

# Digital Logical Design

## Decimal number :-

Decimal number system is a number system that we use in our daily life. It includes the numbers from 0 to 9 and the base of decimal number is 10.

Example :

$$(128)_{10} \text{ or } (256.68)_{10}$$

Decimal value to sum of each value :-

$$(128)_{10}$$

$$= 1 \times 10^2 + 2 \times 10^1 + 8 \times 10^0 \\ = 100 \quad 20 \quad 8$$

$$(256.68)_{10}$$

$$= 2 \times 10^2 + 5 \times 10^1 + 6 \times 10^0 + 6 \times 10^{-1} + 8 \times 10^{-2} \\ = 200 + 50 + 6 + 0.6 + 0.08$$

## Binary Number :-

Binary number system is a number system to represents values. It uses only two numbers <sup>bits</sup> 0 and 1 to represent values. Binary number has base 2.

Example : The number 4 represent in  $\frac{2^3}{1-0}$  4 bit binary code

$$(4)_{10} = (0100)_2 \quad (0100)_2$$

## Decimal to binary Conversion:-

$$82 \quad (82)_{10} = (?)_2$$

$$(82)_{10} = (1001010)_2$$

by LCM Method

2	82	↑
2	41 — 0	
2	20 — 1	
2	10 — 0	
2	5 — 0	
2	2 — 1	
2	1 — 0	

$$(25)_{10} = (?)_2$$

$$25 = 16 + 8 + 1$$

$$= 2^4 + 2^3 + 2^0$$

$$= (11001)_2$$

Sum of weights method

$$(0.625)_{10} = (?)_2$$

$$= 0.5 + 0.125$$

$$= 2^{-1} + 2^{-3}$$

$$= 0.101 \quad - \text{by sum of weight}$$

$$(45.5)_{10} = (?)_2$$

$$45 = (101101)_b$$

$$0.5 = (1.00)_2$$

$$45.5 = 101101.1$$

2	45
2	22-1
2	11-0
2	5-1
2	2-0
	1-0

$$0.5 \times 2 = 1.00\ 1$$

$$(0.3125)_{10} = (?)_2$$

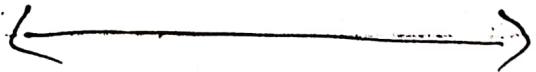
$$(0.0101)_2$$

$$0.3125 \times 2 = 0.625 \quad 0$$

$$0.625 \times 2 = 1.25 \quad 1$$

$$0.25 \times 2 = 0.50 \quad 0$$

$$0.5 \times 2 = 1.00 \quad 1$$



## Binary Arithmetic:-

### 1. Binary addition:-

Some basic rules to add binary digits.

$$0+0 = 0$$

$$0+1 = 1$$

$$1+0 = 1$$

$$1+1 = 10 \rightarrow \text{with carry 1}$$

Example :

$$\begin{array}{r} 111 + 11 \\ \hline 1010 \end{array}$$

$$\begin{array}{r} 110 + 100 \\ \hline 1010 \end{array}$$

Binary Subtraction :-

Same basic rules  
to subtract two binary digits

$$0 - 0 = 0$$

$$1 - 0 = 1$$

$$\text{borrow } 1 \leftarrow 10 - 1 = 1$$

$$1 - 1 = 0$$

Example :-

$$(1) 11 - 10$$

$$\begin{array}{r} 11 \\ - 10 \\ \hline 11 \end{array}$$

$$(2) 101 - 011$$

$$\begin{array}{r} 101 \\ - 011 \\ \hline 010 \end{array}$$

## Binary Multiplication :-

Some basic rules

to multiply binary digits.

$$0 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 0 = 0$$

$$1 \times 1 = 1$$

(i)  $11 \times 11$

$$\begin{array}{r} 11 \\ 11 \\ \hline 011 \\ | | x \\ \hline 1001 \end{array}$$

(ii)  $101 \times 111$

$$\begin{array}{r} 101 \\ 101 \\ \hline 011 \\ 000x \\ 111xx \\ \hline 100011 \end{array}$$

## Binary Division :-

(1)  $110 \div 10$

$$\begin{array}{r} 11 \\ 10 \sqrt{110} \\ \hline 10 \\ 0 \end{array}$$

(2)  $1100 \div 100$

$$\begin{array}{r} 11 \\ 100 \sqrt{1100} \\ \hline 100 \\ 0 \end{array}$$

(3)  $110 \div 11$

$$\begin{array}{r} 10 \\ 11 \sqrt{110} \\ \hline 11 \\ 0 \end{array}$$

# Complements

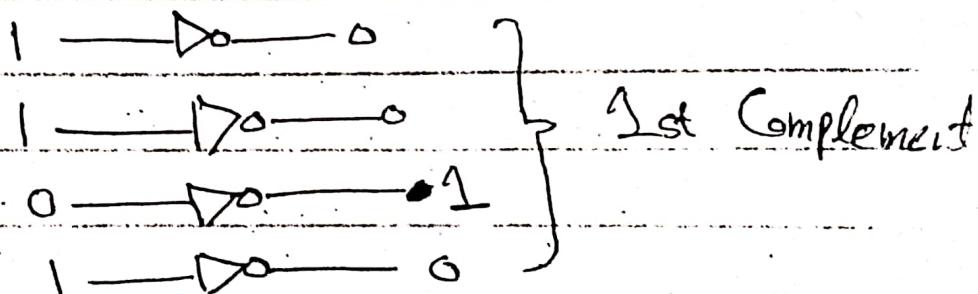
## (1) 1st Complement :-

The first complement of binary number is founded by converting all 1 to 0 and all 0 to 1.

The simplest way to get 1st complement of binary numbers with digital circuit is to use parallel invertors (NOT circuit).

Example

binary number = 1101



1st Complement = 0010

## (2) 2nd Complement :-

The 2nd complement is founded by just adding the 1 to Least Significant bit (LSB) of 1st complement.

Example :

binary number = 10110010

1st Complement = 01001101

$$\begin{array}{r}
 01001101 \\
 + \quad \quad 1 \\
 \hline
 01001110
 \end{array}$$

~~$$\begin{array}{r}
 01001101 \\
 + \quad \quad 1 \\
 \hline
 01001110
 \end{array}$$~~

2nd Complement = 01001110

## Signed Numbers:-

A numbers that consist both sign and magnitude information is called Sign numbers.

The sign indicate whether number is positive or negative and the magnitude indicate its value of its number.

Digital system such as Computer must be able to handle both positive and negative number. These case three forms in which sign integers numbers can be represent in binary are given below

- (1) Sign-Magnitude
- (2) 1st - Complement
- (3) 2nd Complement

Sign Bit:-

The left most bit in sign numbers is called sign bit.  
A 0 sign bit indicates a positive number and 1 sign bit indicates a negative number.

### 1) Sign magnitude forms:

When a binary number is represented in sign magnitude form the left most bit is sign bit and other bits are represented as magnitude bits.

In sign magnitude form a negative number has same magnitude bit as the positive corresponding positive number but the sign bit is 1 rather than 0.

Example :-

25 in 8 bit binary = 00011001,

$$\begin{array}{r} 2 | 25 \\ 2 | 12-1 \\ 2 | 6-0 \\ \hline & 3-0 \\ \text{magnitude bit } 2 & \end{array}$$

↓  
sign bit

-sign bit

-25 in 8 bit binary = 10011001

## 2) 1st Complement form,

Positive numbers

In 1st Complement form are represented same way as positive sign-magnitude numbers. And a negative number is the 1st complement of the corresponding positive numbers.

+25 in 8 bit 1st Complement form

0000 0011

And

-25 in 8 bit 1st Complement form

1111 1100

## 3) 2nd Complement forms

Positive numbers

In 2nd Complement form are represent ad same way in sign-magnitude of 1st Complement form and a negative number is the 2nd complement of the corresponding positive numbers.

+25 in 3bit 2nd Complement form

001

-25 in 3 bit 2nd Complement form

110111

110011

+

110011

# Decimal values of Signed Numbers

Sign magnitude :-

10010101 = ?

~~we~~  $2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$   
0 0 1 0 1 0 1  
magnitude bit

$$\pm 16 + 4 + 1$$

(Show -) sign bit = 21

Decimal value = -21

1st Complement :-

0001011

$2^7 \ 2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$

0 0 0 1 0 1 1

$$16 + 4 + 2 + 1 = +23 \text{ (Decimal)}$$

11101000

$2^7 \ 2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$

1 1 1 0 1 0 0 0

$$-128 + 64 + 32 + 8 = -24$$

Decimal = -24 + 1 (adding 1)

Decimal = -23

## 2nd Complement ,

01010110

$2^7 \ 2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$

0 1 0 1 0 1 1 0

$$64 + 16 + 4 + 2 = + 86$$

1010100110

$2^7 \ 2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$

1 0 1 0 1 0 1 0

$$-128 + 32 + 8 + 2 = -86$$

## Asthetic Operation with Sign Number

### Addition :

(1) Both Positive

00000111 7

+ 00000100 + 4

06001011 11

(2) Positive number larger than Negative

00000111 + 15

+ 11111010 - 6

9

Discard - 100001001

③ Negative numbers : Juggled positive

$$\begin{array}{r} 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \\ + \underline{1 \ 1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0} \\ \hline 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \end{array} \quad \begin{array}{r} 16 \\ + 24 \\ \hline -8 \end{array}$$

④ Both negative

$$\begin{array}{r} 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \\ - 1 \ 1 \ 1 \ 1 \ 0 \ 1 \ 1 \\ \hline 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \end{array} \quad \begin{array}{r} -5 \\ -9 \\ \hline -14 \end{array}$$

Discard — 1 1 1 1 1 0 0 0

Subtraction :

①  $8 - 3 = 8 + (-3) = 5$

$$\begin{array}{r} 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \\ + \underline{1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 1} \\ \hline 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \end{array} \quad \begin{array}{l} \rightarrow 8 \\ \rightarrow 2^{\text{nd}} \text{ Complement } (-3) \end{array}$$

Discard carry ←

②  $12 - (-9) = 12 + 9 = 21$

$$\begin{array}{r} 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \\ + \underline{0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1} \\ \hline 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \end{array} \quad \begin{array}{l} \rightarrow 12 \\ \rightarrow (2^{\text{nd}} \text{ Comp. } + 9) \end{array}$$

# Hexadecimal Number :-

Hexadecimal number system is a 16 numbers system from 0 to 9 and A to F. The base of Hexadecimal number is 16. It has 4 bit binary table.

## TABLE

Decimal	Binary	Hexadecimal
0	0 0 0 0	0
1	0 0 0 1	1
2	0 0 1 0	2
3	0 0 1 1	3
4	0 1 0 0	4
5	0 1 0 1	5
6	0 1 1 0	6
7	0 1 1 1	7
8	1 0 0 0	8
9	1 0 0 1	9
10	1 0 1 0	A
11	1 0 1 1	B
12	1 1 0 0	C
13	1 1 0 1	D
14	1 1 1 0	E
15	1 1 1 1	F

## Binary to Hexadecimal

(a)  $(1100101001010111)_2$

Make Combinations of 4-bits from Right

1100 1010 0101 0111

C A 5 7 using table

## Hexadecimal to Binary

(a)  $10A4_{16}$

$\begin{array}{cccc} 1 & 0 & A & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 0001 & 0000 & 1010 & 0100 \end{array}$

Using table

## Hexadecimal to Decimal

(a)  $(A85)_{16}$

$$= A \times 16^2 + 8 \times 16^1 + 5 \times 16^0$$

$$= 2560 + 128 + 5$$

$$= 2693$$

# Decimal to Hexadecimal

(a)  $(1650)_{10}$

$28A_{16}$

16	650
16	40 - 10-1
16	2 - 8
0	2

## Octal Numbers :-

Octal Number

System is 8 digit numbers

System from 0 to 7. The base of Octal number system is 8. Octal number system is <sup>several</sup> ~~base~~ 3 bit binary code.

Octal numbers 0, 1, 2, 3, 4, 5, 6, 7.

## TABLE

Octal

Binary

0 0 0

1 0 0 1

2 0 1 0

3 0 1 1

4 1 0 0

5 1 0 1

6 1 1 0

7 1 1 1

## Octal to decimal

$$(2374)_8$$

$$= 2 \times 8^3 + 3 \times 8^2 + 7 \times 8^1 + 4 \times 8^0$$

$$= 1024 + 192 + 56 + 4$$

$$= (1276)_{10}$$

## Decimal to Octal

$$(359)_{10}$$

$$(547)_8$$

8	359
8	44 - 7↑
8	5 - 4
	0 - 5

## Binary to Octal

$$(a) \quad 110101$$

$$\begin{matrix} & 1 & 1 & 0 & 1 & 0 & 1 \\ & \downarrow & & \downarrow & & \downarrow & \\ & 6 & & 5 & & & \end{matrix}$$

Using table

$$= (65)_8$$

## Octal to binary

$$(140)_8$$

$$\begin{array}{ccccccc} & & 1 & & 4 & . & 0 \\ & & \downarrow & & \downarrow & & \downarrow \\ & & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$(0011000000)_2$$

## Binary Coded Decimal :-

### Binary Code :-

Decimal is a way to express each of the decimal digit with a binary code. These are only the ten code - group in the BCD system from 0 to 9. It uses 4-bits to express each decimal to binary.

(Decimal)

(BCD)

0

0 0 0 0

1

0 0 0 1

2

0 0 1 0

3

0 0 1 1

4

0 1 0 0

5

0 1 0 1

6

0 1 1 0

7

0 1 1 1

8

1 0 0 0

9

1 0 0 1

## Invalid Codes :-

4-bits binary representation has sixteen numbers but in BCD we uses only first ten numbers and the codes of other six numbers are called Invalid Codes.

Example : Decimal (15)  $\rightarrow$  1111 (invalid code)

## BCD Addition :-

$$\textcircled{a} \quad \begin{array}{r} 0011 \\ + 0100 \\ \hline 0111 \end{array} \quad \begin{array}{r} 0011 \\ + 0100 \\ \hline 11 \end{array}$$

$$\textcircled{b} \quad \begin{array}{r} 1001 \\ + 0100 \\ \hline 1101 \end{array} \quad \begin{array}{r} 1 \\ + 4 \\ \hline 13 \end{array}$$

+ 0110 } invalid BCD add & BCD  
0001   0011    of 6  
+      3

13

## Digital Codes :-

These are many codes used in digital system are given below.

### 1) Gray Code :-

Gray code is unweighted and not an arithmetic code. It is also called RBC (Reflected binary code). It has only single bit change from one code word to next in sequence.

## Binary to Gray Code

Binary	1	+ 0	+ 1	+ 1	+ 0
	↓	↓	↓	↓	↓
Gray	1	1	1	0	1

## Gray Code to binary

Gray	1	1	0	1	1	1
	↓	✗	↓	✗	↓	✗
Binary	1	0	0	1	1	0

## (2) Alphanumeric Code :-

As the name of code represent it includes alphabets and numeric values.

It uses a 6-bit code and represent 64 characters.

## (3) ASCII :-

ASCII stand for "American Standard Code for Information Interchange". ASCII has 7-bit code and represent 128 characters includes alphabets, numeric values and some special characters.

(4) Unicode :-

Unicode stands for "Universal code". It consists of characters of almost all languages in the world. Unicode is mostly used now days in the world.

### Parity Method of Error Detection

Any group of bits contain even or odd numbers of 1s. A parity bit is attached to the group of bits to tell the system the number of 1 in group is even or odd.

Parity bit

BCD

0

0000

1

0001

1

0010

0

0011

1

~~0010100~~

0

0101

0

0110

1

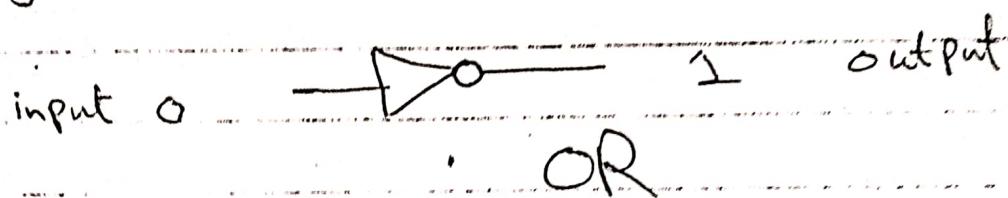
0111

# DLD Outline No 2

## The Inverter:-

The inverter is a circuit performing the operation called inversion or complement. The inverter converts all logical levels to opposite levels. In term of bits it changes 0 to 1, and 1 to 0.

Symbol of inverter



Truth table of inverter

input	Output
0	1
1	0

input	output
Low	High
High	Low

Logical Expression for inverter

$$X = \bar{A}$$

# AND Gate :-

AND is a Logical gate that performs the operation called multiplication between given inputs. AND Gate generate output 1 if and only if all the inputs are 1 otherwise it generate 0 output.

Symbol of And



OR



Truth table of AND

Input		Output
A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

Logical Expression

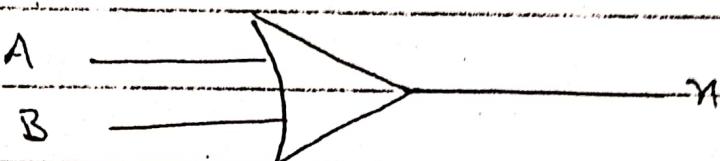
$$X = AB$$

OR Grade :-

OR is a logical gate that performs the operation called logical addition between two or more inputs.

It generates output 0 if and only if all inputs are 0 otherwise it generates 1 - output

### Symbol:



## Truth-table

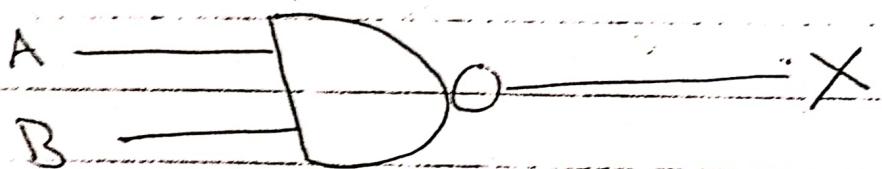
A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

## Logical Expression

$$X = A + B$$

NAND Gate is NAND Gate is the Combination of two Gates AND and NOT. The negative AND gate is also called NAND gate. A from this gate we get inverted outputs of AND gate. It generates 0 when both inputs are 1 otherwise it generates 1 output.

### Symbol



### Truth Table

Input		Output
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

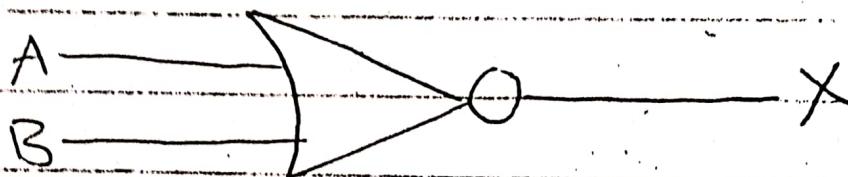
### Logical Expression

$$X = \overline{AB}$$

# NOR Gate :-

NOR gate is a combination of OR and NOT gate. It generates the inverted output of OR gate. It generates 1 when all inputs are 0 otherwise it generate 0.

Symbol



Truth table

input		Output
A	B	X
0	0	1
0	1	0
1	0	0
		0

Logical Expression

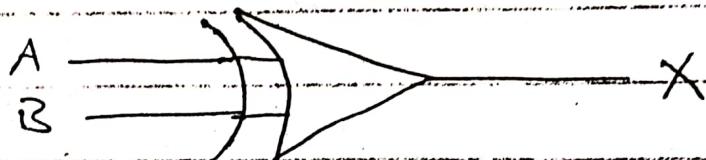
$$X = \overline{A+B}$$

## Exclusive-OR Gate :-

Exclusive-OR

gate also written as XOR gate.  
It generates 0 output when both inputs are same and generate 1 when both are different. It only operate on two inputs.

Symbol



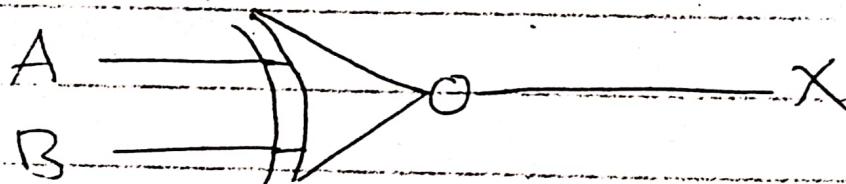
Truth Table

input		Output
A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

# Exclusive-NOR Gate ..

Exclusive-NOR gate also written as XNOR gate. It generates the complemented output of exclusive-OR gate. It generates 1 on both same inputs and 0 on both different inputs.

Symbol



Truth Table

input		output
A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

# DLD Outlin - no 3

## Boolean Operation and Expression

Boolean algebra is the mathematics of digital logic.

### Boolean Addition:

Boolean addition is a OR operation between given inputs.

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10 \rightarrow \text{with 1 carry}$$

### Boolean Multiplication:

Boolean multiplication is equivalent to AND operation.

$$0 \cdot 0 = 0$$

$$0 \cdot 1 = 0$$

$$1 \cdot 0 = 0$$

$$1 \cdot 1 = 1$$

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# Laws of Boolean Algebra

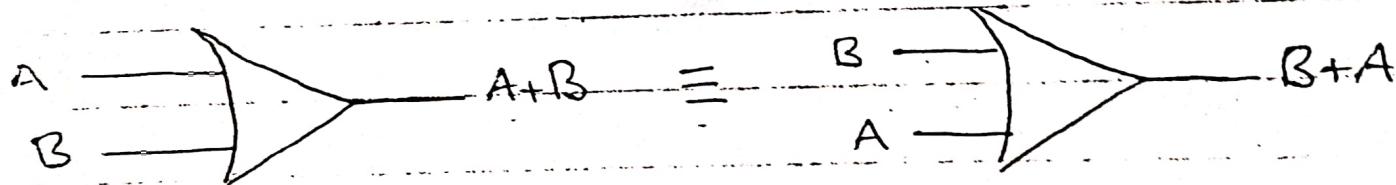
Different laws

of Boolean algebra are given below

## Commutative Laws:-

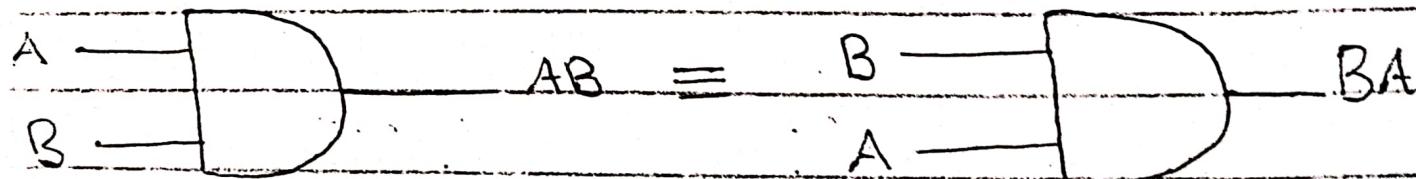
(1) The Commutative law of addition is written as

$$A + B = B + A$$



(2) The Commutative law of multiplication is written as

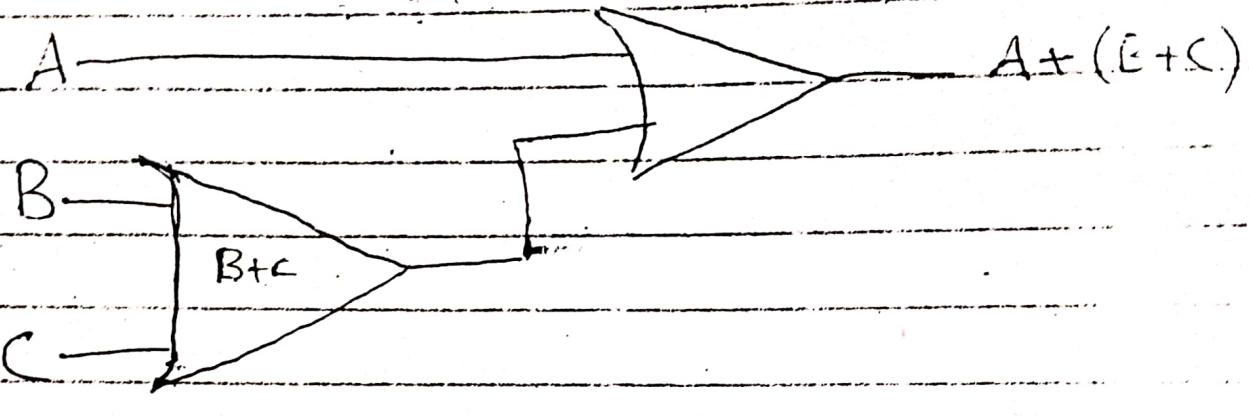
$$AB = BA$$



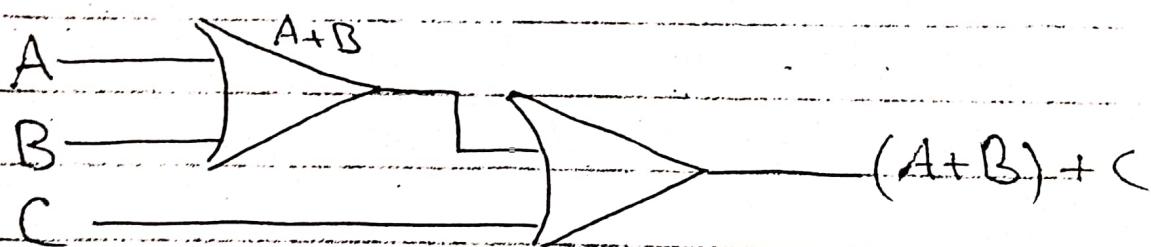
## Associative Laws :-

(1) Associative Law of addition

$$A + (B + C) = (A + B) + C$$

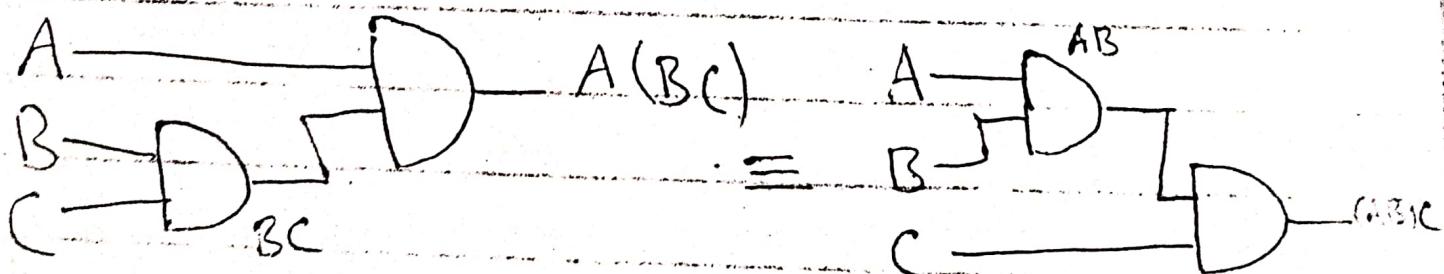


≡



(2) Associative law of multiplication

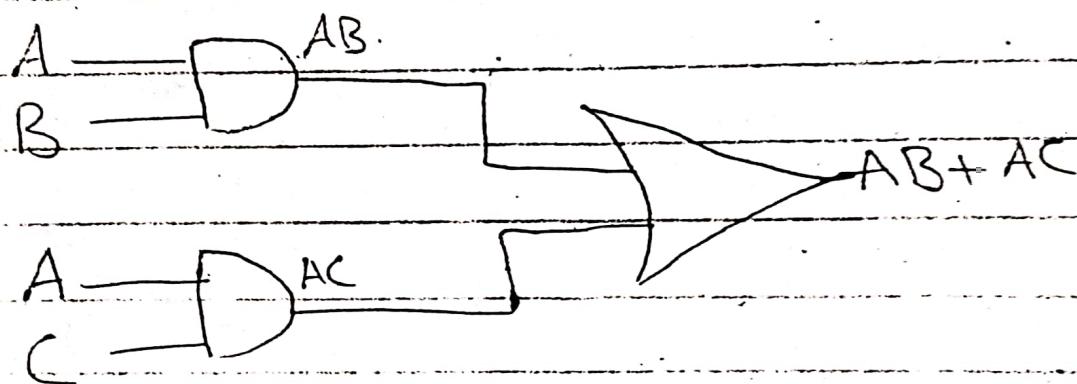
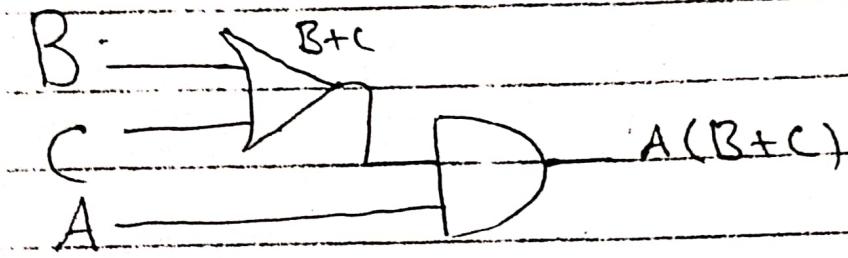
$$A(BC) = (AB)C$$



# Distributive law:

This law implement on three variables

$$A(B+C) = AB + AC$$



## Rules of Boolean Algebra

$$(1) A+0 = A$$

$$(10) A+AB = A$$

$$(2) A+1 = 1$$

$$(11) A+\bar{A}B = A+B$$

$$(3) A \cdot 0 = 0$$

$$(12) (A+B)(A+C) = A+BC$$

$$(4) A \cdot 1 = A$$

$$(5) A+A = A$$

$$(6) A \cdot \bar{A} = 0$$

$$(7) A \cdot A = A$$

$$(8) A \cdot \bar{A} = 0$$

$$(9) \bar{\bar{A}} = A$$

Proof Rule 10  $A + AB = A$

L.H.S.

$$= A + AB$$

$$= A \cdot 1 + AB$$

use Distributive Law  $AB + AC = A(B+C)$

$$= A(1+B)$$

use the Rule  $A+1 = 1$

$$= A(1+1)$$

use Rule  $A \cdot 1 = A$

$$= A$$

Proof Rule 11  $A + \bar{A}B = A + B$

L.H.S.

$$= A + \bar{A}B$$

$$= (A + AB) + \bar{A}B \quad \therefore A + AB = A$$

$$= AA + AB + \bar{A}B \quad \therefore A \cdot A = A$$

$$= AA + AB + A\bar{A} + \bar{A}B \quad \therefore A \cdot \bar{A} = 0$$

$$= A(A+B) + \bar{A}(A+B)$$

$$= (A + \bar{A})(A+B)$$

$$= 1(A+B) \quad \therefore A + \bar{A} = 1$$

$$= A+B$$

Proof Rule 12  $(A+B)(A+C) = A+BC$

L.H.S.  $(A+B)(A+C) = AA + AC + AB + BC$

$$= A \cdot A + AC + AB + BC \quad \therefore A \cdot A = A$$

$$= A(1+C) + B(A+C) + BC$$

$$= A \cdot 1 + AB + BC \quad \therefore A \cdot 1 = 1$$

$$= A(1+B) + BC \quad \therefore A \cdot 1 = 1$$

$$= A \cdot 1 + BC \quad \therefore A \cdot 1 = A$$

# DeMorgan's Theorem

The DeMorgan's

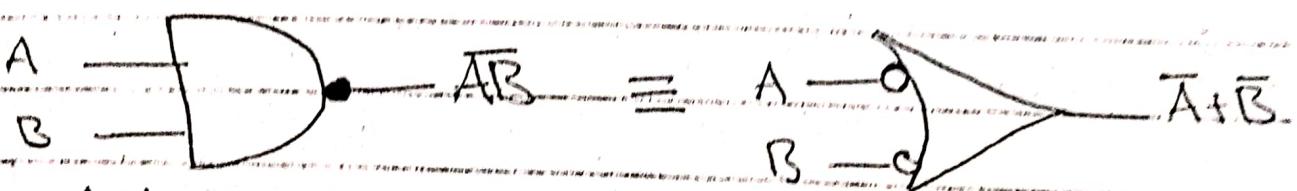
Theorem in term of Boolean algebra is

(a) The Complement of two or more ANDed variables is equal to the OR of ~~two~~ of the complement of individual variables.

(b) The Complement of tow or more ORed variables is equal to the AND of the complements of individuals variables.

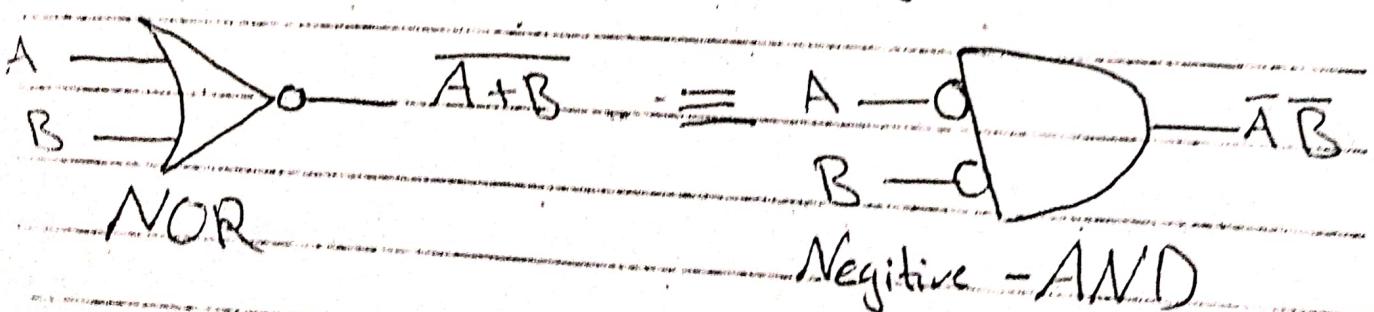
$$\overline{AB} = \overline{A} + \overline{B} \quad (a)$$

$$\overline{A+B} = \overline{A}\overline{B} \quad (b)$$



NAND

Negative-OR



NOR

Negative-AND

$A$	$B$	$\bar{A}$	$\bar{B}$	$AB$	$\bar{AB}$	$\bar{A} + \bar{B}$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0

$\bar{AB}$  is same output as  $\bar{A} + \bar{B}$

$A$	$B$	$\bar{A}$	$\bar{B}$	$A+B$	$\bar{A}+\bar{B}$	$\bar{AB}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

$\bar{A}+\bar{B}$  is same output as  $\bar{AB}$

Demorgan theorem can be implemented on more than 2 variables for example

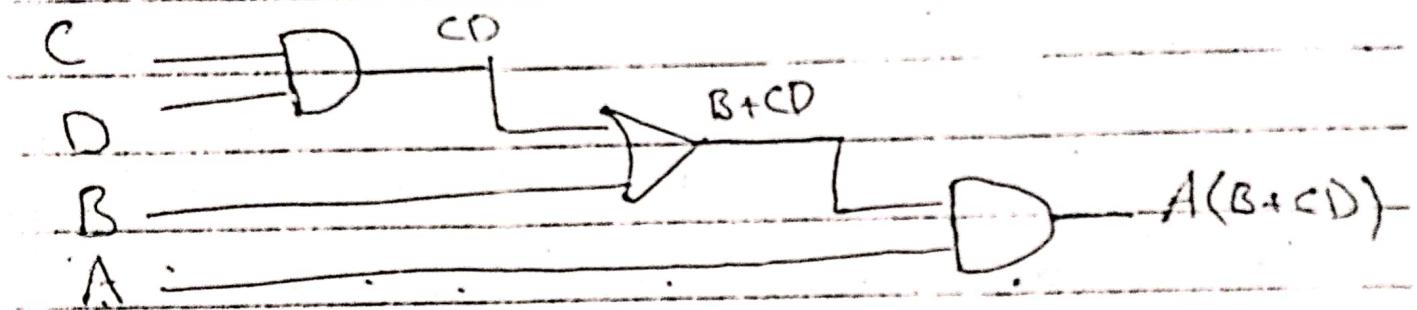
$$\overline{A+B+C} = \bar{A}\bar{B}\bar{C}$$

$$\overline{ABC} = \bar{A} + \bar{B} + \bar{C}$$

# Boolean Expression for a Logic Circuit

In this part of outline we write the Boolean Expression for logic circuit and vice versa.

## Logic circuit



## Logic Expression

$$A(B+CD)$$

## Logic Simplification Using Boolean Algebra

$$(a) AB + A(B+C) + B(B+C)$$

$$= AB + AB + AC + BB + BC$$

$$\Rightarrow \text{using Rule } A \cdot A = A$$

$$\Rightarrow AB + AB + AC + B + BC$$

$$\text{Using rule } A + A = A$$

$$\Rightarrow AB + AC + B + BC$$

$$\Rightarrow AB + AC + B \quad ; \quad A + AB = A$$

$$\Rightarrow B + AC \quad ; \quad A + AB = A$$

$$\begin{aligned}
 (b) & \quad [A\bar{B}(C+BD)+\bar{A}\bar{B}]C \\
 &= A\bar{B}C(C+BD)+A\bar{B}C \\
 &= A\bar{B}C \cdot C + A\bar{B}C \cdot BD + A\bar{B}C \\
 &= A\bar{B}C + A\bar{B} \cdot BCD + A\bar{B}C \quad \therefore A \cdot A = A \\
 &= A\bar{B}C + ACD(C) + A\bar{B}C \quad \therefore A \cdot A = C \\
 &= A\bar{B}C + 0 + A\bar{B}C \\
 &= \bar{B}C(A+\bar{A}) \\
 &= \bar{B}C(1) \quad \therefore A+\bar{A} = 1 \\
 &= \bar{B}C \cdot 1
 \end{aligned}$$

### Standard forms of boolean Expression

These are two forms of standard boolean Expression are given below.

#### The Sop form.

Sop stand for "Sum of Products". In this standard form boolean expression are written as

$$AB + A\bar{B}C \quad \text{OR}$$

$$ABC + C\bar{E}D + \bar{B}CD \quad \text{OR}$$

$$\bar{A}B + \bar{A}\bar{B}\bar{C} + AC$$

(a) convert to Sop form

$$\begin{aligned}
 &= \overline{\overline{A+B+C}} \\
 &= \overline{A+B} + \overline{C} \Rightarrow A+B
 \end{aligned}$$

Convert to SOP form

$$(a) \overline{A+B+C}$$

$$= (\overline{A+B})\bar{C}$$

∴ De Morgan Law

$$= (\overline{A}+\overline{B})\bar{C}$$

i.e.  $\overline{A} = A$

$$= (A+B)\bar{C}$$

$$= A\bar{C} + B\bar{C}$$

Standard SOP form:

The Standard SOP form means each product term must contain all variables used in Boolean Expression.

$$\bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C}D \rightarrow \text{not standard SOP form}$$

Standard SOP form.

$$= \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D}$$

(a)  $A\bar{B}C + \bar{A}\bar{B} + AB\bar{C}\bar{D}$  Convert to standard SOP form

$$= A\bar{B}C(D+\bar{D}) + \bar{A}\bar{B}(C+\bar{C})(D+\bar{D}) + A\bar{B}\bar{C}\bar{D}$$

$$= A\bar{B}CD + A\bar{B}C\bar{D} + (\bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C})(D+\bar{D}) + A\bar{B}\bar{C}\bar{D}$$

$$= A\bar{B}CD + A\bar{B}C\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}$$

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PTO

POS form :-

POS stand for ( Product of sum ) we written boolean expression in POS form as

$$(\bar{A} + B)(\bar{A} + \bar{B} + C) \quad \text{OR}$$

$$(\bar{A} + B + C)(C + \bar{D} + E)(\bar{B} + \bar{C} + D)$$

Standard POS form :-

Standard PCS

form means each sum form must contain all variables use in boolean expression.

$$(A + \bar{B} + C)(A + B + \bar{C})(\bar{A} + B + C)$$

Converting to Standard POS form

$$(a) (A + \bar{B} + C)(\bar{B} + C + \bar{D})(A + \bar{B} + \bar{C} + D)$$

$$= (A + \bar{B} + C + D\bar{D})(A + \bar{A} + \bar{B} + C + \bar{D})(A + \bar{B} + \bar{C} + D)$$

$$= (A + \bar{B} + C + D)(A + \bar{B} + C + \bar{D})(A + \bar{B} + \bar{C} + D)(\bar{A} + \bar{B} + C + \bar{D}) \\ (A + \bar{B} + \bar{C}D)$$

i.e

Boolean Expressions and truth tables

1) SOP to truth table

In SOP if

there is Complement of variable that indicate that there is 0 in truth table and if non-complement variable that indicate that there is 1 in truth table

$$(a) \bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$$

inputs	Output	Product term
a b c		
0 0 0	0	
0 0 1	1 m <sub>1</sub>	$\bar{A}\bar{B}C$
0 1 0	0	
0 1 1	0	
1 0 0	1 m <sub>2</sub>	$A\bar{B}\bar{C}$
1 0 1	0	
1 1 0	0	
1 1 1	1 m <sub>3</sub>	$ABC$

2) POS to truth table :-

In POS if there is complement of variable that indicates 1 in truth table and non-complement variable indicate 0.

$$(a) (A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+B+C)$$

Table on Next page

Input	Output	Sum term
a b c	x	$A + B + C$
0 0 0	0	
0 0 1	1	$A + \bar{B} + C$
0 1 0	0	$A + \bar{B} + \bar{C}$
0 1 1	0	
1 0 0	1	
1 0 1	0	$\bar{A} + B + \bar{C}$
1 1 0	0	$\bar{A} + \bar{B} + C$
1 1 1	1	

### Karnaugh Map :

The karnaugh map is a systematic method for simplifying Boolean expression. Karnaugh map is similar to truth table because in Karnaugh map we use table to present all possible values of input variables and the resulting output for each value.

formula to calculate cell

$\geq 2^n$  → here  $n$  is a numbers of variables.