2.1

Q.1 $A = \begin{bmatrix} 1 & -2 & -3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$

Manner

$A_{11} = \begin{vmatrix} 7 & -1 \\ 1 & 4 \end{vmatrix} = 28+1 = 29$

$A_{12} = \begin{vmatrix} 6 & -1 \\ -3 & 4 \end{vmatrix} = 24-3 = 21$

$A_{13} = \begin{vmatrix} 6 & 7 \\ -3 & 1 \end{vmatrix} = 6+21 = 27$

$A_{21} = \begin{vmatrix} -2 & 3 \\ 1 & 4 \end{vmatrix} = -8-3 = -11$

$A_{22} = \begin{vmatrix} 1 & 3 \\ -3 & 4 \end{vmatrix} = 4+9 = 13$

$A_{23} = \begin{vmatrix} 1 & -2 \\ -3 & 1 \end{vmatrix} = 1-6 = -5$

$A_{31} = \begin{vmatrix} -2 & 3 \\ 7 & -1 \end{vmatrix} = 2-21 = -19$

$A_{32} = \begin{vmatrix} 1 & 3 \\ 6 & -1 \end{vmatrix} = -1-18 = -19$

$A_{33} = \begin{vmatrix} 1 & -2 \\ 6 & 7 \end{vmatrix} = 7+12 = 19$

Coffactors

$A_{11} = (-1)^{1+1} M_{11}$

$A_{11} = (-1)^{1+1} 29 = 29$

$A_{12} = (-1)^{1+2} 21 = -21$

$A_{13} = (-1)^{1+3} 27 = -27$

$A_{21} = (-1)^{2+1} -11 = 11$

$A_{22} = (-1)^{2+2} 13 = 13$

$A_{23} = (-1)^{2+3} -5 = 5$

$A_{31} = (-1)^{3+1} -19 = -19$

$A_{32} = (-1)^{3+2} -19 = +19$

$A_{33} = (-1)^{3+3} 19 = 19$

15. $A = \begin{bmatrix} \lambda-2 & 1 \\ -5 & \lambda+4 \end{bmatrix}$

We know $\det(A) = 0$

$$\begin{vmatrix} \lambda-2 & 1 \\ -5 & \lambda+4 \end{vmatrix} = 0$$

$$(\lambda-2)(\lambda+4) - (-5)(1) = 0$$

$$\lambda^2 + 4\lambda - 2\lambda - 8 + 5 = 0$$

$$\lambda^2 + 2\lambda - 3 = 0$$

$$\lambda^2 + 3\lambda - \lambda - 3 = 0$$

$$\lambda(\lambda+3) - 1(\lambda+3) = 0$$

$$(\lambda-1)(\lambda+3) = 0$$

$\lambda - 1 = 0$ $\qquad$ $\lambda + 3 = 0$

$\boxed{\lambda = 1}$ $\qquad$ $\boxed{\lambda = -3}$

16.) $\begin{bmatrix} \lambda-4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda-1 \end{bmatrix}$

We know that

$$\begin{bmatrix} \lambda-4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda-1 \end{bmatrix} = 0$$

$$\lambda-4 \begin{vmatrix} \lambda & 2 \\ 3 & \lambda-1 \end{vmatrix} + 0 + 0 = 0$$

$$\lambda-4(\lambda(\lambda-1) - 6) = 0$$

$$\lambda-4(\lambda^2 - \lambda - 6) = 0$$

$$\lambda^3 - \lambda^2 - 6\lambda - 4\lambda^2 + 4\lambda + 24 = 0$$

$$\lambda^3 - 5\lambda^2 - 2\lambda + 24 = 0$$

| | | | | |
|---|---|---|---|---|
| | $\lambda^3$ | $-5$ | $-2$ | $24$ |
| 3 | | $3$ | $-6$ | $-24$ |
| | $1$ | $-2$ | $-8$ | $0$ |

$$\lambda^2 - 2\lambda - 8 = 0$$

$$\lambda^2 - 4\lambda + 2\lambda - 8 = 0$$

$$\lambda(\lambda-4) + 2(\lambda-4) = 0$$

$$(\lambda+2)(\lambda-4) = 0$$

$\boxed{\lambda = -2}$ $\quad$ $\boxed{\lambda = 4}$ $\quad$ $\boxed{\lambda = 3}$

Ex 2.2

9. $\begin{bmatrix} 3 & -6 & 9 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix}$

$R_1 \longrightarrow R_1/3$

$\Rightarrow \begin{bmatrix} 1 & -2 & 3 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix}$

$R_2 \longrightarrow R_2 + 2R_1$

$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 3 & 4 \\ 0 & 1 & 5 \end{bmatrix}$

Exchange $R_2$ by $R_3$

$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 3 & 4 \end{bmatrix}$

$R_3 \longrightarrow R_3 - 3R_1$

$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & -11 \end{bmatrix}$

Take determinant

$$\begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & -11 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 5 \\ 0 & -11 \end{vmatrix} - 2\begin{vmatrix} 0 & 5 \\ 0 & -11 \end{vmatrix} + 3\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$$

$$= 1(-11+0)+0+0$$

$$= -11 \text{ Ans.}$$

Q. 10 $\begin{bmatrix} 3 & 6 & -9 \\ 0 & 0 & -2 \\ -2 & 1 & 5 \end{bmatrix}$

$R_1 \longrightarrow R_1/3$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & -2 \\ -2 & 1 & 5 \end{bmatrix}$$

$R_3 \longrightarrow R_3 + 2R_1$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & -2 \\ 0 & 5 & 5-6 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & -2 \\ 0 & 5 & -1 \end{bmatrix}$$

$R_3 \longrightarrow R_3/5$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & -2 \\ 0 & 1 & -1/5 \end{bmatrix}$$

Exchange $R_2$ by $R_3$

$$= \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -1/5 \\ 0 & 0 & -2 \end{bmatrix}$$

determinant

$$= 1\begin{vmatrix} 1 & -1/5 \\ 0 & -2 \end{vmatrix} - 2\begin{vmatrix} 0 & -1/5 \\ 0 & -2 \end{vmatrix} - 3\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$$

$$= 1(-2+0) + 0 + 0$$
$$= -2$$

Q. 12 $\begin{bmatrix} 1 & -3 & 0 \\ -2 & 4 & 1 \\ 5 & -2 & 2 \end{bmatrix}$

$R_2 \rightarrow R_2 + 2R_1$
$R_3 \rightarrow R_3 - 5R_1$

$$= \begin{bmatrix} 1 & -3 & 0 \\ 0 & -2 & 1 \\ 0 & 13 & 2 \end{bmatrix} \quad R_2 \rightarrow R_2/-2$$

$$= \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & -1/2 \\ 0 & 13 & 2 \end{bmatrix}$$

$-13/2$

$R_3 \rightarrow R_3 - 13R_2$

$2 + \frac{13}{2}$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & -1/2 \\ 0 & 0 & 17/2 \end{bmatrix}$$

$\frac{4+13}{2} = \frac{17}{2}$

$2 + \frac{13}{2}$

$\frac{4+13}{2}$

$= 1\begin{vmatrix} 1 & -1/2 \\ 0 & 17/2 \end{vmatrix} + 3\begin{vmatrix} 0 & -1/2 \\ 0 & 17/2 \end{vmatrix} + 0$

$= 17/2$ Ans.

problem

$= \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & -1/2 \\ 0 & 13 & 2 \end{bmatrix}$

$\frac{2}{13} - \frac{1}{2}$

$R_3 \to R_3/13$

$\frac{2}{13} + \frac{1}{2}$

$= \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & -1/2 \\ 0 & 1 & 2/13 \end{bmatrix}$

$\frac{4+13}{26}$

$= \frac{17}{26}$

$R_3 \to R_3 - R_2$

$= \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & -1/2 \\ 0 & 0 & \frac{2}{13} + \frac{1}{2} \end{bmatrix}$

$= \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & -1/2 \\ 0 & 0 & \frac{4+13}{26} \end{bmatrix}$

$\begin{vmatrix} 1 & -3 & 0 \\ 0 & 1 & -1/2 \\ 0 & 0 & 17/26 \end{vmatrix}$

$1\begin{vmatrix} 1 & -1/2 \\ 0 & 17/26 \end{vmatrix} + 0 + 0 \Rightarrow 17/26$

2.2

15
$$\begin{vmatrix} d & e & f \\ g & h & i \\ a & b & c \end{vmatrix}$$

Exchange $R_3$ by $R_2$

$$(-)\begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix}$$

Exchange ~~row~~ $R_2$ by $R_1$

$$(-)(-1)\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$(-)(-1)(-6) = -6$$

17.
$$\begin{vmatrix} 3a & 3b & 3c \\ -d & -e & -f \\ 4g & 4h & 4i \end{vmatrix}$$

$$= 3\begin{vmatrix} a & b & c \\ -d & -e & -f \\ 4g & 4h & 4i \end{vmatrix}$$

$$(-3)\begin{vmatrix} a & b & c \\ d & e & f \\ 4g & 4h & 4i \end{vmatrix} \Rightarrow (4)(-3)\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$-12(-6) = 72$$

19. $\begin{vmatrix} a+g & b+h & c+i \\ d & e & f \\ g & h & i \end{vmatrix}$

$R_1 \longrightarrow R_1 - R_3$

$$= \begin{vmatrix} a+\cancel{g}-\cancel{g} & b+\cancel{h}-\cancel{h} & c+\cancel{i}-\cancel{i} \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -6$$

Exe 2.3

Q.24

$7x_1 - 2x_2 = 3$

$3x_1 + x_2 = 5$

$$A = \begin{bmatrix} 7 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 7 & 3 \\ 3 & 5 \end{bmatrix}$$

$$X_1 = \frac{\det(A_1)}{\det(A)}$$

$$\det(A_1) = \begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix} = 3+10 = 13$$

$$\det(A) = \begin{vmatrix} 7 & -2 \\ 3 & 1 \end{vmatrix} = 7+6 = 13$$

$$X_2 = \frac{\det A_2}{\det(A)}$$

$$\det(A_2) = \begin{vmatrix} 7 & 3 \\ 3 & 5 \end{vmatrix} = 35-9 = 26$$

$$X_1 = \frac{\cancel{13}}{\cancel{13}} = 1 \qquad X_2 = \frac{\cancel{26}\,2}{\cancel{13}}$$

$$\boxed{x_1 = 1} \qquad \boxed{x_2 = 2}$$

25 $4x + 5y + 0 = 2$

$11x + y + 2z = 3$

$x + 5y + 2z = 1$

$$A = \begin{bmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 2 & 5 & 0 \\ 3 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}, \; A_2 = \begin{bmatrix} 4 & 2 & 0 \\ 11 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}, \; A_3 = \begin{bmatrix} 4 & 5 & 2 \\ 4 & 1 & 3 \\ 1 & 5 & 1 \end{bmatrix}$$

$$x = \frac{\det A_1}{\det A} \qquad y = \frac{\det A_2}{\det A} \qquad z = \frac{\det A_3}{\det A}$$

$$\det(A) = \begin{vmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{vmatrix}$$

$$= 4\begin{vmatrix} 1 & 2 \\ 5 & 2 \end{vmatrix} - 5\begin{vmatrix} 11 & 2 \\ 1 & 2 \end{vmatrix} + 0$$

$$= 4(2-10) - 5(22-2) + 0$$

$$= 4(-8) - 5(20)$$

$$= -32 - 100 = \cancel{-124} \; -132$$

$$\det A_1 = \begin{vmatrix} 2 & 5 & 0 \\ 3 & 1 & 2 \\ 1 & 5 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 2 \\ 5 & 2 \end{vmatrix} - 5\begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} + 0$$

$= 2(2-10) - 5(6-2)$

$= 2(-8) - 5(4)$

$= -16 - 20 \Rightarrow -36$

$$\det(A_2) = \begin{vmatrix} 4 & 2 & 0 \\ 11 & 3 & 2 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= 4\begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} 11 & 2 \\ 1 & 2 \end{vmatrix} + 0$$

$= 4(6-2) - 2(22-2) = 16 - 40$

$= -24$

$$\det(A_3) = \begin{vmatrix} 4 & 5 & 2 \\ 11 & 1 & 3 \\ 1 & 5 & 1 \end{vmatrix}$$

$$= 4\begin{vmatrix} 1 & 3 \\ 5 & 1 \end{vmatrix} - 5\begin{vmatrix} 11 & 3 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 11 & 1 \\ 1 & 5 \end{vmatrix}$$

$= 4(1-15) - 5(11-3) + 2(55-1)$

$= -56 - 40 + 108 = 12$

$$x = \frac{\cancel{-36}^{\,3}}{\cancel{-132}_{\,11}} = \frac{3}{11}$$

$$y = \frac{-24}{-132} = \frac{2}{11}$$

$$z = \frac{12}{-132} = \frac{-1}{11}$$