A *set* is an unordered collection of objects, called *elements* or *members* of the set. A set is said to *contain* its elements. We write $a \in A$ to denote that a is an element of the set A. The notation $a \notin A$ denotes that a is not an element of the set A.

It is common for sets to be denoted using uppercase letters. Lowercase letters are usually used to denote elements of sets.

One way to describe a set is to list all the members of a set, when this is possible. We use a notation where all members of the set are listed between braces. For example, the notation $\{a, b, c, d\}$ represents the set with the four elements a, b, c, and d. This way of describing a set is known as the **roster method**.

The set V of all vowels in the English alphabet can be written as $V = \{a, e, i, o, u\}$.

The set O of odd positive integers less than 10 can be expressed by $O = \{1, 3, 5, 7, 9\}$.

Sometimes the roster method is used to describe a set without listing all its members. Some members of the set are listed, and then *ellipses* (. . .) are used when the general pattern of the elements is obvious.

The set of positive integers less than 100 can be denoted by $\{1, 2, 3, \ldots, 99\}$.

Another way to describe a set is to use **set builder** notation. We characterize all those elements in the set by stating the property or properties they must have to be members. For instance, the set O of all odd positive integers less than 10 can be written as

$O = \{x \mid x \text{ is an odd positive integer less than } 10\}$,

or, specifying the universe as the set of positive integers, as

$O = \{x \in \mathbf{Z}^+ \mid x \text{ is odd and } x < 10\}$.

These sets, each denoted using a boldface letter, play an important role in discrete mathematics:

$\mathbf{N} = \{0, 1, 2, 3, \ldots\}$, the set of **natural numbers**

$\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, the set of **integers**

$\mathbf{Z}^+ = \{1, 2, 3, \ldots\}$, the set of **positive integers**

$\mathbf{Q} = \{p/q \mid p \in \mathbf{Z}, q \in \mathbf{Z}, \text{ and } q \neq 0\}$, the set of **rational numbers**

$\mathbf{R}$, the set of **real numbers**

$\mathbf{R}^+$, the set of **positive real numbers**

$\mathbf{C}$, the set of **complex numbers**.

Beware that mathematicians disagree whether 0 is a natural number. We consider it quite natural.

Recall the notation for **intervals** of real numbers. When a and b are real numbers with $a < b$, we write

$[a, b] = \{x \mid a \leq x \leq b\}$

$[a, b) = \{x \mid a \leq x < b\}$

$(a, b] = \{x \mid a < x \leq b\}$

$(a, b) = \{x \mid a < x < b\}$

Note that $[a, b]$ is called the **closed interval** from a to b and (a, b) is called the **open interval** from a to b.

Two sets are ***equal*** if and only if they have the same elements. Therefore, if A and B are sets, then A and B are equal if and only if $\forall x(x \in A \leftrightarrow x \in B)$.We write $A = B$ if A and B are equal sets.

The sets {1, 3, 5} and {3, 5, 1} are equal, because they have the same elements. Note that the order in which the elements of a set are listed does not matter. Note also that it does not matter if an element of a set is listed more than once, so {1, 3, 3, 3, 5, 5, 5, 5} is the same as the set {1, 3, 5} because they have the same elements.

THE EMPTY SET There is a special set that has no elements. This set is called the **empty set**, or **null set**, and is denoted by ∅. The empty set can also be denoted by { } (that is, we represent the empty set with a pair of braces that encloses all the elements in this set). Often, a set of elements with certain properties turns out to be the null set. For instance, the set of all positive integers that are greater than their squares is the null set.

{∅} has one more element than ∅.

A set with one element is called a **singleton set**. A common error is to confuse the empty {∅} has one more element than ∅. set ∅ with the set {∅}, which is a singleton set. The single element of the set {∅} is the empty set itself.

A mathematician named George Cantor's original version of set theory, known as **naive set theory.**

Sets can be represented graphically using Venn diagrams, named after the English mathematician John Venn.

In Venn diagrams the **universal set** *U,* which contains all the objects under consideration, is represented by a rectangle. Inside this rectangle, circles or other geometrical figures are used to represent sets. Sometimes points are used to represent the particular elements of the set

Draw a Venn diagram that represents *V,* the set of vowels in the English alphabet.

We draw a rectangle to indicate the universal set *U*, which is the set of the 26 letters of the English alphabet. Inside this rectangle we draw a circle to represent *V*. Inside this circle we indicate the elements of *V* with points.

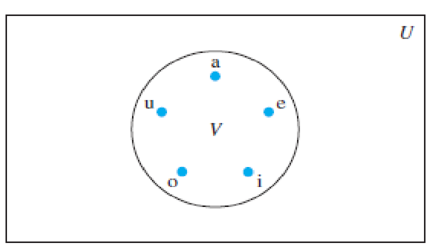


FIGURE 1 Venn Diagram for the Set of Vowels.

It is common to encounter s the elements of a second set.

The set A is a *subset* of B if and only if every element of A is also an element of B. We use the notation $A \subseteq B$ to indicate that A is a subset of the set B. We see that $A \subseteq B$ if and only if the quantification

$\forall x(x \in A \rightarrow x \in B)$

is true.

Showing that A is a Subset of B:

To show that $A \subseteq B$, show that if x belongs to A then x also belongs to B.

Showing that A is Not a Subset of B:

To show that $A \not\subseteq B$, find a single $x \in A$ such that $x \notin B$.

Note that to show that A is not a subset of B we need only find one element $x \in A$ with $x \notin B$.

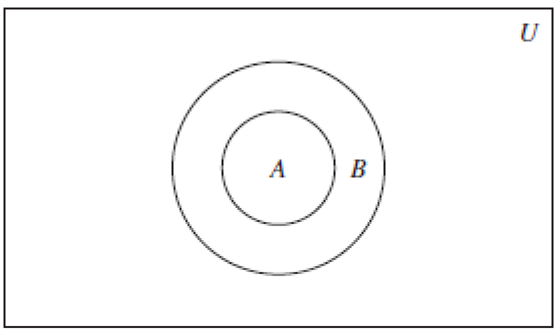


FIGURE 2 Venn Diagram Showing that A Is a Subset of B.

Every nonempty set S is guaranteed to have at least two subsets, the empty set and the set S itself, that is, $\emptyset \subseteq S$ and $S \subseteq S$.

When we wish to emphasize that a set A is a subset of a set B but that $A \neq B$, we write $A \subset B$ and say that A is a **proper subset** of B. For $A \subset B$ to be true, it must be the case that $A \subseteq B$ and there must exist an element x of B that is not an element of A. That is, A is a proper subset of B if and only if

$\forall x(x \in A \rightarrow x \in B) \wedge \exists x(x \in B \wedge x \notin A)$

We can show that if A and B are sets with $A \subseteq B$ and $B \subseteq A$, then $A = B$. That is, $A = B$ if and only if $\forall x(x \in A \rightarrow x \in B)$ and $\forall x(x \in B \rightarrow x \in A)$ or equivalently if and only if $\forall x(x \in A \leftrightarrow x \in B)$, which is what it means for the A and B to be equal.

$A = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

Note that $\{a\}$ belongs to A, but a does not belongs to A.

Let S be a set. If there are exactly n distinct elements in S where n is a nonnegative integer, we say that S is a *finite set* and that n is the *cardinality* of S. The cardinality of S is denoted by $|S|$.

The term *cardinality* comes from the common usage of the term *cardinal number* as the size of a finite set.

Let A be the set of odd positive integers less than 10. Then $|A| = 5$.

Let S be the set of letters in the English alphabet. Then $|S| = 26$.

Because the null set has no elements, it follows that $|\emptyset| = 0$.

A set is said to be *infinite* if it is not finite.

EXAMPLE: The set of positive integers is infinite.

Given a set S, the *power set* of S is the set of all subsets of the set S. The power set of S is denoted by $P(S)$.

What is the power set of the set {0, 1, 2}?

Solution: The power set $P(\{0, 1, 2\})$ is the set of all subsets of {0, 1, 2}. Hence,

$P(\{0, 1, 2\}) = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$.

Note that the empty set and the set itself are members of this set of subsets.

What is the power set of the empty set?

The empty set has exactly one subset $P(\emptyset) = \{\emptyset\}$.

What is the power set of the set $\{\emptyset\}$?

The set $\{\emptyset\}$ has exactly two subsets, namely, $\emptyset$ and the set $\{\emptyset\}$ itself. Therefore,

$P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$.

Empty Set:- Any set which have nothing inside braces
It is represented as ϕ.
So, Null or empty set can be written as:

ϕ or $\{\ \}$

Note that,

(1)

$A = \phi$

Subset of any empty set is the empty set itself i.e

Subset of $A = \phi$

~~Cardin~~ ~~Power Set of A = {ϕ}~~ Cardinality of set $A = 0$

Power Set is nothing but writing the subsets in a set i.e

The order of elements in a collection is often important. Because sets are unordered, a different structure is needed to represent ordered collections. This is provided by **ordered *n*-tuples**.

DEFINITION: The *ordered n-tuple* $(a_1, a_2, \ldots, a_n)$ is the ordered collection that has a_1 as its first element, a_2 as its second element, . . . , and a_n as its nth element.

We say that two ordered n-tuples are equal if and only if each corresponding pair of their elements is equal. In other words, $(a_1, a_2, \ldots, a_n) = (b_1, b_2, \ldots, b_n)$ if and only if $a_i = b_i$, for $i = 1, 2, \ldots, n$. In particular, ordered 2-tuples are called **ordered pairs**. The ordered pairs (a, b) and (c, d) are equal if and only if $a = c$ and $b = d$. Note that (a, b) and (b, a) are not equal unless $a = b$

Cartesian Products

Let A and B be sets. The *Cartesian product* of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. Hence,

$$A \times B = \{(a, b) \mid a \in A \land b \in B\}.$$

What is the Cartesian product of $A = \{1, 2\}$ and $B = \{a, b, c\}$?

Solution: The Cartesian product $A \times B$ is

$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$.

▲ Note that the Cartesian products $A \times B$ and $B \times A$ are not equal, unless $A = \emptyset$ or $B = \emptyset$ (so that $A \times B = \emptyset$) or $A = B$

The Cartesian product $B \times A$ is

$B \times A = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$.

This is not equal to $A \times B$

The *Cartesian product* of the sets $A_1, A_2, \ldots, A_n$, denoted by $A_1 \times A_2 \times \cdots \times A_n$, is the set of ordered n-tuples $(a_1, a_2, \ldots, a_n)$, where a_i belongs to A_i for $i = 1, 2, \ldots, n$. In other words,

$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for } i = 1, 2, \ldots, n\}$.

What is the Cartesian product $A \times B \times C$, where $A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{0, 1, 2\}$?

Solution: The Cartesian product$A \times B \times C$ consists of all ordered triples (a, b, c), where $a \in A$,

$b \in B$, and $c \in C$. Hence,

$A \times B \times C = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2),$

$(1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2)\}$.

$(A \times B) \times C$ is not the same as $A \times B \times C$

We use the notation A^2 to denote $A \times A$, the Cartesian product of the set A with itself. Similarly, $A^3 = A \times A \times A$, $A^4 = A \times A \times A \times A$, and so on.

Suppose that $A = \{1, 2\}$. It follows that $A^2 = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$ and
$A^3 = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)\}$.

A subset of the Cartesian Product $A \times B$ is called a **relation** from the set *A* to the set *B*.

The elements of *R* are ordered pairs, where the first element belongs to *A* and the second to *B*. For example, $R = \{(a, 0), (a, 1), (a, 3), (b, 1), (b, 2), (c, 0), (c, 3)\}$ is a relation from the set $\{a, b, c\}$ to the set $\{0, 1, 2, 3\}$.

A relation from a set *A* to itself is called a relation on *A*.

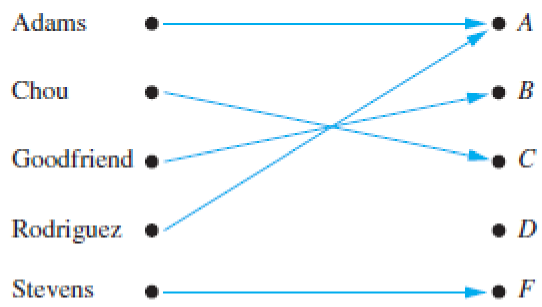
What are the ordered pairs in the less than or equal to relation, which contains (a, b) if $a \leq b$, on the set $\{0, 1, 2, 3\}$?

Solution: The ordered pair (a, b) belongs to *R* if and only if both *a* and *b* belong to $\{0, 1, 2, 3\}$ and $a \leq b$. Consequently, the ordered pairs in *R* are (0,0), (0,1), (0,2), (0,3), (1,1), (1,2), (1,3), (2,2), (2, 3), and (3, 3).

What do the statement $\forall x \in \mathbf{R}\ (x2 \geq 0)$ *mean?*

The statement $\forall x \in \mathbf{R}(x2 \geq 0)$ states that for every real number *x*, $x2 \geq 0$. This statement can be expressed as “The square of every real number is nonnegative.” This is a true statement.

Suppose that each student in a discrete mathematics class is assigned a letter grade from the set $\{A,B,C,D,F\}$. And suppose that the grades are *A* for Adams, *C* for Chou, *B* for Goodfriend, *A* for Rodriguez, and *F* for Stevens.



What are the domain, codomain, and range of the function that assigns grades to students described in the first paragraph of the introduction of this section?

Solution: Let G be the function that assigns a grade to a student in our discrete mathematics class. Note that $G(\text{Adams}) = A$, for instance. The domain of G is the set {Adams, Chou, Goodfriend, Rodriguez, Stevens}, and the codomain is the set $\{A, B, C, D, F\}$. The range of G is the set $\{A, B, C, F\}$, because each grade except D is assigned to some student. ◀

Let *A* and *B* be nonempty sets. A *function f* from *A* to *B* is an assignment of exactly one element of *B* to each element of *A*. We write $f(a) = b$ if *b* is the unique element of *B* assigned by the function *f* to the element *a* of *A*. If *f* is a function from *A* to *B*, we write $f : A \to B$.

Remark: Functions are sometimes also called **mappings** or **transformations**.

If *f* is a function from *A* to *B*, we say that *A* is the *domain* of *f* and *B* is the *codomain* of *f*. If $f(a) = b$, we say that *b* is the *image* of *a* and *a* is a *preimage* of *b*. The *range*, or *image*, of *f* is the set of all images of elements of *A*. Also, if *f* is a function from *A* to *B*, we say that *f maps A* to *B*.

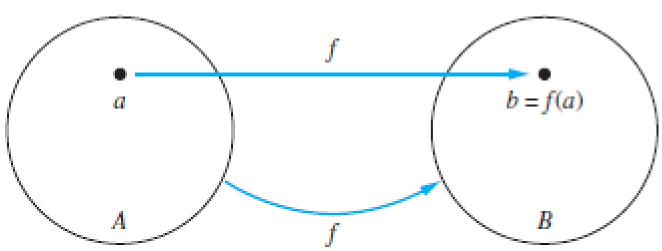


FIGURE 2 The Function *f* Maps *A* to *B*.

Let R be the relation with ordered pairs (Abdul, 22), (Brenda, 24), (Carla, 21), (Desire, 22), (Eddie, 24), and (Felicia, 22). Here each pair consists of a graduate student and this student's age. Specify a function determined by this relation.

Solution: If f is a function specified by R, then $f(\text{Abdul}) = 22$, $f(\text{Brenda}) = 24$, $f(\text{Carla}) = 21$, $f(\text{Desire}) = 22$, $f(\text{Eddie}) = 24$, and $f(\text{Felicia}) = 22$. (Here, $f(x)$ is the age of x, where x is a student.) For the domain, we take the set {Abdul, Brenda, Carla, Desire, Eddie, Felicia}. We also need to specify a codomain, which needs to contain all possible ages of students. Because it is highly likely that all students are less than 100 years old, we can take the set of positive integers less than 100 as the codomain. (Note that we could choose a different codomain, such as the set of all positive integers or the set of positive integers between 10 and 90, but that would change the function. Using this codomain will also allow us to extend the function by adding the names and ages of more students later.) The range of the function we have specified is the set of different ages of these students, which is the set $\{21, 22, 24\}$. ◀

Let f be the function that assigns the last two bits of a bit string of length 2 or greater to that string. For example, $f(11010) = 10$. Then, the domain of f is the set of all bit strings of length 2 or greater, and both the codomain and range are the set $\{00, 01, 10, 11\}$. ◀

Let $f: \mathbf{Z} \to \mathbf{Z}$ assign the square of an integer to this integer. Then, $f(x) = x^2$, where the domain of f is the set of all integers, the codomain of f is the set of all integers, and the range of f is the set of all integers that are perfect squares, namely, $\{0, 1, 4, 9, \dots\}$. ◀

The domain and codomain of functions are often specified in programming languages. For instance, the Java statement

int **floor**(float real){. . .}

and the C++ function statement

int **function** (float x){. . .}

both tell us that the domain of the floor function is the set of real numbers (represented by floating point numbers) and its codomain is the set of integers. ◀

A function is called **real-valued** if its codomain is the set of real numbers, and it is called **integer-valued** if its codomain is the set of integers. Two real-valued functions or two integer-valued functions with the same domain can be added, as well as multiplied.

Let f_1 and f_2 be functions from A to $\mathbf{R}$. Then $f_1 + f_2$ and $f_1 f_2$ are also functions from A to $\mathbf{R}$ defined for all $x \in A$ by

$$(f_1 + f_2)(x) = f_1(x) + f_2(x),$$
$$(f_1 f_2)(x) = f_1(x) f_2(x).$$

Let f_1 and f_2 be functions from $\mathbf{R}$ to $\mathbf{R}$ such that $f_1(x) = x^2$ and $f_2(x) = x - x^2$. What are the functions $f_1 + f_2$ and $f_1 f_2$?

Solution: From the definition of the sum and product of functions, it follows that

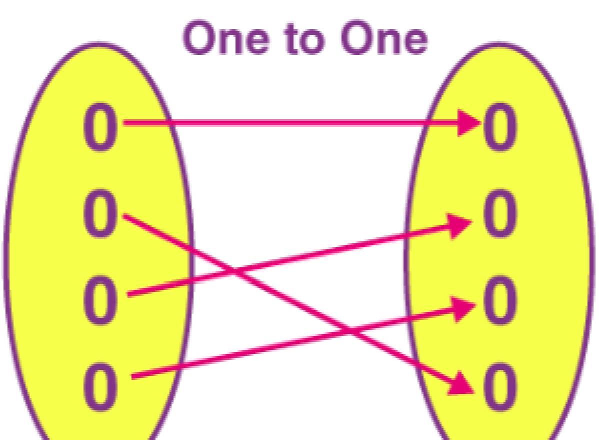
$$(f_1 + f_2)(x) = f_1(x) + f_2(x) = x^2 + (x - x^2) = x$$

and

$$(f_1 f_2)(x) = x^2(x - x^2) = x^3 - x^4.$$

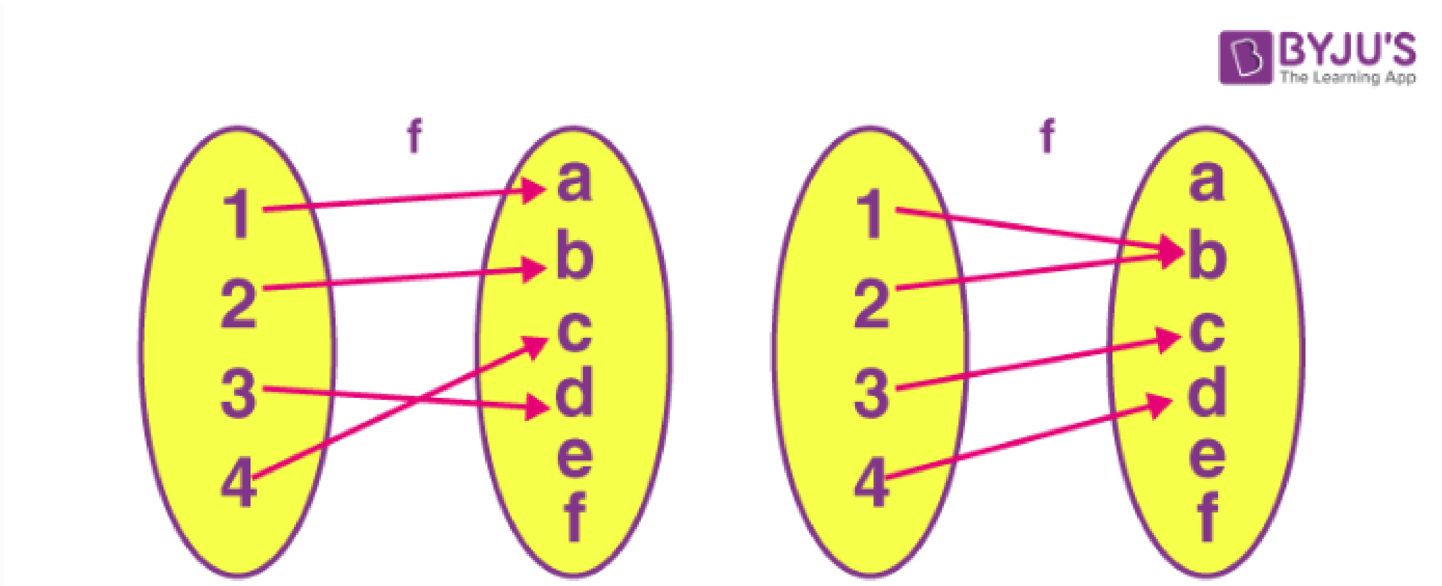
Let $A = \{a, b, c, d, e\}$ and $B = \{1, 2, 3, 4\}$ with $f(a) = 2$, $f(b) = 1$, $f(c) = 4$, $f(d) = 1$, and $f(e) = 1$. The image of the subset $S = \{b, c, d\}$ is the set $f(S) = \{1, 4\}$.

A function g is one-to-one if every element of the range of g corresponds to exactly one element of the domain of g. One-to-one is also written as 1-1.
It is also known as injective function.



It could be defined as each element of Set A has a unique element on Set B.

The below figure shows two functions, where (i) is the injective (one to one) function and (ii) is not an injective, i.e. many-one function.



Examples of Injective Function

- The identity function X → X is always injective.
- If function f: R→ R, then f(x) = 2x is injective.
- If function f: R→ R, then f(x) = 2x+1 is injective.
- If function f: R→ R, then f(x) = x^2 is not an injective function, because here if x = -1, then f(-1) = 1 = f(1). Hence, the element of codomain is not discrete here.
- If function f: R→ R, then f(x) = x/2 is injective.
- If function f: R→ R, then f(x) = x^3 is injective.
- If function f: R→ R, then f(x) = 4x+5 is injective.

Determine whether the function f from $\{a, b, c, d\}$ to $\{1, 2, 3, 4, 5\}$ with $f(a) = 4$, $f(b) = 5$, $f(c) = 1$, and $f(d) = 3$ is one-to-one.

Solution: The function f is one-to-one because f takes on different values at the four elements of its domain. This is illustrated in Figure

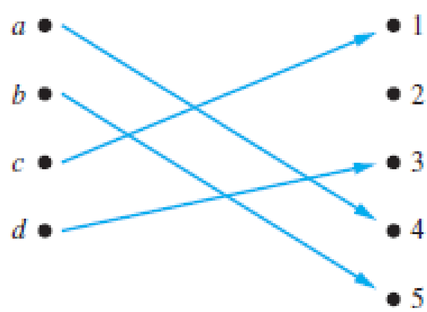


FIGURE 3 **A One-to-One Function.**

Determine whether the function $f(x) = x + 1$ from the set of real numbers to itself is one-to-one.

Solution: The function $f(x) = x + 1$ is a one-to-one function. To demonstrate this, note that $x + 1 \neq y + 1$ when $x \neq y$.

Suppose that each worker in a group of employees is assigned a job from a set of possible jobs, each to be done by a single worker. In this situation, the function f that assigns a job to each worker is one-to-one. To see this, note that if x and y are two different workers, then $f(x) \neq f(y)$ because the two workers x and y must be assigned different jobs.

Note that the function $f(x) = x^2$ with its domain restricted to $\mathbf{Z}^+$ is one-to-one.

Example 1:

Let A = {1, 2, 3} and B = {a, b, c, d}. Which of the following is a one-to-one function?

1. {(1, c), (2, c)(2, c)}
2. {(1, a),(2, b),(3, c)}
3. {(1, b)(1, c)}

The Answer is 2.

Explanation: Here, option number 2 satisfies the one-to-one condition, as elements of set B(range) are uniquely mapped with elements of set A(domain).

Which function is not said to be one to one?
If a horizontal line can intersect the graph of the function, more than one time, then the function is not mapped as one-to-one.

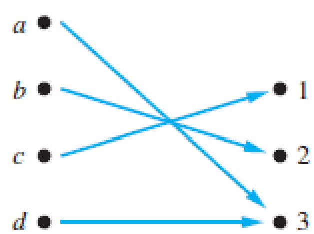


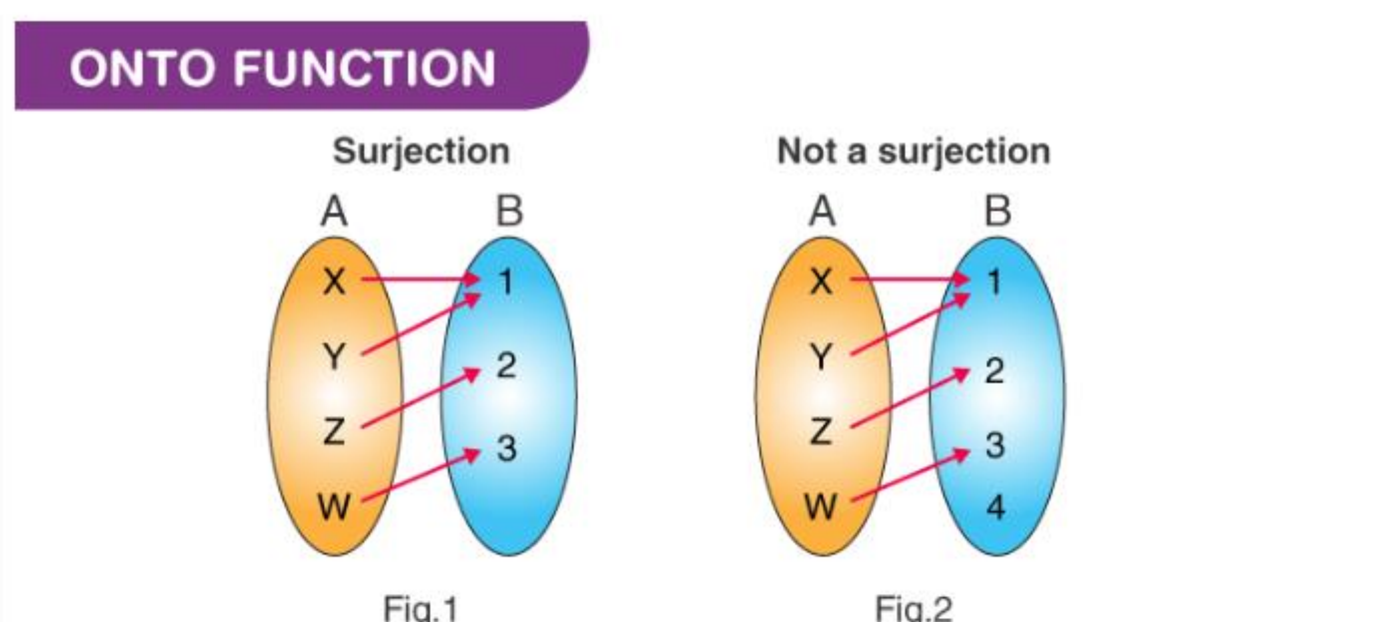
FIGURE 4 **An Onto Function.**

A function *f* whose domain and codomain are subsets of the set of real numbers is called *increasing* if $f(x) \leq f(y)$, and *strictly increasing* if $f(x) < f(y)$, whenever $x < y$ and x and y are in the domain of f. Similarly, f is called *decreasing* if $f(x) \geq f(y)$, and *strictly decreasing* if $f(x) > f(y)$, whenever$x < y$ and x and y are in the domain of f.

A function f from A to B is called *onto,* or a *surjection,* if and only if for every element

Onto Function Definition (Surjective Function)

Onto function could be explained by considering two sets, Set A and Set B, which consist of elements. If for every element of B, there is at least one or more than one element matching with A, then the function is said to be **onto function** or surjective function. The term for the surjective function was introduced by Nicolas Bourbaki.



Example 1:

Let A = {1, 5, 8, 9) and B {2, 4} And f={(1, 2), (5, 4), (8, 2), (9, 4)}. Then prove f is a onto function.

Solution:

From the question itself we get,

A={1, 5, 8, 9)

B{2, 4}

& f={(1, 2), (5, 4), (8, 2), (9, 4)}

So, all the element on B has a domain element on A or we can say element 1 and 8 & 5 and 9 has same range 2 & 4 respectively.

Therefore, f: A ⟶ B is a surjective function.

How to determine if a graph is onto?

The function “f” is onto, if and only if its graph function intersects the horizontal line at least once.

EXAMPLE 12 Let f be the function from $\{a, b, c, d\}$ to $\{1, 2, 3\}$ defined by $f(a) = 3$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$. Is f an onto function?

Extra Examples

Solution: Because all three elements of the codomain are images of elements in the domain, we see that f is onto. This is illustrated in Figure 4. Note that if the codomain were $\{1, 2, 3, 4\}$, then f would not be onto. ◀

EXAMPLE 13 Is the function $f(x) = x^2$ from the set of integers to the set of integers onto?

Solution: The function f is not onto because there is no integer x with $x^2 = -1$, for instance. ◀

What can we call if a function is both injective and surjective?

If a function is both injective and surjective, then the function is called the bijective function, which is also called the one-to-one correspondence.

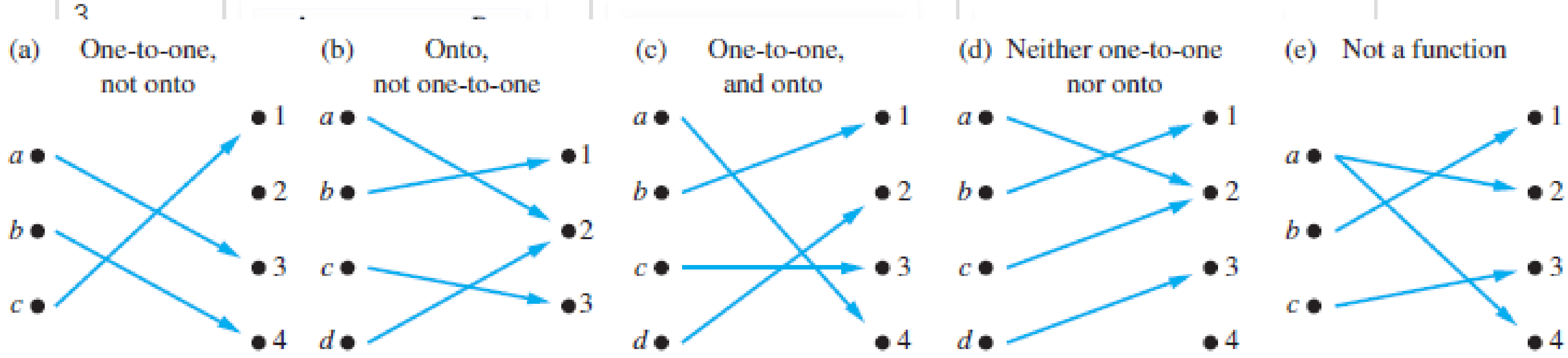
What is Bijective Function?

A function is said to be bijective or bijection, if a function f: A → B satisfies both the injective (one-to-one function) and surjective function (onto function) properties. It means that every element "b" in the codomain B, there is exactly one element "a" in the domain A. such that f(a) = b. If the function satisfies this condition, then it is known as one-to-one correspondence.

Difference between Injective, Surjective, and Bijective Function

The difference between injective, surjective and bijective functions are given below:

S.No	Injective Function	Surjective Function	Bijective Function
1	A function that always maps the distinct element of its domain to the distinct element of its codomain	A function that maps one or more elements of A to the same element of B	A function that is both injective and surjective
2	It is also known as one-to-one function	It is also known as onto function	It is also known as one-to-one correspondence
3			



The function f is a *one-to-one correspondence*, or a *bijection*, if it is both one-to-one and onto. We also say that such a function is *bijective*.

Let f be the function from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$ with $f(a) = 4$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$. Is f a bijection?

Solution: The function f is one-to-one and onto. It is one-to-one because no two values in the domain are assigned the same function value. It is onto because all four elements of the codomain are images of elements in the domain. Hence, f is a bijection. ◀

Let A be a set. The *identity function* on A is the function $\iota_A : A \to A$, where

$$\iota_A(x) = x$$

for all $x \in A$. In other words, the identity function ι_A is the function that assigns each element to itself. The function ι_A is one-to-one and onto, so it is a bijection. (Note that ι is the Greek letter iota.) ◀

Let f be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$. Is f invertible, and if it is, what is its inverse?

Solution: The function f is invertible because it is a one-to-one correspondence. The inverse function f^{-1} reverses the correspondence given by f, so $f^{-1}(1) = c$, $f^{-1}(2) = a$, and $f^{-1}(3) = b$. ◀

Let $f : \mathbf{Z} \to \mathbf{Z}$ be such that $f(x) = x + 1$. Is f invertible, and if it is, what is its inverse?

Solution: The function f has an inverse because it is a one-to-one correspondence,
To reverse the correspondence, suppose that y is the image of x, so

Let f be the function from $\mathbf{R}$ to $\mathbf{R}$ with $f(x) = x^2$. Is f invertible?

Solution: Because $f(-2) = f(2) = 4$, f is not one-to-one. If an inverse function were defined, it would have to assign two elements to 4. Hence, f is not invertible. (Note we can also show that f is not invertible because it is not onto.) ◀

Sometimes we can restrict the domain or the codomain of a function, or both, to obtain an invertible function

A *sequence* is a function from a subset of the set of integers (usually either the set $\{0, 1, 2, \ldots\}$ or the set $\{1, 2, 3, \ldots\}$) to a set S. We use the notation a_n to denote the image of the integer n. We call a_n a *term* of the sequence.

We use the notation $\{a_n\}$ to describe the sequence. (Note that a_n represents an individual

Consider the sequence $\{a_n\}$, where

$$a_n = \frac{1}{n}.$$

The list of the terms of this sequence, beginning with a_1, namely,

$$a_1, a_2, a_3, a_4, \ldots,$$

starts with

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots.$$

A *geometric progression* is a sequence of the form

$$a, ar, ar^2, \ldots, ar^n, \ldots$$

where the *initial term* a and the *common ratio* r are real numbers.

EXAMPLE 2 The sequences $\{b_n\}$ with $b_n = (-1)^n$, $\{c_n\}$ with $c_n = 2 \cdot 5^n$, and $\{d_n\}$ with $d_n = 6 \cdot (1/3)^n$ are geometric progressions with initial term and common ratio equal to 1 and -1; 2 and 5; and 6

DEFINITION An *arithmetic progression* is a sequence of the form

$$a, a + d, a + 2d, \ldots, a + nd, \ldots$$

where the *initial term* a and the *common difference* d are real numbers.

EXAMPLE The sequences $\{s_n\}$ with $s_n = -1 + 4n$ and $\{t_n\}$ with $t_n = 7 - 3n$ are both arithmetic progressions with initial terms and common differences equal to -1 and 4, and 7 and -3, respectively, if we start at $n = 0$. The list of terms $s_0, s_1, s_2, s_3, \ldots$ begins with

$$-1, 3, 7, 11, \ldots,$$

and the list of terms $t_0, t_1, t_2, t_3, \ldots$ begins with

$$7, 4, 1, -2, \ldots.$$

Sequences of the form $a_1, a_2, \ldots, a_n$ are often used in computer science. These finite sequences are also called **strings**. This string is also denoted by $a_1a_2 \ldots a_n$.

The **length** of a string is the number of terms in this string. The **empty string**, denoted by λ, is the string that has no terms. The empty string has length zero.

EXAMPLE The string *abcd* is a string of length four.

A *recurrence relation* for the sequence $\{a_n\}$ is an equation that expresses a_n in terms of one or more of the previous terms of the sequence, namely, $a_0, a_1, \ldots, a_{n-1}$, for all integers n with $n \geq n_0$, where n_0 is a nonnegative integer. A sequence is called a *solution* of a recurrence relation if its terms satisfy the recurrence relation.

EXAMPLE Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-1} + 3$ for $n = 1, 2, 3, \ldots,$ and suppose that $a_0 = 2$. What are a_1, a_2, and a_3?

Solution: We see from the recurrence relation that $a_1 = a_0 + 3 = 2 + 3 = 5$. It then follows that $a_2 = 5 + 3 = 8$ and $a_3 = 8 + 3 = 11$.

EXAMPLE Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-1} - a_{n-2}$ for $n = 2, 3, 4, \ldots,$ and suppose that $a_0 = 3$ and $a_1 = 5$. What are a_2 and a_3?

Solution: We see from the recurrence relation that $a_2 = a_1 - a_0 = 5 - 3 = 2$ and $a_3 = a_2 - a_1 = 2 - 5 = -3$. We can find a_4, a_5, and each successive term in a similar way.

The *Fibonacci sequence*, $f_0, f_1, f_2, \ldots,$ is defined by the initial conditions $f_0 = 0$, $f_1 = 1$, and the recurrence relation

$$f_n = f_{n-1} + f_{n-2}$$

for $n = 2, 3, 4, \ldots.$

Find the Fibonacci numbers f_2, f_3, f_4, f_5, and f_6.

Solution: The recurrence relation for the Fibonacci sequence tells us that we find successive terms by adding the previous two terms. Because the initial conditions tell us that $f_0 = 0$ and $f_1 = 1$, using the recurrence relation in the definition we find that

$$f_2 = f_1 + f_0 = 1 + 0 = 1,$$

$$f_3 = f_2 + f_1 = 1 + 1 = 2,$$

$$f_4 = f_3 + f_2 = 2 + 1 = 3,$$

FOR FACTORIAL:

Suppose that $\{a_n\}$ is the sequence of integers defined by $a_n = n!$, the value of the factorial function at the integer n, where $n = 1, 2, 3, \ldots$. Because $n! = n((n-1)(n-2)\ldots 2 \cdot 1) = n(n-1)! = na_{n-1}$, we see that the sequence of factorials satisfies the recurrence relation $a_n = na_{n-1}$, together with the initial condition $a_1 = 1$.

Find formulae for the sequences with the following first five terms: (a) 1, 1/2, 1/4, 1/8, 1/16 (b) 1, 3, 5, 7, 9 (c) 1, −1, 1, −1, 1.

Solution: (a) We recognize that the denominators are powers of 2. The sequence with $a_n = 1/2^n$, $n = 0, 1, 2, \ldots$ is a possible match. This proposed sequence is a geometric progression with $a = 1$ and $r = 1/2$.

(b) We note that each term is obtained by adding 2 to the previous term. The sequence with $a_n = 2n + 1$, $n = 0, 1, 2, \ldots$ is a possible match. This proposed sequence is an arithmetic progression with $a = 1$ and $d = 2$.

(c) The terms alternate between 1 and −1. The sequence with $a_n = (-1)^n$, $n = 0, 1, 2\ldots$ is a possible match. This proposed sequence is a geometric progression with $a = 1$ and $r = -1$.

How can we produce the terms of a sequence if the first 10 terms are 1, 2, 2, 3, 3, 3, 4, 4, 4, 4?

Solution: In this sequence, the integer 1 appears once, the integer 2 appears twice, the integer 3 appears three times, and the integer 4 appears four times. A reasonable rule for generating this sequence is that the integer n appears exactly n times, so the next five terms of the sequence would all be 5, the following six terms would all be 6, and so on. The sequence generated this way is a possible match.

How can we produce the terms of a sequence if the first 10 terms are 5, 11, 17, 23, 29, 35, 41, 47, 53, 59?

Solution: Note that each of the first 10 terms of this sequence after the first is obtained by adding 6 to the previous term. (We could see this by noticing that the difference between consecutive terms is 6.) Consequently, the nth term could be produced by starting with 5 and adding 6 a total of $n - 1$ times; that is, a reasonable guess is that the nth term is $5 + 6(n - 1) = 6n - 1$. (This is an arithmetic progression with $a = 5$ and $d = 6$.)

How can we produce the terms of a sequence if the first 10 terms are 1, 3, 4, 7, 11, 18, 29, 47, 76, 123?

Solution: Observe that each successive term of this sequence, starting with the third term, What is the value of $\sum_{j=1}^{5} j^2$?

Solution: We have

$$\begin{aligned}\sum_{j=1}^{5} j^2 &= 1^2 + 2^2 + 3^2 + 4^2 + 5^2 \\ &= 1 + 4 + 9 + 16 + 25 \\ &= 55.\end{aligned}$$

What is the value of $\sum_{k=4}^{8} (-1)^k$?

Solution: We have

$$\begin{aligned}\sum_{k=4}^{8} (-1)^k &= (-1)^4 + (-1)^5 + (-1)^6 + (-1)^7 + (-1)^8 \\ &= 1 + (-1) + 1 + (-1) + 1 \\ &= 1.\end{aligned}$$

Key Terms and Results

TERMS

set: a collection of distinct objects

axiom: a basic assumption of a theory

paradox: a logical inconsistency

element, member of a set: an object in a set

roster method: a method that describes a set by listing its elements

set builder notation: the notation that describes a set by stating a property an element must have to be a member

∅ (empty set, null set): the set with no members

universal set: the set containing all objects under consideration

Venn diagram: a graphical representation of a set or sets

$S = T$ (set equality): S and T have the same elements

$S \subseteq T$ (S is a subset of T): every element of S is also an element of T

$S \subset T$ (S is a proper subset of T): S is a subset of T and $S \neq T$

finite set: a set with n elements, where n is a nonnegative integer

infinite set: a set that is not finite

$|S|$ (the cardinality of S): the number of elements in S

$P(S)$ (the power set of S): the set of all subsets of S

$A \cup B$ (the union of A and B): the set containing those elements that are in at least one of A and B

$A \cap B$ (the intersection of A and B): the set containing those elements that are in both A and B.

$A - B$ (the difference of A and B): the set containing those elements that are in A but not in B

$\overline{A}$ (the complement of A): the set of elements in the universal set that are not in A

$A \oplus B$ (the symmetric difference of A and B): the set containing those elements in exactly one of A and B

membership table: a table displaying the membership of elements in sets

function from A to B: an assignment of exactly one element

$\sum_{i=1}^{n} a_i$: the sum $a_1 + a_2 + \cdots + a_n$

$\prod_{i=1}^{n} a_i$: the product $a_1 a_2 \cdots a_n$

cardinality: two sets A and B have the same cardinality if there is a one-to-one correspondence from A to B

countable set: a set that either is finite or can be placed in one-to-one correspondence with the set of positive integers

uncountable set: a set that is not countable

$\aleph_0$ (aleph null): the cardinality of a countable set

$f \circ g$ **(composition of f and g):** the function that assigns $f(g(x))$ to x
$\lfloor x \rfloor$ **(floor function):** the largest integer not exceeding x
$\lceil x \rceil$ **(ceiling function):** the smallest integer greater than or equal to x
partial function: an assignment to each element in a subset of the domain a unique element in the codomain
sequence: a function with domain that is a subset of the set of integers
geometric progression: a sequence of the form $a, ar, ar^2, \ldots,$ where a and r are real numbers
arithmetic progression: a sequence of the form a, $a + d$, $a + 2d, \ldots,$ where a and d are real numbers
string: a finite sequence

$\mathbf{I}_n$ **(identity matrix of order n):** the $n \times n$ matrix that has entries equal to 1 on its diagonal and 0s elsewhere
$\mathbf{A}^t$ **(transpose of A):** the matrix obtained from **A** by interchanging the rows and columns
symmetric matrix: a matrix is symmetric if it equals its transpose
zero–one matrix: a matrix with each entry equal to either 0 or 1
A ∨ B (the join of A and B): see page 181
A ∧ B (the meet of A and B): see page 181
A ⊙ B (the Boolean product of A and B): see page 182

RESULTS

6. Differentiate between subset and proper set.

The set *A* is a *subset* of *B* if and only if every element of *A* is also an element of *B*. We use the notation $A \subseteq B$ to indicate that *A* is a subset of the set *B.*

Let A and B be sets. A is a proper subset of B, if and only if, every element of A is in B but there is at least one element of B that is not in A, and is denoted as $A \subset B$.

7. Is the function $f(x)=x^2$ from the set of integers to the set of integers one to one?

8. Find the formula of this sequence 1, 1/2, 1/3, 1/4…….

The formula of the sequence 1, 1/2, 1/3, 1/4……. Is $\mathbf{a_n = 1/n}$ where n<=1.

11. How many relations are there on set s={a, b, c}?

There are 3x3=9 relations in set S.

12. Differentiate between function and relation.

Relation: Let A and B be sets. A binary relation from A to B is a subset of A × B. Let A and B be sets. The binary relation R from A to B is a subset of A × B. When (a, b) ∈R, we say ‘a’ is related to ‘b’ by R, written aRb.

Function: A function **F** from a set X to a set Y is a relation from X to Y that satisfies the following two properties

15. List the members of the following set.

a. {x|x is real number such that $x^2=4$}

b. {x|x is an integer such that $x^2=2$}

(a) The members of the set {x| x is a real number such that $x^2 = 4$} are {2}.

(b) The members of the set {*x* | *x* is an integer such that $x_2 = 2$} are Ø (√2 is not an integer).

Relation :- If A and B are two sets then any set of A×B is Called relation. It is denoted by $R = \{(a,b), (c,d)\}$

Onto function :- A function $f:A \to B$ is called onto function if Range of function is not repeated.
$f = \{(a,1), (b,3), (c,4)\}$

If $A=\{1,3,5\}$, $B=\{1,2,3\}$, Are A & B disjoint?

No A & B are not

Q.16 Let $\{t_n\}$ be a Sequence where $t_n = 7-3n$ what type of progression is this?

Ans This is an A.P because If we put $n = 1, 2, 3, \ldots$

$t_1 = 7-3$	$t_2 = 7-6$	$t_3 = 7-9$
$t_1 = 4$	$t_2 = 1$	$t_3 = -2$

If $A = \{a, b, c, d, e\}$, $B = \{a, b, c, d, e, f, g, h\}$

i) $B \times A$

$= \{a, b, c, d, e, f, g, h\} \times \{a, b, c, d, e\}$

$= \{(a,a), (a,b), (a,c), (a,d), (a,e), (b,a), (b,b),$
$(b,c), (b,d), (b,e), (c,a), (c,b), (c,c), (c,d),$
$(c,e), (d,a), (d,b), (d,c), (d,d), (d,e), (e,a)$

(2) $A - B$

$= \{a, b, c, d, e\} - \{a, b, c, d, e, f, g, h\}$

$\{\phi\}$ Ans

(11) $$\sum_{i=0}^{2} \sum_{j=0}^{3} i^2 j^3$$

$= \left[(0)^2 + (1)^2 + (2)^2\right]\left[(0)^3 + (1)^3 + (2)^3 + (3)^3\right]$

$= \left[1 + 4\right]\left[1 + 8 + 27\right]$

Q.3 $A = \{0, 1, 2, 3, 4\}$

$A \times A$

$= \{0, 1, 2, 3, 4\} \times \{0, 1, 2, 3, 4\}$

$R_2 = \{(0,4)(1,3)(2,2)(3,1)(4,0)\}$

$R_1 = \{(1,0)(2,0)(2,1)(3,0)(3,1)(3,2)$

$(4,0)(4,1)(4,2)(4,3)\}$

Q.3 How can you Produce the terms of a sequence if first 10 terms are 5, 11, 17, 23, 29, 35, 41, 47, 53, 59 ?

Sol:-

$a_1 = 5$, $d = 6$, $a_{10} = 59$

Q.12 How many subsets with more than two elements does a set with 100 elements have?

Ans If set has 100 elements then the subsets $= 2^{100}$

$2^{100} - 1$ (non empty subsets)

${}^{100}C_1$ & ${}^{100}C_2$ have 2 or less elements

Now subtract from $2^{100} - 1$

8. What is cardinality of each of these sets? {a, {a}, {a, {a}}}

The cardinality of a set is the number of elements it has. {a, {a}, {a, {a}}} has cardinality 3.

12. Let f be the function from {a, b, c, d} to {1, 2, 3} defined by f (a) = 3, f (b) = 2, f (c) = 1, and f (d) = 3 is f an onto function?

Because all three elements of the codomain are images of elements in the domain, we see that f is onto. Note that if the codomain were {1, 2, 3, 4}, then f would not be onto.

Topic :- Sets and Matrices

overview :

1 Sets

2 Set operations.

3 cardinality of sets.

4 Demorgan's law for sets.

5 Matrices

Sets :

A set is a group of objects usually with

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A set is described by either listing out the elements of the set in braces using set builder notation.

- **Example:**

The set v of vowels in English is $v = \{a, e, i, o, u\}$

- **Example:**

The set

- R: the set of real numbers
- ϕ : the empty set (no elements).

Set Equality:

Two sets are equal if and only if they contains the same elements.

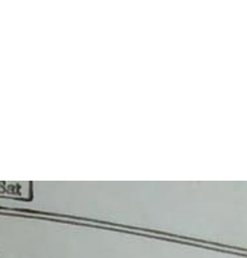
Veen Diagram

A Veen Diagram is a graphical representation of a set.

Subset:

The set A is a sub set of B if and only if every element of A is also an element of the set B we use the notation $A \subseteq B$

$A \subseteq B$



Another way to think about

subset of B. and write it

$$A \subset B$$

$$A = \{1, 2, 3\}$$

$$B = \{x : 0 < x \leq 4\}$$

$$\Rightarrow A \subset B$$

Use subsets to prove equality of sets:

A and B have the

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تاریخ:

<u>Finite set</u>, and a n is the <u>cardinality</u> or S.

The cardinality of S is written $|S|$.

Cardinality : Examples

<u>Example.</u>

Let A be the set of odd Positive integers less than 10.

of all subsets of the set S. The power set of S is written P(S).

Let S be the set $\{0, 1, 2\}$

$P(S) = \{\phi, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\}$

Note The empty set and S (the set itself) are members of the power set.

a bunch of words, but it would be easier to search them if they're sorted or put in a particular order

Order n-tuple:

The **ordered n-tuple** $(a_1, a_2, \dots a_n)$ is the ordered collection that has a_1 as its first

The ordered Pair (a,b) equals the ordered Pair (c,d) if and only if a=c and b=d.
(a,b) only equals (b,a) if a=b.

Cartesian Products

The cartesian product of sets A and B (denoted A×B) is the set of all ordered pairs

A {a, b, c} , B {1, 2, 3}

C {blue, red, green}

A×B×C = {(a, 1, blue), (a, 1, red), (a, 1, green)
(a, 2, blue), (a, 2, red) ----- }

The Cartesian product A×B×C consist of all ordered triples (a, b, c) where a ∈ A, b ∈ B c ∈ C.

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is the set that contain the elements in both A and B

$A \cap B = \{x \mid x \in A \wedge x \in B\}$

Example:

$\{1, 2, 3, 4\} \cap \{x : x \in N\}$

Disjoint:

let A and B be sets.

... sets are disjoint if

A and B twice.

Therefore we need to subtract the number of elements that are in A and B.

$$|A \cup B| = |A| + |B| - |A \cap B|$$

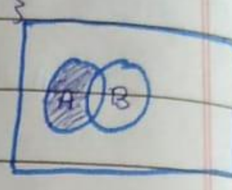
complement of sets:

The complement of a set

minus B) is the set of elements that belong to A but which do not belong to B.

$$A/B = \{x : x \in A,\ x \notin B\}$$

Example:



$\{5, 6, 7, 8, 9, 10\} | \{7, 8, 9\}$

Symmetric difference:

The symmetric difference of

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Demargan's law

تاریخ:

$\neg(p \wedge q) = \neg p \vee \neg q$

$\neg(p \vee q) = \neg p \wedge \neg q$

Example: Let A be the set of students who live within one mile of school, and B the set of student who walk to campus from home. Describe each of these sets of students:

The student who walk to school but live further that a mile away.

Set identities

Identity	Name
$A \cup \emptyset = A$ $A \cap U = A$	identity Laws
$A \cap \emptyset =$	Domination Laws

Worked Problem sets:

1 Let A, B and C be sets. Show that (A-B)-C is not necessarily equal to A-(B-C)

2 Let E denote the set of odd integers and O the set of odd integers Z is the set of all

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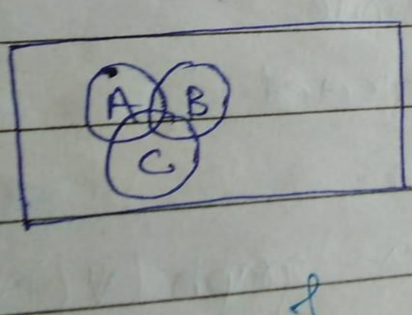
تاریخ:

Venn diagram.

That is $(A \oplus B) \oplus C = A \oplus (B \oplus C)$



$1 \in a$

$2 \in a$

$3 \in a$

$a = \{1, 2, 3\}$