Statistics (ch#6)

1) Probability :- imp

The chance of occurence and not occurence of an element is called Probability.

It can be define as

a) The numerical measure of uncertaintly is called Probability

b) The degree of ~~believe~~ a Person particular statment is called Probability.

2) Experiment :- imp

any process which gives some result is called experiment

3) Random experiment :- imp

Any Process which produces different result given it repeated ~~large~~ number of time under identical condition is called random experiment.

condition and if an event

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Properties Random exp. imp

It has following property.

a) outcomes of each trial should be atleast two

b) The experiment should be repeated at once.

4) Sample space:- imp

A set consists of all possible outcomes of random experiment is called sample space. It is denoted by 'S'.

a) In toss of ~~two~~ coins.

$S = \{H, T\}$

$n(S) = 2$

b) In toss of three coins.

S	HH	HT	TH	TT
H	HHH	HHT	HTH	HTT
T	THH	THT	TTH	TTT

$n(S) = 8$

c) In toss of 4 coins

imp

S	HH	HT	TH	TT
HH	HHHH	HHHT	HHTH	HHTT
HT	HTHH	HTHT	HTTH	HTTT
TH	THHH	THHT	THTH	THTT
TT	TTHH	TTHT	TTHT	TTTT

$n(S) = 16$

5) Size of sample space :-

Number of outcomes in a sample space is called size of sample space. It is denoted by $n(S)$.

a) In throw of dice.

$S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

b) In throw of two dice.

S	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$n(S) = 36$

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6) Playing card:-

Total playing card = 52

52

Red(26) ← → Black(26)

Heart = 13	Diamond = 13	club = 13	Spade = 13
Ace	Ace	Ace	Ace
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9
10	10	10	10
Jack	Jack	Jack	Jack
Queen	Queen	Queen	Queen
King	King	King	King

Total card = 52

Total ace = 4

Total Jack = 4

Total Queen = 4

Total King = 4

Seats = 4

Total colour card = 12

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7) Event :- imp

A subset of sample space is known as event. Event are commonly denoted A, B, C, ... etc

spade=13
Ace
2
3
4
5
6
7
8
9
10
Jack
Queen
King

Example :-

a) Event of odd number in throw of dice

$S = \{1, 2, 3, 4, 5, 6\}$

Let 'A' denote odd number

$A = \{1, 3, 5\}$

b) Make an event same outcomes in toss of two coins.

S	H	T
H	HH	HT
T	TH	TT

Let 'B' ~ same element

$B = \{HH, TT\}$

{∴ ~ = denote}

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c) Make an event of same outcomes in throw of two dice.

S	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Let 'C' ~ Same element

$C = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

8) Size of an event :-

Number of outcomes in an event is known as size of an event. It is denoted as n.

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9) Diff b/w Simple and composite events? imp

Simple event	composite event
An event consist of only one outcomes is Known as simple event. it is also Known as singleton event	An event consists of more than one outcomes is Known as composite event. It is also Known as compound event.
Example $A = \{H\}$ $n(A) = 1$	$A = \{2, 4, 6\}$ $n(A) = 3$

10) **Mutually exclusive event/ DisJoint event :-**

The event having no common element is called mutually exclusive event.

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$

$B = \{2, 4, 6\}$

$A \cap B = \{\ \} = \phi$

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11) Not mutually exclusive/ Joint event:-

The event having common outcomes is known as not mutually exclusive event. It is also known as Joint event.

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2, 5, 6\}$

$B = \{2, 4, 5, 6\}$

$A \cap B = \{2, 5, 6\}$

12) Equally Likely events:-

The events having same chance of occurence is known as equally likely event.

Example:-

1) All side of dice.
2) Both side of coins.

13) Not equally Likely event:-

The event having not same chance of occurence is known as not

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Equally Likely event.
Example:-
1) All side of duster
2) side of Matches.

14) Mathematical/classical defination of Probability:-
If a random experiment consists of n(S) mutually exclusive and equally Likely outcomes from which n(A) outcomes are favourable to event A then

$$P(A) = \frac{n(A)}{n(S)}$$

$$0 \le P(A) \le 1$$

15) Find probability of an event at least two occure in throw of dice?
$S = \{1, 2, 3, 4, 5, 6\}$
$n(S) = 6$
Let 'A' → at least two occure.
$A = \{2, 3, 4, 5, 6\}$

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$n(A) = 5$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{5}{6} \quad \text{Ans.}$$

15) Permutation:- imp

The arrangment of object with some orders is called permutation
It is denoted by

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

n = total object
r = selected object

16) Combination:- imp

The selection of object without any order is called combination.
It is denoted by

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

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Definition of probability:

a) Classical or A Priori defination of probability:-

If a random experiment can produce n mutually exclusive and equally likely outcomes and if m ~~out of these~~ outcomes are ~~considered~~ favourable to the occurence of a certain event A, then the probability of the event A, denoted by $P(A)$.

$$P(A) = \frac{m}{n}$$

b) Relative frequency or A Posteriori defination of probability:-

If a random experiment is repeated a large number of times, say n, under identical condition and if an event

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A is observed to occur m times, then the probability of the event A is defined as the limit of the relative frequency $\frac{m}{n}$ as n tends to infinity.

Symbolically.

$$P(A) = \lim_{n \to \infty} \frac{m}{n}$$

c) Axiomatic defination of Probability :-

Let S be a Sample space with the Sample point $E_1, E_2, \ldots E_i, \ldots E_n$ to each sample point, we assign a real number, denoted by the symbol $P(E_i)$ and called the probability of E_i that must satisfy following basic axioms.

i) For any event E_i, $0 \le P(E_i) \le 1$

ii) $P(S) = 1$ for sure event S

iii) If A and B are mutually

exclusive event, then
$P(A \cup B) = P(A) + P(B)$

Conditional probability:-

The probabilities associated with such a reduced sample are called conditional probability.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Law of probility :-

Theoram 6.1

If ϕ is the impossible event, then
$P(\phi) = 0$

Proof:-

The sure event S and the impossible event

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ϕ are mutually exclusive and their union is

$S \cup \phi = S$

$P(S) = P(S \cup \phi) = P(S) + P(\phi)$

Substruct $P(S)$ both side

$P(S) - P(S) = P(S) + P(\phi) - P(S)$

$P(\phi) = 0$ proved

Theoram 6.2

Law of complement

$P(\bar{A}) = 1 - P(A)$

Proof :-

Since the event A and $\bar{A}$ are mutually exclusive

$A \cup \bar{A} = S$

$P(A \cup \bar{A}) = P(S)$ $\{\because P(S) = 1\}$

$P(A) + P(\bar{A}) = 1$

$P(\bar{A}) = 1 - P(A)$ proved.

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Theoram # 6.4:- If A and B two event then
$P(A\cap\bar{B}) = P(A) - P(A\cap B)$

Proof:-

The events $A\cap\bar{B}$ and $A\cap B$ are mutually exclusive and their union is A
$A = (A\cap\bar{B}) \cup (A\cap B)$
$P(A) = P(A\cap\bar{B}) + P(A\cap B)$
Hence
$P(A\cap\bar{B}) = P(A) - P(A\cap B)$ Proved

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theoram # 6.5 Addition law
$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

Proof:-

The event $A\cup B$ may be written as union of two mutually exclusive events A and $B\cap\bar{A}$

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$A \cup B = A \cup (B \cap \bar{A})$

Then

$P(A \cup B) = P(A) + P(B \cap \bar{A}) \longrightarrow ①$

Again event B also decomposed into two mutually exclusive event as

$B = (A \cap B) \cup (\bar{A} \cap B)$ then

$P(B) = P(A \cap B) + P(\bar{A} \cap B) \longrightarrow ②$

Substruct ① and ②

$P(A \cup B) - P(B) = P(A) - P(A \cap B)$

Hence

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ Prove

Prove $P(A \cup B) = P(A) + P(B)$

Proof :-

Since events A and B are mutually exclusive events, then

$A \cap B = \phi$, $P(A \cap B) = P(\phi) = 0$

Hence

$$P(A \cup B) = \frac{\text{no. Sample Point } A \cup B}{\text{no. Sample Point } S}$$

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$\cup \rightarrow +$

$\cap \rightarrow \times$

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$$P(A \cup B) = \frac{m_1 + m_2}{n}$$

$$= \frac{m_1}{n} + \frac{m_2}{n}$$

$$P(A \cup B) = P(A) + P(B) \quad \text{Proved}$$

Conditional Probability:-

The probabilities associated with such a reduced sample space are called conditional probability. It is denoted by $P(A/B)$ and given as

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \quad P(A) \neq 0$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}, \quad P(B) \neq 0$$

Theoram #6.7 Multiplication Law

$$P(A \cap B) = P(A)\, P(B/A), \quad P(A) \neq 0$$

$$= P(B)\, P(A/B), \quad P(B) \neq 0$$

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Proof :-

Let m_1 number of sample point contained in A and m_2 number of sample point contained in B and m_3 number of sample point belong to A and B. Then

$$P(A \cap B) = \frac{m_3}{n}$$

$$\therefore \frac{m_3}{n} = \frac{m_3}{m_1} \cdot \frac{m_1}{n}$$

$$\therefore P(A) = \frac{m_1}{n}$$

$$\therefore \frac{m_3}{m_1} = \text{conditional probability}$$

Hence

$P(A \cap B) = P(A)P(B/A)$ proved

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Dependent Event:-

Two events are said to be dependent if the occurrence of one event affects the probability of the other event.

Independent Event:-

Two events are said to be independent if the occurrence of one event does not affect the probability of other event.

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