

Addition Laws:

If A and B are any two events defined in a sample space, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proof:

The event $A \cup B$ may be written as the union of two mutually exclusive events A and $B \cap \bar{A}$. that is

$$A \cup B = A \cup (B \cap \bar{A}).$$

Apply Probability on both side

$$P(A \cup B) = P(A) + \frac{P(B \cap \bar{A})}{P(B \cap \bar{A})} \rightarrow \text{eq (i)}$$

Again event B may be decomposed into Two mutually exclusive events.

$$B = (A \cap B) \cup (B \cap \bar{A})$$

Apply probability

$$P(B) = P(A \cap B) + P(B \cap \bar{A}) \rightarrow \text{eq (ii)}$$

$$P(A \cup B) - P(B) = P(A) + P(B \cap \bar{A}) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

1 head occurs?

The SS for this exp is
 $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$$n(S) = 8$$

let A represent event that at least one head occurs.

\bar{A} is the event that no head occurs.

$$\bar{A} = \{TTT\}$$

$$n(\bar{A}) = 1$$

$$P(A) = 1 - P(\bar{A})$$

$$1 - \frac{1}{8} = \frac{7}{8}$$

Question:

If one card is selected at random from a deck of 52 playing cards, what is the probability that card is club or a face card or both

The SS for this exp is

$$S = \binom{52}{1} = 52$$

$$n(S) = 52$$

$$S = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), \\ (6,1), (6,2) \}$$

Let A be the event

$$A = \{ (1,4), (2,3), (3,2), (4,1) \}$$

$$n(A) = 4$$

Let B be the event

$$B = \{ (6,5), (5,6) \}$$

$$n(B) = 2$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{4}{36} + \frac{2}{36} - 0$$

$$= \frac{6}{36}$$

$$= \frac{1}{6}$$

$$P(A \cup B) = \frac{1}{6}$$

Answer:

$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$
 $(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$
 $(6,1), (6,2), \dots, (6,6)$

Let A be the event

$$A = \{(1,4), (2,3), (3,2), (4,1)\}$$

$$n(A) = 4$$

Let B be the event

$$B = \{(6,5), (5,6)\}$$

$$n(B) = 2$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{4}{36} + \frac{2}{36} - 0$$

$$= \frac{6}{36}$$

$$P(A \cup B) = \frac{1}{6}$$

Question:

A class contain 10 men and 20 women's of which half women's have brown eyes Find the probability that a person choose at random is a man has brown eyes?

Question:

Date: —/—/20—

Two coin are Toss. Find the Probability of both are heads
Both faces are Same (i) only 1 is head.
atleast one is head on the first coin.

	H	T
H	HH	HT
T	TH	TT

$$S = \{HH, HT, TH, TT\}.$$

(i) Both are heads.

let A be the event that both are head

$$n(A) = 1$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$$

$$n(S) = 4$$

(ii) Both Faces are Same.

let B be the event that both faces are Same.

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

(iii) Only one is head.

let C is the event that only one is head.

$$P(C) = \frac{n(C)}{n(S)}$$

$$= \frac{2}{4} = \frac{1}{2}$$

iv) Atleast one is head.

Let D is the event that atleast one is head.
 $\{HH, HT, TH\}$.

$$P(D) = \frac{n(D)}{n(S)} = \frac{3}{4}$$

v) head on the first coin.

let E be the event that head on the first coin.

$$n(\bar{E}) = \frac{n(\bar{E})}{n(S)} = \frac{4}{2} = 2$$

Question:

A card is selected from a deck of 52 playing card. Find the probability.

- i) The card is red.
- ii) The card is king.
- iii) The card is of diamond.
- iv) The card is king or queen.
- v) It is a face card.

The SS for this exp is
 $S = \binom{30}{1} = 30$

Let A represent the person chosen,
is a man.

$$n(A) = 10 \text{ men}$$

$$P(A) = \frac{10}{30}$$

Let B be the event that person has
brown eyes.

$$P(B) = \frac{15}{30}$$

$$B = \{(6,5), (5,6)\}$$

, $n(B) = 2$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{10}{30} + \frac{15}{30} - \frac{5}{30} \end{aligned}$$

$$P(A \cup B) = \frac{2}{3}$$

Question:

Let A represent the event that card is club

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{13}{52} = \boxed{0.25}$$

Let B represent the event that card is face card.

$$P(B) = \frac{n(B)}{n(S)} = \frac{12}{52} = \boxed{0.23}$$

$$n(A \cap B) = 3$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52}$$

Question:

A pair of dice thrown find the probability of getting a total of either 5 or 11.

The SS for this exp is

Question:

A coin is Toss 3 Times in Succession what is the probability that atleast 1 head occur?

The SS for this exp is

$$S = \{HHH, HHT, HTH, THH, H\bar{T}, THT, TTH, TTT\}$$

$$n(S) = 8$$

let A represent event that at least one head occurs.

\bar{A} is the event that no head occurs.

$$\bar{A} = \{TTT\}$$

$$n(\bar{A}) = 1$$

$$P(A) = 1 - P(\bar{A})$$

$$1 - \frac{1}{8} = \frac{7}{8}$$

Question:

If one card is selected at random from a deck of 52 playing cards, what is the probability that card is club or a face card or both

The SS is for the experiment.

$$S = \binom{52}{1} = 52.$$

$$n(S) = 52.$$

(i) The card is red.

let A be the event that card is red.

$$A = \binom{13}{52} \quad P(A) = \frac{26}{52}$$

$$\frac{n(A)}{n(S)} = 0.25.$$

(ii) The card is King.

let B be the event that card is King.

$$P(B) = \binom{4}{52} = 0.076$$

(iii) The card is of diamond.

let C be the event that card is diamond.

$$n(C) = 13$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{13}{52} = 0.25$$

Conditional Probability:

The likely hood of an event or outcome occurring, based on the occurrence of a previous event or outcome. is called conditional probability

Example:

we assume that today is 40% chance of raining but this fact condition on many things, such as the probability of.

i) A cold front coming to your area
rain clouds forming.

Another front pushing the rain clouds
a way.

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) \frac{n(A \cap B)}{n(S)} \Rightarrow P(A) = \frac{n(A)}{n(S)}$$

Question:

Two coins are tossed? what is conditional probability that two heads result, given

the likely hood of an event
as outcome occurring, based on the
occurrence of a previous event or
outcome. is called conditional probability.

Example:

we assume that today is 40%
chance of raining but this fact condition
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Another front pushing the rain clouds
a way.

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) \frac{n(A \cap B)}{n(S)} \Rightarrow P(B) = \frac{n(A)}{n(S)}$$

Question:

Two coins are tossed? what is conditional
probability that two heads result, given
that there is at least one head?

- that
- (i) the sum is odd.
 - (ii) the sum is greater than 10.
 - (iii) the two dice had the same outcomes.

The SS experiment is

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}.$$

$$n(S) = 36.$$

let A be the event that the sum is 7.

$$n(A) = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$

$$n(A) = 6.$$

let B be the event that sum is odd.

- (ii) the sum is odd.
 (iii) the two dice had the same outcomes.

The SS experiment is

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}.$$

$$n(S) = 36.$$

let A be the event that the sum is 7.

$$n(A) = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$

$$n(A) = 6.$$

let B be the event that sum is odd.

$$n(B) = 18.$$

let C be the event that sum is greater than six.

$$C = \{(1,6), (2,5), (2,6), (3,4), (3,5), (3,6), \\ (4,3), (4,4), (4,5), (4,6), (5,2), (5,3), (5,4), \\ (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

Question:

what is the probability that a random selected a ~~Real~~ ^{Poker} hand contain ~~at least~~ ^{at least} 3 aces given that it contain ~~at least~~ ^{exactly} 2 aces.

$$N = 52$$

$$K = 4$$

$$n = 5$$

$$P(X=n) = \frac{\binom{K}{n} \binom{N-K}{n-n}}{\binom{N}{n}}$$

$$P(A) = P(X=3) = \frac{\binom{4}{3} \binom{52-4}{5-3}}{\binom{52}{5}}$$

$$P(A) = \frac{\binom{4}{3} \binom{48}{2}}{\binom{52}{5}}$$

let B be the event that at least 2 aces are selected.

$$P(B) = \frac{\binom{4}{2} \binom{48}{3}}{\binom{52}{5}} + \frac{\binom{4}{3} \binom{48}{2}}{\binom{52}{5}} + \frac{\binom{4}{4} \binom{48}{1}}{\binom{52}{5}}$$

$$P(A \cap B) = \frac{\binom{4}{3} \binom{48}{2}}{\binom{52}{5}}$$

TESTING hypothesis of Independence (categorical data)

General Procedure:

are independent

Null hypothesis: H_0 : The Two variable of classification

Alternative hypo: H_1 : The two variables of classification are not independent.

level of Significance:

$\alpha = 5\%, 1\%$ or etc.

Test Statistic:

$$\chi^2 = \frac{(ad - bc)^2}{(a+c)(b+d)(a+b)(c+d)} \text{ with 1 d.f.}$$

Critical Region:

Reject H_0 if $\chi^2_{(cal)} \geq \chi^2_{\alpha}(1)$

Calculation:

Conclusion:

Question: A random sample of 250 men

5. Calculation:

$$\chi^2 = \frac{[(80)(130) - (120)(170)]^2}{(250)(250)(200)(300)} (80 + 120 + 170 + 130)$$

$$\chi^2 = 13.33$$

: The two variable of classification
not independent.

level of significance:

$$\alpha = 0.05$$

Test Statistic:

$$\chi^2 = \frac{(ad - bc)^2}{(a+b+c+d)(a+c)(b+d)(a+b)(c+d)}$$

Critical Region:

Reject H_0 if

$$\chi^2_{(cal)} \geq \chi^2_{(0.05)(1)}$$

$$\chi^2_{(cal)} \geq 3.84$$

Classification	Men	women	Total
want television	80	120	200
Don't want television	170	130	300
Total	250	250	500

Test the hypothesis that desire to own a television set is independent of sex at the 5% level of

Null H_0 : The two variables of classification are independent.

H_1 : The two variables of classification are not independent.

level of significance:

$$\alpha = 0.05$$

Test Statistic:

$$\chi^2 = \frac{(ad - bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

General Procedure:

Null hypothesis: H_0 : The two variables of classification are independent

Alternative hypo: H_1 : The two variables of classification are not independent.

level of Significance:

$\alpha = 5\%, 1\%$ or etc.

Test Statistic:

$$\chi^2 = \frac{(ad - bc)^2}{(a+c)(b+d)(a+b)(c+d)} \text{ with 1 d.f.}$$

Critical Region:

Reject H_0 if $\chi^2_{(n)} \geq \chi^2_{\alpha}(n)$

Calculation:

Conclusion:

Question: A random sample of 250 men and 250 women were polled as to their desire concerning the ownership of television sets. The following data resulted

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\binom{4}{3} \binom{48}{2}}{\binom{52}{5}}$$

$$\frac{\binom{4}{2} \binom{48}{3} + \binom{4}{3} \binom{48}{2} + \binom{4}{4} \binom{48}{0}}{\binom{52}{5}}$$

$$\frac{4512}{108336}$$

$$= \frac{4512}{108336} \Rightarrow 0.0416$$

$$108336$$

Question:

$$n(C) = 21$$

let D be the event the outcomes have Same.

$$D = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$$

$$n(D) = 6$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{6/36}{18/36} = \frac{1}{3}$$

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{6/36}{21/36} = \frac{6}{21}$$

$$P(A|D) = \frac{0}{6/36} = 0$$

$$n(A \cap D) = 0$$

The SS of this experiment is

$$S = \{HH, HT, TH, TT\}.$$

Let A be the event that Two head occur;

$$A = \{HH\}$$

$$n(A) = 1$$

Let B be the event that atleast one head occur.

$$B = \{HH, HT, TH\}.$$

$$n(B) = 3$$

$$A \cap B = \{HH\}$$

$$n(A \cap B) = 1$$

$$P(A \cap B) = \frac{1}{4} = \frac{n(A \cap B)}{n(S)}$$

$$P(B) = \frac{3}{4} = \frac{n(B)}{n(S)}$$

$$P(A|B) = \frac{1/4}{3/4} = 1/3.$$

Question:

A man Toss Two fair dice. what is the conditional probability that the sum of dice will be 7. will be given

The SS of this experiment is

$$S = \{HH, HT, TH, TT\}.$$

Let A be the event that Two head occur:

$$A = \{HH\}$$

$$n(A) = 1$$

Let B be the event that atleast one head occur.

$$B = \{HH, HT, TH\}.$$

$$n(B) = 3$$

$$A \cap B = \{HH\}$$

$$n(A \cap B) = 1$$

$$P(A \cap B) = \frac{1}{4} = \frac{n(A \cap B)}{n(S)}$$

$$P(B) = \frac{3}{4} = \frac{n(B)}{n(S)}$$

$$P(A|B) = \frac{1/4}{3/4}$$

$$= 1/3$$

#12:

DEFINITIONS

Point Estimate:

A Point estimate is a number representing an estimate of the population parameter based on a sample.

Interval Estimate:

An Interval estimate is the range of values within which the value of the parameter is expected to lie.

Error of estimation:

The distance between an estimate and the estimated parameter is called error of estimation.

Unbiased Estimation:

An estimator is unbiased if its expected value is equal to the population parameter being estimated.

Level of Confidence:

The probability that the population parameter is included within the confidence interval is called the level of confidence.

Degree of Freedom:

Degree of Freedom is the number of values that are free to vary after we have placed certain restrictions upon the data.

Statistical Inference:

The process in which we make conclusion about a population based on sample information collected from the population.

Estimation:

Estimation is the process by which we attempt to determine the value of a population parameter from sample information.

Estimate:

Confidence Interval:

A confidence interval is a range of value within which the population parameter is expected to occur.

Confidence Limits:

The two endpoints of a confidence interval are called confidence limits.

The distance between an estimate and the estimated parameter is called error of estimation.

Unbiased Estimation:

An estimator is unbiased if its expected value is equal to the population parameter being estimated.

Biased Estimator:

If the mean of the estimator is not equal to the population parameter the estimator is said to be biased.

Estimate:

An estimate is the numerical value of the estimator.

Estimator:

An estimator is a statistic that specifies how to use the sample data to estimate an unknown parameter of the population.

is a normal distribution with a mean of zero and standard deviation of 1.

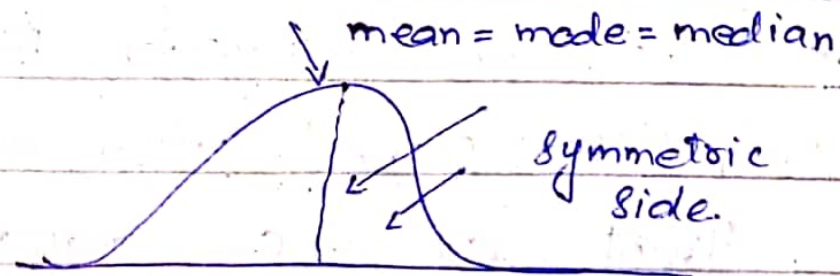
Properties of Normal ———

\Rightarrow The mean, median, and mode are exactly the same.

\Rightarrow It is symmetric. A normal distribution comes with a perfectly shape.

Normal Distribution:

is a probability distribution that is symmetric about the mean, showing that data near the mean are more frequent in occurrence than data far from the mean.



Standard Normal distribution:

The Standard normal distribution is a normal distribution with a mean of zero and standard deviation of 1.