

> `with(Physics) :`
 > `Setup(metric = [5, 29, 1])`

Systems of spacetime coordinates are: $\{X = (t, r, \theta, \phi)\}$

Default differentiation variables for $d_$, $D_$ and dA lembertian are: $\{X = (t, r, \theta, \phi)\}$

The Kerr (1963) metric in coordinates $[t, r, \theta, \phi]$

Parameters: $[a, m]$

Comments: Boyer-Lindquist coordinates

Resetting the signature of spacetime from $(- - + +)$ to $(- + + +)$ in order to match the signature in the database of metrics

$$\left[\begin{aligned} \text{metric} = & \left\{ (1, 1) = \frac{2mr - r^2 - a^2 \cos(\theta)^2}{r^2 + a^2 \cos(\theta)^2}, (1, 4) = -\frac{2mra \sin(\theta)^2}{r^2 + a^2 \cos(\theta)^2}, (2, 2) \right. \\ & = \frac{r^2 + a^2 \cos(\theta)^2}{a^2 - 2mr + r^2}, (3, 3) = r^2 + a^2 \cos(\theta)^2, (4, 4) \\ & = \left. \frac{((a^4 + r^2 a^2) \cos(\theta)^2 + 2mra^2 \sin(\theta)^2 + r^2 a^2 + r^4) \sin(\theta)^2}{r^2 + a^2 \cos(\theta)^2} \right\} \end{aligned} \right] \quad (1)$$

> `Christoffel[]`

$$\Gamma_{\alpha, \mu, \nu} = \begin{cases} 0 & \frac{m(a^2 \cos(\theta)^2 - r^2)}{(r^2 + a^2 \cos(\theta)^2)^2} \frac{2mra^2 \sin(\theta) \cos(\theta)}{(r^2 + a^2 \cos(\theta)^2)^2} \\ -\frac{m(a^2 \cos(\theta)^2 - r^2)}{(r^2 + a^2 \cos(\theta)^2)^2} & 0 \\ -\frac{2mra^2 \sin(\theta) \cos(\theta)}{(r^2 + a^2 \cos(\theta)^2)^2} & 0 \\ 0 & -\frac{ma \sin(\theta)^2 (a^2 \cos(\theta)^2 - r^2)}{(r^2 + a^2 \cos(\theta)^2)^2} - \frac{2mra \sin(\theta) \cos(\theta) (a^2 \cos(\theta)^2 - r^2)}{(r^2 + a^2 \cos(\theta)^2)^2} \end{cases}$$

> #####
 # the Riemann tensor. All non-zero components
 #####

> `Riemann[nonzero]`

$$R_{\alpha, \beta, \mu, \nu} = \left\{ (1, 2, 1, 2) = -\frac{3(a^2 \cos(\theta)^2 - 3a^2 + 4mr - 2r^2) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) rm}{(a^2 - 2mr + r^2) (r^2 + a^2 \cos(\theta)^2)^3}, \right. \quad (3)$$

$$\begin{aligned}
(1, 2, 1, 3) &= -\frac{3 \left(a^2 \cos(\theta)^2 - 3 r^2 \right) \sin(\theta) \cos(\theta) m a^2}{\left(r^2 + a^2 \cos(\theta)^2 \right)^3}, (1, 2, 2, 1) \\
&= \frac{3 \left(a^2 \cos(\theta)^2 - 3 a^2 + 4 m r - 2 r^2 \right) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3} \right) r m}{\left(a^2 - 2 m r + r^2 \right) \left(r^2 + a^2 \cos(\theta)^2 \right)^3}, (1, 2, 2, 4) \\
&= \frac{\left(3 a^2 \cos(\theta)^2 - r^2 \right) \left(3 a^2 - 4 m r + 3 r^2 \right) \sin(\theta)^2 m r a}{\left(a^2 - 2 m r + r^2 \right) \left(r^2 + a^2 \cos(\theta)^2 \right)^3}, (1, 2, 3, 1) \\
&= \frac{3 \left(a^2 \cos(\theta)^2 - 3 r^2 \right) \sin(\theta) \cos(\theta) m a^2}{\left(r^2 + a^2 \cos(\theta)^2 \right)^3}, (1, 2, 3, 4) \\
&= \frac{\left(a^2 \cos(\theta)^2 - 3 r^2 \right) \left(a^2 \cos(\theta)^2 - 3 a^2 - 2 r^2 \right) \sin(\theta) \cos(\theta) a m}{\left(r^2 + a^2 \cos(\theta)^2 \right)^3}, (1, 2, 4, 2) = \\
&- \frac{9 a \left(a^2 - \frac{4}{3} m r + r^2 \right) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3} \right) r m \sin(\theta)^2}{\left(a^2 - 2 m r + r^2 \right) \left(r^2 + a^2 \cos(\theta)^2 \right)^3}, (1, 2, 4, 3) = \\
&- \frac{\left(a^2 \cos(\theta)^2 - 3 r^2 \right) \left(a^2 \cos(\theta)^2 - 3 a^2 - 2 r^2 \right) \sin(\theta) \cos(\theta) a m}{\left(r^2 + a^2 \cos(\theta)^2 \right)^3}, (1, 3, 1, 2) = \\
&- \frac{3 \left(a^2 \cos(\theta)^2 - 3 r^2 \right) \sin(\theta) \cos(\theta) m a^2}{\left(r^2 + a^2 \cos(\theta)^2 \right)^3}, (1, 3, 1, 3) \\
&= \frac{\left(3 a^2 \cos(\theta)^2 - r^2 \right) \left(2 a^2 \cos(\theta)^2 - 3 a^2 + 2 m r - r^2 \right) m r}{\left(r^2 + a^2 \cos(\theta)^2 \right)^3}, (1, 3, 2, 1) \\
&= \frac{3 \left(a^2 \cos(\theta)^2 - 3 r^2 \right) \sin(\theta) \cos(\theta) m a^2}{\left(r^2 + a^2 \cos(\theta)^2 \right)^3}, (1, 3, 2, 4) \\
&= \frac{\left(a^2 \cos(\theta)^2 - 3 r^2 \right) \left(2 a^2 \cos(\theta)^2 - 3 a^2 - r^2 \right) \sin(\theta) \cos(\theta) a m}{\left(r^2 + a^2 \cos(\theta)^2 \right)^3}, (1, 3, 3, 1) =
\end{aligned}$$

$$\begin{aligned}
& - \frac{(3 a^2 \cos(\theta)^2 - r^2) (2 a^2 \cos(\theta)^2 - 3 a^2 + 2 m r - r^2) m r}{(r^2 + a^2 \cos(\theta)^2)^3}, (1, 3, 3, 4) = \\
& - \frac{9 a \left(a^2 - \frac{2}{3} m r + r^2\right) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m \sin(\theta)^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (1, 3, 4, 2) = \\
& - \frac{2 a \cos(\theta) (a^2 \cos(\theta)^2 - 3 r^2) m \sin(\theta) \left(a^2 \cos(\theta)^2 - \frac{3 a^2}{2} - \frac{r^2}{2}\right)}{(r^2 + a^2 \cos(\theta)^2)^3}, (1, 3, 4, 3) \\
& = \frac{(3 a^2 \cos(\theta)^2 - r^2) (3 a^2 - 2 m r + 3 r^2) \sin(\theta)^2 a m r}{(r^2 + a^2 \cos(\theta)^2)^3}, (1, 4, 1, 4) = \\
& - \frac{3 (a^2 - 2 m r + r^2) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m \sin(\theta)^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (1, 4, 2, 3) \\
& = \frac{(a^2 \cos(\theta)^2 - 3 r^2) \sin(\theta) \cos(\theta) a m}{(r^2 + a^2 \cos(\theta)^2)^2}, (1, 4, 3, 2) = \\
& - \frac{(a^2 \cos(\theta)^2 - 3 r^2) \sin(\theta) \cos(\theta) a m}{(r^2 + a^2 \cos(\theta)^2)^2}, (1, 4, 4, 1) \\
& = \frac{3 (a^2 - 2 m r + r^2) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m \sin(\theta)^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (2, 1, 1, 2) \\
& = \frac{3 (a^2 \cos(\theta)^2 - 3 a^2 + 4 m r - 2 r^2) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m}{(a^2 - 2 m r + r^2) (r^2 + a^2 \cos(\theta)^2)^3}, (2, 1, 1, 3) \\
& = \frac{3 (a^2 \cos(\theta)^2 - 3 r^2) \sin(\theta) \cos(\theta) m a^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (2, 1, 2, 1) = \\
& - \frac{3 (a^2 \cos(\theta)^2 - 3 a^2 + 4 m r - 2 r^2) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m}{(a^2 - 2 m r + r^2) (r^2 + a^2 \cos(\theta)^2)^3}, (2, 1, 2, 4) =
\end{aligned}$$

$$-\frac{9 a \left(a^2 - \frac{4}{3} m r + r^2\right) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m \sin(\theta)^2}{(a^2 - 2 m r + r^2) \left(r^2 + a^2 \cos(\theta)^2\right)^3}, (2, 1, 3, 1) =$$

$$-\frac{3 \left(a^2 \cos(\theta)^2 - 3 r^2\right) \sin(\theta) \cos(\theta) m a^2}{\left(r^2 + a^2 \cos(\theta)^2\right)^3}, (2, 1, 3, 4) =$$

$$-\frac{\left(a^2 \cos(\theta)^2 - 3 r^2\right) \left(a^2 \cos(\theta)^2 - 3 a^2 - 2 r^2\right) \sin(\theta) \cos(\theta) a m}{\left(r^2 + a^2 \cos(\theta)^2\right)^3}, (2, 1, 4, 2)$$

$$=\frac{\left(3 a^2 \cos(\theta)^2 - r^2\right) \left(3 a^2 - 4 m r + 3 r^2\right) \sin(\theta)^2 m r a}{(a^2 - 2 m r + r^2) \left(r^2 + a^2 \cos(\theta)^2\right)^3}, (2, 1, 4, 3)$$

$$=\frac{\left(a^2 \cos(\theta)^2 - 3 r^2\right) \left(a^2 \cos(\theta)^2 - 3 a^2 - 2 r^2\right) \sin(\theta) \cos(\theta) a m}{\left(r^2 + a^2 \cos(\theta)^2\right)^3}, (2, 3, 1, 4)$$

$$=\frac{\left(a^2 \cos(\theta)^2 - 3 r^2\right) \sin(\theta) \cos(\theta) a m}{\left(r^2 + a^2 \cos(\theta)^2\right)^2}, (2, 3, 2, 3)$$

$$=\frac{m r \left(3 a^2 \cos(\theta)^2 - r^2\right)}{\left(r^2 + a^2 \cos(\theta)^2\right) (a^2 - 2 m r + r^2)}, (2, 3, 3, 2) =$$

$$-\frac{3 \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m}{\left(r^2 + a^2 \cos(\theta)^2\right) (a^2 - 2 m r + r^2)}, (2, 3, 4, 1) =$$

$$-\frac{\left(a^2 \cos(\theta)^2 - 3 r^2\right) \sin(\theta) \cos(\theta) a m}{\left(r^2 + a^2 \cos(\theta)^2\right)^2}, (2, 4, 1, 2)$$

$$=\frac{\left(3 a^2 \cos(\theta)^2 - r^2\right) \left(3 a^2 - 4 m r + 3 r^2\right) \sin(\theta)^2 m r a}{(a^2 - 2 m r + r^2) \left(r^2 + a^2 \cos(\theta)^2\right)^3}, (2, 4, 1, 3)$$

$$=\frac{\left(a^2 \cos(\theta)^2 - 3 r^2\right) \left(2 a^2 \cos(\theta)^2 - 3 a^2 - r^2\right) \sin(\theta) \cos(\theta) a m}{\left(r^2 + a^2 \cos(\theta)^2\right)^3}, (2, 4, 2, 1) =$$

$$\begin{aligned}
& - \frac{9 a \left(a^2 - \frac{4}{3} m r + r^2 \right) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3} \right) r m \sin(\theta)^2}{(a^2 - 2 m r + r^2) (r^2 + a^2 \cos(\theta)^2)^3}, (2, 4, 2, 4) = \\
& - \frac{1}{(a^2 - 2 m r + r^2) (r^2 + a^2 \cos(\theta)^2)^3} \left(6 \left(a^2 \cos(\theta)^2 - \frac{r^2}{3} \right) r m \sin(\theta)^2 \left(a^2 (a^2 \right. \right. \\
& \left. \left. - 2 m r + r^2) \cos(\theta)^2 - \frac{3 a^4}{2} + 2 m r a^2 - 2 r^2 a^2 - \frac{r^4}{2} \right) \right), (2, 4, 3, 1) = \\
& - \frac{2 a \cos(\theta) (a^2 \cos(\theta)^2 - 3 r^2) m \sin(\theta) \left(a^2 \cos(\theta)^2 - \frac{3 a^2}{2} - \frac{r^2}{2} \right)}{(r^2 + a^2 \cos(\theta)^2)^3}, (2, 4, 3, 4) = \\
& - \frac{3 (a^2 \cos(\theta)^2 - 3 r^2) (a^2 + r^2) m \cos(\theta) \sin(\theta)^3 a^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (2, 4, 4, 2) \\
& = \frac{1}{(a^2 - 2 m r + r^2) (r^2 + a^2 \cos(\theta)^2)^3} \left(6 \left(a^2 \cos(\theta)^2 - \frac{r^2}{3} \right) r m \sin(\theta)^2 \left(a^2 (a^2 \right. \right. \\
& \left. \left. - 2 m r + r^2) \cos(\theta)^2 - \frac{3 a^4}{2} + 2 m r a^2 - 2 r^2 a^2 - \frac{r^4}{2} \right) \right), (2, 4, 4, 3) \\
& = \frac{3 (a^2 \cos(\theta)^2 - 3 r^2) (a^2 + r^2) m \cos(\theta) \sin(\theta)^3 a^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (3, 1, 1, 2) \\
& = \frac{3 (a^2 \cos(\theta)^2 - 3 r^2) \sin(\theta) \cos(\theta) m a^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (3, 1, 1, 3) = \\
& - \frac{(3 a^2 \cos(\theta)^2 - r^2) (2 a^2 \cos(\theta)^2 - 3 a^2 + 2 m r - r^2) m r}{(r^2 + a^2 \cos(\theta)^2)^3}, (3, 1, 2, 1) = \\
& - \frac{3 (a^2 \cos(\theta)^2 - 3 r^2) \sin(\theta) \cos(\theta) m a^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (3, 1, 2, 4) =
\end{aligned}$$

$$-\frac{2 a \cos(\theta) \left(a^2 \cos(\theta)^2 - 3 r^2\right) m \sin(\theta) \left(a^2 \cos(\theta)^2 - \frac{3 a^2}{2} - \frac{r^2}{2}\right)}{\left(r^2 + a^2 \cos(\theta)^2\right)^3}, (3, 1, 3, 1)$$

$$= \frac{(3 a^2 \cos(\theta)^2 - r^2) (2 a^2 \cos(\theta)^2 - 3 a^2 + 2 m r - r^2) m r}{\left(r^2 + a^2 \cos(\theta)^2\right)^3}, (3, 1, 3, 4)$$

$$= \frac{(3 a^2 \cos(\theta)^2 - r^2) (3 a^2 - 2 m r + 3 r^2) \sin(\theta)^2 a m r}{\left(r^2 + a^2 \cos(\theta)^2\right)^3}, (3, 1, 4, 2)$$

$$= \frac{(a^2 \cos(\theta)^2 - 3 r^2) (2 a^2 \cos(\theta)^2 - 3 a^2 - r^2) \sin(\theta) \cos(\theta) a m}{\left(r^2 + a^2 \cos(\theta)^2\right)^3}, (3, 1, 4, 3) =$$

$$- \frac{9 a \left(a^2 - \frac{2}{3} m r + r^2\right) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m \sin(\theta)^2}{\left(r^2 + a^2 \cos(\theta)^2\right)^3}, (3, 2, 1, 4) =$$

$$- \frac{(a^2 \cos(\theta)^2 - 3 r^2) \sin(\theta) \cos(\theta) a m}{\left(r^2 + a^2 \cos(\theta)^2\right)^2}, (3, 2, 2, 3) =$$

$$- \frac{3 \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m}{\left(r^2 + a^2 \cos(\theta)^2\right) (a^2 - 2 m r + r^2)}, (3, 2, 3, 2)$$

$$= \frac{m r (3 a^2 \cos(\theta)^2 - r^2)}{\left(r^2 + a^2 \cos(\theta)^2\right) (a^2 - 2 m r + r^2)}, (3, 2, 4, 1)$$

$$= \frac{(a^2 \cos(\theta)^2 - 3 r^2) \sin(\theta) \cos(\theta) a m}{\left(r^2 + a^2 \cos(\theta)^2\right)^2}, (3, 4, 1, 2)$$

$$= \frac{(a^2 \cos(\theta)^2 - 3 r^2) (a^2 \cos(\theta)^2 - 3 a^2 - 2 r^2) \sin(\theta) \cos(\theta) a m}{\left(r^2 + a^2 \cos(\theta)^2\right)^3}, (3, 4, 1, 3) =$$

$$- \frac{9 a \left(a^2 - \frac{2}{3} m r + r^2\right) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m \sin(\theta)^2}{\left(r^2 + a^2 \cos(\theta)^2\right)^3}, (3, 4, 2, 1) =$$

$$\begin{aligned}
& - \frac{(a^2 \cos(\theta)^2 - 3r^2) (a^2 \cos(\theta)^2 - 3a^2 - 2r^2) \sin(\theta) \cos(\theta) am}{(r^2 + a^2 \cos(\theta)^2)^3}, (3, 4, 2, 4) = \\
& - \frac{3(a^2 \cos(\theta)^2 - 3r^2) (a^2 + r^2) m \cos(\theta) \sin(\theta)^3 a^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (3, 4, 3, 1) \\
& = \frac{(3a^2 \cos(\theta)^2 - r^2) (3a^2 - 2mr + 3r^2) \sin(\theta)^2 amr}{(r^2 + a^2 \cos(\theta)^2)^3}, (3, 4, 3, 4) \\
& = \frac{1}{(r^2 + a^2 \cos(\theta)^2)^3} \left(3(a^2(a^2 - 2mr + r^2) \cos(\theta)^2 - 3a^4 + (2mr - 5r^2)a^2 \right. \\
& \quad \left. - 2r^4) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3} \right) rm \sin(\theta)^2 \right), (3, 4, 4, 2) \\
& = \frac{3(a^2 \cos(\theta)^2 - 3r^2) (a^2 + r^2) m \cos(\theta) \sin(\theta)^3 a^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (3, 4, 4, 3) = \\
& - \frac{1}{(r^2 + a^2 \cos(\theta)^2)^3} \left(3(a^2(a^2 - 2mr + r^2) \cos(\theta)^2 - 3a^4 + (2mr - 5r^2)a^2 \right. \\
& \quad \left. - 2r^4) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3} \right) rm \sin(\theta)^2 \right), (4, 1, 1, 4) \\
& = \frac{3(a^2 - 2mr + r^2) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3} \right) rm \sin(\theta)^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (4, 1, 2, 3) = \\
& - \frac{(a^2 \cos(\theta)^2 - 3r^2) \sin(\theta) \cos(\theta) am}{(r^2 + a^2 \cos(\theta)^2)^2}, (4, 1, 3, 2) \\
& = \frac{(a^2 \cos(\theta)^2 - 3r^2) \sin(\theta) \cos(\theta) am}{(r^2 + a^2 \cos(\theta)^2)^2}, (4, 1, 4, 1) =
\end{aligned}$$

$$\begin{aligned}
& - \frac{3 (a^2 - 2 m r + r^2) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m \sin(\theta)^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (4, 2, 1, 2) = \\
& - \frac{9 a \left(a^2 - \frac{4}{3} m r + r^2\right) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m \sin(\theta)^2}{(a^2 - 2 m r + r^2) (r^2 + a^2 \cos(\theta)^2)^3}, (4, 2, 1, 3) = \\
& - \frac{2 a \cos(\theta) (a^2 \cos(\theta)^2 - 3 r^2) m \sin(\theta) \left(a^2 \cos(\theta)^2 - \frac{3 a^2}{2} - \frac{r^2}{2}\right)}{(r^2 + a^2 \cos(\theta)^2)^3}, (4, 2, 2, 1) \\
& = \frac{(3 a^2 \cos(\theta)^2 - r^2) (3 a^2 - 4 m r + 3 r^2) \sin(\theta)^2 m r a}{(a^2 - 2 m r + r^2) (r^2 + a^2 \cos(\theta)^2)^3}, (4, 2, 2, 4) \\
& = \frac{1}{(a^2 - 2 m r + r^2) (r^2 + a^2 \cos(\theta)^2)^3} \left(6 \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m \sin(\theta)^2 \left(a^2 (a^2 - 2 m r + r^2) \cos(\theta)^2 - \frac{3 a^4}{2} + 2 m r a^2 - 2 r^2 a^2 - \frac{r^4}{2}\right) \right), (4, 2, 3, 1) \\
& = \frac{(a^2 \cos(\theta)^2 - 3 r^2) (2 a^2 \cos(\theta)^2 - 3 a^2 - r^2) \sin(\theta) \cos(\theta) a m}{(r^2 + a^2 \cos(\theta)^2)^3}, (4, 2, 3, 4) \\
& = \frac{3 (a^2 \cos(\theta)^2 - 3 r^2) (a^2 + r^2) m \cos(\theta) \sin(\theta)^3 a^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (4, 2, 4, 2) = \\
& - \frac{1}{(a^2 - 2 m r + r^2) (r^2 + a^2 \cos(\theta)^2)^3} \left(6 \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m \sin(\theta)^2 \left(a^2 (a^2 - 2 m r + r^2) \cos(\theta)^2 - \frac{3 a^4}{2} + 2 m r a^2 - 2 r^2 a^2 - \frac{r^4}{2}\right) \right), (4, 2, 4, 3) = \\
& - \frac{3 (a^2 \cos(\theta)^2 - 3 r^2) (a^2 + r^2) m \cos(\theta) \sin(\theta)^3 a^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (4, 3, 1, 2)
\end{aligned}$$

$$\begin{aligned}
& - \frac{(a^2 \cos(\theta)^2 - 3 r^2) (a^2 \cos(\theta)^2 - 3 a^2 - 2 r^2) \sin(\theta) \cos(\theta) a m}{(r^2 + a^2 \cos(\theta)^2)^3}, (4, 3, 1, 3) \\
& = \frac{(3 a^2 \cos(\theta)^2 - r^2) (3 a^2 - 2 m r + 3 r^2) \sin(\theta)^2 a m r}{(r^2 + a^2 \cos(\theta)^2)^3}, (4, 3, 2, 1) \\
& = \frac{(a^2 \cos(\theta)^2 - 3 r^2) (a^2 \cos(\theta)^2 - 3 a^2 - 2 r^2) \sin(\theta) \cos(\theta) a m}{(r^2 + a^2 \cos(\theta)^2)^3}, (4, 3, 2, 4) \\
& = \frac{3 (a^2 \cos(\theta)^2 - 3 r^2) (a^2 + r^2) m \cos(\theta) \sin(\theta)^3 a^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (4, 3, 3, 1) = \\
& - \frac{9 a \left(a^2 - \frac{2}{3} m r + r^2\right) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m \sin(\theta)^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (4, 3, 3, 4) = \\
& - \frac{1}{(r^2 + a^2 \cos(\theta)^2)^3} \left(3 (a^2 (a^2 - 2 m r + r^2) \cos(\theta)^2 - 3 a^4 + (2 m r - 5 r^2) a^2 \right. \\
& \left. - 2 r^4) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m \sin(\theta)^2 \right), (4, 3, 4, 2) = \\
& - \frac{3 (a^2 \cos(\theta)^2 - 3 r^2) (a^2 + r^2) m \cos(\theta) \sin(\theta)^3 a^2}{(r^2 + a^2 \cos(\theta)^2)^3}, (4, 3, 4, 3) \\
& = \frac{1}{(r^2 + a^2 \cos(\theta)^2)^3} \left(3 (a^2 (a^2 - 2 m r + r^2) \cos(\theta)^2 - 3 a^4 + (2 m r - 5 r^2) a^2 \right. \\
& \left. - 2 r^4) \left(a^2 \cos(\theta)^2 - \frac{r^2}{3}\right) r m \sin(\theta)^2 \right)
\end{aligned}$$

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> #####
## LGT field equations
#####
> eq := D_[nu](Riemann[mu,~nu, alpha, beta])
eq := - \nabla_v (R^\nu_{\mu, \alpha, \beta})
> #####
## show all of the components of the field equations
## All of the components should be zero if the Kerr metric is an exact solution
#####

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(4)

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> Simplify(ArrayElems(TensorArray(eq)))
{(1, 2, 1) = 0, (1, 2, 4) = 0, (1, 3, 1) = 0, (1, 3, 4) = 0, (1, 4, 2) = 0, (1, 4, 3) = 0, (2, 1, 1)
 = 0, (2, 1, 4) = 0, (2, 3, 2) = 0, (2, 3, 3) = 0, (2, 4, 1) = 0, (2, 4, 4) = 0, (3, 1, 1) = 0, (3,
 1, 4) = 0, (3, 2, 2) = 0, (3, 2, 3) = 0, (3, 4, 1) = 0, (3, 4, 4) = 0, (4, 1, 2) = 0, (4, 1, 3) = 0,
 (4, 2, 1) = 0, (4, 2, 4) = 0, (4, 3, 1) = 0, (4, 3, 4) = 0} (5)
> ##### Wrong field equations. replace the covariant derivative with partial derivative and see if the
   components turn out non-zero
#####
> eqWrong := d_[nu](Riemann[mu,~nu, alpha, beta])
   eqWrong := -partial_v(R^v_{ mu, alpha, beta}) (6)
> ##### show all of the components of the wrong field equations
#####
> Simplify(ArrayElems(TensorArray(eqWrong)))
{ (1, 2, 1) = ( 16 m ( 30 cos(4 theta) a^6 - 96 a^4 m r cos(4 theta) - 96 r^2 a^4 cos(4 theta) + 39 a^6 cos(2 theta)
 - 384 a^4 m r cos(2 theta) - 384 r^2 a^4 cos(2 theta) + 1408 a^2 m r^3 cos(2 theta) - 432 a^2 r^4 cos(2 theta)
 + 9 a^6 cos(6 theta) + 18 a^6 - 288 a^4 m r - 288 r^2 a^4 + 1408 a^2 m r^3 - 240 a^2 r^4 - 512 m r^5
 + 192 r^6 ) / ( 10 a^10 cos(8 theta) + 20 a^8 r^2 cos(8 theta) + 120 a^10 cos(4 theta)
 + 560 a^8 r^2 cos(4 theta) + 960 a^6 r^4 cos(4 theta) + 640 a^4 r^6 cos(4 theta) + 210 cos(2 theta) a^10
 + 1120 cos(2 theta) a^8 r^2 + 2400 cos(2 theta) a^6 r^4 + 2560 cos(2 theta) a^4 r^6 + 1280 cos(2 theta) a^2 r^8
 + 45 a^10 cos(6 theta) + 160 a^8 r^2 cos(6 theta) + 160 a^6 r^4 cos(6 theta) + a^10 cos(10 theta) + 126 a^10
 + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^10 ), (1, 2, 4)
 = ( 8 a m ( a^6 cos(8 theta) + 4 cos(4 theta) a^6 - 96 a^4 m r cos(4 theta) - 12 r^2 a^4 cos(4 theta)
 + 704 a^2 m r^3 cos(4 theta) - 32 a^2 r^4 cos(4 theta) + 68 a^6 cos(2 theta) + 48 a^4 m r cos(2 theta)

```

$$\begin{aligned}
& -66 r^2 a^4 \cos(2 \theta) - 128 a^2 r^4 \cos(2 \theta) - 512 m r^5 \cos(2 \theta) - 96 r^6 \cos(2 \theta) \\
& - 4 a^6 \cos(6 \theta) - 48 a^4 m r \cos(6 \theta) + 2 a^4 r^2 \cos(6 \theta) + 59 a^6 + 96 a^4 m r - 52 r^2 a^4 \\
& - 704 a^2 m r^3 - 480 a^2 r^4 + 512 m r^5 - 288 r^6)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) \\
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}),
\end{aligned}$$

$$\begin{aligned}
(1, 3, 1) = & (128 \cos(\theta) \sin(\theta) a^2 m (3 a^4 m \cos(4 \theta) + 9 a^4 r \cos(4 \theta) \\
& + 12 \cos(2 \theta) a^4 m + 12 a^4 r \cos(2 \theta) - 120 a^2 m r^2 \cos(2 \theta) + 9 a^4 m + 3 a^4 r \\
& - 120 a^2 m r^2 - 48 a^2 r^3 + 248 m r^4 - 72 r^5)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) \\
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}),
\end{aligned}$$

$$\begin{aligned}
(1, 3, 4) = & - (64 \sin(\theta) \cos(\theta) m a (6 a^6 r \cos(4 \theta) + 46 m r^2 a^4 \cos(4 \theta) \\
& - 4 a^4 r^3 \cos(4 \theta) + 9 m a^6 \cos(2 \theta) - 33 a^6 r \cos(2 \theta) - 56 m r^2 a^4 \cos(2 \theta) \\
& - 16 a^4 r^3 \cos(2 \theta) - 256 m a^2 r^4 \cos(2 \theta) - 32 a^2 r^5 \cos(2 \theta) - m a^6 \cos(6 \theta)
\end{aligned}$$

$$\begin{aligned}
& + a^6 r \cos(6 \theta) + 8 m a^6 - 38 a^6 r - 102 a^4 m r^2 - 108 a^4 r^3 + 240 a^2 m r^4 - 32 a^2 r^5 \\
& + 112 m r^6)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) \\
& + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} \\
& + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 \\
& + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} \\
& + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}), (1, 4, 2) \\
& = (8 a m (32 r^2 a^2 \cos(4 \theta) + 9 \cos(2 \theta) a^4 - 16 \cos(2 \theta) a^2 r^2 - 48 r^4 \cos(2 \theta) - a^4 \cos(6 \theta) \\
& + 8 a^4 - 48 r^2 a^2)) / (a^8 \cos(8 \theta) + 28 a^8 \cos(4 \theta) + 96 a^6 r^2 \cos(4 \theta) \\
& + 96 a^4 r^4 \cos(4 \theta) + 56 \cos(2 \theta) a^8 + 240 \cos(2 \theta) a^6 r^2 + 384 \cos(2 \theta) a^4 r^4 \\
& + 256 \cos(2 \theta) a^2 r^6 + 8 a^8 \cos(6 \theta) + 16 a^6 r^2 \cos(6 \theta) + 35 a^8 + 160 a^6 r^2 + 288 a^4 r^4 \\
& + 256 a^2 r^6 + 128 r^8), (1, 4, 3) = (32 \sin(\theta) \cos(\theta) m a (a^4 m \cos(4 \theta) \\
& - a^4 r \cos(4 \theta) + 4 \cos(2 \theta) a^4 m + 20 a^4 r \cos(2 \theta) - 56 a^2 m r^2 \cos(2 \theta) \\
& + 32 a^2 r^3 \cos(2 \theta) + 3 a^4 m + 21 a^4 r - 56 a^2 m r^2 - 16 a^2 r^3 + 72 m r^4 - 24 r^5)) / \\
& (a^8 \cos(8 \theta) + 28 a^8 \cos(4 \theta) + 96 a^6 r^2 \cos(4 \theta) + 96 a^4 r^4 \cos(4 \theta) + 56 \cos(2 \theta) a^8 \\
& + 240 \cos(2 \theta) a^6 r^2 + 384 \cos(2 \theta) a^4 r^4 + 256 \cos(2 \theta) a^2 r^6 + 8 a^8 \cos(6 \theta) \\
& + 16 a^6 r^2 \cos(6 \theta) + 35 a^8 + 160 a^6 r^2 + 288 a^4 r^4 + 256 a^2 r^6 + 128 r^8), (2, 1, 1) = \\
& - (16 m (30 \cos(4 \theta) a^6 - 96 a^4 m r \cos(4 \theta) - 96 r^2 a^4 \cos(4 \theta) + 39 a^6 \cos(2 \theta)
\end{aligned}$$

$$\begin{aligned}
& -384 a^4 m r \cos(2 \theta) - 384 r^2 a^4 \cos(2 \theta) + 1408 a^2 m r^3 \cos(2 \theta) - 432 a^2 r^4 \cos(2 \theta) \\
& + 9 a^6 \cos(6 \theta) + 18 a^6 - 288 a^4 m r - 288 r^2 a^4 + 1408 a^2 m r^3 - 240 a^2 r^4 - 512 m r^5 \\
& + 192 r^6) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) \\
& + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} \\
& + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 \\
& + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} \\
& + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}), (2, 1, 4) = \\
& - (8 a m (a^6 \cos(8 \theta) + 4 \cos(4 \theta) a^6 - 96 a^4 m r \cos(4 \theta) - 12 r^2 a^4 \cos(4 \theta) \\
& + 704 a^2 m r^3 \cos(4 \theta) - 32 a^2 r^4 \cos(4 \theta) + 68 a^6 \cos(2 \theta) + 48 a^4 m r \cos(2 \theta) \\
& - 66 r^2 a^4 \cos(2 \theta) - 128 a^2 r^4 \cos(2 \theta) - 512 m r^5 \cos(2 \theta) - 96 r^6 \cos(2 \theta) \\
& - 4 a^6 \cos(6 \theta) - 48 a^4 m r \cos(6 \theta) + 2 a^4 r^2 \cos(6 \theta) + 59 a^6 + 96 a^4 m r - 52 r^2 a^4 \\
& - 704 a^2 m r^3 - 480 a^2 r^4 + 512 m r^5 - 288 r^6)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) \\
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}), \\
& (2, 3, 2) = (32 m r a^2 \sin(\theta) \cos(\theta) (3 a^2 \cos(2 \theta) + 3 a^2 - 10 r^2)) /
\end{aligned}$$

$$\begin{aligned}
& ((6 \cos(4\theta) a^6 + 12 r^2 a^4 \cos(4\theta) + 15 a^6 \cos(2\theta) + 48 r^2 a^4 \cos(2\theta) \\
& + 48 a^2 r^4 \cos(2\theta) + a^6 \cos(6\theta) + 10 a^6 + 36 r^2 a^4 + 48 a^2 r^4 + 32 r^6) (a^2 - 2 m r \\
& + r^2)), (2, 3, 3) = - (4 m (3 a^4 \cos(4\theta) + 12 \cos(2\theta) a^4 - 48 \cos(2\theta) a^2 r^2 + 9 a^4 \\
& - 48 r^2 a^2 + 8 r^4)) / (6 \cos(4\theta) a^6 + 12 r^2 a^4 \cos(4\theta) + 15 a^6 \cos(2\theta) \\
& + 48 r^2 a^4 \cos(2\theta) + 48 a^2 r^4 \cos(2\theta) + a^6 \cos(6\theta) + 10 a^6 + 36 r^2 a^4 + 48 a^2 r^4 \\
& + 32 r^6), (2, 4, 1) = - (16 a m (a^6 \cos(8\theta) - 2 \cos(4\theta) a^6 - 48 a^4 m r \cos(4\theta) \\
& - 30 r^2 a^4 \cos(4\theta) + 352 a^2 m r^3 \cos(4\theta) - 56 a^2 r^4 \cos(4\theta) + 17 a^6 \cos(2\theta) \\
& + 24 a^4 m r \cos(2\theta) - 3 r^2 a^4 \cos(2\theta) + 16 a^2 r^4 \cos(2\theta) - 256 m r^5 \cos(2\theta) \\
& + 48 r^6 \cos(2\theta) - a^6 \cos(6\theta) - 24 a^4 m r \cos(6\theta) - 13 a^4 r^2 \cos(6\theta) + 17 a^6 \\
& + 48 a^4 m r + 14 r^2 a^4 - 352 a^2 m r^3 - 120 a^2 r^4 + 256 m r^5 - 144 r^6)) / (10 a^{10} \cos(8\theta) \\
& + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) + 960 a^6 r^4 \cos(4\theta) \\
& + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 + 2400 \cos(2\theta) a^6 r^4 \\
& + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) \\
& + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 \\
& + 1280 a^2 r^8 + 512 r^{10}), (2, 4, 4) = - (96 m a^8 \cos(6\theta)) / (10 a^{10} \cos(8\theta) \\
& + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) + 960 a^6 r^4 \cos(4\theta) \\
& + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 + 2400 \cos(2\theta) a^6 r^4
\end{aligned}$$

$$\begin{aligned}
& + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) \\
& + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 \\
& + 1280 a^2 r^8 + 512 r^{10}) - (256 m r^8) / (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) \\
& + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) \\
& + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 \\
& + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) \\
& + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& + (192 m a^8) / (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) \\
& + 560 a^8 r^2 \cos(4\theta) + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10} \\
& + 1120 \cos(2\theta) a^8 r^2 + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 \\
& + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} \\
& + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& + (1024 m^2 a^2 r^5 \cos(4\theta)) / (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) \\
& + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) \\
& + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& - (48 m a^6 r^2 \cos(8\theta)) / (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) \\
& + 560 a^8 r^2 \cos(4\theta) + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10}
\end{aligned}$$

$$\begin{aligned}
& + 1120 \cos(2\theta) a^8 r^2 + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 \\
& + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} \\
& + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& - (48 m \cos(2\theta) a^6 r^2) / (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) + \\
& + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) \\
& + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& + (896 m a^2 r^6 \cos(4\theta)) / (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) + \\
& + 560 a^8 r^2 \cos(4\theta) + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10} \\
& + 1120 \cos(2\theta) a^8 r^2 + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 \\
& + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} \\
& + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& + (16 m \cos(6\theta) a^4 r^4) / (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) + \\
& + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) \\
& + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& - (1408 m^2 a^4 r^3 \cos(6\theta)) / (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) \\
& + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) \\
& + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6
\end{aligned}$$

$$\begin{aligned}
& + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) \\
& + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& + (96 m^2 a^6 r \cos(8\theta)) / (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) \\
& + 560 a^8 r^2 \cos(4\theta) + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10} \\
& + 1120 \cos(2\theta) a^8 r^2 + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 \\
& + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} \\
& + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) - (240 m a^6 r^2) / \\
& (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) \\
& + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 \\
& + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) \\
& + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 \\
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) - (192 m a^8 \cos(4\theta)) / \\
& (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) \\
& + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 \\
& + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) \\
& + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 \\
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) - (2720 m a^4 r^4) /
\end{aligned}$$

$$\begin{aligned}
& \left(10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) \right. \\
& + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 \\
& + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) \\
& + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 \\
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10} \Big) + (256 m \cos(2 \theta) r^8) / \\
& \left(10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) \right. \\
& + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 \\
& + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) \\
& + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 \\
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10} \Big) - (2816 m^2 a^4 r^3) / \\
& \left(10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) \right. \\
& + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 \\
& + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) \\
& + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 \\
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10} \Big) + (3072 m^2 a^2 r^5) / \\
& \left(10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) \right. \\
& + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2
\end{aligned}$$

$$\begin{aligned}
& + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) \\
& + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 \\
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) - (1664 m a^2 r^6) / \\
& (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) \\
& + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 \\
& + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) \\
& + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 \\
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) + (288 m^2 a^6 r) / (10 a^{10} \cos(8\theta) \\
& + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) + 960 a^6 r^4 \cos(4\theta) \\
& + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 + 2400 \cos(2\theta) a^6 r^4 \\
& + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) \\
& + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 \\
& + 1280 a^2 r^8 + 512 r^{10}) + (96 m \cos(2\theta) a^8) / (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) \\
& + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) \\
& + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 \\
& + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) \\
& + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10})
\end{aligned}$$

$$\begin{aligned}
& + (48 m a^6 r^2 \cos(6 \theta)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) \\
& + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} \\
& + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 \\
& + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} \\
& + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& + (288 m a^6 r^2 \cos(4 \theta)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& + (1184 m a^4 r^4 \cos(4 \theta)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) \\
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& + (1520 m \cos(2 \theta) a^4 r^4) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) \\
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta)
\end{aligned}$$

$$\begin{aligned}
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& - (4096 m^2 a^2 r^5 \cos(2 \theta)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) \\
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& + (768 m \cos(2 \theta) a^2 r^6) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) \\
& + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} \\
& + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 \\
& + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} \\
& + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& - (384 m^2 a^6 r \cos(4 \theta)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& + (2816 m^2 a^4 r^3 \cos(4 \theta)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) \\
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6
\end{aligned}$$

$$\begin{aligned}
& + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) \\
& + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& + (1408 m^2 a^4 r^3 \cos(2\theta)) / (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) \\
& + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) \\
& + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 \\
& + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) \\
& + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}),
\end{aligned}$$

$$\begin{aligned}
(3, 1, 1) = & - (128 \cos(\theta) \sin(\theta) a^2 m (3 a^4 m \cos(4\theta) + 9 a^4 r \cos(4\theta) \\
& + 12 \cos(2\theta) a^4 m + 12 a^4 r \cos(2\theta) - 120 a^2 m r^2 \cos(2\theta) + 9 a^4 m + 3 a^4 r \\
& - 120 a^2 m r^2 - 48 a^2 r^3 + 248 m r^4 - 72 r^5)) / (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) \\
& + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) \\
& + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 \\
& + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) \\
& + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}),
\end{aligned}$$

$$\begin{aligned}
(3, 1, 4) = & (64 \sin(\theta) \cos(\theta) m a (6 a^6 r \cos(4\theta) + 46 m r^2 a^4 \cos(4\theta) \\
& - 4 a^4 r^3 \cos(4\theta) + 9 m a^6 \cos(2\theta) - 33 a^6 r \cos(2\theta) - 56 m r^2 a^4 \cos(2\theta) \\
& - 16 a^4 r^3 \cos(2\theta) - 256 m a^2 r^4 \cos(2\theta) - 32 a^2 r^5 \cos(2\theta) - m a^6 \cos(6\theta)
\end{aligned}$$

$$\begin{aligned}
& + a^6 r \cos(6 \theta) + 8 m a^6 - 38 a^6 r - 102 a^4 m r^2 - 108 a^4 r^3 + 240 a^2 m r^4 - 32 a^2 r^5 \\
& + 112 m r^6)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) \\
& + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} \\
& + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 \\
& + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} \\
& + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}), (3, 2, 2) = \\
& - (32 m r a^2 \sin(\theta) \cos(\theta) (3 a^2 \cos(2 \theta) + 3 a^2 - 10 r^2)) / ((6 \cos(4 \theta) a^6 \\
& + 12 r^2 a^4 \cos(4 \theta) + 15 a^6 \cos(2 \theta) + 48 r^2 a^4 \cos(2 \theta) + 48 a^2 r^4 \cos(2 \theta) \\
& + a^6 \cos(6 \theta) + 10 a^6 + 36 r^2 a^4 + 48 a^2 r^4 + 32 r^6) (a^2 - 2 m r + r^2)), (3, 2, 3) \\
& = (4 m (3 a^4 \cos(4 \theta) + 12 \cos(2 \theta) a^4 - 48 \cos(2 \theta) a^2 r^2 + 9 a^4 - 48 r^2 a^2 + 8 r^4)) / \\
& (6 \cos(4 \theta) a^6 + 12 r^2 a^4 \cos(4 \theta) + 15 a^6 \cos(2 \theta) + 48 r^2 a^4 \cos(2 \theta) \\
& + 48 a^2 r^4 \cos(2 \theta) + a^6 \cos(6 \theta) + 10 a^6 + 36 r^2 a^4 + 48 a^2 r^4 + 32 r^6), (3, 4, 1) \\
& = (32 \sin(\theta) \cos(\theta) m a (6 m a^6 \cos(4 \theta) + 30 a^6 r \cos(4 \theta) + 40 m r^2 a^4 \cos(4 \theta) \\
& + 20 a^4 r^3 \cos(4 \theta) + 33 m a^6 \cos(2 \theta) + 15 a^6 r \cos(2 \theta) - 320 m r^2 a^4 \cos(2 \theta) \\
& + 80 a^4 r^3 \cos(2 \theta) - 592 m a^2 r^4 \cos(2 \theta) + 16 a^2 r^5 \cos(2 \theta) - m a^6 \cos(6 \theta) \\
& + a^6 r \cos(6 \theta) + 26 m a^6 - 14 a^6 r - 360 a^4 m r^2 - 132 a^4 r^3 + 400 a^2 m r^4 - 176 a^2 r^5 \\
& + 512 m r^6 - 96 r^7)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta)
\end{aligned}$$

$$\begin{aligned}
& + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} \\
& + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 \\
& + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} \\
& + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}), (3, 4, 4) \\
& = (32 m \cos(\theta) \sin(\theta) (104 a^4 r^5 - 176 a^2 r^7 - 344 a^4 m r^4 - 368 a^2 m r^6 - 64 r^9 + 318 a^6 r^3 \\
& + 6 m a^8 \cos(4 \theta) - 3 m \cos(2 \theta) a^8 + 3 m a^6 r^2 \cos(6 \theta) - 114 m a^6 r^2 \cos(4 \theta) \\
& + 40 \cos(4 \theta) a^4 r^5 + 240 \cos(2 \theta) a^4 r^5 + 114 m a^6 r^2 - 3 \cos(2 \theta) a^6 m r^2 - 6 m a^8 \\
& - 152 m a^4 r^4 \cos(4 \theta) + 496 m \cos(2 \theta) a^4 r^4 + 368 m \cos(2 \theta) a^2 r^6 + 102 a^8 r \\
& - 6 a^8 r \cos(4 \theta) + 99 a^8 r \cos(2 \theta) - 3 a^8 r \cos(6 \theta) + 34 \cos(4 \theta) a^6 r^3 \\
& + 163 \cos(2 \theta) a^6 r^3 - 3 \cos(6 \theta) a^6 r^3 + 3 m a^8 \cos(6 \theta) + 176 \cos(2 \theta) a^2 r^7)) / \\
& (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) \\
& + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 \\
& + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) \\
& + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 \\
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}), (4, 1, 2) = \\
& - (8 a m (32 r^2 a^2 \cos(4 \theta) + 9 \cos(2 \theta) a^4 - 16 \cos(2 \theta) a^2 r^2 - 48 r^4 \cos(2 \theta) \\
& - a^4 \cos(6 \theta) + 8 a^4 - 48 r^2 a^2)) / (a^8 \cos(8 \theta) + 28 a^8 \cos(4 \theta) + 96 a^6 r^2 \cos(4 \theta)
\end{aligned}$$

$$\begin{aligned}
& + 96 a^4 r^4 \cos(4 \theta) + 56 \cos(2 \theta) a^8 + 240 \cos(2 \theta) a^6 r^2 + 384 \cos(2 \theta) a^4 r^4 \\
& + 256 \cos(2 \theta) a^2 r^6 + 8 a^8 \cos(6 \theta) + 16 a^6 r^2 \cos(6 \theta) + 35 a^8 + 160 a^6 r^2 + 288 a^4 r^4 \\
& + 256 a^2 r^6 + 128 r^8), (4, 1, 3) = - (\sin(\theta) \cos(\theta) m a (a^4 m \cos(4 \theta) \\
& - a^4 r \cos(4 \theta) + 4 \cos(2 \theta) a^4 m + 20 a^4 r \cos(2 \theta) - 56 a^2 m r^2 \cos(2 \theta) \\
& + 32 a^2 r^3 \cos(2 \theta) + 3 a^4 m + 21 a^4 r - 56 a^2 m r^2 - 16 a^2 r^3 + 72 m r^4 - 24 r^5)) / \\
& (a^8 \cos(8 \theta) + 28 a^8 \cos(4 \theta) + 96 a^6 r^2 \cos(4 \theta) + 96 a^4 r^4 \cos(4 \theta) + 56 \cos(2 \theta) a^8 \\
& + 240 \cos(2 \theta) a^6 r^2 + 384 \cos(2 \theta) a^4 r^4 + 256 \cos(2 \theta) a^2 r^6 + 8 a^8 \cos(6 \theta) \\
& + 16 a^6 r^2 \cos(6 \theta) + 35 a^8 + 160 a^6 r^2 + 288 a^4 r^4 + 256 a^2 r^6 + 128 r^8), (4, 2, 1) \\
& = (16 a m (a^6 \cos(8 \theta) - 2 \cos(4 \theta) a^6 - 48 a^4 m r \cos(4 \theta) - 30 r^2 a^4 \cos(4 \theta) \\
& + 352 a^2 m r^3 \cos(4 \theta) - 56 a^2 r^4 \cos(4 \theta) + 17 a^6 \cos(2 \theta) + 24 a^4 m r \cos(2 \theta) \\
& - 3 r^2 a^4 \cos(2 \theta) + 16 a^2 r^4 \cos(2 \theta) - 256 m r^5 \cos(2 \theta) + 48 r^6 \cos(2 \theta) \\
& - a^6 \cos(6 \theta) - 24 a^4 m r \cos(6 \theta) - 13 a^4 r^2 \cos(6 \theta) + 17 a^6 + 48 a^4 m r + 14 r^2 a^4 \\
& - 352 a^2 m r^3 - 120 a^2 r^4 + 256 m r^5 - 144 r^6)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) \\
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}),
\end{aligned}$$

$$\begin{aligned}
(4, 2, 4) = & \left(96 m a^8 \cos(6\theta) \right) / (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) \right. \\
& + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) \\
& + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 \\
& + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) \\
& + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& + (256 m r^8) / (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) \\
& + 560 a^8 r^2 \cos(4\theta) + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10} \\
& + 1120 \cos(2\theta) a^8 r^2 + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 \\
& + 45 a^{10} \cos(6\theta) + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} \\
& + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) - (192 m a^8) / \\
& (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) \\
& + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 \\
& + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) \\
& + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 \\
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) - (1024 m^2 a^2 r^5 \cos(4\theta)) / \\
& (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) \\
& + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2
\end{aligned}$$

$$\begin{aligned}
& + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) \\
& + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 \\
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) + (48 m a^6 r^2 \cos(8\theta)) / \\
& (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) \\
& + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 \\
& + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) \\
& + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 \\
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) + (48 m \cos(2\theta) a^6 r^2) / \\
& (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) \\
& + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 \\
& + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) \\
& + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2 \\
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) - (896 m a^2 r^6 \cos(4\theta)) / \\
& (10 a^{10} \cos(8\theta) + 20 a^8 r^2 \cos(8\theta) + 120 a^{10} \cos(4\theta) + 560 a^8 r^2 \cos(4\theta) \\
& + 960 a^6 r^4 \cos(4\theta) + 640 a^4 r^6 \cos(4\theta) + 210 \cos(2\theta) a^{10} + 1120 \cos(2\theta) a^8 r^2 \\
& + 2400 \cos(2\theta) a^6 r^4 + 2560 \cos(2\theta) a^4 r^6 + 1280 \cos(2\theta) a^2 r^8 + 45 a^{10} \cos(6\theta) \\
& + 160 a^8 r^2 \cos(6\theta) + 160 a^6 r^4 \cos(6\theta) + a^{10} \cos(10\theta) + 126 a^{10} + 700 a^8 r^2
\end{aligned}$$

$$\begin{aligned}
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) - (16 m \cos(6 \theta) a^4 r^4) / \\
& (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) \\
& + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 \\
& + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) \\
& + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 \\
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) + (1408 m^2 a^4 r^3 \cos(6 \theta)) / \\
& (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) \\
& + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 \\
& + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) \\
& + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 \\
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) - (96 m^2 a^6 r \cos(8 \theta)) / \\
& (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) \\
& + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 \\
& + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) \\
& + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 \\
& + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) + (240 m a^6 r^2) / (10 a^{10} \cos(8 \theta) \\
& + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta)
\end{aligned}$$

$$\begin{aligned}
& + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 \\
& + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) \\
& + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 \\
& + 1280 a^2 r^8 + 512 r^{10}) + (192 m a^8 \cos(4 \theta)) / (10 a^{10} \cos(8 \theta) \\
& + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) \\
& + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 \\
& + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) \\
& + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 \\
& + 1280 a^2 r^8 + 512 r^{10}) + (2720 m a^4 r^4) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) \\
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& - (256 m \cos(2 \theta) r^8) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) \\
& + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} \\
& + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 \\
& + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10}
\end{aligned}$$

$$\begin{aligned}
& + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10} \Big) + (2816 m^2 a^4 r^3) \Big| \\
& \left(10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) \right. \\
& + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 \\
& + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) \\
& + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 \\
& \left. + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10} \right) - (3072 m^2 a^2 r^5) \Big| \\
& \left(10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) \right. \\
& + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 \\
& + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) \\
& + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 \\
& \left. + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10} \right) + (1664 m a^2 r^6) \Big| \\
& \left(10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) \right. \\
& + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 \\
& + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) \\
& + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 \\
& \left. + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10} \right) - (288 m^2 a^6 r) \Big/ (10 a^{10} \cos(8 \theta) \\
& + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta)
\end{aligned}$$

$$\begin{aligned}
& + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 \\
& + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) \\
& + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 \\
& + 1280 a^2 r^8 + 512 r^{10}) - (96 m \cos(2 \theta) a^8) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) \\
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& - (48 m a^6 r^2 \cos(6 \theta)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) \\
& + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} \\
& + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 \\
& + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} \\
& + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& - (288 m a^6 r^2 \cos(4 \theta)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& - (1184 m a^4 r^4 \cos(4 \theta)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta)
\end{aligned}$$

$$\begin{aligned}
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& - (1520 m \cos(2 \theta) a^4 r^4) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) \\
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& + (4096 m^2 a^2 r^5 \cos(2 \theta)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) \\
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& - (768 m \cos(2 \theta) a^2 r^6) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) \\
& + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} \\
& + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8
\end{aligned}$$

$$\begin{aligned}
& + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} \\
& + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& + (384 m^2 a^6 r \cos(4 \theta)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& - (2816 m^2 a^4 r^3 \cos(4 \theta)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) \\
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \\
& - (1408 m^2 a^4 r^3 \cos(2 \theta)) / (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) \\
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}), \\
(4, 3, 1) = & - (32 \sin(\theta) \cos(\theta) m a (6 m a^6 \cos(4 \theta) + 30 a^6 r \cos(4 \theta) \\
& + 40 m r^2 a^4 \cos(4 \theta) + 20 a^4 r^3 \cos(4 \theta) + 33 m a^6 \cos(2 \theta) + 15 a^6 r \cos(2 \theta)
\end{aligned}$$

$$\begin{aligned}
& -320 m r^2 a^4 \cos(2 \theta) + 80 a^4 r^3 \cos(2 \theta) - 592 m a^2 r^4 \cos(2 \theta) + 16 a^2 r^5 \cos(2 \theta) \\
& - m a^6 \cos(6 \theta) + a^6 r \cos(6 \theta) + 26 m a^6 - 14 a^6 r - 360 a^4 m r^2 - 132 a^4 r^3 \\
& + 400 a^2 m r^4 - 176 a^2 r^5 + 512 m r^6 - 96 r^7 \big) \Big/ (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) \\
& + 120 a^{10} \cos(4 \theta) + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) \\
& + 210 \cos(2 \theta) a^{10} + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 \\
& + 1280 \cos(2 \theta) a^2 r^8 + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) \\
& + a^{10} \cos(10 \theta) + 126 a^{10} + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}), \\
(4, 3, 4) = & - (32 m \cos(\theta) \sin(\theta) (104 a^4 r^5 - 176 a^2 r^7 - 344 a^4 m r^4 - 368 a^2 m r^6 \\
& - 64 r^9 + 318 a^6 r^3 + 6 m a^8 \cos(4 \theta) - 3 m \cos(2 \theta) a^8 + 3 m a^6 r^2 \cos(6 \theta) \\
& - 114 m a^6 r^2 \cos(4 \theta) + 40 \cos(4 \theta) a^4 r^5 + 240 \cos(2 \theta) a^4 r^5 + 114 m a^6 r^2 \\
& - 3 \cos(2 \theta) a^6 m r^2 - 6 m a^8 - 152 m a^4 r^4 \cos(4 \theta) + 496 m \cos(2 \theta) a^4 r^4 \\
& + 368 m \cos(2 \theta) a^2 r^6 + 102 a^8 r - 6 a^8 r \cos(4 \theta) + 99 a^8 r \cos(2 \theta) - 3 a^8 r \cos(6 \theta) \\
& + 34 \cos(4 \theta) a^6 r^3 + 163 \cos(2 \theta) a^6 r^3 - 3 \cos(6 \theta) a^6 r^3 + 3 m a^8 \cos(6 \theta) \\
& + 176 \cos(2 \theta) a^2 r^7) \Big/ (10 a^{10} \cos(8 \theta) + 20 a^8 r^2 \cos(8 \theta) + 120 a^{10} \cos(4 \theta) \\
& + 560 a^8 r^2 \cos(4 \theta) + 960 a^6 r^4 \cos(4 \theta) + 640 a^4 r^6 \cos(4 \theta) + 210 \cos(2 \theta) a^{10} \\
& + 1120 \cos(2 \theta) a^8 r^2 + 2400 \cos(2 \theta) a^6 r^4 + 2560 \cos(2 \theta) a^4 r^6 + 1280 \cos(2 \theta) a^2 r^8 \\
& + 45 a^{10} \cos(6 \theta) + 160 a^8 r^2 \cos(6 \theta) + 160 a^6 r^4 \cos(6 \theta) + a^{10} \cos(10 \theta) + 126 a^{10} \\
& + 700 a^8 r^2 + 1600 a^6 r^4 + 1920 a^4 r^6 + 1280 a^2 r^8 + 512 r^{10}) \}
\end{aligned}$$

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