Proceedings of the Twenty-Fifth International Joint Conference on Artificial Intelligence (IJCAI-16)

Improved Heuristic and Tie-Breaking for Optimally Solving Sokoban

André G. Pereira
Federal University of Rio Grande do Sul, Brazil
agpereira@inf.ufrgs.br

Robert Holte and **Jonathan Schaeffer**
University of Alberta, Canada
{holte,jonathan}@cs.ualberta.ca

Luciana S. Buriol and **Marcus Ritt**
Federal University of Rio Grande do Sul, Brazil
{buriol,marcus.ritt}@inf.ufrgs.br

Abstract

We present a novel admissible pattern database heuristic (`D`) and tie-breaking rule (`L`) for Sokoban, allowing us to increase the number of optimally solved standard Sokoban instances from 20 to 28 and the number of proved optimal solutions from 25 to 32 compared to previous methods. The previously best heuristic for Sokoban (`I`) used the idea of an intermediate goal state to enable the effective use of pattern database heuristics in transportation domains, where the mapping of movable objects to goal locations is not fixed beforehand. We extend this idea to allow the use of multiple intermediate goal states and show that heuristic `I` is no longer effective. We solve this problem and show that our heuristic `D` is effective in this situation. Sokoban is a well-known single-agent search domain characterized by a large branching factor, long solution lengths, and the presence of unsolvable states. Given the exponential growth in the complexity of standard Sokoban instances, the increase in the number of optimally solved instances represents a major advance in our understanding of how to search in extremely large search spaces.

1 Introduction

A single-agent search problem is given by the *initial state*, the set of *goal states* (or goal condition) and the set of actions. Each action defines how to transform a state into a successor state with given cost. A solution is a path from the initial state to a goal state – an *optimal solution* has minimum cost. Single-agent heuristic search algorithms such as A* [Hart *et al.*, 1968] and Iterative Deepening A* [Korf, 1985] use the function $f(u) = g(u) + h(u)$ to guide the search, where $g(u)$ is the cost to reach the state u from the initial state and $h(u)$ is an estimate of the cost to reach a goal state from u. If the heuristic function h is admissible these algorithms are guaranteed to find an optimal solution. Culberson and Schaeffer [1996] proposed pattern databases (PDB) as a way to generate high-quality admissible heuristics for many single-agent search domains [Korf, 1997; Edelkamp, 2001; Korf and Felner, 2002; Felner *et al.*, 2004; Holte *et al.*, 2006; Felner *et al.*, 2007].

Sokoban is a PSPACE-complete [Culberson, 1999] single-agent search domain that is harder to solve than other common search domains considering the branching factor, solution length, domain-dependent characteristics and search space size – estimated at 10^{98} [Junghanns and Schaeffer, 2001]. In addition, no domain-independent PDB heuristic can optimally solve the easiest instance in the standard test set. Major progress has been made in solving Sokoban non-optimally [Junghanns and Schaeffer, 2001]. Currently, the best non-optimal method solves 86 [Takahashi, 2016] of the 90 standard test set instances [Myers, 2016].

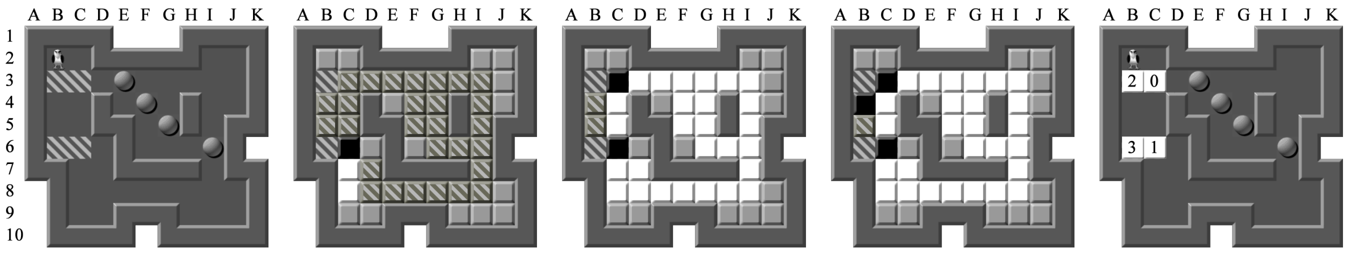
PDB heuristics are ineffective in transportation domains where the mapping of movable objects to goal locations is not fixed beforehand (this is the case in Sokoban). Pereira *et al.* [2015] introduced an admissible PDB heuristic (`I`) that addresses this issue by using an intermediate goal state. The heuristic `I` with the proposed tie-breaking rule (`F`) increases from 10 to 20 the number of optimality solved instances. However, the gap in solving abilities between optimal and non-optimal solvers is still wide.

We extend this idea to allow the use of multiple intermediate goal states. However, the heuristic `I` is the sum of two estimates which are computed independently. As we will show, the fact they are solved independently makes the heuristic ineffective when using multiple intermediate goal states. In addition, the tie-breaking rule `F` is prone to error and can guide the search to parts of the search space that will not lead to an (optimal) solution.

In this paper, we propose a novel PDB heuristic (`D`) that is effective when using multiple intermediate goal states. We also propose a tie-breaking rule (`L`) that solves the main limitations of `F`. Our method increases the number of optimally solved instances from 20 to 28 and the number of proved optimal solutions from 25 to 32. Our heuristic is general and addresses an important limitation of PDB heuristics in a large class of single-agent search domains (i.e., transportation).

2 Background

An instance of Sokoban has k movable blocks (*stones*) and k goal squares placed on a grid-square maze defined by immovable blocks (*walls*) and free squares. There is an additional block called the *man* which is the only block that can be moved directly. The man can traverse free squares and



(a) Instance. (b) One cut square. (c) Two cut squares. (d) Three cut squares. (e) Fill Order (F).

Figure 1: Decompositions with one (1b), two (1c) and three (1d) cut squares of the instance shown in (1a). Cut squares are shown in black, maze zone squares in white, goal zone squares in brown with diagonal lines and dead squares in gray. Tie-breaking rule computed by F (1e).

push (never pull) a stone to an adjacent free square. Figure 1a shows a stone at square $3E$, a goal square at $3B$, the man at square $2B$ and a wall at $1A$. A solution is a sequence of actions that moves the k stones from their initial squares to the k goal squares and an optimal solution has the minimum number of pushes. The moves of the man are not counted. Note that square $9C$ is considered *dead*; once a stone is pushed there it can never reach a goal square.

Let Q be the set of free squares of an instance, $G \subseteq Q$ the set of goal squares and B the set of stones. A state u is a pair $u = (p(B), m)$, where p is a map from the stones to the free squares and m is the position of the man. For a stone at square $p(b)$ let $\delta(p(b), g)$ be the minimum cost to push the stone at $p(b)$ to square g when the man is at m in an instance with only one stone. This cost ignores all other stones. The enhanced minimum matching (EMM) is the standard heuristic of Sokoban, it is based on a minimum cost perfect matching in the complete bipartite graph between stones and goal squares with edge set $\{(b, g) \mid b \in B, g \in G\}$ and weights $\delta(p(b), g)$. In this bipartite graph let M^* be a minimum cost perfect matching. The matching cost is enhanced with linear conflicts that increase the heuristic value by two when a pair of adjacent stones is in the optimal path of each other, each stone can be part of only one linear conflict. Let L be the number of linear conflicts in a state. Then, the value of EMM for a state u is,

$$\texttt{EMM} = \sum_{(b,g)\in M^*} \delta(p(b), g) + 2L.$$

A series of articles [Pereira *et al.*, 2013; 2014; 2015] introduced admissible PDB heuristics to Sokoban. The heuristic I [Pereira *et al.*, 2015] provides the best previous results. It uses an instance decomposition to obtain an intermediate goal state. If all stones on a set of non-dead squares M have to pass over a fixed *cut square* c to be pushed to all reachable goal squares, the cut square c is used as an intermediate goal state. The squares in M are the *maze zone* squares and all other non-dead and non-goal squares are the *goal zone* squares. Figure 1b shows an instance decomposition: the cut square shown at $6C$ is part of the maze zone, the two maze zone squares at $7C$ and $8C$ and for example a goal zone square at $7D$.

The heuristic I is the sum of the cost to solve two independent subproblems that are a relaxation of the original problem. The cut square defines the intermediate goal state and it can store many stones and the man simultaneously (this is the relaxation). The maze subproblem corresponds to the cost to push stones that are in the maze zone to the cut square. The goal subproblem corresponds to the cost to push stones at the cut square and in the goal zone to the goal squares. Let h^M and h^G be respectively the heuristic functions for the maze and goal subproblems. The value of heuristic I for a state u is defined as the sum of h^M and h^G.

A PDB is used to compute h^M. The abstraction used maintains only k' of all k stones in the instance, a PDB-k' uses an abstraction of k' stones. For this PDB, there is a unique abstract goal state that has k' stones placed at the cut square. To build the PDB the algorithm performs a reverse search from the abstract goal state to enumerate all reachable abstract states with stones k' in the maze zone. Let $\delta(p(P))$ be the cost stored in the PDB for an abstract state u' with stones $P \subseteq B$. To compute h^M the algorithm creates a partition $\mathcal{B}$ of all stones in the maze zone into parts P of size k'. Among all possible partitions $\mathcal{B}$ the heuristic tries to find one that maximizes the heuristic value as described in [Pereira *et al.*, 2015]. The value of h^M for a state u is,

$$h^M = \sum_{P\in\mathcal{B}} \delta(p(P)).$$

A modified state is used to compute h^G. Stones in the maze zone are placed at the cut square and the position of the man is changed accordingly, all stones in the goal zone remain the same. A complete description of the procedure can be found in [Pereira *et al.*, 2015]. Let d be this new map from the stones to free squares and position of the man. Let L be the number of linear conflicts in a state considering only stones in the goal zone and at the cut square. Finally, the value of h^G for a state u is defined as,

$$h^G = \sum_{(b,g)\in M^*} \delta(d(b), g) + 2L.$$

In addition to the heuristic I, Pereira *et al.* [2015] introduced the tie-breaking rule called fill order F. It uses the property that there is an order to fill the goal squares respecting the restrictions of the instance. F attempts to approximate this order by placing all stones on goal squares and removing them in an arbitrary order with reverse moves. If a reverse move is possible the stone is removed from the instance and the corresponding goal square receives a priority value equal to the number of stones removed so far. This process continues until all stones have been removed. During the search, nodes with stones on goal squares with higher numbers are preferred. Figure 1e shows the order computed by F.

3 Proposed Pattern Database Heuristic based on Multiple Intermediate Goal States

In this section, we describe a simple method to extend the heuristic I to use multiple intermediate goal states and we show why it is ineffective. We present a more complex approach our novel admissible PDB heuristic D and describe how to compute the it efficiently in Section 3.2.

3.1 Instance Decomposition and Independent Subproblems

We want to find a set of cut squares $\mathcal{C}$ that maximizes the size of the maze zone M. To find such a set we analyze all possible sets of cut squares of fixed size. Given a set $\mathcal{C}$ we perform k reverse searches one for each goal square in which we place a single stone at the goal square. In all searches the squares in $\mathcal{C}$ are blocked for the stone, it cannot be placed at them. Then, all reachable non-dead and non-goal squares in these searches are part of the goal zone, and all other non-dead squares are part of the maze zone. By construction, all stones in the set of non-dead squares M can only be pushed to the goal zone passing over the squares in $\mathcal{C}$.

We place a number in front of the letter I (and later D) to define the number of cut squares used. 1I uses one cut square and 2I uses two cut squares. The PDB construction for heuristic I using more cut squares is the same as the original approach the only difference is that there are more abstract goal states. Each combinations of k' stones on the cut squares generates an abstract goal state. With two cut squares, x stones on one cut square and y on the other, and $k' = 4$ stones, we have five abstract goal states $(x, y) = (0, 4), (1, 3), (2, 2), (3, 1)$ and $(4, 0)$. Each pair (x, y) is a unique abstract goal state. When computing h^G for each stone in the maze zone, it is unknown which cut square will be used in pushing that stone into the goal zone. Then, when placing stones in the maze zone at the cut squares, the stone will be placed at the closest cut square for each goal square. The remainder of the heuristic computation is the same.

An instance decomposition of Figure 1a using one cut square, Figure 1b, finds a maze zone with three squares ($6C$, $7C$ and $8C$). Because of that 1I and plain EMM have the same heuristic value of 27 for that instance. In an instance decomposition using two cut squares (Figure 1c), the maze zone comprises almost the whole instance, a three cut squares decomposition increases the maze zone by one square (Figure 1d). 2I with a PDB-$k' = 4$ provides the optimal solution cost of 23 for the maze subproblem, where all stones are pushed to the cut square on $3C$. When computing h^G, two stones are placed at each cut square and the cost of this subproblem is two. 2I provides the heuristic value of 25 which is lower than plain EMM; because of that small difference, 2I expands 100 times more nodes to solve the instance.

The solution to improve the heuristic value is to solve the subproblems recognizing that they are dependent – the number of stones in each cut square in both subproblems has to be the same. We call this heuristic D. Using it in the instance shown in Figure 1a it provides the heuristic value of 31 – the optimum solution cost.

3.2 PDB Construction and Heuristic Computation

In heuristic D, we have a unique PDB for each abstract goal state generated by the combinations of k' stones on the cut squares. One PDB for each pair (x, y) of the previous example. Let a be an assignment that maps stones in the maze zone to cut squares, stones in the goal zone remain the same. Let A be the set of all possible such assignments. Let $\delta_{a(P)}(p(P))$ be the cost in one of the PDBs for an abstract state with subset of stones P, the number of stones in each cut squares defined by $a(P)$ selects the PDB. To guarantee admissibility of heuristic D we have to find an assignment a that minimizes the sum of h^M and h^G. Thus, the value of heuristic D for a state u is defined as,

$$\mathtt{D} = \min_{\forall a \in A} \left[\sum_{P \in \mathcal{B}} \delta_{a(P)}(p(P)) + \sum_{(b,g) \in M^*} \delta(a(b), g) \right] + 2L.$$

In the following, we describe how to compute the partition $\mathcal{B}$ and an optimal assignment efficiently.

Partitioning Computation

When computing $\mathcal{B}$ it is unknown at the time which cut square will be assigned to each stone. We assume that each subset P will be assigned to the set of cut squares that minimizes its cost and thus selecting the PDB with minimum cost. If $k' = 2$, the optimal $\mathcal{B}$ can be found in polynomial time by a maximum weighted matching. If $k' > 2$, we use a greedy randomized constructive method based on the one proposed by [Pereira *et al.*, 2015]. The method starts by querying the cost of every subset, all $\binom{k}{k'}$ subsets. Then, it ranks the subset according to the number of conflict pushes: the difference between the cost of the subset and the cost to push each stone in the subset individually to its closest cut square. Then, a greedy randomized method selects a disjoint partition trying to maximize the sum of the conflict pushes. Only subsets with all stones in the maze zone are included in $\mathcal{B}$.

Assignment Computation

The assignment computation, in general, is the costly part of the heuristic. If the sum of conflict pushes in $\mathcal{B}$ is zero, we return the value of EMM. We only assign cut squares for stones in parts P with conflict pushes greater than zero, all other parts are removed from $\mathcal{B}$. The intuition is that we just have to select cut squares for stones that are likely to increase the heuristic value. The simplest approach is to compute the cost of all possible assignments. When checking all possible assignments we call the heuristic the exhaustive heuristic $\mathtt{D}^E$.

The difficulty is that the number of stones in $\mathcal{B}$ could be k and thus, we have to check the cost of $|\mathcal{C}|^k$ assignments for a single state. To find an optimal assignment more efficiently we propose the use of a branch and bound computation.

We use a best-first branch and bound computation (BB). At each step in the BB a stone is assigned to a cut square. At the beginning, no stones have cut squares assigned and thus the lower bound is equal to EMM. If all stones in a part have cut squares then we compute the lower bound, otherwise, we just use the lower bound of the parent in the BB tree. The lower bound is defined as the cost of the two subproblems, but only parts where all stones have assigned cut squares use the cost of the PDB. The upper bound is defined as the cost of EMM plus the number of conflict pushes. The heuristic D using the BB is called D^B.

3.3 Admissibility

In this subsection, we show that the heuristic D is admissible. Let h^* be the perfect heuristic. It is witnessed by some optimal sequence of actions S. For each stone in any part of $\mathcal{B}$, consider the corresponding subsequence of S that brings it for the first time to some cut square. Such a subsequence must exist, by definition of the cut squares. The final position of each stone in these subsequences defines an assignment a of the stones in $\mathcal{B}$ to the cut squares. For all the stones in $\mathcal{B}$, there must be a subsequence disjoint from the subsequence above, that brings it from the cut square to some goal square. Similarly, for all remaining stones there must be such a sequence. These subsequences define a matching M of stones to goal squares. Therefore, we have a pair (a, M) and the value of h^* can be defined as $h^*(a, M) = \delta(p(\mathcal{B}), a(\mathcal{B})) + \delta(a(B), M(B))$. Let $h^D(a^*, M^*) = h^M(p(\mathcal{B}), a^*(\mathcal{B})) + h^G(a^*(B), M^*(B))$ be the heuristic D with an optimal assignment a^* that minimizes the total cost given an optimal matching M^*.

Theorem 3.1. *$h^D(a^*, M^*)$ is admissible.*

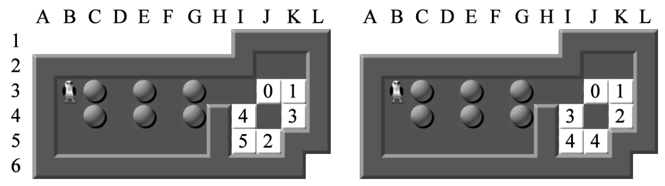
Proof. For any state, we want to show that $h^D(a^*, M^*) \leq h^*(a, M)$.

By definition, the pair (a^*, M^*) minimizes the value of h^D, any other pair (a, M) cannot provide a lower value. Thus, we have that $h^D(a^*, M^*) \leq h^D(a, M)$.

Now, we want to show that $h^D(a, M) \leq h^*(a, M)$ where (a, M) is the pair extracted from the optimal sequence S. First, consider h^M and $\delta(p(\mathcal{B}), a(\mathcal{B}))$ both heuristics compute the cost to push the same set of stones from their original squares $p(\mathcal{B})$ to the same set of cut squares $a(\mathcal{B})$. The value of $\delta(p(\mathcal{B}), a(\mathcal{B}))$ accounts for conflicts between all stones in $\mathcal{B}$, but the value of h^M accounts only for conflicts that occur within each part P and thus it must be a lower bound on $\delta(p(\mathcal{B}), a(\mathcal{B}))$. A similar argument also applies for $\delta(a(B), M(B))$ and h^G. All stones B have the same original $a(B)$ and destination $M(B)$ squares. The value of h^G accounts only for linear conflicts while $\delta(a(B), M(B))$ accounts for all conflicts which include linear conflicts. Thus, h^G must be a lower bound on $\delta(a(B), M(B))$. Therefore,

$$h^D(a^*, M^*) \leq h^D(a, M) \leq h^*(a, M).$$

$\square$



(a) Fill Order F. (b) Level Order L.

Figure 2: Two different tie-breaking rules.

4 Tie-Breaking Rule

Having the f-value equal to the optimal solution length does not necessarily mean a solution is close at hand; there can be a prohibitively large number of states remaining to be expanded. This makes tie-breaking rules important. When comparing two Sokoban states, the one with more stones on goal squares, in general, is closer to being solved. Moreover, there is an optimal order to place stones on goal squares such that a solution can be found or the solution cost is minimized. The tie-breaking rule F [Pereira *et al.*, 2015] explores these ideas to speed up the process of finding solutions. Using F the search expands fewer nodes and solves more instances, but it has three main sources of errors: the total order of the goal squares, the rule used to define the order, and the assignment of partial priorities.

A total order may be attempting too much. When there is insufficient information to define an order to fill the goal squares one should not be preferred to the other. Figure 2a shows an order defined by F. In this instance according to the rule F, the goal square with number three has to receive a stone before the goal square with number two, which is not a feasible solution and thus the order is wrong. The rule used to define the order could be strengthened to avoid this type of error. Also, giving partial priorities may prioritize nodes with stones on goal squares with lower priorities such that those with higher priorities cannot be filled anymore. These problems may cause the search to explore a large portion of the search space without finding a solution.

To solve these problems we propose the tie-breaking rule level order (L). We compute L by placing all stones on goal squares. Then, iteratively we remove a stone with reverse moves. A stone is considered removed if it can reach a square that has a stone on it on the initial state of the instance without moving other stones on goal squares. Goal squares that have their stones removed in the same iteration receive the same priority. If after one iteration no stone can be removed, all the remaining goal squares with stones receive the highest priority. Figure 2b shows the order defined by L. During the search, a node receives a priority of the goal square with a stone only if all the goal squares with higher priority (bigger numbers) already have stones. For example, (a) a node with a single stone on a goal square with priority three will not receive any priority, and (b) a node with two stones on goal squares with priority four and one stone on goal square with priority zero will receive a priority of two.

Table 1: Heuristic values for the initial states of the standard set of 90 instances. Highlighted cells in columns `1I`, `2I` and `2D` have values equal to the best-known solution. Improved values over `1I` are shown in bold. Highlighted cells in column column `UB` show proved optimal solution lengths: when the time limit is reached, the lowest f-value on the open list is equal to the best-known solution. `1I` proves the optimal solution length of 25 instances and $\texttt{2D}^B$ of 32 instances.

#	1I	2I	2D	UB	#	1I	2I	2D	UB	#	1I	2I	2D	UB	#	1I	2I	2D	UB
1	97	93	97	97	24	518	517	**534**	544	47	201	183	201	209	70	329	327	329	333
2	131	123	131	131	25	378	361	378	386	48	200	186	200	200	71	298	**300**	**302**	308
3	134	126	134	134	26	175	162	175	195	49	106	88	**114**	124	72	294	262	294	296
4	355	343	355	355	27	359	350	359	363	50	102	100	100	370	73	441	437	441	441
5	141	133	141	143	28	290	287	290	308	51	118	100	118	118	74	182	176	**190**	212
6	106	100	106	110	29	132	130	132	164	52	379	359	379	421	75	263	**267**	**273**	295
7	80	**86**	**86**	88	30	407	400	407	465	53	186	182	186	186	76	194	**196**	**198**	204
8	220	220	220	230	31	236	233	236	250	54	181	178	181	187	77	360	238	**364**	368
9	231	222	231	237	32	115	**122**	**129**	139	55	120	115	120	120	78	136	136	136	136
10	510	510	510	512	33	152	140	**170**	174	56	193	193	**201**	203	79	170	149	170	174
11	213	209	213	241	34	164	164	164	168	57	217	**219**	**219**	225	80	231	201	231	231
12	206	198	**208**	212	35	368	352	368	378	58	197	189	197	199	81	167	141	**173**	173
13	224	224	224	238	36	511	502	511	521	59	218	**222**	**222**	230	82	137	131	137	143
14	231	231	231	239	37	242	225	**246**	284	60	148	147	**150**	152	83	194	184	194	194
15	100	**106**	**108**	122	38	79	79	79	81	61	253	**255**	**255**	263	84	153	149	**155**	155
16	170	162	170	186	39	658	598	658	672	62	241	236	241	245	85	307	307	307	329
17	203	199	203	213	40	314	306	314	324	63	429	424	429	431	86	124	112	124	134
18	106	103	106	124	41	227	227	227	237	64	381	373	381	385	87	223	217	223	233
19	286	278	286	302	42	208	204	208	218	65	207	203	207	211	88	342	336	342	390
20	450	374	450	462	43	138	130	138	146	66	193	193	193	325	89	361	351	361	379
21	137	126	137	147	44	169	165	169	179	67	395	392	395	401	90	446	**447**	**448**	460
22	308	307	**310**	324	45	290	282	290	300	68	333	329	333	341					
23	432	428	432	448	46	227	217	227	247	69	223	217	223	433					

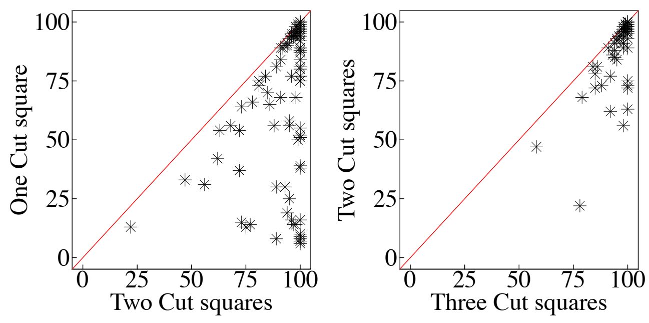
5 Experimental Results

In this section, we compare our proposed heuristic `D` and tie-breaking rule `L` with the state-of-the-art heuristic `I` and tie-breaking rule `F`. As well, we include extending heuristic `I` to multiple intermediate goal states. All experiments were run on a PC with an AMD FX-8150 CPU running at 3.6 GHz with 32 GB of RAM. We use the standard limits for Sokoban of 20 million expanded nodes or one hour of CPU time. In our experiments a search ended either because a solution was found or the time limit was reached, the node limit was never reached. The standard test set *xSokoban* of 90 instances is used.

5.1 Instance Decomposition and PDB Construction

Our first experiment is regarding instance decomposition. We compare the percentage of maze zone squares obtained by one, two, and three cut squares. These results are shown in Figure 3. Over the 90 instances, decompositions with one, two, and three cut squares on average include respectively 66%, 92% and 97% of the squares in the instance are maze zone squares. Using two cut squares has 100% of the maze zone squares in 33 instances, while using one cut has only three instances.

A two cut squares decomposition provides a considerable improvement in the percentage of maze zone squares over one cut square. Because of that we perform experiments to at most two cut squares. PDBs built with $k' = 4$ are the largest that can be built for all instances in one hour, and previously



(a) One vs. two cut squares. (b) Two vs. three cut squares.

Figure 3: Comparison of the percentage of maze zone squares considering different numbers of cut squares: (3a) two cut squares compared to one cut square has a $>10\%$ increase in the size of the maze zone in 48 instances, and (3b) three cut squares compared to two cut squares has it in 14 instances.

provided the best results. Thus, we fix the size of $k' = 4$ in all our experiments. On average, PDB-4 for `2I` has $323,572$ entries and takes 15 seconds to build, while `2D` has $695,606$ entries and takes 108 seconds.

5.2 Heuristics on Initial States and Proved Optimal Solutions

Table 1 shows the heuristic values for the initial states of the 90 instances. Column `UB` shows the best-known upper bound. The first information to be noted is that heuristic `I` when extended from one cut square `1I` to two cut squares `2I` has worse results in general, but it is still able to increase the heuristic value on some instances (e.g. #7). `2D` improves the heuristic value on average by 1.71 compared to `1I` over instances where `1I` doesn't provide the best-known solution. For some instances it may be hard to improve the heuristic value. For example, consider #10: the heuristic has not improved, but `1I` has 32% maze zone squares while `2D` has 100%. Thus during the search `2D` will detect more deadlocks. `2D` improves the heuristic value in 25% of the instances compared to `1I`, including an enormous improvement of 18 for instance #33.

Highlighted entries in Table 1 are the instances for which the optimal solution cost is now known. If the heuristic value of the initial state or the lowest f-value on the open list when the time limit is reached is equal to the best-known solution, then the optimal solution length is known. Considering only the heuristic value for the initial states 14 instances have the optimal solution proved. `1I`, `2I` and `2D` prove respectively 12, 1 and 14. The ones proved by `1I` and `2I` are a subset of `2D`. Considering the f-value when the time limit is reached, 32 instances have their optimal solution cost proven. `2D`^{B} proves 32, and `1I` proves 25, a subset of `2D`^{B}. For some instances, we may not prove the optimality of the solution cost because the upper bound is loose (best human solution). However, we can compare which heuristic is closer to solve the instance: over all algorithms, `1I` has the highest or equal highest f-value on the open list at the end of the search for 64 instances while `2D`^{B} has it for 87 instances.

5.3 Solved Instances

All methods use the same basic code infrastructure and an A* search. We applied an additional improvement to heuristic `I` to make the comparison fairer: only use the cost in the PDB if the whole subset of stones is in the maze zone. Doing this `1I` can solve one more instance.

Table 2 shows all instances solved by at least one of the methods. We use two combinations of tie-breaking rules: `IF` corresponds to the previous best tie-breaking rule using *inertia* [Junghanns and Schaeffer, 2001] as first order rule, and *fill order* as second order rule, and `LI` corresponds to our proposed tie-breaking rule `L` as first level rule, and *inertia* as second order rule. For each heuristic (`1I`, `2I`, `2D`^{E}, `2D`^{B}) there are two columns, one for each tie-breaking rule (`IF`, `LI`). A dot in a specific column indicates that the instance defining the row was solved by that combination of heuristic and tie-breaking rule. With the exception of `2I` all heuristics solve more instances using `LI`. The improvement is more significant in `2D`^{B} solving five more instances, but even `1I` benefits from `LI`. For example in instance #21 it expands 100 times fewer nodes and reduces the time by more than half an hour.

Comparing the heuristics, `2I` can only solve five instances, showing that the extension to multiple intermediate goal states is ineffective with heuristic `I`. Even with a more accurate heuristic, `2D`^{E} cannot solve more instances than `1I` due to the cost of expanding nodes. `2D`^{B} can solve more instances than `1I` with both tie-breaking rules. In computing Table 2, `2D`^{B} on average expands 10 times fewer nodes per second than `1I`, but it solves seven more instances. In large search spaces the comparison of `1I` and `2D`^{B} indicates that a heuristic that is more informed can be beneficial, even if more computationally expensive. Another point highlighted by the results of `2D`^{B} is the effort to prove the optimality and to find the solution. `2D`^{B} can prove the optimality of 32 instances. However, it can only find the optimal solution for 28.

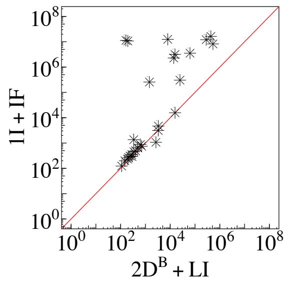
Table 2: Solved instances for different heuristics and tie-breaking rules. Only instances solved by at least one of the methods are shown.

#	**1I**		**2I**		**2D^E**		**2D^B**	
	IF	**LI**	**IF**	**LI**	**IF**	**LI**	**IF**	**LI**
1	●	●	●	●	●	●	●	●
2	●	●			●	●	●	●
3	●	●			●	●	●	●
4	●	●			●	●	●	●
5	●	●			●	●	●	●
6	●	●			●	●	●	●
7	●	●	●	●	●	●	●	●
9	●	●			●	●	●	●
17	●	●	●	●	●	●	●	●
21	●	●					●	●
33								●
38	●	●	●	●	●	●	●	●
43	●	●			●	●	●	●
48		●						●
51	●						●	●
53	●	●			●	●	●	●
55		●				●		●
57								●
60								●
65	●	●			●	●	●	●
73	●	●			●	●	●	●
78	●	●	●	●	●	●	●	●
79	●	●			●	●	●	●
80	●	●			●	●	●	●
81					●	●	●	●
82	●	●			●	●	●	●
83	●	●			●	●	●	●
84							●	●
Tot.	21	22	5	5	20	21	23	**28**

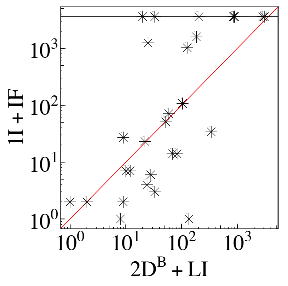
Figure 4 shows the detailed results of `1I+IF`, the previous state-of-the-art, compared to `2D`^{B}`+LI`. `2D`^{B} uses at most 307 seconds more than `1I` to solve any instance (#73). Comparing only the solved instances by both methods, `2D`^{B} uses more time in 13 instances while `1I` uses more time in 15 instances. Regarding time, there is no clear winner. Regarding expanded nodes `2D`^{B} is the clear winner. Comparing only the solved instances by both methods `2D`^{B} expands 200 times fewer nodes on average.

6 Discussion

We have shown how to effectively apply PDB heuristics in transportation domains where the mapping of movable objects to goal locations is not fixed beforehand and where multiple intermediate goal states are helpful. We use our heuristic `D` and tie-breaking rule `L` and solve optimally more instances than previous methods. Domains like



(a) Number of expanded nodes.



(b) Total time in seconds.

Figure 4: Comparison of the heuristics `1I+IF` (previous state-of-the-art) and `2D`^{B}`+LI`.

ATOMIX [Hüffner *et al.*, 2001] and AIRPORT-IPC-4 [Trug *et al.*, 2004] require `D` instead of heuristic `I` because in these domains the heuristic `1I` in general doesn't provide maze zones with effective size. Tie-breaking rules inspired in `L` could also improve the results in these domains. Other domains with similar characteristics like STORAGE and TIDYBOT (both from IPC) are likely to benefit from our techniques.

The main limitation of our approach is that an increase of the number of cut squares will make the heuristic computation more costly. If the number of cut squares is similar to the number of goal locations the heuristic `D` is unlikely to improve the results. It is reasonable to increase the number of cut squares given that we are not using any specific method to prune the `BB` and that many selections of cut squares will not produce an optimal solution. In Sokoban for example, it is often the case that if more than one stone chooses the same cut square the solution is already infeasible. We could detect this infeasibility early without `EMM`. Pruning methods based on these cases could increase the efficiency of our method.

7 Conclusion

We extend the effectiveness of PDB heuristics in transportation domains and use this to increase the number of optimally solved instances of Sokoban. Further improvements in Sokoban could be produced by better strategies to select the subsets and by increasing the number of stones in the PDB. Another improvement could be related to the tie-breaking rule. We have proven the optimal solution cost for 32 instances, but we were not able to find a solution for four of them mainly because of tie-breaking.

Acknowledgments

The first author would like to thanks the financial support of CNPq.

References

[Culberson and Schaeffer, 1996] Joseph C. Culberson and Jonathan Schaeffer. Searching with pattern databases. In *Canadian Conference on Artificial Intelligence*, pages 402–416, 1996.

[Culberson, 1999] Joseph C. Culberson. Sokoban is PSPACE-Complete. In *Fun With Algorithms*, pages 65–76, 1999.

[Edelkamp, 2001] Stefan Edelkamp. Planning with pattern databases. In *European Conference on Planning*, pages 13–24, 2001.

[Felner *et al.*, 2004] Ariel Felner, Richard E. Korf, and Sarit Hanan. Additive pattern database heuristics. *J. Artificial Intelligence Res.*, 22:279–318, 2004.

[Felner *et al.*, 2007] Ariel Felner, Richard E. Korf, Ram Meshulam, and Robert C. Holte. Compressed pattern databases. *J. Artificial Intelligence Res.*, 30:213–247, 2007.

[Hart *et al.*, 1968] Peter E. Hart, Nils J. Nilsson, and Bertram Raphael. A formal basis for the heuristic determination of minimum cost paths. *IEEE Trans. Systems Science and Cybernetics*, 4(2):100–107, 1968.

[Holte *et al.*, 2006] Robert C. Holte, Ariel Felner, Jack Newton, Ram Meshulam, and David Furcy. Maximizing over multiple pattern databases speeds up heuristic search. *Artificial Intelligence*, 170(16-17):1123–1136, 2006.

[Hüffner *et al.*, 2001] Falk Hüffner, Stefan Edelkamp, Henning Fernau, and Rolf Niedermeier. Finding optimal solutions to Atomix. In *KI: Advances in Artificial Intelligence*, pages 229–243, 2001.

[Junghanns and Schaeffer, 2001] Andreas Junghanns and Jonathan Schaeffer. Sokoban: Enhancing general single-agent search methods using domain knowledge. *Artificial Intelligence*, 129(1–2):219–251, 2001.

[Korf and Felner, 2002] Richard E. Korf and Ariel Felner. Disjoint pattern database heuristics. *Artificial Intelligence*, 134(1–2):9–22, 2002.

[Korf, 1985] Richard E. Korf. Depth-first iterative-deepening: An optimal admissible tree search. *Artificial Intelligence*, 27(1):97–109, 1985.

[Korf, 1997] Richard E. Korf. Finding optimal solutions to Rubik's Cube using pattern databases. In *AAAI Conference on Artificial Intelligence*, pages 700–705, 1997.

[Myers, 2016] Andrew Myers. Standard set. www.cs.cornell.edu/andru/xsokoban.html, February 2016.

[Pereira *et al.*, 2013] André Grahl Pereira, Marcus Ritt, and Luciana Salete Buriol. Finding optimal solutions to Sokoban using instance dependent pattern databases. In *Symposium on Combinatorial Search*, pages 141–148, 2013.

[Pereira *et al.*, 2014] André Grahl Pereira, Marcus Ritt, and Luciana Salete Buriol. Solving Sokoban optimally using pattern databases for deadlock detection. In *Encontro Nacional de Inteligência Artificial*, 2014.

[Pereira *et al.*, 2015] André Grahl Pereira, Marcus Ritt, and Luciana Salete Buriol. Optimal Sokoban solving using pattern databases with specific domain knowledge. *Artificial Intelligence*, 227:52 – 70, 2015.

[Takahashi, 2016] Ken'ichiro Takahashi. Takaken Solver. www.ic-net.or.jp/home/takaken/e/soko, February 2016.

[Trug *et al.*, 2004] Sebastian Trug, Jorg Hoffmann, and Bernhard Nebel. Applying automatic planning systems to airport ground-traffic control–a feasibility study. In *KI: Advances in Artificial Intelligence*, pages 183–197. Springer, 2004.