University of Sargodha

BS 2nd Term Examination 2018

Subject: Computer Science **Paper: Discrete Structure (CMP-2211)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (16*2)

i. Construct a truth table for $p \oplus (p \vee q)$.
ii. Define Existential quantifier.
iii. Determine the truth value of following statement if the domain consists of all real numbers $\exists x(x^4 < x^2)$
iv. Define Idempotent Law for sets.
v. Differentiate between proposition and predicate with example.
vi. How many different elements does $A \times B \times C$ have if A has m elements, B has n elements, and C has p elements?
vii. Determine whether the function f from {a, b, c, d} to {1, 2, 3, 4, 5} with f (a) = 4, f (b) = 5, f (c) = 1, and f (d) = 3 is one-to-one.
viii. Let A= {1,2,3,4}, and R is a relation defined by "a divides b". Write R as a set of ordered pair, draw directed graph.
ix. Define the term Halting Problem.
x. How many comparisons are needed for a binary search in a set of 64 elements?
xi. Convert the octal expansion of $(1604)_8$ integers to a binary expansion.
xii. What is pigeonhole principle?
xiii. Define reflexive closure and symmetric closure.
xiv. What is a recurrence relation?
xv. A bag contains 6 white, 5 black and 4 red balls. Find the probability of getting a white or a black ball in a single draw
xvi. What is cyclic graph? Give an example of cyclic graph.

Subjective Part

Attempt any three questions out of five questions (3*16)

Q.2 Prove that following are logically equivalent by developing a series of logically equivalences.

i. $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$
ii. $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$

Q.3 Describe an algorithm based on the linear search for determining the correct position in which to insert a new element in an already sorted list.

Q.4 (a) Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."
(b) Define f: R →R by the rule f(x) = 4x-1 for all x ∈R Is f onto? Prove or give a counter example.

Q.5 Draw a binary search tree by inserting the following numbers from left to right.

11, 6, 8, 19, 4, 10, 5, 17, 43, 49, 31

Determine the order, in which the vertices of the following binary trees will be visited under

i. Preorder traversal. ii. inorder traversal. iii. postorder traversal.

Q.6 In a school, 100 students have access to three software packages, A, B and C 28 did not use any software 8 used only packages A, 26 used only packages B, 7 used only packages C ,10 used all three packages, 13 used both A and B.

i. Draw a Venn diagram with all sets enumerated as for as possible. Label the two subsets which cannot be enumerated as x and y, in any order.
ii. If twice as many students used package B as package A, write down a pair of simultaneous equations in x and y.
iii. Solve these equations to find x and y.
iv. How many students used package C?

BSCS - 2018 DS (CMP - 2211)

Short Answers:

1- Construct a truth table for $p \oplus (p \vee q)$

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

2- Define Existential Quantifier.

A proposition that is true if & only if $P(x)$ is true for at least one value of x in the domain.

→ 1 side must be true. → identify by "$\exists$"

~~Rules~~:-

- **Notation:** $\exists x \, P(x)$

$\exists$ is called exisistential quantifier.

$\exists$ is real as "there exist an element x in domain $P(x)$ is true."

Statement	when is true ?	when is false ?
$\exists x \, P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every/all x.

Example: Let $Q(x)$ denotes "$x = x+1$", what is the truth vallue

The existential quantification $\exists x \, Q(x)$ is false for every real no:s x.

4- Define idempotent law for sets.

Idempotence is the property of certain operations in mathematics that can be applied multiple times without changing the result.

Idempotent law for a set:
Intersection & union of any set with itself revert the same set.

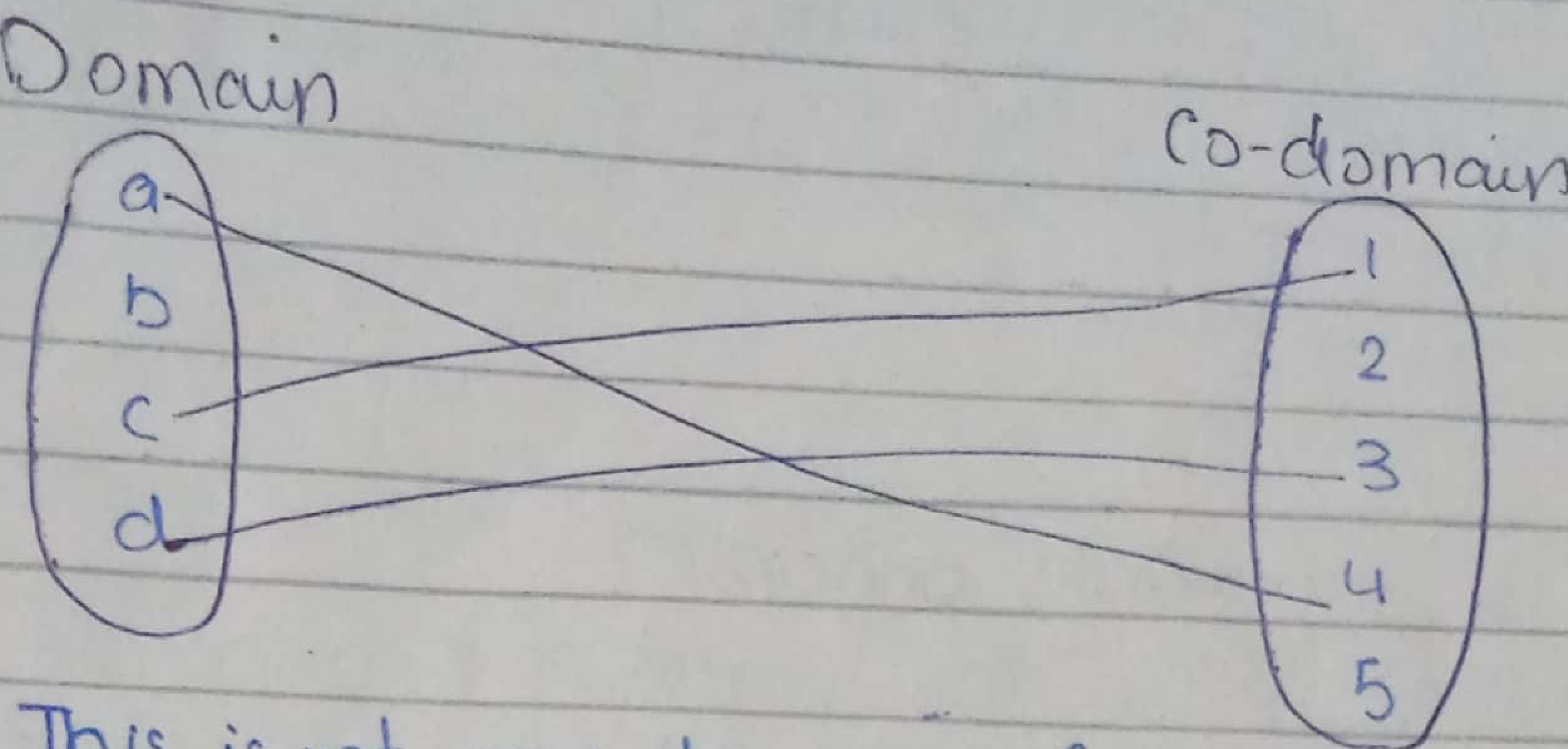
i) $P \wedge T = P$
ii) $P \vee F = P$

P	T	$P \wedge q$
T	T	T
F	T	F

5 Diff. b/w proposition & predicate with example

	Propositional	Predicate
1	It deals with a collection of declarative statements which have a truth value, true or false.	It is an expression consisting of variables with a specified domain. It consists of obj's, relations & functions b/w the obj's.
2	It is the basic & most widely used logic. Also known as Boolean logic	It is an extension of propositional logic covering predicates & quantification
3	It has a specific truth value, either true or false.	It's truth value depends on the variables' value.
4	There should be no command & question in it.	
	Example: $(a+b)^2 = a^2+2ab+b^2$; Sun rises in the east. ~~Sun rises in the west~~; $4+2=6$	Example: $x>3$; $x=4+3$; $x+4=2$

7- Determine whether the function f from {a,b,c,d} to {1,2,3,4,5} with f(a)=4, f(b)=-5, f(c)=1, & f(d) = 3 is one-to-one.



This is not one-to-one function

a-

a- Define the term Halting Problem.

In computability theory, it is the problem of determining, from a description of an arbitrary computer program & an input, whether the program will finish running, or continue to run forever.

The problem of finding out whether, with the given input, a program will halt at some time or continue to run indefinitely

10- How many comparisons are needed for a binary search in a set of 64 elements?

The no. of comparisons needed for a binary search in a set of 64 elements is f(64).

$f(64) = f(32) + 2$

$= f(16) + 2 + 2$

$= f(8) + 2 + 2 + 2$

$= f(4) + 2 + 2 + 2 + 2$

$= f(2) + 2 + 2 + 2 + 2 + 2$

$= f(1) + 2 + 2 + 2 + 2 + 2 + 2$

$= 0 + 2 + 2 + 2 + 2 + 2 + 2 + 2$

$f(16) = 14$

12- what is pigeonhole principle?

In Mathematics, the pigeonhole principle states that if n items are put into m containers, with $n > m$, then at least one container must contain more than one item.

In Discrete Mathematics, the pigeonhole principle states that if we must put N+1 or more pigeons into N Pigeon Holes, then some pigeonholes must contain 2 or more pigeons.

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<u>Example:</u>

If $kn+1$ (where k is a +ve integer) pigeons are distributed among n holes than some hole contains at least $k+1$ pigeons.

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4 13- Define reflexive closure & Symmetric closure.

<u>Reflexive Closure:-</u> of a relation R on A is obtained by adding (a,a) to R for each $a \in A$.

w.r.t, we can obtain closures of relations w.r.t property in the following.

Reflexive Closure – Is the diagonal relation on set

2,6,8,15,11

Symmetric Calosure:-

of R is obtained by adding (b,a) to R for each (a,b) $\in$ R,

w.r.t we can obtain closures of relations w.r.t property in the following.

Symmetric Closure — Let be a relation on set, & let a be the inverse of.

14- What is a recurrence relation?

It is an equation that recursively defines a sequence where the next term is a function of the previous terms.

(Expressing Fn as some Combination of Fi with $i<n$).

Example: {1, 2, 3, 5, 8, 13, 21}

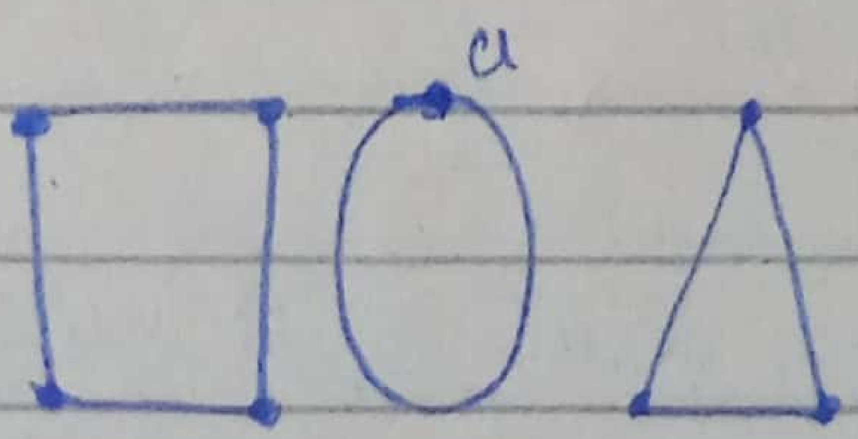
16- What is Cyclic Graph? Give an e.g of cyclic graph.

Cyclic Graph:-

A walk is closed if end points are same. Such graph is called CYCLIC GRAPH

The cycle C_n, $n \geq 3$ consists of n vertices 1, 2 ... n & edges {1,2}, {2,3}... {n-1, n} & {n, 1}.

Example:-



11- Convert the octal expansion of $(1604)_8$ integers to a binary expansion.

1

3- Determine the truth value of following statement if the domain consists of all real no:s

$\exists x (x^4 < x^2)$

False

15- A bag contains 6 white, 5 black & 4 red balls. Find the probability of getting a white or a black ball in a single draw.

White	Black	Red	Total
6	5	4	15

Let A indicate the ball will be white

$n(A) = 6C_1, 5C_0, 4C_0$

$= 6$

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Let B indicate the ball will be black.

$n(B) = 4C_1, 6C_0, 5C_0 = 4$

$n(S) = 15C_1 = 15$

Let D indicate that the ball will be white or black.

$n(D) = n(A) + n(B)$

$$P(D) = P(A) + P(B)$$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)}$$

$$= \frac{6}{15} + \frac{4}{15}$$

$$= \frac{\cancel{10}\,2}{\cancel{15}\,3} \div 5$$

$$= \frac{2}{3}$$