University of Sargodha

BS 3rd Semester/Term Exam 2021

Subject: Information Technology **Paper: Linear Algebra (MATH-203)**

Time Allowed: 02:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. (2*16)

i. For what value of K the vectors $(1,-2,K)$ in R^3 be a linear combination of vectors $(3,0,-2)$ and $(2,-1,-5)$.

ii. Let $S = \{u = (1,2,1), v = (2,9,0), w = (3,3,4)\}$ form bases for R^3. Find the vector v in R^3 whose coordinate vector relative to bases S is $(-1,3,2)$.

iii. Use Wronskian to show that $f_1 = 1, f_2 = e^x$ and $f_3 = e^{2x}$ are linearly independent.

iv. If A is invertible matrix and n is nonnegative integer, then show that $(A^n)^{-1} = (A^{-1})^n$.

v. If B and C are both inverses of the matrix A, then $B = C$.

vi. Define Null space.

vii. Show that matrix P is orthogonal if and only if P^T is orthogonal.

viii. Define characteristic equation.

ix. State Calay's Hamilton theorem.

x. Show that matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ is zero of $g(x) = x^2 + 3x - 10$.

xi. If A is symmetric then show that $(A^{-1})^T = (A^T)^{-1}$.

xii. Normalize the vector $v = (1,2,4,5)$.

xiii. Consider the vector $u = (1,-5,3)$ and find $\|u\|_\infty, \|u\|_1, \|u\|_2$.

xiv. Show that set of all symmetric matrices is subspace of vector space of all $n \times n$ matrices.

xv. If $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$, & $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ find a & b.

xvi. Write the basis and dimension of vector space V of all $M_{2\times2}$ matrices.

Subjective part (3*16)

Q.2. a) Find Eigen values and bases for Eigen spaces of $A = \begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$.

b) Determine whether the vectors in R^4 are linear independent or linear dependent $(1,3,-1,-4), (3,8,-5,7), (2,9,4,23)$.

Q.3. a) Determine whether the vector $v = (3,3,-4)$ is a linear combination of $(1,2,3), y = (2,3,7), z = (3,5,6)$.

b) Find the inverse of matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$.

Q.4. a) Solve the system by Gauss elimination method

$$3x_1 + x_2 - x_3 = -4$$
$$x_1 + x_2 - 2x_3 = -4$$
$$-x_1 + 2x_2 - x_3 = 1.$$

b) Show that the set $\{1, i\}$ in $\mathbb{C}$ is linearly independent over $\mathbb{R}$ but linearly dependent over $\mathbb{C}$.

Q.5. a) Consider the set $v = \mathbb{R}"$ with standard addition and scalar multiplication defined as $rv = 0_V$ for any $v \in \mathbb{R}", r \in \mathbb{R}$, where $F = \mathbb{R}$, Check whether the set V over F forms a vector space or not?

b) Compute the determinant of $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 2 & 7 & 0 & 6 \\ 0 & 6 & 3 & 0 \\ 7 & 3 & 1 & -5 \end{bmatrix}$.

Q 6. a) Apply the Gram Schmidt process to transform the basis vectors $u_1 = (1,1,1), u_2 = (0,1,1)$ and $u_3 = (0,0,1)$ into an orthogonal basis and then normalize the orthogonal basis vectors to obtain an orthonormal basis.

b) Show that the matrix $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ cannot be diagonalized.