① Find Eigen value and Date
Bases for Eigen Space
of $A=\begin{bmatrix}-2 & -1\\ 5 & 2\end{bmatrix}$ (5.4.2) (7.1.4)

② diagonalize $\begin{pmatrix}4 & 2\\ 3 & -1\end{pmatrix}$

③ Matrix $\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$ satisfies its Characteristic Eq.

④ Eigen values and Eigen Vectors of $A=\begin{bmatrix}1 & 2\\ 3 & 2\end{bmatrix}$

⑤ $X=\begin{bmatrix}1\\ 2\end{bmatrix}$ → Eigen vector

$A=\begin{bmatrix}3 & 0\\ 8 & -1\end{bmatrix}$ Eigen value.

$$\begin{bmatrix}3 & 0\\ 8 & -1\end{bmatrix}\begin{bmatrix}1\\ 2\end{bmatrix}=\begin{bmatrix}3\\ 6\end{bmatrix}=3\begin{bmatrix}1\\ 2\end{bmatrix}$$

A X $=\lambda X$

Date:

$$|A - \lambda I| = 0$$

$$\lambda I - A = \begin{bmatrix} \lambda - 3 & 0 \\ -8 & \lambda + 1 \end{bmatrix}$$

$$(\lambda I - A)X = 0$$

$$\begin{bmatrix} \lambda - 3 & 0 \\ -8 & \lambda + 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

for $\lambda = 3$

$$\begin{bmatrix} 3-3 & 0 \\ -8 & 3+1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ -8 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ -8x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-8x_1 + 4x_2 = 0$$

$$8x_1 = 4x_2$$

$$2x_1 = x_2$$

Put $x_1 = t$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ 2t \end{bmatrix}$$

$$= t \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Date:

Eigen Vector Corresponding

to $\lambda = 3$, $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

for $\lambda = -1$

$$\begin{bmatrix} -1-3 & 0 \\ -8 & -1+1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 0 \\ -8 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -4x_1 + 0 \\ -8x_1 + 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4x_1 \\ -8x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-4x_1 = 0$ $\quad x_2 = t$

$x_1 = 0$ $\quad x = \begin{bmatrix} 0 \\ t \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Date:

Eigen

$$(\lambda I - A)X = 0$$

$$\left(\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 6 & 3 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 & 2 \\ -1 & \lambda - 2 & -1 \\ -1 & 0 & \lambda - 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

for $\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 2 \\ -1 & 0 & -1 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Date:

$2x_1 + 0x_2 + 2x_3 = 0$

$-x_1 + 0x_2 - x_3 = 0$

$-x_1 + 0x_2 - x_3 = 0$

$-x_1 - x_3 = 0$

$\boxed{x_1 = -x_3}$

Put $x_3 = t$, $x_2 = s$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -t \\ s \\ t \end{bmatrix}$$

$$= \begin{bmatrix} t \\ 0 \\ t \end{bmatrix} + \begin{bmatrix} 0 \\ s \\ 0 \end{bmatrix}$$

$$= t\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore P_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $P_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$,