Theorem 4.5.3 Plus / Minus Theorem	_
Theorem 4.5.3 Plus / Minus Theorem Let S be a nonempty set of vectors in a vector space V (a) le (ii) a linearly independent set, and y	
vector space V	
(a) by (is a linearly independent set, and y	
v is a vector in V that is outside of	
by inserting v into S is still linearly	
(b) ly v is a vector in S that is expressible	le_
in S. and y S-3v3 denotes the set obtains	1
in S, and ig S-9v3 denotes the set obtained by Lemoving V from S, then S and S-9v3	
Span the same space that is	-
Span the same space that is span (S)= span (S-\(\frac{4}{2}\)\)	
Proof (a) Let 5=3 V1, V2, V23 is a direarly independent	de
sel or vectors in V and V is a vector	
that is octisiale of spaints.	
ove need to show {v,v, , , , v, v} 'as	
a linearly independent set.	
(e) K, V, + K, V2 + - + Kx, V4 + Kx+1 V = 0 - (1)	
Note that Kitt must be zew. be cause.	
0 => KI+1 V = -K, V, -K2 V2K1 V4	
V = -KI V, K2 Vs - K2 Vy KA+1 KA+1	
ie VE Span (S) which is not possible	
Hence [k+1=0] -(E)	
Now Busing this value in 1	
$\Rightarrow k_1 v_1 + k_2 v_2 = 0$	
Combining (2 & (3) inclependent Set)	
$K_1 = K_2 = = K_A = K_{A+1} = 0$	-

which shows that susus is Investige independents (b) Assume that S=\{\frac{9}{1}\times\{5}\time		er met m
(b) Assume that $S = \frac{2}{3}V_{1}, V_{2},, V_{4-1}, V_{4}^{2}$ is a Set of vectors in V and suppose that V_{4} is a linear continuation of $V_{1}, V_{2}, V_{3}, V_{4-1}$ We want to show $V_{4} = V_{4} + V_{4-1} + V_{$	independent	_
V ₁ = C ₁ V ₁ + C ₂ V ₂ + + C ₄ · V ₄ - 1 — (9) We want to show Span (S) = Span (S - 2V ₈) Take w & Span (S) = w' = k ₁ V ₁ + k ₂ V ₂ + + k ₄ V ₄ + k ₄ V ₄ using (Q) = w' = k ₁ V ₁ + k ₂ V ₂ + + k ₄ V ₄ + k ₄ V ₄ i.e. w = (k ₁ + k ₄ C)V ₁ + (k ₂ + k ₃ C)V ₂ + + (k ₄ + k ₄ C ₄ ·)V ₄ = w & Span (S - 2V ₄) Theorem 4.5.5 let S be a finite set of vectors in a finite dimensional sector space V. (a) & S span V but is not a basic for V, then S can be reduced to a basic for V by semoving appropriate vectors from S. (b) & S is a linearly independent Set that is not	(b) Assume that S= {v, v, -, v,	_
Span (S) = Span (S-8V3) Take w E. Span (S) = k_1V_1+k_2V_2 + - + k_4V_4+k_4V_4 using (G) = w_1 = k_1V_1+k_2V_2 + - + k_4V_4+k_4V_4 using (G) = w_2 = k_1V_1+k_2V_2 + - + k_4V_4+k_4V_4 using (G) = w_2 = k_1V_1+k_2V_2 + - + k_4V_4+k_4V_4 = w_2 = k_1V_1+k_2V_2 + k_4V_4+k_4V_4 = w_2 = k_1V_1+k_2V_2 + - + k_4V_4+k_2V_4 = w_2 = k_1V_1+k_2V_2 + - + k_4V_2+k_2V_2 = w	is a linear Combination of 4,2 1/2, 1/4	
Take w & Span(S) W = K_1V_1 + K_2 V_2 + - + K_4V_4 + K_4 V_4 using @ = w' = K_1V_1 + K_2 V_2 + - + K_4 - V_4 - + K_5 (C_1V_1 - + G_1V_4 -) i.e. w = (K_1 + K_4 ())V_1 + (K_2 + K_3 ())V_2 + + (K_3 - + K_4 () -)V_4 - 1 S	V1 = C1V1+(2V2+ - + C1-1V2-1 - (4)	
Theorem 4.55 let S be a finite set of vectors in a finite climensional vector space V. (a) & S sparse V but is not a basis for V, then Semoving appropriate vectors from S. (b) & S is a linearly independent set that is not	Span (S) = Span (S-9V2)	
Hence Span ((-3Vi3) Theorem 4.55 let S be a finite set of vectors in a finite-climensional vector space V. (a) by S spany V but is not a basis for V, then Semoving appropriate vectors from S. (b) by S is a linearly independent Set that is not	Take $\omega \in \text{Span}(S)$ $\Rightarrow \omega = k_1 V_1 + k_2 V_2 + \cdots + k_K + V_{K+1} + k_K V_K$ $using Q \Rightarrow \omega = k_1 V_1 + k_2 V_2 + \cdots + k_{K-1} V_{K-1} + k_K (C_1 V_{1+} \cdots + C_{K-1} V_{K-1})$	
Theorem 4.5.5 let S be a finite set of Vectors in a finite dimensional vector space V. (a) y S spary V but is not a basic for V, then Semoving appropriate vectors from S. (b) y S is a linearly independent set that is not		
in a finite-dimensional vector space V. (a) If S spary V but is not a basis for V, then S can be seduced to a basis for V by semoving appropriate vectors from S. (b) If S is a linearly independent set that is not		
semoving appropriate vectors from S. (b) If S is a linearly independent Set that is not also ady a basis for V. then S can be enlarged to a basis for V by inserting appropriate vectors into S.	in a finite dimensional nector space V	
appropriate vectors into S.	semoving appropriate vectors from S. (b) If S is a linearly independent Set that is not already a basic for V. then S can be.	
	appropriate vectors into S.	

Proof: (a) & s is a set of vectors that spans V but is not a basis for V then
Circles not a basis fol Viano
Six a linearly dependent set. Some vector via in S is a linear Combination
Now Take S'= S- 3v3
Now 19Ke S-5-3V3
then by plus/minus theorem
V
Span(s) = Span(s') = Span (S-4v3) = V
of S is a linearly independent. Then S' is a basis for V.
Of not these come vector V' can be
ly not then some vector V' can be a linear combination of other
verture in V
vectors in V
then by plus /minus theorem
Span (S") = Span (S') = V Ly (" is linearly independent, no are done otherwise and will continue, in
If " is linearly independent, no are
done Otherwise well continue in
Same manner centil we reachead a
linearly independent set that form a
bans for V.
(b) Suppose that down (V)=n.
- y s'is a linearly independent set
that is not already a basis for V
that is not already a basis for V then I fails to Gan V.
- there exist ve V which is not
present in Span(S)
Take ('- Sugri}
Then by plus/minus theorem & is
Still linearly independent.
\boldsymbol{U}

and s' is a basis for V.
and s' is a bout for V.
These exist some vector we which is
There lexist we we which is
not were the continue
Take S' s'u zw?
to get a set st with n linearly
spans , we are come manner
de will continue in same
get a set & with n strang
independent vectors in V.
The set of will be a basil for
- (by he over; "(c) V be an n-dimentional
The set of will be a basil for V (by Theorem; "(ct V be an n-dimensional vector space and (ct & be a cd in V with
- exactly n vectors. Then Su a basis for
- exactly n vectors. Then Su a basis for Vig Supany V or S'is Dinearly independent)
independent.)
Exercise 4.5
9# 10: Find the dimension of the subspace Pa consisting of all polynomials act 0, x, 0, x 0, 0, 2 - for which ac-0
Pa consisting of all polynomials
ac, a, x, a, 20, a, 20 - for which ac-0
Solution . W
An element of above subspace? Dooke like
like
P(x) = a1x + a2x2 + a2x3
-> ρ(x) ε Span {x, x, x, x, x, 3}
All a 2x x2 x3 in Ome only molependent.
Also \(\frac{2}{3}\), \(\frac{2}{3}\) is lone only independent. \(\frac{2}{3}\), \(\frac{2}{3}\) is a basis.
7 12 1 2 3 4 3 4 3 4 3 4 3 4 3 4 3 4 3 4 3 4 3
\rightarrow dim $W=3$.
0 # 11 11 11 11 16 101 11 11 11 11 11
S#11 show that the set W of all polynomials in B, such that P(V)=0 is a subspace of B.
in 1, such that PU-O is a suispace of t.
Find it basis and dimension,
The property of the second sec

Solution To show that W is a subspace of P2. We need to show

b) & f, g \in W \rightarrow f + g \in W

b) & f \in W, \kappa is a scalar than kf \in W (f+g)(1) = f(1) + g(1) = 0+0=0 (asing 0+0) let few => f(1)=0 (b) (kf)(1)= k(f(1))= k(0)=0 => Kf EW Hence Wis a Subspace of Po P(X) E P2 Now for P(x) to be a member of W need P(1)=0 ac+ 9(1)+ 92(1)=0 ac = -9-92 P(21)= -91-92+91X2 = 9(x-1)+ 92(x2-1)

10 Waspar (1-x, 1-x2)
i.e W= Span (x-1, x'-1)
$a_1(x-1) + a_2(x^2-1) = 0$
$\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}$
Congains Coefficient
Congaing Conefficient
Centant; - 9-92-0
$\frac{1}{2}; \frac{1}{12-0}$
=> {x-1, x2-1} is linearly independent.
=> {x-1, x'-1} is a basis for W -> clim W = 2.
\rightarrow $\alpha im W = 2$.
0. H1/1 . 1 / 2
ve tor space V. Show that \(u_1, u_2, u_3 \\ is
also or borsis where 4, - V, o 42 - V, + V2 2
$U_3 = Y_1 + l_2 + V_3$
Solution
Use theorem 454 To prove this
let V be an n-dimensional vector space
vectors. There a set in V with exactly or
Ose theorem 454 To prove this Ox let V be an n-dimensional vector space and let S be a Set in V with exactly or vectors. Then S is a basis for V by S spans V on S is linearly independent?
0. 511 11 3
As 90, 1, 1, 23 is a basis for V => dim Y_3
Now 34, 4, 4, 43 will be a basis if
Participation of the Control of the

