Roll No.

# SHAMBHUNATH INSTITUTE OF ENGINEERING AND TECHNOLOGY, PRAYAGRAJ

**Subject Code:BAS 203**
**Subject: Engineering Mathematics II**

**Course: B.Tech.**
**SEMESTER: II**

### THIRD SESSIONAL EXAMINATION, EVEN SEMESTER, (2024-2025)

#### Common To All

Time –1:30 hrs.
Maximum Marks – 30

1. Attempt any *FIVE* questions.

| Q N | QUESTION | Marks | CO | BL |
|---|---|---|---|---|
| a. | Define Analytic function with an example and state the necessary and sufficient condition for the function to be analytic. | 2 | CO4 | L1 |
| b. | Find the point where the C-R equations are satisfied for the function $f(z) = xy^2 + ix^2y$, where does $f'(z)$ exist? Where is $f(z)$ analytic? | 2 | CO4 | L2 |
| c. | An electrostatic field in the xy-plane is given by the potential function $\varphi = 3x^2y - y^3$. Find the stream function. | 2 | CO4 | L2 |
| d. | Find the image of $2x + y - 3 = 0$ under the transformation $w = z + 2i$. | 2 | CO4 | L2 |
| e. | Define Cross-ratio. | 2 | CO4 | L1 |
| f. | Find the fixed points of the bilinear transformation $w = \frac{z}{z-2}$. | 2 | CO4 | L2 |

2. Attempt any *ONE* of the following.

| Q N | QUESTION | Marks | CO | BL |
|---|---|---|---|---|
| a. | If $f(z) = \frac{x^3y(y-ix)}{x^6 + y^2}$ $z \neq 0$, $f(0) = 0$. Prove that $\frac{f(z)-f(0)}{z} \to 0$ as $z \to 0$ along any radius vector but not as $z \to 0$ in any manner and also that $f(z)$ is not analytic at $z = 0$. | 5 | CO4 | L3 |
| b. | If $u = e^x(x\cos y - y\sin y)$ is a harmonic function. Find an analytic function $f(z) = u + iv$ such that $f(1) = e$. | 5 | CO4 | L4 |
| c. | Find a bilinear transformation which maps the point $i, -i, 1$ of the z-plane into $0, 1, \infty$ of the w- plan respectively. | 5 | CO4 | L3 |

P.T.O.

## 3. Attempt any *FIVE* questions.

| Q N | QUESTION | Marks | CO | BL |
|---|---|---|---|---|
| a. | Evaluate $\int_0^{2+i} (\bar{z})^2 \, dz$, along the real axis from $z = 0$ to $z = 2$ and then along the line parallel to y-axis from $z = 2$ to $z = 2 + i$. | 2 | CO5 | L1 |
| b. | Evaluate $\oint \frac{2z^2+5}{(z+2)^3(z^2+4)} dz$, where C is the square with vertices at $1 + i, 2 + i, 2 + 2i, 1 + 2i$. | 2 | CO5 | L1 |
| c. | Find Taylor's series expansion of $\frac{4z-1}{z^4-1}$ about the point $z = 0$. | 2 | CO5 | L2 |
| d. | Define isolated and non-isolated singular points. | 2 | CO5 | L1 |
| e. | Discuss the nature of singularity of $\frac{\cot \pi z}{(z-a)^2}$ at $z = a$ and $z = \infty$. | 2 | CO5 | L2 |
| f. | Find the value of $\oint z\, e^{1/z}$ around the unit circle. | 2 | CO5 | L2 |

## 4. Attempt any *ONE* of the following.

| Q N | QUESTION | Marks | CO | BL |
|---|---|---|---|---|
| a. | Evaluate $\int \left\{ \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} \right\} dz$ where C is the circle $|z| = 3$. | 5 | CO5 | L3 |
| b. | Find the Laurent's series expansion of $f(z) = \frac{7z-2}{z(z+1)(z-2)}$ in the region <br> (i) $1 < |z+1| < 3$     (ii) $|z+1| > 3$ | 5 | CO5 | L4 |
| c. | Determine the poles of the function and residues at each pole: $f(z) = \frac{z-1}{(z+1)^2(z-2)}$ and hence evaluate $\oint f(z)dz$, where C is the circle $|z - i| = 2$. | 5 | CO5 | L3 |

**Bloom's Taxonomy Level (BL): -**
Remember (L1),     Understanding (L2),     Apply (L3),     Analyze (L4),    Evaluating (L5),    Creating (L6)