Parabola

A parabola is the set of all points P in the plane that are equidistand from a fixed line and a fixed point in the plane. The fixed point does not lie on the fixed line.

The fixed line is called directrix of parabola.

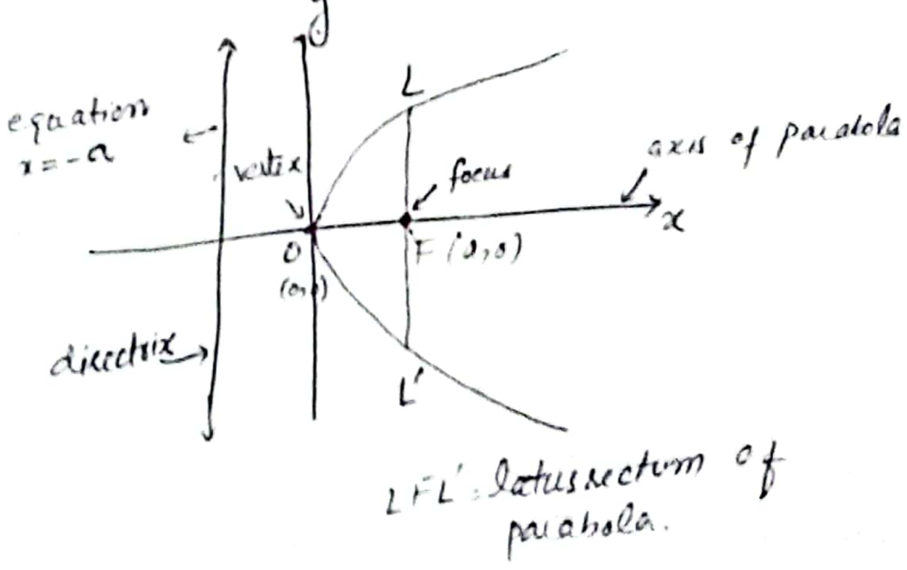
The fixed point is called focus of parabola.

The straight line through the focus and perpendicular to the directrix is called Axis of parabola

The point, where the parabola meets its axis is called vertex of parabola.

There are four standard parabolas.

(i) $y^2 = 4ax$



Coordinates of vertex = (0,0)

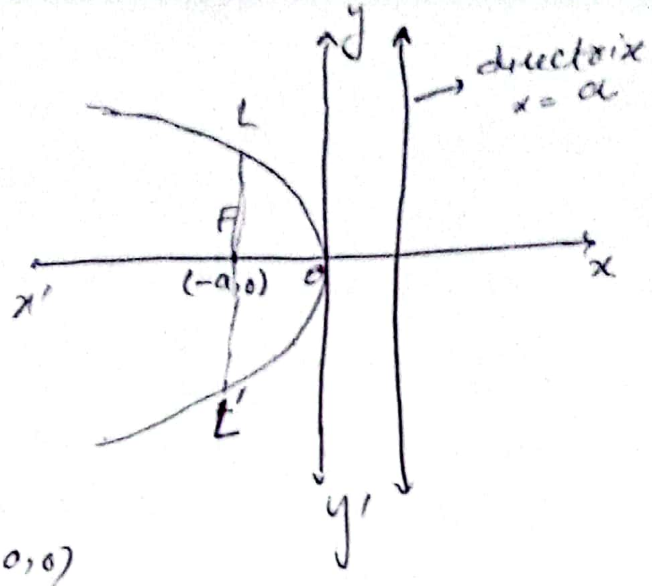
Coordinates of focus = (a,0)

equation of directrix: $x = -a$.

length of latusrectum = $4a$

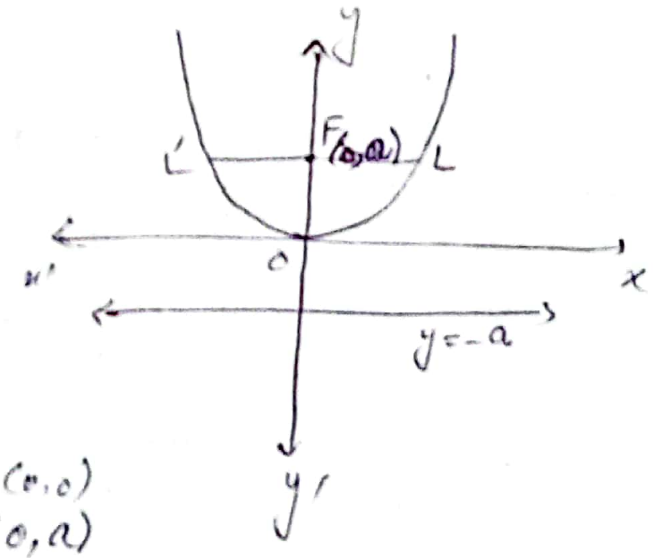
eccentricity = 1

(ii) $y^2 = -4ax$



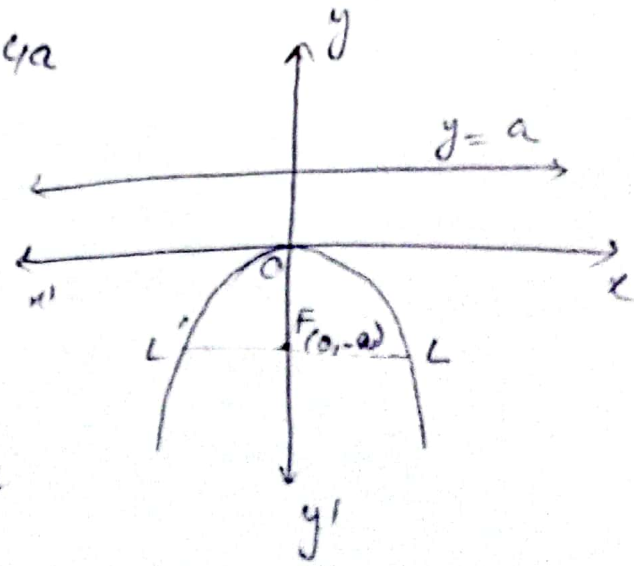
Coordinates of vertex $= (0,0)$
coordinates of focus $= (-a, 0)$
equation of directrix : $x = a$
length of latusrectum $= 4a$
(LFL')

(iii) $x^2 = 4ay$



Coordinates of vertex $= (0,0)$
Coordinates of focus $= (0,a)$
equation of directrix : $y = -a$
length of latusrectum LFL' $= 4a$

(iv) $x^2 = -4ay$



coordinates of vertex $= (0,0)$
coordinates of focus $= (0,-a)$
equation of directrix : $y = a$
length of latusrectum LFL' $= 4a$

Question

Find focus, vertex, equation of directrix for parabola; $x^2 = -6y$. Also draw graph.

Solution

Given parabola

$x^2 = -6y$

Comparing with $x^2 = -4ay$

$4a = 6$

$\Rightarrow a = \frac{6}{4} = \frac{3}{2}$

$\Rightarrow \boxed{a = \frac{3}{2}}$

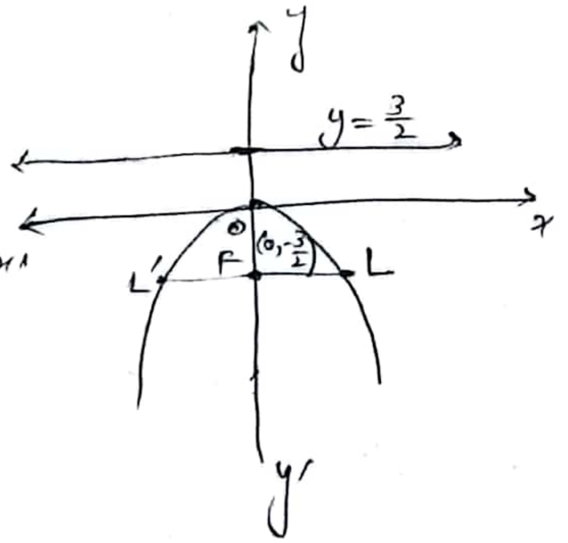
Now

Coordinates of vertex = $(0,0)$

Coordinates of Focus = $(0,-a) = (0,-\frac{3}{2})$

equation of directrix : $y = a \Rightarrow y = \frac{3}{2}$

length of latus rectum $LFL' = 4a = 4 \cdot \frac{3}{2} = 6$



Ellipse

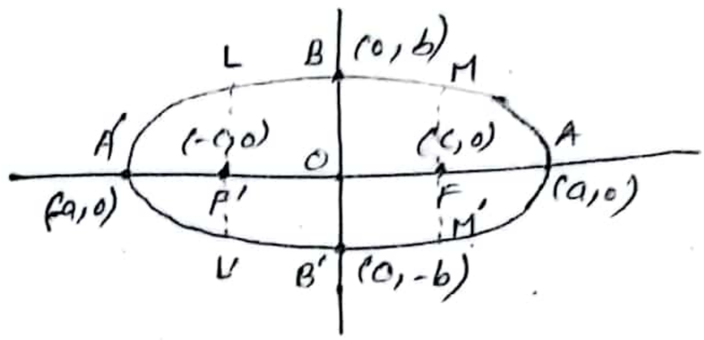
An ellipse is the set of all points in the plane the sum of whose distances from two fixed points is a constant.

The two fixed points are called the foci of ellipse. The point half-way between the foci is the centre of ellipse.

i) equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad ; \quad a > b$$

Graph



The line A'A through foci and across the ellipse is called major axis.

The line B'B through centre and perpendicular to major axis is called minor axis of ellipse.

The point A' and A (ends of major axis) are called vertices of ellipse

The point B' and B (ends of minor axis) are called covertices of ellipse

In above graph F', F are foci of ellipse

Centre of ellipse : (0,0)

Coordinates of foci : $(\pm c, 0)$ where $c^2 = a^2 - b^2$

Coordinates of vertices : $(\pm a, 0)$

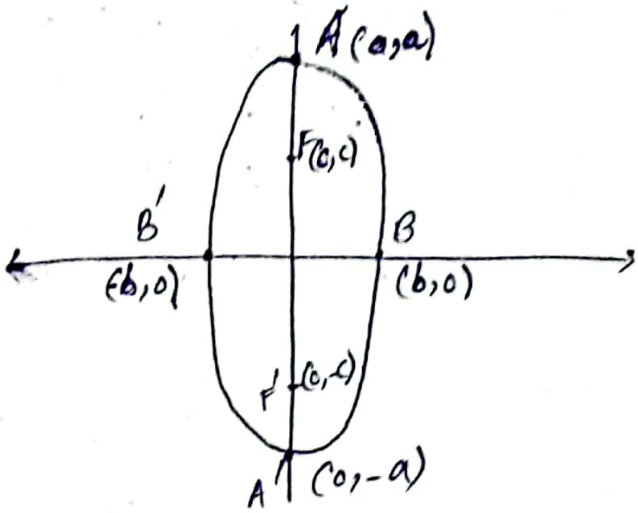
Coordinates of vertices : $(0, \pm b)$

The lines $LF'L'$ and MFM' are called **latera recta** (plural of latus rectum) and length of each is $\frac{2b^2}{a}$.

Length of each latus rectum $= \frac{2b^2}{a}$

eccentricity $= e = \frac{c}{a}$

(ii) **equation** $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$; $a > b$



Centre of ellipse : $(0,0)$

Coordinates of foci : $(0, \pm c)$ where $c^2 = a^2 - b^2$

Coordinates of vertices : $(0, \pm a)$

Coordinates of covertices : $(\pm b, 0)$

Length of each latus rectum $= \frac{2b^2}{a}$

eccentricity $= \frac{c}{a}$

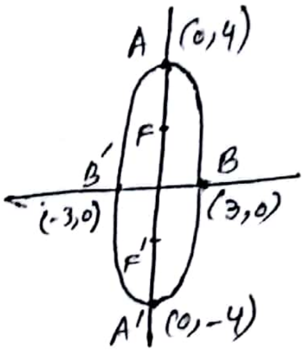
Question

Find coordinates of foci ; vertices, co-vertices, length of latus rectum, and eccentricity for following ellipse.

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

Solution

Rewriting equation of ellipse



$$\frac{y^2}{16} + \frac{x^2}{9} = 1$$

$$\Rightarrow \frac{y^2}{(4)^2} + \frac{x^2}{(3)^2} = 1$$

Here $a = 4$, $b = 3$

Now $c^2 = a^2 - b^2$

$\Rightarrow c^2 = (4)^2 - (3)^2$

$\Rightarrow c^2 = 16 - 9$

$\Rightarrow c^2 = 7$

$\Rightarrow c = \sqrt{7}$

Now

Coordinates of foci $= (0, \pm c) = (0, \pm\sqrt{7})$

coordinates of vertices $= (0, \pm a) = (0, \pm 4)$

coordinates of covertices $= (\pm b, 0) = (\pm 3, 0)$

Length of latusrectum $= \frac{2b^2}{a} = \frac{2(3)^2}{4} = \frac{18}{4} = \frac{9}{2}$

eccentricity $= \frac{c}{a} = \frac{\sqrt{7}}{4}$

The Hyperbola

A hyperbola, is the set of all points in the plane the difference of whose distances from two fixed points is a constant.

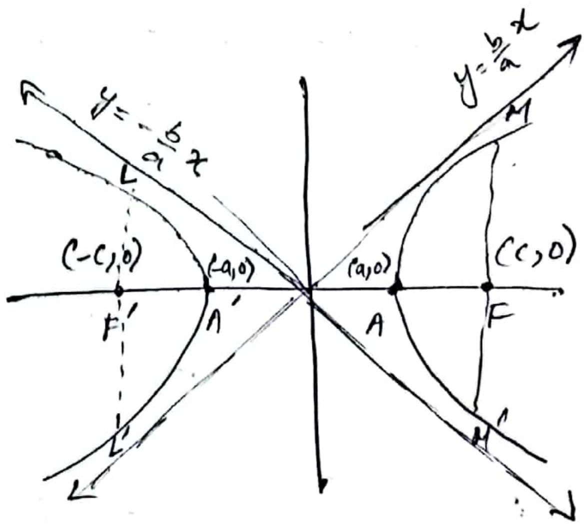
- The two fixed points are called __foci__ and the point half-way between foci is called __centre__, of hyperbola.

The line through foci is called __transverse axis__ (or __focal axis__) and the line through the centre and perpendicular to focal axis is called __Conjugate axis__ of hyperbola.

The points where the hyperbola meets the focal axis are called __vertices__ of hyperbola

Each half of hyperbola is called a __branch__

(i) Equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



Center of Hyperbola = $(0,0)$

Coordinates of foci = $(\pm c, 0)$ where $c^2 = a^2 + b^2$

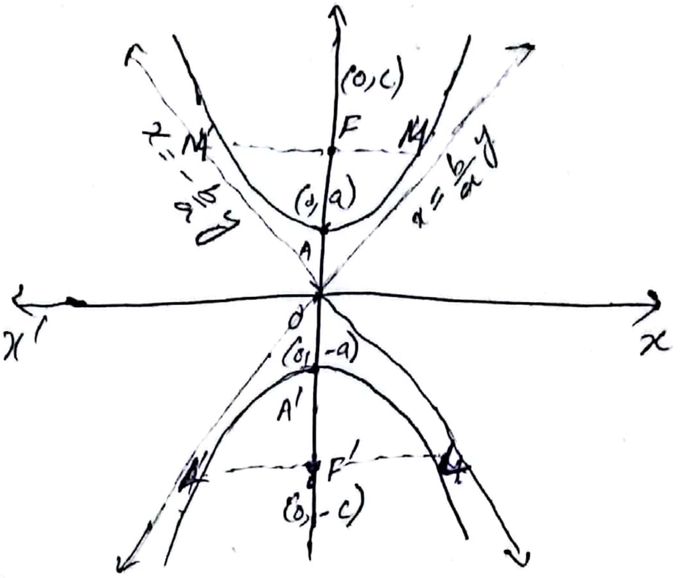
Coordinates of vertices = $(\pm a, 0)$

length of each latusrectum ($LF'L'$ or MFM') = $\frac{2b^2}{a}$

eccentricity = $\frac{c}{a}$

equation of Assymptotes : $y = \pm \frac{b}{a} x$

(iii) Equation $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$



Center of Hyperbola: $(0,0)$

Coordinates of foci: $(0, \pm c)$

where
$c^2 = a^2 + b^2$

Coordinates of vertices: $(0, \pm a)$

Length of each latusrectum $= \frac{2b^2}{a}$

eccentricity $= \frac{c}{a}$

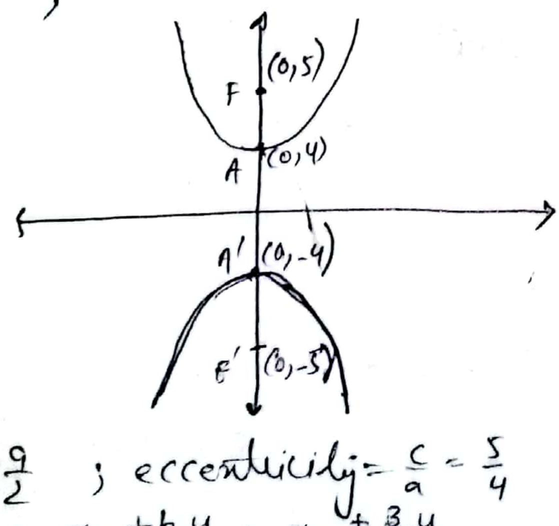
equation of Assymptotes: $x = \pm \frac{b}{a} y$

<u>Question</u>

Graph the Hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = -1$.
Find coordinates of foci and vertices. Also write equations of Assymptotes.

<u>Solution</u> Rewriting $\frac{y^2}{16} - \frac{x^2}{9} = 1$

$\Rightarrow \frac{y^2}{4^2} - \frac{x^2}{3^2} = 1$; Here $a = 4$; $b = 3$



Comparing with $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$\therefore c^2 = a^2 + b^2$
$\Rightarrow c^2 = 4^2 + 3^2 \Rightarrow c^2 = 16 + 9$
$\Rightarrow c^2 = 25 \Rightarrow \boxed{c = 5}$

Now
Coordinates of foci; $(0, \pm c) = (0, \pm 5)$
vertices coordinates; $(0, \pm a) = (0, \pm 4)$
Length of latusrectum $= \frac{2b^2}{a} = \frac{2 \cdot 9}{4} = \frac{18}{4} = \frac{9}{2}$; eccentricity $= \frac{c}{a} = \frac{5}{4}$
equations of Assymptotes; $x = \pm \frac{b}{a} y \Rightarrow x = \pm \frac{3}{4} y$