Pertemuan 13

Basis untuk ruang kolom, Basis untuk ruang baris, Basis yang berasal dari vektor sendiri.

Bilqis

Tujuan

- ➔ beberapa pertemuan yang lalu ➔ basis untuk matrix A
- ➔ Sekarang :
  1. Dapat mencari basis untuk ruang baris A
  2. Dapat mencari basis untuk ruang kolom A
  3. Dapat mencari basis yang di dapat dari vektor vektor dia sendiri A

Pengertian

4.8

1) Kombinasi Linier

$\vec{w} \rightarrow$ kombinasi Linier dr $\vec{v_1}$ $\vec{v_2}$ ... $\vec{v_n}$ jk
$\vec{w}$ dpt diungkapkan dlm bentuk :

$$\vec{w} = k_1 \vec{v_1} + k_2 \vec{v_2} + \dots + k_n \vec{v_n}$$

dimana

$k_1, k_2 \dots k_n \rightarrow$ skalar

ingat !! baris di atas bkn hanya satu baris, tp terdiri dari beberapa baris

or kombinasi Linier $\rightarrow$ ada nilai $\forall$ $k_1\ k_2 \dots k_n$

Merentang = spanning

4.9

2. Merentang :

- $\bar{v}_1 \; \bar{v}_2 \dots \bar{v}_n$ merentang ruang vektor V jk sembarang vektor pd ruang vektor V dpt dinyatakan sbg kombinasi linier dr $\bar{v}_1 \; \bar{v}_2 \dots \bar{v}_n$
- ada banyak nilai u $k_1 \; k_2 \dots k_n$
- det $\neq 0$ (jk bisa dicari det)

ex:

tent. apakah $\bar{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ & $\bar{v}_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ merentang R^2

jwb: merentang jk → sembarang vektor pd R^2 dpt dinyatakan sbg kombinasi linier $\bar{v}_1 \; \bar{v}_2$

Tulis di papan

5.3 Kebebasan Linier

- $\bar{v}_1 \quad \bar{v}_2 \quad \ldots \quad \bar{v}_n$ → bebas Linier jk <u>hanya</u> ada <u>satu</u> pemecahan <u>u</u> persamaan :

$$k_1 \bar{v}_1 + k_2 \bar{v}_2 + \ldots + k_n \bar{v}_n = \bar{0}$$

yaitu $k_1 = k_2 = \ldots = k_n = 0$ //

- det $\neq 0$ (jk bisa dicari det)
- jk ada sebuah or lebih vektor dpt dinyatakan sbg k.l vektor lainnya mk $\neq$ bebas Linier

ex:

Basis

$\{ \bar{v}_1 \ \bar{v}_2 \ldots \bar{v}_n \}$

→ merentang

$\bar{x} = k_1 \bar{v}_1 + k_2 \bar{v}_2 + \ldots + k_n \bar{v}_n$

$\bar{x}$ = sembarang vektor

$k_1 \quad k_2 \quad \ldots \ k_n$ → ada nilai-nya

or

Det $\neq 0$

→ bebas Linier

$\bar{0} = k_1 \bar{v}_1 + k_2 \bar{v}_2 + \ldots + k_n \bar{v}_n$

$k_1 = k_2 = \ldots = k_n = 0$ → hanya satu jawaban

or

Det $\neq 0$

Tujuan

1. Dapat mencari basis untuk ruang baris A
   – Basis ruang baris A = basis ruang baris R
   – Menghasilkan vektor baru
2. Dapat mencari basis untuk ruang kolom A
   – Basis ruang kolom A <> basis ruang kolom R
   – Tapi berkorespondensi
   – Tidak menghasilkan vektor baru
3. Dapat mencari basis yang di dapat dari vektor – vektor dia sendiri A
   – Tidak menghasilkan vektor baru

Theorem 5.5.3. *Elementary row operations do not change the nullspace of a matrix.*

Theorem 5.5.4. *Elementary row operations do not change the row space of a matrix.*

Theorem 5.5.5. *If A and B are row equivalent matrices, then:*

(a) *A given set of column vectors of A is linearly independent if and only if the corresponding column vectors of B are linearly independent.*

(b) *A given set of column vectors of A forms a basis for the column space of A if and only if the corresponding column vectors of B form a basis for the column space of B.*

The following theorem makes it possible to find bases for the row and column spaces of a matrix in row-echelon form by inspection.

Theorem 5.5.6. *If a matrix R is in row-echelon form, then the row vectors with the leading 1's (i.e., the nonzero row vectors) form a basis for the row space of R, and the column vectors with the leading 1's of the row vectors form a basis for the column space of R.*

$$A = \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 3 \end{bmatrix}$$

$$\Downarrow \text{ OBE}$$

$$R = \begin{bmatrix} 1 & 7 & 8 \\ 0 & 1 & 4 \end{bmatrix}$$

A
- basis ruang baris
  - $r_1 = (1 \quad 7 \quad 8)$
  - $r_2 = (0 \quad 1 \quad 4)$
- basis ruang kolom
  - $c_1 = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$
  - $c_2 = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$

Catt: **bukan** hasil sebenarnya

R
- basis ruang baris
  - $r_1 = (1 \quad 7 \quad 8)$
  - $r_2 = (0 \quad 1 \quad 4)$
- basis ruang kolom
  - $c_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
  - $c_2 = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$

- Matrix A = […]
- Matrix R = […]
- ➔ Matrix R ➔ matrix A yang sudah di OBE dan berbentuk eselon baris
- Teori 5.5.4 ➔ OBE tidak merubah ruang baris dari matrix
- Teori 5.5.6 ➔ Jika matrix R berbentuk eselon baris, maka vektor baris yang ada 1 utama membentuk basis untuk ruang baris R, dan vektor kolom yang ada 1 utama membentuk basis untuk ruang kolom R
- Gabungan dari 2 teori ini ➔ basis ruang baris R = basis ruang baris A

- Teori 5.5.4 ➔ OBE tidak merubah ruang baris dari matrix
- Teori 5.5.6 ➔ Jika matrix R berbentuk eselon baris, maka vektor kolom yang ada 1 utama membentuk basis untuk ruang baris R, dan vektor kolom yang ada 1 utama membentuk basis untuk ruang kolom R
- Gabungan dari 2 teori ini ➔ basis ruang baris R = basis ruang baris A
- Teori 5.5.5.b ➔ himpunan vektor kolom yang membentuk basis untuk ruang kolom A berkorespondensi dengan himpunan vektor kolom yang membentuk basis untuk ruang kolom R
- Gabungan teori 5.5.6 dan 5.5.5.b ➔ basis Ruang kolom R berkorespondensi dengan basis ruang kolom A
- Catt ➔
  - Kalau = berarti boleh langsung diambil
  - Kalau berkorespondensi berarti ➔ tidak boleh langsung diambil, tapi disamakan

- Contoh 6 dan 7 ➔ mencari basis untuk ruang baris dan ruang vektor , dimana hasil basisnya nanti bukan berasal dari baris atau vektor yang ada di A. Tapi vektor baru

Example 6 Find bases for the row and column spaces of

Soal 1

$$A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$$

Solution. Since elementary row operations do not change the row space of a matrix, we can find a basis for the row space of A by finding a basis for the row space of any row-echelon form of A. Reducing A to row-echelon form we obtain (verify)

$$R = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 0 & 0 & 1 & 3 & -2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By Theorem 5.5.6 the nonzero row vectors of R form a basis for the row space of R, and hence form a basis for the row space of A. These basis vectors are

$$\mathbf{r}_1 = [1 \quad -3 \quad 4 \quad -2 \quad 5 \quad 4]$$
$$\mathbf{r}_2 = [0 \quad 0 \quad 1 \quad 3 \quad -2 \quad -6]$$
$$\mathbf{r}_3 = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 5]$$

Keeping in mind that A and R may have different column spaces, we cannot find a basis for the column space of A *directly* from the column vectors of R. However, it follows from Theorem 5.5.5*b* that if we can find a set of column vectors of R that forms a basis for the column space of R, then the *corresponding* column vectors of A will form a basis for the column space of A.

The first, third, and fifth columns of R contain the leading 1's of the row vectors, so

$$\mathbf{c}_1' = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{c}_3' = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{c}_5' = \begin{bmatrix} 5 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

form a basis for the column space of R; thus the corresponding column vectors of A, namely,

$$\mathbf{c}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ -1 \end{bmatrix}, \quad \mathbf{c}_3 = \begin{bmatrix} 4 \\ 9 \\ 9 \\ -4 \end{bmatrix}, \quad \mathbf{c}_5 = \begin{bmatrix} 5 \\ 8 \\ 9 \\ -5 \end{bmatrix}$$

form a basis for the column space of A.

basis → row space of A
→ column space of A

$$A = \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ \end{array}$$

$$A = \begin{bmatrix} \boxed{1} & -3 & \boxed{4} & -2 & \boxed{5} & 4 \\ \boxed{2} & & \boxed{9} & & \boxed{8} & \\ \boxed{2} & & \boxed{9} & & \boxed{9} & \\ \boxed{-1} & & \boxed{-4} & & \boxed{-5} & \end{bmatrix}$$

⇓ gauss

$$R = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 0 & 0 & 1 & 3 & -2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{cccccc} k_1 & k_2 & k_3 & k_4 & k_5 & k_6 \end{array}$$

∴ Row space of R = row space of A

⇒ row space of A → vektor baru
→ vektor yg ≠ ada di A

⇒ r_1, r_2, r_3 } row yg ada 1 utama

∴ column space of R ≠ c.s of A

tp → korespondensi

⇒ column space of A → bukan vektor baru
→ vektor ada di A

C_1 C_3 C_5 ⇒ ada di A

k_1 k_3 k_5 ⇒ ada di R tp ≠ di A.

Contoh 6 : • Rowspace (A) = Rowspace (R) → Teorema 5.5.4 .
dimana R adalah matriks A
yang sudah di - O.B.E.
dan berbentuk Eselon Baris

• Terapkan Teorema 5.5.6 , maka akan didapat

Basis Ruang ~~Vektor~~ Baris (A) ~~dan~~ $= \{\vec{r_1}, \vec{r_2}, \vec{r_3}\}$.

dan Basis Ruang kolom (A) $= \{\vec{c_1}, \vec{c_2}, \vec{c_3}\}$

dengan menerapkan Teorema 5.5.5. (b)
dimana A di soal = A di Teorema 5.5.5.
R —"— = B —"— —"—

Example 7 Find a basis for the space spanned by the vectors **Soal 2**

$$\mathbf{v}_1 = (1, -2, 0, 0, 3), \quad \mathbf{v}_2 = (2, -5, -3, -2, 6), \quad \mathbf{v}_3 = (0, 5, 15, 10, 0),$$
$$\mathbf{v}_4 = (2, 6, 18, 8, 6)$$

Solution. Except for a variation in notation, the space spanned by these vectors is the row space of the matrix

$$\begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$$

Reducing this matrix to row-echelon form we obtain

$$\begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The nonzero row vectors in this matrix are

$$\mathbf{w}_1 = (1, -2, 0, 0, 3), \quad \mathbf{w}_2 = (0, 1, 3, 2, 0), \quad \mathbf{w}_3 = (0, 0, 1, 1, 0)$$

These vectors form a basis for the row space and consequently form a basis for the subspace of R^5 spanned by $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$.

Contoh 7: Soal: Basis u/ Ruang yg. direntang $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$

$\downarrow$

"ditransformasi" menjadi

Basis u/ Ruang Baris (A)

- dimana A dibentuk dari $\begin{pmatrix} — \vec{r}_1 = \vec{v}_1 — \\ — \vec{r}_2 = \vec{v}_2 — \\ — \vec{r}_3 = \vec{v}_3 — \\ — \vec{r}_4 = \vec{v}_4 — \end{pmatrix}$

- Gunakan OBE untuk mengubah A ke dalam
- matriks eselon baris
- Kemudian terapkan Teorema 5.5.6 .

Ex.8 hal 267

Soal 3

- Basis untuk row space yang berasal dari vektor baris yang ada di A

Teorema
-5.5.5. b
-5.5.6

Example 8 Find a basis for the row space of

$$A = \begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$$

consisting entirely of row vectors from A.

Solution. We will transpose A, thereby converting the row space of A into the column space of A^T; then we will use the method of Example 6 to find a basis for the column space of A^T; and then we will transpose again to convert column vectors back to row vectors. Transposing A yields

Basis dari vektor baris A sendiri, apakah r1, r2, r3 atau r4

- R1 = [1 -2 0 0 3]
- R2 = [2 -5   -3   -2   6]
- R3 = [0 5   15   10   0]
- R4 = [2 6 18   8   6]

$$A^T = \begin{bmatrix} 1 & 2 & 0 & 2 \\ -2 & -5 & 5 & 6 \\ 0 & -3 & 15 & 18 \\ 0 & -2 & 10 & 8 \\ 3 & 6 & 0 & 6 \end{bmatrix}$$

Reducing this matrix to row-echelon form yields

$$\begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 1 & -5 & -10 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The first, second, and fourth columns contain the leading 1's, so the corresponding column vectors in A^T form a basis for the column space of A^T; these are

$$\mathbf{c}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \quad \mathbf{c}_2 = \begin{bmatrix} 2 \\ -5 \\ -3 \\ -2 \\ 6 \end{bmatrix}, \quad \text{and} \quad \mathbf{c}_4 = \begin{bmatrix} 2 \\ 6 \\ 18 \\ 8 \\ 6 \end{bmatrix}$$

Transposing again and adjusting the notation appropriately yields the basis vectors

$$\mathbf{r}_1 = [\,1 \quad -2 \quad 0 \quad 0 \quad 3\,], \qquad \mathbf{r}_2 = [\,2 \quad -5 \quad -3 \quad -2 \quad 6\,],$$

and

$$\mathbf{r}_4 = [\,2 \quad 6 \quad 18 \quad 8 \quad 6\,]$$

for the row space of A. ▲

find a basis for the row space of A consisting entirely of row vectors from A (using gauss)

	-6	7	-9	7	2
A=	9	4	-3	-3	5
	-4	7	-4	2	7
	7	-3	7	3	-5

	-6	9	-4	7		
At =	7	4	7	-3		
	-9	-3	-4	7		
	7	-3	2	3		
	2	5	7	-5		
	1	-1.5	0.67	-1.17	iterasi ke 1, berapa isi sel A(1,2)	-1.5
	7	4	7	-3		
	-9	-3	-4	7		
	7	-3	2	3		
	2	5	7	-5		
	1	-1.5	0.67	-1.17	iterasi ke 2, berapa isi sel A(2,3)	2.31
	0	14.5	2.31	5.19		
	-9	-3	-4	7		
	7	-3	2	3		
	2	5	7	-5		

1	-1.5	0.67	-1.17	iterasi ke 3, berapa isi sel A(3,4)	-3.53
0	14.5	2.31	5.19		
0	-16.5	2.03	-3.53		
7	-3	2	3		
2	5	7	-5		
1	-1.5	0.67	-1.17	iterasi ke 4, berapa isi sel A(4,3)	-2.69
0	14.5	2.31	5.19		
0	-16.5	2.03	-3.53		
0	7.5	-2.69	11.19		
2	5	7	-5		
1	-1.5	0.67	-1.17	iterasi ke 5, berapa isi sel A(5,2)	8
0	14.5	2.31	5.19		
0	-16.5	2.03	-3.53		
0	7.5	-2.69	11.19		
0	8	5.66	-2.66		
1	-1.5	0.67	-1.17	iterasi ke 6, berapa isi sel A(2,3)	0.16
0	1	0.16	0.36		
0	-16.5	2.03	-3.53		
0	7.5	-2.69	11.19		
0	8	5.66	-2.66		
1	-1.5	0.67	-1.17	iterasi ke 7, berapa isi sel A(3,4)	2.41
0	1	0.16	0.36		
0	0	4.67	2.41		
0	7.5	-2.69	11.19		
0	8	5.66	-2.66		

1	-1.5	0.67	-1.17		iterasi ke 8, berapa isi sel A(4,3)	-3.89
0	1	0.16	0.36			
0	0	4.67	2.41			
0	0	-3.89	8.49			
0	8	5.66	-2.66			
1	-1.5	0.67	-1.17		iterasi ke 9, berapa isi sel A(5,4)	-5.54
0	1	0.16	0.36			
0	0	4.67	2.41			
0	0	-3.89	8.49			
0	0	4.38	-5.54			
1	-1.5	0.67	-1.17		iterasi ke 10, berapa isi sel A(3,4)	0.52
0	1	0.16	0.36			
0	0	1	0.52			
0	0	-3.89	8.49			
0	0	4.38	-5.54			
1	-1.5	0.67	-1.17		iterasi ke 11, berapa isi sel A(4,4)	10.51
0	1	0.16	0.36			
0	0	1	0.52			
0	0	0	10.51			
0	0	4.38	-5.54			

1	-1.5	0.67	-1.17
0	1	0.16	0.36
0	0	1	0.52
0	0	0	10.51
0	0	0	-7.82

1	-1.5	0.67	-1.17
0	1	0.16	0.36
0	0	1	0.52
0	0	0	1
0	0	0	-7.82

1	-1.5	0.67	-1.17
0	1	0.16	0.36
0	0	1	0.52
0	0	0	1
0	0	0	0

iterasi ke 12, berapa isi sel A(5,4) ..... -7.82

The basis for the row space of A consisting entirely of row vectors from A are r1, r2, r3, r4

r1 =	4	6	-9	7	2
r2 =	-6	4	8	-3	5
r3 =	9	6	-4	8	7
r4 =	-5	8	7	3	2

Contoh 8: • $A \xrightarrow{\text{di transpos}} A^T$

$A \downarrow$ $A^T \downarrow$

Ruang Baris (A) = Ruang kolom (A^T)
Ruang kolom (A) = Ruang Baris (A^T)

• Terapkan Teorema 5.5.6 pada matriks A^T

$$A^T \xrightarrow{\text{O.B.E.}} \text{matriks eselon-baris } R = \begin{array}{c} \begin{array}{cccc} \vec{c_1} & \vec{c_2} & \vec{c_3} & \vec{c_4} \end{array} \\ \begin{pmatrix} 1 & 2 & 0 & 2 \\ 0 & 1 & -5 & -10 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \begin{array}{l} \\ \to \vec{r_1} \\ \to \vec{r_2} \\ \to \vec{r_3} \\ \\ \\ \end{array}$$

•

Basis Ruang Baris (A^T) $= \{\vec{r_1}, \vec{r_2}, \vec{r_3}\}$
= Basis Ruang Baris (R) $= \{\vec{r_1}, \vec{r_2}, \vec{r_3}\}$
Teorema 5.5.6.

• Basis Ruang kolom $(R) = \{\vec{c_1}, \vec{c_2}, \vec{c_4}\} \to$ Teorema 5.5.6.

$\hookrightarrow$ Basis Ruang kolom $(A^T) = \{\underset{A^T}{\text{kolom-1}}, \underset{A^T}{\text{kolom-2}}, \underset{A^T}{\text{kolom-4}}\}$

berdasarkan Teorema 5.5.5 (b)

Soal 4

Mencari basis yang direntang oleh vektor itu sendiri
➔
v1 atau v2 atau V3 atau V4 atau V5

Example 9

(a) Find a subset of the vectors

$$\mathbf{v}_1 = (1, -2, 0, 3), \quad \mathbf{v}_2 = (2, -5, -3, 6),$$
$$\mathbf{v}_3 = (0, 1, 3, 0), \quad \mathbf{v}_4 = (2, -1, 4, -7), \quad \mathbf{v}_5 = (5, -8, 1, 2)$$

that forms a basis for the space spanned by these vectors.

(b) Express the vectors not in the basis as a linear combination of the basis vectors.

Solution (*a*). We begin by constructing a matrix that has $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_5$ as its column vectors:

$$\begin{array}{c} \begin{bmatrix} 1 & 2 & 0 & 2 & 5 \\ -2 & -5 & 1 & -1 & -8 \\ 0 & -3 & 3 & 4 & 1 \\ 3 & 6 & 0 & -7 & 2 \end{bmatrix} \\ \begin{array}{ccccc} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 & \mathbf{v}_5 \end{array} \end{array} \tag{7}$$

The first part of our problem can be solved by finding a basis for the column space of this matrix. Reducing the matrix to *reduced* row-echelon form and denoting the column vectors of the resulting matrix by $\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4$, and $\mathbf{w}_5$ yields

$$\begin{array}{c} \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{ccccc} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \mathbf{w}_1 & \mathbf{w}_2 & \mathbf{w}_3 & \mathbf{w}_4 & \mathbf{w}_5 \end{array} \end{array} \tag{8}$$

The leading 1's occur in columns 1, 2, and 4, so that by Theorem 5.5.6

$$\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_4\}$$

is a basis for the column space of (8) and consequently

$$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$$

is a basis for the column space of (7).

Solution (*b*). We shall start by expressing $\mathbf{w}_3$ and $\mathbf{w}_5$ as linear combinations of the basis vectors $\mathbf{w}_1$, $\mathbf{w}_2$, $\mathbf{w}_4$. The simplest way of doing this is to express $\mathbf{w}_3$ and $\mathbf{w}_5$ in terms of basis vectors with smaller subscripts. Thus, we shall express $\mathbf{w}_3$ as a linear combination of $\mathbf{w}_1$ and $\mathbf{w}_2$, and we shall express $\mathbf{w}_5$ as a linear combination of $\mathbf{w}_1$, $\mathbf{w}_2$, and $\mathbf{w}_4$. By inspection of (8), these linear combinations are

$$\mathbf{w}_3 = 2\mathbf{w}_1 - \mathbf{w}_2$$
$$\mathbf{w}_5 = \mathbf{w}_1 + \mathbf{w}_2 + \mathbf{w}_4$$

We call these the ***dependency equations***. The corresponding relationships in (7) are

$$\mathbf{v}_3 = 2\mathbf{v}_1 - \mathbf{v}_2$$
$$\mathbf{v}_5 = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_4$$

Kombinasi Linier

- V3 = k1V1 + k2V2 + k4V4

  ➔ cari k1, k2 dan k4
- V5 = m1v1 + m2v2 + m4v4

  ➔ cari m1, m2 dan m4

Contoh 9: (a)

$$A = \begin{array}{c} \begin{pmatrix} 1 & 2 & 0 & 2 & 5 \\ -2 & -5 & 1 & -1 & -8 \\ 0 & -3 & 3 & 4 & 1 \\ 3 & 6 & 0 & -7 & 2 \end{pmatrix} \\ \begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \vec{v}_1 & \vec{v}_2 & \vec{v}_3 & \vec{v}_4 & \vec{v}_5 \end{array} \end{array}$$

$$\Big\downarrow \text{ di O.B.E.}$$

$$R = \begin{array}{c} \begin{pmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\ \begin{array}{ccccc} \vec{w}_1 & \vec{w}_2 & \vec{w}_3 & \vec{w}_4 & \vec{w}_5 \end{array} \end{array}$$

ada 1 utama

Dengan Teorema 5.5.5 (b)

$$\underbrace{\text{Basis (R)}}_{\text{ruang kolom}} = \{\vec{w}_1, \vec{w}_2, \vec{w}_4\}$$

maka

$$\text{Basis Ruang kolom } (A) = \{\vec{v}_1, \vec{v}_2, \vec{v}_4\}$$

(b): $\vec{v}_3$ kombinasi linier $\{\vec{v}_1, \vec{v}_2, \vec{v}_4\}$

$\vec{v}_5$ —"— —"—

find a subset of the vector v1, v2, v3, v4, v5 that forms a basis for the space spanned by these vectors (using gauss)

v1 =	6	-2	7	6
v2 =	4	-2	4	-3
v3 =	-5	6	-6	-5
v4 =	5	-7	4	-9
v5 =	-4	5	7	-3

6	4	-5	5	-4		
-2	-2	6	-7	5		
7	4	-6	4	7		
6	-3	-5	-9	-3		
1	0.67	-0.83	0.83	-0.67	iterasi ke 1, berapa isi sel A(1,3) …..	-0.83
-2	-2	6	-7	5		
7	4	-6	4	7		
6	-3	-5	-9	-3		
1	0.67	-0.83	0.83	-0.67	iterasi ke 2, berapa isi sel A(2,4) …..	-5.34
0	-0.66	4.34	-5.34	3.66		
7	4	-6	4	7		
6	-3	-5	-9	-3		
1	0.67	-0.83	0.83	-0.67	iterasi ke 3, berapa isi sel A(3,5) …..	11.69
0	-0.66	4.34	-5.34	3.66		
0	-0.69	-0.19	-1.81	11.69		
6	-3	-5	-9	-3		

1	0.67	-0.83	0.83	-0.67	iterasi ke 4, berapa isi sel A(4,2) …..	-7.02
0	-0.66	4.34	-5.34	3.66		
0	-0.69	-0.19	-1.81	11.69		
0	-7.02	-0.02	-13.98	1.02		
1	0.67	-0.83	0.83	-0.67	iterasi ke 5, berapa isi sel A(2,3) …..	-6.58
0	1	-6.58	8.09	-5.55		
0	-0.69	-0.19	-1.81	11.69		
0	-7.02	-0.02	-13.98	1.02		
1	0.67	-0.83	0.83	-0.67	iterasi ke 6, berapa isi sel A(3,4) …..	3.77
0	1	-6.58	8.09	-5.55		
0	0	-4.73	3.77	7.86		
0	-7.02	-0.02	-13.98	1.02	bilqis	

1	0.67	-0.83	0.83	-0.67	iterasi ke 7, berapa isi sel A(4,3) …..	-46.21
0	1	-6.58	8.09	-5.55		
0	0	-4.73	3.77	7.86		
0	0	-46.21	42.81	-37.94		
1	0.67	-0.83	0.83	-0.67	iterasi ke 8, berapa isi sel A(3,5) …..	-1.66
0	1	-6.58	8.09	-5.55		
0	0	1	-0.8	-1.66		
0	0	-46.21	42.81	-37.94		
1	0.67	-0.83	0.83	-0.67	iterasi ke 9, berapa isi sel A(4,4) …..	5.84
0	1	-6.58	8.09	-5.55		
0	0	1	-0.8	-1.66		
0	0	0	5.84	-114.65		
1	0.67	-0.83	0.83	-0.67	Iterasi ke 10, berapa isi sel A(4,5) …	-19.63
0	1	-6.58	8.09	-5.55		
0	0	1	-0.8	-1.66		
0	0	0	1	-19.63	Jadi basis nya adalah V1, V2, V3 dan V4	

- Tugas Kelompok ➔
  - cari 2 soal dan jawaban di internet yang berhubungan dengan materi ppt ini
  - Tulis alamat internetnya
  - Di kirim ke elearning, terakhir ➔
    - Minggu depan
- Format ➔ subject ➔
  - Alin-B-melati
  - Bentuk ➔ ppt ➔ informasi nama kelompok + anggota