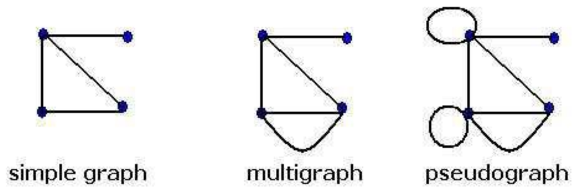
1. Any graph which contains some multiple edges is called a **multigraph**. In a multigraph,no loops are allowed.

   A graph in which loops and multiple edges are allowed is called **pseudograph**.



2.

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

Using the same analysis as in previous exercises, we have two propositional variables and five compound propositions. This means the truth table will have 4 rows and 7 columns.

p	**q**	**p ∧ q**	**¬(p ∧ q)**	**¬p**	**¬q**	**¬p ∨ ¬q**
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Notice that I used the truth table for the conjunction, disjunction, and negation of propositions.

3.

$521 --314++*$

Start from the left most operation symbol. For every operation symbol, the two integers preceding the operation symbol are the symbols on which the operation needs to be performed. Then replace the operation symbol with the two following integers by the result of the operation Then we repeat for the left most operation symbol in the remaining string.

$$5\,2\underbrace{\frac{1}{2-1=1}}_{s}-314++*$$

$$\underbrace{51-}_{5-1=4}3-314++*$$

$$43\underbrace{\ 1\ }_{1+4=5}+*$$

$$4\,\underbrace{35+}_{3+5=8}*$$

$$\underbrace{4\ \ 8\ \ *}_{4*8=32}$$

4.

d) p∨T≡T

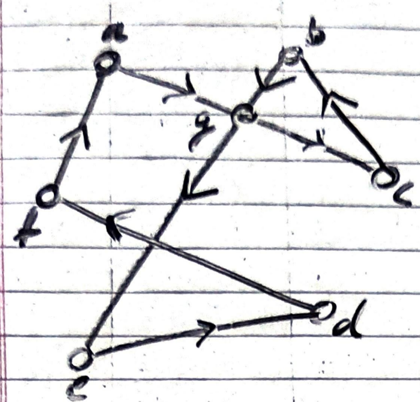
p	p∨T
T	T
F	T

From the truth table above, we can state that p is not logically equivalent to p∨T, because they don't have the same truth values (see the second row).

5. No " the relation R= {(1.1). (1,2), (2,1), (3,2)) on the set A=(1,2,3) is not a reflexive relation because

Reflexive: If a relation has {(a, b)} as its element, then it should also have {(a, a), (b, b)} as its elements too.

6.



Eular Path: (Cover Edge)
a, g, c, b, g, e, d, f, a
Also have Eular circuit.
Hamiltonian Path: (Cover vertex)
e, d, f, a, g, c, b
Hamiltonian circuit does not exist.

7.

Cardinality of sets

$\{a, \{a\}, \{a, \{a\}\}\} = 3$

$\{\{a\}\} = 1$

8.

1. A function $f : Z \to Z$, defined by $f(x) = x + 1$, is one-one as well as onto.

$f(x) = x + 1$,

calculate $f(x1)$:

$f(x1) = x1 + 1$

calculate $f(x2)$:

$f(x2) = x2 + 1$

Now, $f(x1) = f(x2)$

$\Rightarrow x1 + 1 = x2 + 1$

$\Rightarrow x1 = x2$

So, f is one-one function.

Consider $f(x) = y$

$y = x + 1$

$x = y - 1$

$f(y-1) = y - 1 + 1 = y$

f is onto.

9.

Complexity

- Worst case time complexity: `Θ(n^2)`
- Average case time complexity: `Θ(n^2)`
- Best case time complexity: `Θ(n)`
- Space complexity: `Θ(1)`

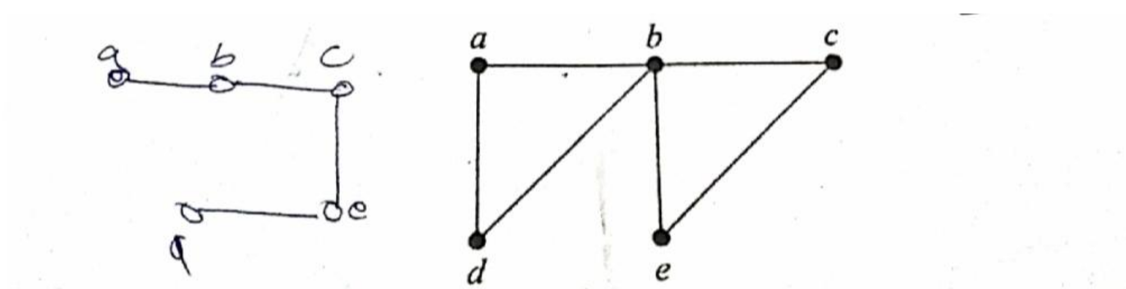
10.

Definition

A recurrence relation is an equation that recursively defines a sequence where the next term is a function of the previous terms (Expressing F_n as some combination of F_i with $i < n$).

Example – Fibonacci series – $F_n = F_{n-1} + F_{n-2}$, Tower of Hanoi – $F_n = 2F_{n-1} + 1$

11.



12.

... replaced with the letter represented by

Example 9 *What is the secret message produced from the message "MEET YOU IN THE PARK" using the Caesar cipher?*

Solution First replace the letters in the message with numbers. This produces

12 4 4 19   24 14 20   8 13   19 7 4   15 0 17 10.

Now replace each of these numbers p by $f(p) = (p + 3)$ **mod** 26. This gives

15 7 7 22   1 17 23   11 16   22 10 7   18 3 20 13.

Translating this back to letters produces the encrypted message "PHHW BRX LQ WKH SDUN." ◀

To recover the original message from a secret message encrypted by the Caesar cipher, the function f^{-1}, the inverse of f, is used. Note that the function f^{-1} sends an integer p from $\{0, 1, 2, \ldots, 25\}$ to $f^{-1}(p) = (p - 3)$ **mod** 26. In other words, to find the original message, each letter is shifted back three letters in the alphabet, with the first three letters sent to the last three letters of the alphabet. The process of determining the original message from the encrypted message is called **decryption.**

There are various ways to generalize the Caesar cipher. For example, instead of shifting each letter by 3, we can shift each letter by k, so that

$$f(p) = (p + k) \textbf{ mod } 26.$$

Such a cipher is called a **shift cipher.** Note that decryption can be carried out using

$$f^{-1}(p) = (p - k) \textbf{ mod } 26.$$

Obviously, Caesar's method and shift ciphers do not provide a high level of security. There are various ways to enhance this method. One approach that slightly enhances the security is to use a function of the form

$$f(p) = (ap + b) \textbf{ mod } 26,$$

where a and b are integers, chosen such that f is a bijection. (Such a mapping is called an *affine transformation.*) This provides a number of possible encryption systems. The use of one of these systems is illustrated in Example 10.