LAB 3: Fourier Series Analysis Using MATLAB

Objective

In this experiment, you will use Fourier series in the analysis and synthesis of periodic signals while continuing to learn how to use MATLAB effectively. You will explore characteristics of Fourier series, investigate how the period of a signal affects Fourier series coefficients, and study the effects of series truncation on signal reconstruction.

Introduction

Fourier series is an alternate way of representing periodic signals. Specifically, Fourier series allows a periodic signal to be expressed as a linear combination of harmonically related complex exponentials. Let $x(t)$ be be a periodic signal with period T_0 and the corresponding fundamental frequency $\omega_0 = 2\pi/T_0$. Using Fourier series expansion, $x(t)$ can be expressed by the *synthesis* equation:

$$x(t) = \sum_{n=-\infty}^{+\infty} D_n e^{jn\omega_0 t}, \tag{1}$$

where $\{D_n, n = 0, \pm 1, \pm 2, \ldots\}$ is the set of Fourier coefficients that define the periodic signal $x(t)$. We can calculate Fourier coefficients using the analysis equation:

$$D_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jn\omega_0 t} dt, \quad n = 0, \pm 1, \pm 2, \ldots \tag{2}$$

where the integral can be evaluated over any interval of T_0 duration.

Preparation

- Read *Lathi, Chapter 6*, specifically sections 6.1, 6.2 and 6.3, pp. 594–636.
- Work through Computer Example C6.1 on page 602 of the text.
- Work through Computer Example C6.3 on page 626 of the text.

Lab Assignment

In this experiment you will write a function that will first generate the Fourier coefficients of a periodic signal. You will plot the magnitude and phase spectra of the signal generated by the Fourier coefficients and then reconstruct the original signal from its Fourier coefficients.

Problem A.1 [1 Mark] Given the periodic signal $x_1(t)$:

$$x_1(t) = \cos \frac{3\pi}{10} t + \frac{1}{2} \cos \frac{\pi}{10} t, \tag{3}$$

derive an expression for the Fourier coefficients D_n.

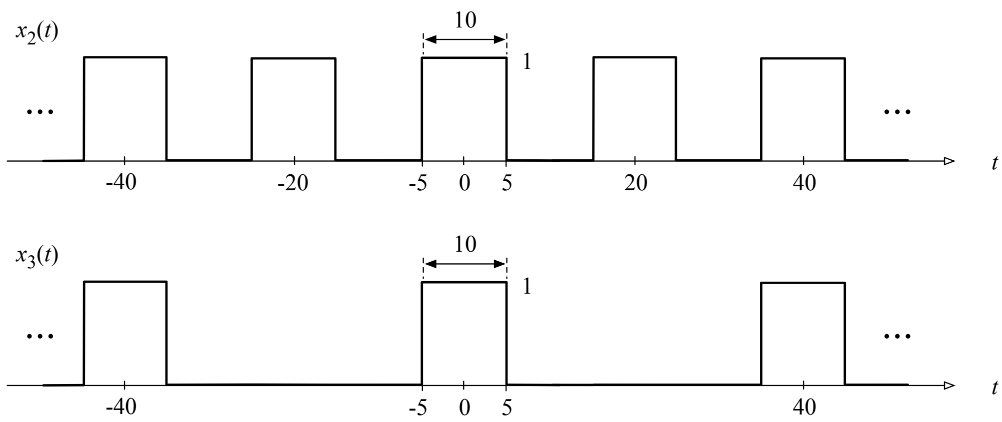

Figure 1: Periodic functions $x_2(t)$ and $x_3(t)$.

Problem A.2 [1 Mark] Repeat Problem A.1 for the periodic signals $x_2(t)$ and $x_3(t)$ shown in Figure 1.

Problem A.3 [3 Marks] Now that you have an expression for D_n, write a MATLAB function that generates D_n for a user specified range of values of n.

Problem A.4 [3 Marks] Generate and plot the **magnitude** and **phase** spectra of $x_1(t)$, $x_2(t)$ and $x_3(t)$ (using the **`stem`** command) from their respective D_n sets for the following index ranges:

(a) $-5 \leq n \leq 5$;

(b) $-20 \leq n \leq 20$;

(c) $-50 \leq n \leq 50$;

(d) $-500 \leq n \leq 500$.

Note: You can use the MATLAB commands **`abs`** and **`angle`** to determine the magnitude and phase of a complex number.

Problem A.5 [3 Marks] Write a MATLAB function that takes a MATLAB generated D_n set and reconstructs the original time-domain signal from which the Fourier coefficients had been derived. For example, given the set of truncated Fourier coefficients $\{D_n,\ n = 0,\ \pm 1,\ \ldots,\ \pm 20\}$, your code should reconstruct the time-domain signal from this set using Equation (1). **Note:** Use the time variable t defined with the MATLAB command `t=[-300:1:300]`.

Problem A.6 [3 Marks] Reconstruct the time-domain signals $x_1(t)$, $x_2(t)$ and $x_3(t)$ with the Fourier coefficient sets you generated in Problem A.4. Plot each reconstructed signal.

Discussion

Problem B.1 [1 Mark] Determine the fundamental frequencies of $x_1(t)$, $x_2(t)$ and $x_3(t)$.

Problem B.2 [1 Mark] What is the main difference between the Fourier coefficients of $x_1(t)$ and $x_2(t)$?

Problem B.3 [1 Mark] Signals $x_2(t)$ and $x_3(t)$ have the same rectangular pulse shape but different periods. How are these characteristics reflected in their respective Fourier coefficients?

Problem B.4 [2 Marks] The Fourier coefficient D_0 represents the DC value of the signal. Let $x_4(t)$ be the periodic waveform shown in Figure 2. Derive D_0 of $x_4(t)$ from D_0 of $x_2(t)$.

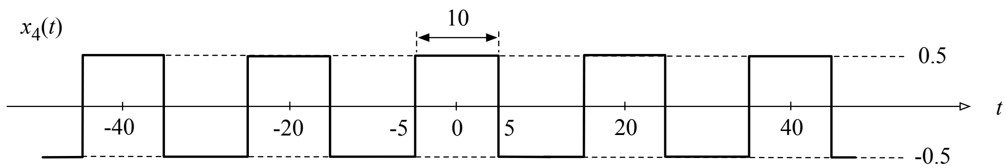

Figure 2: Periodic function $x_4(t)$.

Problem B.5 [2 Marks] Using the results of Problem A.6, explain how the reconstructed signal changes as you increase the number of Fourier coefficients used in the reconstruction. Discuss for both $x_1(t)$ and $x_2(t)$.

Problem B.6 [2 Marks] How many Fourier coefficients do you need to **perfectly** reconstruct the periodic waveforms discussed in this lab experiment?

Problem B.7 [2 Marks] Let $x(t)$ be an arbitrary periodic signal. Instead of storing $x(t)$ on a computer, we consider storing the corresponding Fourier coefficients. When we need to access $x(t)$, we read the Fourier coefficients stored on the computer hard drive and reconstruct the signal. Is this a viable scenario? Explain your answer.