LAB 4: Discrete Time Fourier Transform

Objective

In Lab 4, you will learn about discrete-time Fourier transform (DTFT). You will learn how to use FFT to calculate DTFT of a signal and examine the time convolution property of DTFT. Also, you will design an FIR high-pass filter and investigate the difference between an ideal filter and a realizable filter.

Discrete-Time Fourier Transform

The Discrete-time Fourier transform for a signal $x[n]$ is defined as follows,

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(\Omega)e^{jn\Omega}d\Omega$$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

Where, the signal $x[n]$is discrete, but its DTFT is a continuous signal in frequency domain.

Preparation

- Read chapter 9 from *Linear Signals and Systems by B.P. Lathi*.
- Work through Examples 9.5 and 9.7-3 of the text.

Lab Assignment

A. Discrete-Time Fourier Transform (DTFT)

In this assignment, we will use *fft* to calculate discrete-time Fourier transform of a signal. The Fast Fourier transform algorithm (fft) computes the discrete Fourier transform of an N-point signal.

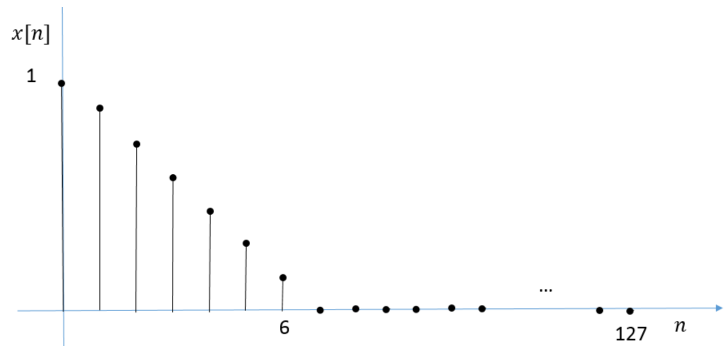

Figure 1

1) Use *fft* command from MATLAB to compute the DTFT of the signal $x[n]$, depicted in Figure 1. The length of the signal $x[n]$ is 128 points. Plot magnitude and phase of $X(\Omega)$ in $(-\pi, \pi)$. Use *fftshift* commamd to center the zero frequency component.
2) Compute FTDT of $x[n]$ by hand. Use MATLAB to plot the magnitude and phase of the result. Are the results consistent with part (1).
3) Use *ifft* command from MATLAB to reconstruct the signal in the time domain from it spectrum $X(\Omega)$ and plot the result. Is the obtained result the same as $x[n]$?

B. Time Convolution

In this assignment, you will examine the convolution property of discrete-time Fourier transform. To implement DTFT, you can use a matrix-based approach similar to computer Example 9.7-1 of the textbook. The following code computes the DTFT of a signal $x[n]$,

omega= linspace(-pi,pi,1001);

*W_omega = exp(-j).^((0:length(x)-1)'*omega);*

*X = (x*W_omega);*

1) Find and plot the DTFT of the signal $x[n] = \sin(\frac{2\pi n}{10})(u[n] - u[n-10])$.
2) Find and plot the DTFT of the signal $h[n]$ as shown in Figure 2.

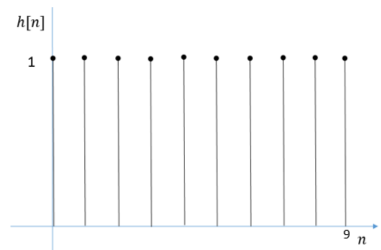

Figure 2

3) Find and plot the result of $X(\Omega)H(\Omega)$.
4) Use *conv* command from MATLAB to compute $y[n]$ by convolving $x[n]$ and $h[n]$.
5) Find and plot the DTFT of the signal $y[n]$.
6) Did you get the same results from part 3 and 5? Explain why.

C. FIR Filter Design by Frequency Sampling

1) Design a high pass FIR filter with a cutoff frequency $\Omega_0 = \frac{2\pi}{3}$. Use *ifft* command from MATLAB to compute and sketch the impulse response of the filter ($h[n]$) when the filter length is 35 points.
2) Use the *freqz* command from MATLAB to find the frequency response of the filter from $h[n]$. Plot the magnitude of $H(\Omega)$.
   *H = freqz(h,1, 0:2*pi/1001:2*pi)*
3) How is the result in part (2) different from the ideal filter you started with?
4) Increase the number of points to 71 and then sketch $h[n]$ and $|H(\Omega)|$.
5) How does the resulting change when the length of the filter is increased?