Pertemuan 15

Eigen value,
Eigen vektor,

Bilqis

nilai eigen dan vektor eigen

$$\boxed{A_{n\times n} \; X_{n\times 1} = \lambda \,.\, X_{n\times 1}}$$

nilai eigen / sebenarnya

vektor eigen

Diket $X = \begin{bmatrix}1\\2\end{bmatrix}$ adalah vektor eigen dari $A = \begin{bmatrix}3 & 0\\8 & -1\end{bmatrix}$

? ⇒ berapakah nilai eigen ? ?

jwb:

$$\begin{bmatrix}3 & 0\\8 & -1\end{bmatrix} \cdot \begin{bmatrix}1\\2\end{bmatrix} = \lambda \,. \begin{bmatrix}1\\2\end{bmatrix}$$

$$\begin{bmatrix}3\\6\end{bmatrix} = \lambda \begin{bmatrix}1\\2\end{bmatrix}$$

$$\therefore \; \lambda = 3$$

pers karakteristik A :

$$\boxed{\det(\lambda . I - A) = 0}$$

Mencari Eigen Value

polinom karakteristik A : ⇝ menghasilkan persamaan

$$\det(\lambda . I - A) = \lambda^n + c_1 \lambda^{n-1} + c_2 \lambda^{n-2} + \dots + c_n$$

variabel ↲ ↳ konstanta

x:

Cari nilai eigen dari $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$

jawab:

- polinom karakteristik A :

$$\det(\lambda . I - A) = \det\left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}\right)$$

$$= \det \begin{bmatrix} \lambda - 3 & -2 \\ 1 & \lambda \end{bmatrix}$$

$$= \lambda^2 - 3\lambda + 2$$

- pers. karakteristik A :

$$\det(\lambda . I - A) = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$(\lambda - 1)(\lambda - 2) = 0$$

- ∴ $\lambda = 1$ dan $\lambda = 2$ adalah nilai-nilai eigen dari A

6.2

3 : Carilah nilai eigen dari $A = \begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$

Jawab :

- polinom karakteristik A :

$$\det(\lambda . I - A) = \det\left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}\right)$$

$$= \det \begin{pmatrix} \lambda + 2 & 1 \\ -5 & \lambda - 2 \end{pmatrix}$$

$$= \lambda^2 - 4 + 5$$

$$= \lambda^2 + 1$$

- pers. karakteristik A

$$\det(\lambda . I - A) = 0$$

$$\lambda^2 + 1 = 0$$

$$\lambda = \sqrt{-1}$$

- $\therefore$ krn $\lambda = \sqrt{-1}$, mk tidak ada nilai eigen untuk A

:4 : Carilah nilai eigen dari : $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$

jawab:

pers. karakteristik A: $\det(\lambda . I - A) = 0$

$$\det\left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}\right) = 0$$

$$\det\begin{pmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -4 & 17 & \lambda - 8 \end{pmatrix} = 0$$ → using kofaktor kolom pertama

$$\lambda \underbrace{\begin{bmatrix} \lambda & -1 \\ 17 & \lambda - 8 \end{bmatrix}}_{\det} - 4 \underbrace{\begin{bmatrix} -1 & 0 \\ \lambda & -1 \end{bmatrix}}_{\det} = 0$$

$$\lambda(\lambda^2 - 8\lambda + 17) - 4\,(1) = 0$$

$$\lambda^3 - 8\lambda^2 + 17\lambda - 4 = 0$$

→ kemungkinan $\boxed{-4}$ → ± 1, ± 2, ± 4 } coba satu² mana yg benar

mk ⇒ $\lambda = 4$

→ $\lambda^3 - 8\lambda^2 + 17\lambda - 4 = (\lambda - 4)(\lambda^2 - 4\lambda + 1)$

$= (\lambda - 4)$

↓ pakai rumus ABC

kmd ketemu nilai² eigennya adalah ⇒ $\lambda = 4$

$\lambda = 2 + \sqrt{3}$

$\lambda = 2 - \sqrt{3}$

teorema 1

jk $A_{n \times n}$ mk pernyataan berikut ekivalen satu sama lain:

- a) $\lambda \rightarrow$ nilai eigen dari A
- b) $(\lambda . I - A) . X = 0$ memp. pemecahan yang tak trivial (banyak pemecahan)
- c) ada vektor tak nol X sehingga $A . X = \lambda . X$
- d) λ adalah pemecahan riil dari pers karakteristik $\det(\lambda . I - A) = 0$

: 5 :

Cari basis u ruang eigen $A = \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

↳ cari vektor eigen

Jawab:

1) Cari λ → pers karakteristik $A \Rightarrow (\lambda - 1)(\lambda - 5)^2 = 0$

→ ∴ $\lambda = 1$, $\lambda = 5$ } buktikan

→ ∴

2) Cari X $\Rightarrow (\lambda I - A) . X = 0$

$$\left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}\right) . \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} \lambda-3 & 2 & 0 \\ 2 & \lambda-3 & 0 \\ 0 & 0 & \lambda-5 \end{pmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ jk $\lambda = 5$ mk menj :

$$\begin{bmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

maka $\Rightarrow$ $\left.\begin{aligned} x_1 &= -s \\ x_2 &= s \\ x_3 &= t \end{aligned}\right\}$

catt : - index besar terletak di kiri =
- index besar mrp komb lin dr kecil

$$x = \begin{bmatrix} -s \\ s \\ t \end{bmatrix} = \begin{bmatrix} -s \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix}$$

$$= s\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ vektor eigen $\underline{u}$ $\lambda = 5 \longrightarrow \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ dan $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

→ jk $\lambda = 1$ mk menj :

$$\begin{bmatrix} -2 & 2 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

mk $\Rightarrow$ $x_1 = t$

$x_2 = t$

$x_3 = 0$

$$x = \begin{bmatrix} t \\ t \\ 0 \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore$ vektor eigen $\underline{u}$ $\lambda = 1 \longrightarrow \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

Carilah nilai eigen dan vector eigen dari matrix A

$$A = \begin{vmatrix} 0 & -1 & -3 \\ 2 & 3 & 3 \\ -2 & 1 & 1 \end{vmatrix}$$

nilai eigen

det (ʎ . I - A) = 0

$$\det \begin{vmatrix} ʎ & 0 & 0 \\ 0 & ʎ & 0 \\ 0 & 0 & ʎ \end{vmatrix} - \begin{vmatrix} 0 & -1 & -3 \\ 2 & 3 & 3 \\ -2 & 1 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} ʎ & 1 & 3 \\ -2 & ʎ - 3 & -3 \\ 2 & -1 & ʎ - 1 \end{vmatrix} = 0$$

baris pertama => kofaktor

ʎ * ((ʎ-3)*(ʎ-1)-3) +

- 1 * (- 2 * (ʎ - 1) + 6) +

3 * (2 - (ʎ - 3)*2) =

ʎ =	-2
ʎ =	2
ʎ =	4

vektor eigen

$(ʎ . I - A) . X = 0$

$$\begin{vmatrix} ʎ & 0 & 0 \\ 0 & ʎ & 0 \\ 0 & 0 & ʎ \end{vmatrix} - \begin{vmatrix} 0 & -1 & -3 \\ 2 & 3 & 3 \\ -2 & 1 & 1 \end{vmatrix} * \begin{vmatrix} X1 \\ X2 \\ X3 \end{vmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$$\begin{vmatrix} ʎ & 1 & 3 \\ -2 & ʎ - 3 & -3 \\ 2 & -1 & ʎ - 1 \end{vmatrix} * \begin{vmatrix} X1 \\ X2 \\ X3 \end{vmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

Jika $\lambda =$ -2

$$\begin{bmatrix} -2 & 1 & 3 \\ -2 & -5 & -3 \\ 2 & -1 & -3 \end{bmatrix} * \begin{vmatrix} X1 \\ X2 \\ X3 \end{vmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

Carilah nilai X1, X2 dan X3 dengan Gauss

1	-0.5	-1.5	0
-2	-5	-3	0
2	-1	-3	0

1	-0.5	-1.5	0
0	-6	-6	0
2	-1	-3	0

1	-0.5	-1.5	0
0	-6	-6	0
0	0	0	0

1	-0.5	-1.5	0
0	1	1	0
0	0	0	0

$$\begin{matrix} x1 = & t & & 1 \\ x2 = & -t & = t & -1 \\ x3 = & t & & 1 \end{matrix}$$

Jadi vektor eigen untuk λ = - 2 adalah

$$x = \begin{vmatrix} 1 \\ -1 \\ 1 \end{vmatrix}$$

Jika Λ = 2

2	1	3	*	X1		0
-2	-1	-3		X2	=	0
2	-1	1		X3		0

Carilah nilai X1, X2 dan X3 dengan Gauss

1	0.5	1.5	0
-2	-1	-3	0
2	-1	1	0
1	0.5	1.5	0
0	0	0	0
2	-1	1	0
1	0.5	1.5	0
0	0	0	0
0	-2	-2	0
1	0.5	1.5	0
0	0	0	0
0	1	1	0

x1 =	-t		-1
x2 =	-t	= t	-1
x3 =	t		1

Jadi vektor eigen untuk Λ = 2 adalah

	-1
x =	-1
	1

Jika Λ = 4

$$\begin{bmatrix} 4 & 1 & 3 \\ -2 & 1 & -3 \\ 2 & -1 & 3 \end{bmatrix} * \begin{vmatrix} X1 \\ X2 \\ X3 \end{vmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

Carilah nilai X1, X2 dan X3 dengan Gauss

1	0.25	0.75	0
-2	1	-3	0
2	-1	3	0

1	0.25	0.75	0
0	1.5	-1.5	0
2	-1	3	0

1	0.25	0.75	0
0	1.5	-1.5	0
0	-1.5	1.5	0

1	0.25	0.75	0
0	1	-1	0
0	-1.5	1.5	0

1	0.25	0.75	0
0	1	-1	0
0	0	0	0

$$\begin{matrix} x1 = & -t & & -1 \\ x2 = & t & = t & 1 \\ x3 = & t & & 1 \end{matrix}$$

Jadi vektor eigen untuk Λ = 4 adalah

$$x = \begin{vmatrix} -1 \\ 1 \\ 1 \end{vmatrix}$$

Theorem 7.1.1. *If A is an $n \times n$ triangular matrix (upper triangular, lower triangular, or diagonal), then the eigenvalues of A are the entries on the main diagonal of A.*

Example 4 By inspection, the eigenvalues of the lower triangular matrix

$$A = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & \frac{2}{3} & 0 \\ 5 & -8 & -\frac{1}{4} \end{bmatrix}$$

are $\lambda = \frac{1}{2}$, $\lambda = \frac{2}{3}$, and $\lambda = -\frac{1}{4}$.

Example 5 Find bases for the eigenspaces of

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

Solution. The characteristic equation of A is $\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$, or in factored form, $(\lambda - 1)(\lambda - 2)^2 = 0$ (verify); thus, the eigenvalues of A are $\lambda = 1$ and $\lambda = 2$, so there are two eigenspaces of A.

By definition,

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

is an eigenvector of A corresponding to λ if and only if $\mathbf{x}$ is a nontrivial solution of $(\lambda I - A)\mathbf{x} = \mathbf{0}$, that is, of

$$\begin{bmatrix} \lambda & 0 & 2 \\ -1 & \lambda - 2 & -1 \\ -1 & 0 & \lambda - 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

If $\lambda = 2$, then (3) becomes

$$\begin{bmatrix} 2 & 0 & 2 \\ -1 & 0 & -1 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this system yields (verify)

$$x_1 = -s, \qquad x_2 = t, \qquad x_3 = s$$

Thus, the eigenvectors of A corresponding to $\lambda = 2$ are the nonzero vectors of the form

$$\mathbf{x} = \begin{bmatrix} -s \\ t \\ s \end{bmatrix} = \begin{bmatrix} -s \\ 0 \\ s \end{bmatrix} + \begin{bmatrix} 0 \\ t \\ 0 \end{bmatrix} = s\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Since

$$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

are linearly independent, these vectors form a basis for the eigenspace corresponding to $\lambda = 2$.

If $\lambda = 1$, then (3) becomes

$$\begin{bmatrix} 1 & 0 & 2 \\ -1 & -1 & -1 \\ -1 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this system yields (verify)

$$x_1 = -2s, \qquad x_2 = s, \qquad x_3 = s$$

Thus, the eigenvectors corresponding to $\lambda = 1$ are the nonzero vectors of the form

$$\begin{bmatrix} -2s \\ s \\ s \end{bmatrix} = s\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

so that

$$\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

is a basis for the eigenspace corresponding to $\lambda = 1$.

Power matrix A^k

Teori 7.1.3

jika $k \to$ int $(+)$

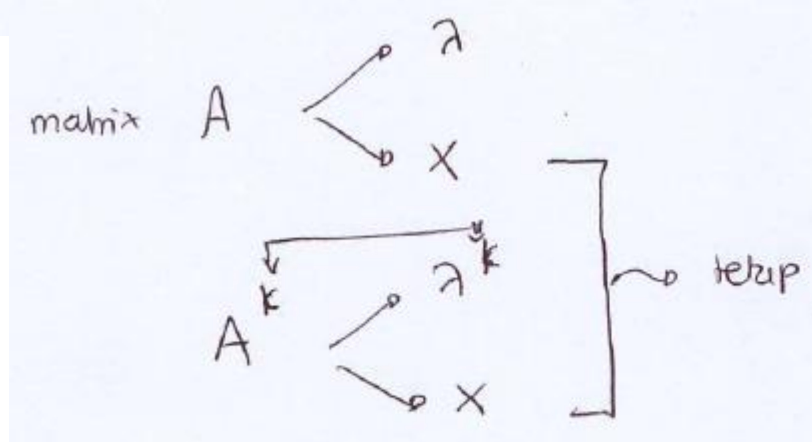
$\lambda \to$ eigen value matrix A

$x \to$ eigen vektor $\to A$

maka $\lambda^k \to$ eigen value $\to A^k$.

$x \to$ eigen vektor $\to A$

misal : $A^{13} = A \cdot A \cdots A$



Contoh

contoh :

jika diketahui $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

mempunyai → $\lambda = 2$ → eigen vektor → $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

→ $\lambda = 1$ → $\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

berapakah → eigen value, → eigen vektor } dari matrix A^k

jika $k = 7$ → A^7

jawab:

eigen value $\to \lambda^k \Rightarrow 2^k \Rightarrow 2^7 = 128$

$\to$ tapi eigen vektor tetap $\to \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\to \lambda^k = 1^7 = 1$

$\to$ eigen vektor tetap $\to \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

•) mencari A^k dpt dgn cara → biasa → $A . A . A . A$

atau dgn rumus : $\boxed{A^k = P . D^k . P^{-1}}$

P → matrix diagonalizes A

→ kumpulan dr eigen vektor A

$D = P^{-1} . A . P$ → menghasilkan matrix diagonal

D^k → matrix D dimana, tiap item pd diagonalnya dipangkat k contoh :

Temukan A^{13} u matrix $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

P dari matrix A adalah

$$P = \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$D = P^{-1} . A . P = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

maka

$$A^{13} = P \cdot D^{13} \cdot P^{-1}$$

$$= \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2^{13} & 0 & 0 \\ 0 & 2^{13} & 0 \\ 0 & 0 & 1^{13} \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \\ -1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -8190 & 0 & -16382 \\ 8191 & 8192 & 8191 \\ 8191 & 0 & 16383 \end{bmatrix}$$

2. (Nilai 72) Diketahui matrix A sebagai berikut

$$A = \begin{vmatrix} 3 & 8 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 8 \end{vmatrix}$$

Ditanya :

1. Nilai eigen. Catt : carilah determinan dengan menggunakan kofaktor kolom ke-2
2. Vektor Eigen
3. Carilah A^3 dengan menggunakan vektor eigen

   Catt :

   a. Urutan matrix P adalah :
      1. Kolom 1 ➔ nilai eigen terkecil
      2. Kolom 2 ➔ nilai eigen terkecil berikutnya
      3. Kolom 3 ➔ nilai eigen terbesar

   b. Cari P^{-1} dengan menggunakan OBE

   c. $A^k = P \,.\, (D^k \,.\, P^{-1})$

      Hitung dulu $D^k \,.\, P^{-1}$, setelah itu baru dikalikan dengan P

   d. $D = P^{-1} \,.\, (A \,.\, P)$

      Hitung dulu A . P, setelah itu baru di kalikan P^{-1}

$$A = \begin{bmatrix} 3 & 8 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

i) Nilai eigen $\Rightarrow \lambda$

$$\det(\lambda . I - A) = \emptyset$$

$$\det\left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 8 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}\right) = \emptyset$$

$$\det\left(\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 3 & 8 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}\right) = \emptyset$$

3

$$\det. \begin{bmatrix} \lambda - 3 & -8 & 0 \\ -3 & \lambda - 1 & 0 \\ 0 & 0 & \lambda - 8 \end{bmatrix} = \emptyset$$

cari det dgn menggo. kofaktor kolom ke **2** (2)

$$8.\left(-3(\lambda-8)\right)+(\lambda-1)(\lambda-3)(\lambda-8)=0$$ 3

$$8\,(-3\lambda+24)+(\lambda-1)(\lambda^2-11\lambda+24)=0$$

$$-24\lambda+192+(\lambda-1)(\lambda^2-11\lambda+24)=0$$

$$-24\lambda+192+\lambda^3-11\lambda^2+24\lambda-\lambda^2+11\lambda-24=0$$

$$\lambda^3-12\lambda^2+11\lambda+168=0$$ 3

$$(\lambda+3)(\lambda-7)(\lambda-8)=0$$ 3

$\therefore$ nilai eigen $\longrightarrow$ $\lambda=-3$

$\lambda=7$

$\lambda=8.$

2) Vektor eigen

$\Rightarrow$ Cari x $\Rightarrow$ $(\lambda . I - A) . \bar{x} = \bar{O}$

$$\left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 8 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix} \right) . \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

3

$$\begin{bmatrix} \lambda - 3 & -8 & 0 \\ -3 & \lambda - 1 & 0 \\ 0 & 0 & \lambda - 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ jk $\lambda = -3$

$$\begin{bmatrix} -6 & -8 & 0 \\ -3 & -4 & 0 \\ 0 & 0 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

mk $\Rightarrow$ $-6x_1 - 8x_2 = \varnothing$

$$\boxed{x_2 = t}$$

$$-6x_1 - 8t = \varnothing$$

$$-6x_1 = 8t$$

$$x_1 = -\frac{8t}{6}$$

$$\boxed{x_1 = -\frac{4}{3}t}$$

$$-11x_3 = \varnothing$$

$$\boxed{x_3 = \varnothing}$$

$$x = \begin{bmatrix} -4/3\,t \\ t \\ \varnothing \end{bmatrix} = t \begin{bmatrix} -4/3 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore$ vektor eigen y $\lambda = -3$ adalah $\begin{bmatrix} -4/3 \\ 1 \\ 0 \end{bmatrix}$

→ jk $\lambda = 7$

$$\begin{bmatrix} 4 & -8 & 0 \\ -3 & 6 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

mk ⇒ $4X_1 - 8X_2 = \varnothing$

$$\boxed{X_2 = t}$$

$$4X_1 - 8t = \varnothing$$

$$4X_1 = 8t$$

$$\boxed{X_1 = 2t}$$

$$-X_3 = \varnothing$$

$$\boxed{X_3 = \varnothing}$$

$$X = \begin{bmatrix} 2t \\ t \\ \varnothing \end{bmatrix} = t \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

∴ vektor eigen u $\lambda = 7$ adalah $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

→ jk $\lambda = 8$

$$\begin{bmatrix} 5 & -8 & 0 \\ -3 & 7 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

mk → $5x_1 - 8x_2 = \emptyset \quad | \times 3$

$-3x_1 + 7x_2 = \emptyset \quad | \times 5$

$0 x_3 = \emptyset$

$\boxed{x_3 = t}$

$15x_1 - 24x_2 = \emptyset$

$-15x_1 + 35x_2 = \emptyset$ +

$11 x_2 = \emptyset$

$\boxed{x_2 = \emptyset}$

$-3x_1 + 7x_2 = \emptyset$

$-3x_1 + 7.0 = \emptyset$

$-3x_1 = \emptyset$

$\boxed{x_1 = 0}$

$$x = \begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix} = t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

∴ vektor eigen y $\lambda = 8$ adalah $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

(3) A^3 dgn menggo vektor eigen (6)

$$A^3 = P \cdot D^k \cdot P^{-1}$$

P → kumpulan vektor eigen

D → $P^{-1} \cdot A \cdot P$

→ matrix diagonal

→ kumpulan nilai eigen

D^k = matrix D yg tiap item pd diagonalnya dipangkatkan k

P^{-1} dicari dgn OBE

$$P = \begin{bmatrix} -4/3 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3

	-1,3333	2	0			
P =	1	1	0			
	0	0	1			
	-1,3333	2	0	1	0	0
	1	1	0	0	1	0
	0	0	1	0	0	1
	1,00	-1,50	0,00	-0,75	0,00	0,00
	1,00	1,00	0,00	0,00	1,00	0,00
	0,00	0,00	1,00	0,00	0,00	1,00
	1,00	-1,50	0,00	-0,75	0,00	0,00
	0,00	2,50	0,00	0,75	1,00	0,00
	0,00	0,00	1,00	0,00	0,00	1,00
	1,00	-1,50	0,00	-0,75	0,00	0,00
	0,00	1,00	0,00	0,30	0,40	0,00
	0,00	0,00	1,00	0,00	0,00	1,00
	1,00	0,00	0,00	-0,30	0,60	0,00
	0,00	1,00	0,00	0,30	0,40	0,00
	0,00	0,00	1,00	0,00	0,00	1,00

	-0,30	0,60	0,00		3	8	0		-1,33333	2	0,00
D =	0,30	0,40	0,00	x	3	1	0	x	1	1	0
	0,00	0,00	1,00		0	0	8		0	0	1
	-0,30	0,60	0,00		4	14	0,00				
D =	0,30	0,40	0,00		-3	7	0				
	0,00	0,00	1,00		0	0	8				
	-3	0	0								
D =	0	7	0								
	0	0	8								
	-27	0	0								
D 3 =	0	343	0								
	0	0	512								

	-1,3333	2	0		-27	0	0		-0,30	0,60	0,00
A3 =	1	1	0	x	0	343	0	x	0,30	0,40	0,00
	0	0	1		0	0	512		0,00	0,00	1,00
	-1,3333	2	0		8,1	-16,2	0				
A 3 =	1	1	0	x	102,9	137,2	0				
	0	0	1		0	0	512				
	195	296	0								
A 3 =	111	121	0								
	0	0	512								

- Tugas Kelompok ➔
  - cari 2 soal dan jawaban di internet yang berhubungan dengan materi ppt ini
  - Tulis alamat internetnya
  - Di kirim ke elearning, terakhir ➔
    - Minggu depan
- Format ➔ subject ➔
  - Alin-B-melati
  - Bentuk ➔ ppt ➔ informasi nama kelompok + anggota