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Lab Report No.	2

Report Title	Performance of Control Systems under Proportional, PI, PD, and PID Control

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(Note: remove the first 4 digits from your student ID)

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Lab # 2 Report: Performance of Control Systems under Proportional, PI, PD, and PID Control

<table>
<tr><td colspan="2">Your Parameter Set Number: #040</td></tr>
<tr><td>Partner 1 (name):
Ahmad Fahmy</td><td>Partner 2 (name):
Vanessa Hoang</td></tr>
<tr><td colspan="2">PRE-LAB (INDIVIDUAL): Lab Instructors, please check your records for each partner if he/she demonstrated to you their Pre-Lab simulation that consist of a functional SIMULINK simulation diagram for the Proportional Control, and when marking, please check off the appropriate box.</td></tr>
<tr><td>□ YES □ NO</td><td>□ YES □ NO</td></tr>
<tr><td colspan="2">Please note that the Pre-Lab check-in is suspended for Winter 2022</td></tr>
<tr><td>Pre-Lab Penalty =</td><td>Pre-Lab Penalty =</td></tr>
<tr><td>EXECUTIVE SUMMARY</td><td>/10</td></tr>
<tr><td>Proportional Control – Performance Analysis, Benchmarking, Establishing Performance Specifications</td><td>/14</td></tr>
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<tr><td>Total Mark for the Collaborative Part of the Report</td><td>/80</td></tr>
<tr><td>Deductions for lack of clarity, poor writing style, grammar or spelling mistakes, layout of the report, etc., will be recorded here:</td><td></td></tr>
<tr><td>Interview mark
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<tr><td colspan="2">Please note that the interviews are suspended for Winter 2022, and replaced with 10-minute D2L Lab Quizzes. Lab 2 Quiz is open on Monday, March 7, 2022 from 8:00 am till 11:45 pm.</td></tr>
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Executive Summary

The purpose of this experiment was to analyze the performance of systems under Proportional (P), Proportional + Integral (PI), Proportional + Derivative (PD), and Proportional + Integral + Derivative (PID) control and compare the transient state response and steady state errors. In Part 1 of the lab, a benchmarked was established to improve system performance. By changing the gain and time constants, the changes P, PI, and PD controls could be examined to find the best settings that matched specifications. In Part 2, a PID controller was designed using the "Trial and Error" and "Ziegler- Nichols" Approach to improve response specifications.

A system under proportional control was created using SIMULINK simulation, and the proportional controller gains were experimented with to determine the effects on steady state error and transient response. Three data sets of various sizes (1,3,5) for K_p were recorded, and it was found that as gain increases, the rise time and steady state error (step) decreases, while percent overshoot and settling time increases. For our system under proportional control, it was a type 0 system, thus $e_{ss(ramp)} = \infty$. A high gain also increases oscillations which can increase stability, and saturate the controller, while a low gain creates an overdamped system response. A benchmark for the system performance was created using the "Quarter Decay Response" formula in the appendix, where $K_p = \frac{K_{crit}}{2}$. Since we used the same data set at Lab 1, we determined that $K_{crit} = 6.22$, thus $K_p = 3.11$. At this gain, the transient response and steady state errors were recorded. System performance specifications were established, where $e_{ss(step)\%} = 0\%$, $e_{ss(ramp)} = 0$, $PO\% < 15\%$, and $t_{s(\pm 2\%)} < 10.31s$. Proportional control will not meet specifications, so PI and PD control must also be explored to see their effects on controller parameters.

In Part 1.2, the simulation diagram is connected to PI control and transient response and steady state errors are analyzed. The benchmarked system was improved by adjusting parameters K_p and τ_i , and the best settings occurred at $K_p = 1.5$ and $\tau_i = 8$ allowing $PO\% < 15\%$, and $t_{s(\pm 2\%)} < 10.31s$. Since the integral time constant changes, the controller into a type 1 system, the steady state step error will be 0%. However, the ramp error will never reach 0% from the specifications. To summarize the PI results, a smaller τ_i means faster integration, thus, gain margin decreases, percent overshoot increases, and wind-up effect may occur. In Part 1.3, the simulation diagram is connected to PD control and transient response and steady state errors are analyzed again. The benchmarked system was improved by adjusting parameters K_p and τ_d , and the best settings occurred at $K_p = 3$ and $\tau_d = 2$ allowing $PO\% < 15\%$, and $t_{s(\pm 2\%)} < 10.31s$. Since the derivative time constant does not change the system type from type 0, the unit step and ramp error will never reach 0% from the specifications. For PD results, it has no effect on steady state error tracking, but a higher τ_d means a stronger action, resulting in an increased gain margin, smaller percent overshoot, however noise may be created.

In Part 2, PID control was used to improve the system response and meet the performance specifications established in Part 1. There were two methods used to tune the systems: "Trial and Error Approach" and "Ziegler-Nichols 'Ultimate Gain' Method". The proportional control is the main source of control, as it can be used to create a stable and accurate response. PI control improves steady state tracking, whereas PD control increases damping. In the Trial-and-Error Approach, the parameters were set to have a K_p and $\tau_d$based on previously used PD controller, and τ_i from the PI controller, then the values were adjusted for fine tuning. Since the PID controller creates a type 1 system, to lower $e_{ss(ramp)}$, τ_i must decrease while K_p increases, resulting in instability. To minimize the ramp response, we know that a higher proportional gain would results in a lower steady state error. However, a main problem of increasing the gain too much would result in increased oscillations and instability. In comparison, for the Ziegler-Nichol's approach, the parameters were calculated based on the "Ultimate Gain" formulas from the appendix. However, we were not able to meet all system specifications for both methods since for PID control as well $e_{ss(ramp)}$ did not equal 0 and percent overshoot was too high. Overall, we could not reach all specifications such as $e_{ss(ramp)}$, which is due to the system type, as well as errors in calculating the Ziegler-Nichol's approach causing errors in percent overshoot.

Pre-Lab: Creating SIMULINK Simulation Diagram
SIMULINK Simulation Diagram for Proportional Control
Ahmad Fahmy

In the Pre-Lab, a SIMULINK diagram is created for 3 modes of control: P, PI and PD, similar to the diagram in Lab 1 and displayed in Figure 1a and 2a. To demonstrate the functionality of each controller, they the $\tau_i = 5$, and $\tau_d = 2$, and K_p is set to the K_{crit} for each controller type. For P, PI, and PD, the K_{crit} is 6.22, 4.75, and 25.97 respectively. The steady state response graph is shown in Figure 1b and 2b.

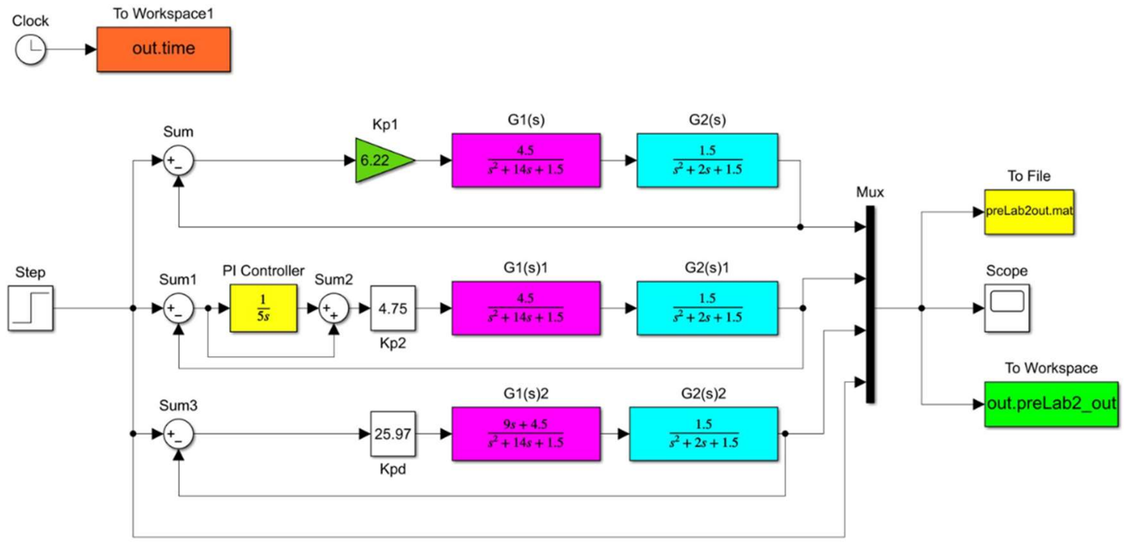

Figure 1a. Simulink Diagram for Comparisons between Various Modes of PID Control

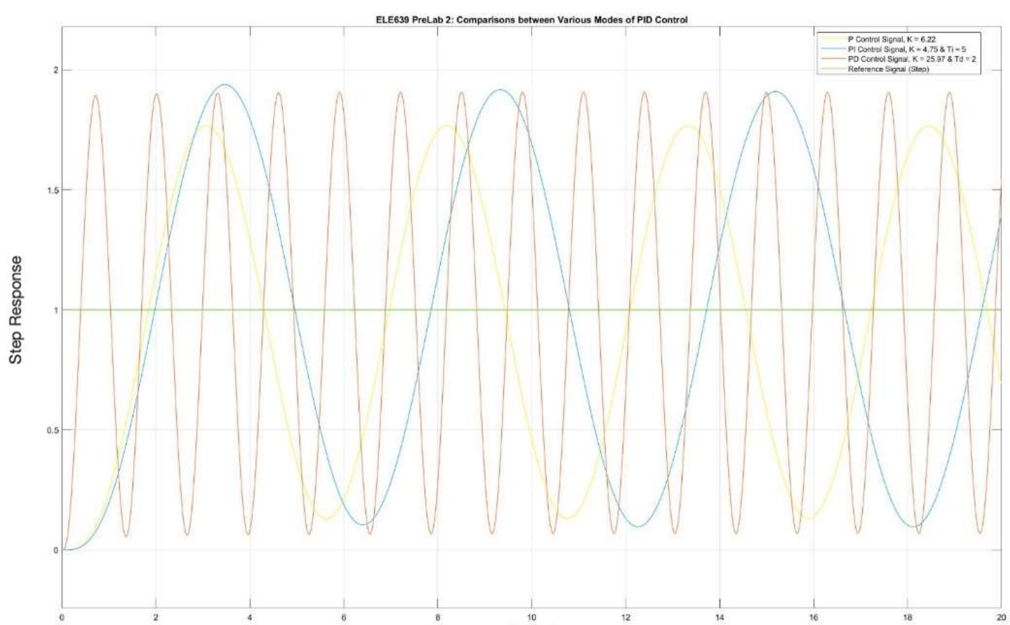

Figure 1b. Signal Waveforms for Various Modes of PID Control (P, PI, and PD)

Pre-Lab: Creating SIMULINK Simulation Diagram

SIMULINK Simulation Diagram for Proportional Control

Vanessa Hoang

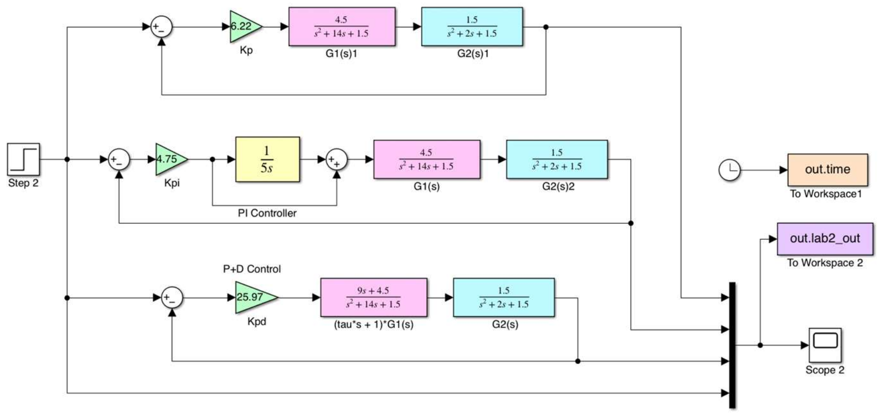

Figure 2a. Simulink Diagram for Comparisons between Various Modes of PID Control

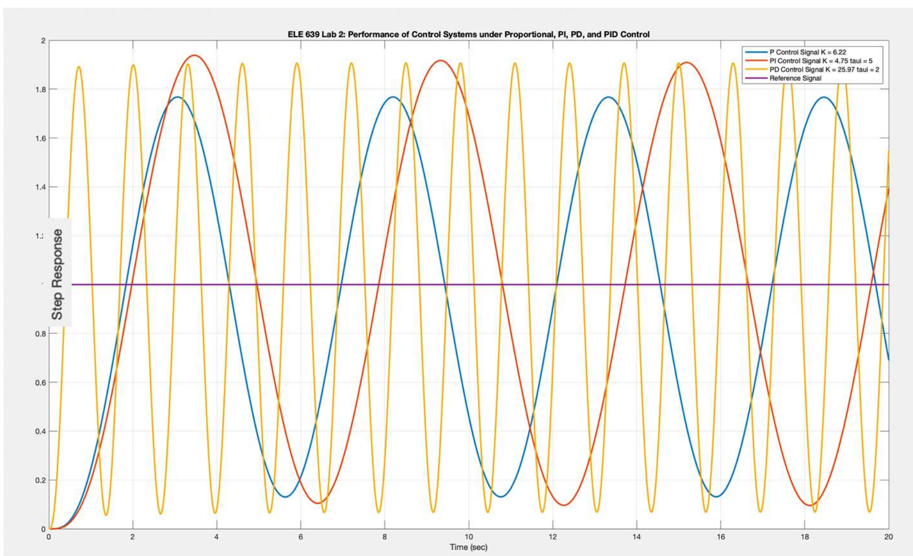

Figure 2b. Waveform for the Comparisons between Various Modes of PID Control

Part 1: Exploring Control Modes (P, PI and PD)

Data Set #040

$$G_1(s) = \frac{4.5}{s^2 + 14s + 1.5} \qquad G_2(s) = \frac{1.5}{s^2 + 2s + 1.5}$$

Part 1.1: Proportional Control

In the first part of the lab, the effect of gain changes on steady state errors using unit step $e_{ss(step)}$ and unit ramp $e_{ss(ramp)}$ and transient response are analyzed. To do this, a SIMULINK simulation diagram is created with the provided data set and only P control is connected to the output, as seen in Figure 1a. The simulation is run, and data is collected and saved for low, medium, and high gain in Table 1 below. The following time response characteristics can be calculated by looking at the step response graph in figure 1b and 1c. A sample calculation of transient response parameters is found for K_p = 1.

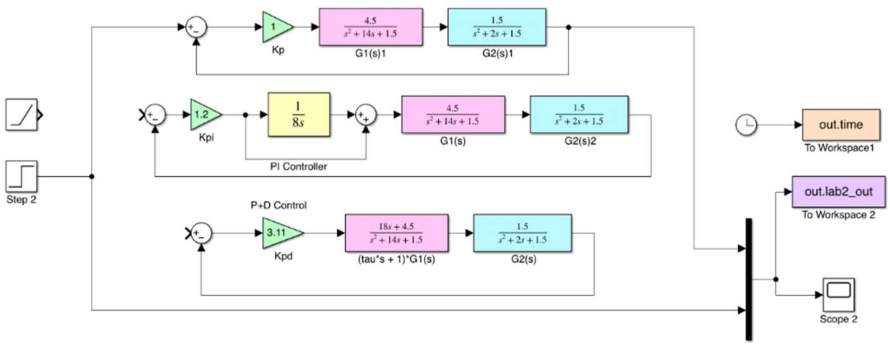

Figure 1a. Simulation Diagram for Proportional Control

$\boldsymbol{t_{r(0-100\%)}}$ can be found in Figure 1b. and seeing the time it takes for the signal to go from 0 to its steady state response. As seen in figure 1, the steady state response is 0.75, thus the $t_{r(0-100\%)} = 5.33s$.

Percent overshoot %O.S measures how smooth or oscillatory the signal is, where y_{max} is the maximum point in the step response, and y_{ss} is the steady state value.

$$PO = \frac{y_{max} - y_{ss}}{y_{ss}} \times 100\%$$
$$PO = \frac{0.78 - 0.75}{0.75} \times 100\%$$
$$PO = 4.83\%$$

Settling time $\boldsymbol{t_{s(\pm 2\%)}}$ measure how quickly the signal reaches steady state. It is found by looking on the graph and seeing where the signal crosses either $\pm 2\%$ of the steady state value y_{ss}. As seen in figure 1, this value is 8.74s

Steady-state error $\boldsymbol{e_{ss(step)}}$ measures the difference between the desired steady state of 1 and the actual steady state, where r_{ss} is the reference steady state, and y_{ss} is the signal steady state value. It can be measures 2 ways.

$$e_{ss(step)} = \frac{r_{ss} - y_{ss}}{r_{ss}} \times 100\%$$
$$e_{ss(step)} = \frac{1 - 0.75}{0.75} \times 100\%$$
$$e_{ss(step)} = 25.0\%$$

$$K_p = \lim_{s \to 0} G_{op}(s) = 1\left(\frac{4.5}{s^2+14s+1.5}\right)\left(\frac{1.5}{s^2+2s+1.5}\right) = 3 \text{ and } e_{ss(step)} = \frac{1}{1+K_p} = \frac{1}{1+3} = 0.25.$$

Steady-state error $\boldsymbol{e_{ss(ramp)}}$ is the difference between the desired steady state and the actual steady state at a ramp input. Since the lines on Figure 1c are not parallel the ramp steady state error is infinity.

$$K_v = \lim_{s \to 0} sG(s) = 1\left(\frac{4.5}{s^2+14s+1.5}\right)\left(\frac{1.5}{s^2+2s+1.5}\right) = 0 \text{ and } e_{ss(ramp)} = \frac{1}{K_v} = \frac{1}{0} = \infty$$

The above calculations can be repeated to find percent overshoot, settling time, rise time, and steady state errors for K_p = 3 in Figure 2a and 2b and K_p = 5 in Figure 3a and 3b. These sample calculations are also used throughout the rest of the report. All data is displayed in Table 1.

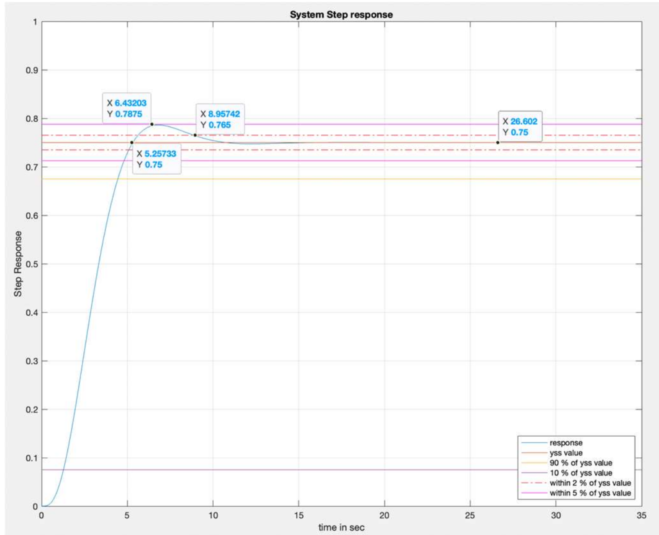

Figure 1b. Step response when K_p = 1.

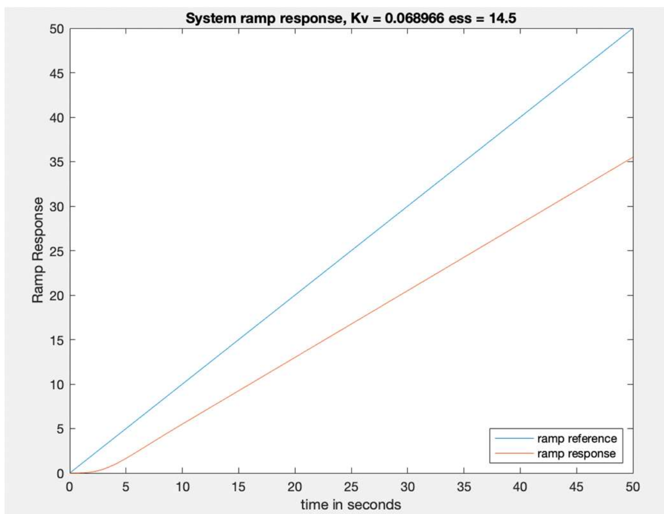

Figure 1c. Ramp response when K_p = 1.

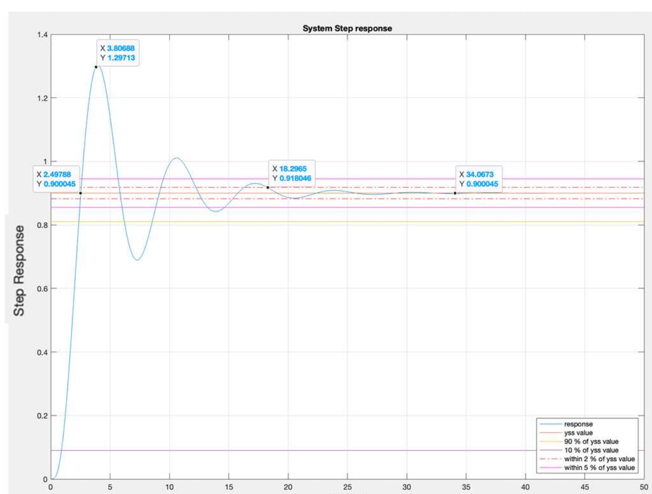

Figure 2a. Step response when K_p = 3.

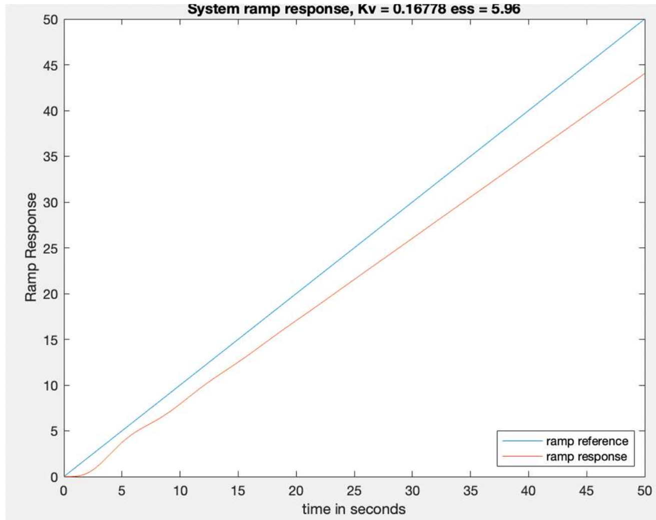

Figure 2b. Ramp response when K_p = 3.

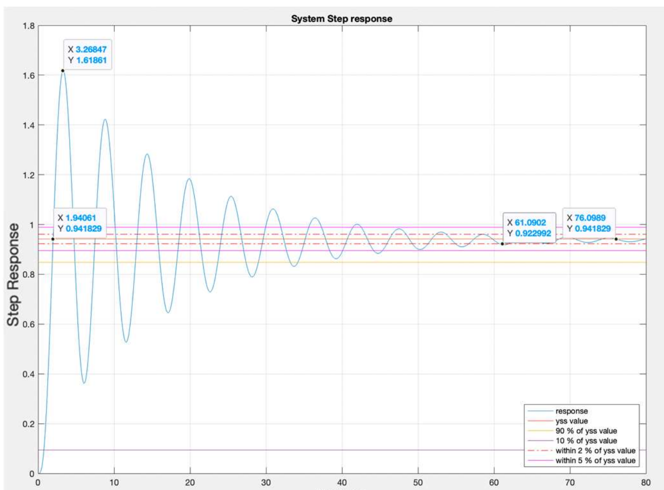

Figure 3a. Step response when K_p = 5.

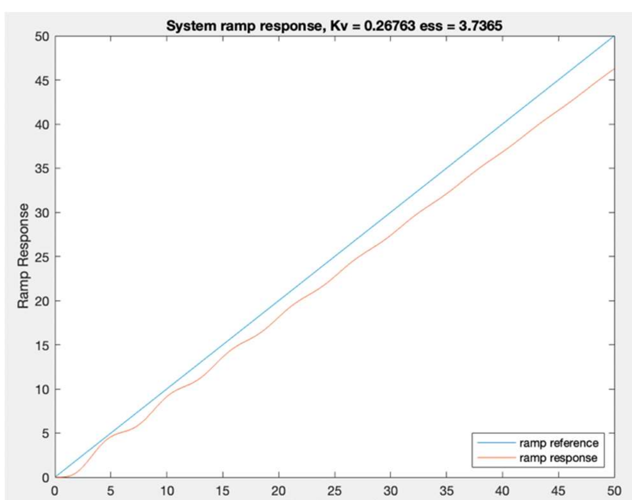

Figure 3b. Ramp response when K_p = 5.

Proportional Control	Proportional Gain K_p	Rise-time $t_{r(0-100\%)}$	Maximum overshoot %O.S.	Settling-time $t_{s(\pm 2\%)}$	Steady-state error $e_{ss(step)}$	Steady-state error $e_{ss(ramp)}$
Low-Gain	1	5.33 s	4.83%	8.74s	25.0%	∞
Medium-Gain	3	2.54 s	44.35%	18.13 s	10.00%	∞
High-Gain	5	2.0115 s	72.9844%	59.21 s	6.43%	∞

Table 1. Time response characteristics for unit feedback system under proportional control.

Based on the results, we can see that as the proportional gain increases, the rise time and steady state error decreases, while the maximum percent overshoot and settling time $e_{ss(step)}$ increases due to increased oscillations. Since the system type is 0, the $e_{ss(ramp)}$ should be infinite since the slopes of the graphs are not parallel and diverge. From Table 1, we can determine that the higher the proportional gain, is the, the lower the steady state error. High gain also increases oscillations which can increase stability, and saturate the controller, while a low gain creates an overdamped system response.

“Benchmarking” the System

The benchmarked proportional control of the signal was found using the Ziegler-Nichols “Quarter Decay” method. This benchmark is established, allowing us to improve the system performance using different control modes. We find the benchmark by setting the K_p to half of K_{crit} found in Lab 1. In lab 1, the $K_{crit} = 6.22$, thus $K_p = 3.11$ [1], which we set in Figure 4a, to see the system responses in Figure 4b and 4c. All data is recorded in Table 2.

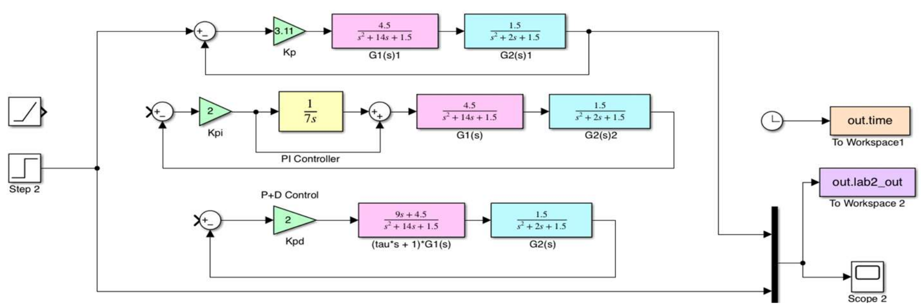

Figure 4a. Benchmarked Proportional Control Gain Simulation Diagram

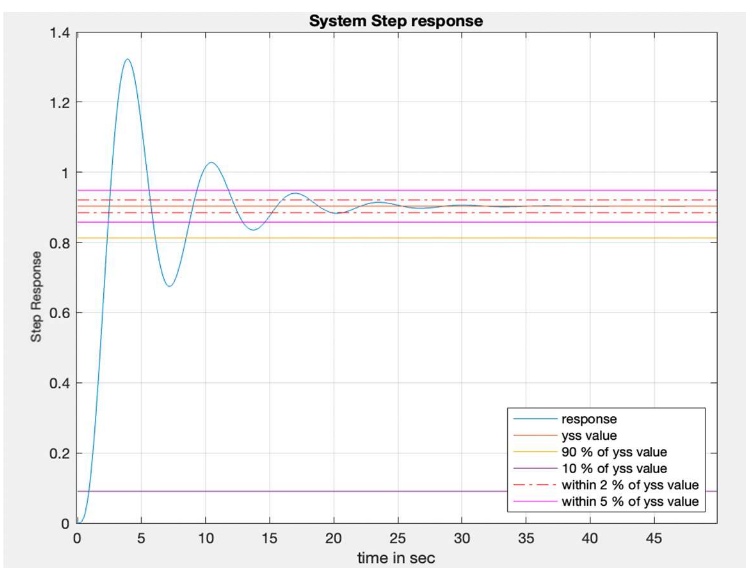

Figure 4b. Benchmarked Proportional Control Gain

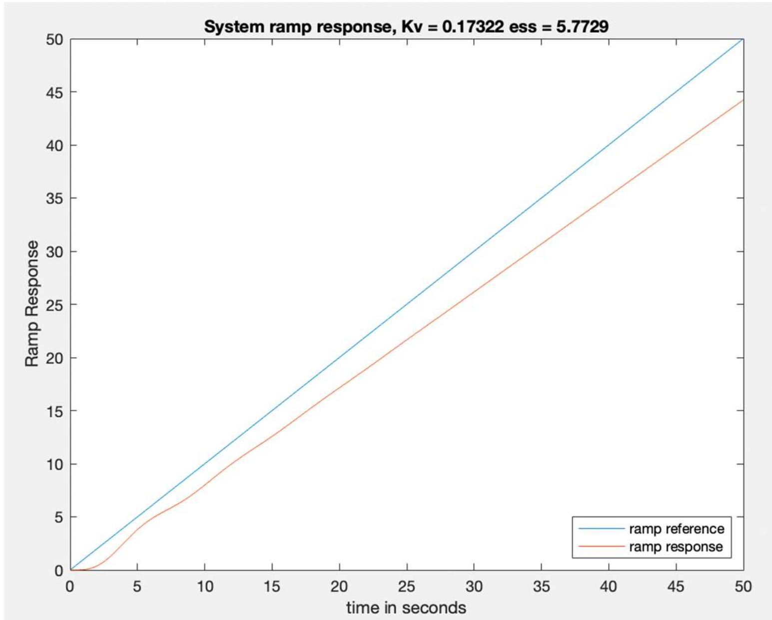

Figure 4c. Benchmarked Proportional Control Ramp Response

Benchmarked Proportional Control	**Proportional Gain K_p**	**Rise-time $t_{r(0-100\%)}$**	**Maximum overshoot %O.S.**	**Settling time $t_{s(\pm 2\%)}$**	**Steady-state error $e_{ss(step)}$**	**Steady-state error $e_{ss(ramp)}$**
Ziegler-Nichols "Quarter Decay"	3.11	2.49 s	46.45%	20.62 s	9.68%	∞

Table 2. Time response characteristics for unit feedback system under proportional control tuned by Ziegler-Nichols Ultimate Gain Method ("Quarter Decay")

Establishing System Performance Specifications

From Table 2, we must use the values and try to meet desired specifications for the system response. Therefore, the summarize of our required specifications are listed below:

- **Steady State Error:** $e_{ss(step)\%} = 0\%$
- **Steady State Error:** $e_{ss(ramp)} = 0$
- **Percent Overshoot:** $PO\% < 15\%$
- **Settling Time:** $t_{s(\pm 2\%)} < 10.31s$
-

We know proportional control will not allow us to meet these requirements at the same time, as seen in Table 1. This is because adding a proportional control does not change the system type from 0, so $e_{ss(ramp)}$ will always equal infinity. However, PID control can be used to design a controller that can help to meet these specifications. Thus, we examine PI and PD control separately in order understand the effects of the integral and derivative time constant on the controller parameters.

Part 1.2: Proportional + Integral (PI) Control

Controller parameters K_p and τ_i can be adjusted to determine the effect of the controller parameters on steady state errors (with inputs unit step and unit ramp) and transient response. The PI controller parameters are adjusted, and samples of the PI system responses are saved in Table 3.

Proportional Gain K_p	**Time Constant** τ_i	**Rise-time** $t_{r(0-100\%)}$	**Maximum overshoot %O.S.**	**Settling-time** $t_{s(\pm 2\%)}$	**Steady-state error** $e_{ss(step)}$	**Steady-state error** $e_{ss(ramp)}$
3.11	5	2.47	68.5	35.6	0%	53.0%
3.11	7	2.56	58.4	28.5	0%	75.0%
2	7	3.28	35.5	15.1	0%	116%

Table 3. Transient Response and Steady State error of PI Control

In comparison to the benchmarked system of the Proportional only control, we can see that if the time constant $\tau_i = 5$ is added, the rise time decreases, while the maximum percent overshoot and settling time increases by a great amount. However, the effect of τ_i on the steady state unit step error helps to meet specifications of 0%. To improve the system characteristics, more adjustments are made to controller parameters, and we can determine their effects. Ultimately, as K_p decreases or τ_i increases, the rise time increases, while maximum overshoot and settling time decrease. Using this information, the controller parameters can keep being adjusted to meet required specifications, as seen in Figure 5a, 5b, and 5c. The best response is recorded in Table 4.

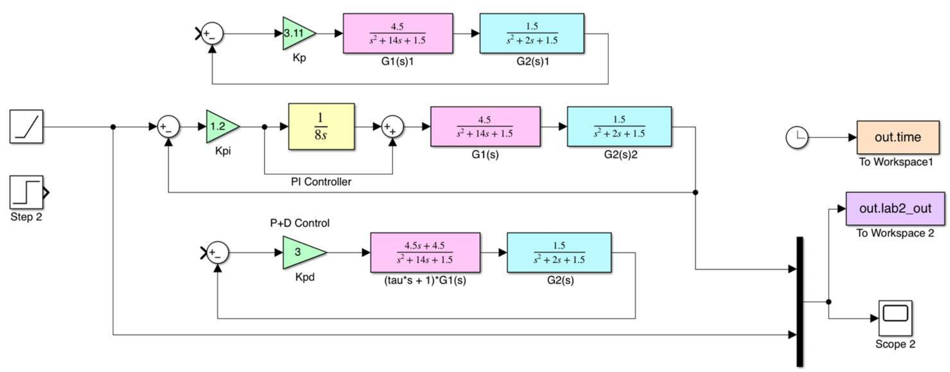

Figure 5a. Best PI Control Parameters Simulation Diagram

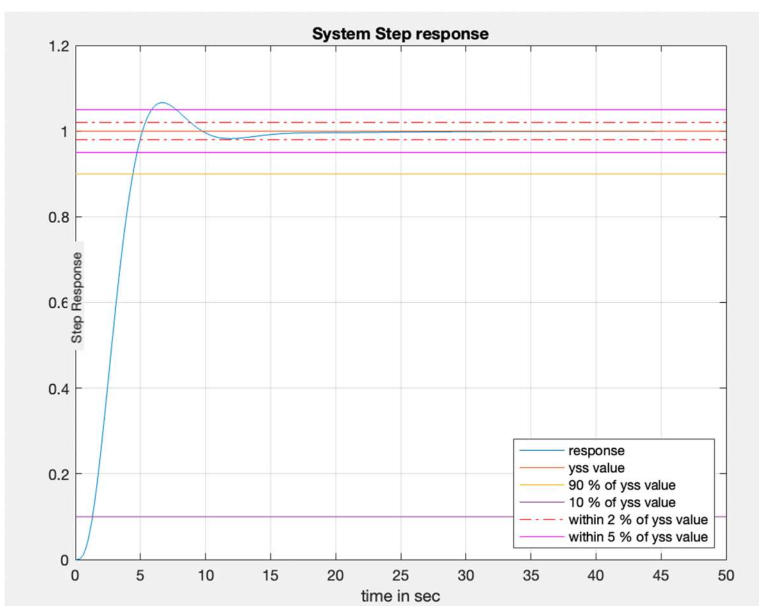

Figure 5b. Best PI Control Parameters Steady State Response

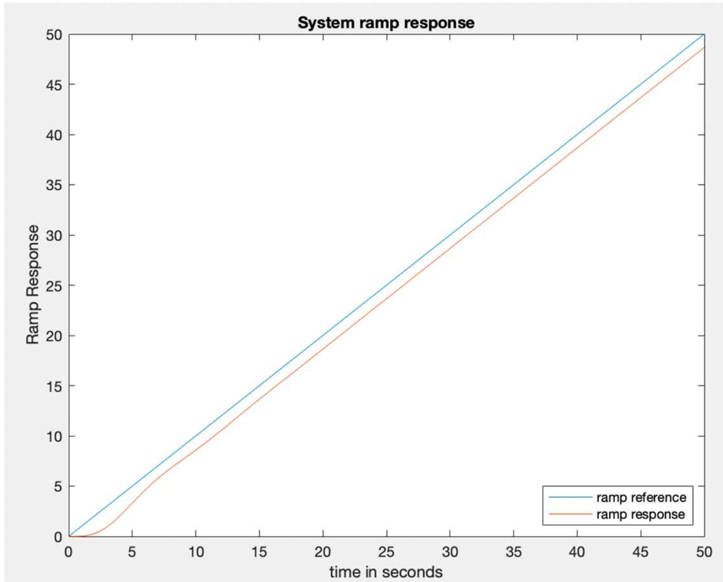

Figure 5c. Best PI Control Parameters Ramp Response

Proportional Integral Control	Proportional Gain K_p	Time Constant τ_i	Rise-time $t_{r(0-100\%)}$	Maximum overshoot %O.S.	Settling-time $t_{s(\pm 2\%)}$	Steady-state error $e_{ss(step)}$	Steady-state error $e_{ss(ramp)}$
PI Controller	1.2	8	5.08	11.7	10.4	0%	22.0%

Table 4: Time Response Characteristics for Unit Feedback System under PI Control

Based on the benchmarked system, we can see that the rise time, percent overshoot, settling time, and steady state error are able to meet the benchmarked specifications. Since the integral time constant changes, the controller into a type 1 system, the steady state step error will be 0%. However, the steady state error from the unit ramp becomes an issue, as the ramp error will never reach 0% from the specifications. From Table 4, we can determine that PI control will result in 0 steady state unit step error, and a smaller τ_i means faster integration. Thus, gain margin decreases, percent overshoot increases, and wind-up effect may occur.

Part 1.3: Proportional + Derivative (PD) Control

Controller parameters K_p and τ_d can be adjusted to determine the effect of the controller parameters on steady state errors (with inputs unit step and unit ramp) and transient response. The PD controller parameters are adjusted, and samples of the PD system responses are saved in Table 5.

Proportional Gain K_p	Time Constant τ_d	Rise-time $t_{r(0-100\%)}$	Maximum overshoot %O.S.	Settling-time $t_{s(\pm 2\%)}$	Steady-state error $e_{ss(step)}$	Steady-state error $e_{ss(ramp)}$
3.11	2	1.32	14.35	4.58	9.68%	∞
3.11	4	0.87	30.87	7.6	9.68%	∞
2	2	2.1	1.63	6.4	14.3%	∞

Table 5. Transient Response and Steady State error of PD Control

In comparison to the benchmarked system of the Proportional only control, we can see that if the derivative time constant $\tau_d = 2$ is added, the rise time, maximum percent overshoot, and settling time decreases. However, the effect of τ_i on the steady state unit step error does not help the system meet a unit and ramp steady state error of 0% since the system type is still 0. To improve the system characteristics, more adjustments are made to controller parameters, and we can determine their effects. Ultimately, as K_p decreases, the rise time and settling time increases, while maximum overshoot decreases. On the other hand, as τ_d decreases, the rise time increases, while the maximum overshoot and settling time decreases. Using this information, the controller parameters can keep being adjusted to meet required specifications, as seen in Figure 6a, 6b, and 6c and Table 6.

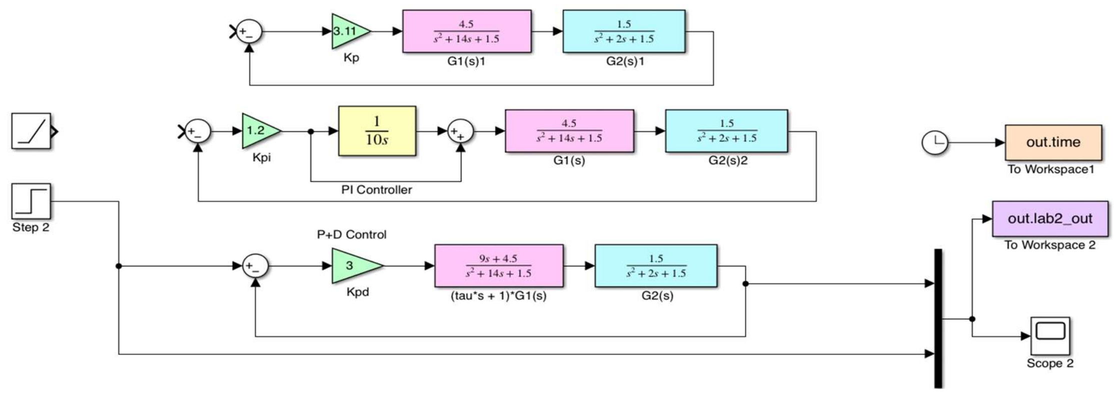

Figure 6a. Best PD Control Parameters Simulation Diagram

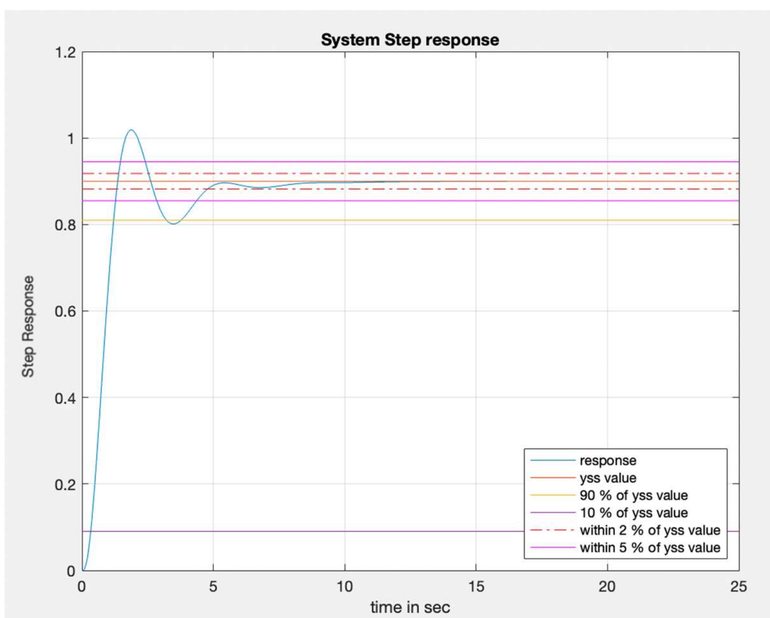

Figure 6b. Best PD Control Parameters Steady State Response

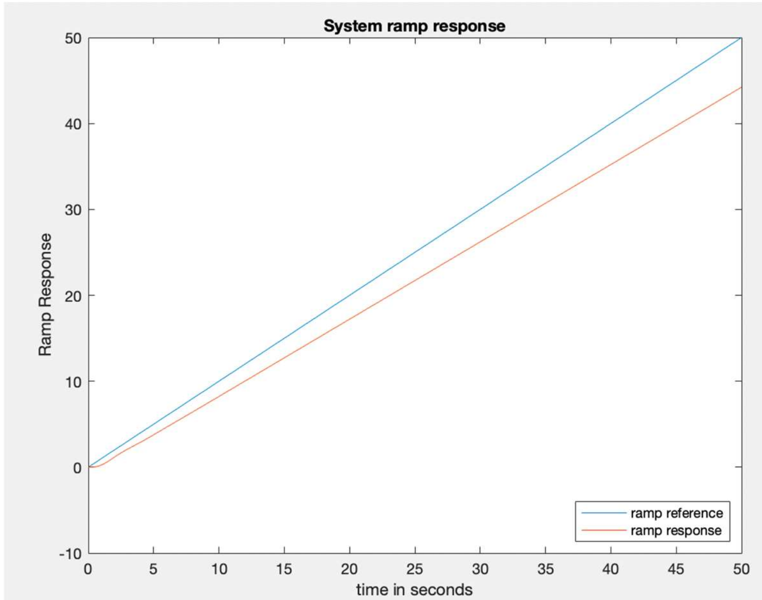

Figure 6c. Best PD Control Parameters Ramp Response

Proportional Derivative Control	Proportional Gain K_p	Time Constant τ_d	Rise-time $t_{r(0-100\%)}$	Maximum overshoot %O.S.	Settling-time $t_{s(\pm 2\%)}$	Steady-state error $e_{ss(step)}$	Steady-state error $e_{ss(ramp)}$
PD Controller	3	2	1.41	13.23	4.77	10.0%	∞

Table 6: Time Response Characteristics for Unit Feedback System under PD Control

Based on the benchmarked system, we can see that the rise time, percent overshoot, and settling time are able to meet the benchmarked specifications. Since the derivative time constant does not change the system type, the steady state unit step and ramp error will not be 0% from the required specifications. From Table 6, we can determine that PD control has no effect on steady state error tracking, but a higher τ_d means a stronger action, resulting in an increased gain margin, smaller percent overshoot, however noise may be created.

Part 2: PID Control

PID control can be used to improve the system response and meet the performance specifications established in part 1. There are two methods that will be used to tune the systems: "Trial and Error Approach" and "Ziegler-Nichols Method". We know that the proportional control is the main source of control, as it can be used to create a stable and accurate response. From Part 1, we know PI control improves steady state tracking, whereas PD control increases damping. Thus, combining them together in the parallel PID structure can be implemented.

Method 1: "Trial and Error" Approach

The tuning process is started by setting the PID controller parameters based on the PI and PD controllers from part 1. K_p is selected based on PD, and the system response is found using MATLAB and recorded in Table 5. To complete this approach, we first increased K_p at a low level such that the system is stable. τ_d and K_p are increased until oscillations can be seen. τ_i is set to 100 and slowly reduced until oscillations are observed. Finally, τ_d and K_p are adjusted for fine tuning. The best parameters are found in Figure 7a, 7b, and 7c and Table 7.

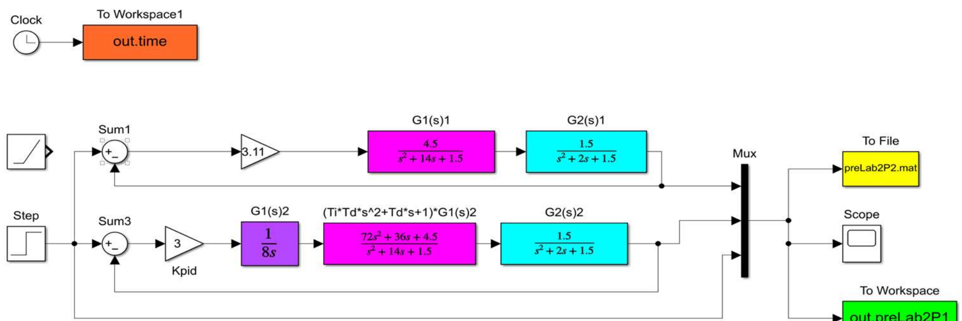

Figure 7a. Simulation Diagram for Trial & Error Method

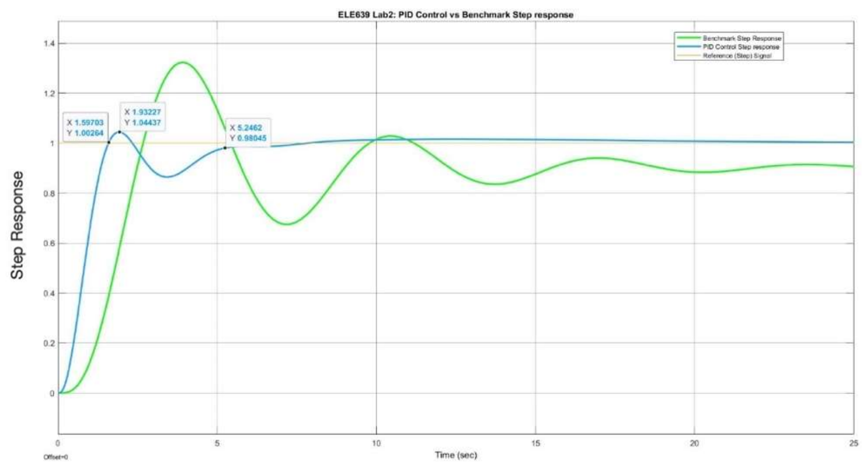

Figure 7b. Simulation Plot for Trial & Error Method

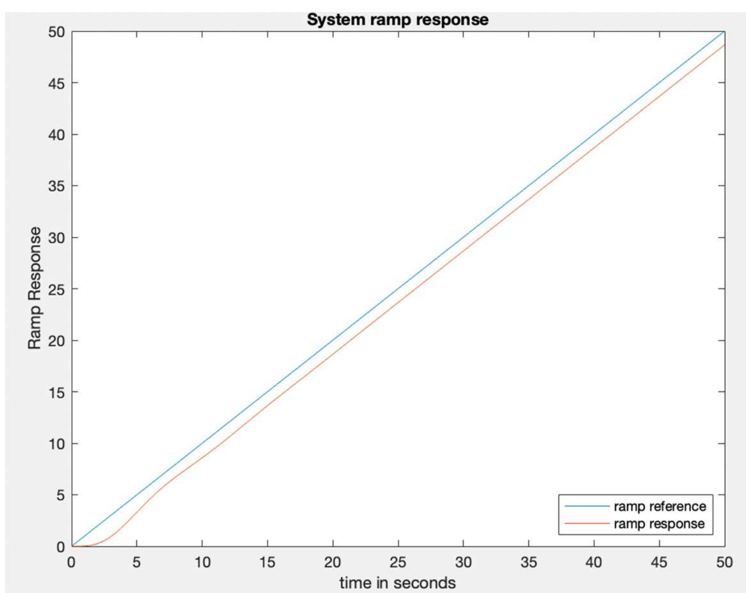

Figure 7c. Ramp Response for Trial and Error Method

$K_p = \lim_{s\to 0} G_{op}(s) = (K_P + \frac{1}{\tau_i s} + \tau_d s)\left(\frac{4.5}{s^2+14s+1.5}\right)\left(\frac{1.5}{s^2+2s+1.5}\right) = \infty$ and $e_{ss(step)} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0.$

$K_v = \lim_{s\to 0} sG(s) = s(K_P + \frac{1}{\tau_i s} + \tau_d s)\left(\frac{4.5}{s^2+14s+1.5}\right)\left(\frac{1.5}{s^2+2s+1.5}\right) = 2.1$ and $e_{ss(ramp)} = \frac{1}{K_v} = \frac{1}{2.1} = 0.47$

Method 2: Ziegler-Nichols Approach
In this approach, the system bandwidth must first be established to estimate the open loop system dynamics. From the appendix, it is noted that at critical gain, the frequency is equal to bandwidth. Thus, we set the controller to P only control, and find the critical gain K_u and period of oscillations T_u. Since we use the same parameters from lab 1, $K_u = 6.22$ and $T_u = 5.13\ s$. The recommended Ziegler-Nichols Quarter Decay settings from Table 2 in the appendix is then used to find the controller parameter settings in parallel form. The Quarter Decay response is used as the gain is only increased such that the amplitude of each oscillation is ¼ the previous one. The best parameters are found in Figure 8a, 8b, and 8c and Table 7.

$$K_p = K_u = K_{crit} = 6.22$$
$$T_u = \left(\frac{2\pi}{\omega_{critP}}\right) = \left(\frac{2\pi}{1.22}\right) = 5.15$$

$$\tau_i = \frac{T_u}{1.5} = \frac{5.15}{1.5} = 3.43$$
$$\tau_d = \frac{T_u}{6} = \frac{5.15}{6} = 0.858$$

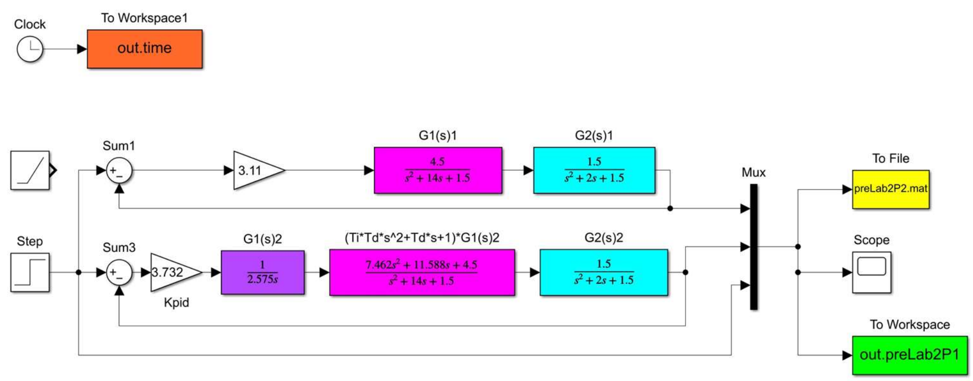

Figure 8a. Simulation Diagram for Ziegler-Nichols Method

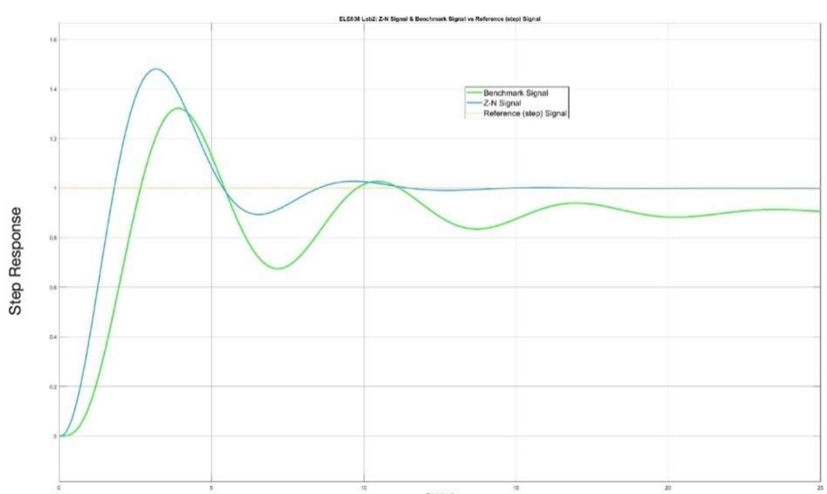

Figure 8b. Simulation Plot for Ziegler-Nichols Method

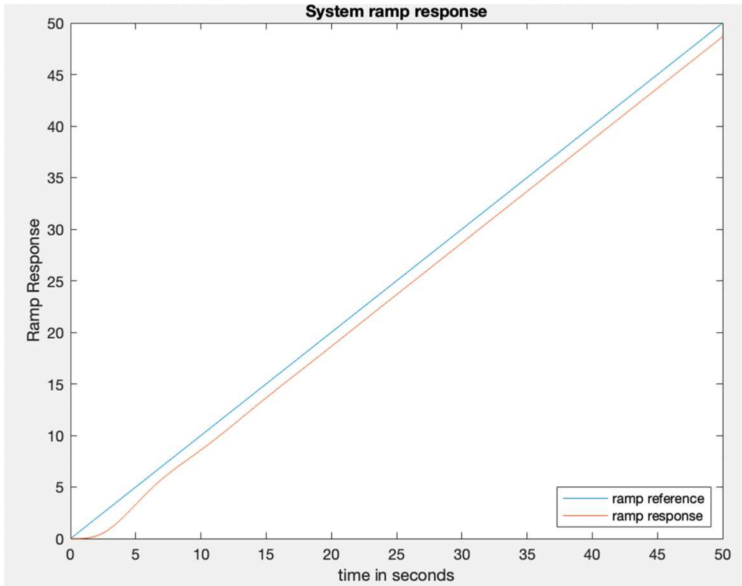

Figure 8c. Ramp Response for Ziegler-Nichols Method

$$K_p = \lim_{s \to 0} G_{op}(s) = (K_P + \frac{1}{\tau_i s} + \tau_d s)\left(\frac{4.5}{s^2+14s+1.5}\right)\left(\frac{1.5}{s^2+2s+1.5}\right) = \infty \; e_{ss(step)} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0.$$

$$K_v = \lim_{s \to 0} sG(s) = s(6.22 + \frac{1}{3.43s} + 0.858s)\left(\frac{4.5}{s^2+14s+1.5}\right)\left(\frac{1.5}{s^2+2s+1.5}\right) = 4.5 \; e_{ss(ramp)} = \frac{1}{K_v} = \frac{1}{4.34} = 0.23$$

Proportional Integral Derivative Control	Proportional Gain K_p	Derivative Time Constant τ_d	Integral Time Constant τ_i	Rise-time $t_{r(0-100\%)}$	Maximum overshoot %O.S.	Settling-time $t_{s(\pm 2\%)}$	Steady-state error $e_{ss(step)}$	Steady-state error $e_{ss(ramp)}$
PID Controller "Trial & Error" Approach	3	2	8	1.67	4.4%	5.4	0	47%
PID Controller Ziegler-Nichols Approach	6.22	3.43	0.858	1.43	39%	6.89	0	23%

Table 7: Time Response Characteristics for Unit Feedback System under PID Control

As seen in the above results, the Trial-and-Error approach will help meet all specifications except $e_{ss(ramp)}$, and the lowest amount that can be reached before instability is 47%. This is because to lower $e_{ss(ramp)}$, proportional gain mist be increased while τ_i is decreased. However, changing these parameters too much may result in instability. However, the specifications for maximum overshoot, settling time, and $e_{ss(step)}$ are significantly improved, as they are much lower than what was asked to meet. In comparison, the Ziegler-Nichols Approach used also improves specifications, as we can see in Table 5 that rise time, settling time, $e_{ss(step)}$ also meet required specifications. Although $e_{ss(ramp)} = 23\%$ instead of 0, it is much lower than the trial-and-error approach. This very good since the PID system is type is 1, so it will be difficult for $e_{ss(ramp)}$ to be 0. The greatest error in the Ziegler-Nichols Approach is that it does not meet percent overshoot specifications of $< 15\%$ since the quarter decay method will result in each amplitude being ¼ less than the previous one. Although this method drastically improves the rise and settling time, it does not help improve the percent overshoot.

Discussion

1. The steady-state error e_{ss} measures the difference between the desired steady state of 1 and the actual steady state. In P control, the system type is 0, thus the unit step steady state error will be a constant while the ramp step steady state error is infinity. Increasing the gain K_p results in a smaller steady state unit step error since $K_p = \lim_{s\to 0} G(s) = constant$ and $e_{ss(step)} = \frac{1}{1+K_p} = constant$. However, since $K_v = \lim_{s\to \infty} sG(s) = 0$ and $e_{ss(ramp)} = \frac{1}{K_v} = \infty$ for P control, the unit step errors can be improved, but the ramp errors will always be infinity. The implications of choosing the best parameters for P control are that by increasing the gain too much, the controller may become saturated and result in instability. In comparison, for PI control, the system type becomes 1 due to the integrator being added, resulting in a unit step steady state error always being 0 since $K_p = \lim_{s\to 0} G(s) = \infty$ and $e_{ss(step)} = \frac{1}{1+K_p} = 0$. However, the unit ramp error is a constant since $K_v = \lim_{s\to 0} sG(s) = constant$ and $e_{ss(ramp)} = \frac{1}{K_v} = constant$. Thus, to improve PI control, the ramp error can be improved if K_p increases, whereas $e_{ss(step)} = 0$ always. The implications of choosing the best parameters for PI control is that there may be a possible wind-up effect. Finally, in PD control, since the system type is 0, the system unit step response will be a constant and the ramp response is infinity, which has the same explanation from P control. Thus, for PD control, the unit step errors can be improved by increasing K_p, but the ramp errors will always be infinity. Therefore, there are no effects of steady state tracking using PD, however, possible noise issues can be present by finding the best parameters.

2. The percent overshoot measures how smooth or oscillatory the signal is, where y_{max} is the maximum point in the step response, and y_{ss} is the steady state value. For P control, the percent overshoot decreases when the gain decreases. This is because percent overshoot is the difference from the maximum point and the reference. By increasing the gain, the maximum point will increase as well, Therefore, if K_p decreases, then percent overshoot decreases. For PI control, if τ_i increases and K_p decreases, then percent overshoot will decrease. This is because a higher τ_i results in a stronger integration, so the gain margin will increase, and the percent overshoot will decrease. Finally, for PD control, the percent overshoot decreases when the τ_d decreases and K_p decreases. This is because a higher τ_d results in a stronger integration, so the gain margin will increase, and the percent overshoot will decrease. The implications of choosing the best settings are that you must make sure the K_p is not too low when trying to reduce percent overshoot, since it can result in an overdamped system.

3. The settling time measure how quickly the signal reaches steady state. It is found by looking on the graph and seeing where the signal crosses either $\pm 2\%$ of the steady state value y_{ss}. In P control, the settling time increases as K increases. This is because the gain increase causes the gain margin to increase. As a result, it will take longer for the signal to reach steady state from a higher K_p and gain. For PI control, when τ_i increases and K_p decreases, the settling time will also decrease. This is because a smaller τ_i results in a stronger integration, thus, the gain margin decreases, and the setting time decreases. Finally, for PD control, when τ_d decreases and K_p increases, the settling time will decrease. This is because a larger τ_d results in the gain margin increasing, thus the settling time will increase. The implications of choosing the best parameters for settling time are that trying to reduce the settling time with a low gain may over damp and slow down the response.

4. The trial and error and Ziegler-Nichol's approach were both used to design a PID controller that would meet the following specifications that were benchmarked: $e_{ss(step)\%} = 0\%$, $e_{ss(ramp)} = 0$, $PO\% < 15\%$, and $t_{s(\pm 2\%)} < 10.31s$. From the theory learned in class, we know that the proportional control is the main source of control, as it can be used to create a stable and accurate response. In comparison, PI control improves steady state tracking, whereas PD control increases damping. From this information, we were able to apply it to the PID parameters such that the system would meet all specifications. In the Trial-and-Error Approach, the parameters were set to have a K_p and τ_d based on previously used PD controller, and τ_i from the PI controller, then the values were adjusted for fine tuning. In the Trial-and-Error Approach, $e_{ss(ramp)} = 0$ was not met as the system was of type 1, meaning the $e_{ss(ramp)}$ would not be 0, but would be a constant number instead. To minimize the ramp response, we know that a higher proportional gain would results in a lower steady state error. However, a main problem of increasing the gain too much would result in increased oscillations and instability.

In comparison, for the Ziegler-Nichol's approach, the parameters were calculated based on the "Ultimate Gain" formulas from the appendix. We were not able to meet all system specifications for both methods since for PID control as well $e_{ss(ramp)}$ did not equal 0 and percent overshoot was too high. Although τ_d can be used for damping, resulting in a smaller percent overshoot, and settling time due to faster response, there are possible noise issues which can cause distortion in the signal. In addition, since τ_i was used to improve steady state errors, it was essential in helping our system to reach $e_{ss(step)\%} = 0\%$, By decreasing τ_i, the $e_{ss(ramp)}$ would also be decreased due to faster integration and smaller action. However, reducing the τ_i too much may result in a wind-up effect. Overall, we could not reach all specifications such as $e_{ss(ramp)}$, which is due to the system type, as well as errors in calculating the Ziegler-Nichol's approach causing errors in percent overshoot.

References

[1] Fahmy, A & Hoang, V. (2022) *ELE 639 Lab 1.* Ryerson University.