

**1) Define Existential quantifier?**

**Ans:** A formal expression used in asserting that something exists of which a stated general proposition can be said to be true. We use the notation  $\exists xP(x)$  for the existential quantification of  $P(x)$ . Here  $\exists$  is called the *existential quantifier*.

**2) Define predicate?**

**Ans:** A predicate is an expression of one or more variables defined on some specific domain. A predicate with variables can be made a proposition by either assigning a value to the variable or by quantifying the variable. The following are some examples of predicates –

Let  $E(x, y)$  denote " $x = y$ "

Let  $X(a, b, c)$  denote " $a + b + c = 0$ "

Let  $M(x, y)$  denote " $x$  is married to  $y$ "

**3) Define reflexive relation?**

**Ans:** A relation  $R$  on a set  $A$  is called reflexive if  $(a, a) \in R$  for every element  $a \in A$ .

**4) Define Euler path?**

**Ans:** Euler circuit in a graph  $G$  is a simple circuit containing every edge of  $G$ . Euler path in  $G$  is a simple path containing every edge of  $G$ .

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**6) Give an indirect proof to the theorem: "if  $3n+2$  is odd, then  $n$  is odd"**

**Ans:** Assume  $n$  is even, that is  $n = 2k$ , where  $k$  is an integer.

$$3n+2 = 3(2k+2)$$

$$= 6k+2$$

$$= 2(3k+1)$$

Therefore  $3n+2$  is even that contradicts the assumption that  $3n+2$  is odd.

**7) Cardinality of a Set?**

**Ans:** Let  $S$  be a set. If there are exactly  $n$  distinct elements in  $S$  where  $n$  is a nonnegative integer, We say that  $S$  is a finite set and that  $n$  is the cardinality of  $S$ . The cardinality of  $S$  is denoted by  $|S|$

**8) Monotonic Function?**

**Ans:** A monotonic function is a function which is either entirely non increasing or non decreasing. A function is monotonic if its first derivative does not change sign.

**9) Define Transitive Closure?**

**Ans:** Let  $R$  be a relation on a set  $A$ . The *connectivity relation*  $R^*$  consists of the pairs  $(a, b)$  such that there is a path of length at least one from  $a$  to  $b$  in  $R$ .

**10) Proof by contradiction with example?**

**Ans:** A classic proof by contradiction from mathematics is the proof that the square root of 2 is irrational. If it were rational, it could be expressed as a fraction  $a/b$  in lowest terms, where  $a$  and  $b$  are integers, at least one of which is odd. But if  $a/b = \sqrt{2}$ , then  $a^2 = 2b^2$ . Therefore,  $a^2$  must be even.

**11) What is bipartite graph?**

**Ans:** bipartite graph (or bigraph) is a graph whose vertices can be divided into two disjoint and independent sets and such that every edge connects a vertex in to one in... Equivalently, a bipartite graph is a graph that does not contain any odd-length cycles.

**12) Define Euler path?**

**Ans:** Euler path, in a graph or multigraph, is a walk through the graph which uses every edge exactly once. Euler circuit is a Euler path which starts and stops at the same vertex. Our goal is to find a quick way to check whether a graph (or multigraph) has a Euler path or circuit.

**13) What is worst case complexity of bubble sort?**

**Ans:** The bubble sort always uses many comparisons, because it continues even if the list becomes completely sorted at some intermediate step. Consequently, the bubble sort uses  $(n-1)n/2$  comparisons, so it has  $\_ (n^2)$  worst-case complexity in terms of the number of comparisons used.

**14) What is the space complexity of linear search algorithm?**

**Ans:** in computer science, a linear search or sequential search is a method for finding an **element** within a list. It sequentially checks each element of the list until a match is found or the whole list has been searched.

Worst complexity:  $O(n)$

Average complexity:  $O(n)$

Space complexity:  $O(1)$

**15) Define reflexive function?**

**Ans:** A relation  $R$  on a set  $A$  is called *reflexive* if  $(a, a) \in R$  for every element  $a \in A$ .

**16) Complete graph?**

**Ans:** A complete graph is a graph in which each pair of graph vertices is connected by an edge. The complete graph with graph vertices is denoted and has undirected edges. Complete graphs are sometimes called universal graphs.

**17) Define a tree?**

**Ans:** Tree is a discrete structure that represents hierarchical relationships between individual elements or nodes. A tree in which a parent has no more than two children is called a binary tree.

**18) Define graph?**

**Ans:** A graph  $G = (V, E)$  consists of  $V$ , a nonempty set of *vertices* (or *nodes*) and  $E$ , a set of *edges*. Each edge has either one or two vertices associated with it, called its *endpoints*. An edge is said to *connect* its endpoints.

**19) Cardinality of a set?**

**Ans:** Let  $S$  be a set. If there are exactly  $n$  distinct elements in  $S$  where  $n$  is a nonnegative integer, we say that  $S$  is a finite set and that  $n$  is the cardinality of  $S$ . The cardinality of  $S$  is denoted by  $|S|$ .

**20) What is the minimum no of students required in a class to be sure that at least 6 will receive the same grade if there are five possible grades A, B, C, D, and F?**

**Ans:** The smallest such integer is  $= 5 \cdot 5 + 1 = 26$ . If you have only 25 students, it is possible for there to be five who have received each grade so that no six students have received the same grade. Thus, 26 is the minimum number of students needed to ensure that at least six students will receive the same grade.

**21) Intersection of sets?**

**Ans:** The intersection of the sets  $A$  and  $B$ , denoted by  $A \cap B$ , is the set Containing those elements in both  $A$  and  $B$ . An element  $x$  belongs to the intersection of the sets  $A$  and  $B$  if and only if  $x$  belongs to  $A$  and  $x$  belongs to  $B$ . This tells us that  $A \cap B = \{x \mid x \in A \wedge x \in B\}$

**22) Briefly explain the types of quantifiers?**

**Ans:** In general, a quantification is performed on formulas of predicate logic, such as  $x > 1$  or  $P(x)$ , by using quantifiers on variables. There are two types of quantifiers: universal quantifier and existential quantifier.

**23) What is the quotient and remainder when 101 is divided by 11?**

**Ans:** The quotient when 101 is divided by 11 is  $9 = 101 \text{ div } 11$ , and the remainder is  $2 = 101 \text{ mod } 11$ .

**24) What is the GCD of 45 and 60?**

**Ans:** Find the prime factorization of  $45 = 3 \times 3 \times 5$

Find the prime factorization of  $60 = 2 \times 2 \times 3 \times 5$

To find the gcd, multiply all the prime factors common to both numbers:  
Therefore,  $GCF = 3 \times 3 = 9$

**25) Among any group of 367 people, there must be at least two with the same birthday. Why?**

**Ans:** To use pigeonhole principle, first find boxes and objects.

Suppose that for each day of a year, we have a box That contains a birthday that occurs on that day.

The number of boxes is 366 and the number of objects is 367. By the pigeonhole principle, at least one of these boxes Contains two or more birthdays. So, there must be at least two people with the same Birthday.

**26) Differentiate between permutation and combination with the help of example?**

**Permutation:**

A selection of objects in which the order of the objects matters.

Example: The permutations of the letters in the set {a, b, c} are:

abc, bca, bac, cab, cba

A formula for the number of possible permutations of k objects from a set of n. This is usually written  $nPk$ .

Formula:

$${}_nP_k = \frac{n!}{(n-k)!} = n(n-1)(n-2) \cdots (n-k+1)$$

**Combination**

The number of possible combinations of r objects from a set on n objects.

$$\binom{n}{r} \text{ or } {}_nC_r \text{ or } C(n, r) \text{ or occasionally } C_r^n$$

$$\binom{n}{r} = \frac{{}_nP_r}{r!}$$

**27) Differentiate between function and relation with example?**

**Ans:**

**RELATION:**

Two sets of elements called input and output, where the input is related to the output in some way.

**FUNCTION:**

A relation in which no input relates to than one output.

**28) Differentiate between simple and multigraph with example?**

**Ans:** A graph which has neither loops nor multiple edges i.e. where each edge connects two distinct vertices and no two edges connects the same pair of vertices is called a simple graph. Any graph which contains some multiple edges is called a multigraph. In a multi graph, no loops are allowed.

**29) Differentiate between path and circuit in a graph with the help of example?**

**Ans:** A circuit is path that begins and ends at the same vertex. A circuit that doesn't repeat vertices is called a cycle. A graph is said to be connected if any two of its vertices are joined by a path. A graph that is not connected is a disconnected graph.

**30) Define bipartite graph with the help of example?**

**Ans:** A bipartite graph, also called a bigraph, is a set of graph vertices decomposed into two disjoint sets such that no two graph vertices within the same set are adjacent. A bipartite graph is a special case of a k-partite graph with.... All acyclic graphs are bipartite.

**31) Write the names of an algorithm property?**

**Ans:** An algorithm must satisfy the following properties:

Input: The algorithm must have input values from a specified set. Output: The algorithm must produce the output values from a specified set of input values. ... Finiteness: For any input, the algorithm must terminate after a finite number of steps.

**32) What is the decimal expansion of the number with hexadecimal expansion (2AE0B) 16?**

**Ans:**  $(2AE0B)_{16} = 2 \cdot 16^4 + 10 \cdot 16^3 + 14 \cdot 16^2 + 0 \cdot 16^1 + 11 \cdot 16^0 = (175627)_{10}$ .

**33) Define partial ordering with example?**

**Ans:** In mathematics, especially order theory, a partially ordered set (also poset) formalizes and generalizes the intuitive concept of an ordering, sequencing, or arrangement of the elements of a set. A poset consists of a set together with a binary relation indicating that, for certain pairs of elements in the set, one of the elements precede the other in the ordering. The relation itself is called a "partial order.

**34) Define the spanning tree of a graph?**

**Ans:** In the mathematical field of graph theory, a spanning tree T of an undirected graph G is a sub graph that is a tree which includes all of the vertices of G, with minimum possible number of edges.

**35) How can you produce the terms of a sequence if the first 10 terms are 5, 11, 17, 23, 29, 35, 41, 47, 53, 59...**

**Ans:** 5, 11, 17, 23, The numbers increase by 6  
so, 5, 11, 17, 23, 29, 35, 41, 47, 53, 59.

Formula:  $a + (n - 1)d$

$a = 5$

$d = 6$

$5 + (10 - 1)6$

10th term = 59

**36) How many elements with more than two elements does a set with 100 elements have?**

**Ans:** Thus, there are  $2(n - 1)$  such functions.

Solution The total number of subsets of a set with 100 elements is 2100.

**37) What are the connected components of a graph?**

**Ans:** In graph theory, a component, sometimes called a connected component, of an undirected graph is a subgraph in which any two vertices are connected to each other by paths, and which is connected to no additional vertices in the super graph. For example, the graph shown in the illustration has three components.

**38) What is the height of the rooted tree?**

**Ans:** The height of a rooted tree is the maximum of the levels of vertices. In other words, the height of a rooted tree is the length of the longest path from the root to any vertex. A rooted m-ary tree of height h is balanced if all leaves are at levels h or h - 1.

**39) Logically equivalent?**

**Ans:** In logic, statements p and q are logically equivalent if they have the same logical content. That is, if they have the same truth value in every model. The logical Equivalence of p and q is sometimes expressed as  $p \equiv q$ ,  $E_{pq}$ , or  $p \leftrightarrow q$

**40) Pseudo code?**

**Ans:** Pseudo code is a term which is often used in programming and algorithm-based fields. It is a methodology that allows the programmer to represent the implementation of an algorithm.

**41) Recursive algorithm?**

**Ans:** A recursive algorithm is an algorithm which calls itself with "smaller" input values, and which obtains the result for the current input by applying simple operations to the returned value for the smaller input.

**42) Permutation?**

**Ans:** In mathematics, permutation is the act of arranging the members of a set into a sequence or order, or, if the set is already ordered reordering its elements this process called permuting.

**43) Random variable?**

**Ans:** A discrete random variable is one which may take on only a countable number of distinct values such as 0, 1, 2, 3, 4, ..... Discrete random variables are usually (but not necessarily) counts. If a random variable can take only a finite number of distinct values, then it must be discrete. Examples of discrete random variables include the number of children in a family, the Friday night attendance at a cinema, the number of patients in a doctor's surgery, the number of defective light bulbs in a box of ten.

**44) Derangement?**

**Ans:** In combinatorial mathematics, a derangement is a permutation of the elements of a set, such that no element appears in its original position. The number of derangements of a set of size n is known as the subfactorial of n or the n-th derangement number or n-th de Montmort number.

**45) Multi-graph?**

**Ans:** The term multigraph refers to a graph in which multiple edges between nodes are either permitted.

**46) Tree traversal?**

**Ans:** Traversing means to visit all the nodes of the tree. There are three standard methods to traverse the binary trees. These are as follows:

- 1) Preorder Traversal
- 2) Post order Traversal
- 3) In order Traversal

**47) Full Adder?**

**Ans:** A full adder is a digital circuit that performs addition. Full adders are implemented with logic gates in hardware. A full adder adds three one-bit binary numbers, two operands and a carry bit. The adder outputs two numbers, a sum and a carry bit

**48) Pascal's Triangle?**

**Ans:** In mathematics, Pascal's triangle is a triangular array of the binomial coefficients.

**49) Recurrence Relation?**

**Ans:** A recurrence relation is an equation that recursively defines a sequence where the next term is a function of the previous terms.

**50) Leaf?**

**Ans:** A leaf of an unrooted tree is a node of vertex degree 1. Or a leaf is a node in tree which have no child is known as leaf node.

**51) Greedy algorithm?**

**Ans:** A greedy algorithm is a simple, algorithm that is used in optimization problems. The algorithm makes the optimal choice at each step as it attempts to find the overall optimal way to solve the entire problem. However, in many problems, a greedy strategy does not produce an optimal solution.

**52) K-map?**

**Ans:** A Karnaugh map is a planar area subdivided into  $2^n$  equal cells each representing point for functions of  $n$  variables. Each variable  $x$  is used to split the area into two equal halves in a different way, i.e., one for  $x$  and other for  $x'$ . The cells corresponding to the arguments for which the function has the value 1 contains 1.

**53) Postulate?**

**Ans:** A statement, also known as an axiom, which is taken to be true without proof. Postulates are the basic structure from which lemmas and theorems are derived. The whole of Euclidean geometry, for example, is based on five postulates known as Euclid's postulates.

**54) Sequence?**

**Ans:** In mathematics, a sequence is an enumerated collection of objects in which repetitions are allowed. Formally, a sequence can be defined as a function whose domain is either the set of the natural numbers or the set of the first  $n$  natural numbers.

**55) Algorithm?**

**Ans:** An algorithm is a set of procedures for solving a problem. Example: Describe an algorithm for finding the maximum value in a finite sequence of integers.

**56) Boolean expression?**

**Ans:** The variables which can have two discrete values 0 (False) and 1 (True) and the operations of logical significance are dealt with Boolean algebra. Boolean algebra is widely accepted in switching theory, building basic electronic circuits and designing of the digital computers.

**57) Forest?**

**Ans:** A forest is an undirected graph in which any two vertices are connected by at most one path. Equivalently, a forest is an undirected acyclic graph. Equivalently, a forest is an undirected graph, all of whose connected components are trees; in other words, the graph consists of a disjoint union of trees.

**58) Combination?**

**Ans:** A combination is a way of choosing elements from a set in which order does not matter. A wide variety of counting problems can be cast in terms of the simple concept of combinations therefore this topic serves as a building block in solving a wide range of problems.

**59) Undirected Graph?**

**Ans:** A graph whose edges are not directed. An undirected graph is a graph in which there is at most one edge between each pair of vertices, and there are no loops, which is an edge from a vertex to itself.

**60) Halting problem?**

**Ans:** A decision problem is any arbitrary yes/no question on an infinite set of inputs. A Turing machine is a mathematical model of computation. Turing machine can be halting as well as non halting and it depends on algorithm and input associated with the algorithm.

**61) Binomial Theorem?**

**Ans:** The binomial theorem (or binomial expansion) is a result of expanding the powers of binomials or sums of two terms. The theorem and its generalizations can be used to prove results and solve problems in combinatorics, algebra, calculus, and many other areas of mathematics.

**62) Bernoulli Trial?**

**Ans:** A Bernoulli trial is a probabilistic experiment that has two outcomes: success or failure (e.g., heads or tails). We suppose that  $p$  is the probability of success, and  $q = (1 - p)$  is the probability of failure. We can of course have repeated Bernoulli trials.

**63) Half Adder?**

**Ans:** A **half adder** is a type of **adder**, an electronic circuit that performs the addition of numbers. The **half adder** is able to add two single binary digits and provide the output plus a carry value. It has two inputs, called A and B, and two outputs S (sum) and C (carry).

**64) Pseudo Graph?**

**Ans:** A pseudograph is a non-simple graph in which both graph loops and multiple edges are permitted.

**65) Circuit or cycle in a digraph?**

**Ans:** A path that start and ends at the same vertex.

**66) Difference between tree and graph?**

**Ans: Tree:** A connected undirected graph with no simple circuits

**Graph:** A graph is a tree only and only if there is a unique simple path between every pairs of its vertices. A tree with  $n$  vertices has  $n-1$  edges.

**67) Connected component of a graph:**

**Ans:** In a connected graph component of undirected is a sub graph in which any two vertices are connected to each other by path and which is connected to no additional vertices in the sub graphs.

**68) Partial ordering with example;**

**Ans:** A relation  $R$  on set  $S$  is called a partial ordering or partial order if it is reflexive, anti symmetric and transitive.

**69) Difference between precondition and post conditions:**

**Ans:** The statements that describe valid input are known as precondition and the conditions that the output should satisfy when the program has run are known as post conditions.

**70) Universal quantifier**

**Ans:** The *universal quantification* of  $P(x)$  is the statement " $P(x)$  for all values of  $x$  in the domain."

The notation  $\forall xP(x)$  denotes the universal quantification of  $P(x)$ . Here  $\forall$  is called the **Universal quantifier**. We read  $\forall xP(x)$  as "for all  $xP(x)$ " or "for every  $xP(x)$ ." An element for which  $P(x)$  is false is called a **counterexample** of  $\forall xP(x)$ .

**71) What is an argument, premises and conclusion?**

**Ans:** An *argument* in propositional logic is a sequence of propositions. All but the final proposition in the argument are called *premises* and the final proposition is called the *conclusion*. An argument is *valid* if the truth of all its premises implies that the conclusion is true.

**72) Define Floor Function**

**Ans:** The *floor function* assigns to the real number  $x$  the largest integer that is less than or equal to  $x$ . The value of the floor function at  $x$  is denoted by  $\lfloor x \rfloor$ . The *ceiling function* assigns to the real number  $x$  the smallest integer that is greater than or equal to  $x$ . The value of the ceiling function at  $x$  is denoted by  $\lceil x \rceil$ .



**73) Prolog Rules?**

**Ans:** Prolog programs include a set of declarations consisting of two types of statements, **Prolog facts** and **Prolog rules**. Prolog facts define predicates by specifying the elements that satisfy these predicates. Prolog rules are used to define new predicates using those already defined by Prolog facts.

**74) "Every student in this class has studied calculus." Translate into mathematical form?**

**Ans:** "For every student  $x$  in this class,  $x$  has studied calculus."

Continuing, we introduce  $C(x)$ , which is the statement " $x$  has studied calculus." Consequently, if the domain for  $x$  consists of the students in the class, we can translate our statement as  $\forall x C(x)$ . See example 23 page no 48 for detail

**75) Let  $Q(x)$  denote the statement " $x = x + 1$ ." What is the truth value of the quantification  $\exists x Q(x)$ , where the domain consists of all real numbers?**

**Ans:** Because  $Q(x)$  is false for every real number  $x$ , the existential quantification of  $Q(x)$ , which is  $\exists x Q(x)$ , is false. See example 15 page no 43 for detail

**76) Determine whether the integers 10, 17 and 21 are pair wise relatively prime?**

**Ans:** Because  $\gcd(10, 17) = 1$ ,  $\gcd(10, 21) = 1$ , and  $\gcd(17, 21) = 1$ , we conclude that 10, 17, and 21 are pairwise relatively prime.

Because  $\gcd(10, 24) = 2 > 1$ , we see that 10, 19, and 24 are not pairwise relatively prime. Example no 13 page no 266.

**77) Encrypt the message WATCH YOUR STEP by translating the letters into numbers, applying the given encryption function, and then translating the numbers back into letters.  $F(p) = (14p+21) \bmod 26$ .**

**Ans:** Number Theory and Cryptography exercise question no 3

$$F(p) = (14p+21) \bmod 26$$

Let us apply the function to 22 0 19 27 24 14 20 17 18 19 4 15

$$17 \ 21 \ 1 \ 23 \ 15 \quad 19 \ 9 \ 15 \ 25 \quad 13 \ 1 \ 25 \ 23$$

Let A=0, B=1, C=2, D=3, E=4, F=5, G=6, H=7, I=8, J=9, K=10, L=11, M=12, N=13, O=14, P=15, Q=16, R=17, S=18, T=19, U=20, V=21, W=22, X=23, Y=24, Z=25

RVBXP TJPZ NBZX

**78) How many permutation of the letters ABCDEFG contain the string CFGA?**

**Ans:** We treat the mentioned string as one single letter, thus we have then the possible letters CFGA ,B, D, E, we then have 4 letters of which we want to select 4 letters.

$$N=4$$

$$R=4$$

Evaluate the definition of permutation

$$P(4,4) = 4!/(4-4!) = 4!/0! = 4! = 24$$

**79) Find recurrence relation of the sequence  $S(n) = 5^n$**

**Ans:** The sequence is as  $b_n = 5^n$  substituting  $k$  for  $n$  we get  $b_k = 5^k$

Substituting  $k-1$  for  $n$  we get  $b_{k-1} = 5^{k-1}$

Multiplying both sides by 5 we get

$$5 \cdot b_{k-1} = 5 \cdot 5^{k-1} \Rightarrow 5^k = b_k \text{ .....by using (1)}$$

$$b_k = 5b_{k-1}$$

**80) Write the names of an algorithm properties?**

- The inputs must be specified.
- The outputs must be specified.
- Definiteness.
- Effectiveness.
- Finiteness.

81) Define modes ponen rule of inference?

Ans: In propositional logic, **modus ponens** also known as **modus ponendo ponens** or implication elimination or affirming the antecedent, is a deductive argument form and **rule of inference**. It can be summarized as "P implies Q. P is true.

82) What are the symmetric matrix?

Ans: If a matrix 'A' is said to be symmetric if  $A^t = A$ . For example  $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  then  $A^t = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

83) What is cartesian product of  $A = \{a, b\}$  and  $B = \{1, 2, 3, 4\}$ .

Ans:  $A * B = \{a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4\}$

84) In a certain country, the car number plate is formed by 4 digits from the digits 1,2,3,4,5,6,7,8 and 9 followed by three letters from the alphabet. How many number plates can be formed if neither the digits nor the letters are repeated?

Ans: There are  $9^4 * 26^3 = 115316136$  different plates possible.

85) How many comparisons are needed for a binary search in a set of 64 elements?

**Binary search** divides each list into two sublists and determines in which sublist the elements will need to be present (since the initial list is sorted).

Let  $f(n)$  represent the number of comparisons in a set of  $n$  elements.

Binary search thus makes 2 comparisons in each step (to the two sublists), while the number of elements in the new list is half the number of elements of the original list:

$$f(n) = f(n/2) + 2$$

**Note: divide-and-conquer recurrence relation.**

Repeatedly apply the previous relation with  $n = 64$ :

$$\begin{aligned} f(64) &= f(32) + 2 \\ &= f(16) + 4 \\ &= f(8) + 6 \\ &= f(4) + 8 \\ &= f(2) + 10 \\ &= f(1) + 12 \\ &= 2 + 12 \\ &= 14 \end{aligned}$$

**Note: When the list contains 1 element, then we require 2 comparisons.**

86) Define this function  $f(x) = (x+1)/(x^2-4)$  onto or one to one, Domain consist of all integers.

Ans:

87) What is a recurrence relation?

Ans: A recurrence relation is an equation that recursively defines a sequence where the next term is a function of the previous terms

88) Construct truth table for (1)  $(p \vee q) \vee \sim p$ . (2)  $p \vee \sim q$

Try second yourself

p	q	$p \vee q$	$\sim p$	$(p \vee q) \vee \sim p$
T	T	T	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	T

89) Find the least integer n such that  $f(x)$  is  $O(x^n)$  for each of these functions.  $F(x) = (x^4 + 5 \log x)/(x^4 + 1)$ .



(d) Given:

$$f(x) = \frac{x^3 + 5 \log x}{x^4 + 1}$$

When  $x > 0$ , then  $\log x \leq x$ .

$$\begin{aligned} |f(x)| &= \left| \frac{x^3 + 5 \log x}{x^4 + 1} \right| \\ &\leq \left| \frac{x^3 + 5x}{x^4 + 1} \right| \end{aligned}$$

When  $x > 3$ , we have the property  $x^3 + 5x < x^4 + 1$  (which is noticeable in the following graph).

**90) Difference between Cycle and wheel in a graph.**

Ans: If the degree of each vertex in the graph is two, then it is called a **Cycle Graph**.

A **wheel graph** is obtained from a cycle graph  $C_{n-1}$  by adding a new vertex. That new vertex is called a Hub which is connected to all the vertices of  $C_n$ .

**91) State the idempotent law.**

Ans: The idempotent law states that  $x \text{ OR } x$  is  $x$  and  $x \text{ AND } x$  is  $x$ . The theorems of boolean algebra can be proved using Huntington postulates. Each postulate and theorem of boolean algebra has two parts; one is dual of another. If one part is proved the other one can be proved using duality principle.

**92) What is the prefix notation of the given expression  $(a / (b * c + d) \wedge f - e)$ .**

Ans:  $/ a - \wedge + * b c d f e$ .

**93) What is proposition?**

Ans: A statement which is either true or false is called propositions. Propositions are usually denoted by  $p, q, r$ , etc.

**94) What is Conjunction ( $\wedge$ )?**

Ans: Let  $p$  and  $q$  be two propositions. The conjunctions of  $p$  and  $q$ , denoted by  $p \wedge q$  is true when both  $p$  and  $q$  are true, otherwise false. The truth table of conjunction is,

P	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

**95) What is Disjunction ( $\vee$ )?**

Ans: Let  $p$  and  $q$  be two propositions. The disjunctions of  $p$  and  $q$ , denoted by  $p \vee q$  is false when both  $p$  and  $q$  are false, otherwise true. The truth table of disjunction is,

P	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

**96) What is Conditional Statement? Or implication? ( $\rightarrow$ )**

Ans: Let  $p$  and  $q$  be two propositions. Thus implication or conditional of  $p$  and  $q$ , denoted by  $(p \rightarrow q)$ , is a proposition which is false, when  $p$  is true and  $q$  is false and otherwise always true.

The truth table of condition statement is,

P	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

**97) What is Bi-Conditional Or Equivalence? ( $\leftrightarrow$ )**

**Ans:** Let p and q be two propositions. Thus bi-conditional of p and q, denoted by  $(p \leftrightarrow q)$ , is a proposition which is true when both p and q have same truth values, and otherwise false, the Truth table of Bi-condition statement is,

P	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

**Exclusive or? XOR ( $\oplus$ )**

**Ans:** Let p and q be two propositions. Thus, XOR of p and q, denoted by  $(p \oplus q)$ , is a proposition which is false when both p and q have same truth values, and otherwise true, the Truth table of Bi-condition statement is,

P	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

**98) What is Inverse, converse and contrapositive?**

- $q \rightarrow p$  is called the converse of  $p \rightarrow q$
- $\sim p \rightarrow \sim q$  is called the inverse of  $p \rightarrow q$
- $\sim q \rightarrow \sim p$  is called the contrapositive of  $p \rightarrow q$

**99) What is a Tautology?**

**Ans:** A statement which is true for all cases is called tautology.

**100) What is an absurdity Or Contradiction?**

**Ans:** A statement which is already false is called an absurdity or contradiction.

**101) What is Contingency?**

**Ans:** A statement which can be true or false depending upon the truth values of the variables is called a contingency.

**88) Find  $A \oplus B$ .**

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\text{Ans: } \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

**103) Show the equivalence  $(A \vee B) \rightarrow A = B \rightarrow A$**

**Ans:**

**104) Prove that  $p \rightarrow q(q \rightarrow r)$  and  $(p \vee \sim r) \rightarrow \sim q$  are logically equivalent?**

**Ans:**

**105) Let  $\{t_n\}$  be a sequence where  $t_n = 7 + 3n$ , what is common ratio/difference and what are terms of sequence?**

**Ans:** The sequences  $\{t_n\}$  with  $t_n = 7 + 3n$  are both arithmetic progressions with initial terms and common differences equal to 7, 3, respectively if we start at  $n=0$ . The list of terms to,  $t_1, t_2, t_3, \dots$  begins with  $t_0 = 7 + 3(0) = 7$ ,  $7 + 3(1) = 10$  and so on series will be 7, 10, 13, 16, 19.... And the common ratio is the 3 between the terms of the sequence.

**106) Write the converse of this statement  $p \rightarrow q$**

**Ans:**  $q \rightarrow p$  is the converse of this statement.

**107) if  $P(x): x > 3$  What are the truth values of  $P(4)$  and  $P(2)$ ?**

**Ans:** for  $p(4)$  it will be true because  $4 > 3$ . While for  $p(2)$  it will be false because  $2 > 3$  which is not correct.

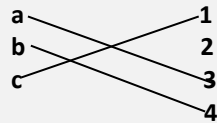
**108) Using truth table determine whether  $(\sim q \wedge (\rightarrow q) \rightarrow \sim p)$  is a tautology?**

**Ans:** After making its truth table if all the values of final statement are true then it will be tautology otherwise not.

**109) Translate into English and determine its truth value  $\exists x \in \mathbb{R} (x^2 = x)$ ?**

**Ans:**

110) Is this function is onto?



Ans: This function is one to one but not onto because A function  $f: A \rightarrow B$  is called an onto function if the range of  $f$  is  $B$ . In other words, if each  $b \in B$  there exists at least one  $a \in A$  such that.

$f(a) = b$ , then  $f$  is an on-to function. **An onto function is also called surjective function.**

Let  $A = \{a_1, a_2, a_3\}$  and  $B = \{b_1, b_2\}$  then  $f: A \rightarrow B$ .

111) If  $A = \{1, 3, 5\}$ ,  $B = \{1, 2, 3\}$  Are A and B disjoint?

Ans: A and B are not disjoint because for disjoint we have the elements common in both sets.

112) Let  $\{t_n\}$  be a sequence where  $t_n = 7 + 3n$ , what type of progression is this?

Ans: The sequences  $\{t_n\}$  with  $t_n = 7 + 3n$  are both arithmetic progressions with initial terms and common differences of 3.

113) Verify whether  $(p \wedge q) \rightarrow (p \vee q)$  is tautology or not?

Ans:

114) Suppose that the domain of the propositional function  $p(x)$  consist of the integers 1,2,3,4, and 5, Express these students without using quantifiers  $\sim \exists x p(x) \sim (p(1) \wedge p(2) \wedge p(3) \wedge p(4) \wedge p(5))$  Write in symbolic form "Some students have not id cards".

a) There exists a value between 1 and 5 such that  $P(x)$  is true, this is thus equivalent with  $P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5)$

b) For every value between 1 and 5 we need  $P(x)$  is true, this is thus equivalent with  $P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5)$

c) This statement is the negation of the statement in a, and is thus equivalent with  $\neg(P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5))$

d) This statement is the negation of the statement in b, and is thus equivalent with  $\neg(P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5))$

e) The statement is equivalent with  $(P(1) \wedge P(2) \wedge P(4) \wedge P(5)) \vee (\neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4) \vee \neg P(5))$ , because the first statement means that  $P(x)$  is true when  $x$  is not equal to 3 and the second statement means that there has to be a value for  $x$  where  $P(x)$  is false and thus  $\neg P(x)$  is true.

115) Find the conjunction of the proposition  $p$  and  $q$  where  $p$  is the proposition "Today is Holiday" and  $q$  is the proposition "It is not raining today".

Ans: Today is a holiday and it is not raining today.  $P \wedge q$

116) What is the difference between an r-combination and an r-permutation of a set with  $n$  elements?

Ans:

### TERMS CHAPTER NO FIVE

**product rule for counting:** The number of ways to do a procedure that consists of two tasks is the product of the number of ways to do the first task and the number of ways to do the second task after the first task has been done.

**product rule for sets:** The number of elements in the Cartesian product of finite sets is the product of the number of elements in each set.

**sum rule for counting:** The number of ways to do a task in one of two ways is the sum of the number of ways to do these tasks if they cannot be done simultaneously.

**sum rule for sets:** The number of elements in the union of pairwise disjoint finite sets is the sum of the numbers of elements in these sets.

**subtraction rule for counting or inclusion-exclusion for**

**sets:** If a task can be done in either  $n_1$  ways or  $n_2$  ways, then the number of ways to do the task is  $n_1 + n_2$  minus the number of ways to do the task that are common to the two different ways.

**subtraction rule or inclusion–exclusion for sets:** The number of elements in the union of two sets is the sum of the number of elements in these sets minus the number of elements in their intersection.

**division rule for counting:** There are  $n/d$  ways to do a task if it can be done using a procedure that can be carried out in  $n$  ways, and for every way  $w$ , exactly  $d$  of the  $n$  ways correspond to way  $w$ .

**division rule for sets:** Suppose that a finite set  $A$  is the union of  $n$  disjoint subsets each with  $d$  elements. Then  $n = |A|/d$ .

**the pigeonhole principle:** When more than  $k$  objects are placed in  $k$  boxes, there must be a box containing more than one object.

**the binomial theorem:**  $(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$

$k=0 \dots n$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

There are  $n^r$   $r$ -permutations of a set with  $n$  elements when repetition is allowed.

There are  $C(n + r - 1, r)$   $r$ -combinations of a set with  $n$  elements when repetition is allowed.

There are  $n!/(n_1! n_2! \dots n_k!)$  permutations of  $n$  objects of  $k$  types where there are  $n_i$  indistinguishable objects of type  $i$  for  $i = 1, 2, 3, \dots, k$ .

the algorithm for generating the permutations of the set  $\{1, 2, \dots, n\}$

### TERMS CHAPTER NO SEVEN

**relation on A:** a binary relation from  $A$  to itself (i.e., a subset of  $A \times A$ )

**reflexive:** a relation  $R$  on  $A$  is reflexive if  $(a, a) \in R$  for all  $a \in A$

**symmetric:** a relation  $R$  on  $A$  is symmetric if  $(b, a) \in R$  whenever  $(a, b) \in R$

**antisymmetric:** a relation  $R$  on  $A$  is antisymmetric if  $a = b$  whenever  $(a, b) \in R$  and  $(b, a) \in R$

**transitive:** a relation  $R$  on  $A$  is transitive if  $(a, b) \in R$  and  $(b, c) \in R$  implies that  $(a, c) \in R$

**directed graph or digraph:** a set of elements called vertices and ordered pairs of these elements, called edges

**closure of a relation  $R$  with respect to a property  $P$ :** the relation  $S$  (if it exists) that contains  $R$ , has property  $P$ , and is contained within any relation that contains  $R$  and has property  $P$

**path in a digraph:** a sequence of edges  $(a, x_1), (x_1, x_2), \dots, (x_{n-2}, x_{n-1}), (x_{n-1}, b)$  such that the terminal vertex of each edge is the initial vertex of the succeeding edge in the sequence

**circuit (or cycle) in a digraph:** a path that begins and ends at the same vertex  $R$

### GRAPH TERMS CHAPTER NO EIGHT

**undirected edge:** an edge associated to a set  $\{u, v\}$ , where  $u$  and  $v$  are vertices

**directed edge:** an edge associated to an ordered pair  $(u, v)$ , where  $u$  and  $v$  are vertices

**multiple edges:** distinct edges connecting the same vertices

**multiple directed edges:** distinct directed edges associated with the same ordered pair  $(u, v)$ , where  $u$  and  $v$  are vertices

**loop:** an edge connecting a vertex with itself

**undirected graph:** a set of vertices and a set of undirected edges each of which is associated with a set of one or two of these vertices

**simple graph:** an undirected graph with no multiple edges or loops

**multigraph:** an undirected graph that may contain multiple edges but no loops

**pseudograph:** an undirected graph that may contain multiple edges and loops

**directed graph:** a set of vertices together with a set of directed edges each of which is associated with an ordered pair of vertices

**directed multigraph:** a graph with directed edges that may contain multiple directed edges

**simple directed graph:** a directed graph without loops or multiple directed edges

**adjacent:** two vertices are adjacent if there is an edge between them

**adjacency matrix:** a matrix representing a graph using the adjacency of vertices

**incidence matrix:** a matrix representing a graph using the incidence of edges and vertices

**isomorphic simple graphs:** the simple graphs  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  are isomorphic if there exists a one-to-one correspondence  $f$  from  $V_1$  to  $V_2$  such that

$\{f(v_1), f(v_2)\} \in E_2$  if and only if  $\{v_1, v_2\} \in E_1$  for all  $v_1$  and  $v_2$  in  $V_1$

**circuit:** a path of length  $n \geq 1$  that begins and ends at the same vertex

**connected graph:** an undirected graph with the property that there is a path between every pair of vertices

**connected component of a graph  $G$ :** a maximal connected sub graph of  $G$

**strongly connected directed graph:** a directed graph with the property that there is a directed path from every vertex to every vertex

**Euler path:** a path that contains every edge of a graph exactly once

**Euler circuit:** a circuit that contains every edge of a graph exactly once

**Hamilton path:** a path in a graph that passes through each vertex exactly once

**Hamilton circuit:** a circuit in a graph that passes through each vertex exactly once

**weighted graph:** a graph with numbers assigned to its edges

**planar graph:** a graph that can be drawn in the plane with no crossings

**homeomorphic:** two undirected graphs are homeomorphic if they can be obtained from the same graph by a sequence of elementary subdivisions

**graph coloring:** an assignment of colors to the vertices of a graph so that no two adjacent vertices have the same color

### TREE TERMS CHAPTER NO NINE

**tree:** a connected undirected graph with no simple circuits

**forest:** an undirected graph with no simple circuits

**rooted tree:** a directed graph with a specified vertex, called the root, such that there is a unique path to every other vertex from this root

**sub tree:** a sub graph of a tree that is also a tree

**leaf:** a vertex with no children

**height of a tree:** the largest level of the vertices of a tree

**binary tree:** an  $m$ -ary tree with  $m = 2$  (each child may be designated as a left or a right child of its parent)

**balanced tree:** a tree in which every leaf is at level  $h$  or  $h - 1$ , where  $h$  is the height of the tree

**binary search tree:** a binary tree in which the vertices are labeled with items so that a label of a vertex is greater than the labels of all vertices in the left sub tree of this vertex and is less than the labels of all vertices in the right sub tree of this vertex

**preorder traversal:** a listing of the vertices of an ordered rooted tree defined recursively—the root is listed, followed by the first sub tree, followed by the other sub trees in the order they occur from left to right

**in order traversal:** a listing of the vertices of an ordered rooted tree defined recursively—the first sub tree is listed, followed by the root, followed by the other sub trees in the order they occur from left to right

**post order traversal:** a listing of the vertices of an ordered rooted tree defined recursively—the sub trees are listed in the order they occur from left to right, followed by the root

**infix notation:** the form of an expression (including a full set of parentheses) obtained from an in order traversal of the binary tree representing this expression

**prefix (or Polish) notation:** the form of an expression obtained from a preorder traversal of the tree representing this expression

**postfix (or reverse Polish) notation:** the form of an expression obtained from a post order traversal of the tree representing this expression

**minimum spanning tree:** a spanning tree with smallest possible sum of weights of its edges

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