Faculty of Engineering, Architecture and Science

Department of Electrical and Computer Engineering

Course Number	**ELE532**
Course Title	**Signals and Systems I**
Semester/Year	Fall 2020

Instructor	**Javad Alirezaie**

ASSIGNMENT No.	**2**

Assignment Title	*System Properties and Convolution*

Submission Date	**October 25th, 2020**
Due Date	**October 25th, 2020**

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A Impulse response

Problem A.1 [0.5 Marks] Complete *Lathi, Section 2.7-1 Script Files*, page 213. Use MATLAB command **poly** to generate the characteristic polynomial from the characteristic values specified by **lambda**.

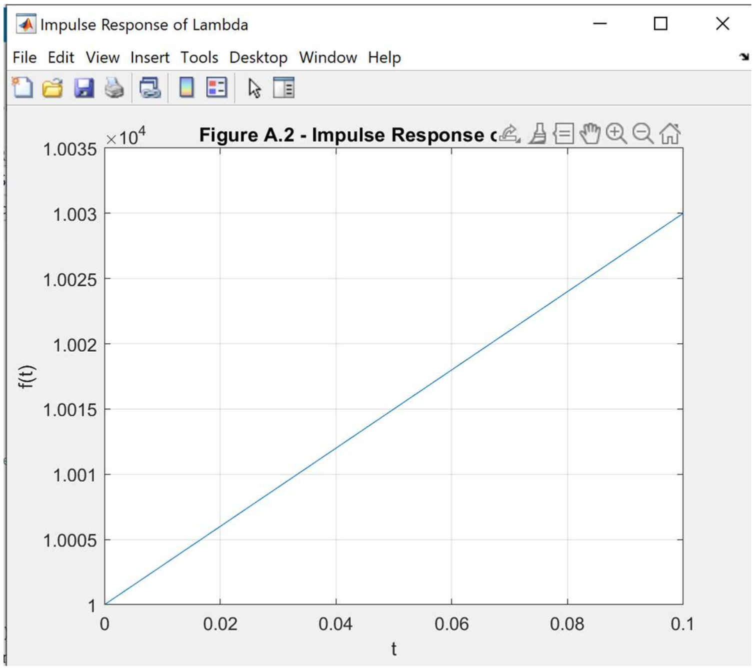
```
% Problem A.1

% CH2MP1.m : Chapter 2, MATLAB Program 1
% Script M-file determines characteristic roots of op-amp circuit.
% Set component values:
R = [1e4, 1e4, 1e4];
C = [1e-6, 1e-6];
% Determine coefficients for characteristic equation:
A = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
% Determine characteristic roots:
lambda = roots(A)
eq1 = poly(lambda);
```

```
lambda =

 -261.8034
  -38.1966
```

Problem A.2 [2 Marks] Plot the impulse response of the system in Problem A.1 for `t = [0:0.0005:0.1]`.

AssignemntB.m | AssignmentA.m | CH2MP1.m | CH2MP2.m | lab2conv.m

```
%Assignment A.2
lab2conv;
CH2MP1;

f = @(t) poly(lambda);
t = (0:0.0005:0.1);

%plot results
figure('Name', 'Impulse Response of Lambda','NumberTitle','off');
plot(t, polyval(f(t), t));
xlabel('t'); ylabel('f(t)'); grid;
```



Problem A.3 [0.5 Marks] Complete *Lathi, Section 2.7-2 Function M-Files*, page 214.

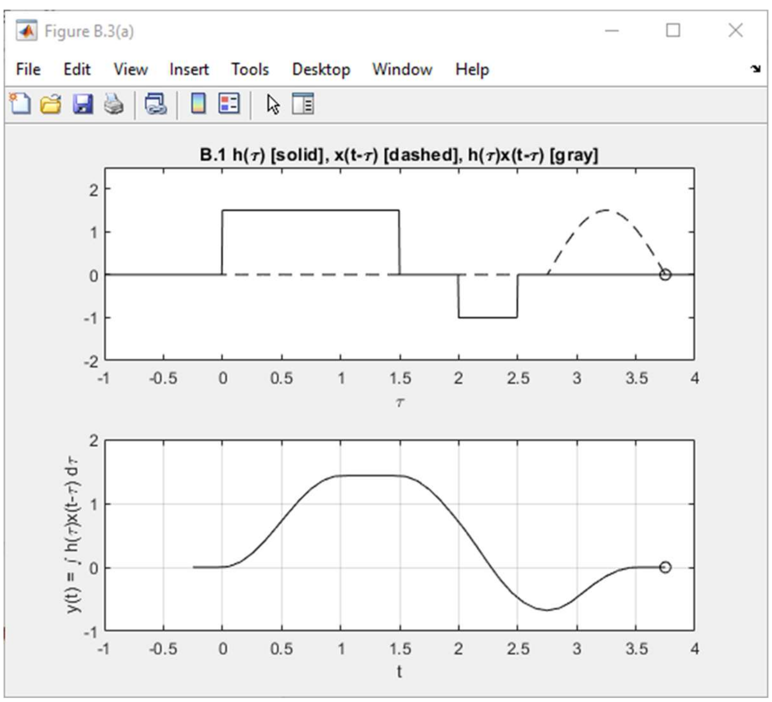
lab2conv.m | CH2MP1.m | CH2MP2.m | AssignmentA.m | AssignemntB.m | ProblemA.m

```
function [lambda] = CH2MP2(R,C)
% CH2MP2.m : Chapter 2, MATLAB Program 2
% Function M-file finds characteristic roots of op-amp circuit.
% INPUTS: R = length-3 vector of resistances
% C = length-2 vector of capacitances
% OUTPUTS: lambda = characteristic roots
% Determine coefficients for characteristic equation:
A = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
% Determine characteristic roots:
lambda = roots(A);
```

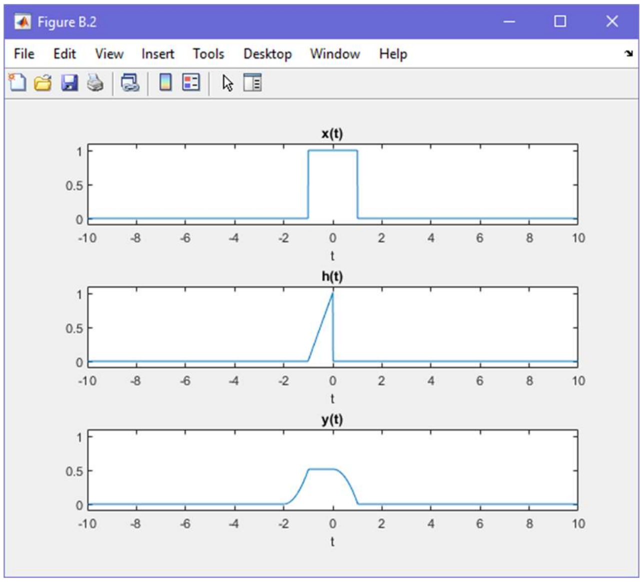
B. Convolution

Problem B.1 [3 Marks] Exercise M2.4 *Lathi, Matlab Session 2*, page 232. Plot $y(t)$ at step $t = 2.25$ as shown in Figure M2.4 on page 233. Use the MATLAB command **pause** instead of **drawnow** to observe the steps of the convolution operation slowly.

```
% CH2MP4.m : Chapter 2, MATLAB Program 4
% Script M-file graphically demonstrates the convolution process.
figure(1) % Create figure window and make visible on screen
u = @(t) 1.0*(t>=0);
x = @(t) 1.5*sin(pi*t).*(u(t)-u(t-1));
h = @(t) 1.5*(u(t)-u(t-1.5))-u(t-2)+u(t-2.5);
dtau = 0.005;
tau = -1:dtau:4;
ti = 0;
tvec = -.25:.1:3.75;
y = NaN*zeros(1,length(tvec));
% Pre-allocate memory
for t = tvec
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau);
    lxh = length(xh);
    y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],'edgecolor','none');
    xlabel('\tau');
    title('B.1 h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
    xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
    axis([tau(1) tau(end) -1.0 2.0]); grid;
    %drawnow;
    pause;
end
%pause;
```

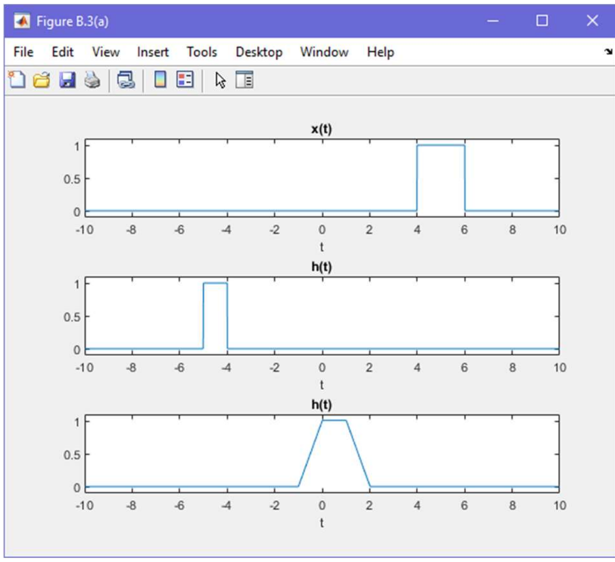


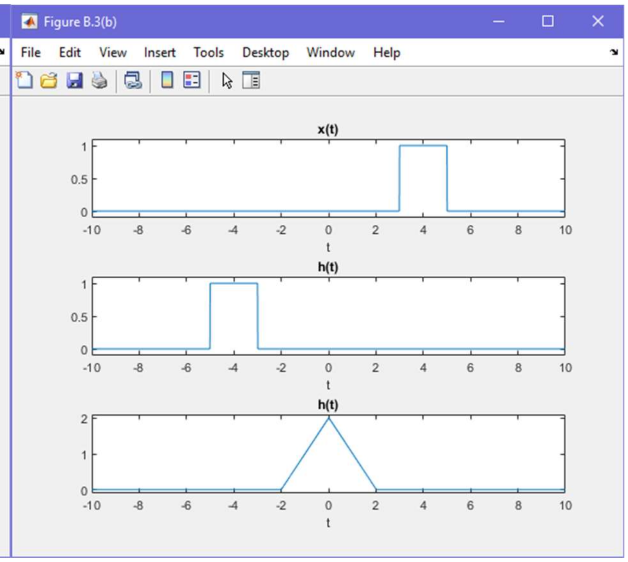
Problem B.2 [4 Marks] Perform the convolution of the signal $x(t)$ in **Figure P2.4-19(a)** with $h(t)$ in **Figure P2.4-21**. Plot all signals and results.




```
clear;
t1=-10;
t2=10;
N=2000;
Delta_t=(t2-t1)/N;
t = (t1:Delta_t:t2);

x = zeros(size(t));
x(t>=-1 & t<=1)=1;

f = zeros(size(t));
f(t>=-1 & t<=0)=1;

for i=1:length(t)
    if (t(i) < -1 || t(i) > 0)
        h(i)=0;
    else
        h(i)=h(i-1)+.01;
    end
end

x1=x*Delta_t;

y=conv(x1,h);

figure('Name','Figure B.2','NumberTitle','off');

subplot(3,1,1);
plot(t,x); axis([t1 t2 -0.1 1.1])
title('x(t)');xlabel('t');

subplot (3,1,2);
plot(t,h);  axis([t1 t2 -0.1 1.1])
title('h(t)');xlabel('t');

subplot (3,1,3);
plot(t,y(1000:3000));  axis([t1 t2 -0.1 1.1])
title('y(t)');xlabel('t');
```

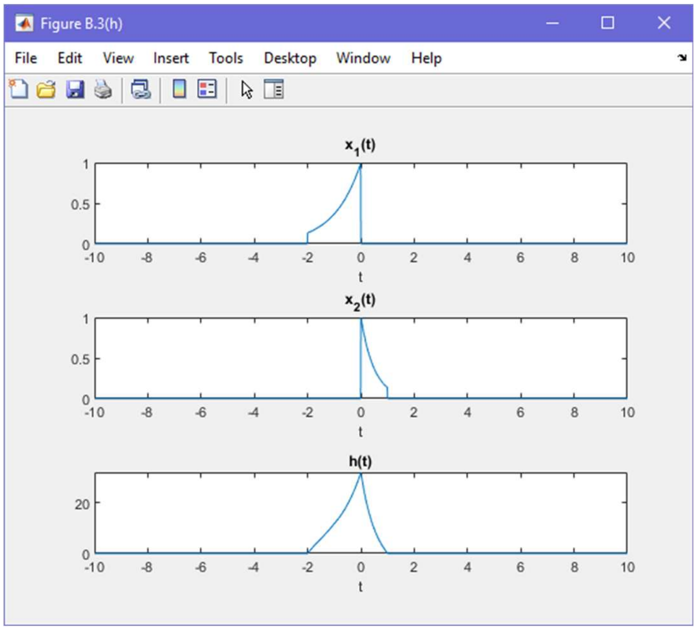
Problem B.3 [5 Marks] Perform the convolution of the signal $x_1(t)$ and $x_2(t)$ in **Figure P2.4-18(a)**, **(b)** and **(h)**. Plot all signals and results.






```
x_1 = zeros(size(t));
x_1(t>=4 & t<=6)=1;

x_2 = zeros(size(t));
x_2(t>=-5 & t<=-4)=1;


x_1b=x_1.*.1;
x_2b=x_2.*.1;


f=conv(x_1b,x_2b);

figure('Name','Figure B.3(a)','NumberTitle','off');

subplot(3,1,1);
plot(t,x_1); axis([t1 t2 -0.1 1.1])
title('x(t)');xlabel('t');

subplot (3,1,2);
plot(t,x_2); axis([t1 t2 -0.1 1.1])
title('h(t)');xlabel('t');

subplot (3,1,3);
plot(t,f(1000:3000));axis([t1 t2 -0.1 1.1])
title('h(t)');xlabel('t');
```

```
x_1 = zeros(size(t));
x_1(t>=3 & t<=5)=1;

x_2 = zeros(size(t));
x_2(t>=-5 & t<=-3)=1;


x_1b=x_1.*.1;
x_2b=x_2.*.1;


f=conv(x_1b,x_2b);

figure('Name','Figure B.3(b)','NumberTitle','off');

subplot(3,1,1);
plot(t,x_1); axis([t1 t2 -0.1 1.1])
title('x(t)');xlabel('t');

subplot (3,1,2);
plot(t,x_2); axis([t1 t2 -0.1 1.1])
title('h(t)');xlabel('t');

subplot (3,1,3);
plot(t,f(1000:3000));axis([t1 t2 -0.1 2.1])
title('h(t)');xlabel('t');
```



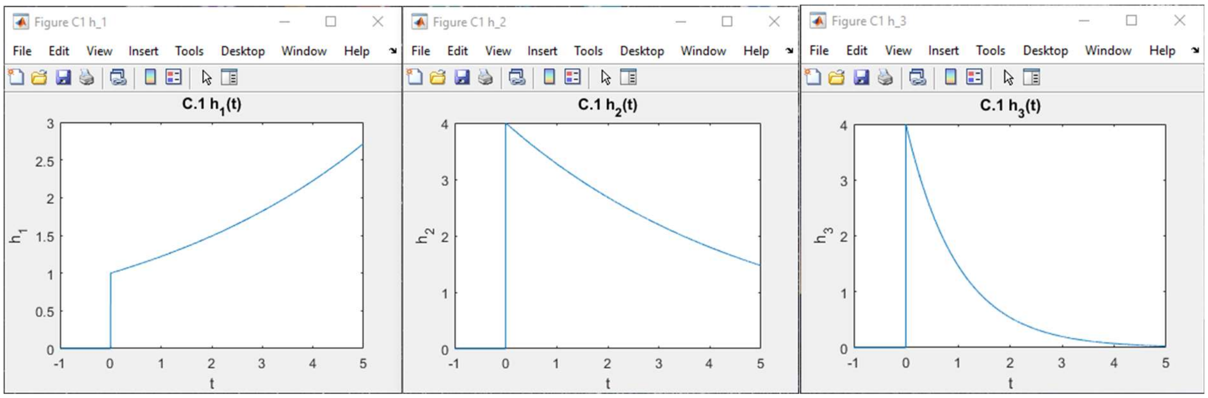

```
u = @(t) 1.0*(t>=0);
f_1 = @(t) exp(t).*(u(t+2)-u(t)); %x1
f_2 = @(t) exp(-2*t).*(u(t)-u(t-1)); %x2

f_1a=f_1(t);
f_2a=f_2(t);

h1=conv(f_1a,f_2a);

figure('Name','Figure B.3(h)','NumberTitle','off');

subplot(3,1,1);
plot(t,f_1a);
title('x_1(t)');xlabel('t');

subplot (3,1,2);
plot(t,f_2a);
title('x_2(t)');xlabel('t');

subplot (3,1,3);
plot(t,h1(1000:3000));
title('h(t)');xlabel('t');
```

C. System behavior and stability

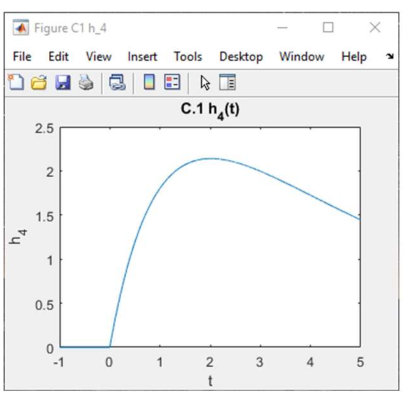
Problem C.1 [4 Marks] Consider the LTI systems S1, S2, S3 and S4 represented by their respective unit impulse response functions given as follows:

$$h_1(t) = e^{\frac{t}{5}}u(t); \tag{3}$$
$$h_2(t) = 4e^{-\frac{t}{5}}u(t); \tag{4}$$
$$h_3(t) = 4e^{-t}u(t); \tag{5}$$
$$h_4(t) = 4\big(e^{-\frac{t}{5}} - e^{-t}\big)u(t); \tag{6}$$

Plot each unit impulse response function for `t = [-1:0.001:5]`.




```
u = @(t) 1.0*(t>=0);
h_1 = @(t)  exp(t/5).*u(t);
h_2 = @(t)  4*exp(-t/5).*u(t);
h_3 = @(t)  4*exp(-t).*u(t);
h_4 = @(t)  4*(exp(-t/5)-exp(-t)).*u(t);

t=[-1:0.001:5];

figure('Name','Figure C1 h_1','NumberTitle','off');
plot(t,h_1(t));
title('C.1 h_1(t)');
ylabel('h_1');
xlabel('t');
pause;
figure('Name','Figure C1 h_2','NumberTitle','off');
plot(t,h_2(t));
title('C.1 h_2(t)');
ylabel('h_2');
xlabel('t');
pause;
figure('Name','Figure C1 h_3','NumberTitle','off');
plot(t,h_3(t));
title('C.1 h_3(t)');
ylabel('h_3');
xlabel('t');
pause;
figure('Name','Figure C1 h_4','NumberTitle','off');
plot(t,h_4(t));
title('C.1 h_4(t)');
ylabel('h_4');
xlabel('t');
```



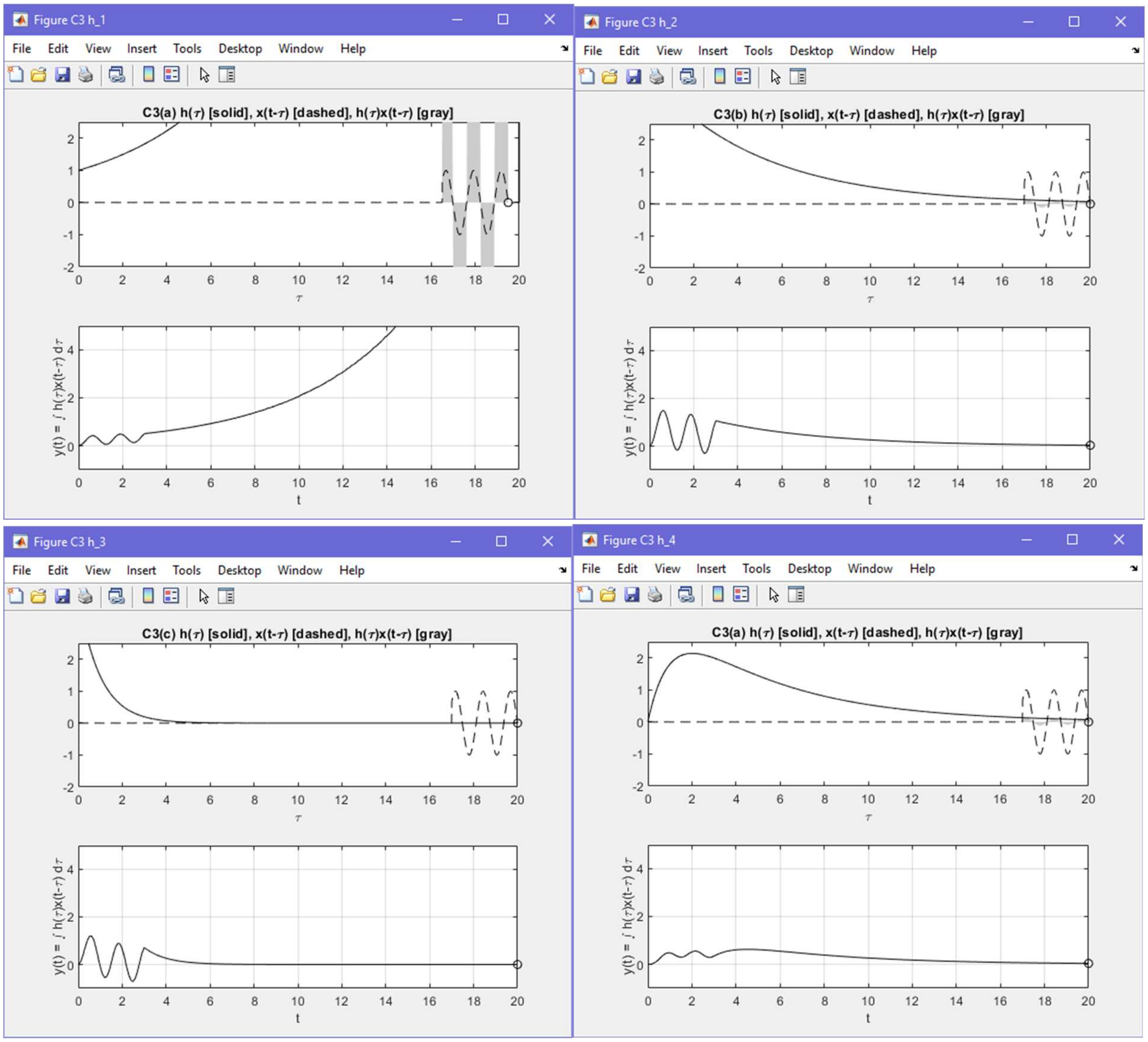
Problem C.2 [1 Mark] Determine the characteristic values (eigenvalues) of systems S1–S4.

$\lambda h1=\frac{1}{5}$

λh2=-⅕
λh3=-1
λh4=-⅕, -1

Problem C.3 [5 Marks] Truncate the impulse response functions $h_1(t), \dots, h_4(t)$ such that they are nonzero only for $0 \leq t \leq 20$. Determine the convolution of the truncated impulse response functions with the input signal $x(t) = \left[u(t) - u(t-3)\right] \sin 5t$ using the M-file in **Problem B.1** with the following changes **tau = [0:dtau:20]** and **tvec = [0:0.1:20]**. Plot the output of each system. State and explain your observations. Is there any relationship between the outputs of systems S2, S3, and S4? Explain.

```
h2_1=@(t) h_1(t).*(u(t)-u(t-20));
h2_2=@(t) h_2(t).*(u(t)-u(t-20));
h2_3=@(t) h_3(t).*(u(t)-u(t-20));
h2_4=@(t) h_4(t).*(u(t)-u(t-20));

figure('Name','Figure C3 h_1','NumberTitle','off');
% Create figure window and make visible on screen
u = @(t) 1.0*(t>=0);
x = @(t) sin(5*t).*(u(t)-u(t-3));
h = @(t) h2_1(t);
dtau = 0.005;
tau = 0:dtau:20;
ti = 0;
tvec = 0:.1:20;
y = NaN*zeros(1,length(tvec));
% Pre-allocate memory
for t = tvec
    ti = ti+1; % Time index
    xh = x(t-tau).*h(tau);
    lxh = length(xh);
    y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
    subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
    axis([tau(1) tau(end) -2.0 2.5]);
    patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
        [zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
        [.8 .8 .8],'edgecolor','none');
    xlabel('\tau');
    title('C3(a) h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
    c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
    subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
    xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
    axis([tau(1) tau(end) -1.0 5.0]); grid;
    drawnow;
    %pause;
end
pause;
```



After performing the convolution, you can see that h2, h3, and h4 have similar graphs. Each one has a wave the length of x(t) and then transitions to h(t) which approaches 0.

D. Discussion

Problem D.1 [2 Marks] Calculate the results of **Problems A.1, A.2** and **A.3** by hand and compare to those obtained with your MATLAB code.

Problem D.2 [1 Mark] What can you say about the width/duration of the signal resulting from the convolution of two signals?

The Width/duration of the convolution of 2 signals is the addition of their individual widths/durations.