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Assignment:

Linear algebra.

7/10

Roll no: 2043.

Q26: Determine values of a for which the system has no-solutions, exactly one solution, or infinitely many solutions.

$$x+2y+z = 2$$
$$2x-2y+3z = 1$$
$$x+2y-(a^2-3)z = a$$

Augmented matrix is given by

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & -2 & 3 & 1 \\ 1 & 2 & -(a^2-3) & a \end{array}\right]$$

Converting in row echelon form.

$$\underset{R}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -6 & 1 & -3 \\ 0 & 0 & -a^2+2 & a-2 \end{array}\right] \begin{array}{l} \\ R_2-2R_1 \\ R_3-R_1 \end{array}$$

$$\underset{R}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 1/-6 & 1/2 \\ 0 & 0 & -a^2+2 & a-2 \end{array}\right] \frac{1}{-6} \times R_2$$

i- The system will have unique solution if $-a^2+2 \neq 0$.

$\Rightarrow a^2 \neq 2 \quad , \quad a \neq \pm\sqrt{2}$

ii. The system will have infinitely many solutions if $-a^2+2=0$ and $a-2=0$.

$\Rightarrow a^2 = 2 \Rightarrow a = \pm\sqrt{2}$ and $a = 2$ which is not satisfied at same time

iii. The system will have no solution if $-a^2+2=0$ and $a-2 \neq 0$.

i.e. $a = \pm\sqrt{2}$ and $a \neq -2$ not satisfied simultaneously. No such value of a exist.

Q28: In what condition, a, b, c satisfy for linear system to be consistent.

$$x + 3y + z = a$$
$$-x - 2y + z = b$$
$$3x + 7y - z = c$$

Augmented matrix is given by:

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & a \\ -1 & -2 & 1 & b \\ 3 & 7 & -1 & c \end{array}\right]$$

Converting in row echelon form.

$$R \left[\begin{array}{ccc|c} 1 & 3 & 1 & a \\ 0 & 1 & 2 & a+b \\ 0 & -2 & -4 & c-3a \end{array}\right] \begin{array}{l} \\ R_2 + R_1 \\ R_3 - 3R_1 \end{array}$$

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$$\underset{R}{\sim}\begin{bmatrix} 1 & 3 & 1 & a \\ 0 & 1 & 2 & a+b \\ 0 & 0 & 0 & -a+2b+c \end{bmatrix} R_3 + 2R_2$$

- The system will be consistent if we have:

$$-a+2b+c=0.$$

* if $A = \begin{bmatrix} 2 & 5 & 4 \\ 3 & -7 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 5 \\ 3 & 7 \\ 1 & 5 \end{bmatrix}$

Prove that $AB \neq BA$.

L.H.S $= AB$.

$$AB = \begin{bmatrix} 2 & 5 & 4 \\ 3 & -7 & 1 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ 3 & 7 \\ 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} (2)(2)+(3)(5)+(4)(1) & (2)(5)+(5)(7)+(4)(5) \\ (3)(2)+(-7)(3)+(1)(1) & (3)(5)+(-7)(7)+(1)(5) \end{bmatrix}$$

$$= \begin{bmatrix} 4+15+4 & 10+35+20 \\ 6-21+1 & 15-49+5 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & 65 \\ -14 & -29 \end{bmatrix}$$

$AA^{-1} = 1$

$B^{-1}B =$

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R.H.S = BA

$$BA = \begin{bmatrix} 2 & 5 \\ 3 & 7 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 & 5 & 4 \\ 3 & -7 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (2)(2)+(5)(3) & (2)(5)+(5)(-7) & (2)(4)+(5)(1) \\ (3)(2)+(7)(3) & (3)(5)+(7)(-7) & (3)(4)+(7)(1) \\ (1)(2)+(5)(3) & (1)(5)+(5)(-7) & (1)(4)+(5)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 4+15 & 10-35 & 8+5 \\ 6+21 & 15-49 & 12+7 \\ 2+15 & 5-35 & 4+5 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & -25 & 13 \\ 27 & -34 & 19 \\ 17 & -30 & 9 \end{bmatrix}$$

Hence prove $L.H.S \neq R.H.S$

$AB \neq BA.$

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* Show that $A \neq 0$, $B \neq 0$ but $AB = 0$.

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 7 \\ 0 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} (0)(3)+(1)(0) & (0)(7)+(1)(0) \\ (0)(3)+(2)(0) & (0)(7)+(2)(0) \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

* Theorem:

If A and B are invertible with same size then the Product AB is also inversible and $(AB)^{-1} = B^{-1}A^{-1}$

Proof:

$(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AIA^{-1} = AA^{-1} = I$ — ①

$(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}IB = B^{-1}B = I$ — ②

using ① & ②

$(AB)(B^{-1}A^{-1}) = (B^{-1}A^{-1})(AB) = I$

$(AB)^{-1} = B^{-1}A^{-1}$

* **Theorem:**

if A is invertible and n is a non-negative integer then

i- A^{-1} is invertible and $(A^{-1})^{-1} = A$

ii- A^n is invertible and $(A^{+n})^{-1} = A^{-n}$

iii- kA is invertible for any scalar k and $(kA)^{-1} = k^{-1}A^{-1}$ $k \neq 0$.

Proof:

i- $A(A^{-1}) = (A^{-1})A = I \Rightarrow (A^{-1})^{-1} = A$

ii- $(A^n)(A^{-1})^n = (AA^{-1})^n = (I)^n = I$

iii- $(kA)(k^{-1}A^{-1}) = k^{-1}(kA)A^{-1}$ Since k and k^{-1} are scalars.

$= (k^{-1}k)AA^{-1} = (1)I$ — ① $= I$

Similarly-

$(k^{-1}A^{-1})(kA) = I$ — ②

from ① & ② $(kA)^{-1} = k^{-1}A^{-1}$

ii $A^n \cdot A^{-n} = \underbrace{A.A....A}_{n\text{ times}} \underbrace{A^{-1}A^{-1}....A^{-1}}_{n\text{ times}}$

$= \underbrace{A.A...A}_{(n-1)\text{ times}} AA^{-1} \underbrace{A^{-1}....A^{-1}}_{(n-1)\text{ times}}$

$= \underbrace{A.A...A}_{(n-1)\text{ times}} I \underbrace{A^{-1}....A^{-1}}_{(n-1)\text{ times}}$

$= \underbrace{A.A...A}_{(n-1)\text{ times}} . \underbrace{A^{-1}....A^{-1}}_{(n-1)\text{ times}}$

After $(n-1)$ steps

$= I$ — ①

Similarly

$A^{-n}A^{n} = I$ — ②

from ① & ②

$(A^n)^{-1} = A^{-n}$

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- **Theorem:**

if A is invertible, A^T is also matrix invertible, and

$$(A^T)^{-1} = (A^{-1})^T$$

Proof:

We want → $A^T(A^{-1})^T = (A^{-1})^T A^T = I$

Now $A^T(A^{-1})^T = (A^{-1}A)^T = I^T = I$ ①

$\because I^T = I$

$(A^{-1})^T A^T = (AA^{-1})^T = I^T = I$ ②

from ① & ②

$(A^T)^{-1} = (A^{-1})^T$

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$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & -2 & 3 & 1 \\ 1 & 2 & -a^2+3 & a \end{array}\right]$$

$$R \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -6 & 1 & -3 \\ 0 & 0 & -a^2+2 & a-2 \end{array}\right]$$

$$R \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & +1 & -\frac{1}{6} & \frac{1}{2} \\ 0 & 0 & 2\ a^2 & a-2 \end{array}\right]$$