

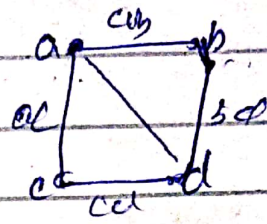
# Graphs

are non-linear data structure consists of a pair of Vertices & Edges  $(V, E)$

$$G = (V, E)$$

$$V = \{a, b, c, d\}$$

$$E = \{ab, bc, cd, ad\}$$



Size = 4 no of edges



A graph with an infinite vertex set or an infinite edges is called infinite graph.

A graph with a finite vertex set or a finite edges is called finite graphs.

A graph in which each edge connects two different vertices and where no two edges connects the same pair of vertices is a simple graph.

Graphs that may have multiple edges connecting the same vertices are called multigraphs.

An edge that connect a vertex to itself is called loop (edge).



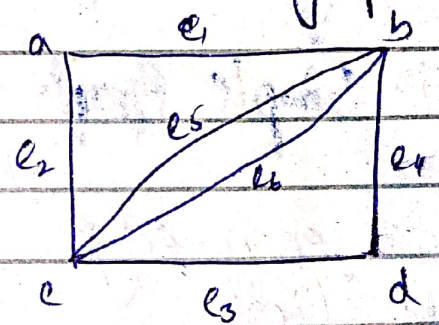
Graphs that may include loops and multiple edges connecting the same pair of vertices are called pseudographs.

Graphs containing multiple edges connected to same pair of vertices and having no direction are called undirected.

A graph consists of a nonempty set of vertices  $V$  and a set of directed edges is called directed graph.

A graph that does not have multiple directed edges connected to same pair of vertices is called simple directed graph.

A graph with multiple undirected and directed edges is called mixed graph.



$$V = \{a, b, c, d\}$$

$$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$$

Parallel edges

Edges having same starting and ending vertices

$e_5$  &  $e_6$  are parallel edges



## Adjacent Vertex

The vertices connected to a vertex are its adjacent vertices.

$a \rightarrow b, c$

$b \rightarrow a, c, d$

$c \rightarrow a, b, d$

$d \rightarrow c, b$

## Degree of Vertex

The number of vertices connected to a vertex is its degree

$$\deg(a) = 2$$

$$\deg(b) = 3$$

$$\deg(c) = 3$$

$$\deg(d) = 2$$

## Types of Graphs

### (i) Simple Graph

The graph that don't include loop and each edge connects two diff pair of vertices.



### Cyclic Graph

A walk is closed if end points are same. Such graph is called cyclic graph

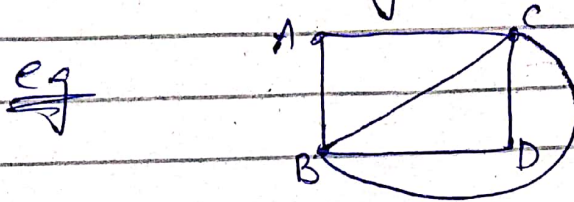
e.g.



Directed Graph: Null Graph  
Complete Directed Graph: Disconnected Graph  
Sub-Graph:

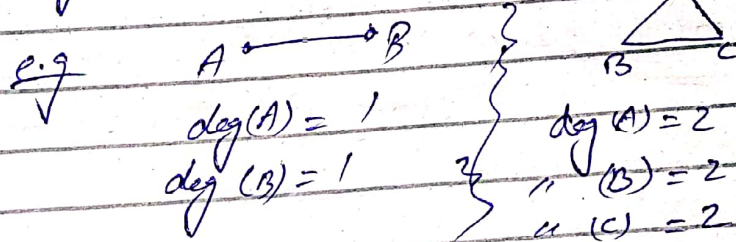
## Planner Graph

In planner graph there is no cross edges



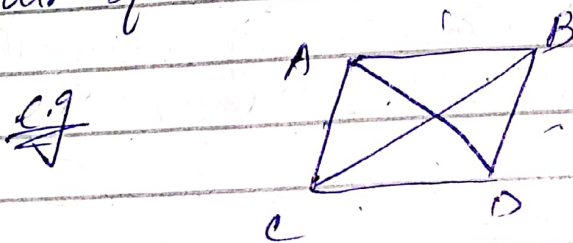
## Regular Graph

The graph that have equal degree of vertices



## Complete Graph

A graph in which each pair of vertices is connected by an edge



Edges of complete graph:

Total no of edges in a complete graph is given by

$$\text{edges} = \frac{n(n-1)}{2}$$

$\because n = \text{no of vertices}$



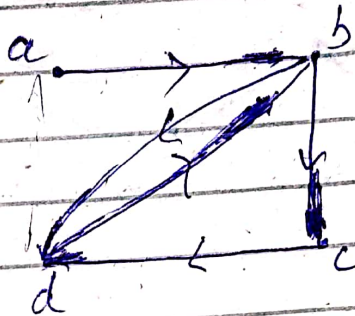
# Euler & Hamilton Path

Euler Path:-

1) Euler path visits every edge once.

$\Rightarrow$  An Euler path is possible, if exactly two vertices have odd degree  
~~and graph is directed graph~~

Example



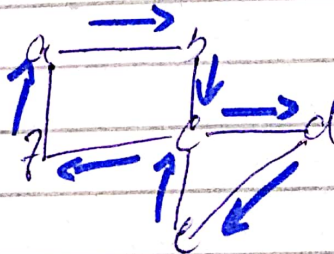
Euler path:

$a \rightarrow b \rightarrow c \rightarrow d \rightarrow b \rightarrow d$

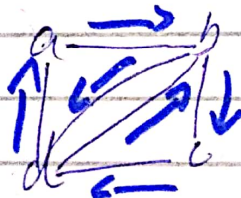
Euler circuit/cycle:-

An Euler circuit or cycle visits every edge once and "begins" and "ends" at the same vertex.

Example 1



Example 2



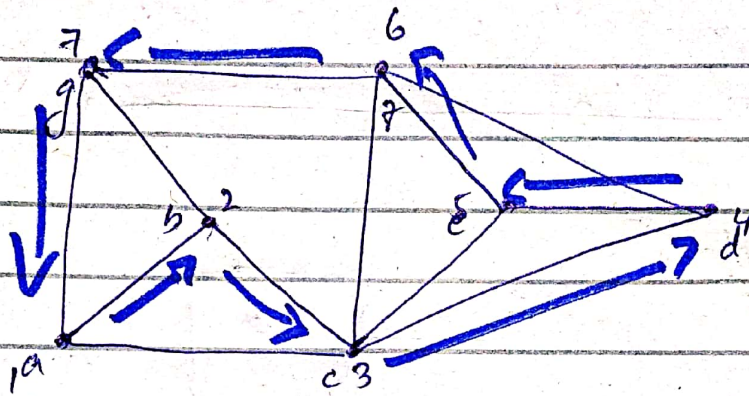
$\Rightarrow$  An Euler circuit is possible if it has even edges.



## Hamilton Path

A hamilton path visits every vertex once

Example:-



Hamilton Path:

$a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow f \rightarrow g \rightarrow h \rightarrow i$