(iii) charactarstic Root method:

Example:
$$a_n - 6a_{n-1} + 8a_{n-2} = 0$$
$$a_0 = 4, \quad a_1 = 10.$$

STEP-1: write char Equ^n.

STEP-2: Find roots and let roots be $r_1, r_2, \cdots, r_k$.

Solve: Put $a_n = r^n$.

$$r^n - 6r^{n-1} + 8r^{n-2} = 0.$$

Divide B.S by r^{n-2}

STEP-3: (a) If all roots are diff. then G.S is:

$$r^2 - 6r + 8 = 0.$$
$$r^2 - 4r - 2r + 8 = 0$$
$$r(r-4) - 2(r-4) = 0$$
$$(r-4)(r-2) = 0$$

$$a_n = c_1 r_1^n + c_2 r_2^n + \cdots + c_k r_k^n$$

(b) If two of its roots are same then G.S is:

$$r - 4 = 0; \quad r - 2 = 0$$
$$r = 4; \quad r = 2.$$

$$a_n = (c_1 + c_2 n) r_1^n + c_3 r_2^n + \cdots + c_k r_k^n.$$

$$r = 2, 4.$$

$$\boxed{a_n = c_1 2^n + c_2 4^n} \rightarrow ①$$
G.S.

put $n = 0$. in ①:
$$a_0 = c_1 2^0 + c_2 4^0$$
$$a_0 = c_1 + c_2$$
$$\boxed{4 = c_1 + c_2} \rightarrow ②$$

put $n = 1$ in ①:
$$a_1 = c_1 2^1 + c_2 4^1$$
$$10 = 2c_1 + 4c_2$$
Divide by 2 on B.S

$$\boxed{5 = c_1 + 2c_2} \rightarrow ③$$

Eq ③ - ②.
$$\boxed{1 = c_2} \quad \text{put in ②}$$

$$4 = c_1 + 1$$
$$\boxed{c_1 = 3}$$

put c_1 & c_2 in ①:
$$\boxed{a_n = 3 \cdot 2^n + 1 \cdot 4^n}$$