

Time Allowed: 2:30 Hours

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

(2*16)

- Q.1.** Write short answers of the following in 2-3 lines each.
- Define zero vector space.
 - Compute $u+v$ and ku for $u = (-1, 2)$, $v = (3, 4)$, and $k=3$.
 - Define Linearly independent.
 - Show that a finite set that contains 0 is linearly independent.
 - Explain why the following form linearly dependent set of vectors.
 $u_1 = (-1, 2, 4)$ and $u_2 = (5, -10, -20)$
 - Write standard basis for \mathbb{R}^n .
 - Define dimension of a vector space.
 - Define row space.
 - What is the relation between rank, nullity and dimension of a vector space.
 - Define characteristic equation.
 - Find eigen values of the matrix $A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$
 - Define unit vector.
 - Define orthogonal vectors.
 - If u and v are orthogonal vectors in a real inner product space, then
 $\|u+v\|^2 = \|u\|^2 + \|v\|^2$
 - Find the cosine of the angle between the vectors w.r.t Euclidean inner product
 $u = (1, -3)$, $v = (2, 4)$
 - Determine whether the vectors are orthogonal w.r.t the Euclidean inner product
 $u = (-1, 3, 2)$, $v = (4, 2, -1)$

Subjective Part (3*16)

- Q.2.** (a) State and prove Cauchy Schwarz Inequality.
(b) State and prove Triangle Inequality.
- Q.3.** (a) Show that the vectors
 $v_1 = (1, 2, 1)$, $v_2 = (2, 9, 0)$, $v_3 = (3, 3, 4)$
form a basis for \mathbb{R}^3
(b) Write the procedure for diagonalizing an $n \times n$ matrix.
- Q.4.** Find the inverse of

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 8 & 8 \end{bmatrix}$$

- Q.5.** (a) Evaluate $\det(A)$ where $A = \begin{bmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{bmatrix}$

- Q.6.** (b) Evaluate the expression $\langle 2v-w, 3u+2w \rangle$
(a) If u , v , and w are vectors in a real inner product space V , and if k is any scalar, then
 $\|u+v\| \leq \|u\| + \|v\|$

(b) Use Cramer's rule to solve

$$\begin{aligned} x_1 + 2x_3 &= 6 \\ -3x_1 + 4x_2 + 6x_3 &= 30 \\ -x_1 - 2x_2 + 3x_3 &= 8 \end{aligned}$$