

(11) Consider $u = (1, 1, 1)$, $v = (1, 2, -3)$ and $w = (1, -4, 3)$
which vectors are orthogonal

orthogonal vectors $= \vec{u} \cdot \vec{v} = 0$

$$u \cdot v = (1, 1, 1) \cdot (1, 2, -3)$$

$$= 1 + 2 - 3 = 0$$

\therefore ~~u & v are~~ orthogonal

$$u \cdot w = (1, 1, 1) \cdot (1, -4, 3)$$

$$= 1 - 4 + 3$$

$$= 0$$

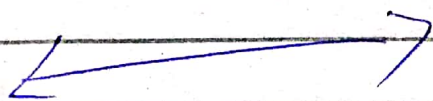
u & w are orthogonal

$$v \cdot w = (1, 2, -3) \cdot (1, -4, 3)$$

$$= 1 - 8 - 9$$

$$\neq 0$$

v & w are not orthogonal



(ix) Consider the vector

$$v = (1, -5, 3)$$

find $\|v\|_1 = ?$, $\|v\|_2$, $\|v\|_\infty$

$$\begin{aligned}\|v\|_1 \text{ (first norm)} &= \sum_{i=1}^n |x_i| \\ &= 1 + 5 + 3 \\ &= 9\end{aligned}$$

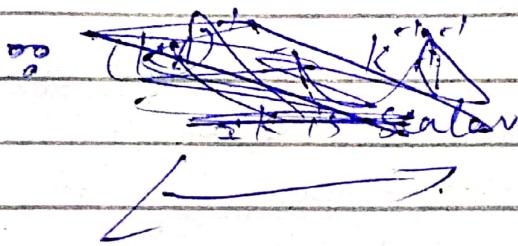
$$\begin{aligned}\|v\|_2 \text{ (2nd norm)} &= \left[\sum_{i=1}^n (x_i)^2 \right]^{\frac{1}{2}} \\ &= \sqrt{1^2 + (-5)^2 + 3^2} \\ &= \sqrt{1^2 + 25 + 9} \\ &= \sqrt{35}\end{aligned}$$

$$\begin{aligned}\|v\|_\infty \text{ (infinity norm)} &= \max [1, -5, 3] \\ &= 5\end{aligned}$$



$$(VII) \quad (A^n)^{-1} = (A^{-1})^n$$

$$\begin{aligned} (A^n)^{-1} &= \cancel{A \cdot A \cdot A \cdot A \cdot A} \cdot (A \cdot A \cdot A \cdot \dots \cdot A_n)^{-1} \\ &= A^{-1} \cdot A^{-1} \cdot A^{-1} \cdot \dots \cdot A_n^{-1} \\ &= (A^{-1})^n \end{aligned}$$



$$(VIII) \quad (A^T)^{-1} = (A^{-1})^T$$

$$\Rightarrow \bar{A} \cdot (A^{-1})^T = (A^{-1} A)^T = (I)^T = I$$

$$\Rightarrow (A^{-1})^T A^T = (A A^{-1})^T = I^T = I$$

$$\text{So } A^T (A^{-1})^T = (A^{-1})^T A^T = I$$

which completes the proof