Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

Course Title:	Signals and Systems II
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Semester/Year (e.g.F2016)	W2022

Instructor:	Dimitri Androutsos

Assignment/Lab Number:	2
Assignment/Lab Title:	Time-Domain Analysis of Discrete-Time Systems - Part 2

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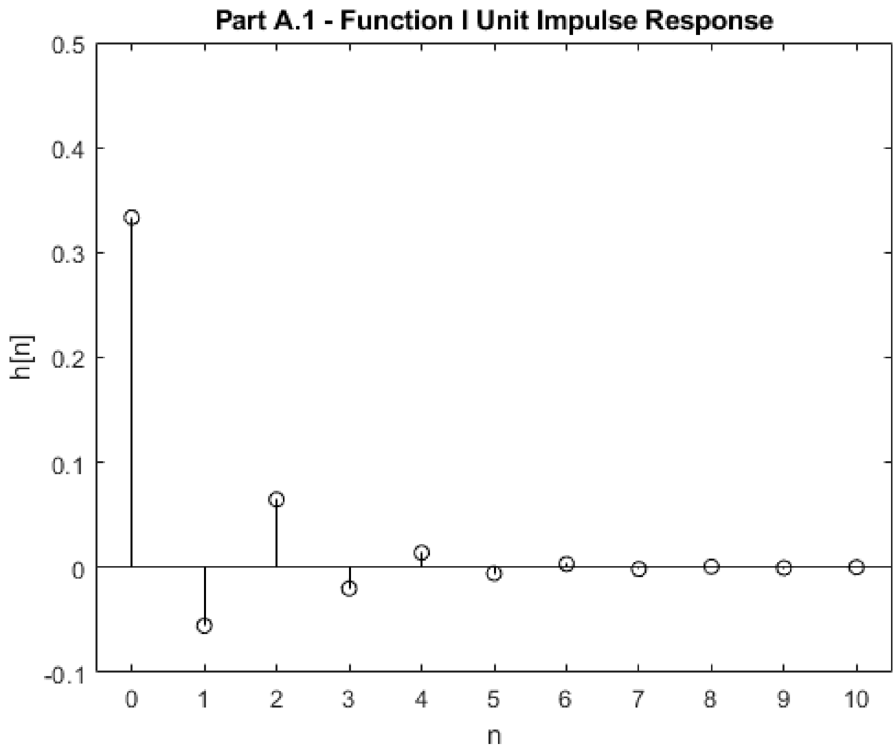
Appendix

A) Unit Impulse Response

Part 1 - Filter to Receive Unit Impulse Response

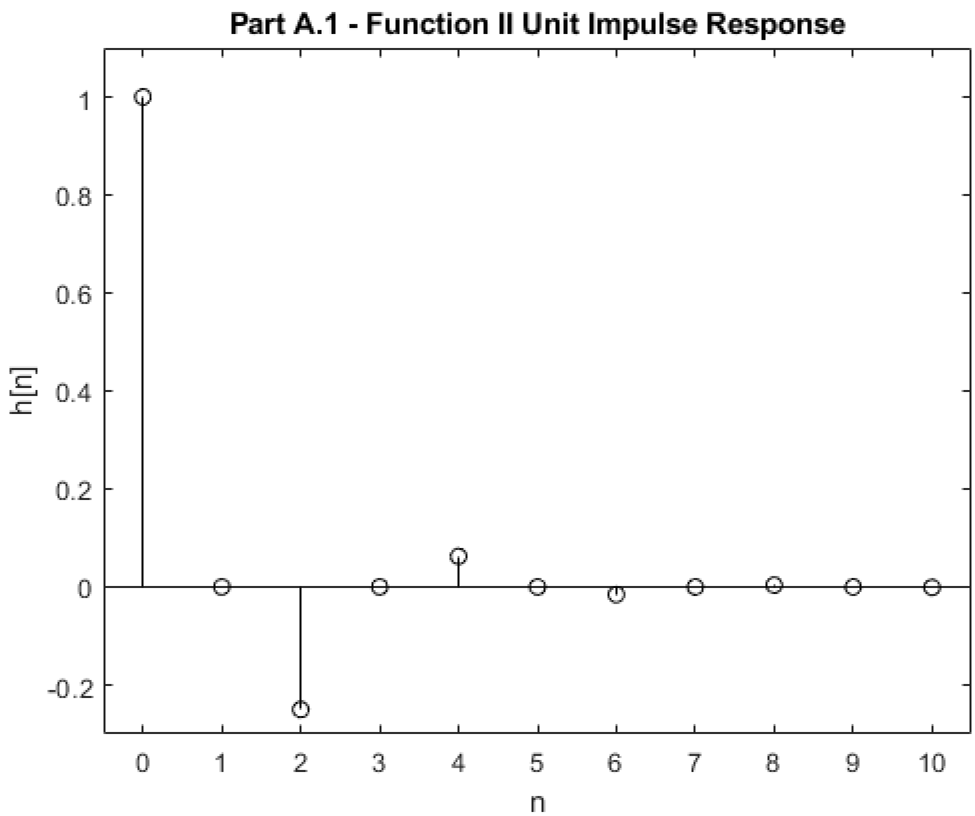
Function I

```
n =  0:10;

b = [1/3 0 0];
a = [1 1/6 -1/6];
% delta = 1.0.*((n)==0);
delta = (n)==0;
% figure;
% stem(n, delta);
% title('Part A.1 - Unit Impulse')
% axis([-0.5 0.5 -0.1 1.2])
% xlabel('n')



h = filter(b, a, delta);

figure;
stem(n, h, 'k');
title('Part A.1 - Function I Unit Impulse Response')
axis([-0.5 10.5 -0.1 0.5])
xlabel('n')
ylabel('h[n]')
```



Function II

```
n =  0:10;

b = [1 0 0];
a = [1 0 1/4];
delta = (n)==0;

h = filter(b, a, delta(n));
clf;

stem(n, h, 'k');
title('Part A.1 - Function II Unit Impulse Response')
axis([-0.5 10.5 -0.3 1.1])
xlabel('n')
ylabel('h[n]')
```



Part A.1 - Function II Unit Impulse Response
h[n]
n
1
0.8
0.6
0.4
0.2
0
-0.2
0
1
2
3
4
5
6
7
8
9
10

Part 2 – Hand Calculations

A.2 - Function I

$\gamma^2 + \frac{1}{6}\gamma - \frac{1}{6} = 0$

$\gamma_{1,2} = \frac{-1/6 \pm \sqrt{(1/6)^2 - 4(-1/6)}}{2}$

$= \frac{-1/6 \pm 5/6}{2}$

$\gamma_{1,2} = 1/3 \ , \ -1/2$

$y[n] = C_1 (1/3)^n + C_2 (-1/2)^n$

$h[n] + 1/6\ h[n-1] - 1/6\ h[n-2] = 1/3\ \delta[n]$

$h[-1] = h[-2] = 0$

@n=0 $h[0] + 1/6\ h[n-1] - 1/6\ h[n-2] = \frac{1}{3}(1)$

$h[0] = 1/3$

@n=1 $h[1] + 1/6\ h[0] - 1/6\ h[n-1]$

$h[1] + 1/6(1/3) - 0 = 1/3\ (0)$

$h[1] = -1/18$

$h[n] = C_1 (1/3)^n + C_2 (-1/2)^n$

$h[0] = 1/3, \quad h[1] = -1/18$

@ n=0 $\quad 1/3 = C_1 + C_2 \qquad (1)$

$-1/18 = C_1(1/3) + C_2(-1/2) \qquad (2)$

$\Rightarrow C_1 = 1/3 - C_2 \qquad (3)$

$(1) \rightarrow (2) \Rightarrow -1/18 = 1/9 - 1/3\, C_2 - 1/2\, C_2$

$-\frac{5}{6} C_2 = -1/6$

$C_2 = 1/5$

into (3) $\quad C_1 = 1/3 - 1/5$

$C_1 = 2/15$

$$\boxed{\therefore\ h[n] = \left[2/15 (1/3)^n + 1/5(-1/2)^n\right] u[n]}$$

A.2 - Function II

$$\gamma^2 + \frac{1}{4} = 0$$

$$\gamma_{1,2} = \frac{0 \pm \sqrt{0 - 4(1/4)}}{2}$$

$$\gamma_{1,2} = \frac{1}{2}j \,,\ \frac{-1}{2}j$$

$$y[n] = C_1(0.5j)^n + C_2(-0.5j)^n$$

$$h[n] + \frac{1}{4}h[n-2] = \delta[n]$$

@ $n=0$ $\quad h[0] + \frac{1}{4}h[-2] = 1$

$\bullet\ h[-2] = 0$

$\Rightarrow h[0] = 1$

@ $n=1$ $\quad h[1] + \frac{1}{4}h[-1] = 0 \quad \bullet\ h[-1] = 0$

$h[1] = 0$

$$h[n] = C_1(0.5j)^n + C_2(-0.5j)^n$$

@ $n=0$ $\quad h[0] = C_1(0.5j)^0 + C_2(-0.5j)^0$

$1 = C_1 + C_2 \Rightarrow C_1 = 1 - C_2$

@ $n=1$ $\quad h[1] = C_1(0.5j)^1 + C_2(-0.5j)^1$

$0 = (1 - C_2)(0.5j) + C_2(-0.5j)$

$0 = 0.5j + C_2(-0.5j) + C_2(-0.5j)$

$-0.5j = 2C_2(-0.5j)$

$(2)\ \frac{-0.5j}{-0.5j} = C_2 \Rightarrow$

$\Rightarrow C_2 = 1/2$

$C_1 = 1 - 1/2$

$C_1 = 1/2$

$$\boxed{\therefore h[n] = \left[\frac{1}{2}(0.5j)^n + \frac{1}{2}(-0.5j)^n\right]u[n]}$$

Part 3 – Comparison of Hand & MATLAB Calculations

A.2 - Function I

$$h[n] = \left[\frac{2}{15}(1/3)^n + 1/5\,(-1/2)^n\right] u[n]$$

$$h[3] = \frac{2}{15}(1/3)^3 + 1/5\,(-1/2)^3$$

$$= \frac{2}{15}\left(\frac{1}{27}\right) + \frac{1}{5}\left(\frac{-1}{8}\right)$$

$$\boxed{h[3] = -13/648}$$

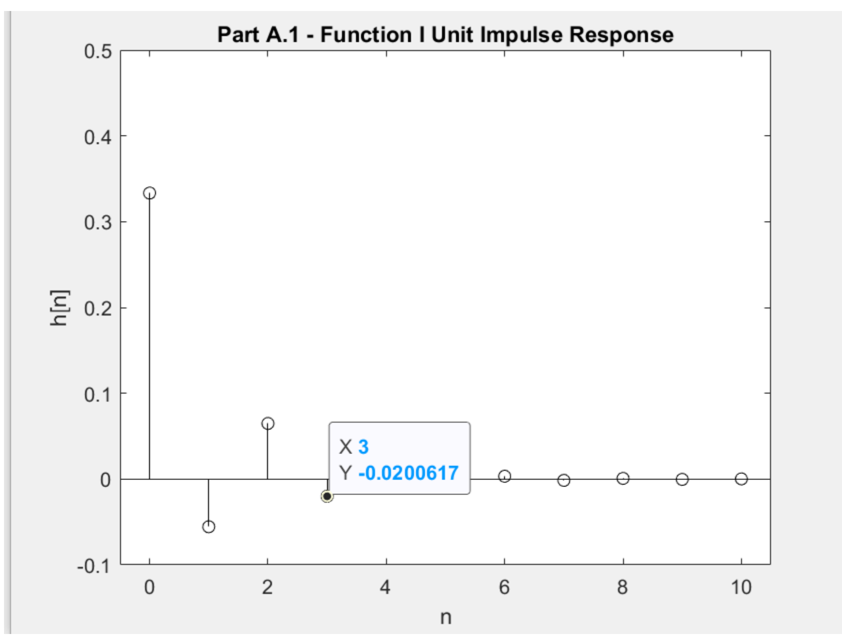
A.2 - Function II

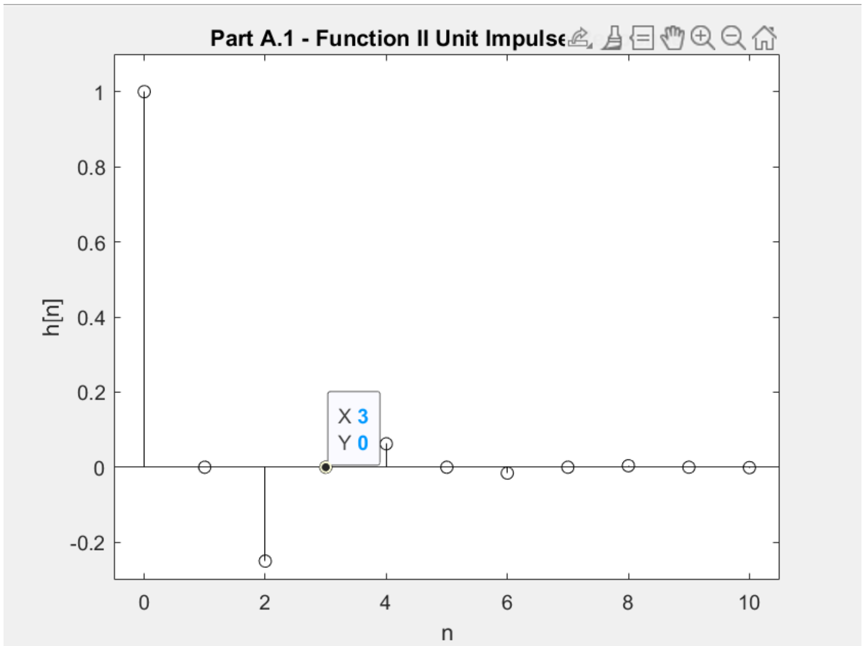
$$h[n] = \left[\frac{1}{2}(0.5j)^n + \frac{1}{2}(-0.5j)^n\right] u[n]$$

$$h[3] = \frac{1}{2}(0.5j)^3 + \frac{1}{2}(-0.5j)^3$$

$$= -1/16j + 1/16j$$

$$\boxed{h[3] = 0}$$

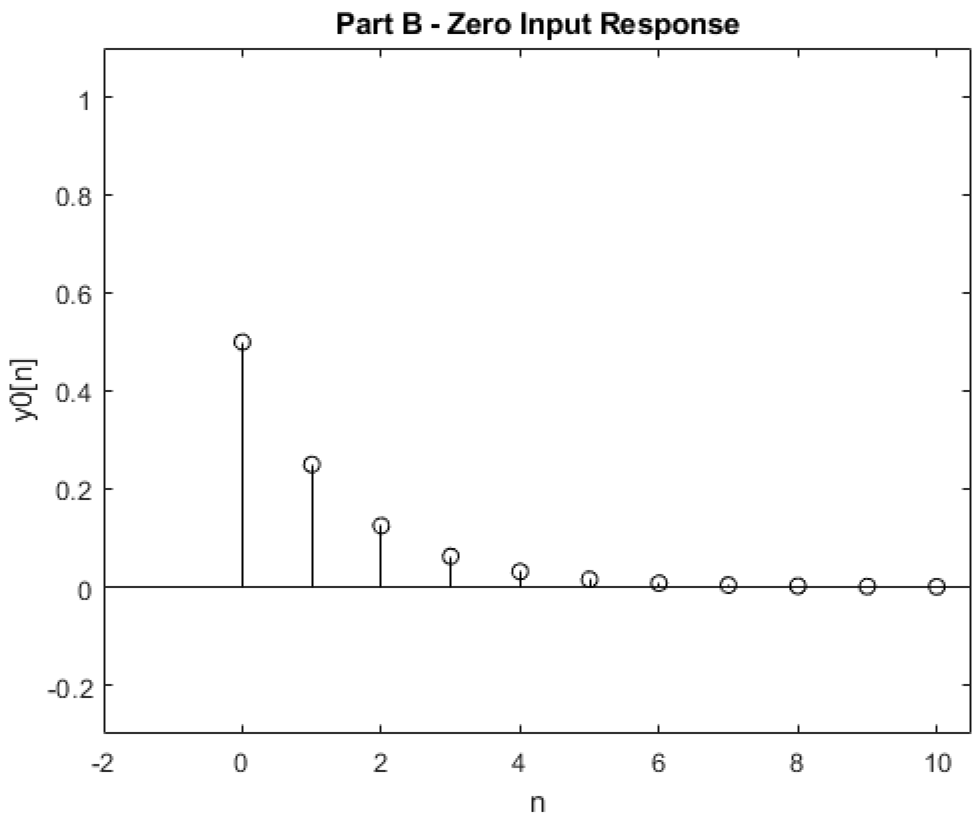




Therefore, the h[3] values match accordingly.

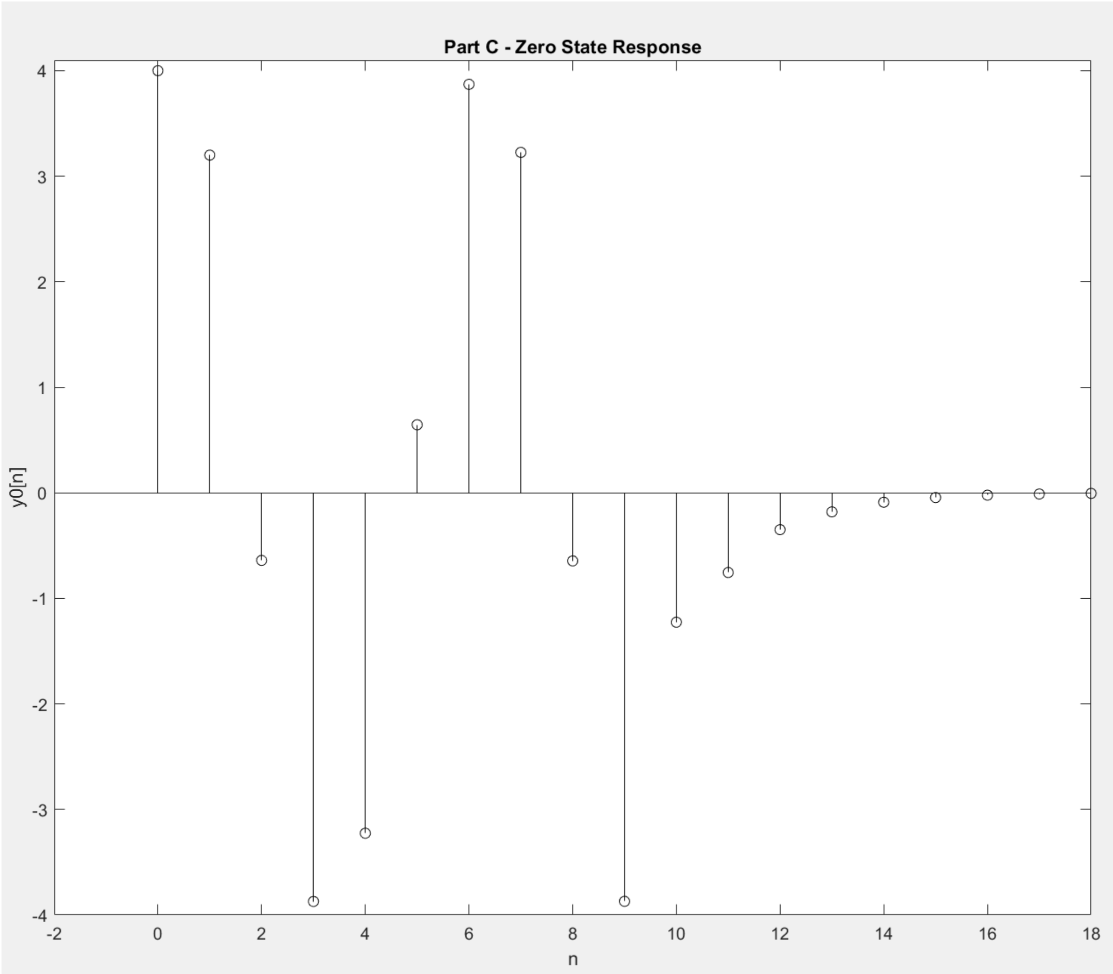
B) Zero Input Response

```
n =  0:50;

b = [2 0 0];
a = [1 -3/10 -1/10];



z_i = filtic(b, a, [1 2]);
y_0 = filter(b, a, zeros(size(n)), z_i);

figure
stem(n, y_0, 'k');
title('Part B - Zero Input Response')
axis([-2 10.5 -0.3 1.1])
xlabel('n')
ylabel('y0[n]')
```



C) Zero State Response

```
n =  -50:50;
u = (mod(n,1)==0)*1.0.*(n>=0);
u2 = (mod(n,1)==0)*1.0.*(n>=10);

b = [2 0 0];
a = [1 -3/10 -1/10];
x = 2.*cos((2.*pi.*n)./6).*(u-u2);

z_i = filtic(b, a, 0);
y_0 = filter(b, a, x, z_i);

figure
stem(n, y_0, 'k');
title('Part C - Zero State Response')
axis([-2 18 -4 4.1])
xlabel('n')
ylabel('y0[n]')
```

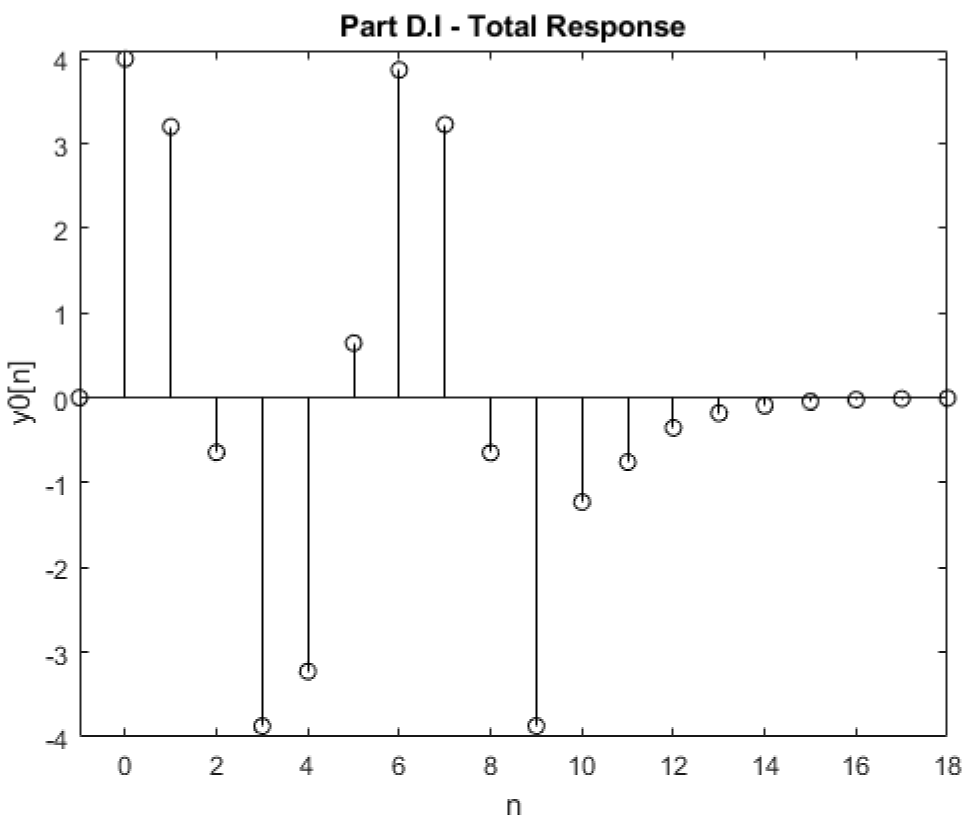


D) Total Response

Part I - Total response

```
n =  -50:50;
u = (mod(n,1)==0)*1.0.*(n>=0);
u2 = (mod(n,1)==0)*1.0.*(n>=10);

b = [2 0 0];
a = [1 -3/10 -1/10];
x = 2.*cos((2.*pi.*n)./6).*(u-u2);

z_i = filtic(b, a, [1 2]);
y_0 = filter(b, a, x, z_i);

figure
stem(n, y_0, 'k');
title('Part D.I - Total Response')
axis([-1 18 -4 4.1])
```

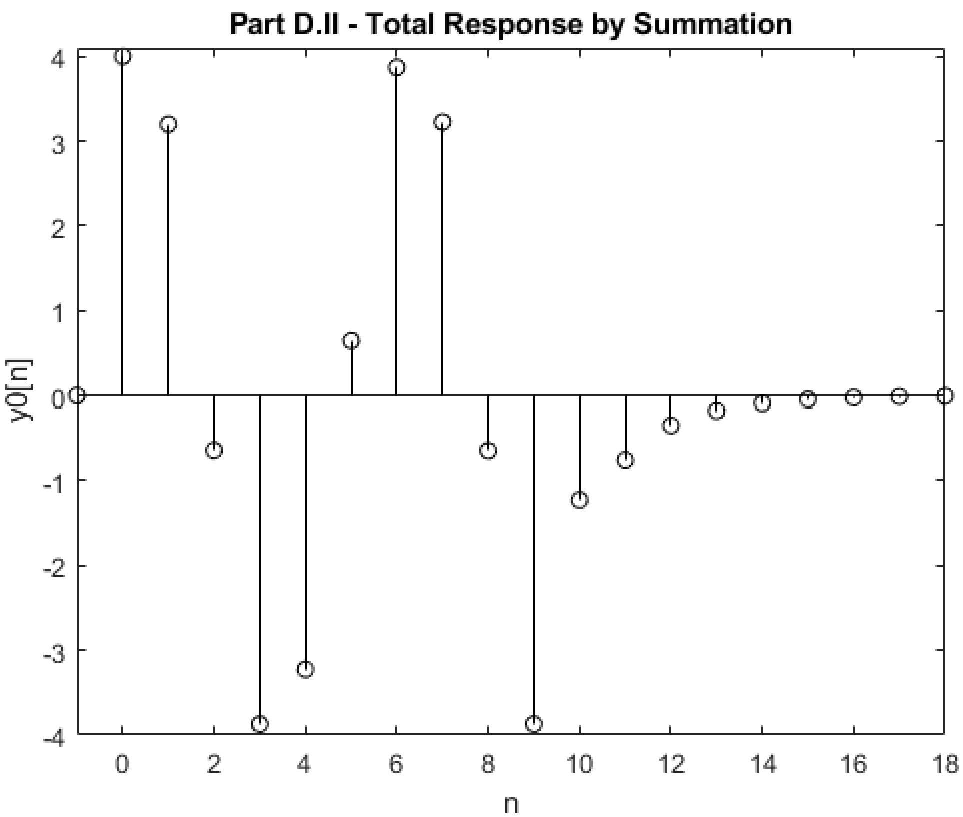
```
xlabel('n')
ylabel('y0[n]')
```



Part II - Total response by Summation of Zero Input and Zero state Responses

```
n =  -50:50;
u = (mod(n,1)==0)*1.0.*(n>=0);
u2 = (mod(n,1)==0)*1.0.*(n>=10);
b = [2 0 0];
a = [1 -3/10 -1/10];
x = 2.*cos((2.*pi.*n)./6).*(u-u2);

z_i = filtic(b, a, [1 2]);
y_0i = filter(b, a, zeros(size(n)), z_i);

z_i2 = filtic(b, a, 0);
y_0 = filter(b, a, x, z_i2);

yt = y_0 + y_0i;
```

```
figure
stem(n, y_0, 'k');
title('Part D.II - Total Response by Summation')
axis([-1 18 -4 4.1])
xlabel('n')
ylabel('y0[n]')
```



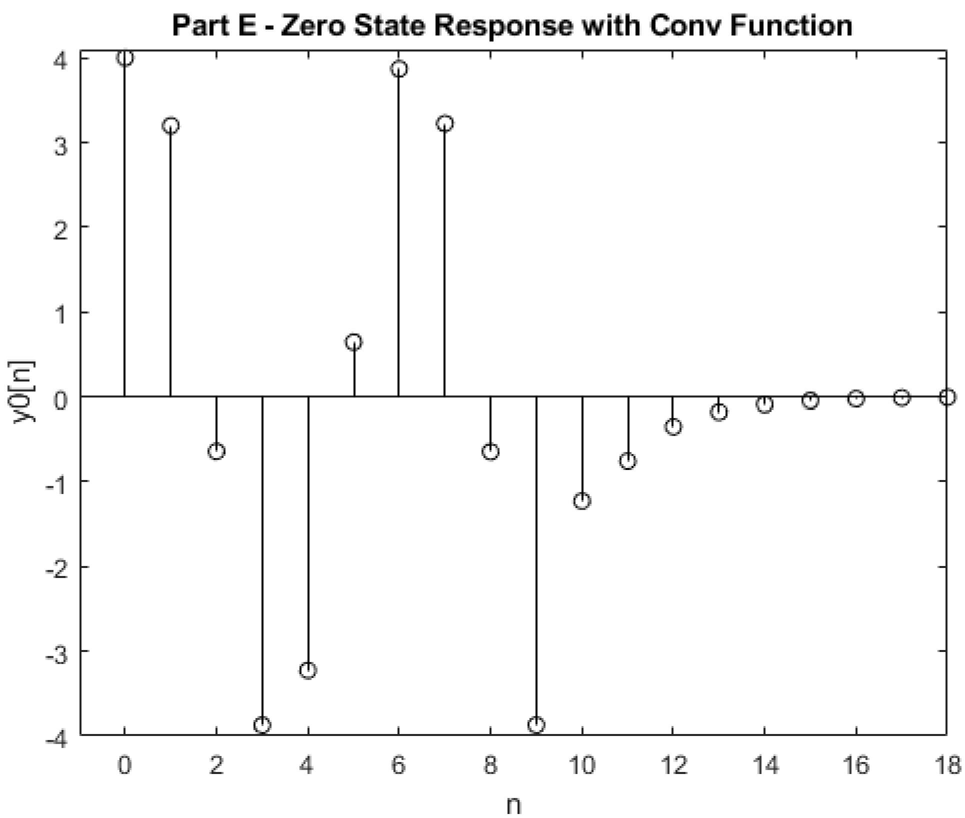
E) Convolution and System Stability

Part I - conv command

```
n =  0:50;
u = (mod(n,1)==0)*1.0.*(n>=0);
u2 = (mod(n,1)==0)*1.0.*(n>=10);
b = [2 0 0];
a = [1 -3/10 -1/10];
x = 2.*cos((2.*pi.*n)./6).*(u-u2);
%delta = @(n) 1.0.*(n==0);
```

```
delta = (n)==0;


h = filter(b, a, delta(n));
```

```
y = conv(x, h);

n =  0:100;

figure
stem(n, y, 'k');
title('Part E - Zero State Response with Conv Function')
axis([-1 18 -4 4.1])
xlabel('n')
ylabel('y0[n]')
```



Part II

Yes, the results in the plots “Part E – Zero State Response with Conv Function” and “Part C – Zero State Response” are the same.

Part III

Yes, the system is asymptomatically stable, this is because the system's Characteristic Equation has two real roots ($E^2 - (3/10)E - (1/10)$ → $E_1 = 0.5$ and $E_2 = -0.2$) within the unit circle in the Complex Plane which is shown as a decreasing cone shaped trajectory in the plot "Part E – Zero State Response with Conv Function".

F) Moving Average Filter

Part I - Constant Coefficient Difference Equation with h[n] Impulse Response

Part F.1

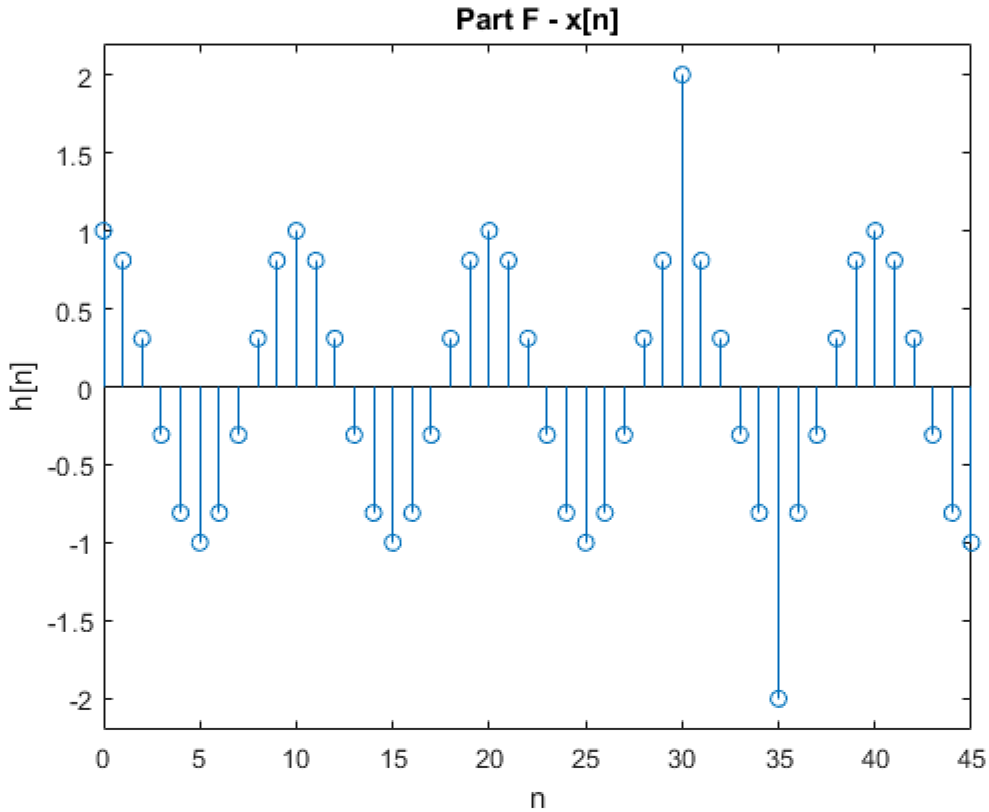
$$y[n] = \frac{1}{N}\Big((1)\,x[n] + (1)\,x[n-1] + \ldots + (1)\,x[n-(N-1)]\Big)$$

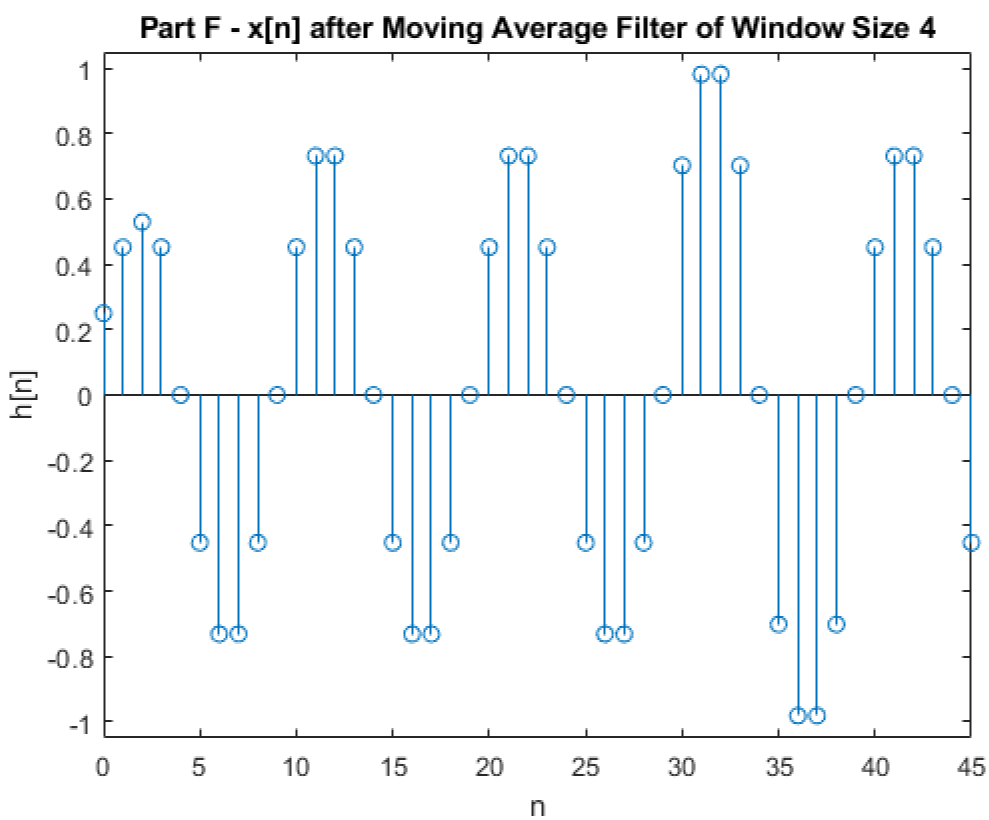
Part II & III - MATLAB Function for N-point moving-average filter

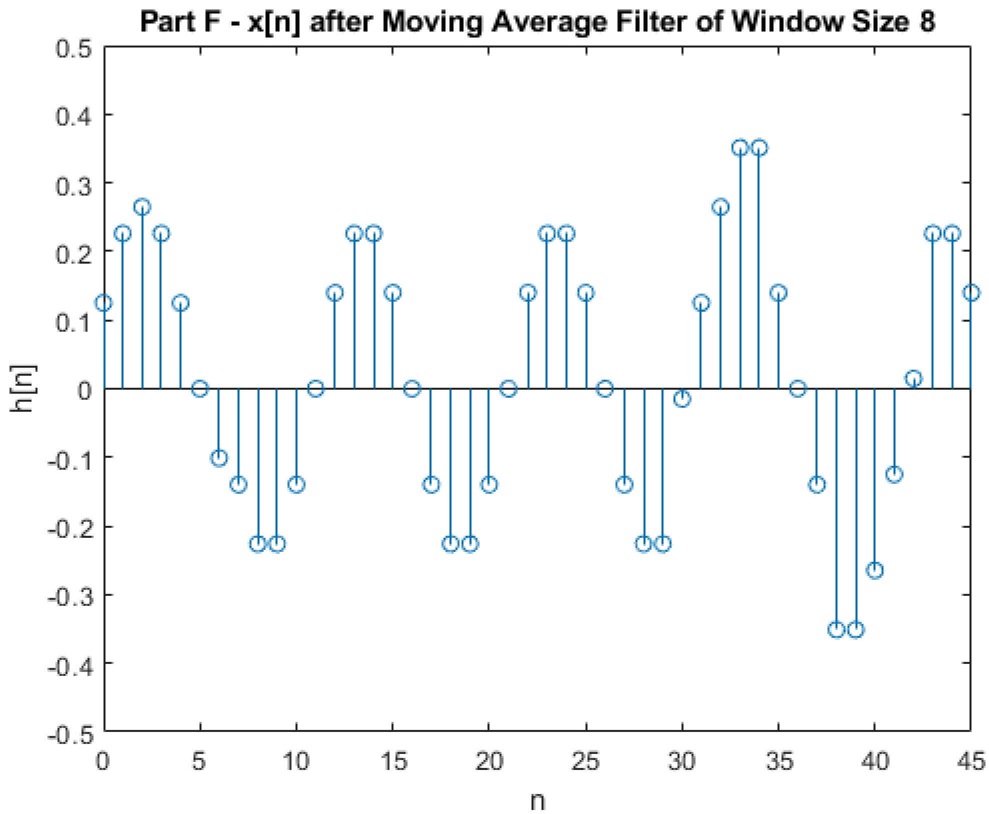
```
n = 0:1:45;
d = (n-30)==0;
d2 = (n-35)==0;
a = 1;

x = cos((pi.*n)./5)+d-d2;

%original function
figure
stem(n, x);
title('Part F - x[n]')
axis([0 45 -4 4.1])
xlabel('n')
ylabel('h[n]')

%N=4
filterN = 4;

b = (1/filterN)*ones(1, filterN);
h = filter(b, a, x);
figure
stem(n, h);
title('Part F - x[n] after Moving Average Filter of Window Size 4')
axis([0 45 -4 4.1])
```

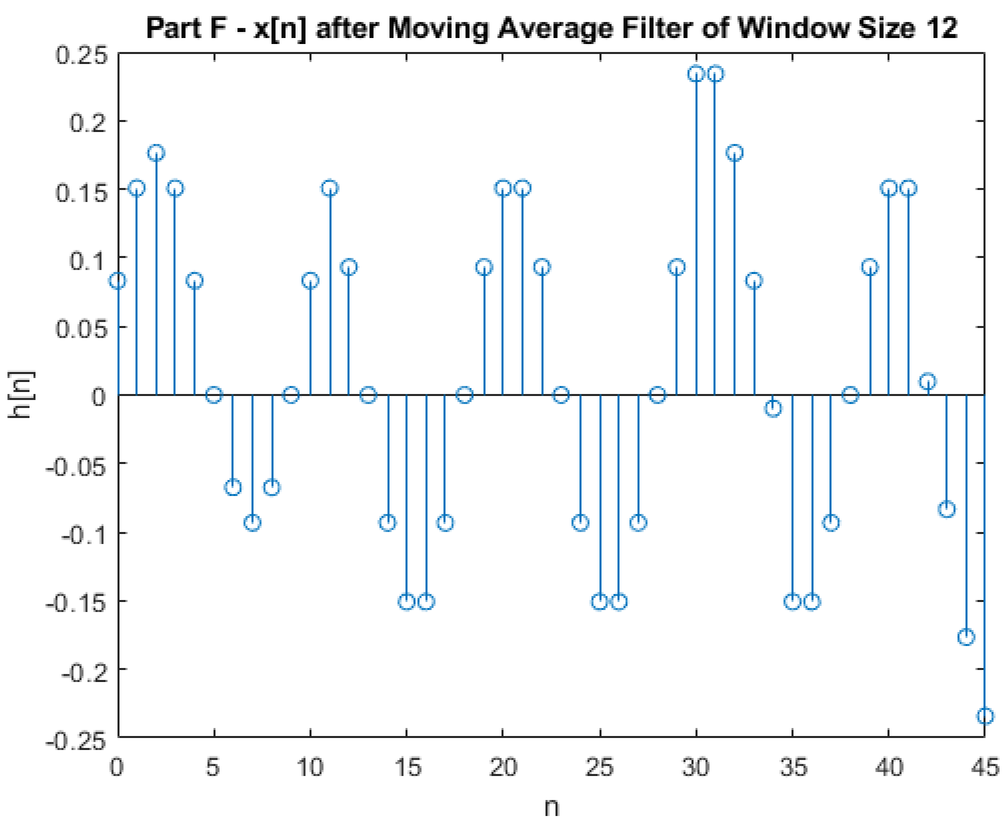
```
xlabel('n')
ylabel('h[n]')

%N=8
filterN = 8;
b = (1/filterN)*ones(1, filterN);
h = filter(b, a, x);
figure
stem(n, h);
title('Part F - x[n] after Moving Average Filter of Window Size 8')
axis([0 45 -4 4.1])
xlabel('n')
ylabel('h[n]')

%N=12
filterN = 12;
b = (1/filterN)*ones(1, filterN);
h = filter(b, a, x);
figure
stem(n, h);
title('Part F - x[n] after Moving Average Filter of Window Size 12')
axis([0 45 -4 4.1])
xlabel('n')
ylabel('h[n]')
```

As the window size (filterN) of the moving-average filter increases the amplititude of the resulting sinusoidal signal decreases. This is due to the filter taking ana verage over a larger number of data points while the number of repeting y-values and the number of y-value that cancel out increases. This is a common method to smooth out noisy data (outlier data) that would otherwise give an incorrect representation of trajectory of the actual signal.


Part F - x[n]
h[n]
n
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
5
10
15
20
25
30
35
40
45


Part F - x[n] after Moving Average Filter of Window Size 4
h[n]
n
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
20
25
30
35
40
45


Part F - x[n] after Moving Average Filter of Window Size 8
h[n]
n
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
5
10
15
20
25
30
35
40
45


Part F - x[n] after Moving Average Filter of Window Size 12
h[n]
n
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
5
10
15
20
25
30
35
40
45