

Outline no: 7

GRAPHS

Graphs:- It is a collection of vertices & edges.

OR

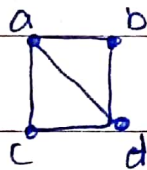
Graphs are non-linear data structure consists of a pair of vertices & edges (V, E) .

Vertex / node / path.

Edge / line

They are used to model various systems in the real world.

Example:



$$G(V) = \{a, b, c, d\}$$

$$G(E) = \{ab, ac, ad, ba, bd, cd, da, db, dc\}$$

$$G(V) = \{4\}$$

$$G(E) = \{10\}$$

Types of graph:

Infinite graphs: A graph with infinite vertex set or an infinite edges is called INFINITE GRAPH.

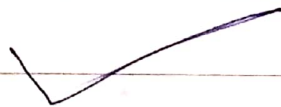
set of a finite edges is called FINITE GRAPH.

iii- Simple graph: A graph in which edge connects 2 different vertices & where no two edges connects the same pair of vertices is a simple graph.

(Example:) OR

The graph that don't include loop & each edge connects 2 diff. pair of vertices.

Example:



iv- Directed graph: A graph consists of a non-empty set of vertices (V) & a set of directed edges (E) is called directed graph.
at least 1 direction is compulsory

OR

In which loop exist & tell direction.

Example:

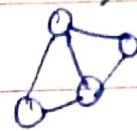
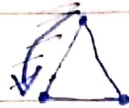


→ multiple edges connected to same pair of vertices

v-

Undirected graph:- Graphs containing multiple edges connected to same pair of vertices & having no direction are called **UNDIRECTED GRAPH**.

Example:

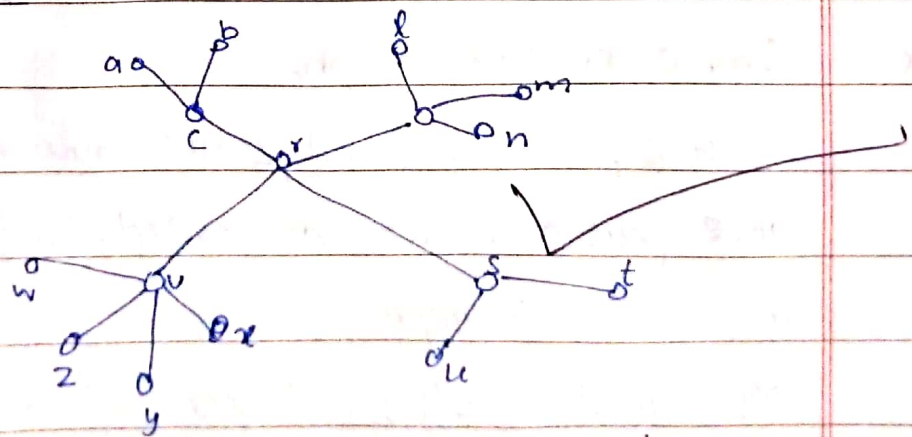


undirected.

vi-

Web graph:- The world wide web can be modeled as a directed graph where each webpage is represented by a vertex & where an edge starts at the webpage A & ends at the webpage B, if there is a link on A pointing B. Because new webpages are created & another removed somewhere on the web almost every second, the web graph changes on an almost continual basis

Example:

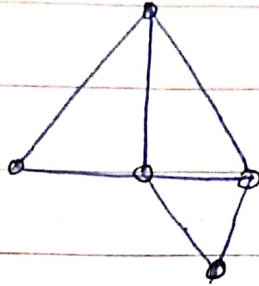


→ Each node denotes a webpage, & each edge from page P to Q. Q denotes a link on page P pointing to page Q. It is a directed graph.

vii- Hollywood Graph-

viii- Graph coloring Problem-

→ no. of std. courses, vertices.



if no. of stds problem comes the graph should be directed.

Problem: are solved by directed graph

1- 1std = 6std

i- case 1: std diff.

case 2: course diff.

ix- Simple directed Graph:-

No loop, no multiple edges are connected to the same pair of vertices also called directed graph.

multiple/multi graph

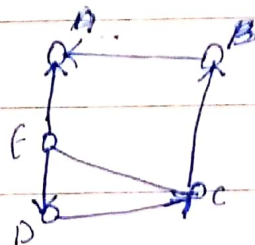
loop, multiple edges are connected to the same pair of vertices.

Pseudo graph

- loop
- all possible multiple edges are connected to same pair of vertices
- directed graph.

Find vertex degree

- in degree $(-)$ (\leftarrow)
- out degree $(+)$ (\rightarrow)



$$G(A)^- = \{2\}, G(B)^- = \{1\}, G(C)^- = \{1\}, G(D)^- = \{1\}, G(E)^- = \{0\}$$

$$G(A)^+ = \{0\}, G(B)^+ = \{1\}, G(C)^+ = \{1\}, G(D)^+ = \{1\}, G(E)^+ = \{1\}$$

Special type of Graph.

- cyclic graph: A walk is closed if end points are same. Such graph is called CYCLIC GRAPH

$\rightarrow C_n \Rightarrow$ Mathematically indication \Rightarrow Symbol
 \rightarrow no. of edges & vertices are countable.

\rightarrow Complete Graph: $\text{کُلّی اور واسطہ دار}$

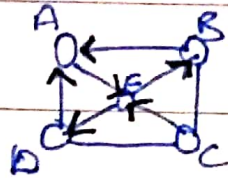
1st Condition = direction should be present

2nd " =

3rd " = 1 Edge 1 vertex per mark karta ho

K = complete, n = no. of vertices

Type of Complete Graph



$$\bar{in} = \text{in-degree}$$

$$out^+ = \text{out-degree}$$

$$A = A \leftarrow B, A \leftarrow D \quad \bar{A} = 2 \quad A^+ = 1$$

$$\bar{B} = 1 \quad B^+ = 1$$

$$\bar{C} = 0 \quad C^+ = 1$$

$$\bar{D} = 1 \quad D^+ = 1$$

$$\therefore \bar{E} = 2 \quad E^+ = 2$$

Graphs:-

- MultiGraphs:-

Graphs that may have multiple edges connecting the same vertices are called multigraphs.

Loop(edge):-

An edge that connects a vertex to itself is called loop(edge)

Pseudograph

- Pseudo Graphs:-

Graphs that may include loops & multiple edges connecting the same pair of vertices are called pseudo graph

→ loop

→ all possible multiple edges are connected to same pair of vertices.

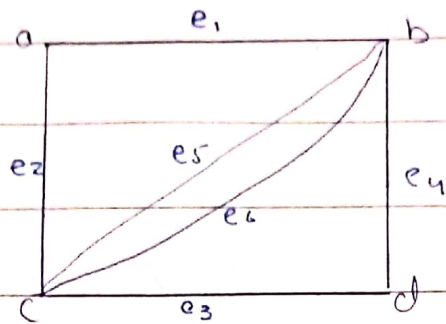
→ directed graph.

- Simple directed graph:-

A graph that does not have multiple directed edges connected to same pair of vertex is called simple directed graph.

- Mixed Graph:

A graph with multiple undirected & directed edges is called mixed graph.



$$V = \{a, b, c, d\}$$

$$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$$

- Parallel edges:

Edges having same starting & ending vertices.

e_5 & e_6 are parallel edges.

- Adjacent Vertex:

The vertices connected to a vertex are its adjacent vertices

$$a \rightarrow b, c$$

$$b \rightarrow a, c, d$$

$$c \rightarrow a, b, d$$

$$d \rightarrow c, b$$

- Degree of Vertex:

The no: of vertices connected to a vertex is

calculate its degree.

$$\deg(a) = 2$$

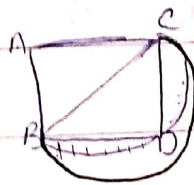
$$n(b) = 3$$

$$n(c) = 3$$

$$n(d) = 2$$

• Planner Graph:-

In this graph there is no cross edges.



• Regular Graph:

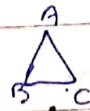
The graph that have equal degree

of vertices

$$A \leftrightarrow B$$

$$\deg(A) = 1$$

$$\deg(B) = 1$$



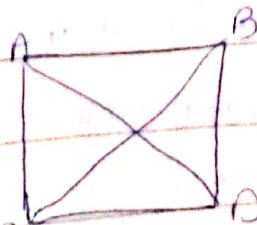
$$\deg(A) = 2$$

$$\deg(B) = 2$$

$$\deg(C) = 2$$

• Complete Graphs:

A graph in which each pair of vertices is connected by an edge



Edges of complete graphs:-

Total no: of edges in a complete graph is given by

$$\text{edges} = \frac{n(n-1)}{2}$$

$\therefore n = \text{no: of vertices}$

Type	Edges	Multiple edges allowed?	Loops Allowed
Simple graph	Undirected	No	No
Multi graph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Both	Yes	Yes

• Hollywood Graph:-

Each node denotes an actor or an actress, & each edge b/w P & Q denotes that P, Q worked together in some movie. It is an undirected graph.

• Planner Graph:-

is a graph which does not have any edge which is crossing AND the graph which consists of edges that are crossing each other is said to be