Pertemuan 10

Basis; Kombinasi Linier, Merentang, Bebas linier

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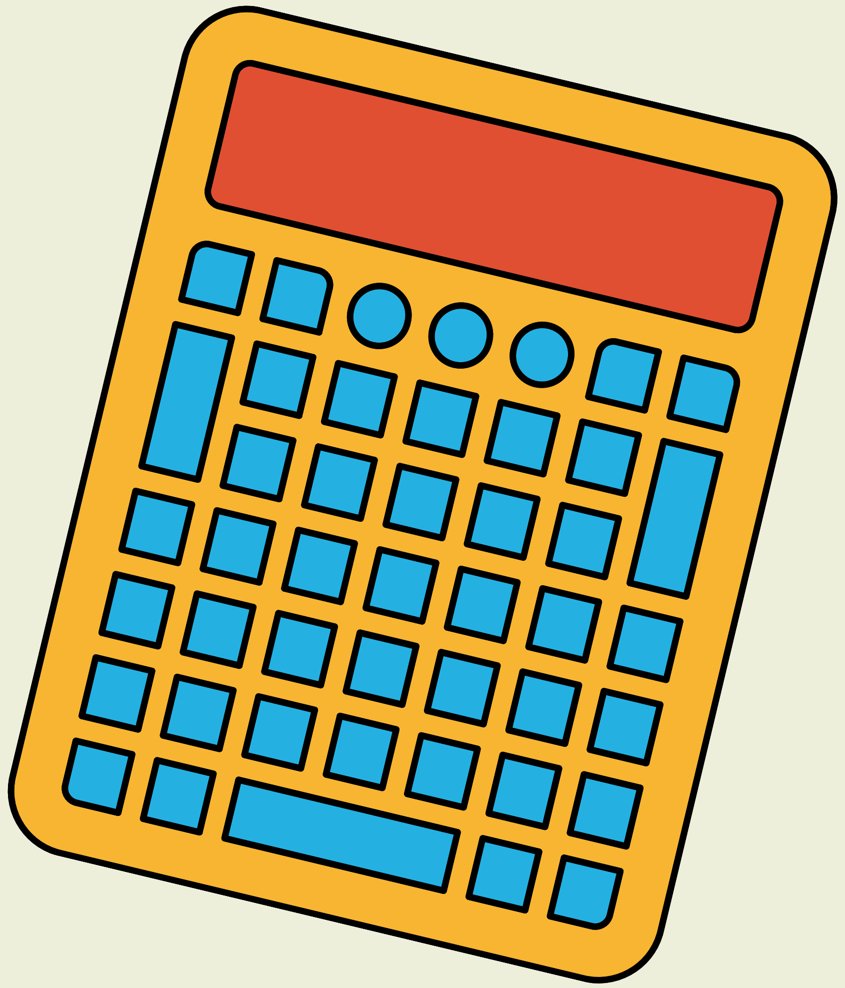
Materi yang akan dibahas

- Kombinasi Linier
- Merentang
- Bebas Linier
- **Basis**



TUJUAN INSTRUKSIONAL KHUSUS

Setelah menyelesaikan pertemuan ini mahasiswa diharapkan :

- Dapat mengetahui apakah suatu vektor merentang dan bebas linier
- Dapat mencari basis dari suatu SPL

Ilustrasi -> Kombinasi Linier

Contoh Kombinasi Linier -> Pencampuran 2 warna

Pink = 2 putih + 2 merah

Hijau = x hitam + x merah

x = tidak ada

Pengertian

1) kombinasi Linier

$\overline{w}$ → kombinasi Linier dr $\overline{v_1}$ $\overline{v_2}$ ... $\overline{v_n}$ jk
$\overline{w}$ dpt diungkapkan dlm bentuk :

$$\overline{w} = k_1 \overline{v_1} + k_2 \overline{v_2} + \dots + k_n \overline{v_n}$$

dimana
k_1 , k_2 ... k_n → skalar

ingat !! baris di atas bkn hanya satu baris, tp terdiri dari beberapa baris

or kombinasi Linier → ada nilai u k_1 k_2 ... k_n

ex:

$$\vec{v_1} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad \vec{v_2} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Cari kombinasi linier (nilai k_1 & k_2) u vektor $\vec{w} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$$\vec{w} = k_1 \vec{v_1} + k_2 \vec{v_2}$$

$$\begin{bmatrix} 4 \\ 5 \end{bmatrix} = k_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + k_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Apakah W adalah kombinasi linier (KL) Dari V1 dan V2 ? (jawab dengan gauss-jordan)

$$2k_1 + 3k_2 = 4 \quad \Rightarrow \quad 2k_1 + 3k_2 = 4$$

$$(k_1 + 2k_2 = 5) \times 2 \quad \Rightarrow \quad 2k_1 + 4k_2 = 10 \quad -$$

$$\boxed{k_2 = 6}$$

$$k_1 + 2.6 = 5$$

$$\boxed{k_1 = -7}$$

$$\therefore \ \vec{w} = -7\vec{v_1} + 6\vec{v_2}$$

Contoh soal yang bukan kombinasi linier

Apakah W adalah kombinasi linier (KL)

Dari V1 dan V2 ? (jawab dengan gauss-jordan)

4.10

ex : tent apakah $\bar{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\bar{v}_2 = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

merentang R^2

jwb:

det $A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$

det $A = 0 \rightsquigarrow \neq$ merentang

coba 2 dt sbg k.L

$$\begin{bmatrix} 4 \\ 5 \end{bmatrix} = k_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + k_2 \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$2k_1 + 4k_2 = 4 \quad \rightarrow \quad 2k_1 + 4k_2 = 4$

$(k_1 + 2k_2 = 5) * 2 \quad \rightarrow \quad 2k_1 + 4k_2 = 10$

$-$

$0 = -6$

?!....

$\therefore$ tidak ada nilai u k_1 & k_2

krn $\neq$ sembarang vektor pd R^2 yg dpt dinyatakan sbg k.L $\bar{v}_1$ $\bar{v}_2$, mk $\bar{v}_1$ $\bar{v}_2$ tidak merentang R^2

Soal 1

Nyatakan (65, 17, - 21) sebagai kombinasi linier dari (8, 5, - 3) ; (- 3, - 4, 6) ; (4, - 5,3).
Carilah nilai k1, k2 dan k3 dengan menggunakan Gauss-Jordan.

8	-3	4	65
5	-4	-5	17
-3	6	3	-21

1	-0.38	0.5	8.13
5	-4	-5	17
-3	6	3	-21

Pada iterasi ke 1, berapa isi sel A(1,4) **8.13**

1	-0.38	0.5	8.13
0	-2.1	-7.5	-23.65
-3	6	3	-21

Pada iterasi ke 2, berapa isi sel A(2,3) ..... **-7.5**

1	-0.38	0.5	8.13
0	-2.1	-7.5	-23.65
0	4.86	4.5	3.39

Pada iterasi ke 3, berapa isi sel A(3,2) **4.86**

Soal 1

1	-0.38	0.5	8.13
0	1	3.57	11.26
0	4.86	4.5	3.39

Pada iterasi ke 4, berapa isi sel A(2,4) **11.26**

1	-0.38	0.5	8.13
0	1	3.57	11.26
0	0	-12.9	-51.33

Pada iterasi ke 5, berapa isi sel A(3,4) ..... **-51.33**

1	-0.38	0.5	8.13
0	1	3.57	11.26
0	0	1	3.99

Pada iterasi ke 6, berapa isi sel A(3,4) **3.99**

1	-0.38	0.5	8.13
0	1	0	-2.98
0	0	1	3.99

Pada iterasi ke 7, berapa isi sel A(2,4) ..... **-2.98**

Soal 1

1	-0.38	0	6.14
0	1	0	-2.98
0	0	1	3.99

Pada iterasi ke 8, berapa isi sel A(1,4) **6.14**

1	0	0	5.01
0	1	0	-2.98
0	0	1	3.99

Pada iterasi ke 9, berapa isi sel A(1,4) **5.01**

Ex. 9 hal 226

Example 9 Consider the vectors $\mathbf{u} = (1, 2, -1)$ and $\mathbf{v} = (6, 4, 2)$ in R^3. Show that $\mathbf{w} = (9, 2, 7)$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$ and that $\mathbf{w}' = (4, -1, 8)$ is *not* a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

Solution. In order for $\mathbf{w}$ to be a linear combination of $\mathbf{u}$ and $\mathbf{v}$, there must be scalars k_1 and k_2 such that $\mathbf{w} = k_1\mathbf{u} + k_2\mathbf{v}$; that is,

$$(9, 2, 7) = k_1(1, 2, -1) + k_2(6, 4, 2)$$

or

$$(9, 2, 7) = (k_1 + 6k_2, 2k_1 + 4k_2, -k_1 + 2k_2)$$

Equating corresponding components gives

$$\begin{aligned} k_1 + 6k_2 &= 9 \\ 2k_1 + 4k_2 &= 2 \\ -k_1 + 2k_2 &= 7 \end{aligned}$$

Solving this system yields $k_1 = -3$, $k_2 = 2$, so

$$\mathbf{w} = -3\mathbf{u} + 2\mathbf{v}$$

Similarly, for $\mathbf{w}'$ to be a linear combination of $\mathbf{u}$ and $\mathbf{v}$, there must be scalars k_1 and k_2 such that $\mathbf{w}' = k_1\mathbf{u} + k_2\mathbf{v}$; that is,

$$(4, -1, 8) = k_1(1, 2, -1) + k_2(6, 4, 2)$$

or

$$(4, -1, 8) = (k_1 + 6k_2, 2k_1 + 4k_2, -k_1 + 2k_2)$$

Equating corresponding components gives

$$\begin{aligned} k_1 + 6k_2 &= 4 \\ 2k_1 + 4k_2 &= -1 \\ -k_1 + 2k_2 &= 8 \end{aligned}$$

This system of equations is inconsistent (verify), so no such scalars k_1 and k_2 exist. Consequently, $\mathbf{w}'$ is not a linear combination of $\mathbf{u}$ and $\mathbf{v}$. ▲

Merentang = spanning

4.9

2. Merentang :

- $\bar{v}_1\ \bar{v}_2 \ldots \bar{v}_n$ merentang ruang vektor V jk sembarang vektor pd ruang vektor V dpt dinyatakan sbg kombinasi linier dr $\bar{v}_1\ \bar{v}_2 \ldots \bar{v}_n$
- ada banyak nilai u $k_1\ k_2 \ldots k_n$
- det $\neq 0$ (jk bisa dicari det)

ex:

tent. apakah $\bar{v}_1 = \begin{bmatrix}2\\1\end{bmatrix}$ & $\bar{v}_2 = \begin{bmatrix}3\\2\end{bmatrix}$ merentang R^2

jwb: merentang jk → sembarang vektor pd R^2 dpt dinyatakan sbg kombinasi linier $\bar{v}_1\ \bar{v}_2$

$$\det A = \begin{array}{cc} \bar{V_1} & \bar{V_2} \\ \end{array}$$
$$\det A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

det A = 1 → merentang

coba 2 data sbg kombinasi linier (KL)

ex:

$$\begin{bmatrix} 4 \\ 5 \end{bmatrix} = k_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + k_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$k_1 = -7$
$k_2 = 6$

ex:

$$\begin{bmatrix} 23 \\ 14 \end{bmatrix} = k_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + k_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$k_1 = 4$
$k_2 = 5$

∴ krn sembarang vektor pd R^2 dpt dinyatakan sbg K.L $\bar{V_1}$ $\bar{V_2}$

jd $\bar{V_1}$ & $\bar{V_2}$ merentang R^2

Apakah W adalah (KL) Dari V1 dan V2 ?
(jawab dengan gauss-jordan)

ex : tent apakah $\bar{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\bar{v}_2 = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ merentang R^2

jwb:

det $A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$

det $A = 0 \rightarrow \not$ merentang

coba 2 dll sbg k.l

$$\begin{bmatrix} 4 \\ 5 \end{bmatrix} = k_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + k_2 \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$\begin{aligned} 2k_1 + 4k_2 &= 4 & &\rightarrow & 2k_1 + 4k_2 &= 4 \\ (k_1 + 2k_2 &= 5) \times 2 & &\rightarrow & 2k_1 + 4k_2 &= 10 \\ & & & & \hline 0 &= -6 \;\; - \end{aligned}$$

?! ....

$\therefore$ tidak ada nilai u k_1 & k_2

krn ~~ada~~ sebarang vektor pd R^2 yg dpt dinyatakan sbg k.l $\bar{v}_1$ $\bar{v}_2$, mk $\bar{v}_1$ $\bar{v}_2$ tidak merentang R^2

Ex. 12 hal 229

Example 12 Determine whether $\mathbf{v}_1 = (1, 1, 2)$, $\mathbf{v}_2 = (1, 0, 1)$, and $\mathbf{v}_3 = (2, 1, 3)$ span the vector space R^3.

Solution. We must determine whether an arbitrary vector $\mathbf{b} = (b_1, b_2, b_3)$ in R^3 can be expressed as a linear combination

$$\mathbf{b} = k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + k_3\mathbf{v}_3$$

of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$. Expressing this equation in terms of components gives

$$(b_1, b_2, b_3) = k_1(1, 1, 2) + k_2(1, 0, 1) + k_3(2, 1, 3)$$

or

$$(b_1, b_2, b_3) = (k_1 + k_2 + 2k_3,\ k_1 + k_3,\ 2k_1 + k_2 + 3k_3)$$

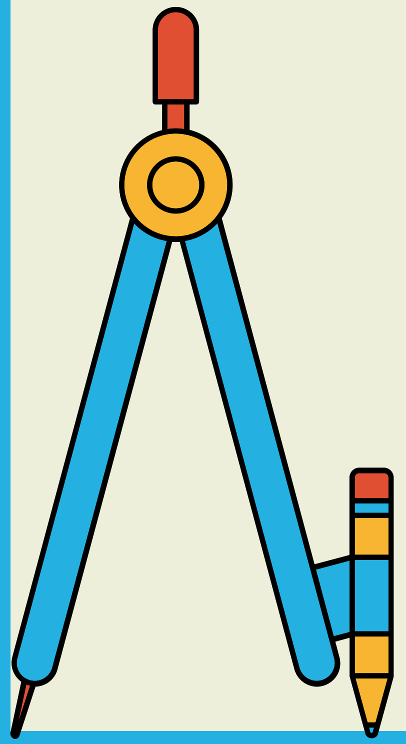
or

$$\begin{aligned} k_1 + k_2 + 2k_3 &= b_1 \\ k_1 \qquad + k_3 &= b_2 \\ 2k_1 + k_2 + 3k_3 &= b_3 \end{aligned}$$

The problem thus reduces to determining whether this system is consistent for all values of b_1, b_2, and b_3. By parts (*a*) and (*e*) of Theorem 4.3.4, this system is consistent for all b_1, b_2, and b_3 if and only if the coefficient matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

is invertible. But $\det(A) = 0$ (verify), so that A is not invertible; consequently, $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ do not span R^3.



5.3 Kebebasan Linier

- $\bar{v}_1\ \bar{v}_2\ \dots\ \bar{v}_n \rightarrow$ bebas Linier jk hanya ada satu pemecahan u persamaan :

$$k_1 \bar{v}_1 + k_2 \bar{v}_2 + \dots + k_n \bar{v}_n = \bar{0}$$

yaitu $k_1 = k_2 = \dots = k_n = 0$ //

- det $\neq 0$ (jk bisa dicari det)
- jk ada sebuah or lebih vektor dpt dinyatakan sbg k.l vektor lainnya mk $\neq$ bebas Linier

ex:

Kombinasi Linier (k.l): **w** k.l. $S = \{\mathbf{v1}, \mathbf{v2}, \mathbf{v3}, \ldots, \mathbf{vr}\}$ jika
$\mathbf{w} = k1\mathbf{v1} + k2\mathbf{v2} + k3\mathbf{v3} \ldots + kr\mathbf{vr}$ $k1, k2, k3, \ldots, kr$ ada nilainya

Independensi Linier:

$$S = \{\mathbf{v1}, \mathbf{v2}, \mathbf{v3}, \ldots, \mathbf{vr}\}$$

disebut himpunan bebas linier (***linearly independent***) jika solusi Sistem Persamaan Linier Homogen

$$k1\mathbf{v1} + k2\mathbf{v2} + k3\mathbf{v3} \ldots + kr\mathbf{vr} = \mathbf{0}$$

adalah solusi trivial $k1, k2, k3, \ldots, kr = 0$

Independensi Linier:

$S = \{v1, v2, v3, ..., vr\}$

disebut himpunan bebas linier / tidak-bergantung linier (***linearly independent***) jika solusi Sistem Persamaan Linier Homogen

$k1\mathbf{v1} + k2\mathbf{v2} + k3\mathbf{v3} .... + kr\mathbf{vr} = \mathbf{0}$

adalah solusi trivial $k1, k2, k3, ..., kr = 0$

Dependensi Linier:

$S = \{v1, v2, v3, ..., vr\}$

disebut himpunan tidak-bebas linier / bergantung linier (***linearly dependent***) jika solusi Sistem Persamaan Linier Homogen

$k1\mathbf{v1} + k2\mathbf{v2} + k3\mathbf{v3} .... + kr\mathbf{vr} = \mathbf{0}$

adalah solusi non-trivial $k1, k2, k3, ..., kr = 0$

dan ada $kj \neq 0$ **(j = 1 .. r)**

Diketahui : himpunan S = **{v1, v2, v3, ..., vr }**

Ditanyakan : apakah S ***linearly independent*** atau ***linearly dependent***?

Jawab:

1. Bentuk SPL Homogen *k1***v1**+ *k2***v2** + *k3***v3**.... + *kr***vr** = **0**
2. Tentukan solusinya
3. Jika solusinya trivial *k1*, *k2*, *k3*, ..., *kr* = 0
   maka S ***linearly independent***
4. Jika solusinya **non-trivial** maka S ***linearly dependent***

$\neq$ bebas Linier

ex:

tent. apakah $\overline{v_1} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ & $\overline{v_2} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ beben Linier ?

jwb:

det $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

det $A = 1$

$k_1 \overline{v_1} + k_2 \overline{v_2} = \overline{0}$

$k_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + k_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2k_1 + 3k_2 = 0 \rightarrow 2k_1 + 3k_2 = 0$

$(k_1 + 2k_2 = 0) \times 2 \rightarrow 2k_1 + 4k_2 = 0$ $-$

$-k_2 = 0$

$\boxed{k_2 = 0}$

$k_1 + 2.0 = 0$

$\boxed{k_1 = 0}$

$\therefore$ terbukti hanya ada 1 pemecahan u k_1 & k_2 yaitu 0 //

Apakah V1 dan V2 bebas linier

Soal 3

Apakah V1, V2 dan V3 bebas linier ?
Kerjakan dengan gauss-jordan

contoh:

$S = \{ \bar{v}_1 , \bar{v}_2 , \bar{v}_3 \}$ dimana

$\bar{v}_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 3 \end{bmatrix} \quad \bar{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 5 \\ -1 \end{bmatrix} \quad V_3 = \begin{bmatrix} 7 \\ -1 \\ 5 \\ 8 \end{bmatrix}$

mk ? $\bar{v}_1$ $\bar{v}_2$ & $\bar{v}_3$ bebas linier ?

jwb: if ada nilai u k_1 k_2 k_3 selain $\varnothing$, mk
$\neq$ bebas linier ... so... cari nilai k_1 k_2 & k_3

$k_1 \overline{v_1} + k_2 \overline{v_2} + k_3 \overline{v_3} = 0$

$$k_1 \begin{bmatrix} 2 \\ -1 \\ 0 \\ 3 \end{bmatrix} + k_2 \begin{bmatrix} 1 \\ 2 \\ 5 \\ -1 \end{bmatrix} + k_3 \begin{bmatrix} 7 \\ -1 \\ 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$2k_1 + k_2 + 7k_3 = 0$ | $\times 1 \rightarrow$ $2k_1 + k_2 + 7k_3 = 0$

$-k_1 + 2k_2 - k_3 = 0$ | $\times 2 \rightarrow$ $-2k_1 + 4k_2 + -2k_3 = 0$ $+$

$5k_2 + 5k_3 = 0$

$3k_1 + -k_2 + 8k_3 = 0$

$5k_2 + 5k_3 = 0$

$5k_2 + 5k_3 = 0$ $-$

$0 = 0$

jawaban 1 $\longrightarrow$ $\boxed{k_1 = 0}$ $\boxed{k_2 = 0}$ $\boxed{k_3 = 0}$

jawaban 2 $\longrightarrow$ misal : $\boxed{k_3 = t}$

$5k_2 + 5k_3 = 0$

$5k_2 + 5t = 0$

$\boxed{k_2 = -t}$

$-k_1 + 2k_2 - k_3 = 0$

$2k_{\cancel{1}} - k_3 = k_1$

2 $\boxed{k_1 = -3t}$

u membuktikan → misal $t = 1$ → $k_1 = -3$, $k_2 = -1$, $k_3 = 1$ } masukkan ke pers. awal

$\overline{v_1}\ \overline{v_2}\ \overline{v_3}$ ≠ bebas linier krn:

→ ada bermacam² nilai u k_1 k_2 & k_3

↳ satu persamaan dpt dinyatakan sbg kombinasi dr pers lain

→ satu vektor dpt dinyatakan sbg kombinasi vektor lain

misal : $-3\overline{v_1} - \overline{v_2} + \overline{v_3} = \overline{0}$

$$\overline{v_1} = \frac{-\overline{v_2} + \overline{v_3}}{3}$$

Ex 1 hal 232

Example 1 If $\mathbf{v}_1 = (2, -1, 0, 3)$, $\mathbf{v}_2 = (1, 2, 5, -1)$, and $\mathbf{v}_3 = (7, -1, 5, 8)$, then the set of vectors $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent, since $3\mathbf{v}_1 + \mathbf{v}_2 - \mathbf{v}_3 = \mathbf{0}$.

Ex. 6 hal 235

Example 6 In Example 1 we saw that the vectors

$$\mathbf{v}_1 = (2, -1, 0, 3), \qquad \mathbf{v}_2 = (1, 2, 5, -1), \qquad \text{and} \qquad \mathbf{v}_3 = (7, -1, 5, 8)$$

form a linearly dependent set. It follows from Theorem 5.3.1 that at least one of these vectors is expressible as a linear combination of the other two. In this example each vector is expressible as a linear combination of the other two since it follows from the equation $3\mathbf{v}_1 + \mathbf{v}_2 - \mathbf{v}_3 = \mathbf{0}$ (see Example 1) that

$$\mathbf{v}_1 = -\tfrac{1}{3}\mathbf{v}_2 + \tfrac{1}{3}\mathbf{v}_3, \qquad \mathbf{v}_2 = -3\mathbf{v}_1 + \mathbf{v}_3, \qquad \text{and} \qquad \mathbf{v}_3 = 3\mathbf{v}_1 + \mathbf{v}_2 \quad \blacktriangle$$

Ex. 2 hal 233

Example 2 The polynomials

$$\mathbf{p}_1 = 1 - x, \quad \mathbf{p}_2 = 5 + 3x - 2x^2, \quad \text{and} \quad \mathbf{p}_3 = 1 + 3x - x^2$$

form a linearly dependent set in P_2 since $3\mathbf{p}_1 - \mathbf{p}_2 + 2\mathbf{p}_3 = \mathbf{0}$. ∠

Ex. 3 hal 233

Example 3 Consider the vectors $\mathbf{i} = (1, 0, 0)$, $\mathbf{j} = (0, 1, 0)$, and $\mathbf{k} = (0, 0, 1)$ in R^3. In terms of components the vector equation

$$k_1\mathbf{i} + k_2\mathbf{j} + k_3\mathbf{k} = \mathbf{0}$$

becomes

$$k_1(1, 0, 0) + k_2(0, 1, 0) + k_3(0, 0, 1) = (0, 0, 0)$$

or equivalently,

$$(k_1, k_2, k_3) = (0, 0, 0)$$

This implies that $k_1 = 0$, $k_2 = 0$, and $k_3 = 0$, so the set $S = \{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$ is linearly independent. A similar argument can be used to show that the vectors

$$\mathbf{e}_1 = (1, 0, 0, \ldots, 0), \quad \mathbf{e}_2 = (0, 1, 0, \ldots, 0), \quad \ldots, \quad \mathbf{e}_n = (0, 0, 0, \ldots, 1)$$

form a linearly independent set in R^n. ▲

Ex. 4 hal 233

Example 4 Determine whether the vectors

$$\mathbf{v}_1 = (1, -2, 3), \quad \mathbf{v}_2 = (5, 6, -1), \quad \mathbf{v}_3 = (3, 2, 1)$$

form a linearly dependent set or a linearly independent set.

Solution. In terms of components the vector equation

$$k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + k_3\mathbf{v}_3 = \mathbf{0}$$

becomes

$$k_1(1, -2, 3) + k_2(5, 6, -1) + k_3(3, 2, 1) = (0, 0, 0)$$

or equivalently,

$$(k_1 + 5k_2 + 3k_3, -2k_1 + 6k_2 + 2k_3, 3k_1 - k_2 + k_3) = (0, 0, 0)$$

Equating corresponding components gives

$$\begin{aligned} k_1 + 5k_2 + 3k_3 &= 0 \\ -2k_1 + 6k_2 + 2k_3 &= 0 \\ 3k_1 - k_2 + k_3 &= 0 \end{aligned}$$

Thus, $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ form a linearly dependent set if this system has a nontrivial solution, or a linearly independent set if it has only the trivial solution. Solving this system yields

$$k_1 = -\tfrac{1}{2}t, \quad k_2 = -\tfrac{1}{2}t, \quad k_3 = t$$

Thus, the system has nontrivial solutions and $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ form a linearly dependent set. Alternatively, we could show the existence of nontrivial solutions without solving the system by showing that the coefficient matrix has determinant zero and consequently is not invertible (verify). ▲

5.4 Basis & Dimensi

ex → garis → berdimensi satu → $S = \{\bar{v}_1\}$
→ bidang → berdimensi dua → $S = \{\bar{v}_1, \bar{v}_2\}$
→ ruang → berdimensi tiga → $S = \{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$

∴ dimensi → jumlah vektor pada suatu himpunan

Basis : → contoh 30, 32, 37, 38

→ himp S yang terdiri dari beberapa vektor ⇒ $S = \{\bar{v}_1, \bar{v}_2 \ldots \bar{v}_n\}$
dinamakan basis if → S bebas linier } det ≠ 0 (if det bisa dicari)
→ S merentang

→ tiap persamaan dapat dicari basis & dimensinya

Basis:

V adalah Ruang Vektor

S = {**v1**, **v2**, **v3**, …., **vn** } di mana v1, v2, v3, …. , vn $\in$ V

maka S disebut Basis dari V jika

1. **S *linearly independent***
2. **S merupakan rentang (*span*) dari V**

Dimensi

V adalah Ruang Vektor

S = {**v1**, **v2**, **v3**, …., **vn** } basis dari V

Dimensi dari V = n (banyaknya vektor di S)

Basis

$\{\bar{v}_1\ \bar{v}_2 \ldots \bar{v}_n\}$

merentang

$\bar{x} = k_1\bar{v}_1 + k_2\bar{v}_2 + \ldots + k_n\bar{v}_n$

$\bar{x}$ = sembarang vektor

$k_1 \quad k_2 \quad \ldots k_n \rightarrow$ ada nilai-nya

or

Det $\neq 0$

bebas Linier

$\bar{0} = k_1\bar{v}_1 + k_2\bar{v}_2 + \ldots + k_n\bar{v}_n$

$k_1 = k_2 = \ldots = k_n = 0 \rightarrow$ hanya satu jawaban

or

Det $\neq 0$

Soal 4

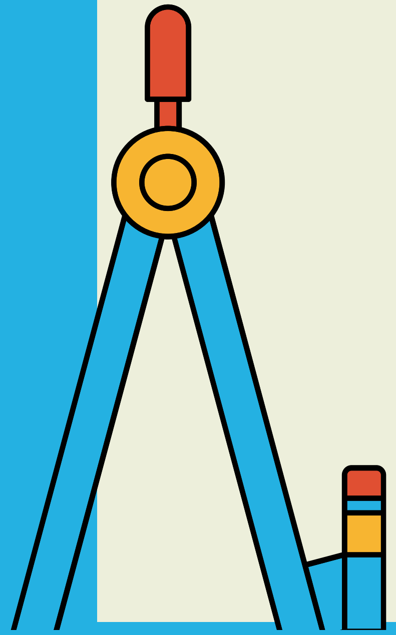
Kerjakan dengan gauss-jordan

contoh : 30

$\bar{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ $\bar{v}_2 = \begin{pmatrix} 2 \\ 9 \\ 0 \end{pmatrix}$ $\bar{v}_3 = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}$

? → basis , buktikan → merentang

→ bebas linier



⇒ merentang

→ merentang jika ada sembarang vektor $\bar{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ yang mrp kombinasi Linier dari $\bar{v_1}$, $\bar{v_2}$ & $\bar{v_3}$

→ $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = k_1 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} 2 \\ 9 \\ 0 \end{pmatrix} + k_3 \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}$

→ $\det(A) = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 9 & 0 \\ 3 & 3 & 4 \end{bmatrix}$

$\det(A) = -1$ ⇒ $\det(A) \neq 0$

→ buktikan u → $\bar{b} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ ①

→ $\bar{b} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$ ②

① $\bar{b} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ → $\begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} = k_1 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} 2 \\ 9 \\ 0 \end{pmatrix} + k_3 \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}$

→ $k_1 = -107$ $k_2 = 15$ $k_3 = 27$

→ ada nilai u k_1, k_2 & k_3

② $\bar{b} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$ ⇒ $\begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} = k_1 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} 2 \\ 9 \\ 0 \end{pmatrix} + k_3 \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}$

→ $k_1 = 85$ $k_2 = -12$ $k_3 = -20$

→ ada nilai u k_1, k_2 & k_3

∴ terbukti bahwa sembarang vektor $\bar{b}$ mrp kombinasi Linier dari $\bar{v_1}$ $\bar{v_2}$ & $\bar{v_3}$

└→ bebas Linier if

$$k_1 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} 2 \\ 9 \\ 0 \end{pmatrix} + k_3 \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

└→ if $k_1 = k_2 = k_3 = 0$, if $\neq$ jawaban lain, mk bebas Linier

→ buktikan

Apakah V1, V2 dan V3 bebas linier ?

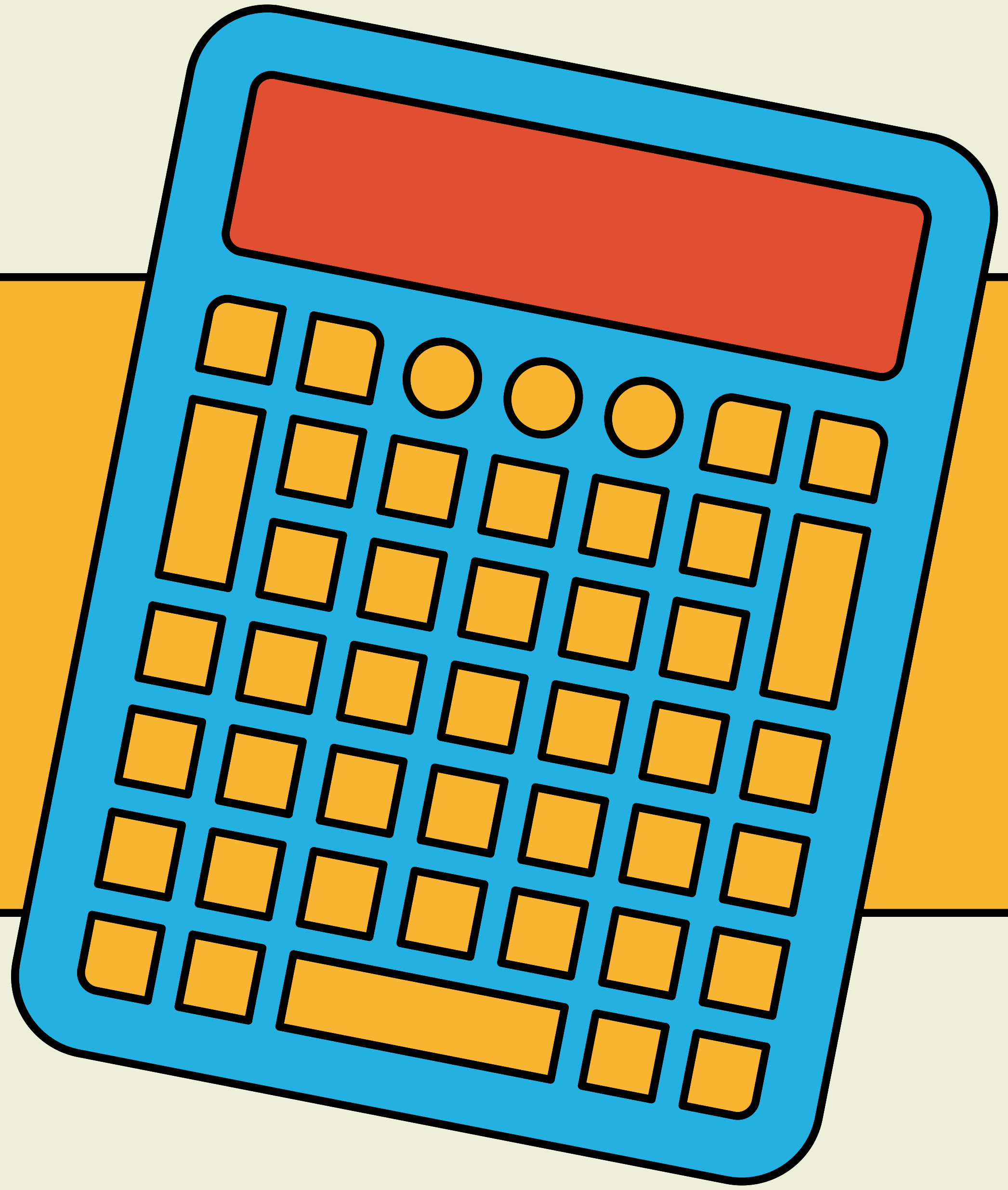
Tugas Kelompok

- Cari 2 soal dan jawaban di internet yang berhubungan dengan materi ppt ini
- Tulis alamat internetnya
- Di kirim ke elearning, terakhir -> **Minggu Depan**

Format -> Subject

- Alin-B-melati
- Bentuk PPT -> informasi nama kelompok + anggota

Any Questions?



THANK YOU