

Qno 10
 Let W be a subspace of \mathbb{R}^5 spanned by vectors $u_1 = (1, 2, -1, 3)$, $u_2 = (2, 4, 1, -2)$, $u_3 = (3, 6, 3, -7)$, $u_4 = (1, 2, -4, 11)$, $u_5 = (2, 4, -5, 14)$. Find a basis for W and its dimension.

Sol:-

$$= \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \\ 1 & 2 & -4 & 11 \\ 2 & 4 & -5 & 14 \end{bmatrix}$$

$$R_2 - 2R_1 \Rightarrow R_2$$

$$R_4 - R_1 \Rightarrow R_4$$

$$; R_3 - 3R_1 \Rightarrow R_3$$

$$; R_5 - 2R_1 \Rightarrow R_5$$

$$= \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 6 & -16 \\ 0 & 0 & -3 & 8 \\ 0 & 0 & -3 & 8 \end{bmatrix}$$

$$R_5 - R_4 \Rightarrow R_5$$

$$= \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 6 & -16 \\ 0 & 0 & -3 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_4 + R_2 \Rightarrow R_4$$

$$= \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 6 & -16 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_3 - 2R_2 \Rightarrow R_3$$

$$= \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

The non-zero rows in the matrix are:

$$w_1 = (1, 2, -1, 3) \quad \text{and} \quad w_2 = (0, 0, 3, -8)$$

These rows form basis of row space and consequently form basis of subspace.

It's dimensions is 2-