Solve Past Paper 2022.

"Short Answers"

(i)

$$f(x,y,z) = e^{\sqrt{z-x^2-y^2}}, \quad f(2,-1,6) = ?$$

Solve:

$$f(2,-1,6) = e^{\sqrt{6-(2)^2-(-1)^2}}$$

$$= e^{\sqrt{6-4-1}}$$

$$= e^{\sqrt{6-5}}$$

$$f(2,-1,6) = e^{\sqrt{1}} = e$$

(ii)

$$r(t) = 2\cos t\,\hat{i} + \sin t\,\hat{j} + 2t\,\hat{k}$$

$$\int_0^{\pi/2} r(t)\,dt$$

Solve: $$\int_0^{\pi/2} 2\cos t\,\hat{i} + \sin t\,\hat{j} + 2t\,\hat{k} \cdot (dt)$$

$$\int_0^{\pi/2} 2\cos t\,\hat{i}\,dt + \int_0^{\pi/2} \sin t\,\hat{j}\,dt + 2\int_0^{\pi/2} t\,\hat{k}\,dt$$

$$= 2\left[\sin t\,\hat{i}\right]_0^{\pi/2} + \left[-\cos t\,\hat{j}\right]_0^{\pi/2} + 2\left[\frac{t^2}{2}\hat{k}\right]_0^{\pi/2}$$

$$= 2\left[\sin\frac{\pi}{2}\hat{i} - \sin 0\,\hat{i}\right] + \left[-\cos\frac{\pi}{2}\hat{j} + \cos(0)\hat{j}\right] + \left[\left(\frac{\pi}{2}\right)^2 - (0)\right]$$

$$= (\sin\pi\,\hat{i}) - (\cos 90\,\hat{j} + 1\hat{j}) + (8100\,\hat{k})$$

$$= (0) - \{0 + 1\hat{j} + 8100\,\hat{k}$$

$$= 1\hat{j} + 8100\,\hat{k}$$ Ans.

(vii)

Let $f(x,y,z) = zxy + x$, find $f(xy, \frac{y}{x}, xz)$.

Solve:

$$f(xy, \frac{y}{z}, xz) = \left[(xz)(xy)\left(\frac{y}{x}\right) + xy\right]$$

$$= \frac{\cancel{x^2}y^2z}{\cancel{x}} + xy$$

$$= xy^2z + xy$$

$$f(xy, \frac{y}{z}, xz) = xy(yz+1) \text{ Ans}$$

(ix)

Find Domain ?

Solve $r(t) = \cos \pi t\,\hat{i} - \ln t\,\hat{j} + \sqrt{t-2}\,\hat{k}$

At $r(0) = 1 - \infty - \sqrt{-2}$

At $r(1) = \cos 180 - 0 + \sqrt{-1}$

At $r(2) = \cos 360 - 0.69 + \sqrt{2-2}$

$= 1 - 0.69 + 0$

$= 0.31$

At $r(3) = 0.93 - 1.09 + 1$

$= 0.84$

At $r(-1) = \cos 180(-1) - \ln(-1) + \sqrt{-1-2}$

$= 1 - \infty + \sqrt{-3}$

At $r(\frac{1}{2}) = \cos \frac{180}{2} + 0.69 + \sqrt{\frac{-3}{2}}$

So Domain is $[2, \infty)$ Ans.

(xi)

Give precise definition of:

(i) $\lim_{x \to 2^-} f(x) = 5$ (ii) $\lim_{x \to 2^+} f(x) = 5$

Ans: From (i)

Left hand limit of the function is "5"

From (2), "4" Right hand limit of function is "5".

" LONG Question"

"LONG Question"

QNo2: (a)

$$f(x) = \begin{cases} x+2 & x \leq -1 \\ c+2 & x > 1 \end{cases}$$, find c so $\lim_{x \to -1} f(x)$ exist.

Solve:

L.H.S.

$$\lim_{x \to -1} f(x) = x+2$$

$$= -1+2$$

R.H.S.

$$= 1$$

$$\lim_{x \to 1^+} f(x) = c+2$$

$$L.H.S. = R.H.S.$$

$$1 = c+2$$

$$1-2 = c$$

$$\boxed{-1 = c}$$

Q No 2:

(b) $\int_{3\pi/8}^{\pi/2} \sec^2(\pi-2\theta)\,d\theta$

Let $u = \pi - 2\theta$

$du = 0 - 2(1)d\theta$

$du = -2\,d\theta$

$\frac{du}{-2} = d\theta$

$\pi/2 \longrightarrow u = \pi - \cancel{2}(\frac{\pi}{\cancel{2}})$

$u = \pi - \pi = 0$

$\frac{3\pi}{8} \longrightarrow u = \pi - 2(\frac{3\pi}{8})$

$= \pi - \frac{6\pi}{8}$

$= \frac{8\pi - 6\pi}{8} = \frac{\cancel{2}\pi}{\cancel{8}_4}$

$= \pi/4$

$$\int_{\pi/4}^{0} \frac{\sec^2(u)\,du}{-2}$$

$$\frac{-1}{2}\int_{\pi/4}^{0} \sec^2(u)\,du.$$

$$= -\frac{1}{2}\left[\tan u\right]_{\pi/4}^{0}$$

$$= -\frac{1}{2}\left[\tan(0) - \tan\frac{\pi}{4}\right].$$

$$= -\frac{1}{2}\left[(0) - \cancel{\tan}\ 1\right].$$

$$= -\frac{1}{2} \times -1$$

$$= \frac{1}{2} \text{ Ans.}$$

$(x+1)^{2/3}($

Q NO 3: (a)

$$y = 2\sqrt{x} \qquad 1 \le x \le 2$$

put $y = 0$

$$0 = 2\sqrt{x}$$

$$x = 0, \quad x = 1, \quad x = 2$$

$$= \left| \int_0^1 2\sqrt{x} \right| + \left| \int_1^2 2\sqrt{x} \right|$$

$$= \left| 2\int_0^1 x^{1/2} \right| + \left| 2\int_1^2 x^{1/2} \right|$$

$$= 2\left[\frac{2}{3}(x)^{3/2} \right]_0^1 + 2\left[\frac{2}{3}(x)^{3/2} \right]_1^2$$

$$= 2\left[\frac{2}{3}(1)^{3/2} \right] + 2\left[\frac{2}{3}(2)^{3/2} - \frac{2}{3}(1)^{3/2} \right]$$

$$= 2\left(\frac{2}{3} \right) + 2\left(\frac{2}{3}(8)^{1/2} - \frac{2}{3} \right)$$

$$= \frac{4}{3} + 2\left(\frac{2}{3}(2\sqrt{2}) - \frac{2}{3} \right)$$

$= -2($

$= -2($

$= -16$

Qno3 (a) Remaining.

$$= \frac{4}{3} + 2\left(\frac{4}{3}\sqrt{2} - \frac{2}{3}\right)$$

$$= \frac{\cancel{4}}{\cancel{3}} + \frac{8\sqrt{2}}{3} - \frac{\cancel{4}}{\cancel{3}}$$

$$= \frac{8\sqrt{2}}{3}$$ Ans.

Q No 3.

(b)

$$y'' = ?$$

$$x^2 + 4y^2 = 4$$

$$\frac{d}{dx}(x^2 + 4y^2) = \frac{d}{dx}(4)$$

$$2x + 4 \cdot 2y \frac{dy}{dx} = 0$$

$$2x + 8y \frac{dy}{dx} = 0$$

$$8y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-\cancel{2}x}{\cancel{8}4y}$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{4y}}$$

Now y''

$$\frac{d^2y}{dx^2} = \frac{-x}{4y} = \frac{4y \, \frac{d}{dx}(-x) - \left[(-x)\frac{d}{dx}4y\right]}{(4y)^2}$$

$$= \frac{4y(-1) - (-x) \cdot 4(1)\, \frac{dy}{dx}}{(4y)^2}$$

$$= \frac{-4y + 4x\left(-\frac{x}{4y}\right)}{(4y)^2} = \frac{-4y + -\frac{x^2}{y}}{(4y)^2}$$

$$= \frac{\frac{-4y^2 - x^2}{y}}{(4y)^2} = \frac{-4y^2 - x^2}{16y^3} \text{ Ans.}$$

Q No 5 (a):

$$f(x) = x^3(4-x)$$
$$= 4x^3 - x^4$$
$$f'(x) = 12x^2 - 4x^3$$
$$f'(x) = 4x^2(3-x) \longrightarrow (i)$$
$$f''(x) = 24x - 12x^2$$
$$f''(x) = 12x(2-x) \text{ --- (ii)}$$

For increasing and decreasing

$$f'(x) = 4x^2(3-x)$$

put $x = 4$

$$f'(x) = -64 < 0.$$

put $x = 5$

$$f'(x) = -200 < 0$$

function is decreasing in $(+3, \infty)$

function is increasing in $(-\infty, 3)$.

→ For concave upward and concave downward

$$f''(x) = 12x(2-x)$$

function is concave up at $(-\infty, 2)$

function is concave down at $(2, \infty)$.

Q.No5: (b) $y = \dfrac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$

$$\ln y = \ln x + \frac{1}{2}\ln(x^2+1) - \frac{2}{3}\ln(x+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{\cancel{2}}\cdot\frac{1}{x^2+1}\cdot\cancel{2}x - \frac{2}{3}\frac{1}{x+1}(1)$$

$$\frac{1}{y}\frac{dy}{dx} = \left[\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3x+3}\right]$$

$$\frac{dy}{dx} = y\left[\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3x+3}\right]$$

$$\frac{dy}{dx} = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}\left[\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3x+3}\right]$$

QNo6 (a)

Find length of astroid.

Solve: $x = \cos^3 t$, $y = \sin^3 t$

$0 \le t \le \pi$

$$\left(\frac{dx}{dt}\right)^2 = \left[3\cos^2 t\,(-\sin t)\right]^2$$

$$= 9\cos^4 t \sin^2 t$$

$$\left(\frac{dy}{dt}\right)^2 = \left[3\sin^2 t\,(\cos t)\right]^2$$

$$= 9\sin^4 t \cos^2 t$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{9\cos^2 t \sin^2 t\,(\cos^2 t + \sin^2 t)}$$

$$= \sqrt{9\cos^2 t \sin^2 t}$$

$$= 3\left[\cos t \sin t\right]$$

$$\int_0^{\pi/2} 3\cos t \sin t\, dt \rightarrow \text{Length of first quadrant}$$

$$= \frac{3}{2}\int_0^{\pi/2} 2\sin t \cos t\, dt$$

$$= \frac{3}{2}\int_0^{\pi/2} \sin t\,(dt) = \frac{3}{2}\left[-\frac{\cos 2t}{2}\right]_0^{\pi/2}$$

$$= -\frac{3}{2\times 2}\left[\cos 2\pi/2 - \cos 0\right]$$

$$= -\frac{3}{4}\left[-1-1\right]$$

$$= -\frac{3}{4}(-2) = \frac{6}{4} = \frac{3}{2}$$

The length of astroid is 2 times this:

$$= 2\left(\frac{3}{2}\right) = \frac{6}{2} = 3 \text{ Ans.}$$