Calculus Paper 2017 (CS)

Question No. 1 - Short Questions

III Solve inequality $4(x-1) < 5(x-2)$

$4x - 4 < 5x - 10$

$4x < 5x - 6$ Add $+4$

$\cancel{4x < 5}$ $4x + 6 < 5x$ Sub $+4x$

$6 < x$ $[6, \infty)$

6

II If $\left|\frac{x}{5}\right| \le 1$ Then find value of x

$-1 \le \frac{x}{5} \le 1$ $\cancel{\frac{x}{5} \le 1}$

$\cancel{\frac{x}{5} \le 1}$

$-1 \le \frac{x}{5} \le 1$

$-5 \le x \le 5$

$[-5, 5]$

XIII Find Eq. of Tangent line at $(0, b)$, $m = k$

$P(0, b)$ $m = k$
$x_1 \; y_1$

$y - y_1 = m(x - x_1)$

$y - b = k(x - 0)$

$y - b = kx$

$y = kx + b$

vi $y = \dfrac{x}{x^2-1}$ even or odd? not?

$f(x) = \dfrac{x}{x^2-1}$ Odd fn

$f(-x) = \dfrac{-x}{(-x)^2-1} = \dfrac{-x}{x^2-1} = -\left(\dfrac{x}{x^2-1}\right)$

vii $y = \sqrt{x-1}$

Domain $= [1, \infty)$

Range $= [0, \infty)$

viii Even, Odd functions

A function $f(x)$ is an

→ even fn of x if $f(-x) = f(x)$

→ odd fn of x if $f(-x) = -f(x)$

v Slope of vertical line

$m = \tan\theta = \tan 90 = \infty$

Slope of vertical line $= \pm\infty$

xv Evaluate $\lim\limits_{x\to 5} \dfrac{x-5}{\sqrt{x}-\sqrt{5}}$

$\lim\limits_{x\to 5} \dfrac{(\sqrt{x})^2-(\sqrt{5})^2}{(\sqrt{x}-\sqrt{5})} = \lim\limits_{x\to 5} \dfrac{\cancel{(\sqrt{x}-\sqrt{5})}(\sqrt{x}+\sqrt{5})}{\cancel{(\sqrt{x}-\sqrt{5})}}$

$\lim\limits_{x\to 5} \sqrt{x}+\sqrt{5} = \sqrt{5}+\sqrt{5}$

$= 2\sqrt{5}$

iv Third derivative $y = 5x^4 - 3x^3 + 8$

$\frac{dy}{dx} = 5(4)x^3 - 3(3)x^2 + 0$

$= 20x^3 - 9x^2$

$\frac{d^2y}{dx^2} = 20(3)x^2 - 9(2)x$

$= 60x^2 - 18x$

$\frac{d^3y}{dx^3} = 60(2)x - 18(1)$

$= 120x - 18$ —— Ans

xiv $f(x) = \sqrt{x} \Rightarrow f'(25) = ?$

$f(x) = (x)^{1/2}$

$f'(x) = \frac{1}{2}(x)^{\frac{1-2}{2}}(1)$

$= \frac{1}{2}(x)^{-1/2} = \frac{1}{2\sqrt{x}}$

$f'(25) = \frac{1}{2\sqrt{25}} = \frac{1}{2(5)}$

$f'(25) = \frac{1}{10}$ —— Ans.

Q4 Evaluate $\int \frac{\cos\sqrt{\theta}}{\sqrt{\theta}\,\sin^2\sqrt{\theta}}\,d\theta$

~~u = cosec~~

$$= \int \frac{1}{\sqrt{\theta}} \frac{\cos\sqrt{\theta}}{\sin\sqrt{\theta}} \cdot \frac{1}{\sin\sqrt{\theta}}\,d\theta$$

$$= \int \frac{\cot\sqrt{\theta}\,\operatorname{cosec}\sqrt{\theta}}{\sqrt{\theta}}\,d\theta$$

$\Rightarrow$ Let $u = \operatorname{cosec}\sqrt{\theta} \rightarrow (1)$

$$\frac{du}{d\theta} = -\operatorname{cosec}\sqrt{\theta}\cot\sqrt{\theta}\left(\frac{1}{2\sqrt{\theta}}\right)$$

$$\frac{du}{d\theta} = -\frac{1}{2\sqrt{\theta}}\operatorname{cosec}\sqrt{\theta}\cot\sqrt{\theta}$$

$$-2\,du = \frac{1}{\sqrt{\theta}}\operatorname{cosec}\sqrt{\theta}\cot\sqrt{\theta}\,d\theta \rightarrow (2)$$

$$= \int \frac{\cot\sqrt{\theta}\,\operatorname{cosec}\sqrt{\theta}}{\sqrt{\theta}}\,d\theta$$

$$= \int -2\,du = -2u + c$$

$$= -2(\operatorname{cosec}\sqrt{\theta}) + c$$

$$= -2\operatorname{cosec}\sqrt{\theta} + c \quad \text{Ans}$$

$$\frac{d(\sqrt{\theta})}{d\theta} = (\theta)^{1/2}$$

$$\cancel{(\theta)^{1/2}}$$

$$= \frac{1}{2}(\theta)^{\frac{1-2}{2}}$$

$$= \frac{1}{2}(\theta)^{-\frac{1}{2}}$$

Q3 $f(x) = 2x^3 - 15x^2 + 36x - 10$

Extreme values?

$f'(x) = 2(3)x^2 - 15(2)x + 36$

$= 6x^2 - 30x + 36$

For x values

$6x^2 - 30x + 36 = 0$

$a = 6 \quad b = -30 \quad c = 36$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{+30 \pm \sqrt{900 - 4(6)(36)}}{12}$

$x = \frac{30 \pm \sqrt{36}}{12}$

$x = \frac{30 + 6}{12} = 3$ | $x = \frac{30 - 6}{12} = 2$

$f'(x) = 6x^2 - 30x + 36$

$f''(x) = 6(2)x - 30$

$f''(x) = 12x - 30$

$12x - 30$
$12(2) - 30$
$24 - 30 = -6$

$f''(3) = 12(3) - 30 = 36 - 30$

$= 6 > 0$ relative minima.

$f''(2) = 12(2) - 30 = 24 - 30$

$= -\cancel{12}\,6 < 0$ relative maxima.

Q6 $y = (\sin^{-1}x)^2$ prove $(1-x^2)y'' - xy' - 2 = 0$

$y' = \frac{dy}{dx} = 2\sin^{-1}x \frac{d}{dx}(\sin^{-1}x)$

$y' = 2\sin^{-1}x\left(\frac{1}{\sqrt{1-x^2}}\right)$

$y'' = \frac{d}{dx}\left(\frac{2\sin^{-1}x}{\sqrt{1-x^2}}\right)$

$y'' = \frac{\sqrt{1-x^2}\frac{d}{dx}(2\sin^{-1}x) - 2\sin^{-1}x\frac{d}{dx}(\sqrt{1-x^2})}{(\sqrt{1-x^2})^2}$

$y'' = \frac{\sqrt{1-x^2}(2/\sqrt{1-x^2}) - 2\sin^{-1}x\left(\frac{1}{2}(1-x^2)^{-1/2}(-2x)\right)}{(1-x^2)}$

$(1-x^2)y'' = \sqrt{1-x^2}(2/\sqrt{1-x^2}) - 2\sin^{-1}x\left(\frac{1}{2}(1-x^2)^{-1/2}(-2x)\right)$

$(1-x^2)y'' = \frac{2\sqrt{1-x^2}}{\sqrt{1-x^2}} + 2\sin^{-1}x \frac{x}{\sqrt{1-x^2}}$

$(1-x^2)y'' = 2 + \frac{2x\sin^{-1}x}{\sqrt{1-x^2}}$

$(1-x^2)y'' = 2 + xy'$

$(1-x^2)y'' - xy' - 2 = 0$

Hence Proved ——— Ans

Evaluate $\int x^2 \tan^{-1}x \, dx$

$$= \tan^{-1}x \int x^2 dx - \int \frac{d}{dx} \tan^{-1}x \int x^2 dx$$

$$= \tan^{-1}x \, \frac{x^3}{3} - \int \frac{1}{1+x^2} \cdot \frac{x^3}{3} dx$$

$$= \tan^{-1}x \cdot \frac{x^3}{3} - \frac{1}{3} \int \frac{x^3}{1+x^2}$$

$$1+x^2 \overline{\big) \; x^3 \;}\; x$$
$$\underline{-x^3 \mp x}$$
$$-x$$

So,

$$= \tan^{-1}x \, \frac{x^3}{3} - \frac{1}{3} \int x - \frac{x}{1+x^2}$$

$$= \tan^{-1}x \, \frac{x^3}{3} - \frac{1}{3} \left[\int x \, dx - \frac{1}{2} \int \frac{2x}{1+x^2} dx \right]$$

$$= \tan^{-1}x \, \frac{x^3}{3} - \frac{1}{3} \frac{x^2}{2} - \frac{1}{6} \ln(1+x^2) + c$$

(i) Question Short (4): Evaluate $\lim_{x \to \infty} x \sin \frac{1}{x}$

$$\lim_{x \to \infty} \left(x \sin \frac{1}{x} \right)$$ Page 297

Put $x = \frac{1}{h} \Rightarrow \lim_{h \to 0} \left(\frac{1}{h} \sin h \right)$

$= 1$ —— Ans.

Q5 Evaluate $\int \frac{x}{1+x^4} dx$

$= \int \frac{x}{(1)^2+(x^2)^2} dx$

Let $x^2 = \tan\theta$

$2x\,dx = \sec^2 d\theta$

$x\,dx = \frac{1}{2} \sec^2\theta\, d\theta$

$\int \frac{\frac{1}{2}\sec^2\theta\, d\theta}{1+(\tan\theta)^2}$

$\int \frac{\frac{1}{2}\cancel{\sec^2\theta}\, d\theta}{\cancel{\sec^2\theta}\, d\theta}$

$\frac{1}{2}\int d\theta \Rightarrow \frac{1}{2}\theta + C$

$x^2 = \tan\theta \Rightarrow \theta = \tan^{-1} x^2$

Puting value

$= \frac{1}{2} \tan^{-1} x^2 + C$

$= \frac{\tan^{-1} x^2}{2} + C$ — Ans.

Area between curve $= \int_a^b f(x) - f(g)\; dx$