Ch#8 Discrete P.D

Binomial Probability distrib:-

Many experiment consist of repeated independent trial each trial having two possible complemently outcomes called Binomial P.D.

Example:-

The two possible outcomes of a trial to may be head and tail.

P.d.F:-

$$f(x) = p(X=x) = \binom{n}{x} p^{x} q^{n-x}$$

$X = 0, 1, 2, \ldots n$

$\therefore p+q = 1$, $\mu = np$, $\sigma^2 = npq$

$p =$ probability sucess

$q =$ probability failure

Parameter:-

The binomial p.d has two parameter n and p generally denoted by $b(x; n, p)$

Date:

Properties of binomial dist:-

i) Two possible outcomes

ii) The probability of sucess remains constant

iii) The succesive trial are all independent.

iv) The experiment is repeated ~~large~~ a fixed number of times, say n.

Drive mean and variance of binomial distribution:-

Mean = $u = np$

Proof:-

$$u = E(x) = \sum_{0}^{n} x f(x)$$

$$= \sum_{0}^{n} x \binom{n}{x} p^{x} q^{n-x}, \quad x = 0, 1, 2, \ldots n$$

$$= 0q^{n-1} + 1\binom{n}{1} p^{1} q^{n-1} + \ldots + np^{n}$$

$$u = np\left(q^{n-1} + \binom{n-1}{1} q^{n-2} p + \ldots p^{n-1}\right)$$

$$u = np(q+p)^{n-1} \qquad \{\because p+q=1\}$$

$$\boxed{u = np}$$

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Variance $= \sigma^2 = npq$

Proof :-

$$\sigma^2 = E(x-\mu)^2 = E(x^2) - [E(x)]^2 \longrightarrow ①$$

$$\therefore E(x^2) = E(x(x-1)) + E(x)$$

$$= \sum_{i=0}^{n} x(x-1)\binom{n}{x} p^x q^{n-x} + np$$

$$= \sum_{i=2}^{n} \frac{n(n-1)(n-2)!}{(x-2)(x-n)!} p^{x-2} p^2 q^{n-x} + np$$

$$= n(n-1)p^2(q+p)^{n-2} + np$$

$$= n(n-1)\, p^2(1) + np$$

$$E(x^2) = n(n-1)\, p^2 + np \longrightarrow \text{put } ①$$

$$\sigma^2 = n(n-1)p^2 + np - (np)^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$= np - np^2$$

$$= np(1-p)$$

$$\boxed{\sigma^2 = npq}$$

Moment Generation Function B.D:-

$$M_0(t) = E(e^{tx}) \Rightarrow \sum_{x=0}^{n} e^{tx}\binom{n}{x} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \binom{n}{x} (pe^t)^x q^{n-x} = (q+pe^t)^n$$

2. Hypergeometric P.D :-

There are many experiment in which the condition of independence is violated and the probability of sucess does not remain constant for all trial. Such experiment called hypergeometric distribution.

P.d.F :-

$$P(X=x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} \quad , X=0,1,2,\ldots n$$

$$\boxed{p = \frac{K}{N}}$$

Parameter :-

Three parameter N, n, K.

Properties :-

i) Two possible outcomes
ii) The probability of success changes on each trial.
iii) The succesive trials are dependent

iv) The experiment is repeated a fixed number of times.

3. Poisson distribution :-

A limiting approximation of the binomial distribution $b(x;n,p)$ when p the probability of success is very small but n, the number of trial is so large that the product $np = \mu$ is a moderate size.

P.d.F :-

$$\lim_{\substack{n\to\infty \\ p\to 0}} b(x;n,p) = \frac{\mu^x e^{-\mu}}{x!}$$

$x = 0, 1, 2, \ldots \infty$

$e = 2.71828$

Parameter :-

only one parameter $\mu > 0$ denoted by $P(x;\mu)$

Poisson Frequency distribution:-

When the poisson distribution is multiplied by N. The number of sets of experiment, each of n trials, the resulting distribution is Known as poisson frequency distribution.

$$f(x) = N \cdot \frac{e^{-\mu}\mu^{x}}{x!} \quad , x = 0,1,2, \ldots \infty$$

Calculate Mean and Variance of poisson distribution:-

Mean $= E(x) = \mu$

Variance $= Var(X) = \mu$

Proof :-

$$E(x) = \sum_{0}^{\infty} x f(x)$$

$$= \sum_{0}^{\infty} x . p(x; \mu)$$

$$= \sum_{0}^{\infty} x . \frac{e^{-\mu}\mu^{x}}{x!} \quad , x = 0,1, \ldots \infty$$

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$$E(X) = 0 \cdot e^{-\mu} + 1 \cdot e^{-\mu}\mu + \frac{2e^{-\mu}\mu^2}{2!}$$

$$+ \frac{3e^{-\mu}\mu^3}{3!} + \cdots$$

$$E(X) = \mu e^{-\mu}\left[1 + \mu + \frac{\mu^2}{2!} + \cdots\right]$$

$$= \mu e^{-\mu} \cdot e^{\mu} \Rightarrow \mu e^{-\mu+\mu}$$

$$= \mu e^0 \qquad \{e^0 = 1\}$$

$$\boxed{E(X) = \mu}$$

$$\text{var}(x) = E(x^2) - [E(x)]^2 \longrightarrow ①$$

$$E(x^2) = E(x(x-1)+x) = E(x) + E(x(x-1))$$

$$E(x^2) = \sum_0^\infty x \frac{e^{-\mu}\mu^x}{x!} + \sum_0^\infty x(x-1)\frac{e^{-\mu}\mu^x}{x!}$$

$$= \mu + \sum_0^\infty \cancel{x(x-1)} \frac{e^{-\mu}\mu^x}{\cancel{x(x-1)}(x-2)!}$$

$$= \mu + \sum_0^\infty \frac{e^{-\mu}\mu^x}{(x-2)!}$$

x starts at 2

$$E(x^2) = \mu + \mu^2 e^{-\mu} \sum_{x=2}^{\infty} \frac{\mu^{x-2}}{(x-2)!}$$

Let $y = x-2$ then $y = 0, 1, 2, \ldots \infty$

$$E(x^2) = \mu + \mu^2 e^{-\mu} \sum_{0}^{\infty} \frac{\mu^y}{y!}$$

$$= \mu + \mu^2 e^{-\mu} e^{+\mu}$$

$$E(x^2) = \mu + \mu^2 e^{-2\mu} \Rightarrow \mu + \mu^2$$

put $E(x^2)$ in eq ①

$$\text{var}(x) = E(x^2) - [E(x)]^2$$

$$= \mu + \cancel{\mu^2} - \cancel{\mu^2}$$

$$\boxed{\text{var}(x) = \mu}$$

Moment Generating Function of Poisson distribution:-

Moment generation function

$$M_0(t) = E(e^{tx})$$

$$= \sum_{0}^{\infty} e^{tx} f(x)$$

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$$M_0(t) = \sum_0^\infty e^{tx} \frac{\mu^x e^{-\mu}}{x!}$$

$$= e^{-\mu} \sum_0^\infty \frac{e^{tx} \mu^x}{x!}$$

$$= e^{-\mu} \sum_0^\infty \frac{(e^t \mu)^x}{x!}$$

$$= e^{-\mu} e^{\mu e^t}$$

$$= e^{\mu(e^t - 1)}$$

$$\boxed{M_0(t) = e^{\mu(e^t - 1)}}$$

✓ Cumulant Generating function (c.g.f) of Poisson distribution:-

$$c.g.f = K(t) = \log_e M_0(t)$$

$$K(t) = \log_e e^{\mu(e^t - 1)}$$

$$= \mu(e^t - 1)$$

$$= \mu\left[t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots + \frac{t^r}{r!} + \cdots\right]$$

K_r = coefficient $\frac{t^r}{r!}$ in $K(t) = \mu$

Hence all cumulant of Poisson distribution equal to μ

Negative Binomial distribution:-

In the binomial experiments, the number of successes varies and the number of trial is fixed. But there are experiments in which the number of successes is fixed and the number of trials varies to produce the fixed number of successes. Such experiment are called negative binomial experiment.

P.D.f :-

$$P(X=x) = \binom{x-1}{k-1} p^k q^{x-k}$$

$x = k, k+1, k+2, \ldots$

Parameter:-

Negative binomial distribution has two parameter K and $p>0$

Properties:-

i) Two possible outcomes.
ii) The probability of success remains constant for all trials.
iii) The successive trials are all independent.
iv) The experiment is repeated a variable number of times.

Geometric Distribution:-

When an experiment consists of independent trials with probability p of success and the trials are repeated untill the first success occurs is called geometric distribution.

P.D.f :-

$$P(X=x) = q^{x-1}p \quad , \quad x = 1, 2, \ldots \infty$$

Properties :-

i) Two possible outcomes.
ii) The probability of success (p) remains constant for all trials.
iii) The successive trial are all independent.
iv) The experiment is repeated a variable number of times.

Multinomial distribution :-

A binomial experiment becomes a multinomial experiment when there are more than two possible outcomes of each trial.

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