The basic building blocks of logic—propositions.

A **proposition** is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.

## EXAMPLE:

All the following declarative sentences are propositions.

1. Washington, D.C., is the capital of the United States of America.

2. Toronto is the capital of Canada.

3. 1 + 1 = 2.

4. 2 + 2 = 3.

Propositions 1 and 3 are true, whereas 2 and 4 are false.

## EXAMPLE:

Consider the following sentences.

1. What time is it?

2. Read this carefully.

3. $x + 1 = 2$.

4. $x + y = z$.

Sentences 1 and 2 are not propositions because they are not declarative sentences. Sentences and 4 are not propositions because they are neither true nor false. Note that each of sentences 3 and 4 can be turned into a proposition if we assign values to the variables. We will also discuss other ways to turn sentences such as these into propositions later.

---

The conventional letters used for propositional variables are *p, q, r, s, . . . .* The **truth value** of a proposition is true, denoted by T, if it is a true proposition, and the truth value of a proposition is false, denoted by F, if it is a false proposition.

The area of logic that deals with propositions is called the **propositional calculus** or **propositional logic**. It was first developed systematically by the Greek philosopher Aristotle more than 2300 years ago.

We now turn our attention to methods for producing new propositions from those that we already have. These methods were discussed by the English mathematician George Boole in 1854 in his book *The Laws of Thought.* Many mathematical statements are constructed by combining one or more propositions. New propositions, called **compound propositions**, are formed from existing propositions using logical operators.

---

Let $p$ be a proposition. The *negation of p*, denoted by $\neg p$ (also denoted by $p$), is the statement

"It is not the case that $p$."

The proposition $\neg p$ is read "not $p$." The truth value of the negation of $p$, $\neg p$, is the opposite of the truth value of $p$.

## EXAMPLE:

Find the negation of the proposition

"Michael's PC runs Linux"

and express this in simple English.

## *Solution:*

The negation is

"It is not the case that Michael's PC runs Linux."

This negation can be more simply expressed as

"Michael's PC does not run Linux."

## EXAMPLE:

Find the negation of the proposition

"Vandana's smartphone has at least 32GB of memory"

and express this in simple English.

## *Solution:*

The negation is

"It is not the case that Vandana's smartphone has at least 32GB of memory."

This negation can also be expressed as

"Vandana's smartphone does not have at least 32GB of memory"

or even more simply as

"Vandana's smartphone has less than 32GB of memory."

**TABLE 1 The Truth Table for the Negation of a Proposition.**

| $p$ | $\neg p$ |
|---|---|
| T | F |
| F | T |

---

Logical operators are also called **connectives**.

Let $p$ and $q$ be propositions. The *conjunction* of $p$ and $q$, denoted by $p \land q$, is the proposition "$p$ and $q$." The conjunction $p \land q$ is true when both $p$ and $q$ are true and is false otherwise.

## EXAMPLE:

Find the conjunction of the propositions $p$ and $q$ where $p$ is the proposition "Rebecca's PC has more than 16 GB free hard disk space" and $q$ is the proposition "The processor in Rebecca's PC runs faster than 1 GHz."

## *Solution:*

The conjunction of these propositions, $p \land q$, is the proposition "Rebecca's PC has more than 16 GB free hard disk space, and the processor in Rebecca's PC runs faster than 1 GHz." This conjunction can be expressed more simply as "Rebecca's PC has more than 16 GB free hard disk space, and its processor runs faster than 1 GHz." For this conjunction to be true, both conditions given must be true. It is false, when one or both of these conditions are false.

**TABLE 2 The Truth Table for the Conjunction of Two Propositions.**

| $p$ | $q$ | $p \land q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

---

Let $p$ and $q$ be propositions. The *disjunction* of $p$ and $q$, denoted by $p \lor q$, is the proposition "$p$ or $q$." The disjunction $p \lor q$ is false when both $p$ and $q$ are false and is true otherwise.

The use of the connective *or* in a disjunction corresponds to one of the two ways the word *or* is used in English, namely, as an **inclusive or**. A disjunction is true when at least one of the two propositions is true. For instance, the inclusive or is being used in the statement

"Students who have taken calculus or computer science can take this class."

Here, we mean that students who have taken both calculus and computer science can take the class, as well as the students who have taken only one of the two subjects. On the other hand, we are using the **exclusive or** when we say

"Students who have taken calculus or computer science, but not both, can enroll in this class."

Here, we mean that students who have taken both calculus and a computer science course cannot take the class. Only those who have taken exactly one of the two courses can take the class.

Similarly, when a menu at a restaurant states, "Soup or salad comes with an entrée," the restaurant almost always means that customers can have either soup or salad, but not both. Hence, this is an exclusive, rather than an inclusive, or.

**TABLE 3** The Truth Table for the Disjunction of Two Propositions.

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

---

Let $p$ and $q$ be propositions. The *exclusive or* of $p$ and $q$, denoted by $p \oplus q$, is the proposition that is true when exactly one of $p$ and $q$ is true and is false otherwise.

As was previously remarked, the use of the connective *or* in a disjunction corresponds to one of the two ways the word *or* is used in English, namely, in an inclusive way. Thus, a disjunction is true when at least one of the two propositions in it is true. Sometimes, we use *or* in an exclusive sense. When the exclusive or is used to connect the propositions $p$ and $q$, the proposition "$p$ or $q$ (but not both)" is obtained. This proposition is true when $p$ is true and $q$ is false, and when $p$ is false and $q$ is true. It is false when both $p$ and $q$ are false and when both are true.

**TABLE 4** The Truth Table for the Exclusive Or of Two Propositions.

| $p$ | $q$ | $p \oplus q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

---

Let $p$ and $q$ be propositions. The *conditional statement* $p \rightarrow q$ is the proposition "if $p$, then $q$." The conditional statement $p \rightarrow q$ is false when $p$ is true and $q$ is false, and true otherwise. In the conditional statement $p \rightarrow q$, $p$ is called the *hypothesis* (or *antecedent* or *premise*) and $q$ is called the *conclusion* (or *consequence*).



**TABLE 5** The Truth Table for the Conditional Statement $p \rightarrow q$.

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|

The statement $p \rightarrow q$ is called a conditional statement because $p \rightarrow q$ asserts that $q$ is true on the condition that $p$ holds. A conditional statement is also called an **implication**.

The truth table for the conditional statement $p \rightarrow q$ is shown in Table 5. Note that the statement $p \rightarrow q$ is true when both $p$ and $q$ are true and when $p$ is false (no matter what truth value $q$ has).

If a person $A$ lives at Lahore, then he lives in Pakistan.

Because conditional statements play such an essential role in mathematical reasoning, a variety of terminology is used to express $p \rightarrow q$. You will encounter most if not all of the following ways to express this conditional statement:

"if $p$, then $q$"

"$p$ implies $q$"

"if $p$, $q$"

"$p$ only if $q$"

"$p$ is sufficient for $q$"

"a sufficient condition for $q$ is $p$"

"$q$ if $p$"

"$q$ whenever $p$"

"$q$ when $p$"

"$q$ is necessary for $p$"

"a necessary condition for $p$ is $q$"

"$q$ follows from $p$"

"$q$ unless $\neg p$"

## EXAMPLE:

Let $p$ be the statement "Maria learns discrete mathematics" and $q$ the statement "Maria will find a good job." Express the statement $p \rightarrow q$ as a statement in English.

## *Solution:*

From the definition of conditional statements, we see that when $p$ is the statement "Maria learns discrete mathematics" and $q$ is the statement "Maria will find a good job," $p \rightarrow q$ represents the statement

"If Maria learns discrete mathematics, then she will find a good job."

There are many other ways to express this conditional statement in English. Among the most natural of these are:

“Maria will find a good job when she learns discrete mathematics.”

“For Maria to get a good job, it is sufficient for her to learn discrete mathematics.”

and

“Maria will find a good job unless she does not learn discrete mathematics.”

---

The proposition $q \rightarrow p$ is called the **converse** of $p \rightarrow q$. The **contrapositive** of $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$. The proposition $\neg p \rightarrow \neg q$ is called the **inverse** of $p \rightarrow q$.

*A conditional statement and its contrapositive are equivalent.*

*The converse and the inverse of a conditional statement are also equivalent*

| | | | | Given conditional | Converse | Inverse | Contrapositive |
|---|---|---|---|---|---|---|---|
| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \rightarrow q$ | $q \rightarrow p$ | $\sim p \rightarrow \sim q$ | $\sim q \rightarrow \sim p$ |
| T | T | F | F | T | T | T | T |
| T | F | F | T | F | T | T | F |
| F | T | T | F | T | F | F | T |
| F | F | T | F | T | T | T | T |

---

What are the contrapositive, the converse, and the inverse of the conditional statement

“The home team wins whenever it is raining?”

*Solution:* Because “$q$ whenever $p$” is one of the ways to express the conditional statement $p \rightarrow q$, the original statement can be rewritten as

“If it is raining, then the home team wins.”

Consequently, the contrapositive of this conditional statement is

“If the home team does not win, then it is not raining.”

The converse is

“If the home team wins, then it is raining.”

The inverse is

“If it is not raining, then the home team does not win.”

Only the contrapositive is equivalent to the original statement.

Let $p$ and $q$ be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition "*p* if and only if *q*." The biconditional statement $p \leftrightarrow q$ is true when $p$ and $q$ have the same truth values, and is false otherwise. Biconditional statements are also called *bi-implications*.

There are some other common ways to express $p \leftrightarrow q$:

"*p* is necessary and sufficient for *q*"

"if *p* then *q*, and conversely"

"*p* iff *q*."

The last way of expressing the biconditional statement $p \leftrightarrow q$ uses the abbreviation "iff" for "if and only if." Note that $p \leftrightarrow q$ has exactly the same truth value as $(p \rightarrow q) \land (q \rightarrow p)$.

**TABLE 6** The Truth Table for the Biconditional $p \leftrightarrow q$.

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |

| $p$ | $q$ | $p \rightarrow q$ | $q \rightarrow p$ | $p \leftrightarrow q$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | F |

Let $p$ be the statement "You can take the flight," and let $q$ be the statement "You buy a ticket."

Then $p \leftrightarrow q$ is the statement

"You can take the flight if and only if you buy a ticket."

---

We have now introduced four important logical connectives—conjunctions, disjunctions, conditional statements, and biconditional statements—as well as negations

**TABLE 7** The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

| $p$ | $q$ | $\neg q$ | $p \vee \neg q$ | $p \wedge q$ | $(p \vee \neg q) \rightarrow (p \wedge q)$ |
|---|---|---|---|---|---|
| T | T | F | T | T | T |
| T | F | T | T | F | F |
| F | T | F | F | F | T |
| F | F | T | T | F | F |

---

**TABLE 8** Precedence of Logical Operators.

| *Operator* | *Precedence* |
|---|---|
| $\neg$ | 1 |
| $\wedge$ | 2 |
| $\vee$ | 3 |
| $\rightarrow$ | 4 |
| $\leftrightarrow$ | 5 |

A variable is called a **Boolean variable** if its value is either true or false.

| *Truth Value* | *Bit* |
|---|---|
| T | 1 |
| F | 0 |

Computers represent information using bits. A bit is a symbol with two po 0 (zero) and 1 (one). This meaning of the word bit comes from binary digit, because zeros and ones are the digits used in binary representations of numbers.

A bit can be used to represent a truth value, because there are two truth values, namely, true and false. As is customarily done, we will use a 1 bit to represent true and a 0 bit to represent false. That is, 1 represents T (true), 0 represents F (false). A variable is called a Boolean variable if its value is either true or false.

Computer bit operations correspond to the logical connectives. By replacing true by a one and false by a zero in the truth tables for the operators ∧, ∨, and ⊕, shown in Tables for the corresponding bit operations are obtained. We will also use the notation OR, AND, and XOR for the operators ∨, ∧, and ⊕, as is done in various programming languages.

**TABLE 9 Table for the Bit Operators *OR*, *AND*, and *XOR*.**

| $x$ | $y$ | $x \vee y$ | $x \wedge y$ | $x \oplus y$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |

A *bit string* is a sequence of zero or more bits. The *length* of this string is the number of bits in the string.

101010011 is a bit string of length nine. ◀

Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of the bit strings 01 1011 0110 and 11 0001 1101. (Here, and throughout this book, bit strings will be split into blocks of four bits to make them easier to read.)

*Solution:* The bitwise *OR*, bitwise *AND*, and bitwise *XOR* of these strings are obtained by taking the *OR*, *AND*, and *XOR* of the corresponding bits, respectively. This gives us

**Propositional Equivalences:**

An important type of step used in a mathematical argument is the replacement of a statement with another statement with the same truth value.

### 2.7.3 Tautologies

i) A statement which is true for all the possible values of the variables involved in it is called a tautology, for example, $p \to q \leftrightarrow (\sim q \to \sim p)$ is a *tautology*. (are already verified by a truth table).

ii) A statement which is always false is called an **absurdity** or a contradiction e.g., $p \to \sim p$



iii) A statement which can be true or false depending upon the truth values of the variables involved in it is called a **contingency** e.g., $(p \to q) \wedge (p \vee q)$ is a contingency.
(You can verify it by constructing its truth table).

### 2.7.4 Quantifiers

The words or symbols which convey the idea of quantity or number are called quantifiers.

In mathematics two types of quantifiers are generally used.

i) **Universal quantifier** meaning for all

Symbol used : $\forall$

ii) **Existential quantifier:** There exist (some or few, at least one) symbol used: $\exists$

**Logically Equivalent:**

Compound propositions that have the same truth values in all possible cases are called logically equivalent.

The compound propositions p and q are called logically equivalent if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes that p and q are logically equivalent.

The symbol $\equiv$ is not a logical connective, and $p \equiv q$ is not a compound proposition but rather is the statement that $p \leftrightarrow q$ is a tautology.

The symbol $\Leftrightarrow$ is sometimes used instead of $\equiv$ to denote logical equivalence.

**TABLE 1 Examples of a Tautology and a Contradiction.**

| $p$ | $\neg p$ | $p \vee \neg p$ | $p \wedge \neg p$ |
|---|---|---|---|
| T | F | T | F |
| F | T | T | F |

**TABLE 2 De Morgan's Laws.**

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

**TABLE 3 Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.**

| $p$ | $q$ | $p \vee q$ | $\neg(p \vee q)$ | $\neg p$ | $\neg q$ | $\neg p \wedge \neg q$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | F | T | F | F | T | F |
| F | T | T | F | T | F | F |
| F | F | F | T | T | T | T |

Show that $p \to q$ and $\neg p \vee q$ are logically equivalent.

*Solution:* We construct the truth table for these compound propositions in Table 4. Because the truth values of $\neg p \vee q$ and $p \to q$ agree, they are logically equivalent. ◀

**TABLE 4** **Truth Tables for $\neg p \vee q$ and $p \to q$.**

| $p$ | $q$ | $\neg p$ | $\neg p \vee q$ | $p \to q$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

**TABLE 5** **A Demonstration That $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ Are Logically Equivalent.**

| $p$ | $q$ | $r$ | $q \wedge r$ | $p \vee (q \wedge r)$ | $p \vee q$ | $p \vee r$ | $(p \vee q) \wedge (p \vee r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | F | T | T | T | T |
| T | F | T | F | T | T | T | T |
| T | F | F | F | T | T | T | T |
| F | T | T | T | T | T | T | T |
| F | T | F | F | F | T | F | F |
| F | F | T | F | F | F | T | F |
| F | F | F | F | F | F | F | F |

Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent. This is the *distributive law* of disjunction over conjunction.

Table 6 contains some important equivalences. In these equivalences, **T** denotes the compound proposition that is always true and **F** denotes the compound proposition that is always

**TABLE 6 Logical Equivalences.**

| *Equivalence* | *Name* |
|---|---|
| $p \wedge \mathbf{T} \equiv p$<br>$p \vee \mathbf{F} \equiv p$ | Identity laws |
| $p \vee \mathbf{T} \equiv \mathbf{T}$<br>$p \wedge \mathbf{F} \equiv \mathbf{F}$ | Domination laws |
| $p \vee p \equiv p$<br>$p \wedge p \equiv p$ | Idempotent laws |
| $\neg(\neg p) \equiv p$ | Double negation law |
| $p \vee q \equiv q \vee p$<br>$p \wedge q \equiv q \wedge p$ | Commutative laws |
| $(p \vee q) \vee r \equiv p \vee (q \vee r)$<br>$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ | Associative laws |
| $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$<br>$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ | Distributive laws |
| $\neg(p \wedge q) \equiv \neg p \vee \neg q$<br>$\neg(p \vee q) \equiv \neg p \wedge \neg q$ | De Morgan's laws |
| $p \vee (p \wedge q) \equiv p$<br>$p \wedge (p \vee q) \equiv p$ | Absorption laws |
| $p \vee \neg p \equiv \mathbf{T}$<br>$p \wedge \neg p \equiv \mathbf{F}$ | Negation laws |

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

$$\begin{aligned} \neg(p \rightarrow q) &\equiv \neg(\neg p \vee q) \\ &\equiv \neg(\neg p) \wedge \neg q \\ &\equiv p \wedge \neg q \end{aligned}$$

Show that $\neg(p \lor (\neg p \land q))$ and $\neg p \land \neg q$ are logically equivalent by developing a series of logical equivalences.

*Solution:*

t

$$
\begin{aligned}
\neg(p \lor (\neg p \land q)) &\equiv \neg p \land \neg(\neg p \land q) && \text{by the second De Morgan law}\\
&\equiv \neg p \land [\neg(\neg p) \lor \neg q] && \text{by the first De Morgan law}\\
&\equiv \neg p \land (p \lor \neg q) && \text{by the double negation law}\\
&\equiv (\neg p \land p) \lor (\neg p \land \neg q) && \text{by the second distributive law}\\
&\equiv \mathbf{F} \lor (\neg p \land \neg q) && \text{because } \neg p \land p \equiv \mathbf{F}\\
&\equiv (\neg p \land \neg q) \lor \mathbf{F} && \text{by the commutative law for disjunction}\\
&\equiv \neg p \land \neg q && \text{by the identity law for } \mathbf{F}
\end{aligned}
$$

Consequently $\neg(p \lor (\neg p \land q))$ and $\neg p \land \neg q$ are logically equivalent. ◀

Show that $(p \land q) \rightarrow (p \lor q)$ is a tautology.

*Solution:* To show that this statement is a tautology, we will use logical equivalences to demonstrate that it is logically equivalent to **T**. (*Note:* This could also be done using a truth table.)

$$
\begin{aligned}
(p \land q) \rightarrow (p \lor q) &\equiv \neg(p \land q) \lor (p \lor q) && \text{by Example 3}\\
&\equiv (\neg p \lor \neg q) \lor (p \lor q) && \text{by the first De Morgan law}\\
&\equiv (\neg p \lor p) \lor (\neg q \lor q) && \text{by the associative and commutative laws for disjunction}\\
&\equiv \mathbf{T} \lor \mathbf{T} && \text{by Example 1 and the commutative law for disjunction}\\
&\equiv \mathbf{T} && \text{by the domination law}
\end{aligned}
$$

◀

Discrete

## Objectives

The main aim of lecture is define the notation Proposition or statment.
compound proposition
conjunction, disjunction and negation.

## Logic and Proportion calculs:

Many algorithm and Proofs use logical expressions such:
IF P THEN q" or "If $P_1$ AND $P_2$ ~~FE~~ THEN $q_1$ OR $q_2$"
Therefore it is necessary to know the cases in which these expressions are TRUE or FALS, that is, to know the ~~thtuth~~ "truth value" of such expression.

## Proposition or Statement:

A proposition (or statement)

contengens

is declarative stetment which is true or false, but not both.

**Example:**

consider, for example, the following six sentences:

1 Ice floats in water

2 china in Europe.

3 $2+2=4$

4 $2+2=5$

5 where are you going.

6 Do your home work.

The first four are Proposition, the last two are not. Also (i) and (iii) are true, but (ii) and (iv) are false

**Compound:**

Many Proposition are composite, that is, composed of sub Proposition and various connectives discussed Subsequently. Such composite Proposition are called compamd Proposition.

A Proposition is said to be Primitive if it cannot be broken dawn into simpler Proposition that is, if it is not composite.

**Example:**

The above Proposition (i) through (iv) are Primitive Propositions. On the other hand, the following two Proposition are composite:

(i) "Roses are red and violets are

(ii) blue" and "jhon is smart or

(iii) he studies every night".

**Remarks:**

The fundamental Property of a compound proposition is that its true value is is completely determined by the truth value of its subpropositions together with the

which they are connected

to from the compound
Proposition.

**Basic Logical Operations:**

Now we discuss the three basic logical operations of ~~conj~~ conjunction, disjunction, and negation which correspond, respectively, to the English words "AND" "OR" and "Not".

Examples:

(i) Roses are red and violets are blue.

(ii) John is smart or he studies every night

(iii) China is not in Europe.

**conjunction, $P \wedge q$**

Any two propositions can be combined by the word "and" to form a compound proposition called the conjunction of the original propositions. Symbolically,

the conjunction of p and q.
Since $p \wedge q$ is a proposition it has a truth value, and this truth value depends only on the truth values of p and q defined as follows:

Definition

If p and q are true, then $p \wedge q$ is true; otherwise $p \wedge q$ is false.

Examples:

Consider the following statments:

(i) Ice floats in water and 2+2=4 (True)

(ii) China is in Europe and 2+2 =4 (False)

Disjunction $p \vee q$

Any two propositions can be combined by the word "or" to form a compound propositions called the disjunction of the original denotes the disjunction of $p$ and $q$. The truth value of $p \vee q$ depends only on the truth values of $p$ and $q$ as follows:

**Definition:**

If $p$ and $q$ are false, then $p \vee q$ is false; otherwise $p \vee q$ is true.

**Example:** consider the following four statements:

Ice floats in water or 2+2 = 4 (True)

Ice floats in water or 2+2 = 5 (True)

china is in Europe and 2+2 = 5 (False)

**Negation, $\neg p$**

Given any Proposition $p$, another proposition, called the negation of $p$, can be formed by writing "It

read "not p," is denoted by $\neg p$
The truth value of $\neg p$ depends on the truth value of p as follows:

**Defination:**

If p is true, the $\neg p$ is false; and if p is false then $\neg p$ is true.

**Example:**

$(a_1)$ Ice floats in water $(a_2)$ It is false that ice ~~floats in wat~~ er.
$(a_3)$ Ice does not float in water.
$(b_1)$ $2+2=5$ $(b_2)$ It is false that $2+2=5$
$(b_3)$ $2+2\neq5$
Then $(a_2)$ and $(a_3)$ are each the negation of $(a_1)$; and $(b_2)$ and $(b_3)$ are each the negation of $(b_1)$
since $(a_1)$ is true $(a_2)$ and $(a_3)$

Mon Tue Wed Thu Fri Sat

## Question No # 1

$(A \cap B)' = A' \cup B'$

$\sim(p \wedge q) = \sim p \vee \sim q$

| p | q | ~p | ~q | p∧q | ~(p∧q) | ~p∨~q |
|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F |
| T | F | F | T | F | T | T |
| F | T | T | F | F | T | T |
| F | F | T | T | F | T | T |

## Question No # 2:

$(A \cup B) \cup C$ and $A \cup (B \cup C)$

$(p \vee q) \vee r = p \vee (q \vee r)$

| p | q | r | p∨q | (p∨q)∨r | (q∨r) | p∨(q∨r) |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |

| $p$ | $q$ | $r$ | $p \vee q$ | $(p \vee q) \vee r$ | $(q \vee r)$ | $p \vee (q \vee r)$ |
|---|---|---|---|---|---|---|
| F | F | T | F | T | T | T |
| F | F | F | F | F | F | F |

# Question No #3

$(A \cap B) \cap C = A \cap (B \cap C)$

$$(p \wedge q) \wedge r = p \wedge (q \wedge r)$$

| $p$ | $q$ | $r$ | $p \wedge q$ | $(p \wedge q) \wedge r$ | $(q \wedge r)$ | $p \wedge (q \wedge r)$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | F | T | F | F | F |
| T | F | T | F | F | F | F |
| T | F | F | F | F | F | F |
| F | T | T | F | F | T | F |
| F | T | F | F | F | F | F |
| | | | [illegible] | F | F | F |

| $p$ | $q$ | $r$ | $(q \wedge r)$ | $p \vee (q \wedge r)$ | $(p \vee q)$ | $(p \vee r)$ | $(p \vee q)$ [illegible] |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | F | F | F | T | T | T | T |
| T | F | T | F | T | T | T | T |
| T | F | F | F | T | T | T | T |
| F | T | T | T | T | T | T | T |
| F | T | F | F | F | T | F | F |
| F | F | T | F | F | F | T | F |
| F | F | F | F | F | F | F | F |

## Past Paper : 2014

$(p \vee q) \vee \sim p$

| $p$ | $q$ | $p \vee q$ | $\sim p$ | $(p \vee q) \vee \sim p$ |
|---|---|---|---|---|
| T | T | T | F | T |

| $p$ | $q$ | $r$ | $q \to r$ | $p \to (q \to r)$ | $(p \land \neg r)$ | $\neg r$ | $\neg q$ | $(p \land \neg r) \to \neg q$ |
|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | F | F | F | T |
| T | T | F | F | F | T | T | F | F |
| T | F | T | T | T | F | F | T | T |
| T | F | F | T | T | T | T | T | T |
| F | T | T | T | T | F | F | F | T |
| F | T | F | F | T | F | T | F | T |
| F | F | T | T | T | F | F | T | T |
| F | F | F | T | T | F | T | T | T |

write converse of $p \to q$ ?

| $p$ | $q$ | $p \to q$ | $q \to p$ |
|---|---|---|---|
| T | T | T | T |

## Universal Quantifiers

$P(x)$ is a preposition which is true for all values of $x$ than its notation will be

$$\forall x, P(x) \text{ is true}$$

## Existential Quantifiers

If $P(x)$ is a ~~statement~~ or preposition "There exist an element $x$ in the universe of discourse such that $P(x)$ is true."

Example

$$X = \{x_1, x_2, x_3, \ldots, x_n\}$$

$$\exists x P(x) = P(x_1) \vee P(x_2) \ldots P(x_n)$$

$P(x)$ is true for some $x$.

$$(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$$

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \rightarrow q$ | $\sim q \wedge (p \rightarrow q)$ | $(\sim q \wedge (p \rightarrow q))$ |
|---|---|---|---|---|---|---|

2. Verify whether $(P \wedge Q) \rightarrow (P \vee Q)$ is tautology or not?

Solution:-

| $P$ | $Q$ | $P \wedge Q$ | $P \vee Q$ | $(P \wedge Q) \rightarrow (P \vee Q)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | F | T | T |
| F | F | F | F | T |

Hence $(P \wedge Q) \rightarrow (P \vee Q)$ is tautology.

Let $Q(x)$ denote the statement "$x = x+1$". what are the truth values of quantification $\exists x Q(x)$, where the domain consists all real number?

If $x$ belongs ...

"$x = x+1$" are "false" for all real numbers.

16 Suppose the domain of propositinal function $p(x)$ consiststs integers 1, 2, 3, 4, & 5 ............... "Some students have no ID cards"?

(i) $\sim \exists x\, p(x) \sim (p(1) \wedge p(2) \wedge p(3) \wedge p(4) \wedge p(5))$

Ans "There is no $x$ for which $p(x)$ is not true".

(ii) Some students have no ID card

Ans:- $\{\exists x \sim p(x)\}$

where $p(x)$ denotes ID Card.

**1. What is Compound Proposition? Give an example.**

Mathematical statements that is constructed by combining one or more propositions is called compound propositions.
Example: “3 + 2 = 5” **and** “Lahore is a city in Pakistan”.

**3.What are the contrapositive, the converse, and the inverse of conditional statement? “The home team wins whenever it is raining”?**

Because “q whenever *p*” is one of the ways to express the conditional statement $p \to q$, the original statement can be rewritten as “If it is raining, then the home team wins.” Consequently, the contrapositive of this conditional statement is “If the home team does not win, then it is not raining.”

The converse is “If the home team wins, then it is raining.”

The inverse is “If it is not raining, then the home team does not win.”

Only the contrapositive is equivalent to the original statement.

**4. What is Bi-implication? State with an example.**

Bi-implication: Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition “p if and only if q.” The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called bi-implications.

**Example:** “You can take the flight if and only if you buy a ticket.” is bi-implication/ biconditional

**5. Show that $\neg (p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.**

Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.

| $p$ | $q$ | $p \vee q$ | $\neg(p \vee q)$ | $\neg p$ | $\neg q$ | $\neg p \wedge \neg q$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | F | T | F | F | T | F |
| F | T | T | F | T | F | F |
| F | F | F | T | T | T | T |

**1. Differentiate between bound and free variable with the help of an example?**

When a quantifier is used on the variable *x*, we say that this occurrence of the variable is **bound**. An occurrence of a variable that is not bound by a quantifier or set equal to a particular value is said to be **free**. For **Example**: In the statement $\exists x(x + y = 1)$, the variable *x* is bound by the existential quantification $\exists x$, but the variable *y* is free because it is not bound by a quantifier and no value is assigned to this variable.

**2. Find the negation of this statement $\forall x \forall y( (x > 0) \wedge (y<0) \to (xy < 0))$**

The negation of the statement $\forall x \forall y( (x > 0) \wedge (y<0) \to (xy < 0))$ is

What is a vacuous statement?

In logic, **statements of type if P, then Q are said to be vacuously true when the proposition P is false**. For example, the statement, if sun rises in the north then everyone gets 100 percent in final exam, is a true statement since the proposition "sun rises in the north" is false.

What is the resolution rule?

Resolution rule

The resolution rule in propositional logic is **a single valid inference rule that produces a new clause implied by two clauses containing complementary literals**. A literal is a propositional variable or the negation of a propositional variable.

**16. Use truth table to determine whether $(\neg p \land (p \rightarrow q)) \rightarrow \neg q$ is tautology.**

$(\neg p \land (p \rightarrow q)) \rightarrow \neg q$ is Tautology

| p | q | $\neg p$ | $\neg q$ | $p \rightarrow q$ | $\neg p \land (p \rightarrow q)$ | $(\neg p \land (p \rightarrow q)) \rightarrow \neg q$ |
|---|---|---|---|---|---|---|
| F | F | T | T | T | T | T |
| F | T | T | F | T | T | T |
| T | F | F | T | F | F | T |
| T | T | F | F | T | F | T |

**Example 1** All the following declarative sentences are propositions.

Extra Examples

1. Washington, D.C., is the capital of the United States of America.
2. Toronto is the capital of Canada.
3. $1 + 1 = 2$.
4. $2 + 2 = 3$.

Propositions 1 and 3 are true, whereas 2 and 4 are false.

**Example 2** Consider the following sentences.

1. What time is it?
2. Read this carefully.
3. $x + 1 = 2$.

Sentences 1 and 2 are not propositions because they are not declarative sentences. Sentences 3 and 4 are not propositions because they are neither true nor false. Note that each of sentences 3 and 4 can be turned such as these into propositions in Section 1.3.

**Example 3** *Find the negation of the proposition*

Extra Examples

*"Today is Friday."*

*and express this in simple English.*

**Solution** The negation is

"It is not the case that today is Friday."

This negation can be more simply expressed by

"Today is not Friday,"

or

"It is not Friday today."

**Example 4** *Find the negation of the proposition*

*"At least 10 inches of rain fell today in Miami."*

*and express this in simple English.*

**Solution** The negation is

"It is not the case that at least 10 inches of rain fell today in Miami."

This negation can be more simply expressed by

"Less than 10 inches of rain fell today in Miami."

**Example 5** *Find the conjunction of the propositions $p$ and $q$ where $p$ is the proposition "Today is Friday" and $q$ is the proposition "It is raining today."*

**Solution** The conjunction of these propositions, $p \wedge q$, is the proposition "Today is Friday and it is raining today." This proposition is true on rainy Fridays and is false on any day that is not a Friday and on Fridays when it does not rain.

**Example 6** *What is the disjunction of the propositions $p$ and $q$ where $p$ and $q$ are the same propositions as in Example 5?*

**Solution** The disjunction of $p$ and $q$, $p \vee q$, is the proposition

"Today is Friday or it is raining today."

Extra

**Example 7** *Let $p$ be the statement "Maria learns discrete mathematics" and $q$ the statement "Maria will find a good job." Express the statement $p \to q$ as a statement in English.*

**Solution** From the definition of conditional statements, we see that when $p$ is the statement "Maria learns discrete mathematics" and $q$ is the statement "Maria will find a good job," $p \to q$ represents the statement

Extra Examples

"If Maria learns discrete mathematics, then she will find a good job."

**Example 8** *What is the value of the variable x after the statement*

*if* $2 + 2 = 4$ *then* $x := x + 1$

*if $x = 0$ before this statement is encountered? (The symbol $:=$ stands for assignment. The statement $x := x + 1$ means the assignment of the value of $x + 1$ to $x$.)*

**Solution** Because $2 + 2 = 4$ is true, the assignment statement $x := x + 1$ is executed. Hence, $x$ has the value $0 + 1 = 1$ after this statement is encountered. ◀

**Example 9** *What are the contrapositive, the converse, and the inverse of the conditional statement*

*"The home team wins whenever it is raining."?*

**Solution** Because "$q$ whenever $p$" is one of the ways to express the conditional statement $p \to q$, the original statement can be rewritten as

Extra Examples

"If it is raining, then the home team wins."

Consequently, the contrapositive of this conditional statement is

"If the home team does not win, then it is not raining."

The converse is

"If the home team wins, then it is raining."

The inverse is

"If it is not raining, then the home team does not win."

Only the contrapositive is equivalent to the original statement. ◀

**Example 10** Let $p$ be the statement "You can take the flight" and let $q$ be the statement "You buy a ticket." Then $p \leftrightarrow q$ is the statement

"You can take the flight if and only if you buy a ticket."

This statement is true if $p$ and $q$ are either both true or both false, that is, if you buy a ticket and can take the flight or if you do not buy a ticket and you cannot take the flight. It is false