1

* Find Inverse of $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$A^{-1} = \frac{Adj}{|A|}$$

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \Rightarrow Adj\, A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$|A| = (\cos\theta)(\cos\theta) - (\sin\theta)(-\sin\theta)$

$|A| = \cos^2\theta + \sin^2\theta$

$|A| = 1$

$$A^{-1} = \frac{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}{1}$$

Inverse of A is : $A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Answer

Solved by Talha Shahab

* Explain Eigen Values..

If $A$ is $n \times n$ matrix over $\mathbb{R}$ then a Scalar $\lambda \in R$ is Called an "Eigen Values of $A$"

Each eigen values has eigen Vectors
$$[A - \lambda I][x] = 0$$

* What is meant by Linearly independent give example..

Let $v$ be a vector space over a field $F$. The vector $v_1, v_2, v_3, \dots v_m \in V$ are Said to be Linearly independent over $F$ if

$$a_1 v_1 + a_2 v_2 + a_3 v_3 \dots + a_m v_m = 0$$

$i = 1, 2, 3 \dots n$

In this Case $\{v_1, v_2, v_3, \dots v_3\}$ is Linearly Independent Set

each $a_i = 0$

e.g vectors $\{(1,0,0), (0,1,0), (0,0,1)\}$

$$a_1 v_1 + a_2 v_2 + a_3 v_3 = 0$$

$$a_1(1\ 0\ 0) + a_2(0\ 1\ 0) + a_3(0\ 0\ 1) = (0\ 0\ 0)$$

$a_1 + 0 + 0 = 0$ , $a_2 + 0 + 0 = 0$ , $a_3 + 0 + 0 = 0$

$\boxed{a_1 = 0}$ , $\boxed{a_2 = 0}$ , $\boxed{a_3 = 0}$

So. $a_1 v_1 + a_2 v_2 + a_3 v_3 = 0$

$$0 + 0 + 0 = 0$$

$$0 = 0 \quad \text{Satify}$$

Linearly Independent

* Differentiate between Singular Matrix and non Singular Matrix.

→ Singular Matrix

If the determinant of Matrix is equal to Zero than Matrix is Singular

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow |A| = ad - cb = 0$$

Singular

→ Non Singular Matrix

If the determinant of Matrix is not equal to Zero than Matrix is Non Singular

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow |A| = ad - cb \neq 0$$

Non Singular

* Define Hermitian Matrix

Let $A$ be a Matrix if

$$(\bar{A})^t = A$$

then $A$ is Called Hermitian Matrix

* Define Skew Symmetric Matrix

Let '$A$' be a ~~Matrix~~ Square Matrix if

$$A^t = -A$$

then $A$ is Said to be a Skew Symmetric Matrix

* find $M_{12}$, $M_{22}$, $M_{33}$ for the Matrix $A = \begin{bmatrix} 4 & 4 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$M_{12} = \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix}$ . $M_{22} = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$

$M_{33} = \begin{bmatrix} 4 & 4 \\ -1 & 1 \end{bmatrix}$

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* What is trace of a Matrix

The trace of a Matrix is defined to be the Sum of the Diagonal element, from the upper left to the Lower right of the Matrix.

e.g

$$A = \begin{bmatrix} \underline{3} & 6 & 2 & -3 & 0 \\ -2 & \underline{5} & 1 & 0 & 7 \\ 0 & -4 & \underline{-2} & 8 & 6 \\ 7 & 1 & -4 & \underline{9} & 0 \\ 8 & 3 & 7 & 5 & \underline{4} \end{bmatrix}$$

So, trace of Matrix A = ~~3+5+~~

$A = 3+5+(-2)+9+4$

Trace A = 19

* Define Linear Span

The Linear Span is also Called "Linear hull or Just span" of a Set of Vectors in a vector space is the intersection of all Linear Subspaces which each Contain every vector in that Set. The Linear Span of a set of vectors is therefore a Vector Space. Spans

Can be generalized to matroids and modules.

* Solve the augmented Matrix which is reduced to row echelon form.

$$\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 10 & 13 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad R_1 + 3R_2$$

$$\begin{bmatrix} 1 & 0 & 0 & -37 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad R_1 - 10R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & -37 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad R_2 - 2R_3$$

Answer

Find $P(A)$ for $P(x) = x^2 - 2x - 3$ and $A = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$

$P(A) = ?$

$P(x) = x^2 - 2x - 3$

Putting value of A.

$$P\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}^2 - 2\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix} - 3$$

Sol

$$P(A) = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 0 & 6 \end{bmatrix} - 3$$

$$P(A) = \begin{bmatrix} +1+0 & -2+6 \\ 0+0 & 0+9 \end{bmatrix} - \begin{bmatrix} -2-3 & 4-3 \\ 0-3 & 6-3 \end{bmatrix}$$

$$P(A) = \begin{bmatrix} 1 & 4 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} -5 & 1 \\ -3 & 3 \end{bmatrix}$$

$$P(A) = \begin{bmatrix} 1-(-5) & 4-1 \\ 0-3 & 9-3 \end{bmatrix}$$

$$P(A) = \begin{bmatrix} 6 & 3 \\ -3 & 6 \end{bmatrix}$$

Answer

* Write two application areas of eigen Vectors.

1- The Google Page Rank algorithm : The Largest eigenvector of the graph of the internet is how the page are ranked.

2- Low rank factorization for Collaborative prediction : This what Netflix does (or once did) to predict what rating you'll have for a movie you have not yet watched. It uses the SVD, and throws away the smallest eigen values of $A^TA$.

* what is LU Decomposition.

LU Decomposition (where "LU" stand for "lower-upper", and also called LU factorization) factors a matrix as the product of a lower triangular matrix and a upper triangular Matrix.

* Find 'a' and 'b' if

$$\begin{bmatrix} a+3 & 1 \\ -3 & 3b-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

$a+3=2$ , $3b-4=2$

$a=2-3$ , $3b=2+4$

$a=-1$    $3b=6$

$b=6/3$

$b=2$

$\left.\begin{matrix} a=-1 \\ b=2 \end{matrix}\right\}$ Answer.

* $A=\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ , $B=\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ find $AB$ , $BA$

$AB=?$

$$AB=\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$$

$$AB=\begin{bmatrix} 6+3 & 10+4 \\ 3+5 & 5+10 \end{bmatrix}$$

$$AB=\begin{bmatrix} 9 & 14 \\ 8 & 15 \end{bmatrix}$$

$BA = ?$

$$BA = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$$

$$BA = \begin{bmatrix} 6+5 & 9+25 \\ 2+2 & 3+10 \end{bmatrix}$$

$$BA = \begin{bmatrix} 11 & 34 \\ 4 & 13 \end{bmatrix}$$

So, $AB = \begin{bmatrix} 9 & 14 \\ 8 & 15 \end{bmatrix}$ ; $BA = \begin{bmatrix} 11 & 34 \\ 4 & 13 \end{bmatrix}$ Answer

★ How L.U factors can be computed for Rectangular Matrix?

* Verify $det(A) = det(A^t)$

$$A = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 0 & 4 \\ 5 & -3 & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 0 & 4 \\ 5 & -3 & 6 \end{bmatrix}$$

$$det A = |A| = 3\begin{vmatrix} 0 & 4 \\ -3 & 6 \end{vmatrix} - 1\begin{vmatrix} 1 & 4 \\ 5 & 6 \end{vmatrix} - 2\begin{vmatrix} 1 & 0 \\ 5 & -3 \end{vmatrix}$$

$det A = 3(0+12) - 1(6-20) - 2(-3+0)$

$det A = 3(12) - 1(-14) - 2(-3)$

$det A = 36 + 14 + 6$

$\boxed{det A = 56}$

$$A = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 0 & 4 \\ 5 & -3 & 6 \end{bmatrix} \Rightarrow A^t \begin{bmatrix} 3 & 1 & 5 \\ 1 & 0 & -3 \\ -2 & 4 & 6 \end{bmatrix}$$

$$det(A^t) = |A|^t = 3\begin{vmatrix} 0 & -3 \\ 4 & 6 \end{vmatrix} - 1\begin{vmatrix} 1 & -3 \\ -2 & 6 \end{vmatrix} + 5\begin{vmatrix} 1 & 0 \\ -2 & 4 \end{vmatrix}$$

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$= 3(0+12) - 1(6-6) + 5(4+0)$

$= 3(12) - 0 + 5(4)$

$= 36 + 20$

$\det(A)^{T} = 56$

$\det(A) = \det(A)^{T}$

$56 = 56$ Prove

* find $A^{-1}$, if $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{bmatrix}$

$A^{-1} = \frac{\text{Adj} A}{|A|}$

$\Rightarrow$ Adj A

Cofactor of $A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

$C = (-1)^{i+j} M_{ij}$

☆ $C_{11} = (-1)^{1+1} M_{11}$

$= (-1)^2 \begin{vmatrix} 1 & 0 \\ -3 & 5 \end{vmatrix}$

$= (1)(15)$

$\boxed{C_{11} = 15}$

☆ $C_{12} = (-1)^{1+2} M_{12}$

$= (-1)^3 \begin{vmatrix} 1 & 0 \\ 2 & 5 \end{vmatrix}$

$= (-1)^3 (5) \Rightarrow -5$ $\boxed{C_{12} = -5}$

☆ $C_{13} = (-1)^{1+3} M_{13}$

$= (-1)^4 \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix}$

$= 1(-3-2)$

$\boxed{C_{13} = -5}$

$\cancel{C_{14} = (-1)^{1+4} M_{14}}$

$\cancel{= (-1)^5}$

☆ $C_{21} = (-1)^{2+1} M_{21}$

$C_{21} = (-1)^3 \begin{vmatrix} 1 & 0 \\ -3 & 5 \end{vmatrix}$

$C_{21} = (-1)(5)$

$\boxed{C_{21} = -5}$

☆ $C_{22} = (-1)^{2+2} M_{22}$

$= (-1)^4 \begin{vmatrix} 2 & 0 \\ 2 & 5 \end{vmatrix}$

$C_{22} = 1(10)$

$\boxed{C_{22} = 10}$

☆ $C_{23} = (-1)^{2+3} M_{23}$

$= (-1)^5 \begin{vmatrix} 2 & 1 \\ 2 & -3 \end{vmatrix}$

$= (-1)(-6-2)$

$= (-1)(-8)$

$\boxed{C_{23} = 8}$

☆ $C_{31} = (-1)^{3+1} M_{31}$

$= (-1)^4 \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}$

$= (-1)^4 (0)$

$\boxed{C_{31} = 0}$

☆ $C_{32} = (-1)^{3+2} M_{32}$

$= (-1)^5 \begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix}$

$= (-1)^5 (0)$

$\boxed{C_{32} = 0}$

☆ $C_{33} = (-1)^{3+3} M_{33}$

$= (-1)^6 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$

$= (1)(2-1)$

$= 1(1)$

$\boxed{C_{33} = 1}$

$$A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

putting values

$$A = \begin{bmatrix} 15 & -5 & -5 \\ -5 & 10 & 8 \\ 0 & 0 & 1 \end{bmatrix}$$

Taking Transpose

$$Adj\,A = \begin{bmatrix} 15 & -5 & 0 \\ -5 & 10 & 0 \\ -5 & 8 & 1 \end{bmatrix}$$

$|A| = ?$

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{bmatrix}$$

$$|A| = 2\begin{vmatrix} 1 & 0 \\ -3 & 5 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} + 0$$

$|A| = 2(5) - 1(-3-2)$

$|A| = 10 - 1(-5)$

$|A| = 10 + 5$

$|A| = 15$

$$A^{-1} = \frac{Adj\,A}{|A|}$$

$$A^{-1} = \frac{\begin{bmatrix} 15 & -5 & 0 \\ 5 & 10 & 0 \\ -5 & 8 & 1 \end{bmatrix}}{15}$$

$$A^{-1} = \begin{bmatrix} 15/15 & -5/15 & 0/15 \\ 5/15 & 10/15 & 0/15 \\ -5/15 & 8/15 & 1/15 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & \frac{-1}{3} & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{-1}{3} & \frac{8}{15} & \frac{1}{15} \end{bmatrix}$$

Answer

Find LU-Decomposition of $A = \begin{bmatrix} 4 & 4 & 0 \\ 8 & 6 & 2 \\ -4 & -10 & 8 \end{bmatrix}$

$$L = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix}, \quad U = \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = LU$$

$$\begin{bmatrix} 4 & 4 & 0 \\ 8 & 6 & 2 \\ -4 & -10 & 8 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 4 & 0 \\ 8 & 6 & 2 \\ -4 & -10 & 8 \end{bmatrix} = \begin{bmatrix} L_{11}+0+0 & L_{11}U_{12}+0+0 & L_{11}U_{13}+0+0 \\ L_{21}+0+0 & L_{21}U_{12}+L_{22}+0 & L_{21}U_{13}+L_{22}U_{23}+0 \\ L_{31}+0+0 & L_{31}U_{12}+L_{32}+0 & L_{31}U_{13}+L_{32}U_{23}+L_{33} \end{bmatrix}$$

$$\begin{bmatrix} 4 & 4 & 0 \\ 8 & 6 & 2 \\ -4 & -10 & 8 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{11}U_{12} & L_{11}U_{13} \\ L_{21} & L_{21}U_{12}+L_{22} & L_{21}U_{13}+L_{22}U_{23} \\ L_{31} & L_{31}U_{12}+L_{32} & L_{31}U_{13}+L_{32}U_{23}+L_{33} \end{bmatrix}$$

Here, $\underline{L_{11} = 4}$, $\underline{L_{21} = 8}$, $\underline{L_{31} = -4}$

$L_{11}U_{12} = 4 \rightarrow (1)$ , $L_{31}U_{12} + L_{32} = -10 \rightarrow (3)$

$L_{21}U_{12} + L_{22} = 6 \rightarrow (2)$ , $L_{11}U_{13} = 0 \rightarrow (4)$

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$L_{21} U_{13} + L_{22} U_{23} = 2 \longrightarrow (5)$
$L_{31} U_{13} + L_{32} U_{23} + L_{33} = 8 \longrightarrow (6)$

→ from eq 1

$L_{11} U_{12} = 4$
$4U_{12} = 4$
$\underline{U_{12} = 1}$

→ from eq 2

$L_{21} U_{12} + L_{22} = 6$
$(8)(1) + L_{22} = 6$
$8 + L_{22} = 6$
$L_{22} = 6 - 8$
$\underline{L_{22} = -2}$

→ from eq 3

$L_{31} U_{12} + L_{32} = -10$
$(-4)(1) + L_{32} = -10$
$-4 + L_{32} = -10$
$L_{32} = -10 + 4$
$\underline{L_{32} = -6}$

→ from eq 4

$L_{11} U_{13} = 0$
$4U_{13} = 0$
$\underline{U_{13} = 0}$

→ from eq 5

$L_{21} U_{13} + L_{22} U_{23} = 2$
$(8)(0) + (-2) U_{23} = 2$
$-2U_{23} = 2$
$\underline{U_{23} = -1}$

→ from eq 6

$L_{31} U_{13} + L_{32} U_{23} + L_{33} = 8$
$(-4)(0) + (-6)(-1) + L_{33} = 8$
$0 + 6 + L_{33} = 8$
$L_{33} = 8 - 6$
$\underline{L_{33} = 2}$

So, LU Decomposition is

$$\begin{bmatrix} 4 & 4 & 0 \\ 8 & 6 & 2 \\ -4 & -10 & 8 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 8 & -2 & 0 \\ -4 & -6 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

* For what value of 'a' is the vector $(a^2, a, 1)$ in span $\{(1,2,3), (1,1,1), (0,1,2)\}$

$$V = a_1V_1 + a_2V_2 + a_3V_3$$

$$(a^2, a, 1) = a_1(1, 2, 3) + a_2(1, 1, 1) + a_3(0, 1, 2)$$

$$(a^2, a, 1) = (a_1, 2a_1, 3a_1) + (a_2, a_2, a_2) + (0, a_3, 2a_3)$$

$$a_1 + a_2 = a^2 \rightarrow eq\ 1$$

$$2a_1 + a_2 + a_3 \longrightarrow eq\ 2$$

$$3a_1 + a_2 + 2a_3 = 1 \rightarrow eq\ 3$$

x by 3 in eq 1 and subtract eq 3

$$3(a_1 + a_2 = a^2) \Rightarrow \quad 3a_1 + 3a_2 = 3a^2$$

$$\cancel{3}a_1 + a_2 + 2a_3 = 1$$

$$2a_2 - 2a_3 = 3a^2 - 1 \rightarrow eq\ 4$$

x by 3 in eq 2, x by 2 in eq 3 and Subtract

$$3(2a_1 + a_2 + a_3 = a) \Rightarrow \cancel{6a_1} + 3a_2 + 3a_3 = 3a$$

$$2(3a_1 + a_2 + 2a_3 = 1) \Rightarrow \cancel{6a_1} + 2a_2 + 4a_3 = +2$$

$$a_2 - a_3 = 3a - 2 \Rightarrow eq\ 5$$

eq(2) and subtract from eq(1)

$a^2 + 3a - 2) \Rightarrow 2a_1 - 2a_1 + 6a - 4$

$2a_1 - 2a_1 - 3a^2 - 1$

$2a_1 - 2a_1 - 6a - 4$

$0 = 3a^2 - 6a - 1 + 4$

$3a^2 - 6a + 3 = 0$

$3a^2 - 6a = -3$

~~3(a^2 - 2a)~~

$3(a^2 - 2a) = -3$

$a^2 - 2a = \frac{-3}{3}$

$a^2 - 2a = -1$

$-2a = -1 - a^2$

$-2a = -(1 + a^2)$

$2a = 1 + a^2$

$a = \left(\frac{1 + a^2}{2}\right)$ Answer

$2(a_1 - a$

* For what value of '$\lambda$' is set

$\{t+3,\ 2t+\lambda^2+2\}$

Linear Independent.

$$V = a_1V_1 + a_2V_2 + a_3V_3 + \cdots$$

$$(0,0,0) = a_1(t+3) + a_2(2t+\lambda^2+2)$$

$$(0,0,0) = (a_1t + 3a_1) + (2a_2t + a_2\lambda^2 + 2a_2)$$

$$a_1t + 2a_2t = 0 \rightarrow (1)$$

$$3a_1 + a_2\lambda^2 = 0 \rightarrow (2)$$

$$2a_2 = 0 \rightarrow (3)$$

$$\Downarrow$$

$$\boxed{a_2 = 0}$$

put in eq (1)

$$a_1t + 2a_2t = 0$$

$$a_1t + 2(0)t = 0$$

$$a_1t = 0$$

$$\boxed{a_1 = 0}$$

Putting Values in eq (2)

$$3a_1 + a_2\lambda^2 = 0$$

$$\cancel{3(0)+(0)} \quad 3a_1 = -a_2\lambda^2$$

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$$-\frac{3a_1}{a_0} = \lambda^2$$

$$\lambda^2 = \frac{-3(0)}{(0)}$$

$$\lambda^2 = 0$$

So, $\boxed{\lambda = 0}$ Answer

Define Transpose of a Matrix.

Let $A = [a_{ij}]$ be an $m \times n$ matrix over a field F. The transpose of A denoted by $A^T$, is an $n \times n$ matrix obtain by interchanging row and columns of A. Thus $A^T = [b_{ij}]$ where $b_{ij} = a_{ji}$. In other words $(i,j)$th element of $A^T = (j,i)$th element of A.

Express the Matrix equation as a system of linear equation.

$$\begin{bmatrix} 5 & 6 & -7 \\ -1 & -2 & 3 \\ 0 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$

$$5x_1 + 6x_2 - 7x_3 = 2$$

$$-x_1 - 2x_2 + 3x_3 = 0$$

$$+4x_2 - x_3 = 3$$

Solve the matrix equation ~~as a system~~ for a, b, c, d.

$$\begin{bmatrix} 3 & a \\ 1 & a+b \end{bmatrix} = \begin{bmatrix} b & c-2d \\ c+2d & 0 \end{bmatrix}$$

find a, b, c, d = ?

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$$\begin{bmatrix} 3 & a \\ 1 & a+b \end{bmatrix} = \begin{bmatrix} b & c-2d \\ c+2d & 0 \end{bmatrix}$$

$\boxed{3 = b} \rightarrow (1)$

$a = c - 2d \rightarrow (2)$

$1 = c + 2d \rightarrow (3)$

$a + b = 0 \rightarrow (4)$

put $b = 3$ in eq (4)

$a + b = 0$

$a + 3 = 0$

$\boxed{a = -3}$

put $a = -3$ in eq (2)

$a = c - 2d$

$-3 = c - 2d$

$-3 + 2d = c \rightarrow$ eq (5)

eq 5 put in eq (3)

$1 = c + 2d$

$\cancel{1 = (-3)}$

$1 = -3 + 2d + 2d$

$1 = -3 + 4d$

$1 + 3 = 4d \Rightarrow \cancel{\frac{3}{4} = d}$ $\quad 4 = 4d \; \boxed{d = 1}$

value of d put in eq 5

$-3 + 2d = c$

$-3 + 2(1) = c$

$-3 + 2 = c$

$\boxed{c = -1}$

So,

$a = -3$

$b = 3$

$c = -1$

$d = 1$

Answer

Prove that $(AB)^{-1} = B^{-1}A^{-1}$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

Let $(AB)^{-1} = ?$

$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 3+0 & 1+4 \\ 9+0 & 3+8 \end{bmatrix}$

$AB = \begin{bmatrix} 3 & 5 \\ 9 & 11 \end{bmatrix}$

$(AB)^{-1} = \frac{Adj[AB]}{|AB}$

$Adj\,AB = \begin{bmatrix} 11 & -5 \\ -9 & 3 \end{bmatrix}$

$|AB| = (3)(11) - 45$

$|AB| = 33 - 45$

$|AB| = -12$

$$|AB|^{-1} = \frac{\begin{bmatrix} 11 & -5 \\ -9 & 3 \end{bmatrix}}{-12}$$

$$|AB|^{-1} = \begin{bmatrix} \frac{-11}{12} & \frac{-5}{-12} \\ -9/_{12} & 3/_{12} \end{bmatrix}$$

$$|AB|^{-1} = \begin{bmatrix} \frac{-11}{12} & \frac{5}{12} \\ \frac{3}{4} & -\frac{1}{4} \end{bmatrix}$$

<u>For $B^{-1} \cdot A^{-1}$</u>

$B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ . $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\Downarrow$ $\Downarrow$

$B^{-1} \Rightarrow$ $B^{-1} = \frac{adj\,B}{|B|}$

$B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \Rightarrow Adj^B = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$

$|B| = (6)$

$B^{-1} = \dfrac{\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}}{6}$

$B^{-1} = \begin{bmatrix} 2/6 & -1/6 \\ 0/6 & 3/6 \end{bmatrix}$

$B^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{-1}{6} \\ 0 & \frac{1}{2} \end{bmatrix}$

$A^{-1} \Rightarrow A^{-1} = \dfrac{adj\, A}{|A|}$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow adj\, A \cdot \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$|A| = (4 - 6)$

$|A| = (-2)$

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$$A^{-1} = \frac{\operatorname{adj} A}{|A|}$$

$$A^{-1} = \frac{\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}}{-2}$$

$$A^{-1} = \begin{bmatrix} 4/-2 & -2/-2 \\ -3/-2 & 1/-2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{+3}{+2} & \frac{1}{-2} \end{bmatrix}$$

So,

$$B^{-1}A^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{-1}{6} \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -2 & 1 \\ +3/2 & -1/2 \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} -\frac{2}{3} + \left(\frac{-1}{6}\right)\left(\frac{3}{2}\right) & \left(\frac{1}{3}\right) + \left(\frac{-1}{6}\right)\left(\frac{-1}{2}\right) \\ 0 + \left(\frac{1}{2}\right)\left(\frac{3}{2}\right) & 0 + \left(\frac{1}{2}\right)\left(\frac{-1}{2}\right) \end{bmatrix}$$

$$\begin{bmatrix} -\frac{2}{3} - \frac{3}{12} & \frac{1}{3} - \frac{1}{12} \\ \frac{3}{4} & \frac{-1}{4} \end{bmatrix} = \begin{bmatrix} \frac{-11}{12} & \frac{5}{12} \\ \frac{3}{4} & \frac{-1}{4} \end{bmatrix}$$

Prove

Is given Matrix Symmetric:

$$\begin{bmatrix} 2 & 3a & 0 \\ 3a & 1 & b \\ 0 & b & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3a & 0 \\ 3a & 1 & b \\ 0 & b & 0 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 2 & 3a & 0 \\ 3a & 1 & b \\ 0 & b & 0 \end{bmatrix}$$

So, $A^t = A$ So, This Matrix is Symmetric

Find $\lambda$ for the Matrix $A = \begin{bmatrix} \lambda-4 & 4 & 0 \\ -1 & \lambda & 0 \\ 0 & 0 & \lambda-5 \end{bmatrix}$

$$|A| = \lambda-4 \begin{vmatrix} \lambda & 0 \\ 0 & \lambda-5 \end{vmatrix} - 4 \begin{vmatrix} -1 & 0 \\ 0 & \lambda-5 \end{vmatrix} = 0$$

$$= (\lambda-4)(\lambda(\lambda-5)) - 4(-1(\lambda-5)) = 0$$

$\lambda - 4 (\lambda^2 - 5\lambda) - 4(-\lambda + 5) = 0$

$\lambda(\lambda^2 - 5\lambda) - 4(\lambda^2 - 5\lambda) - 4(-\lambda + 5) = 0$

$\lambda^3 - 5\lambda^2 - 4\lambda^2 + 20\lambda + 4\lambda - 20 = 0$

$\lambda^3 - 9\lambda^2 + 24\lambda - 20 = 0$

| | | | | |
|---|---|---|---|---|
| 2 | 1 | -9 | 24 | -20 |
| | ↓ | 2 | -14 | 20 |
| | 1 | -7 | 10 | 0 |

$\lambda = 2$ , $\lambda^2 - 7\lambda + 10 = 0$

$\lambda^2 - 5\lambda - 2\lambda + 10 = 0$

$\lambda(\lambda - 5) - 2(\lambda - 5) = 0$

$(\lambda - 5)(\lambda - 2) = 0$

$(\lambda - 5) = 0 \quad \lambda - 2 = 0$

$\lambda = 5 , \quad \lambda = 2$

So, $\lambda = 2, 2, 5$ Answer

Evaluate determinant.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$|A| = 1 \begin{vmatrix} 2 & 2 & 2 \\ 0 & 3 & 3 \\ 0 & 0 & 4 \end{vmatrix} - 0 + 0 - 0$$

$$|A| = 1(2 \begin{vmatrix} 3 & 3 \\ 0 & 4 \end{vmatrix} - 0 + 0$$

$$|A| = 2(12)$$

$$|A| = \underline{\underline{24}} \Rightarrow \text{Determinant}$$

If $U = (5, -1, 2)$ find Norm of $u$.

$$\|U\| = \sqrt{(5)^2 + (-1)^2 + (2)^2} \Rightarrow \sqrt{25 + 1 + 4}$$

$$\|U\| = \sqrt{30} \Rightarrow \text{Norm of } u.$$

find angle between U and v, where
$U = (2, 0, 1, -2)$
$v = (1, 5, -3, 2)$

~~$\cos\theta = U$~~

$U \cdot V = |U||V| \cos\theta$.

$$\cos\theta = \frac{U \cdot V}{|U||V|}$$

$U \cdot V = (2, 0, 1, -2)(1, 5, -3, 2)$

$U \cdot V = 2 + 0 - 3 - 4$

$U \cdot V = -5$

$U = (2, 0, 1, -2)$

$|U| = \sqrt{(2)^2 + (0)^2 + (1)^2 + (-2)^2}$

$|U| = \sqrt{4 + 1 + 4}$

$|U| = \sqrt{9}$

$|U| = 3$

$|V| = (1, 5, -3, 2)$

$|V| = \sqrt{(1)^2 + (5)^2 + (-3)^2 + (2)^2}$

$|V| = \sqrt{1 + 25 + 9 + 4}$

$= \sqrt{39}$

$$\cos\theta = \frac{U.V}{|U||V|}$$

$$\cos\theta = \frac{-5}{3\sqrt{39}}$$

$$\theta = \cos^{-1}\left(\frac{-5}{3\sqrt{39}}\right)$$

$$\theta = \cos^{-1}(0.266880)$$

$$\theta = 74.52129$$ Answer

+ Define Symmetric Matrix

Let A be a Matrix if $A^t = A$ Then A is Said to be Symmetric Matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad A^t \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \Rightarrow A = A^t$$

* What is an invertible Matrix

An n-by-n square Matrix A is Called Invertible if there exists an n-by-n Square matrix B Such that where I denotes the n-by-n identity matrix and the multiplication used is Ordinary Matrix Multiplication.

The Invertible is also "Non-Singular or Nondegenerate"

$AB = BA = I_n$

Not Invertible Matrix

→ A Square Matrix that is Not Invertible is Called "Singular or Degenerate"

* What is LU Decomposition

LU Demposition (where "LU" Stand for "Lower - Upper" and also Called L-U factorization) factors a matrix as the Product of a Lower triangular matrix and a upper triangular matrix

* Determine whether A is invertible or not if $A = \begin{bmatrix} 2 & 0 & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 3 \end{bmatrix}$

Taking Determinant

$|A| = 2\begin{vmatrix} 3 & 3 \\ 4 & 3 \end{vmatrix} - 0\begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} + 3\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}$

$|A| = 2(9-12) - 0 + 3(4-6)$

$|A| = 2(-3) + 3(-2)$

$|A| = -6 - 6$

$|A| = -12 \neq 0$

This Matrix is Not invertible because it is Not Singular

* Find a Non-Zero Vector $u$ with terminal point $P(-1, 3, -5)$ Such that $u$ has the Same direction as $v = (6, 7, -3$

Solve the linear System of equation by Guass - Jordan elimination.

$$x_1 + 2x_2 - 3x_3 = 6$$
$$2x_1 - x_2 + 4x_3 = 1$$
$$x_1 - x_2 + x_3 = 3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 2 & -1 & 4 & 1 \\ 1 & -1 & 1 & 3 \end{array}\right] \begin{array}{l} R_2 - 2R_1 \\ R_3 - R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -3 & 6 \\ 0 & 3 & 2 & 11 \\ 0 & 1 & 4 & -3 \end{array}\right] R_1 + 2R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 3 & 2 & 11 \\ 0 & 1 & 4 & -3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & -6 & 17 \\ 0 & 1 & 4 & -3 \end{array}\right] R_2 - 2R_3$$

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$$\left[\begin{array}{ccc;c} 1 & 0 & 5 & 0 \\ 0 & 1 & -6 & 17 \\ 0 & 0 & 10 & 20 \end{array}\right] R_3 - R_2$$

$$\left[\begin{array}{ccc;c} 1 & 0 & 5 & 0 \\ 0 & 1 & -6 & 17 \\ 0 & 0 & 1 & 2 \end{array}\right] \frac{1}{10} R_3$$

$$\left[\begin{array}{ccc;c} 1 & 0 & 0 & -10 \\ 0 & 1 & -6 & 17 \\ 0 & 0 & 1 & 2 \end{array}\right] R_1 - 5R_3$$

$$\left[\begin{array}{ccc;c} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & 17 \\ 0 & 0 & 1 & 2 \end{array}\right] R_2 + 6R_3$$

So, ~~$x = -10$, $y = 17$, $z = 2$~~

$x_1 = -10$, $x_2 = 17$, $x_3 = 2$

Answer

find eigen values and eigen vectors

$$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

$$[A - \lambda I] = 0$$

$$A - \lambda I = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 3-\lambda & 0 \\ 8 & -1-\lambda \end{bmatrix}$$

Taking determinant

$$\cancel{3-\lambda} \left[ (3-\lambda)(-1-\lambda) - (0)(8) \right] = 0$$

$$[3(-1-\lambda) - \lambda(-1-\lambda) - 0] = 0$$

$$[-3 - 3\lambda + \lambda + \lambda^2] = 0$$

$$[\lambda^2 - 2\lambda - 3] = 0$$

$$[\cancel{\lambda^2 + 3\lambda - \lambda - 3}] = 0$$

$$(\lambda^2 - 3\lambda + \lambda - 3) = 0$$

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$\lambda(\lambda-3)+1(\lambda-3)=0$

$(\lambda+1)(\lambda-3)=0$

$\lambda+1=0 \qquad \lambda-3=0$

$\lambda=-1 \quad , \quad \lambda=3$

So, Eigen values are $\lambda=-1, 3$

~~For $\lambda=$~~ $-1$

Each eigen value has eigen vector.

$[A-\lambda I][x]=0$

$$\begin{bmatrix} 3-\lambda & 0 \\ 8 & -1-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For $\lambda=-1$

$$\begin{bmatrix} 3-(-1) & 0 \\ 8 & -1-(-1) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3+1 & 0 \\ 8 & -1+1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 \\ 8 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

So, $4x = 0$

$8y = 0$

So, $x = 0$ eigen vectors

$y = 0$

$4x + 0 = 0$

$8x + 0 = 0$

$\boxed{x = 0}$

Find Minors and Cofactors.

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$$

Cofactors $= ?$ $C_{ij} = (-1)^{i+j} M_{ij}$

$C_{11} = (-1)^{1+1} M_{11}$

$= (-1)^2 \begin{vmatrix} 7 & -1 \\ 1 & 4 \end{vmatrix}$

$= (1)(28 - (-1))$

$= (28 + 1)$

$\boxed{C_{11} = 29}$

$C_{12} = (-1)^{1+2} M_{12}$

$= (-1)^3 \begin{vmatrix} 6 & -1 \\ -3 & 4 \end{vmatrix}$

$= (-1)(24 - 3)$

$= (-1)(21)$

$\boxed{C_{12} = -21}$

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$$C_{13} = (-1)^{1+3} M_{13}$$
$$= (-1)^4 \begin{vmatrix} 6 & 7 \\ -3 & 1 \end{vmatrix}$$
$$= (1)(6+21)$$
$$\boxed{C_{13} = (27)}$$

$$C_{21} = (-1)^{2+1} M_{21}$$
$$= (-1)^3 \begin{vmatrix} -2 & 3 \\ 1 & 4 \end{vmatrix}$$
$$= (-1)(-8-3)$$
$$= (-1)(-11)$$
$$\boxed{C_{21} = 11}$$

$$C_{22} = (-1)^{2+2} M_{22}$$
$$= (-1)^4 \begin{vmatrix} 1 & 3 \\ -3 & 4 \end{vmatrix}$$
$$= (+1)(4+9)$$
$$\boxed{C_{22} = 13}$$

$$C_{23} = (-1)^{2+3} M_{23}$$
$$= (-1)^5 \begin{vmatrix} 1 & -2 \\ -3 & 1 \end{vmatrix}$$
$$= (-1)(1-6)$$

$= (-1)(-5)$

$\boxed{C_{23} = 5}$

$C_{31} = (-1)^{3+1} M_{31}$

$= (-1)^4 \begin{vmatrix} -2 & 3 \\ 7 & -1 \end{vmatrix}$

$= (1)(2 - 21)$

$\boxed{C_{31} = -19}$

$C_{32} = (-1)^{3+2} M_{32}$

$= (-1)^5 M_{32}$

$= (-1) \begin{vmatrix} 1 & 3 \\ 6 & -1 \end{vmatrix}$

$= (-1)(-1 - 18)$

$= (-1)(-19)$

$\boxed{C_{32} = 19}$

$C_{33} = (-1)^{3+3} M_{33}$

$= (-1)^6 \begin{vmatrix} 1 & -2 \\ 6 & 7 \end{vmatrix}$

$= 1(7 + 12)$

$\boxed{C_{33} = 19}$

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Minors are.

$M_{11} = \begin{bmatrix} 7 & -1 \\ 1 & 4 \end{bmatrix}$ , $M_{21} = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}$

$M_{12} = \begin{bmatrix} 6 & -1 \\ -3 & 4 \end{bmatrix}$ , $M_{22} = \begin{bmatrix} 1 & 3 \\ -3 & 4 \end{bmatrix}$

$M_{13} = \begin{bmatrix} 6 & 7 \\ -3 & 1 \end{bmatrix}$ , $M_{23} = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$

$M_{31} = \begin{bmatrix} -2 & 3 \\ 7 & -1 \end{bmatrix}$ . $M_{32} = \begin{bmatrix} 1 & 3 \\ 6 & -1 \end{bmatrix}$

$M_{33} = \begin{bmatrix} 1 & -2 \\ 6 & 7 \end{bmatrix}$

Cofactor are

$C_{11} = 29$ , $C_{12} = -21$ , $C_{13} = 27$

$C_{21} = 11$ , $C_{22} = 13$ , $C_{23} = 5$

$C_{31} = -19$ , $C_{32} = 19$ , $C_{33} = 19$

* check wheter A is Singular or not =?

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

its Determinent is zero because has one Column and one row is zero

So, $|A| = 0$

And it is a Singular Matrix because $|A| = 0$

* find the angle between U and V where $U = (1, -5, 1)$, $V = (0, 0, -1)$

$$U \cdot V = |U||V| \cos\theta$$

$$\cos\theta = \frac{U \cdot V}{|U||V|}$$

$$U \cdot V = (1, -5, 1)(0, 0, -1)$$

$$U \cdot V = (0 + 0 + (-1)) \Rightarrow U \cdot V = -1$$

$$|U| = \sqrt{(1)^2 + (-5)^2 + (1)^2} \Rightarrow \sqrt{1 + 25 + 1} = \sqrt{27}$$

$$|V| = \sqrt{(0)^2 + (0)^2 + (-1)^2} \Rightarrow \sqrt{1} = 1$$

$$\cos\theta = \frac{(0, 0, -1)}{5.1966}$$

$$\cos\theta = \frac{U \cdot V}{|U||V|}$$

$$\cos\theta = \frac{-1}{\sqrt{27}}$$

$$\theta = \cos^{-1}(0.19245)$$

$$\theta = 78.90419 \quad \text{Ans}$$

* Define Eigen Vector

If $A$ is $n \times n$ matrix over $\mathbb{R}$ then a Scalar $\lambda \in \mathbb{R}$ is Called an "Eigen values of $A$" If there exist a Non Zero Column vector $v \in \mathbb{R}^n$, Such that $Av = \lambda v$. In this Case $v$ is Called "eigen vector of $A$" Corresponding to the "eigen value $\lambda$"

$$Av = \lambda v$$

$$Av - \lambda I v = 0$$

$$(A - \lambda I)v = 0$$

* If $A = \begin{bmatrix} 2 & 3a & 0 \\ 3a & 1 & b \\ 0 & b & 0 \end{bmatrix}$ find $C_{21}$

$$C_{21} = (-1)^{2+1} M_{21}$$

$$C_{21} = (-1)^{3} \begin{vmatrix} 3a & 0 \\ b & 0 \end{vmatrix}$$

$$C_{21} = (-1)(0)$$

$$\boxed{C_{21} = 0}$$ Answer

* Find Similar Matrices

OR

What are Similar Matrix

If A and B are two Matrices over R. Then A is Said to be Similar to B.

1) If there exist a Non-Singular Matrix $P \Rightarrow |P| \neq 0$

2) If $B = P^{-1}AP$

* What is Reduced Echelon form of A Matrix

The Conditions of Reduced Echelon form is

1) First Non-Zero element of each row Should be 1
2) All elements under this 1 should be Zero
3) All element above leading entry (1) Should be Zero

e.g $A = \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 5 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

* Prove $(AB)^T = B^T A^T$

Suppose $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$

Let $(AB)^T = ?$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2+6 & 1+0 \\ 6+12 & 3+0 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 18 & 3 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 8 & 18 \\ 1 & 3 \end{bmatrix}$$

Let $B^T A^T = ?$

$$B = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} \Rightarrow B^T = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 2+6 & 6+12 \\ 1+0 & 3+0 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 8 & 18 \\ 1 & 3 \end{bmatrix}$$

So, Prove

$$(AB)^T = B^T A^T$$

$$\begin{bmatrix} 8 & 18 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 18 \\ 1 & 3 \end{bmatrix}$$

Prove

* 17 $U = (5, -1, 2)$ find Norm of u.

$$\|U\| = \sqrt{(5)^2 + (-1)^2 + (2)^2}$$

$$\|U\| = \sqrt{25+1+4}$$

$$\|U\| = \sqrt{30}$$ Norm of u.

* Is given Matrix Symmetric?

$$\begin{bmatrix} 2 & 3a & 0 \\ 3a & 1 & b \\ 0 & b & 0 \end{bmatrix}$$

Let

$$A = \begin{bmatrix} 2 & 3a & 0 \\ 3a & 1 & b \\ 0 & b & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 3a & 0 \\ 3a & 1 & b \\ 0 & b & 0 \end{bmatrix}$$

So,

$A^T = A$ So, given Matrix is Symmetric

* Find $\lambda$ for the Matrix $A = \begin{bmatrix} -4 & 4 & 0 \\ -1 & \lambda & \lambda-0 \\ 0 & 0 & \lambda-5 \end{bmatrix}$ then $A = 0$

$$|A| = -4 \begin{vmatrix} \lambda & \lambda-0 \\ 0 & \lambda-5 \end{vmatrix} - 4 \begin{vmatrix} -1 & \lambda-0 \\ 0 & \lambda-5 \end{vmatrix} + 0 = 0$$

$$= -4(\lambda(\lambda-5)-0) - 4(-1(\lambda-5)-0) = 0$$

$$-4(\lambda^2 - 5\lambda - 0) - 4(-\lambda+5) = 0$$

$-4\lambda^2 + 20\lambda + 4\lambda - 20 = 0$

$-4\lambda^2 + 24\lambda - 20 = 0$

$4\lambda^2 - 24\lambda + 20 = 0$

$4(\lambda^2 - 6\lambda + 5) = 0$

$\lambda^2 - 6\lambda + 5 = 0$

$\lambda^2 - 5\lambda - \lambda + 5 = 0$

$\lambda(\lambda - 5) - 1(\lambda - 5) = 0$

$(\lambda - 1)(\lambda - 5) = 0$

$\lambda - 1 = 0 \quad , \quad \lambda - 5 = 0$

$\lambda = 1 \quad , \quad \lambda = 5$

So, $(\lambda = 1, 5)$ Answer

* Give example of Augmented Matrix

Def:- An augmented matrix for a System of equations is a matrix of Numbers in which each row representated the Constants from one equation (both the coefficients and the Constant on the other side of the equal sign) and each Column represents all the coefficients for a Single Variable.

e.g

$$x - 2y + 3z = 7$$
$$2x + y + z = 4$$
$$-3x + 2y - 2z = -10$$

Augmented Matrix is

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & +7 \\ 2 & 1 & 1 & 4 \\ -3 & 2 & -2 & -10 \end{array}\right]$$

e.g

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 5 & 2 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

Augmented Matrix is

$$(A|B) = \left[\begin{array}{ccc|c} 1 & 3 & 2 & 4 \\ 2 & 0 & 1 & 3 \\ 5 & 2 & 2 & 1 \end{array}\right]$$

* Let $A$ be the Matrix $\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$
find $P(A)$
where $P(x) = x^3 - 2x - 4$

$$P(x) = x^3 - 2x - 4$$

$$P\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}^3 - 2\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} - 4$$

$$P(A) = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 8 & 2 \end{bmatrix} - 4$$

$$P(A) = \begin{bmatrix} 4+0 & 0+0 \\ 8+4 & 0+1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 4-4 & 0-4 \\ 8-4 & 2-4 \end{bmatrix}$$

$$P(A) = \begin{bmatrix} 4 & 0 \\ 12 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -4 \\ 4 & -2 \end{bmatrix}$$

$$P(A) = \begin{bmatrix} 8+0 & 0+0 \\ 24+4 & 0+1 \end{bmatrix} - \begin{bmatrix} 0 & -4 \\ 4 & -2 \end{bmatrix}$$

$$P(A) = \begin{bmatrix} 8 & 0 \\ 28 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -4 \\ 4 & -2 \end{bmatrix}$$

$$P(A) = \begin{bmatrix} 8-0 & 0+4 \\ 28-4 & 1+2 \end{bmatrix} \Rightarrow P(A) = \begin{bmatrix} 8 & 4 \\ 24 & 3 \end{bmatrix}$$

Answer

For what value of a is

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 3 \\ 0 & 1 & a \end{bmatrix} + \begin{bmatrix} 0 & a & 1 \\ 1 & 3a & 0 \\ -2 & a & 2 \end{bmatrix} = 14$$

$$\begin{bmatrix} 2+0 & 1+a & 0+1 \\ 0+1 & -1+3a & 3+0 \\ 0-2 & 1+a & a+2 \end{bmatrix} = 14$$

$$\begin{bmatrix} 2 & 1+a & 1 \\ 1 & 3a-1 & 3 \\ -2 & 1+a & a+2 \end{bmatrix} = 14$$

$\rightarrow$ $1+a = 14$ , $\rightarrow$ $3a-1 = 14$ $\rightarrow$ $1+a = 14$

$a = 14-1$ , $3a = 14+1$ $a = 13$

$a = 13$ , $3a = 15$

$a = 5$

$\rightarrow$ $a+2 = 14$

$a = 14-2$

$a = 12$

So, Value of 'a' is

13, 5, 13, 12

* What are Different types of Distributions..

Distribution is one of the four elements of the marketing mix. Distribution is the Process of making a Product or Service available for the Consumer or business users that needs it. This can be done directly by the Producer or Service Provider or using indirect channels with Distributors or intermediaries

There are three types of Distribution

1- Intensive Distribution
2- Selective Distribution
3- Exclusive Distribution

Intensive Distribution:-

Intensive Distribution aims to Provide Saturation Coverage of the market by using all available outlets. for many Products, Total Sales are directly linked to the number of outlets Used (e.g... Cigarettes, beer). Intensive Distribution is usually required where Customers have a range of acceptable brands to choose from. In other words, if one brand is not available a Customer with will Simply choose another

## Selective Distribution.

Selective Distribution involves a producer using a limited no. of outlets in a geographical area to sell products. An advantages of this approach is that the producer can choose the most appropriate or best performing outlets and focus effort (e.g. Training) on them. Selective Distribution works best when consumers are prepared to "shop around" - in other words - they have a preference for a particular brand or price and will search out the outlets that supply.

## Exclusive Distribution.

Exclusive Distribution is an extreme form of Selective Distribution in which only one wholesaler, retailer or Distributor is used in a specific geographical area.

When the firm Distributes its brand through just one or two major outlets in the Market, who exclusively deal in it and Not all Competing brands. it is Said that the firm is using an exclusive Distribution strategy. This is Common form of Distribution in Products and brands that seek a high, Prestigious image.

* What is Characteristics Equation.

The Characteristics equation is the equation which is solved to find a matrix's eigen values, also Called the "Characteristic ~~Equation~~ Polynomial." For a general $K \times K$ Matrix $A$, the Characteristics equation in variable $\lambda$ is defined by

$$\det(A - \lambda I) = 0$$

* What are Dependent vectors give example

Solve the linear System of equation by Gaussian Elimination.

$$x_1 - 2x_2 - 3x_3 = 6$$
$$2x_1 - x_2 - 4x_3 = 1$$
$$x_1 - x_2 + x_3 = 3$$

$$\left[\begin{array}{ccc:c} 1 & -2 & -3 & 6 \\ 2 & -1 & -4 & 1 \\ 1 & -1 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & -2 & -3 & 6 \\ 0 & 1 & -6 & -5 \\ 1 & -1 & +1 & 3 \end{array}\right] \quad R_2 - 2R_1$$

$$\left[\begin{array}{ccc:c} 1 & -2 & -3 & 6 \\ 0 & 1 & -6 & -5 \\ 0 & 1 & 4 & -3 \end{array}\right] \quad R_3 - R_1$$

$$\left[\begin{array}{ccc:c} 1 & -2 & -3 & 6 \\ 0 & 1 & -6 & -5 \\ 0 & 0 & 10 & 2 \end{array}\right] \quad R_3 - R_2$$

$x - 2y - 3z = 6 \rightarrow (1)$

$y - 6z = -5 \rightarrow (2)$

$10z = 2 \rightarrow (3)$

from eq (3)

$$10z = 2$$

$$z = \frac{\cancel{2}\,1}{\cancel{10}\,5}$$

$$\boxed{z = \frac{1}{5}}$$

Put in eq (2)

$$y - 6z = -5$$

$$y - 6\left(\frac{1}{5}\right) = -5$$

$$y - \frac{6}{5} = -5$$

$$y = \frac{6}{5} - 5$$

$$y = \frac{6 - 25}{5}$$

$$\boxed{y = \frac{19}{5}}$$

All values put in eq ①

$$x - 2y - 3z = 6$$

$$x - 2\left(\frac{19}{5}\right) - 3\left(\frac{1}{5}\right) = 6$$

$$x - \frac{38}{5} - \frac{3}{5} = 6$$

$$x = 6 + \frac{38}{5} + \frac{3}{5}$$

$$x = \frac{30 + 38 + 3}{5}$$

$$x = \frac{71}{5}$$

So, $\left( x = \frac{71}{5}, \; y = \frac{19}{5}, \; z = \frac{1}{5} \right)$

Answer

For checking values is True

$$x - 2y - 3z = 6$$

$$\frac{71}{5} - 2\left(\frac{19}{5}\right) - 3\left(\frac{1}{5}\right) = 6$$

$$\frac{71 - 38 - 3}{5} = 6$$

$$\frac{30}{5} = 6 \qquad 6 = 6 \text{ prove}$$

**Solved by Talha Shahab**

Find all the Minors and Co factors of given Matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 3 & 6 \\ 0 & 1 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 3 & 6 \\ 0 & 1 & 4 \end{bmatrix}$$

★ $C_{11} = (-1)^{1+1} M_{11}$

$= (-1)^2 \begin{vmatrix} 3 & 6 \\ 1 & 4 \end{vmatrix}$

$= (1)(12-6)$

$\boxed{C_{11} = 6}$

★ $C_{12} = (-1)^{1+2} M_{12}$

$= (-1)^3 \begin{vmatrix} 3 & 6 \\ 0 & 4 \end{vmatrix}$

$= (-1)(12)$

$\boxed{C_{12} = -12}$

★ $C_{13} = (-1)^{1+3} M_{13}$

$= (-1)^4 \begin{vmatrix} 3 & 3 \\ 0 & 1 \end{vmatrix}$

$= (1)(3)$

$\boxed{C_{13} = 3}$

★ $C_{21} = (-1)^{2+1} M_{21}$

$C_{21} = (-1)^3 M_{21}$

$= (-1) \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix}$

$= (-1)(4-2)$

$= (-1)(2)$

$\boxed{C_{21} = -2}$

Solved by Talha Shahab

☆ $C_{22} = (-1)^{2+2} M_{22}$

$= (-1)^4 \begin{vmatrix} 3 & 6 \\ 0 & 4 \end{vmatrix}$

$= (1)(12)$

$\boxed{C_{22} = 12}$

☆ $C_{22} = (-1)^{2+2} M_{22}$

$C_{22} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix}$

$C_{22} = (1)(4)$

$\boxed{C_{22} = 4}$

$C_{23} = (-1)^{2+3} M_{23}$

☆ $C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$

$= (-1)^5 (1)$

$\boxed{C_{23} = -1}$

☆ $C_{33} = (-1)^{3+3} M_{33}$

$= (-1)^6 \begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix}$

$= (+1)(3-3)$

$\boxed{C_{33} = 0}$

☆ $C_{31} = (-1)^{3+1} M_{31}$

$= (-1)^4 \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix}$

$= (1)(6 - 6)$

$\boxed{C_{31} = 0}$

★ $C_{32} = (-1)^{3+2} M_{32}$

$= (-1)^5 \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix}$

$= (-1)(6 - 6)$

$\boxed{C_{32} = 0}$

So, Cofactors are $C_{11} = 6$

$C_{12} = -12$, $C_{21} = -2$, $C_{23} = -1$

$C_{13} = 3$, $C_{22} = 4$, $C_{31} = 0$

$C_{32} = 0$, $C_{33} = 0$

So, Minors are

$M_{11} = \begin{bmatrix} 3 & 6 \\ 1 & 4 \end{bmatrix}$, $M_{12} = \begin{bmatrix} 3 & 6 \\ 0 & 4 \end{bmatrix}$

$M_{13} = \begin{bmatrix} 3 & 3 \\ 0 & 1 \end{bmatrix}$, $M_{21} = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$

$$M_{22} = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$$

$$M_{23} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$M_{31} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$$

$$M_{32} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$$

$$M_{33} = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$$

Express $(6, 11, 6)$ as Linear Combination of $u = (2, 1, 4)$, $v = (1, -1, 3)$ and $w = (3, 2, 5)$

* find characteristics equation and eigen values of $A = \begin{bmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 19 & 5 & -4 \end{bmatrix}$

Characteristic Equation of A

$$|A - \lambda I| = 0$$

$$A - \lambda I = \begin{bmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 19 & 5 & -4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 19 & 5 & -4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} -2-\lambda & 0 & 1 \\ -6 & -2-\lambda & 0 \\ 19 & 5 & -4-\lambda \end{bmatrix}$$

Taking Determinant

$$-2-\lambda \begin{vmatrix} -2-\lambda & 0 \\ 5 & -4-\lambda \end{vmatrix} - 0 + 1 \begin{vmatrix} -6 & -2-\lambda \\ 19 & 5 \end{vmatrix} = 0$$

$$(-2-\lambda)\big((-2-\lambda)(-4-\lambda) - 0\big) + 1\big((-6)(5) - (-2-\lambda)(19)\big) = 0$$

Solved by Talha Shahab

$$-2-\lambda[-2(-4-\lambda)-\lambda(-4-\lambda)]+1(-30-(-38-19\lambda)]=0$$

$$-2-\lambda[8+2\lambda+4\lambda+\lambda^2]+1[-30+38+19\lambda]=0$$

$$-2-\lambda[\lambda^2+6\lambda+8]+1[19\lambda+8]=0$$

$$-2\cancel{\lambda}[\lambda^2+6\lambda+8]-\lambda[\lambda^2+6\lambda+8]+1[19\lambda+8]=0$$

$$-2\lambda^2-12\lambda-16-\lambda^3-6\lambda^2-8\lambda+19\lambda+8=0$$

$$-\lambda^3-8\lambda^2-\lambda-8=0$$

$$\lambda^3+8\lambda^2+\lambda+8=0$$

$$\lambda^2(\lambda+8)+1(\lambda+8)=0$$

$$(\lambda^2+1)(\lambda+8)=0$$

$\lambda^2+1=0$ , $\lambda+8=0$

$(\lambda)^2+(1)^2=0$ , $\lambda=-8$

$(\lambda+1)(\lambda+1)=0$ ,

$\lambda=-1,-1$

$\lambda=-1,-1,-8$ Answer

Eigen Values