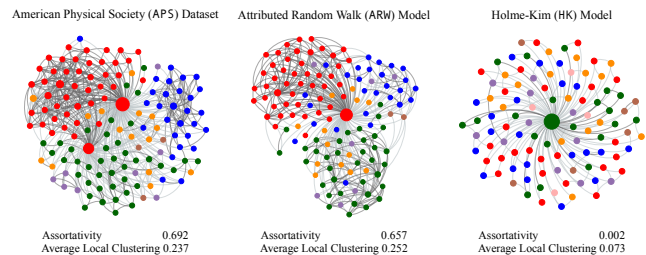|	Assortativity	Average Local Clustering
APS	0.692	0.237
ARW	0.657	0.252
HK	0.002	0.073

Figure 1: We contrast our proposed model, Attributed Random Walk (ARW), with a non-attributed growth model [20] to underscore the importance of using attributes for network growth.

mechanisms that incorporate sociological factors to jointly explain the emergence of multiple structural properties. Additionally, accurate network growth models are useful for synthesizing networks and extrapolating existing real-world networks.

Well-known models of network growth tend to make unrealistic assumptions about how individuals form edges. Consider a simple stylized example: the process of finding a set of papers to cite when writing an article. In preferential attachment [3] or fitness [5, 10, 43] based models, a node making *m* citations would pick papers from the *entire* network in proportion to their in-degree or fitness respectively. This process assumes that individuals possess complete knowledge of in-degree or fitness of every node in the network. An equivalent formulation—vertex copying [25]—induces preferential attachment: for every citation, a node would pick a paper uniformly at random from *all* papers, and either cite it or copy its citations. Notice that vertex copying assumes individuals have complete access to the network and form each edge independently. Although these models explain the emergence of power law degree distributions, they are unrealistic: preferential attachment and vertex copying require global node-level knowledge and complete network access respectively. Additionally, they do not account for the role of assortative mixing [35] via nodal attributes (e.g., venue of paper, political interests of Facebook users) in network formation.

Recent papers account for resource constraints [32, 44, 45] and nodal attributes [12, 17]. However, the former disregard attributes and the latter do not provide a realistic representation of edge formation under constraints. Furthermore, both sets of models do not jointly preserve multiple structural properties.

We aim to develop a growth model that accounts for resource constraints and sociological phenomena influencing edge formation in addition to preserving global network structure. We make three key contributions. First, we propose a simple and accurate model of attributed network growth. Second, our model is based on local processes to form edges, without recourse to global network information. Third, our model unifies multiple sociological phenomena—bounded rationality; structural constraints; triadic

closure; attribute homophily; preferential attachment—to jointly model global network structure and attribute mixing patterns.

The proposed model—Attributed Random Walk (ARW)—jointly explains the emergence of in-degree distribution, local clustering, clustering-degree relationship and attribute mixing patterns through a resource constrained mechanism based on random walks (see Figure 1). In particular, the model relies entirely on local information to grow the network, without access to information of all nodes. In ARW, incoming nodes select a seed node based on attribute similarity and initiate a biased random walk: at each step of the walk, the incoming node either jumps back to its seed or chooses an outgoing link or incoming link to visit another node; it links to each visited node with some probability and halts after it has exhausted its budget to form connections. Our experiments on six large-scale network datasets indicate that the proposed growth model outperforms eight state-of-the-art network growth models by a statistically significant margin of 2.5–10×.

The rest of the paper is organized as follows. We begin by defining the problem statement in Section 2. In Section 3, we outline six network datasets, describe key structural properties of real-world networks and discuss insights from sociological studies. Then, in Section 4, we describe the network growth model. Then, We present experiments in Section 5, discuss related work in Section 6 and conclude in Section 7.

2 PROBLEM STATEMENT

Consider an attributed directed network $G = (V, E, B)$, where V & E are sets of nodes & edges and each node has an attribute value $b \in B$. The goal is to develop a directed network growth model that preserves structural and attribute based properties observed in G. The growth model should be normative, accurate and parsimonious:

(1) **Normative**: The model should account for multiple sociological phenomena that influence how individuals form edges under constraints of limited global information and partial network access.
(2) **Accurate**: The model should preserve key structural and attribute based properties: degree distribution, local clustering, degree-clustering relationship and attribute mixing patterns.
(3) **Parsimonious**: The model should be able to generate networks with tunable structural properties, while having few parameters.

Next, we present empirical analysis on real-world datasets to motivate our attributed random walk model.

3 EMPIRICAL ANALYSIS

We first describe six large-scale network datasets. Then, we describe global network properties, insights from empirical studies and common assumptions in network modeling.

3.1 Datasets

We consider six citation networks of different scales (size, time) from diverse sources: research articles, utility patents and judicial cases. Table 1 lists their summary statistics and global network properties. Three of the six datasets are attributed networks; that is, each node has a categorical attribute value.

We focus on citation networks for two reasons. First, since nodes in citation networks form all outgoing edges to existing nodes at the time of joining the network, these datasets provide a clean basis to study edge formation in attributed networks. Second, the node-level, temporal information in datasets that span long time periods (e.g. USSC) enables us to study structural properties at different time stages via network snapshots. Next, we study the structural and attribute properties of these networks.

3.2 Global Network Properties

Statistical descriptors of network properties [33] such as degree distribution, local clustering, and attribute assortativity quantify the extent to which edge formation shapes global network structure.

Degree distribution: Real-world networks tend to exhibit heavy tailed degree distributions in which a small but significant fraction of the nodes turn into high-degree hubs. We observe that Lognormal fits, with parameters listed in Table 1, well describe the in-degree distributions, consistent with Broido and Clauset's [9] observation that scale-free, real-world networks are rare.

Local Clustering: Real-world networks exhibit high local clustering (LCC), as shown in Table 1. Local clustering can arise from triadic closure [34, 39], where nodes with common neighbor(s) have an increased likelihood of forming a connection. The coefficient of node i equals the probability with which two randomly chosen neighbors of the node i are connected. In directed networks, the neighborhood of a node i can refer to the nodes that link to i, nodes that i links to or both. We define the neighborhood to be the set of all nodes that link to node i. In Figure 2, we show that (a) average

Table 1: Summary statistics & global properties of six network datasets: $|V|$ nodes join the networks and form edges $|E|$ over time period T. In attributed networks, each node has a categorical attribute value that belongs to set A of size $|A|$. The networks exhibit lognormal (LN) in-degree distribution with mean μ and standard deviation σ, high average local clustering (LCC) & attribute assortativity (AA) coefficient and densify over time with densification power law (DPL) exponent α.

| Network	Description	$|V|$	$|E|$	T	$A, |A|$	LN (μ, σ)	DPL α	Avg. LCC	AA r
USSC [14]	U.S. Supreme Court cases	30,288	216,738	1754-2002	-	(1.19, 1.18)	2.32	0.12	-
HEP-PH [15]	ArXiv Physics manuscripts	34,546	421,533	1992-2002	-	(1.32, 1.41)	1.67	0.12	-
Semantic [2]	Academic Search Engine	7,706,506	59,079,055	1991-2016	-	(1.78, 0.96)	1.58	0.06	-
ACL [36]	NLP papers	18,665	115,311	1965-2016	VENUE, 50	(1.93, 1.38)	1.43	0.07	0.07
APS [1]	Physics journals	577,046	6,967,873	1893-2015	JOURNAL, 13	(1.62, 1.20)	1.26	0.11	0.44
Patents [27]	U.S. NBER patents	3,923,922	16,522,438	1975-1999	CATEGORY, 6	(1.10, 1.01)	1.94	0.04	0.72
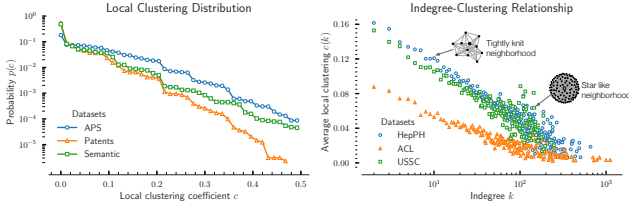
Figure 2: Real-world networks exhibit skewed local clustering distribution (left subplot) and a negatively correlated relationship between in-degree and average local clustering (right subplot).

Figure 3: Attributed networks ACL, APS and Patents exhibit homophily w.r.t attributes Venue ($r = 0.07$), Journal ($r = 0.44$) and Category ($r = 0.72$) respectively.

local clustering is not a representative statistic of the skewed local clustering distributions and (b) real-world networks exhibit a negative correlation between in-degree and clustering. That is, low in-degree nodes have small, tightly knit neighborhoods and high in-degree nodes tend have large, star-shaped neighborhoods.

Homophily: Attributed networks tend to exhibit homophily [29], the phenomenon where similar nodes are more likely to be connected than dissimilar nodes. The assortativity coefficient [35] $r \in [-1, 1]$, quantifies the level of homophily in an attributed network. Intuitively, assortativity compares the observed fraction of edges between nodes with the same attribute value to the expected fraction of edges between nodes with same attribute value if the edges were rewired randomly. In Fgiure 3, we show that attributed networks ACL, APS and Patents exhibit varying level of homophily with assortativity coefficient ranging from 0.07 to 0.72.

Increasing Out-degree over Time: The out-degree of nodes that join real-world networks tends to increase as functions of network size and time. This phenomenon densifies networks and can shrink effective diameter over time. Densification tends to exhibit a power law relationship [27] between the number of edges $e(t)$ and nodes $n(t)$ at time t: $e(t) \propto n(t)^{\alpha}$. Table 1 lists the densification power law (DPL) exponent α of the network datasets.

To summarize, citation networks tend to be homophilic networks that undergo accelerated network growth and exhibit regularities in structural properties: heavy tailed in-degree distribution, skewed local clustering distribution, negatively correlated degree-clustering relationship, and varying attribute mixing patterns.

3.3 Insights from Sociological Studies

Sociological studies on network formation seek to explain how individuals form edges in real-world networks.

Interplay of Triadic Closure and Homophily: Empirical studies [6, 24] that analyze the interplay between triadic closure and homophily indicate that *both* structural proximity and homophily are statistically significant factors that simultaneously influence edge formation. Homophilic preferences [29] induce edges between

similar nodes, whereas structural factors such as network distance limit edge formation to proximate nodes (e.g. friend of a friend).

Bounded Rationality: Extensive work [16, 28, 40] on decision making shows that individuals are boundedly rational actors; constraints such as limited information, cognitive capacity and time impact decision making. This suggests that resource-constrained individuals that join networks are likely to employ simple rules to form edges using limited information and partial network access.

Current preferential attachment and fitness-based models [3, 13, 41] make two assumptions that are at variance with these findings. First, by assuming that successive edge formations are independent, these models disregard the effect of triadic closure and structural proximity. Second, as discussed in section 1, these models require complete network access or knowledge of node-level properties.

Insights from sociological studies indicate that edge formation in real-world networks comprises biases towards nodes that are similar, well-connected or structurally proximate. Coupled with empirical analyses, it also motivates the need to model how resource-constrained edge formation processes collectively shape global network properties of large-scale networks over time.

4 ATTRIBUTED RANDOM WALK MODEL

We propose an Attributed Random Walk (ARW) model to explain the emergence of key structural properties of real-world networks through entirely local edge formation mechanisms.

Consider a stylized example of how a researcher might go about finding relevant papers to cite. First, the researcher broadly identifies one or more relevant papers, possibly with the help of external information (e.g. Google Scholar). These initial set of papers act as seed nodes. Then, acting under time and information constraints, she will examine papers cited by the seed and papers that cite the seed. Thus, she navigates a chain of backward and forward references to identify similar, relevant papers. Next, through careful analysis, she will cite a subset of these papers. Similarly, users in online social networks might form new friendships by navigating their social circle (e.g., friends of friends) to find similar others.

ARW grows a directed network as new nodes join the network. The mechanism is motivated by the stylized example: an incoming node selects a seed node and initiates a random walk to explore the network by navigating through neighborhoods of existing nodes. It halts the random walk after connecting to a few visited nodes.

In this section, we describe the edge formation mechanisms underlying ARW, explain how ARW unifies multiple sociological phenomena, and discuss methods required to fit ARW to network data.

4.1 Model Description

The Attributed Random Walk (ARW) model grows a directed network $\{\hat{G}_t\}_{t=1}^{T}$ in T time steps. At every discrete time step t, a new node u, with attribute value $B(u)$, joins the network $\hat{G}_t$. After joining the network, node u forms $m(t)$ edges to existing nodes.

The edge formation mechanism consists of two components: SELECT-SEED and RANDOM-WALK. As shown in Figure 4, an incoming node u with attribute value $B(u)$ that joins the network at time t first selects a seed node using SELECT-SEED.

SELECT-SEED accounts for homophilic preferences of incoming nodes using parameters p_{same} and p_{diff} to tune attribute preferences.
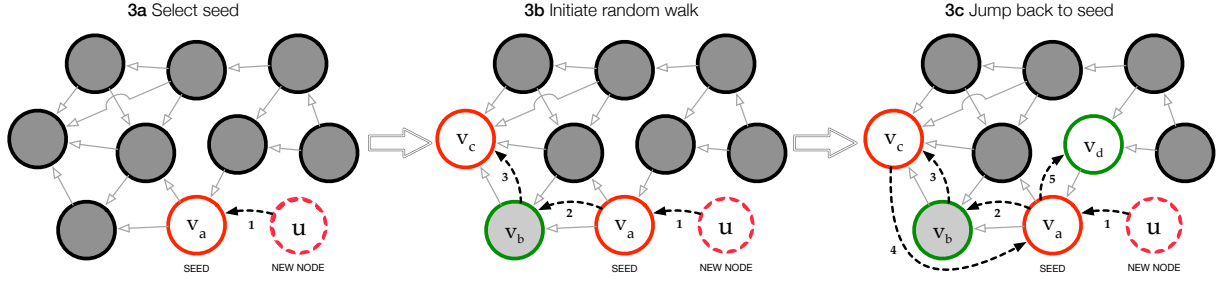
Figure 4: Edge formation in ARW: consider an incoming node u with outdegree $m = 3$ and attribute value $B(u) = \text{RED} \in \{\text{RED}, \text{GREEN}\}$. In fig. 3a, u joins the network and selects seed v_a via SELECT-SEED. Then, in fig. 3b, u initiates a RANDOM-WALK and traverses from v_a to v_b to v_c. Finally, u jumps back to its seed v_a and restarts the walk, as shown in fig. 3c. Node u halts the random walk after linking to v_a, v_c & v_d.

In Figure 4, node u selects a seed node and initiates a random walk using RANDOM-WALK to form $m(t)$ links.

SELECT-SEED

(1) With probability $p_{\text{same}}/p_{\text{same}}+p_{\text{diff}}$, randomly select a seed node from existing nodes that have the same attribute value, $B(u)$.

(2) Otherwise, with probability $p_{\text{diff}}/p_{\text{same}}+p_{\text{diff}}$, randomly select a seed node from existing nodes that do *not* have the same attribute value, $B(u)$.

The RANDOM-WALK mechanism consists of four parameters: attribute-based parameters p_{same} & p_{diff} model edge formation decisions and the jump parameter p_{jump} & out-link parameter p_{out} characterize random walk traversals:

RANDOM-WALK

(1) At each step of the walk, new node u visits node v_i.
  - If $B(u) = B(v_i)$, u links to v_i with probability p_{same}
  - Otherwise, u links to v_i with probability p_{diff}

(2) Then, with probability p_{jump}, u jumps back to seed s_u.

(3) Otherwise, with probability $1 - p_{\text{jump}}$, u continues to walk. It picks an outgoing edge with prob. p_{out} *or* an incoming edge with prob. $1 - p_{\text{out}}$ to visit a neighbor of v_i.

(4) Steps 1-3 are repeated until u links to $m(t)$ nodes.

When attribute data is absent, ARW simplifies further. A single link parameter p_{link} replaces both attribute parameters p_{same} & p_{diff}. SELECT-SEED reduces to uniform seed selection and in RANDOM-WALK, the probability of linking to visited nodes equals p_{link}.

Note that ARW has two exogenous parameters: the out-degree $m(t)$ and attribute $B(u)$ of incoming nodes. The attribute distribution varies with time as new attribute values (e.g., journals) crop up, necessitating an exogenous parameter. The parameter $m(t)$ is the mean-field value of out-degree m at time t in the observed network. While it is straightforward to model $m(t)$ endogenously by incorporating a densification power-law DPL exponent, exogenous factors (e.g., venue, topic) may influence node out-degree.

Next, we explain how each parameter is necessary to conform to normative behavior of individuals in evolving networks.

4.2 ARW and Normative Behavior

The Attributed Random Walk model unifies multiple sociological phenomena into its edge formation mechanisms.

Phenomenon 1. *(Limited Resources) Individuals are boundedly rational [16, 28, 40] actors that form edges under constraints of limited information and partial network access.*

As shown in Figure 4, RANDOM-WALK only requires information only about the 1-hop neighborhood of a few visited nodes, thereby accounting for the constraints of limited information and partial network access.

Phenomenon 2. *(Structural Constraints) Network distance act as a constraint that limits long-range connections. [24]*

We incorporate structural constraints using p_{jump}, the probability with which a new node jumps back to its seed node after every step of the random walk. This implies that the probability with which the new node is at most k steps from its seed node is $(1 - p_{\text{jump}})^k$; as a result, p_{jump} controls the extent to which nodes' random walks explore the network to form edges.

Phenomenon 3. *(Triadic Closure) Nodes with common neighbors have an increased likelihood of forming a connection. [39]*

When attribute data is absent, ARW controls the effect of triadic closure on link formation using p_{link} because with probability proportional to p_{link}^2, a new node u closes a triad through its random traversal by linking to both, a visited node and its neighbor,

Phenomenon 4. *(Attribute Homophily) Nodes that have similar attributes are more likely to form a connection. [29]*

The attribute parameters p_{same} and p_{diff} modulate attribute assortativity. When $p_{\text{same}} > p_{\text{diff}}$, nodes are more likely to connect if they share the same attribute value, thereby resulting in a homophilic network over time. Similarly, $p_{\text{same}} < p_{\text{diff}}$ and $p_{\text{same}} = p_{\text{diff}}$ make edge formation heterophilic and attribute agnostic respectively.

Phenomenon 5. *(Preferential Attachment) Nodes tend to link to high degree nodes that have more visibility. [3]*

ARW controls preferential attachment by adding structural bias to the random walk traversal using outlink parameter p_{out}, instead of relying on the global degree distribution. Random walks that traverse outgoing edges only (i.e., $p_{\text{out}} = 1$) eventually visit old nodes that tend to have high in-degree. Similarly, random walks that traverse incoming edges only (i.e., $p_{\text{out}} = 0$) visit recently joined nodes that tend to have low indegree. As a result, we use p_{out} to adjust bias towards node degree.

To summarize: ARW incorporates five well-known sociological phenomena—limited resources; structural constraints; triadic closure; attribute homophily; preferential attachment—into a single edge formation mechanism based on random walks.

4.3 Model Fitting

We now briefly describe methods to estimate model parameters, initialize $\hat{G}$, densify $\hat{G}$ over time and sample nodes' attribute values.

Parameter Estimation. We use grid search to estimate the four parameters using evaluation metrics and selection criterion described in Subsection 5.1.

Initialization. ARW is sensitive to a large number of weakly connected components (WCCs) in initial network $\hat{G}_0$ because incoming nodes only form edges to nodes in the same WCC. To ensure that $\hat{G}_0$ is weakly connected, we perform an undirected breadth-first search on the observed, to-be-fitted network G that starts from the oldest node and halts after visiting 0.1% of the nodes. The initial network $\hat{G}_0$ is the small WCC induced from the set of visited nodes.

Node Out-degree. Node out-degree increases non-linearly over time in real-world networks. We coarsely mirror the growth rate of observed network G as follows. Each incoming node u that joins $\hat{G}$ at time t corresponds to some node that joins the observed network G in year $y(t)$; the number of edges $m(t)$ that u forms is equal to the average out-degree of nodes that join G in year $y(t)$.

Sampling Attribute Values. The distribution over nodal attribute values $P_G(B)$ tends to change over time. The change in the attribute distribution over time is an exogenous factor and varies for every network. Therefore, we sample the attribute value $B(u)$ of node u, that joins $\hat{G}$ at time t, from $P_G(B \mid \text{year} = y(t))$, the observed attribute distribution conditioned on the year of arrival of node u.

To summarize, ARW uses four parameters —p_{same}, p_{diff}, p_{jump}, p_{out}— to incorporate resource-constrained individuals' biases towards similar, proximate and high degree nodes. Next, we discuss our experiments on the performance of ARW in preserving global properties of real networks.

5 MODELING NETWORK STRUCTURE

In this section, we evaluate ARW's efficacy in preserving observed network structure relative to well-known growth models.

5.1 Setup

In this subsection, we introduce eight representative growth models and describe evaluation metrics used to fit models to the datasets.

State-of-the-art Growth Models. We compare ARW to eight models representative of the key edge formation mechanisms: preferential attachment, fitness, triangle closing and random walks. Two of the eight models account for attribute homophily and preserve attribute mixing patterns, as listed in Table 2.

Ensuring Fair Comparison. To ensure fair comparison, we modify existing models in three ways. First, for DMS, SAN, KA do not have an explicitly defined initial graph, so we use initialization method used for ARW, described in subsection 4.3. Second, we extend models that use constant node outdegree m by increasing outdegree over time $m(t)$ using the method described in subsection 4.3. In the absence of model-specific parameter estimation methods, we use grid search to estimate the parameters of every network model, including ARW, using evaluation metrics and selection criterion described below.

Evaluation Metrics. We evaluate the model fit by comparing four properties of G & $\hat{G}$: degree distribution, local clustering distribution, degree-clustering relationship and attribute assortativity. We

Table 2: We evaluate the performance of our model ARW relative to three preferential attachment (PA) models, two pref. attachment & triangle closing (PA+TC) models and 3 random walk (RW) models.

Model	Abbreviation	Type	Attributed?
Dorogovtsev et al. [13]	DMS	PA	✗
Relay Linking [41]	RL	PA	✗
Kim-Altmann [22]	KA	PA	✓
Social Attribute Network [17]	SAN	PA+TC	✓
Holme-Kim [20]	HK	PA+TC	✗
Herera-Zufiria [19]	HZ	RW	✗
Saramaki-Kaski [37]	SK	RW	✗
Forest Fire [27]	FF	RW	✗

use Kolmogorov-Smirnov (KS) statistic to compare univariate distributions and Weighted Relative Error (WRE) for the degree-clustering relationship. WRE aggregates the relative error between the average local clustering $c(k)$ and $\hat{c}(k)$ of nodes with in-degree k in G and $\hat{G}$ weighted by the fraction of nodes with indegree k in G.

Jointly preserving multiple structural properties is a multi-objective optimization problem. Therefore, for each model, the selection criterion for the grid search parameter estimation method chooses the model parameters that minimizes the ℓ^2-norm of the aforementioned evaluation metrics. We normalize the metrics before computing the ℓ^2-norm to prevent unwanted bias towards any particular metric. We note that the parameter sensitivity of the Forest Fire (FF) model necessitates a manually guided grid search method.

5.2 Results

Now, we evaluate the performance of ARW relative to eight models on the datasets introduced in Subsection 3.1. For every pair of model and dataset, Figure 5 tabulates the evaluation metrics described in Subsection 5.1. These metrics are averaged over 100 runs and measure the accuracy with which the fitted models preserve key global network properties: degree distribution, local clustering distribution, degree-clustering relationship and attribute assortativity.

We use one-sided permutation tests [18] to evaluate the relative performance of ARW. If ARW performs better than a model on a dataset with significance level $\alpha = 0.01$ or $\alpha = 0.001$, the corresponding cells in Figure 5 are shaded gray (▨) or dark gray (▨) respectively. We also group models that have similar edge formation mechanisms by color-coding the corresponding rows in Figure 5. We use green ticks in Figure 5 to annotate models that preserve attribute assortativity up to two decimal places.

Existing models fail to *jointly* preserve multiple properties because they either do not account for triadic closure and homophily or are not flexible enough to generate networks with tunable structural properties. Preferential attachment models—DMS, RL, KA—preserve in-degree distributions (Figure 5A) but not clustering because they do not account for triadic closure (Figure 5B & Figure 5C). Models that use triangle closing mechanisms—HK, SAN—lead to considerable improvement over DMS and KA in modeling local clustering, but perform poorly w.r.t. degree-clustering relationship.

Figure 5: Modeling network structure. Tables 5A, 5B and 5C measure the accuracy of eight models in fitting key properties of real-world networks. Our model, ARW, jointly preserves all four properties accurately and often performs considerably better than existing models: the cells are shaded gray or dark gray if the proposed model ARW performs better at significance level $\alpha = 0.01$ (■) or $\alpha = 0.001$ (■) respectively.

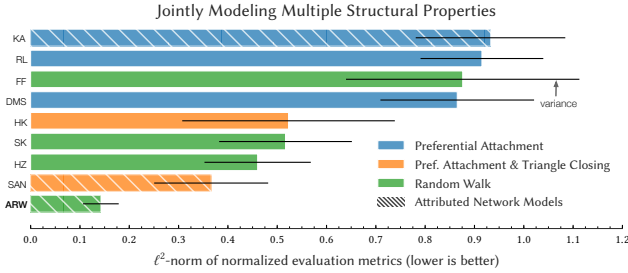|	A: Indegree Distribution (KS Stat)						B: Local Clustering Distribution (KS Stat)						C: Indegree & Clustering Relationship (WRE)							
DMS	0.03	0.03	0.05	0.09	0.04	0.02	0.80	0.82	0.56	0.63	0.83	0.50	1.00	1.00	1.00	1.00	1.00	1.00	✗
KA	0.11	0.19	0.22	0.26	0.13	0.06	0.80	0.82	0.56	0.63	0.82	0.50	1.00	1.00	1.00	1.00	1.00	1.00	✓
RL	0.12	0.12	0.17	0.15	0.07	0.15	0.79	0.82	0.56	0.62	0.83	0.50	0.99	1.00	1.00	0.99	1.00	1.00	✗
HK	0.11	0.19	0.22	0.26	0.13	0.05	0.39	0.55	0.15	0.08	0.52	0.05	0.59	0.74	0.08	0.25	0.73	0.17	✗
SAN	0.12	0.18	0.19	0.24	0.11	0.05	0.12	0.05	0.12	0.16	0.05	0.19	0.13	0.14	0.34	0.31	0.15	1.28	✓
FF	0.16	0.17	0.14	0.12	0.46	0.32	0.53	0.54	0.33	0.69	0.19	0.40	1.64	1.74	0.54	4.11	0.15	0.73	✗
SK	0.19	0.22	0.25	0.27	0.13	0.13	0.15	0.29	0.26	0.34	0.34	0.11	0.14	0.46	0.74	0.41	0.51	0.38	✗
HZ	0.18	0.22	0.23	0.26	0.13	0.13	0.08	0.29	0.10	0.07	0.34	0.03	0.18	0.45	0.21	0.22	0.51	0.04	✗
ARW	0.07	0.06	0.07	0.09	0.07	0.08	0.08	0.04	0.05	0.05	0.05	0.09	0.14	0.10	0.05	0.13	0.08	0.08	✓
	USSC	HepPH	Semantic	ACL	APS	Patents	USSC	HepPH	Semantic	ACL	APS	Patents	USSC	HepPH	Semantic	ACL	APS	Patents	Assortativity $\lvert r - \bar{r}\rvert < \epsilon$

Figure 6: ARW outperforms existing models in jointly preserving key structural properties of node in-degree and local clustering by a significant margin of 2.5x-10x.

Existing random walk models—FF, SK, HZ— do not account for homophily and attribute mixing patterns. FF, in particular, considerably overestimates local clustering because of its recursive edge formation process.

Figure 5 clearly indicates the effectiveness of ARW in jointly preserving multiple global network properties. ARW preserves observed in-degree distributions by adjusting nodes' bias towards high degree nodes using p_{out}. ARW matches the local clustering distribution (Figure 5B) and in-degree & clustering relationship (Figure 5C) with high accuracy using p_{jump} and p_{link}. ARW also preserves attribute assortativity using the attribute parameters p_{same} and p_{diff}.

To summarize, ARW unifies sociological phenomena into a single mechanism to jointly preserve key network properties significantly better than existing models, as shown in Figure 6. Code pertinent to ARW is available online [1]. Please refer to the extended version of our paper [38] for more information.

6 RELATED WORK

Preferential attachment and fitness-based models [4, 5, 10, 30] can preserve heavy-tailed degree distribution, small diameter [8] and temporal dynamics [43] of real-world networks. Extensions of preferential attachment [32, 44, 45] that account for partial network access disregard properties such as clustering and mixing patterns. Models [20, 23, 26] couple preferential attachment with triangle

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[1]Code: https://github.com/CrowdDynamicsLab/ARW

closing to incorporate triadic closure. This increases average local clustering by forming edges between nodes with one or more common neighbors, as shown in Section 5.

Models [12, 17, 21, 46] that account for attribute mixing can be largely categorized as (a) fitness-based model that define fitness as a function of attribute similarity and (b) "microscopic" growth models that require complete temporal information about edge insertions & deletion. In Subsection 5.2, we show that attributed network models SAN and KA preserve mixing patterns but do not account for other structural properties of real-world networks.

First introduced by Vazquez [42], random walk models are inherently local. Models [7] in which new nodes only link to terminal nodes of short random walks generate networks with power law degree distributions [11] and small diameter [31] but do not preserve clustering. Models such as SK [37] and HZ [19], in which new nodes probabilistically link to each visited nodes incorporate triadic closure but are not flexible enough to preserve skewed local clustering, as shown in Subsection 5.2. Recursive random walk models such as FF [27] preserve temporal properties such as shrinking diameter but considerably overestimate local clustering. Furthermore, existing random walk models do not account for nodal attributes.

To summarize, existing models do not accurately explain how resource constrained processes shape well-defined global properties of attributed networks over time. Please refer to the extended version of the paper [38] for a detailed review of existing work.

7 CONCLUSION

In this paper, we proposed a simple, interpretable model of attributed network growth. ARW grows a directed network in the following manner: an incoming node selects a seed node based on attribute similarity, initiates a biased random walk to explore the network by navigating through neighborhoods of existing nodes, and halts the random walk after connecting to a few visited nodes. To the best of our knowledge, ARW is the first model that unifies multiple sociological phenomena—bounded rationality; structural constraints; triadic closure; attribute homophily; preferential attachment—into a single local process to model global network structure *and* attribute mixing patterns. We explored the parameter space of the model

to show how each parameter intuitively controls one or more key structural properties. Our experiments on six large-scale citation networks showed that ARW outperforms relevant and recent existing models by a statistically significant factor of 2.5–10×.

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