Conjugate prior of exponential family

- Any exponential family distribution has conjugate prior which also belongs to the exponential family. More precisely, consider

$$p(x|\eta) = h(x)\exp(\eta^\top T(x) - A(\eta))$$

It is easy to verify that following is its conjugate prior

$$p(\eta|\lambda) = \tilde{h}(\eta)\exp(\lambda_1^\top \eta - \lambda_2 A(\eta) - \tilde{A}(\lambda))$$

- Then, we have

$$p(\eta|x^n, \lambda) \propto p(\eta|\lambda)\prod_{i=1}^n p(x_i|\eta)$$

$$= \left(\tilde{h}(\eta)\prod_{i=1}^n h(x_i)\right)\exp\left(\underbrace{\left(\lambda_1 + \sum_{i=1}^n T(x_i)\right)^\top}_{\lambda_1 \leftarrow \lambda_1 + \sum_{i=1}^n T(x_i)} \eta - \underbrace{(\lambda_2 + n)}_{\lambda_2 \leftarrow \lambda_2 + n} A(\eta) - \tilde{A}(\lambda)\right)$$

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