1.2

Q-5

$$x_1 + x_2 + 2x_3 = 8$$
$$-x_1 - 2x_2 + 3x_3 = 1$$
$$3x_1 - 7x_2 + 4x_3 = 10$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix}$$

$R_2 \rightarrow R_2 + R_1$

$R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix}$$ $R_1 \rightarrow R_1 + R_2$

$R_3 \rightarrow R_3 - 10R_2$

$$\begin{bmatrix} 1 & 0 & 7 & 17 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -52 & -14-90 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 7 & 17 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -52 & -104 \end{bmatrix}$$ $R_3 \rightarrow R_3/52$

$$= \begin{bmatrix} 1 & 0 & 7 & 17 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

$-x_3 = -2$

$-x_2 + 5x_3 = 9$

$x_1 + 0 + 7x_3 = 17$

$\boxed{x_3 = 2}$

$-x_2 + 5(2) = 9$

$-x_2 + 10 = 9$

$-x_2 = 9 - 10$

$-x_2 = -1$

$\boxed{x_2 = 1}$

$x_1 + 7x_3 = 17$

$x_1 + 7(2) = 17$

$x_1 + 14 = 17$

$\boxed{x_1 = 3}$

Jordan

$$\begin{bmatrix} 1 & 0 & 7 & 17 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

$R_2 \to R_2 + 5R_3$

$R_1 \to R_1 + 7R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 17-14 \\ 0 & -1 & 0 & 9-10 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

$R_2 \to \times$ "-", R_3

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$= \boxed{x_1 = 3}$

$\boxed{x_2 = 1}$ $x_3 = 2$

1.4

Q.10

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Let

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$|A| = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$$

$$= (\cos\theta)(\cos\theta) - (\sin\theta)(-\sin\theta)$$

$$= \cos^2\theta + \sin^2\theta$$

we know that $\cos^2\theta + \sin^2\theta = 1$

$$|A| = 1$$

$$\text{adj } A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$A^{-1} = \frac{1}{1} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$ Ans.

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Q.31

$(A+B)(A-B) \neq A^2 - B^2$

Let suppose that

$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

prove = ?

L.H.S $\neq$ R.H.S

L.H.S $\Rightarrow (A+B)(A-B)$

$A+B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0+1 & 0+1 \\ 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$A-B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0-1 & 0-1 \\ 0-0 & 1-0 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}$

$(A+B)(A-B) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1+0 & -1+1 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ = L.H.S

$A^2 =$

$R.H.S = A^2 - B^2$

$A^2 = A \cdot A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = A^2$

$B^2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 1+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$A^2 - B^2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0-1 & 0-1 \\ 0-0 & 1-0 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}$

Hence prove that

$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \neq \begin{bmatrix} -1 & -\cancel{0} \\ 0 & 1 \end{bmatrix}$

b)

Using theorem

$(A+B)(A-B) = A(A-B)+B(A-B)$

$= AA - AB + BA - BB$

$= A^2 - AB + BA - B^2$

c)

It is only possible when

the $AB = BA$

$(A+B)(A-B) = A^2 - B^2$

39

$(AB)^{-1}(AC^{-1})(D^{-1}C^{-1})^{-1}D^{-1}$

By using theorem

$(AB)^{-1} = B^{-1}A^{-1}$

$(B^{-1}A^{-1})(AC^{-1})(D^{-1}C^{-1})^{-1}D^{-1}$

$(D^{-1}C^{-1})^{-1} = (C^{-1})^{-1}(D^{-1})^{-1}$

$(B^{-1}A^{-1})(AC^{-1})(C^{-1})^{-1}(D^{-1})^{-1}D^{-1}$

we know that (using theorem)

$(A^{-1})^{-1} = A$

$= (C^{-1})^{-1} = C \quad (D^{-1})^{-1} = D$

$= (B^{-1}A^{-1})(AC^{-1})(CD)D^{-1}$

we know that

OR using the

theorem (properties)

$A(BC) = (AB)C$

$= B^{-1}(A^{-1}A)(C^{-1}C)DD^{-1}$

$A^{-1}A = I$

$= B^{-1}(I)(I)(I)$

$= B^{-1}III \Rightarrow B^{-1}$

Ex 1.5

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

By using inversion algorithm

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \longrightarrow R_2 - 2R_1$

$R_3 \longrightarrow R_3 - R_1$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 5-4 & 3-6 & -2 & 1 & 0 \\ 1-1 & 0-2 & 8-3 & -1 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right]$$

$R_1 \longrightarrow R_1 - 2R_2$

$R_3 \longrightarrow R_3 + 2R_2$

$$= \left[\begin{array}{ccc|ccc} 1-0 & 2-2 & 3+6 & 1+4 & 0-2 & 0-0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0-0 & -2+2 & 5-6 & -1-4 & 0+2 & 1+0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right]$$

$R_3 \to \times(-) R_3$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & +5 & -2 & -1 \end{array}\right]$$

$R_2 \longrightarrow R_2 + 3R_1$

$R_1 \longrightarrow R_1 - 9R_3$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5-45 & -2+18 & 0+9 \\ 0 & 1 & 0 & -2+15 & 1-6 & -3 \\ 0 & 0 & 1 & +5 & -2 & -1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

The inverse of

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix} \text{Ans.}$$

Ex - 1.6

Q-2

$4x_1 - 3x_2 = -3$

$2x_1 - 5x_2 = 9$.

$$\underbrace{\begin{bmatrix} 4 & -3 \\ 2 & -5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} -3 \\ 9 \end{bmatrix}}_{B}$$

$AX = B$

$X = A^{-1}B$.

$$A^{-1} = \frac{1}{|A|} \text{Adj } A$$

$$|A| = \begin{vmatrix} 4 & -3 \\ 2 & -5 \end{vmatrix} = (-5)(4) - (-3)(2)$$

$$= -20 + 6$$

$$= -14$$

$$\text{Adj } A = \begin{bmatrix} 4 & -3 \\ 2 & -5 \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ -2 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -5 & 3 \\ -2 & 4 \end{bmatrix} \times \frac{-1}{14}$$

$$A^{-1} = \begin{bmatrix} -5/-14 & 3/-14 \\ -2/-14 & 4/-14 \end{bmatrix} = \begin{bmatrix} 5/14 & -3/14 \\ 1/7 & -\frac{2}{7} \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} 5/14 & -3/14 \\ 1/7 & -2/7 \end{bmatrix} \begin{bmatrix} -3 \\ 9 \end{bmatrix}$$

$$= \begin{bmatrix} 5/14 \times -3 + \frac{-3}{14} \times 9 \\ -3/7 - \frac{18}{7} \end{bmatrix}$$

$$= \begin{bmatrix} -15/14 + -27/14 \\ \frac{-3}{7} - 18/7 \end{bmatrix}$$

$$\begin{array}{r} 27 \\ 15 \\ \hline 42 \end{array}$$

$$= \begin{bmatrix} \frac{-15-27}{14} \\ \frac{-3-18}{7} \end{bmatrix} = \begin{bmatrix} -\cancel{42}^{3}/\cancel{14} \\ -\cancel{21}^{3}/\cancel{7} \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$$

$$X = A^{-1}B \Longrightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$$

$\boxed{x_1 = -3}$ $\boxed{x_2 = -3}$

Q.4 $5x_1 + 3x_2 + 2x_3 = 4$

$3x_1 + 3x_2 + 2x_3 = 2$

$x_2 + x_3 = 5$

$$\underbrace{\begin{bmatrix} 5 & 3 & 2 \\ 3 & 3 & 2 \\ 0 & 1 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}}_{B}$$

$AX = B$

$X = A^{-1}B$

$A^{-1} = \frac{1}{|A|} \operatorname{Adj} A$

By using Inversion algorithm

$$\left[\begin{array}{ccc|ccc} 5 & 3 & 2 & 1 & 0 & 0 \\ 3 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

same $\Rightarrow$ privous question

$A^{-1} = \frac{1}{|A|} \operatorname{Adj} A$

$\downarrow$

$$|A| = \begin{vmatrix} 5 & 3 & 2 \\ 3 & 3 & 2 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 5\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} - 3\begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} + 2\begin{vmatrix} 3 & 3 \\ 0 & 1 \end{vmatrix}$$

$$= 5(3-2) - 3(3-0) + 2(3-0)$$

$$= 5(1) - 9 + 6$$

$$= 5 - 9 + 6 = 11 - 9 = 2$$

Adj A $\Longrightarrow$ By using coffactor

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = +(3-2) = 1$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} = -(3-0) = -3$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 3 & 3 \\ 0 & 1 \end{vmatrix} = +(3-0) = 3$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = -(3-2) = -1$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 5 & 2 \\ 0 & 1 \end{vmatrix} = +(5-0) = 5$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 5 & 3 \\ 0 & 1 \end{vmatrix} = -(5-0) = -5$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix} = +(6-6) = 0$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 5 & 2 \\ 3 & 2 \end{vmatrix} = -(10-6) = -4$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 5 & 3 \\ 3 & 3 \end{vmatrix} = +(15-9) = 6$$

$$= \begin{bmatrix} 1 & -3 & 3 \\ -1 & 5 & -5 \\ 0 & -4 & 6 \end{bmatrix}^t$$

$$\text{Adj } A = \begin{bmatrix} 1 & -1 & 0 \\ -3 & 5 & -4 \\ 3 & -5 & 6 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 5 & -4 \\ 3 & -5 & 6 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 0 \\ 3/2 & 5/2 & -2 \\ 3/2 & -5/2 & 3 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} 1/2 & -1/2 & 0 \\ -3/2 & 5/2 & -2 \\ 3/2 & -5/2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} 2-1+0 \\ -6+5-10 \\ 6 \quad -5 \quad 15 \end{bmatrix} = \begin{bmatrix} 1 \\ -11 \\ 16 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -11 \\ 16 \end{bmatrix}$$

$\boxed{x_1 = 1}$ $\boxed{x_2 = -11}$ $\boxed{x_3 = 16}$