Linear Algebra

Q25

$$A = \begin{bmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{bmatrix}$$

Solution:-

$$|A| = \begin{vmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{vmatrix}$$

Expand by R_1

$$= 3\begin{vmatrix} 3 & 0 & -2 \\ 1 & -3 & 0 \\ 10 & 3 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & 0 & -2 \\ 4 & -3 & 0 \\ 2 & 3 & 2 \end{vmatrix} + 0\begin{vmatrix} 2 & 2 & -2 \\ 4 & 1 & 0 \\ 2 & 10 & 2 \end{vmatrix} - 5\begin{vmatrix} 2 & 2 & 0 \\ 4 & 1 & -3 \\ 2 & 10 & 3 \end{vmatrix}$$

$$= 3\left(2\begin{vmatrix} -3 & 0 \\ 3 & 2 \end{vmatrix} - 0\begin{vmatrix} 1 & 0 \\ 10 & 2 \end{vmatrix} - 2\begin{vmatrix} 1 & -3 \\ 10 & 3 \end{vmatrix}\right) - 3\left(2\begin{vmatrix} -3 & 0 \\ 3 & 2 \end{vmatrix} - 0\begin{vmatrix} 4 & 0 \\ 2 & 2 \end{vmatrix} - 2\begin{vmatrix} 4 & -3 \\ 2 & 3 \end{vmatrix}\right) + 0 5\left(2\begin{vmatrix} 1 & -3 \\ 10 & 3 \end{vmatrix} - 2\begin{vmatrix} 4 & -3 \\ 2 & 3 \end{vmatrix} + 0\begin{vmatrix} 4 & 1 \\ 2 & 10 \end{vmatrix}\right)$$

$$= 3[2(-6-0) - 0(2-0) - 2(3+30)] - 3[2(-6-0) - 0(8-0) - 2(12+$$

$$-5[2(3+30) - 2(12+6) + 0(40-2)]$$

$$= 3(-12-0-66) - 3(-12-0-36) - 5[66-36+0]$$

$$= 3(-78) - 3(-48) - 5(30)$$

$$= -234 + 144 - 150$$

$$= -240 \text{ Ans.}$$

Algebra
Chapter #2
Ex 2.1

Determinants:-

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible

if and only if $ad - bc \neq 0$ and that the expression $ad-bc$ is called the determinant of matrix A. Written as

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$
$$= ad - bc$$

Minor of entry a_{ij} :-

If A is a square matrix, then the minor of entry a_{ij} is denoted by M_{ij} and is defined to be the determinant of the submatrix that remain after the ith row & jth column are delete from A

Cofactor of entry a_{ij} :- (Include Minor def in th

The number $(-1)^{i+j} M_{ij}$ is denoted by C_{ij} and is called the cofactor of entry a_{ij}

Question 1 to 4 same.

Find the minor and cofactor of matrix A

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$$

vivo Y20 2021

Q18

$$A = \begin{bmatrix} \lambda-1 & 0 \\ 2 & \lambda+1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} \lambda-1 & 0 \\ 2 & \lambda+1 \end{vmatrix}$$

$$= [(\lambda-1)(\lambda+1)] - (0-2)$$
$$= (\lambda^2+\lambda-\lambda-1) - (-2)$$
$$= (\lambda^2-1) + (2)$$
$$= \lambda^2-1+2$$
$$|A| = \lambda^2+1$$

according to given condition $\det(A)=0$

$$0 = \lambda^2+1$$
$$-1 = \lambda^2$$
$$\boxed{\lambda = \sqrt{-1}}$$

Q19 & 20 (same)

Evalute the determinant in Q13 by a cofactor expansion along (a) & (b) (First row & First column)

$$\begin{vmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{vmatrix}$$

Sol:- let

$$|A| = \begin{vmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{vmatrix}$$

Expansion by R_1

2022.09.16 10:09

$$= 1 \begin{vmatrix} k & k^2 \\ k & k^2 \end{vmatrix} - k \begin{vmatrix} 1 & k^2 \\ 1 & k^2 \end{vmatrix} + k^2 \begin{vmatrix} 1 & k \\ 1 & k \end{vmatrix}$$

$$= 1(k^3 - k^3) - k(k^2 - k^2) + k^2(k - k)$$

$$= 1(0) - k(0) + k^2(0)$$

$$= 0 - 0 + 0$$

$$\boxed{|A| = 0}$$

Q24

$$A = \begin{bmatrix} k+1 & k-1 & 7 \\ 2 & k-3 & 4 \\ 5 & k+1 & k \end{bmatrix}$$

Soln

$$|A| = \begin{vmatrix} k+1 & k-1 & 7 \\ 2 & k-3 & 4 \\ 5 & k+1 & k \end{vmatrix}$$

Expand by C_1

$$= k+1 \begin{vmatrix} k-3 & 4 \\ k+1 & k \end{vmatrix} - 2 \begin{vmatrix} k-1 & 7 \\ k+1 & k \end{vmatrix} + 5 \begin{vmatrix} k-1 & 7 \\ k-3 & 4 \end{vmatrix}$$

$$= k+1(k^2 - 3k - 4k - 4) - 2(k^2 - k - 7k - 7) + 5(4k-$$

$$= k+1(k^2 - 7k - 4) - 2(k^2 - 8k - 7) + 5(3k + 17)$$

$$= (k^3 - 7k^2 - 4k + k^2 - 7k - 4) - (2k^2 - 16k - 14) + (-15k + 8$$

$$= k^3 - 6k^2 - 11k - 4 - 2k^2 + 16k + 14 + 85 - 15k$$

$$\boxed{|A| = k^3 - 8k^2 - 10k + 95}$$

Scanned with CamScanner

Q9 to 14 (Same)

$$\begin{vmatrix} a-3 & 5 \\ -3 & a-2 \end{vmatrix}$$

Soln

let

$$A = \begin{vmatrix} a-3 & 5 \\ -3 & a-2 \end{vmatrix}$$

$$|A| = \begin{vmatrix} a-3 & 5 \\ -3 & a-2 \end{vmatrix}$$

Also write in this way

$$\begin{vmatrix} a-3 & 5 \\ -3 & a-2 \end{vmatrix} = \begin{vmatrix} a-3 & 5 \\ -3 & a-2 \end{vmatrix}$$

then

$$= [(a-3)(a-2)] - (5 \times -3)$$

$$= (a^2 - 2a - 3a + 6) - (-15)$$

$$= a^2 - 5a + 6 + 15$$

$$|A| = a^2 - 5a + 21$$

Q11

$$\begin{bmatrix} -2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{bmatrix}$$

Soln

$$\text{let } A = \begin{bmatrix} -2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{bmatrix}$$

$$|A| = \left|\begin{array}{ccc|cc} -2 & 1 & 4 & -2 & 1 \\ 3 & 5 & -7 & 3 & 5 \\ 1 & 6 & 2 & 1 & 6 \end{array}\right|$$

$$[20 - 7 + 72] - [20 + 84 + 6]$$

Q3

$$A = \begin{bmatrix} 4 & -1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{bmatrix}$$

(a)

M_{13} & C_{13}

Sol:-

$$M_{13} = \begin{vmatrix} 4 & -1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & -3 \\ 4 & 1 & 0 \\ 4 & 1 & 3 \end{vmatrix}$$

By R_1 expansion

$$= 0 \begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} - 0 \begin{vmatrix} 4 & 0 \\ 4 & 3 \end{vmatrix} - 3 \begin{vmatrix} 4 & 1 \\ 4 & 1 \end{vmatrix}$$

$$= 0(3-0) - 0(12-0) - 3(4-4)$$

$$= 0(3) - 0(12) - 3(0)$$

$$= 0 - 0 - 0$$

$$\boxed{M_{13} = 0}$$

The C_{13}

$$C_{13} = (-1)^{1+3} M_{13}$$

$$= (-1)^4 (0)$$

$$= (1)(0)$$

$$\boxed{C_{13} = 0}$$

Linear Algebra

Ex 1.1

Q3 (c)

two equation in four unknown

Sol:-

$$2x_1 + 3x_2 + 4x_3 + 5x_4 = 1$$
$$3x_1 + 4x_2 + 5x_3 + 5x_4 = 2$$

Question no 4

Augmented Matrix:-

(a)

Sol:-

$$\begin{bmatrix} a_{11} & a_{12} & : b_1 \\ a_{21} & a_{22} & : b_2 \end{bmatrix}$$

(b)

Sol:-

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & : b_1 \\ a_{21} & a_{22} & a_{23} & : b_2 \\ a_{31} & a_{32} & a_{33} & : b_3 \end{bmatrix}$$

Question no 5 & 6 (Same)

Find a linear system in the unknown $x_1, x_2, x_3, ----$, that corresponds to given augmented matrix

Q5 (a)

$$\begin{bmatrix} 2 & 0 & 0 \\ 3 & -4 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Linear Algebra

Ex 1.1

$$6x - 9y = 3$$
$$-6x - 9y = 3$$
$$0 = 0$$

Already ~~given~~ mentioned Linear system has infinite solution So we use parametric equation

$$2x - 3y = 1$$
$$2x = 1 + 3y$$
$$x = \frac{1}{2} + \frac{3}{2}y$$

let

$$t = y$$

$$\boxed{x = \frac{1}{2} + \frac{3}{2}t}$$

Q16

(b)

$$2x - y + 2z = -4$$
$$6x - 3y + 6z = -12$$
$$-4x + 2y - 4z = 8$$

Sol:- Already mentioned system has infinite solution directly find Parametric equation or check solution

Multiply by 2 eq(i)

$$\cancel{4x} - \cancel{2y} + \cancel{4z} = -\cancel{8}$$
$$6x - 3y + 6z = -12$$
$$\cancel{-4x} + \cancel{2y} - \cancel{4z} = \cancel{8}$$

$$6x - 3y + 6z = -12$$

Alternative

Pick one equation

$$2x - y + 2z = -4$$
$$6x - 3y + 6z = -12$$

Multiply by 3 eq(i)

$$6x - 3y + 6z = -12$$
$$-6x + 3y - 6z = +12$$

$$0 = 0$$

18 (b)

$$\begin{bmatrix} 7 & -4 & -2 & 2 \\ 3 & -1 & 8 & 1 \\ -6 & 3 & -1 & 4 \end{bmatrix}$$

Sol:-

$R_1 \to R_1 + R_3$

$$= \begin{bmatrix} 7-6 & -4+3 & -2-1 & 2+4 \\ 3 & -1 & 8 & 1 \\ -6 & 3 & -1 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -3 & 6 \\ 3 & -1 & 8 & 1 \\ -6 & 3 & -1 & 4 \end{bmatrix}$$

Q19 to 20

$$\begin{bmatrix} 1 & k & -4 \\ 4 & 8 & 2 \end{bmatrix}$$

Sol:-

$x_1 + kx_2 = -4$

$4x_1 + 8x_2 = 2$

Multiply by 4 eq (i)

$4x_1 + 4kx_2 = -16$

$-4x_1 \pm 8x_2 = -2$

$4kx_2 - 8x_2 = -18$

$4x_2(k-2) = -18$

Q11 (b)

$2x - 4y = 1$ —— (i)

$4x - 8y = 2$ —— (ii)

Sol:-

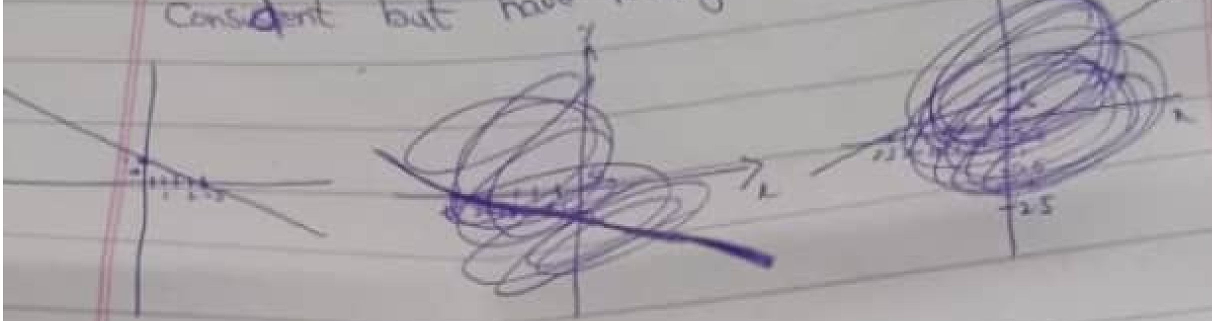
Multiply by 2 (eq (i))

$\cancel{4x} - \cancel{8y} = \cancel{2}$

$-\cancel{4x} \mp \cancel{8y} = \cancel{2}$

$0 = 0$

Consistent but have many solution.



$4x - 8y = 2$

let

$y = t$

$4x - 8t = 2$

$4x = 2 + 8t$

$x = \frac{2}{4} + \frac{8}{4}t$

$\boxed{x = \frac{1}{2} + 2t}$ ⟶ End

Extra For graph

when $t = 1$

$x = \frac{1}{2} + 2$

$\boxed{x = \frac{5}{2}}$ or $\boxed{x = 2.5}$

Put in eq (ii)

$\cancel{2}4\left(\frac{5}{\cancel{2}}\right) - 8y = 2$

$10 - 8y = 2$

$-8y = -8$

$\boxed{y = 1}$

Question 2

(a)

$2^{1/3}x + \sqrt{3}y = 1$

Sol:-

The given equation is a Linear Equation

(c)

$\cos\left(\frac{\pi}{7}\right)x - 4y = \log 3$

Sol:-

The given equation is non-linear Because in linear equation Trignometric and Logarithmic function is not used.

(e)

$xy = 1$

Sol:-

The given equation is non linear Because in linear equation product of two variable with each other not involved or allowed.

Linear System:-

A finite set of Linear Equations is called system of Linear Equation or Linear System

For Example:- $\left.\begin{aligned} 2x + 4y &= 1 \\ 3x + 6y &= 6 \end{aligned}\right\} \rightarrow$ Linear System

Question 2

(a)

$2^{1/3}x + \sqrt{3}\,y = 1$

Sol:-
The given equation is a Linear Equation.

(c)

$\cos\left(\frac{\pi}{7}\right)x - 4y = \log 3$

Sol:-
The given equation is non-linear Because in linear equation Trignometric and Logarithmic function is not used.

(e)

$xy = 1$

Sol:-
The given equation is non linear Because in linear equation product of two variable with each other not involved or allowed.

Linear System:-
A finite set of Linear Equations is called system of linear Equation or Linear System

For Example:- $2x + 4y = 1$ → linear

vivo Y20 2021 2022.09.16 10:

Scanned with CamScanner

Unknown:-

The variables are called unknowns.

or

The variables used in linear system is called unknown variables.

For Example:-

$$2x + 7y = 2$$
$$3x + 4y = 4$$

x, y in linear system are unknown variables.

Question 3:-

Write down the general linear system of

(a)

two equation in two unknowns.

Sol:-

$$3x + 5y = 1$$
$$4x + 6y = 2$$

(b)

three Equation in three unknowns.

Sol:-

$$2x + y + 5z = 1$$
$$3x + 2y + 4z = 2$$
$$4x + 3y + 3z = 3$$

(c)

$2x_2 \quad -3x_4 + x_5 = 0$

$-3x_1 - x_2 + x_3 \quad = -1$

$6x_1 + 2x_2 - x_3 + 2x_4 - 3x_5 = 6$

Sol:-

$$\begin{bmatrix} 0 & 2 & 0 & -3 & 1 & 0 \\ -3 & -1 & 1 & 0 & 0 & -1 \\ 6 & 2 & -1 & 2 & -3 & 6 \end{bmatrix} \rightarrow \text{Augmented Matrix.}$$

Question 8 same as 7

Q8(c)

$x_1 \quad = 1$

$x_2 \quad = 2$

$x_3 = 3$

Sol:-

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \rightarrow \text{Augmented Matrix}$$

Solution:-

A solution of linear system in n-unknown $x_1, x_2, \ldots x_n$ in a sequence of n number $S_1, S_2, \ldots S_n$ for which substitution make each equation a true statement

ordered n-tuple:-

A solution $x_1 = S_1$, $x_2 = S_2$, ~~...~~ S_n of a linear system in n unknown can be written as $(s_1, s_2, \cdots, s_n)$ which is called an ordered n-tuple.

Question no 9, 10 Same

In Each part, determine whether the given 3-tuple is a solution of linear system.

$$2x_1 - 4x_2 - x_3 = 1$$
$$x_1 - 3x_2 + x_3 = 1$$
$$3x_1 - 5x_2 - 3x_3 = 1$$

Q9(a)

$(\underset{x_1}{3}, \underset{x_2}{1}, \underset{x_3}{1})$

Sol:-

$$2(3) - 4(1) - 1 = 1$$
$$3 - 3(1) + 1 = 1$$
$$3(3) - 5(1) - 3(1) = 1$$

$$6 - 4 - 1 = 1$$
$$3 - 3 + 1 = 1$$
$$9 - 5 - 3 = 1$$

$$1 = 1$$
$$1 = 1$$
$$1 = 1$$

So the given 3-tuple is a solution of the given linear system.

$= 5 \frac{45 - 10}{7} = 5$

$21/7 = 3$

$7/7 = 1$

$35/7 = 5$

$3 = 3$

$1 = 1$

$5 = 5$

So the given 3-tuple $(5/7, 10/7, 2/7)$ is a solution of the given linear system.

Question 11

(a)

$3x - 2y = 4$ —— (i)

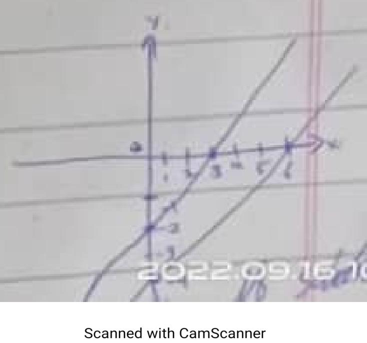
$6x - 4y = 9$ —— (ii)

Multiply by 2 eq (i) then subtract

~~eq (ii) of~~

$$\begin{array}{r} 6x - 4y = 4 \\ - 6x \mp 4y = -9 \\ \hline 0 = -5 \end{array}$$

So the system is inconsistent or no solution



Parametric Equation:-

Q13-14 Same

Use parametric equation to describe the solution set of the linear Equation.

(a)

$7x - 5y = 3$

Sol:-

$7x = 3 + 5y$

let

$t = y$

then

$$\boxed{x = \frac{3}{7} + \frac{5}{7}t}$$

(c)

$-8x_1 + 2x_2 - 5x_3 + 6x_4 = 1$

Sol:-

$-8x_1 = 1 - 2x_2 + 5x_3 - 6x_4$

$x_1 = -\frac{1}{8} + \frac{2}{8}x_2 - \frac{5}{8}x_3 - \frac{6}{8}x_4$

let

$x_2 = t$, $x_3 = u$, $x_4 = v$

vivo Y20 2021 2022.09.16 10:17

$$\boxed{x = -\frac{1}{8} + \frac{1}{7}t - \frac{5}{8}u - \frac{3}{4}v}$$

Q14 (b)

$x_1 + 3x_2 - 12x_3 = 3$

Sol:-

$x_1 = 3 - 3x_2 + 12x_3$

let

$x_2 = t$, $x_3 = v$

then

$$\boxed{x_1 = 3 - 3t + 12v}$$

Q14 (d)

$v + w + x - 5y + 7z = 0$

Sol:-

$v = -w - x + 5y - 7z$

let

$w = a$, $x = b$, $y = c$, $z = d$

then

$$\boxed{v = -a - b - 5c - 7d}$$

Q15, 16 (Same)

(a)

$2x - 3y = 1$

$6x - 9y = 3$

Sol:-

$2x - 3y = 1$

$6x - 9y = 3$

Multiply by 3 eq(i)

vivo Y20 2021 2022.09.16 10:17

Q17 (b)

$$\begin{bmatrix} 0 & -1 & -5 & 0 \\ 2 & -9 & 3 & 2 \\ 1 & 4 & -3 & 3 \end{bmatrix}$$

Sol:-

$$= \begin{bmatrix} 0 & -1 & -5 & 0 \\ 2 & -9 & 3 & 2 \\ 1 & 4 & -3 & 3 \end{bmatrix}$$

Interchange R_1 with R_3

$$\Rightarrow \begin{bmatrix} 1 & 4 & -3 & 3 \\ 2 & -9 & 3 & 2 \\ 0 & -1 & -5 & 0 \end{bmatrix}$$

Q18 (a)

$$\begin{bmatrix} 2 & 4 & -6 & 8 \\ 7 & 1 & 4 & 3 \\ -5 & 4 & 2 & 7 \end{bmatrix}$$

Sol:-

$$= \begin{bmatrix} 2 & 4 & -6 & 8 \\ 7 & 1 & 4 & 3 \\ -5 & 4 & 2 & 7 \end{bmatrix}$$

R_1 divided by 2

$$\Rightarrow = \begin{bmatrix} 1 & 2 & -3 & 4 \\ 7 & 1 & 4 & 3 \\ -5 & 4 & 2 & 7 \end{bmatrix}$$

$$x + 2\left(\frac{0}{2}\right) = -1$$

$$x + \frac{0}{2} = -1$$

$$\boxed{x = -1}$$

if $k = 2$

then

$$(k-2)y = 0$$

$$(2-2)y = 0$$

$$0 = 0$$

So the linear system of augmented matrix

$$\begin{bmatrix} 1 & k & -1 \\ 4 & 8 & -4 \end{bmatrix}$$ is consistent

for all value of k.

Q20 (a)

$$\begin{bmatrix} 3 & -4 & K \\ -6 & 8 & 5 \end{bmatrix}$$

Sol:-

Divided R_2 by -2

$$\begin{bmatrix} 3 & -4 & K \\ -3 & +4 & +5/2 \end{bmatrix}$$

$3x - 4y = K$

$-3x + 4y = +5/2$

$0 = K + 5/2$

$0 = \frac{2K+5}{2} \Rightarrow 0 = 2K+5$

$$\boxed{K = \frac{-5}{2}}$$

Q20(b)

$$\begin{bmatrix} K & 1 & -2 \\ 4 & -1 & 2 \end{bmatrix}$$

Sol:-

$Kx + \cancel{y} = \cancel{-2}$

$-4x - \cancel{y} = \cancel{-2}$

$Kx - 4x = 0$

$x(K-4) = 0$ —— (i)

$4x - y = 2$ —— (ii)

$K - 4 = 0$

$\boxed{K = 4}$

if $K = 4$

$x(4-4) = 0$

$x(0) = 0$

$0 = 0$

So the linea[r]

system of

augmented

matrix

is consistent

if $K = 2$

$(K-2)y = -9/2$

$(2-2)y = -9/2$

$0 = -9/2$

which is false

So if and only if $K \neq 2$

Linear system with augmented matrix $\begin{bmatrix} 1 & K & -4 \\ 4 & 8 & 2 \end{bmatrix}$ is

consistent

19(b)

$\begin{bmatrix} 1 & K & -1 \\ 4 & 8 & -4 \end{bmatrix}$

Solr

Divided R_2 by 4

$\begin{bmatrix} 1 & K & -1 \\ 1 & 2 & -1 \end{bmatrix}$

$x + Ky = -1$

$\underline{-x \pm 2y = -1}$

$Ky - 2y = 0$

$(K-2)y = 0$ —— (i)

$x + 2y = -2$ —— (ii)

if $K - 2 \neq 0$

$y = 0$

Linear Algebra

Ex 1.1

Q19 (a)

$$\begin{bmatrix} 1 & k & -4 \\ 4 & 8 & 2 \end{bmatrix}$$

Sol:-

Dividing R_2 by 4

$$\begin{bmatrix} 1 & k & -4 \\ 1 & 2 & 1/2 \end{bmatrix}$$

$$\cancel{x} + ky = -4$$
$$-\cancel{x} \pm 2y = -1/2$$

$$ky - 2y = -4 - \frac{1}{2}$$

$$ky - 2y = -9/2$$
$$(k-2)y = -9/2 \quad \text{—(i)}$$
$$x + 2y = 1/2 \quad \text{—(ii)}$$

if $k - 2 \neq 0$ then $y = -\frac{9}{2} \times \frac{1}{(k-2)}$

So $y = -\frac{9}{2k-4}$

then y value put in eq(ii)

$$x + \cancel{2}\left(\frac{-9}{\cancel{2}(k-2)}\right) = \frac{1}{2}$$

$$x - \frac{9}{k-2} = \frac{1}{2}$$

$$\boxed{x = \frac{1}{2} + \frac{9}{k-2}}$$

Already mentioned So we also directly find parametric equation

$$0 = 0$$

$$x = \frac{-12}{6} + \frac{3}{6}y - \frac{6}{6}z$$

$$x = -2 + \frac{1}{2}y - 1$$

let

$y = t$, $z = a$

$$\boxed{x = -2 + \frac{1}{2}t - a}$$

Q17, 18 (same)
(a)

$$\begin{bmatrix} -3 & -1 & 2 & 4 \\ 2 & -3 & 3 & 2 \\ 0 & 2 & -3 & 1 \end{bmatrix}$$

Solr

Add $R_1 + R_2$

$$\begin{bmatrix} -3+2 & -1-3 & 2+3 & 4+2 \\ 2 & -3 & 3 & 2 \\ 0 & 2 & -3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -4 & 5 & 6 \\ 2 & -3 & 3 & 2 \\ 0 & 2 & -3 & 1 \end{bmatrix}$$

Multiply by (-1) R_1

$$\Rightarrow \begin{bmatrix} 1 & 4 & -5 & -6 \\ 2 & -3 & 3 & 2 \\ 0 & 2 & -3 & 1 \end{bmatrix}$$

Linear Algebra

Ex 1.1

Q11 (c)

$x - 2y = 6$ —— (i)

$x - 4y = 8$ —— (ii)

Solr

Multiply by 2 eq (i)

$2x - \cancel{4y} = 0$

$-x \mp \cancel{4y} = -8$

$\boxed{x = -8}$

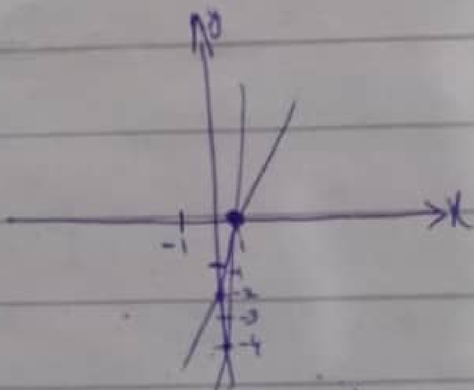
Having only one solution and also consistent

put x value in eq (i)

$-8 - 2y = 6$

$-2y = 8$

$\boxed{y = -4}$



(one solution).

Sol:-

$5+2(8)-2(3)=3$

$3(5)-8+3=1$

$-5+5(8)-5(3)=5$

$5+16-6=3$

$15-8+3=1$

$-5+40-15=5$

$15 \neq 3$

$10 \neq 1$

$20 \neq 5$

So the given 3-tuple $(5,8,3)$ is not a solution of the given linear system.

(d)

$\left(\frac{5}{7}, \frac{10}{7}, \frac{2}{7}\right)$

Sol:-

$\frac{5}{7}+2\left(\frac{10}{7}\right)-2\left(\frac{2}{7}\right)=3$

$3\left(\frac{5}{7}\right)-\frac{10}{7}+\frac{2}{7}=1$

$-\frac{5}{7}+5\left(\frac{10}{7}\right)-5\left(\frac{2}{7}\right)=5$

$\frac{5}{7}+\frac{20}{7}-\frac{4}{7}=3$

$\frac{15}{7}-\frac{10}{7}+\frac{2}{7}=1$

$-\frac{5}{7}+\frac{50}{7}-\frac{10}{7}=5$

20 2021 2022.09.16 10:15

Q12

$2x - 3y = a$
$4x - 6y = b$

Sol:

let $a = 1$, $b = 2$

$2x - 3y = 1$
$4x - 6y = 2$

Multiply by (2) eq (i)

$\cancel{4x} - \cancel{6y} = \cancel{2}$
$- \cancel{4x} \mp \cancel{6y} = \cancel{-2}$

$0 = 0$ (consident but infinite solution)

when we suppose $a=1$ & $b=2$ in linear system then after solving the system. No of solution occur of linear system.

When we put $a=2$, $b=2$ in linear system.

$2x - 3y = 2$
$4x - 6y = 2$

multiply by 2 (i)

$\cancel{4x} - \cancel{6y} = 4$
$- \cancel{4x} \mp \cancel{6y} = -2$

$0 = 2$

when put $a=2, b=2$ in linear system then no solution found of this linear system.

Ex 1.1

Q9 (c)

(17, 7, 5)

Sol:-

$$2x_1 - 4x_2 - x_3 = 1$$
$$x_1 - 3x_2 + x_3 = 1$$
$$3x_1 - 5x_2 - 3x_3 = 1$$

$$2(17) - 4(7) - 5 = 1$$
$$17 - 3(7) + 5 = 1$$
$$3(17) - 5(7) - 3(5) = 1$$

$$34 - 28 - 5 = 1$$
$$17 - 21 + 5 = 1$$
$$51 - 35 - 15 = 1$$

$$1 = 1$$
$$1 = 1$$
$$1 = 1$$

The given 3-tuple (17, 7, 5) is a solution of the given linear system.

Q10

$$x + 2y - 2z = 3$$
$$3x - y + z = 1$$
$$-x + 5y - 5z = 5$$

(c)

(5, 8, 1)

$$\left.\begin{array}{l} 2x_1 \qquad = 0 \\ 3x_1 - 4x_2 = 0 \\ \qquad x_2 = 1 \end{array}\right] \text{— Linear system.}$$

Q5(b)

$$\begin{bmatrix} 3 & 0 & -2 & 5 \\ 7 & 1 & 4 & -3 \\ 0 & -2 & 1 & 7 \end{bmatrix}$$

Sol:-

$$\left.\begin{array}{l} 3x_1 \qquad -2x_3 = 5 \\ 7x_1 + x_2 + 4x_3 = -3 \\ \qquad -2x_2 + x_3 = 7 \end{array}\right] \text{— Linear system.}$$

Q6(a)

$$\begin{bmatrix} 0 & 3 & -1 & -1 & -1 \\ 5 & 2 & 0 & -3 & -6 \end{bmatrix}$$

Sol:-

$$\left.\begin{array}{l} \qquad 3x_2 - x_3 - x_4 = -1 \\ 5x_1 + 2x_2 \qquad -3x_4 = -6 \end{array}\right] \text{— Linear system.}$$

Question no 7 & 8 (Same)

Q7(a)

Find the augmented matrix for the linear system.

$-2x_1 = 6$

$3x_1 = 8$

$9x_1 = -3$

Sol:-

$$\begin{bmatrix} -2 & 6 \\ 3 & 8 \end{bmatrix} \longrightarrow \text{augmented Matrix}$$

2022.09.16 10

Scanned with CamScanner

(ii)

$x_1 + 3x_2 + x_1x_2 = 2$

Sol:-

The given equation is not linear Because two variable not product in Linear equation.

(c)

$x_1 = -7x_2 + 3x_3$

Sol:-

The given equation is Linear.

(d)

$x_1^{-2} + x_2 + 8x_3 = 5$

Sol:-

The given equation is non-linear Because in Linear equation power of variable is only one and do not appear.

(e)

$x_1^{3/5} - 2x_2 + x_3 = 4$

Sol:-

The given equation is non-linear. Because in Linear Equation power of variable occur only one.

(f)

$\pi x_1 - \sqrt{2}\, x_2 = 7^{1/3}$

Sol:-

The given equation is Linear.

vivo Y20 2021 2022.09.16 10:14

Linear Algebra
Chapter # 1

Ex 1.1
Q1 & 2 same

Linear Equation:-
Equation does not involve any product or root of variables. All variable occur only to the first power and do not appear. Such types of Equation is called Linear equation. (Not involve Trignometric, logarithmic or exponential function)

for example:-

$$2x + 3y = 4$$

is a Linear Equation.

Question:-
In each part, determine whether the equation is linear in x_1, x_2, x_3.

(i)

$$x_1 + 5x_2 - \sqrt{2}\, x_3 = 1$$

Sol:-
The given Equation is Linear.

(b) Q2

$$A = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 3 & 6 \\ 0 & 1 & 4 \end{vmatrix}$$

Find M_{23} & C_{23}

Sol:-

$$M_{23} = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 3 & 6 \\ 0 & 1 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= (1\times1) - (1\times0)$$

$$= 1-0$$

$$\boxed{M_{23} = 1}$$

Then Find C_{23}

Using Formula.

$$C_{23} = (-1)^{2+3} M_{23}$$

$$= (-1)^5 (1)$$

$$= (-1)(1)$$

$$\boxed{C_{23} = -1}$$

(5)

M_{22} and C_{22}

Sol:-

$$A = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 3 & 6 \\ 0 & 1 & 4 \end{vmatrix}$$

For M_{22}

$$M_{22} = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 3 & 6 \\ 0 & 1 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix}$$

$$= (1\times4) - (2\times0)$$

$$= 4-0$$

$$\boxed{M_{22} = 4}$$

Then Find C_{22}

Using formula

$$C_{22} = (-1)^{2+2} M_{22}$$

$$= (-1)^4 (4)$$

$$= (1)(4)$$

$$\boxed{C_{22} = 4}$$

vivo Y20 2021

Q4(c)

M_{41} and C_{41}

Soln

$$M_{41} = \begin{vmatrix} 2 & 3 & -1 & 1 \\ -3 & 2 & 0 & 3 \\ 3 & -2 & 1 & 0 \\ 3 & -2 & 1 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ -2 & 1 & 0 \end{vmatrix}$$

Expand by R_1

$$= 3\begin{vmatrix} 0 & 3 \\ 1 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} + 1\begin{vmatrix} 2 & 0 \\ -2 & 1 \end{vmatrix}$$

$= 3(0-3) + 1(0+6) + 1(2-0)$

$= 3(-3) + 1(6) + 1(2)$

$= -9 + 6 + 2$

$\boxed{M_{41} = -1}$

Then Find C_{41}

$C_{41} = (-1)^{4+1} M_{41}$

$= (-1)^5(-1)$

$= (-1)(-1)$

$\boxed{C_{41} = 1}$

Q4

$$A = \begin{bmatrix} 2 & 3 & -1 & 1 \\ -3 & 2 & 0 & 3 \\ 3 & -2 & 1 & 0 \\ 3 & 2 & 1 & 4 \end{bmatrix}$$

Find (a)

M_{32} and C_{32}

Soln-

$$M_{32} = \begin{vmatrix} 2 & 3 & -1 & 1 \\ -3 & 2 & 0 & 3 \\ 3 & -2 & 1 & 0 \\ 3 & 2 & 1 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & -1 & 1 \\ -3 & 0 & 3 \\ 3 & 1 & 4 \end{vmatrix}$$

Expand by R_1

$$= 2\begin{vmatrix} 0 & 3 \\ 1 & 4 \end{vmatrix} - (-1)\begin{vmatrix} -3 & 3 \\ 3 & 4 \end{vmatrix} + 1\begin{vmatrix} -3 & 0 \\ 3 & 1 \end{vmatrix}$$

$= 2(0-3) + 1(-12-9) + 1(-3-0)$

$= 2(-3) + 1(-21) + 1(-3)$

$= -6 - 21 - 3$

$\boxed{M_{32} = -30}$

The Find C_{32}

$C_{32} = (-1)^{3+2} M_{32}$

$= (-1)^5 (-30)$

$= (-1)(-30)$

$\boxed{C_{32} = 30}$

vivo Y20 2021

2022.09.16 10:12

Q33

(a)

$$\begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix}$$

Sol:-

$$= \begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix}$$

$$= (\sin\theta \cdot \sin\theta) - (\cos\theta \cdot -\cos\theta)$$

$$= \sin^2\theta - (-\cos^2\theta)$$

$$= \sin^2\theta + \cos^2\theta$$

$$= 1$$

(b)

$$\begin{vmatrix} \sin\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ \sin\theta-\cos\theta & \sin\theta+\cos\theta & 1 \end{vmatrix}$$

Sol:-

$$= \begin{vmatrix} \sin\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ \sin\theta-\cos\theta & \sin\theta+\cos\theta & 1 \end{vmatrix}$$

Expand by R_1

$$= \sin\theta \begin{vmatrix} \sin\theta & 0 \\ \sin\theta+\cos\theta & 1 \end{vmatrix} - \cos\theta \begin{vmatrix} -\cos\theta & 0 \\ \sin\theta-\cos\theta & 1 \end{vmatrix} + 0 \begin{vmatrix} -\cos\theta & \sin\theta \\ \sin\theta-\cos\theta & \sin\theta+\cos\theta \end{vmatrix}$$

$$= \sin\theta(\sin\theta - 0) - \cos\theta(-\cos\theta - 0) + 0$$

$$= \sin\theta(\sin\theta) - \cos\theta(-\cos\theta)$$

$$= \sin^2\theta + \cos^2\theta$$

$$= 1 \text{ Ans.}$$

$= -20 - 7 + 72 - 20 - 84 - 6$

$= -65$ Ans

Q14

$$\begin{vmatrix} c & -4 & 3 \\ 2 & 1 & c^2 \\ 4 & c-1 & 2 \end{vmatrix}$$

Sol:

By using arrow method to find determinant.

$$\begin{vmatrix} c & -4 & 3 \\ 2 & 1 & c^2 \\ 4 & c-1 & 2 \end{vmatrix} = \left|\begin{array}{ccc|cc} c & -4 & 3 & c & -4 \\ 2 & 1 & c^2 & 2 & 1 \\ 4 & c-1 & 2 & 4 & c-1 \end{array}\right|$$

$= [2c - 16c^2 + 6c - 6] - [12 + c^4 - c^3 - 16]$

$= 2c - 16c^2 + 6c - 6 - 12 - c^4 + c^3 + 16$

$= 8c - 16c^2 + c^3 - c^4 - 2$ Ans

Q15 — 18 (same

Find all value of λ for which $\det(A) = 0$.

$$A = \begin{bmatrix} \lambda-2 & 1 \\ -5 & \lambda+4 \end{bmatrix}$$

Sol:

$$|A| = \begin{vmatrix} \lambda-2 & 1 \\ -5 & \lambda+4 \end{vmatrix}$$

$= [(\lambda-2)(\lambda+4)] - (1 \times (-5))$

$= (\lambda^2 + 4\lambda - 2\lambda - 8) - (-5)$

$= \lambda^2 + 2\lambda - 8 + 5$

$= \lambda^2 + 2\lambda - 3$

The $\det(A) = 0$

Then

$0 = \lambda^2 + 2\lambda - 3$

$0 = \lambda^2 + 3\lambda - \lambda - 3$

$0 = \lambda(\lambda+3) - 1(\lambda+3)$

$0 = (\lambda+3)(\lambda-1)$

Then

$\lambda + 3 = 0$ or $\lambda - 1 = 0$

$\lambda = -3$ or $\lambda = 1$

λ value are $(-3, 1)$

$$= 3\begin{vmatrix} -1 & 5 \\ 9 & -4 \end{vmatrix} - 0\begin{vmatrix} 2 & 5 \\ 1 & -4 \end{vmatrix} + 0\begin{vmatrix} 2 & -1 \\ 1 & 9 \end{vmatrix}$$

$$= 3(4-45) - 0(-8-5) + 0(18+1)$$

$$= 3(-41) - 0(-13) + 0(19)$$

$$= -123 - 0 + 0$$

$$\boxed{|A| = -123}$$

Q19(b)

By First column

Let

$$|A| = \begin{vmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{vmatrix}$$

Expansion by c_1

$$= 3\begin{vmatrix} -1 & 5 \\ 9 & -4 \end{vmatrix} - 2\begin{vmatrix} 0 & 0 \\ 9 & -4 \end{vmatrix} + 1\begin{vmatrix} 0 & 0 \\ -1 & 5 \end{vmatrix}$$

$$= 3(4-45) - 2(0-0) + 1(0-0)$$

$$= 3(-41) - 2(0) + 1(0)$$

$$= -123 - 0 + 0$$

$$\boxed{|A| = -123}$$

vivo Y20 2021 2022.09.16 10:11

Q20

Evalute the determine in Q12 by a cofactor exponsion along.

$$\begin{vmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{vmatrix}$$

(e)

the third row

Solr

$$|A| = \begin{vmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{vmatrix}$$

Expand by R_3

$$= 1\begin{vmatrix} 1 & 2 \\ 0 & -5 \end{vmatrix} - 7\begin{vmatrix} -1 & 2 \\ 3 & -5 \end{vmatrix} + 2\begin{vmatrix} -1 & 1 \\ 3 & 0 \end{vmatrix}$$

$$= 1(-5-0) - 7(+5-6) + 2(0-3)$$

$$= 1(-5) - 7(-1) + 2(-3)$$

$$= -5 + 7 - 6$$

$$= 7 - 11$$

$$\boxed{|A| = -4}$$

(f)

the third column

Solr

$$|A| = \begin{vmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{vmatrix} \rightarrow \text{SHEET \# 5}$$

Q28

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Solⁿ

let

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

By expand R_1

$$= 2\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ 0 & 2 \end{vmatrix} + 0\begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix}$$

$$= 2(4-0) - 0(0-0) + 0(0-0)$$

$$= 2(4) - 0 + 0$$

$$= 8 \text{ Ans-}$$

vo Y20 2021 2022.09.16 10:11

Q27

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sol: By arrow Method.

$$= \left|\begin{matrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{matrix}\right| \begin{matrix} 1 & 0 \\ 0 & -1 \\ 0 & 0 \end{matrix}$$

$= [-1+0+0] - [0+0+0]$

$= -1-0$

$\boxed{= -1}$ Ans

OR

let $\left|\begin{matrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{matrix}\right|$

By Expand R_1

$$= 1\left|\begin{matrix} -1 & 0 \\ 0 & 1 \end{matrix}\right| - 0\left|\begin{matrix} 0 & 0 \\ 0 & 1 \end{matrix}\right| + 0\left|\begin{matrix} 0 & -1 \\ 0 & 0 \end{matrix}\right|$$

$= 1(-1-0) - 0(0-0) + 0(0-0)$

$= 1(-1) - 0(0) + 0(0)$

$= -1-0+0$

$= -1$ Ans

Remaining Q20 (1)

Solt

Expanding by C_2

$$= 2\begin{vmatrix} 3 & 0 \\ 1 & 7 \end{vmatrix} - (-5)\begin{vmatrix} -1 & 1 \\ 1 & 7 \end{vmatrix} + 2\begin{vmatrix} -1 & 1 \\ 3 & 0 \end{vmatrix}$$

$= 2(21-0) + 5(-7-1) + 2(0-3)$

$= 2(21) + 5(-8) + 2(-3)$

$= 42 - 40 - 6$

$= 42 - 46$

$\boxed{|A| = -4}$

Q21 to 26 (same)

Evalute det(A) by a cofactor expansion along a row or column of your choice

Q23

$$A = \begin{bmatrix} 1 & K & K^2 \\ 1 & K & K^2 \\ 1 & K & K^2 \end{bmatrix}$$

Solt

$$|A| = \begin{vmatrix} 1 & K & K^2 \\ 1 & K & K^2 \\ 1 & K & K^2 \end{vmatrix}$$

Expand by R_1

Q#2

Q16

$$A = \begin{bmatrix} \lambda-4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda-1 \end{bmatrix}$$

Sol:

$$|A| = \begin{vmatrix} \lambda-4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda-1 \end{vmatrix}$$

Expand by R_1

$$= \lambda-4 \begin{vmatrix} \lambda & 2 \\ 3 & \lambda-1 \end{vmatrix} - 0 \begin{vmatrix} 0 & 2 \\ 0 & \lambda-1 \end{vmatrix} + 0 \begin{vmatrix} 0 & \lambda \\ 0 & 3 \end{vmatrix}$$

$$= \lambda-4((\lambda^2-\lambda)-6)-0(0-0)+0(0-0)$$

$$= \lambda-4(\lambda^2-\lambda-6)-0+0$$

$$= \lambda^3-\lambda^2-6\lambda-4\lambda^2+4\lambda+24$$

$$|A| = \lambda^3-5\lambda^2-2\lambda+24$$

According to given condition $\det(A)=0$

$$0 = \lambda^3-5\lambda^2-2\lambda+24$$

$$-24 = (\lambda^3-5\lambda^2-2\lambda)$$

Then

$\lambda^3 = -24$ or $-5\lambda^2 = -24$ or $-2\lambda = -$

$\lambda = \sqrt{-24}$ or $\lambda = \sqrt{24/5}$ or $\lambda = 1$

So the value of λ are $(\sqrt{-24}, \sqrt{24/5}$

Q8

$$\begin{bmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{bmatrix}$$

Sol:

let $A = \begin{bmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{bmatrix}$

then

$$|A| = \begin{vmatrix} \sqrt{2} & \sqrt{8} \\ 4 & \sqrt{3} \end{vmatrix}$$

$$= (\sqrt{2} \times \sqrt{3}) - (\sqrt{6} \times 4)$$

$$= 6 - 4\sqrt{6}$$

$$|A| = 2(3 - 2\sqrt{6}) \Rightarrow = -3.7$$

So the square matrix is invertible.

Then also find A^{-1}

$$A = \begin{bmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} \sqrt{3} & -\sqrt{6} \\ -4 & \sqrt{2} \end{bmatrix}$$

Then

$$A^{-1} = \frac{1}{|A|} \text{adj} A$$

$$= \frac{1}{-3.7} \begin{vmatrix} \sqrt{3} & -\sqrt{6} \\ -4 & \sqrt{2} \end{vmatrix}$$

$$A^{-1} = \begin{vmatrix} -\sqrt{3}/3.7 & -\sqrt{6}/3.7 \\ 4/3.7 & -\sqrt{2}/3.7 \end{vmatrix}$$

vivo Y20 2021 2022.09.16 10:09

Linear Algebra

CH # 2

Q5 (a)

$$\begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}$$

Solt

let

$$A = \begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}$$

Find determinant

$$|A| = \begin{vmatrix} 3 & 5 \\ -2 & 4 \end{vmatrix}$$

$$= (12 - (-10))$$

$$= 12 + 10$$

$$\boxed{|A| = 22}$$

Yes the given matrix is invertible.

Then we find adj A

$$A = \begin{vmatrix} 3 & 5 \\ -2 & 4 \end{vmatrix}$$

$$\text{adj}A = \begin{vmatrix} 4 & -5 \\ 2 & 3 \end{vmatrix}$$

Then find A^{-1} Using Formula.

$$A^{-1} = \frac{1}{|A|} \text{adj } A \quad \text{—(ii)}$$

Put value in eq (ii)

$$A^{-1} = \frac{1}{\;} \begin{vmatrix} 4 & -5 \end{vmatrix}$$

$$A^{-1} = \begin{vmatrix} 4/22 & -5/22 \\ 2/22 & 3/22 \end{vmatrix}$$

So

$$A^{-1} = \begin{vmatrix} 2/11 & -5/22 \\ 1/11 & 3/22 \end{vmatrix}$$

Q(b)

$$\begin{bmatrix} 4 & 1 \\ 8 & 2 \end{bmatrix}$$

Solt

let

$$A = \begin{bmatrix} 4 & 1 \\ 8 & 2 \end{bmatrix}$$

Find determinant to check given matrix is invertible.

$$|A| = \begin{vmatrix} 4 & 1 \\ 8 & 2 \end{vmatrix}$$

$$= (4\times2) - (1\times8)$$

$$= 8 - 8$$

$$= 0$$

So the given square matrix is not invertible.

Q3(c)

Find M_{22} and C_{22}

Given Matrix is

$$A = \begin{bmatrix} 4 & 1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{bmatrix}$$

Sol:-

$$M_{22} = \begin{vmatrix} 4 & 1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{vmatrix}$$

ON Right Side →

$$\cancel{M_{22} = \begin{vmatrix} 4 & 1 & 6 \\ 4 & 0 & 14 \\ 4 & 3 & 2 \end{vmatrix}}$$

~~Expand by R_1~~

$$\cancel{= 4\begin{vmatrix} 0 & 14 \\ 3 & 2 \end{vmatrix} - (1)\begin{vmatrix} 4 & 14 \\ 4 & 2 \end{vmatrix} + 6\begin{vmatrix} 4 & 0 \\ 4 & 3 \end{vmatrix}}$$

$$\cancel{= 4(0-42) + 1(8-56) + 6(12-0)}$$

$$\cancel{= 4(-42) + 1(-48) + 6(12)}$$

$$\cancel{= -168 - 48 + 72}$$

$$\cancel{M_{22} = -108}$$

$$M_{22} = \begin{vmatrix} 4 & 1 & 6 \\ 4 & 0 & 14 \\ 4 & 3 & 2 \end{vmatrix}$$

Expand by R_1

$$= 4\begin{vmatrix} 0 & 14 \\ 3 & 2 \end{vmatrix} - 1\begin{vmatrix} 4 & 14 \\ 4 & 2 \end{vmatrix} + 6\begin{vmatrix} 4 & 0 \\ 4 & 3 \end{vmatrix}$$

$$= 4(0-42) - 1(8-56) + 6(12-0)$$

$$= 4(-42) - 1(-48) + 6(12)$$

$$= -168 + 48 + 72$$

$$\boxed{M_{22} = -48}$$

Then Find C_{22}

Using Formula

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$C_{22} = (-1)^{2+2} M_{22}$$

$$= (-1)^4 (-48)$$

$$= (1)(-48)$$

$$\boxed{C_{22} = -48}$$

vivo Y20 2021

2022.09.16 10:09

Linear Algebra
CH #2
Ex 2.1
Q3 (b)

M_{23} and C_{23}
Given Matrix is

$$A = \begin{bmatrix} 4 & -1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{bmatrix}$$

Sol:-

$$M_{23} = \begin{vmatrix} 4 & -1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & -1 & 6 \\ 4 & 1 & 14 \\ 4 & 1 & 2 \end{vmatrix}$$

Expand by R_1

$$= 4\begin{vmatrix} 1 & 14 \\ 1 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 4 & 14 \\ 4 & 2 \end{vmatrix} + 6\begin{vmatrix} 4 & 1 \\ 4 & 1 \end{vmatrix}$$

$$= 4(2-14) + 1(8-56) + 6(4-4)$$

$$= 4(-12) + 1(-48) + 6(0)$$

$$M_{23} = -48 - 48 + 0$$

$$\boxed{M_{23} = -96}$$

Then Find C_{23}

$$C_{23} = (-1)^{2+3} M_{23}$$

$$= (-1)^5(-96)$$

$$= (-1)(-96)$$

$$\boxed{C_{23} = 96}$$

Q1 (a)

M_{13} and C_{13}

Soln

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$$

$$M_{13} = \begin{vmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} 6 & 7 \\ -3 & 1 \end{vmatrix}$$

$$= (6 \times 1) - (7 \times -3)$$
$$= 6 - (-21)$$
$$= 6 + 21$$

$$\boxed{M_{13} = 27}$$

Then

C_{13}

$$C_{13} = (-1)^{1+3} M_{13}$$
$$= (-1)^4 (27)$$
$$= (1)(27)$$

$$\boxed{C_{13} = 27}$$

Q1 (a)

M_{21} and C_{21}

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$$

Find M_{21}

then

$$M_{21} = \begin{vmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} -2 & 3 \\ 1 & 4 \end{vmatrix}$$

$$= (-2 \times 4) - (1 \times 3)$$
$$= -8 - 3$$

$$\boxed{M_{21} = -11}$$

then

C_{21}

formula

$$C_{21} = (-1)^{2+1} M_{21}$$
$$= (-1)^3 (-11)$$
$$= (-1)(-11)$$

$$\boxed{C_{21} = 11}$$

2022.09.16 10:09

Ex2.1 Q31

$$\begin{bmatrix} 1 & 2 & 7 & -3 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

Sol:

$$\begin{vmatrix} 1 & 2 & 7 & -3 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & 3 \end{vmatrix}$$

Pick the row that contain maximum zero for expansion.

Expand by R_4

$$= 0\begin{vmatrix} 2 & 7 & -3 \\ 1 & -4 & 1 \\ 0 & 2 & 7 \end{vmatrix} - 0\begin{vmatrix} 1 & 7 & -3 \\ 0 & -4 & 1 \\ 0 & 2 & 7 \end{vmatrix} + 0\begin{vmatrix} 1 & 2 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 7 \end{vmatrix} - 3\begin{vmatrix} 1 & 2 & \\ 0 & 1 & \\ 0 & 0 & \end{vmatrix}$$

$$= 0 - 0 + 0 - 3\left[1\begin{vmatrix} 1 & -4 \\ 0 & 2 \end{vmatrix} - 2\begin{vmatrix} 0 & -4 \\ 0 & 2 \end{vmatrix} + 7\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}\right]$$

$$= -3\left[1(2-0) - 2(0-0) + 7(0-0)\right]$$

$$= -3\left[1(2) - 2(0) + 7(0)\right]$$

$$= -3(2 - 0 + 0)$$

$$= -3(2)$$

$$= -6 \text{ Ans.}$$

2022.09.16 10:36

Scanned with CamScanner

Q27 to 32 (same)

Pick the row that contain maximum zero for expansion.

Q30

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{vmatrix}$$

Soln

Expand by R_4

$$= 0\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 0 & 3 & 3 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 3 & 3 \end{vmatrix} + 0\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{vmatrix} - 4\begin{vmatrix} 1 & 1 \\ 0 & 2 \\ 0 & 0 \end{vmatrix}$$

$$= 0 - 0 + 0 - 4\left[1\begin{vmatrix} 2 & 2 \\ 0 & 3 \end{vmatrix} - 1\begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix} + 1\begin{vmatrix} 0 \\ 0 \end{vmatrix}\right.$$

$$= -4\left[1(6-0) - 1(0-0) + 1(0-0)\right]$$

$$= -4(6 - 0 + 0)$$

$$= -4(6)$$

$$= -24$$

vivo Y20 2021