

Model the Complex Dependence Structures of Financial Variables by Using Canonical Vine

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ABSTRACT

Financial variables such as asset returns in the massive market contain various hierarchical and horizontal relationships forming complicated dependence structures. Modeling and mining of these structures is challenging due to their own high structural complexities as well as the stylized facts of the market data. This paper introduces a new canonical vine dependence model to identify the asymmetric and non-linear dependence structures of asset returns without any prior independence assumptions. To simplify the model while maintaining its merit, a partial correlation based method is proposed to optimize the canonical vine. Compared with the original canonical vine, the new model can still maintain the most important dependence but many unimportant nodes are removed to simplify the canonical vine structure. Our model is applied to construct and analyze dependence structures of European stocks as case studies. Its performance is evaluated by measuring portfolio of Value at Risk, a widely used risk management measure. In comparison to a very recent canonical vine model and the ‘full’ model, our experimental results demonstrate that our model has a much better quality of Value at Risk, providing insightful knowledge for investors to control and reduce the aggregation risk of the portfolio.

Categories and Subject Descriptors

J.1 [Computer Applications]: Financial

General Terms

Economics

Keywords

Financial Variables, Dependence Structure, Canonical Vine

1. INTRODUCTION

Modeling the complex dependence structures of financial variables is a fundamental research problem in the financial domain useful for a wide range of applications including price prediction and risk measurement. Its extreme importance has been partially demonstrated in the 2007 global financial crisis. That financial turmoil was originated from the subprime mortgage market in the United States (US), and it quickly spread to every cell in the US and global financial system. The bankrupt of Lehman Brothers in September, 2008 marked a peak point of the crisis—the failure of the fourth largest investment company was the largest bankrupt in US history. The European financial market was heavily impacted by the collapse of Lehman Brothers as the European financial market was not isolated. If early precautions measures were taken according to the fundamental understanding of the global financial dependence, some of the crisis may be avoided.

There are various hierarchical and horizontal coupling relations in the stock markets [4]. We take European stocks as an example to explain the concept of dependence. ^STOXX50E is a composite index of European stocks, composing of 50 European stocks from six countries. Siemens and Allianz are two companies in Germany; France Telecom and Societe Generale GRP are two companies from France. It is easily understood that the price index of Siemens is directly dependent on its national index of Germany. Although sometimes not easily visible, its price index is also dependent on the composite index ^STOXX50E , the national index of France, the price index of Allianz, that of France Telecom, or that of Societe Generale GRP (see the dashed lines in Figure 1). Thus, the price of an individual stock can be

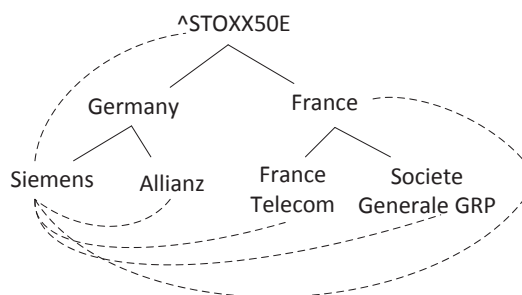


Figure 1: An Example of Dependence Structure

affected not only by the composite index and its national index, but also by other national indices and the price of in-

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dividual stocks from other countries. So, the price index of two stocks even from different countries should be dependent to each other which is however assumed as independent by past models. Our concept of this comprehensive dependence structure without prior independence assumptions can give investors an insightful understanding of the financial market. In particular, it is useful for portfolio investors to control the aggregation risk [2] of the portfolio with higher precision.

To model the price and composite index in stock markets, the price's log difference of every two consecutive trading days is usually taken, which are called returns. Early techniques in modeling dependence of returns in stock markets is by the Pearson's correlation method. It uses the average deviation from mean, ignoring the small or large returns as well as negative and positive returns. However, it is unable to explain the asymmetric correlation of stock markets as shown in [17]. Another classical idea in modeling dependence is through the use of the Capital Asset Pricing Model (CAPM). CAPM belongs to the family of factor models, in which CAPM is the simplest version with only one factor to control market. The normality assumption makes CAPM inappropriate in modeling returns of stocks, as the returns of stock markets are not normally distributed.

Copula model is a powerful tool in modeling the dependence structure for the returns of stock markets. This is mostly attributed to that copula model can separate the dependence structure from the marginal distribution. Therefore, the selection of copula functions is not constrained by the choice of marginal distributions. Another key point of copula model is its consideration of both the dependence of the portfolio of stock returns as well as correlations of individual stock returns at the same time. Recently, Heinen and Valdesogo [10] proposed a new copula model, named Canonical Vine Autoregressive (CAVA) model, which introduces three different levels of variables: market indices, sector indices and individual stock returns. It integrates these different levels of stock returns as a whole to conduct dependence analysis. However, one limit of the CAVA model is that it imposes two independence assumptions on the dependence structure. With this restriction on dependence structures, it is definitely not a perfect model to understand the complex relationships of financial variables.

The challenging task to investigate the dependence structure in stock returns contains other three difficulties. Firstly, it is difficult to build an appropriate model to describe and capture the dependence. The dependence structure will be very much complicated as the data dimension is quite large. If the dependence is constructed by using canonical vine (which is discussed in Section 2 in detail), the number of nodes increases exponentially as the the number of variables grows. It is difficult to construct and optimize the model when the variables is quite large. Second, financial variables has its own characteristics, which is called stylized facts. The most important stylized facts are volatility clustering, fat tails and asymmetry. Volatility clustering refers to the variance of returns, namely low values of volatility tend to be followed by low values; and high values of volatility followed by high values [3]. The fat tail means that extreme values in stock return occur more frequent than the implied number by a normal distribution. Asymmetry means that positive and negative returns of the same magnitude have different impact on the variance. For example, stock return will have stronger correlation in bear market downturn than

in bull market [7]. Third, daily returns are not normally distributed. Even after the daily returns being standardized by dependence model, the standardized residuals are still not normally distributed. Therefore, a new idea in modeling the dependence structure is demanded to consider both the dependence between stock returns as well as the stylized facts of financial time series at the same time.

To fulfill this need, we propose a new canonical vine based dependence model, called *Canonical Vine Dependence Model* (CVDM). CVDM can capture various and important hierarchical and horizontal dependence. In order to address the high-dimensionality issue, we use an idea of partial correlation to optimize the canonical vine structure. The canonical vine, when it is constructed and optimized by the partial correlation method, is able to model high-dimensional dependence structures. It is capable of maintaining the most important dependence, while reducing the complexity of the dependence structure remarkably, especially for high-dimensional input. In addition, we also take an ARMA-GARCH model with skewed student t distribution for marginal models to capture the stylized facts in financial variables, such as volatility clustering and fat tail.

The main contribution made by this work is the new partial correlation method to construct and optimize the canonical vine in our model CVDM. It is practically needed to optimize the canonical vine as the number of parameters in canonical vine is quit large for high dimensional data. The optimal canonical vine can reduce the number of parameters and simplify the canonical vine structure. The optimal vine can capture the most important dependence of data, without big affect on the structure of canonical vine. In addition, compared with CAVA model, the partial correlation based canonical vine model, CVDM, does not impose any independence assumption on the structure, totally eliminating any prior bias for modeling the dependence structure in stock returns. CVDM not only summarizes the various dependence with a single number, but also contains much information which is helpful for a deeper understanding of dependence structure. For example, CVDM can be used to determine the correlation between two stock returns as well as multivariate stock returns. It can be also used to calculate the conditional copula of stock return, such as the correlation of two stock returns conditioning on market indices. Further, the rich information in CVDM can be utilized to compute risk measures such as Value at Risk.

The rest of the paper is organized as follows. In Section 2, we provide a short introduction to copula and present related works. Section 3 describes the framework of our *Canonical Vine Dependence Model* in detail, including construction and optimization of canonical vine. Section 4 discusses the selection of marginal distributions. Section 5 provides simulation steps for calculating the portfolio of Value at Risk. In Section 6, we apply the Canonical Vine Dependence Model to capture the dependence of returns in European stock markets, and evaluate our model by comparing with the performance of CAVA and full canonical vine model. Finally, Section 7 concludes the paper.

2. RELATED WORK

Recently, copula theories have been rapidly developed. Copula is a powerful tool in modeling the dependence of variables. It can capture complicated correlations between variables, including linear or non-linear ones. According to

Sklar's theorem [18], a copula function is defined to connect univariate functions to form a multivariate distribution function. The definition of a copula function is given by:

$$F(x_1, \dots, x_n) = C(F_1(x_1), F_2(x_2), \dots, F_n(x_n)) \quad (1)$$

where, $x = [x_1, x_2, \dots, x_n]$ is a random variable vector, F is a joint distribution and F_1, F_2, \dots, F_n are the marginal distributions of the corresponding variables respectively. The differential of Equation (1) is:

$$f(x_1, \dots, x_n) = \prod_{i=1}^n f_i(x_i) \cdot c(F_1(x_1), F_2(x_2), \dots, F_n(x_n)) \quad (2)$$

where, c is the density copula function and $f_i(x_i)$ is the density function of marginal distributions. A useful property of copula is that given any set of marginal distribution $(F_1(x_1), F_2(x_2), \dots, F_n(x_n))$ and copula function C , we can obtain a joint distribution via Equation (1). Equation (2) implies another important property: copula function can separate dependence from marginal distributions. This indicates that the choice of copula functions is not constrained by the choice of marginal distributions and the marginal distributions do not have to be the same—the marginal distribution can even be selected from mixed families. This property makes copula flexible and be a powerful tool for modeling dependence.

One way to build high-dimensional copula models is to construct multivariate copula model based on canonical vine defined by Aas et.al [1] as:

$$c(F_1(x_1), F_2(x_2), \dots, F_n(x_n)) = \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{j,j+i|1,\dots,j-1}(F(x_j|x_1, \dots, x_{j-1}), F(x_{j+i}|x_1, \dots, x_{j-1})) \quad (3)$$

where, $c(\cdot, \cdot)$ is the pair copula density function. In Equation (3), the jointly copula function can be decomposed into many conditional pair copula functions. Canonical vine is hierarchical in nature. The basic scheme for modeling dependence with canonical vine model is to decompose multivariate density functions into many conditional pair copulas. These pair copulas are bivariate copulas in one time. The multivariate model based on canonical vine transforms one high dimensional model into multiple two-dimensional models, which makes canonical vine model the complex dependence structure with high dimensional variables.

Chollete et al. [5] proposed a multivariate copula regime-switching model based on vine copula to capture the asymmetric dependence in stock markets. Berg and Aas [1] compared multivariate model based on canonical vine with the nested Archimedean construction. The result suggested that canonical vine is better in fitting the data based on empirical study with two four-dimensional data sets. Min and Czado [16] proposed a Bayesian inference for canonical vine. They developed a Markov chain Monte Carlo (MCMC) algorithm, which can reveal unconditional and conditional independence in data sets. Smith et al [19] proposed a Bayesian method for the estimation of parameters in high-dimensional model based on canonical vine.

Recently, Heinen and Valdesogo [10] proposed a Canonical Vine Autoregressive (CAVA) model based on canonical vine. Suppose that there are four stock returns (r_{A1}, r_{A2}, r_{B1} and

r_{B2}) from two different sectors (S_A and S_B). The return of market is r_M , and the returns of sector A and B are r_A and r_B respectively. The independence assumptions imposed by the CAVA model are:

- the stocks are only dependent on their own sector returns conditioned on the market, but they are independent to all of the other sector returns. It means that r_{A1} and r_{A2} are independent to r_B conditioned on r_M , and r_{B1} and r_{B2} are independent to r_A conditioned on r_M . This leads to $c_{r_A, r_{B1}|r_M}(\cdot, \cdot) = 1$, $c_{r_A, r_{B2}|r_M}(\cdot, \cdot) = 1$, $c_{r_B, r_{A1}|r_M}(\cdot, \cdot) = 1$, and $c_{r_B, r_{A2}|r_M}(\cdot, \cdot) = 1$;
- the returns of sector conditioned on market are independent to each other. It means that r_A and r_B conditioned on r_M are independent. This leads to $c_{r_A, r_B|r_M}(\cdot, \cdot) = 1$

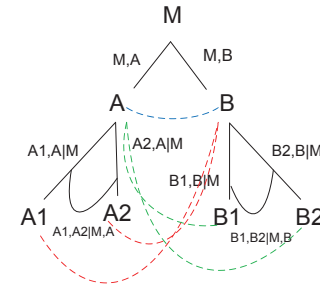


Figure 2: CAVA Dependence Structure

Thus, we can obtain the joint density function (Equation (4)) by rewriting Equation (2)

$$f(r_M, r_A, r_B, r_{A1}, r_{A2}, r_{B1}, r_{B2}) = f(r_M) \cdot f(r_A) \cdot f(r_B) \cdot f(r_{A1}) \cdot f(r_{A2}) \cdot f(r_{B1}) \cdot f(r_{B2}) \quad (4)$$

where $c(r_M, r_A, r_B, r_{A1}, r_{A2}, r_{B1}, r_{B2})$ are the copula function, which can be decomposed into the following Equation:

$$c(r_M, r_A, r_B, r_{A1}, r_{A2}, r_{B1}, r_{B2}) = c_{r_M, r_A}(F(r_M), F(r_A)) \cdot c_{r_M, r_B}(F(r_M), F(r_B)) \cdot c_{r_M, r_{A1}}(F(r_M), F(r_{A1})) \cdot c_{r_M, r_{A2}}(F(r_M), F(r_{A2})) \cdot c_{r_M, r_{B1}}(F(r_M), F(r_{B1})) \cdot c_{r_M, r_{B2}}(F(r_M), F(r_{B2})) \cdot c_{r_A, r_{A1}|r_M}(F(r_A, |r_M), F(r_{A1}|r_M)) \cdot c_{r_A, r_{A2}|r_M}(F(r_A, |r_M), F(r_{A2}|r_M)) \cdot c_{r_{A1}, r_{A2}, r_{B1}, r_{B2}|r_M, r_A, r_B}(\cdot, \cdot) \quad (5)$$

The dependence structure modeled by CAVA is described in Figure 2, where a dashed line indicates that the correlation is assumed to be independent. It is a biased assumption, not an assumption based on data analysis. It is implausible.

3. OUR CANONICAL VINE DEPENDENCE MODEL

Figure 3 shows the flow chart of our Canonical Vine Dependence Model (CVDm). It consists of two separated parts: canonical vine and marginal distributions. For the part of

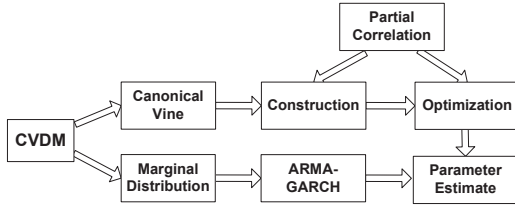


Figure 3: CVDM Work Flow

canonical vine, the first step is to construct the canonical vine, and then optimize the canonical vine. Both steps are based on partial correlation. For the second part, we take the ARMA-GARCH model for the marginal distribution with the skew student t distribution for error distribution. The parameters of both parts are estimated by using maximize likelihood estimation. A parameterized CVDM is therefore capable of measuring the risk or trends of the financial market.

3.1 Canonical Vine

As highlighted in Introduction, our CVDM is centered on the canonical vine. It is constructed by a large number of conditional pair copula functions, making it flexible and powerful for modeling the complex dependence structures of high-dimensional financial variables. We do not impose any independence assumption on the CVDM, which is more appropriate in modeling the dependence structure of stock returns. We continue the example from the Introduction to describe the main ideas of CVDM.

Suppose that there is one market M with four stocks in two sectors (A and B). The return of M is denoted by r_M , returns of sectors A and B are denoted by r_A and r_B respectively. The returns of stocks r_{A1} and r_{A2} belong to sector A , and the returns of stocks r_{B1} and r_{B2} belong to sector B . Mapping to the example discussed in Introduction, M (Market) stands for ^STOXX50E (market index), A and B stand for Germany and France respectively. $A1$, $A2$, $B1$ and $B2$ stand for Siemens, Allianz, France Telecom and Societe Generale GRP respectively. The joint density function of

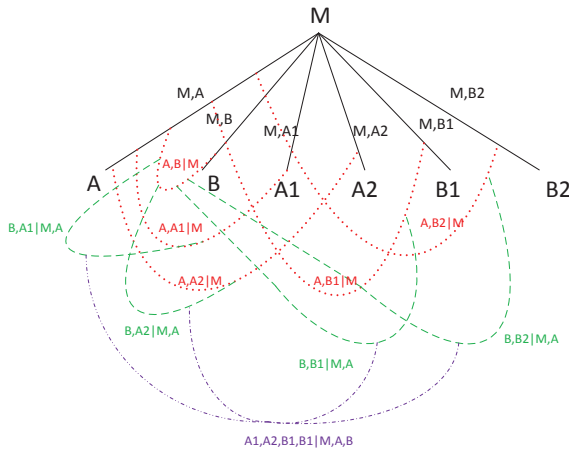


Figure 4: CVDM Dependence Structure

the CVDM model is given by:

$$f(r_M, r_A, r_B, r_{A1}, r_{A2}, r_{B1}, r_{B2}) = c(r_M, r_A, r_B, r_{A1}, r_{A2}, r_{B1}, r_{B2}) \cdot \prod f(\cdot) \quad (6)$$

where $\prod f(\cdot) = f(r_M) \cdot f(r_A) \cdot f(r_B) \cdot f(r_{A1}) \cdot f(r_{A2}) \cdot f(r_{B1}) \cdot f(r_{B2})$ are marginal distributions, which will be described in Section 4. $c(r_M, r_A, r_B, r_{A1}, r_{A2}, r_{B1}, r_{B2})$ is the copula function defined by Equation (7):

$$\begin{aligned} c(r_M, r_A, r_B, r_{A1}, r_{A2}, r_{B1}, r_{B2}) = & c_{r_M, r_A}(F(r_M), F(r_A)) \cdot c_{r_M, r_B}(F(r_M), F(r_B)) \\ & c_{r_M, r_{A1}}(F(r_M), F(r_{A1})) \cdot c_{r_M, r_{A2}}(F(r_M), F(r_{A2})) \\ & c_{r_M, r_{B1}}(F(r_M), F(r_{B1})) \cdot c_{r_M, r_{B2}}(F(r_M), F(r_{B2})) \\ & c_{r_A, r_B|r_M}(F(r_A|r_M), F(r_B|r_M)) \\ & c_{r_A, r_{A1}|r_M}(F(r_A, |r_M), F(r_{A1}|r_M)) \\ & c_{r_A, r_{A2}|r_M}(F(r_A, |r_M), F(r_{A2}|r_M)) \\ & c_{r_A, r_{B1}|r_M}(F(r_A, |r_M), F(r_{B1}|r_M)) \\ & c_{r_A, r_{B2}|r_M}(F(r_A, |r_M), F(r_{B2}|r_M)) \\ & c_{r_B, r_{A1}|r_M, r_A}(F(r_B, |r_M, r_A), F(r_{A1}|r_M, r_A)) \\ & c_{r_B, r_{A2}|r_M, r_A}(F(r_B, |r_M, r_A), F(r_{A2}|r_M, r_A)) \\ & c_{r_B, r_{B1}|r_M, r_A}(F(r_B, |r_M, r_A), F(r_{B1}|r_M, r_A)) \\ & c_{r_B, r_{B2}|r_M, r_A}(F(r_B, |r_M, r_A), F(r_{B2}|r_M, r_A)) \\ & c_{r_{A1}, r_{A2}, r_{B1}, r_{B2}|r_M, r_A, r_B}(\cdot, \cdot) \end{aligned} \quad (7)$$

where c_{r_M, r_A} stands for the copula between the return of market and the return of sector A , $c_{r_M, r_{A1}}$ is the copula between the return of market and the return of stock 1 in sector A . $c_{r_{A1}, r_{A2}, r_{B1}, r_{B2}|r_M, r_A, r_B}$ represents a four-dimensional conditional copula, meaning the dependence of the four stocks conditioned on the market and the two sectors. The term $c_{r_{A1}, r_{A2}, r_{B1}, r_{B2}|r_M, r_A, r_B}(\cdot, \cdot)$ can be decomposed into 6 conditional pair copulas.

This dependence structure is displayed in Figure 4, where the term (M, A) stands for the correlation between the market and sector A measured by the copula function c_{r_M, r_A} . The term $(A, B|M)$ means the conditional correlation between sector A and sector B conditioned on M , measured by the conditional copula function $c_{r_A, r_B|r_M}$. In the example, the canonical vine consists of 6 trees and 21 nodes. All trees and nodes (explained in Section 3.2) are shown as Figure 5. If the dependence is modeled by CAVA which imposes two independence assumptions, then the following copula functions $c_{r_A, r_B|r_M}(\cdot, \cdot)$, $c_{r_A, r_{B1}|r_M}(\cdot, \cdot)$, $c_{r_A, r_{B2}|r_M}(\cdot, \cdot)$, $c_{r_B, r_{A1}|r_M, r_A}(\cdot, \cdot)$, $c_{r_B, r_{A2}|r_M, r_A}(\cdot, \cdot)$, $c_{r_B, r_{B1}|r_M, r_A}(\cdot, \cdot)$ and $c_{r_B, r_{B2}|r_M, r_A}(\cdot, \cdot)$ are all ignored in CAVA. However, the CVDM does not make any independence assumption in the structure. The conditional independence or dependence between financial variables are judged by data instead.

3.2 Canonical Vine Dependence Construction and Optimization

The key step to construct a canonical vine is to determine the root variables for all trees. We take the example in section 3.1 to explain the root variable. In Figure 5, from first tree to sixth tree, the corresponding root variables are M , A , B , $A1$, $A2$, $B1$, $B2$ respectively. When the root variable of each tree is identified, the whole canonical vine structure is then determined. Nodes in a canonical vine are defined as the relations in this paper. Each tree has different number of nodes, and each node can only be allocated to one

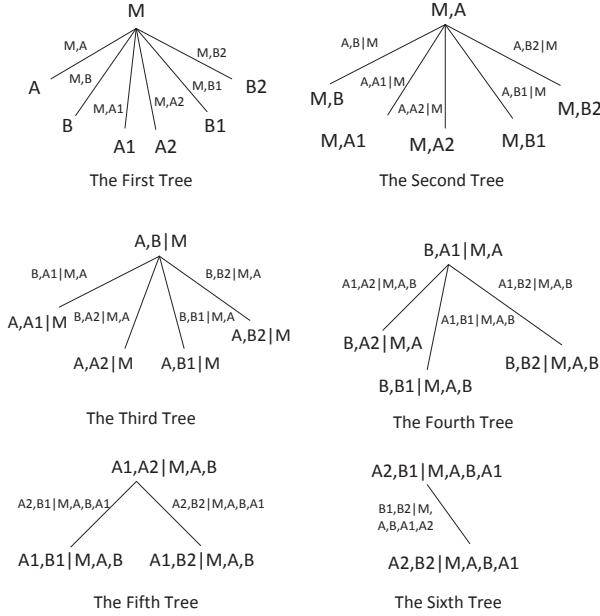


Figure 5: CVDM Trees

conditional pair copula. We take the example in Section 3.1 to explain nodes. In Figure 5, for the first tree, there are 6 nodes, and each node are allocated to one bivariate copula, including c_{r_M, r_A} , c_{r_M, r_B} , ... and $c_{r_M, r_{B2}}$. For the sixth tree, there is only one node, which is allocated to one conditional copula $c_{r_{B1}, r_{B2} | r_M, r_A, r_B, r_{A1}, r_{A2}}$. It is obvious that the number of nodes in a canonical vine will increase exponentially as the number of variables increases. Since each node has to be allocated to one conditional copula, the number of parameters will be double for the two-parameter conditional pair copulas. For ease of comprehensibility, it is essential to simplify and optimize the canonical vine.

In the Canonical Vine Dependent Model, the principle for canonical vine construction and optimization is to reduce the complexity of model without significantly affecting or changing the original structure. It means that the optimal canonical vine should capture the most important dependence, ignoring the weak dependence. Based on this principle, we use a partial correlation method to construct the canonical vine dependence structure. In CVDM, the construction of canonical vine is based on conditional copula. The new method is to build a canonical vine based on partial correlation, which has same structure as canonical vine based on conditional copula. Then, we optimize the canonical vine based partial correlation by setting the small value of partial correlation (the absolute values of partial correlation less than significance value τ) to zero, in order to decrease the number of nodes and reduce the complexity of canonical vine. The optimal canonical vine based on partial vine can be used to mapping into the canonical vine based on conditional copula, since canonical vine based on conditional copula has the same structure as the canonical vine based on partial correlation, which is explained in [13].

The reason for using partial correlation is that it is not easy to obtain the conditional copula in high-dimensional model, but partial correlations can be easily obtained from

the correlation matrix. The definition of a partial correlation is as follows:

$$\rho_{12;3,\dots,n} = \frac{\rho_{12;3,\dots,n-1} - \rho_{1n;3,\dots,n-1} \cdot \rho_{2n;3,\dots,n-1}}{\sqrt{1 - \rho_{1n;3,\dots,n-1}^2} \cdot \sqrt{1 - \rho_{2n;3,\dots,n-1}^2}} \quad (8)$$

Obviously, $\rho_{1,2}$ is equal to the 'Kendall' correlation with two variables. For elliptical distributions, partial correlations are equal to conditional copula [12]. Therefore, we can build dependent structure via partial correlations. Then, the canonical vine based on partial correlation is optimized by setting small values to zero as the small values (weak partial correlation) can be ignored. The canonical vine based on partial correlation will correspond to the canonical vine dependence structure based on conditional copula. The method to construct a canonical vine based on partial correlation is described in Algorithm 1, where the function of $\log(D)$ is explained in Equation (9).

Algorithm 1 Canonical Vine Construction and Optimization

Require: observations of n variables

- 1: Calculate all values of partial correlation, and then allocate the smallest absolute value of partial correlation to the node of last tree.
 - 2: **for** $k = 1, \dots, n - 2$ **do**
 - 3: For those trees exceeding the k th tree ($> k$) excluding the last tree, find an appropriate root variables for each tree which can minimize the function $\sum |\rho_{c;d}|$.
 - 4: For those trees from the first to the k th tree ($\leq k$), find an appropriate root variables for each tree which can minimize the function of $\sum \log(1 - \rho_{c;d}^2)$.
 - 5: **end for**
 - 6: There will be $(n - 2) - 1$ canonical vines as $k = 1, \dots, n - 2$. Calculate the function $-\log(D)$ of all of the canonical vines based on partial correlation, and choose the maximum value of the function as the 'best' canonical vine.
 - 7: For the 'best' canonical vine, the small absolute values of partial correlation, which are less than significance value τ , are set to zero.
 - 8: The optimal canonical vine based on conditional copula is corresponding to the canonical vine based on partial correlation.
 - 9: **return** The optimal canonical vine dependence structure
-

We take the example in Section 3.1 to explain how to construct a canonical vine based on the partial correlation. There are one market variable (M), two sector variables (A, B), and four stocks ($A1, A2, B1, B2$), totally 7 variables. There will have 6 trees and 21 nodes in both canonical vine structure based on partial correlation and conditional copula. Each node can be allocated to one bivariate copula or one partial correlation. For constructing the canonical vine based on conditional copula, we build the canonical vine based on partial correlation at first. The partial correlation can be obtained via the Equation 8. For these 7 variables, there are totally 21 partial correlations, including $\rho_{M,A;B,A1,A2,B1,B2}, \rho_{M,B;A,A1,A2,B1,B2}, \dots, \rho_{B1,B2;M,A,B,A1,A2}$. The smallest absolute value of these partial correlations is allocated to the root node of last tree (the sixth tree in Figure 5) as the last tree only has one node. Suppose the selected partial correlation in the last tree is $\rho_{B1,B2;M,A,B,A1,A2}$. The variables in last tree are variables $B1$ and $B2$. The sets $c_7 = \{B1, B2\}$ and $d_7 = \{M, A, B, A1, A2\}$ are called conditioned set and conditioning set respectively. For the selection of the root variable

of the second to last tree (the fifth tree in Figure 5), there are two nodes which can be allocated as two partial correlations. We have to select one root variable for the second to last tree (the fifth tree) from d_7 and generate 2 new conditioned sets. If the selected root variable of the second to last tree is A_2 , then the two new conditioned sets are $c_6 = \{A_2, B_1\}$ and $c'_6 = \{A_2, B_2\}$. The corresponding conditioning set for c_6 and c'_6 is $d_6 = \{M, A, B, A_1\}$. The partial correlations allocated to the two nodes are $\rho_{A_2, B_1; M, A, B, A_1}$ and $\rho_{A_2, B_2; M, A, B, A_1}$. If the chosen root variable is A_1 , the two new conditioned sets will be $c_6 = \{A_1, B_1\}$ and $c'_6 = \{A_1, B_2\}$. The corresponding conditioning set for c_6 and c'_6 is $d_6 = \{M, A, B, A_2\}$. The partial correlation allocated to the two nodes are $\rho_{A_1, B_1; M, A, B, A_2}$ and $\rho_{A_1, B_2; M, A, B, A_2}$. When the selected root variable of the second to last tree is M , A or B , the process is similar to A_1 and A_2 . The next step is to determine k , where k is a tree-broken level. For trees beyond the k th tree ($> k$), the appropriate root variable must minimize the value of function $\sum |\rho_{c;d}|$. For trees within the k th tree ($\leq k$), the appropriate root variables must minimize the value of function $\sum \log(1 - \rho_{c;d}^2)$. For example, suppose k is 3 in the example. For the first, second and third trees, the selected root variables for each tree must minimize the value of function $\sum |\rho_{c;d}|$. For the fourth, fifth and sixth trees, the chosen root variables for each tree must minimize the value of function $\sum \log(1 - \rho_{c;d}^2)$. The parameter k can choose different values, such as $k = 1, 2, 3, 4, 5$. Therefore, there should totally have 5 canonical vines. The 'Best' canonical vine should maximize the value of function of $-\log(D)$, where D is the determinant. The determinant (D) can be calculated by using:

$$D = \prod_{\{i,j\}} (1 - \rho_{i,j;d(i,j)}^2) \quad (9)$$

where $d(i, j)$ is the conditioning set excluding variable i, j . The corresponding conditioned set is i, j . The small absolute values of partial correlation in the 'Best' canonical vine, which are less than significance value τ , will be set to zero. Finally, the optimal canonical vine structure based on partial correlation is built. Since the canonical vine based on conditional vine has a similar structure, we can construct the optimal canonical vine based conditional copula by using the structure based on partial correlation.

3.3 Parameter Estimation

Parameters of CVDM are estimated by optimizing the following log-likelihood function:

$$L(\xi : x) := \sum_{j=1}^N \left\{ \sum_{i=1}^p \log f_i(x_{i,j}; \phi_i) + \log(c(F_1(x_{1,n}), \dots, F_p(x_{p,n}); \theta)) \right\} \quad (10)$$

where $\xi = (\phi, \theta)$ is a vector covering all parameters of the marginal distributions $\phi = (\phi_1, \dots, \phi_p)$ and the copula parameters θ . Then, we decompose Equation (10) into two parts, marginal distribution log likelihood function and copula log likelihood function. The marginal distribution log-likelihood can be written as:

$$L_m(\phi; x) = \sum_{i=1}^p \sum_{j=1}^N \log(f_{i,j}; \phi_i) \quad (11)$$

and the copula log likelihood is:

$$L_c(\theta; u, \phi) = \log(c(F_1(x_{1,n}), \dots, F_p(x_{p,n}); \theta)) \quad (12)$$

where $u = (F_1(x_1), \dots, F_p(x_p))$. Therefore, the parameters of the whole model can be estimated via two steps: marginal step and copula step. For the marginal step, parameters are estimated by optimizing the marginal log likelihood:

$$\hat{\phi} = \arg \max_{\phi} L_m(\phi; x) \quad (13)$$

For the copula step, parameters are estimated by optimizing the copula log likelihood $L_c(\theta; u, \phi)$ which is conditional on the estimated parameters ϕ for marginal distributions:

$$\hat{\theta} = \arg \max_{\theta} L_c(\theta; u, \phi) \quad (14)$$

4. MARGINAL MODELS SPECIFICATION

Volatility clustering is one of the most important characteristic of stocks, referring to the variance of returns. It says that low values of volatility tend to be followed by low values, and that high values of volatility is followed by high values. As ARMA representation of GARCH models can capture this characteristic of volatility clustering, we choose ARMA-GARCH model as the marginal distribution of our CVDM to capture the volatility of daily returns.

4.1 ARMA-GARCH Model

ARMA(p,q)-GARCH(1,1) model is defined as follows:

$$X_t = c_0 + \sum_{i=1}^p \delta_i X_{t-i} + \sum_{j=1}^q \gamma_j \varepsilon_{t-j} + \varepsilon_t \quad (15)$$

$$\varepsilon_t = \sigma_t e_t \quad (16)$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (17)$$

where $\omega > 0, \alpha \geq 0, \beta \geq 0, \alpha + \beta < 1$, X_t is actual stock return and σ_t is volatility of return on day t . The constraint $\alpha + \beta < 1$ is to maintain the GARCH weak stationary. e_t is the error as discussed in next subsection. In general, a GARCH (1, 1) model with three parameters is adequate to fit the financial time series. Hansen and Lunde [9] provided evidence that it is difficult to find a volatility model which outperforms the GARCH (1, 1) model. In this paper, we take GARCH (1, 1) to interpret how GARCH models capture volatility clustering. The GARCH coefficient is to be constant and positive. Given the value of ω , it is obvious that a small value of σ_{t-1}^2 will result in a small value of σ_t^2 , and a large value of σ_{t-1}^2 will result in a large value of σ_t^2 . Therefore, we select the ARMA-GARCH for the marginal distributions in CVDM.

4.2 Error Types

The simplest choice of an error is the standard normal distribution, such as $e_t \sim N(0, 1)$. The ARMA-GARCH with normal distribution errors indicates a symmetric distribution for observations $\{x_t\}$. However, the distribution of financial time series is not normal with negative skewness and excess kurtosis. It also has been demonstrated that the distribution of financial time series has fat tails beyond normal distribution [3]. Fat tail means that extreme values occur more frequently than that produced by a normal distribution. The ARMA-GARCH with normal error cannot take account of the asymmetry in the distribution of financial data. Therefore, the normal distribution should be excluded. It is more appropriate to use a distribution which has fatter tails than Gaussian distribution. The most commonly used fat tail distributions for fitting ARMA-GARCH

model are the skew student t distribution. Hansen [9] proposed the skew student t distribution to fit the financial time series. Its density function is as follows:

$$f_{Skewt}(e_t) = \frac{2\gamma}{1+\gamma^2} [f_t(\gamma e_t)I(e_t < 0) + f_t(\frac{e_t}{\gamma})I(e_t > 0)] \quad (18)$$

where $I(\cdot)$ is the indicator function, $\gamma > 0$ and $f_t(\cdot)$ is the density of the student t distribution with v degrees of freedom. When $\gamma = 1$, the skew student t distribution becomes student t distribution. The skew student t distribution is to skew the symmetric student t distribution by combining together two differently scaled halves of the symmetric distributions. The advantage of skew student t distribution is that it can fit the fat-tailed data very well as it has two tails behaving as polynomials. Therefore, we select the skew student t error distribution in our application.

5. PORTFOLIO OF VALUE AT RISK: A WIDELY USED MEASURE FOR RISK MANAGEMENT

Value at Risk (VaR) is a widely used risk measurement on a specific portfolio of stock returns [2]. The performance of models can be evaluated by estimating the value at risk. A good model can generate good estimates of VaR. The quality of VaR generated by different models can be judged by using backtesting methods, including unconditional coverage test [11] and the conditional coverage test [6]. These tests are based on likelihood ratios and the null hypothesis is that VaR should exhibit a conditional or unconditional coverage equal to the normal significance level α (0.05 in general). The details of these tests can be found in [8]. The portfolio of stock returns is denoted by:

$$r_{portfolio} = \sum_{i=1}^n \mu_i r_i \quad (19)$$

where r_i is the return for $i = 1, \dots, n$ and μ_i is the weight.

The process for computing a Value at Risk is as follows:

- The AR(1)-GARCH(1,1) with skewed t student distribution is specified for the marginal distribution. Then, the parameters of AR(1)-GARCH(1,1) are estimated for each stock and indices;
- Use the parameter estimations to calculate the one day forward GARCH variance forecast for each stock by using Equation (17);
- The standardized residuals obtained from AR(1)-GARCH(1,1) are transformed to uniform data by using skew student t cumulative distribution function;
- Fit a CVDM by using these data and estimated parameters by using Equation (10);
- Use the estimated parameters of copula functions to simulate a sample for each financial time series. Then, the sample is transformed to standard residuals by using the inverse skew student t cumulative probability distribution functions;
- Calculate the one day forecast return for each stocks and indices by using the estimated AR(1) model and one day forward GARCH variance which is calculated in Step (ii);

- The portfolio return is calculated by using Equation (19). Then, we repeat Steps (v) and (vi) T times (e.g. $T = 10000$). Then we can determine 99%, 95%, or 90% VaR of portfolio return.

The backtesting used to evaluate the performance of the portfolio of VaR by using the CVDM. If the VaR forecast is accurate, the VaR forecast should possess two properties. The first property is that the exceedances has to occur independently. The second property is that the proportion of exceedances should approximately equal to the significance level α . The hit variable of ex-post exceedances is given by:

$$I_t = \begin{cases} 1, & \text{if } r_{t,p} < VaR_t(1 - \alpha); \\ 0, & \text{otherwise.} \end{cases} \quad (20)$$

where $r_{t,p}$ denote the ex-post observed portfolio return at time t . If the VaR forecast is accurate, the I_t should be equal to α that is the significance level in the backtesting.

6. DEPENDENCE ANALYSIS ON EUROPEAN STOCK RETURNS

6.1 Data and Marginal Model Specification

To evaluate the performance of CVDM on real financial data, we used the log returns of 45 stocks from ^STOXX50E, a Europe-wide composite index, corresponding to the market variable (M) mentioned in the example of Section 3.1. The sector variables are the five national leading stock indices corresponding to the home country of our chosen stocks such as ^FCHI, ^GDAXIP, ^AEX, FTSEMIB.MI and ^IBEX, namely the national indices of France, Germany, Netherland, Spain and Italy. Some of these national leading stock indices are referred as the sector variables A and B by the example of Section 3.1. There are 51 variables involved in our experiment in total. The data were download from yahoo finance (<http://finance.yahoo.com>), it spans 970 days from 22/05/2006 to 30/06/2010. These stocks and indices in our experiment are listed in Table 1. The returns of these indices and stocks are calculated by taking the log difference of the prices on every two consecutive trading days.

Table 1: Indices and Stocks

Indices	Netherlands	Spain	Italy
^STOXX50E	AGN.AS	BBVA.MC	ENEL.MI
^AEX	INGA.AS	IBE.MC	ENI.MI
^IBEX	PHIA.AS	REP.MC	G.MI
^FTSEMIB.MI		SAN.MC	ISP.MI
^GDAXIP		TEF.MC	TIT.MI
^FCHI			
Germany		France	
ALV.DE	SIE.DE	ACA.PA	FTE.PA
BAYN.DE	SAP.DE	AI.PA	GLE.PA
DAI.DE		ALO.PA	GSZ.PA
DB1.DE		BN.PA	MC.PA
DBK.DE		BNP.PA	OR.PA
DTE.DE		CA.PA	SAN.PA
EOAN.DE		CS.PA	SGO.PA
MUV2.DE		DG.PA	SU.PA
RWE.DE		FP.PA	UL.PA
		VIV.PA	

We carried out experiments and obtained descriptive statistics of the data as shown in Table 2. Skewness (Skew) is a measure of asymmetry of financial time series, and Kurtosis (Kurt) is to measure the "peakedness". We can see that

all of the variables have positive skewness, excluding the national indices \hat{AEX} and \hat{IBEX} . All of the variables exhibit an excess kurtosis, indicating that they do not follow any normal distribution. This is suggestive of that different and appropriate models should be selected for their marginal distributions. As described in the method section, AR (1) - GARCH(1,1) is considered as the marginal distribution model to capture the asymmetry. The Ljung Box Q test [15] was then used for examining the existence of residual autocorrelation for each time series.

We conducted the Ljung Box Q test for each marginal model to ensure that residuals do not have autocorrelation. When marginal distributions fail the Q test, we increased the value of p and q in ARMA (p,q) - GARCH (1,1) model until all marginal distributions pass the Q test. The Ljung Q Test results of the composition index $\hat{STOXX50E}$ and the five national leading indices are shown in Table 3. The 45 stocks are not listed due to the limit of pages. In Table 3, p is the corresponding p value. The results indicate that the six indices do not have autocorrelation as all of the p values are bigger than the significance level 0.05.

Table 2: Descriptive Statistics for the Indices

	$\hat{STOXX50E}$	\hat{GDAXIP}	\hat{FCH}
Min	-0.081	-0.0975	-0.947
Max	0.1211	0.1080	0.1059
Skew	0.1178	0.2168	0.7650
Kurt	9.8754	10.1641	9.5348
	\hat{AEX}	\hat{IBEX}	FTSEMIB.MI
Min	-0.0959	-0.0959	-0.0860
Max	0.1003	0.1023	0.1088
Skew	-0.1723	-0.1330	0.0014
Kurt	10.0045	8.9012	9.1447

Table 3: Results of Ljung Box Q Test

	$\hat{STOXX50E}$	\hat{GDAXIP}	\hat{FCH}
statistics	12.7841	14.6496	11.4877
p	0.8865	0.7961	0.9326
	\hat{AEX}	\hat{IBEX}	FTSEMIB.MI
statistics	20.2718	19.0624	18.4021
p	0.4447	0.5178	0.5609

6.2 Canonical Vine Dependence Model: A Case Study

A case study was conducted to compare the detailed performance of our CVDM with a recent CAVA model and the 'full' canonical vine model. The 'full' canonical vine model is constructed by taking the 'full' canonical vine. The 'full' canonical vine can be constructed via Algorithm 1 without the optimization step. Given the 51-variable data set above, the canonical vine contains 50 trees and 1275 nodes. In the case study, all nodes are t copulas. For $k = 1, 2, 3, \dots, 49$, there are totally 49 canonical vines based on partial correlation. We choose the canonical vine based on partial correlation which can maximize the value of function $-\log(D)$, where D is the determinant of partial correlation. The result shows that when $k = 47$, the canonical vine has the maximum value of the function $-\log(D)$, which is the 'best' canonical vine. The selected root variables of the first five trees of the canonical vine are $\hat{STOXX50E}$, \hat{FCHI} , FTSEMIB.MI, \hat{AEX} and \hat{GDAXIP} . The two variables in

the last tree (the fiftieth tree) are SU.PA and CS.PA. Then, the selected canonical vine is optimized based on partial correlation. The value of partial correlation, which is less than significance value τ , is set to zero. In our experiment, we considered to use different significance values to optimize the canonical vine, and then compare the optimal canonical vine with the 'full' canonical vine. The comparison between canonical vines with different significance value τ is based on the function $-\log(D)$, where D is the determinant of partial correlation as mentioned in Section 3.2. The function is to calculate the determinant canonical vine based on partial correlation, which can also be used to compare the similarity of vine structure based on partial correlation [14].

Table 4 shows the determinants and numbers of pair copulas under various significance value τ . When τ is equal to 0, all values that are less than 0 were set to zero to indicate that the canonical vine is the 'full' canonical vine without any simplification or optimization. The value of $-\log(D)$ means the strength of a canonical vine based on partial correlation. When τ is equal to 0.1, the value of $-\log(D)$ is 27.43, which is not significantly different from the value of the 'full' canonical vine. Compared with the number of nodes in the 'full' canonical vine, the number of nodes of the canonical vine with $\tau=0.1$ is 217, decreasing the complexity of the canonical vine a lot. If the conditional pair copulas for each node are selected from t copula family, the gap of parameters will double as the t conditional pair copula has two parameters. When τ takes the value of 0.2, 0.25 or 0.3, the number of nodes decreased to 90, 69 and 59 respectively, however, the value of $-\log(D)$ are significantly different from the 'full' model. It indicates that these canonical vines with high value of τ are obviously different from the 'full' canonical vine model. We also constructed the optimal canonical vine based on conditional copula. Since the canonical vine based on conditional copula has a similar structure with that of the canonical vine based on partial correlation, we can construct the canonical vine based on conditional copula and then built our CVDM. In the experiment, we built two CVDMs with $\tau = 0.1$ and 0.2, in order to compare the performance of Value at Risk in the following section. CVDM ($\tau=0.1$) has similar determinant as the 'full' model, but the number of nodes is significantly decreased. CVDM ($\tau=0.2$) has less number of nodes, but the determinant is obviously different from the 'full' model. The performance of the two CVDMs is further evaluated in the section below.

Table 4: Determinants and Numbers of Pair Copulas

τ	0	0.01	0.02	0.03	0.04
$-\log(D)$	29.02	29.02	28.97	28.88	28.74
No. nodes	1275	1081	906	755	643
τ	0.05	0.06	0.07	0.08	0.09
$-\log(D)$	28.52	28.31	28.04	27.81	27.43
No. nodes	534	465	399	358	307
τ	0.1	0.15	0.20	0.25	0.30
$-\log(D)$	27.10	25.10	23.65	22.59	21.82
No. nodes	271	139	90	69	59

6.3 Experiments on Value at Risk

The performance of our model was further evaluated by measuring the Value at Risk (VaR). The backtesting tests discussed in Section 5 were used for validating VaR forecast generated by different models. A moving window of 970 observations, corresponding to appropriately 4 years of daily returns from 22/05/2006 to 30/6/2010, was used to construct the model. The test period was from 01/07/2010 to 01/03/2012 with 500 observations of daily returns. We calculated three VaRs: 90%, 95%, and 99% VaR. The process is explained in Section 5. The VaR of portfolio returns of 51 variables (45 stocks, 5 leading national indices and 1 composite index) in the test period was calculated by using Equation (19). We compared our CVDM ($\tau = 0.1$ and 0.2) with the 'full' model and CAVA model.

It is important to examine whether VaR are consistent with subsequently realized return given a significant level. Kupiec [11] introduced a percentage of failure of unconditional coverage to examine the quality of VaR. Christoffersen [6] proposed a more complete test to examine the conditional coverage and independence of VaR. These tests were used as backtesting for examining the quality of VaR here. The exceedances result is presented in Table 5, where POF stands for the percentage of failure. It can be seen that CVDM ($\tau = 0.1$) performs very good as the two levels of POF are in the level 95% and 90%, lower than α . At the level 99%, the exceedances is slightly increased in comparison to expected ones. Compared with CVDM ($\tau=0.1$), the 'full' canonical vine model has a similar performance. However, the CAVA model does not have a good performance, as all of the three levels of exceedances are higher than the expected one. The performance of CVDM ($\tau= 0.2$) is worse than CAVA. The corresponding display of these VaR forecasts is shown in Figure 6.

Table 6 presents statistics of a conditional coverage test, unconditional coverage test and an independent coverage test, where LRCC, LRUC, and LRIC are short for the likelihood ratio of conditional coverage, likelihood ratio of unconditional coverage and likelihood ratio of independent coverage respectively. The values in the parentheses are the corresponding P values of these tests. The smaller the statistics is, the bigger the corresponding P value is, indicating that the VaR forecast is more accurate. If the P value is bigger than 0.05, it means that the null hypothesis of backtesting can be accepted, and the VaR forecasts are accurate and reliable. For CVDM ($\tau=0.1$), it can be seen that the VaR forecasting at all of the three levels are sufficiently accurate, as the null hypotheses of the conditional, unconditional and independent cannot be rejected according to these tests. The 'full' canonical vine model has similar performance in backtesting. It is evident that the model based on our optimal canonical vine performs as good as the 'full' model, suggesting that these weak correlations can be ignored without affecting or changing the whole dependence structure. It means that our new method for constructing and optimizing canonical vine is excellent and reliable. On the other hand, the tests on CAVA suggest that VaR forecasting at all of the three levels is not accurate, since the null hypotheses of conditional, unconditional and independent at all of the three levels are rejected. The reason is that CAVA imposes two independence assumptions to ignore some strong correlations in the canonical vine. CVDM ($\tau=0.2$) has similar performance with CAVA, and the reason

is that the number of nodes is too small to capture the most important dependence in canonical vine.

We have conducted similar analysis for CVDM under different significance value (τ) listed in Table 4. The results show that for those CVDMs with τ greater than 0.1, their VaR performance are not accepted. For those CVDMs with τ less than 0.1, their performance are as good as the 'full' model, however, the number of nodes are quite large. When τ is equal to 0.1, the nodes is small enough, and the performance is accepted. We also conduct analysis for other five portfolios of European stocks, the results show that CVDM ($\tau=0.1$) performs as good as the 'full' canonical vine model, and better than CAVA's.

Table 5: Number of Exceedances of VaR Forecast

Model	VaR	α	Expected	Actual	POF
CVDM $\tau=0.1$	99%	1%	5	7	1.4%
	95%	5%	25	21	4.2%
	90%	10%	50	41	8.2%
CVDM $\tau=0.2$	99%	1%	5	16	3.2%
	95%	5%	25	43	8.6%
	90%	10%	50	96	19.2%
CAVA	99%	1%	5	13	2.6%
	95%	5%	25	39	7.8%
	90%	10%	50	82	16.4%
'Full'	99%	1%	5	7	1.4%
	95%	5%	25	20	4%
	90%	10%	50	40	8%

Table 6: The Results of VaR Backtesting

Model	VaR	LRCC	LRUC	LRIC
CVDM $\tau=0.1$	99%	3.81(0.149)	0.72(0.397)	0.91(0.44)
	95%	2.01(0.367)	0.71(0.399)	0.74(0.39)
	90%	2.16(0.340)	0.88(0.77)	0.77(0.379)
CVDM $\tau=0.2$	99%	13.21(0.001)	15.47(0.000)	6.32(0.012)
	95%	13.75(0.001)	15.04(0.000)	5.68(0.017)
	90%	14.60(0.001)	16.18(0.000)	6.03(0.014)
CAVA	99%	11.96(0.003)	10.99(0.001)	5.16(0.023)
	95%	11.17(0.004)	10.19(0.001)	3.98(0.046)
	90%	11.73(0.003)	10.45(0.001)	4.64(0.031)
'Full'	99%	3.8(0.149)	0.72(0.397)	0.77(0.381)
	95%	2.00(0.367)	0.71(0.399)	0.84(0.361)
	90%	2.28(0.320)	0.77(0.379)	0.84(0.361)

7. CONCLUSION

This work proposes a new canonical vine, as optimized based on partial correlation, to model the complex dependence structures of financial variables. It has been demonstrated useful for analyzing the complicated dependence structure of a big portfolio of European stocks. Compared with CAVA, our CVDM shows a much better performance in producing the VaR forecasts of portfolio returns. CVDM also has a similar performance of VaR forecasting as that of the 'full' canonical vine model although with a size much smaller. As a future work, we will examine in detail the strong correlations identified by CVDM to understand more about dependence structures of European/Asian/US stocks.

8. ACKNOWLEDGEMENTS

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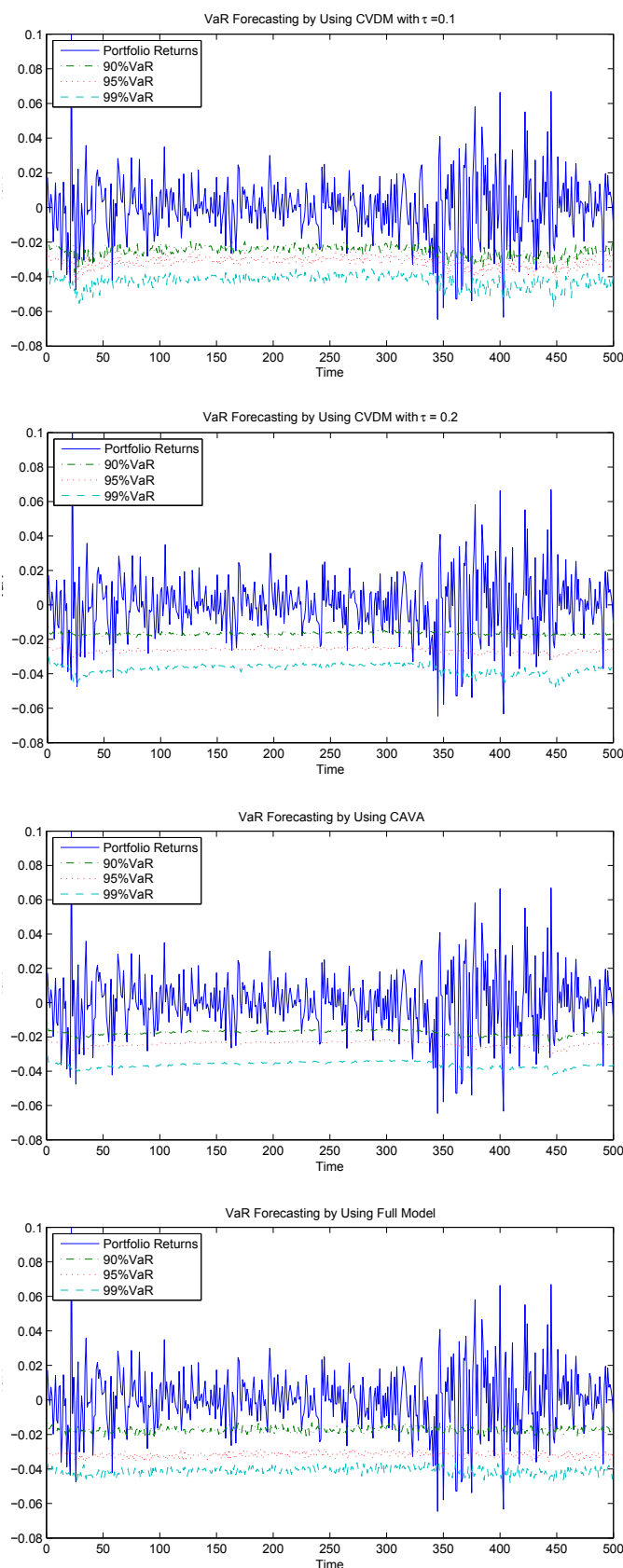


Figure 6: The VaR Forecast of Portfolio Returns

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