

Ex 9.3

$$T_0 = 2\pi, \quad \omega = \frac{2\pi}{2\pi} = 1$$

Find $f(t)$

$$(0,0), (2\pi, 1)$$

$$x - x_1 \frac{(y_2 - y_1)}{(x_2 - x_1)} = y - y_1$$

$$y - 0 = (x - 0) \left(\frac{1-0}{2\pi-0} \right)$$

$$y = x / 2\pi$$

Replace $xy = f(t)$ by $x = (t)$

$$\text{Then } f(t) = \frac{t}{2\pi}$$

$$D = \frac{1}{T_0} \int_0^{2\pi} f(t) dt$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{2\pi} dt$$

$$= \frac{1}{4\pi^2} \int_0^{2\pi} t dt = \frac{1}{4\pi^2} \left[\frac{t^2}{2} \right]_0^{2\pi}$$

$$= \frac{1}{4\pi^2} \left[\frac{4\pi^2}{2} \right] = 1/2$$

$$\boxed{D = 1/2}$$

$$\begin{aligned}
 D_n &= \frac{1}{T_0} \int_{T_0} x(t) e^{-j\omega_0 n t} dt \\
 &= \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{2\pi} t e^{-j n t} dt \\
 &= \frac{1}{4\pi^2} \int_0^{2\pi} t e^{-j n t} dt
 \end{aligned}$$

$$\int t e^{-at} dt = -\frac{(1+a)t e^{-at}}{a^2}$$

$$a = jn$$

$$\begin{aligned}
 D_n &= \frac{1}{4\pi^2} \left[-\frac{(1+jnt) e^{-jnt}}{(jn)^2} \right]_0^{2\pi} \\
 &= \frac{1}{4\pi^2} \left[-\frac{(1+jnt) e^{-jnt}}{n^2} \right]_0^{2\pi}
 \end{aligned}$$

$$D_n = \frac{1}{4\pi^2 n^2} (1+j2\pi n) e^{-j2\pi n}$$

Then Fourier Series

$$f(t) = D_0 + \sum_{n=1}^{\infty} D_n e^{j n \omega_0 t} + D_n e^{-j n \omega_0 t}$$

$$f(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{(1+j2\pi n) e^{-j2\pi n}}{4\pi^2 n^2} + \frac{(1-j2\pi n) e^{j2\pi n}}{4\pi^2 n^2}$$

$n \leq \pi$

$\leq 1/2$

$1 < t \leq 3/2$

