Q#1

i) Solve the inequility

$$\frac{6}{x-1} \geq 5$$

$$6 \geq 5(x-1)$$

$$6 \geq 5x-5$$

$$6+5 \geq 5x \Rightarrow 11 \geq 5x$$

$$\frac{11}{5} \geq x$$

$$x \leq 11/5 \quad [11/5, \infty)$$

vii) Transcendental function:-

These are functions that are not algebric. They include the trignometric, inverse trignometric, exponential and log functions and many other functions. (hyberbolic)

example

v) Define local extreme values.

Let f be a function with domain D. Then f has an absolute maximum value on D at point c if.

$$f(x) \leq f(c) \text{ for all } x \text{ in } D$$

and an absolute minimum on D if

$$f(x) \geq f(c)$$

iii) Define Sandwitch theorem:-

Suppose that $g(x) \leq f(x) \leq h(x)$ for all x in Some open interval containing c, except possibly at $x=c$ itself. Suppose also that

$$\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L.$$

Then, $\lim_{x \to c} f(x) = L$.

It is also called the sqveeze or pinching theorem.

iv) Define Ellipse:-

An ellipse is a set of points in a plan whose ~~to~~ distance from the two fined points in the plane have a Constant Sum.

Two fin points are foci of ellipse.

ii) Define constant function.

A function $f: x \to x$ defined by $f(x) = k$ $\forall x \in x$ where k is Some constant is called constant function.

$f(x) = 3$ for all x in the domain of f.

ix) Inverse derivative:-

$$g(x) = x-2 \qquad x=1$$

Inverse derivative $= \int (x-2)\,dx$

$$\left| \frac{x^2}{2} - 2x \right|$$

at $x=1$ $\quad \left(\frac{1}{2} - 2(1)\right) = \frac{1}{2} - 2$

$$= \frac{1-4}{2} = -\frac{1}{2}$$

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viii) Evaluate $\int_{-\pi/2}^{0} \sec x \tan x\,dx$

$$= \left| \sec x + c \right|_{-\pi/2}^{0}$$

$$= \left[\sec(0) - \sec(-\pi/2) \right] + C$$

$$= (1-0) + C$$

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xi) Find $\int \ln x\,dx$

$$= \int \ln x \cdot 1\,dx$$

$$= \ln x \int 1\,dx - \int \left(\frac{d}{dx} \ln x \int 1\,dx\right) dx$$

$$= \ln(x) - \int \left(\frac{1}{x} \cdot x\right) dx$$

$$= x \ln x - \int 1\,dx$$

$$= x \ln x - x + C \quad \text{Ans}$$

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xiii) Find polar Equation of Circle

$$(x+2)^2 + (y-3)^2 = 3$$

x) Prove that $\frac{d}{du}(\cosh u) = \sinh u \frac{du}{dx}$

L.H.S $= \cosh u$

Let $x = \cosh u$

$= \left(\frac{e^u + e^{-u}}{2}\right)$

$\frac{dx}{du} = \frac{d}{dx}\left(\frac{e^u - e^{-u}}{2}\right) \Rightarrow \frac{dx}{du} = \left(\frac{e^u - e^{-u}}{2}\right)$

$\frac{d}{du}(\cosh u) = \frac{e^u - e^{-u}}{2}$ $\qquad \frac{e^u \cdot e^{-u}}{2} = \sinh u.$

$\frac{d}{du}(\cosh u) = \sinh u \cdot \frac{du}{dx}$

$x = r\cos\theta \quad , \quad y = r\sin\theta$

$x^2 + 4x + 4 + y^2 - 6x + 9 = 3$

$x^2 + y^2 + 4x - 6y + 13 = 3$

$x^2 + y^2 + 4x - 6y = 3 - 13$

$r^2\cos^2\theta + r^2\sin^2\theta + 4r\cos\theta - 6r\sin\theta = -10$

$r^2 + 4r\cos\theta - 6r\sin\theta = -10$ Ans.

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xiv) Define projection of a vector

vector projection of u into v

$$\text{Proj}_v u = \left[\frac{u \cdot v}{|v|^2} \right] v$$

vector projection along v into u

$$u \cdot v = |u||v|\cos\theta$$

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xvi) Two properties of dot product

→ if u, v, w are vectors and c is a scalar then

$$u \cdot v = v \cdot u$$

$$(cu) \cdot v = u \cdot (cv) = c(u \cdot v)$$

$$u \cdot u = |u|^2$$

$$0 \cdot u = 0$$

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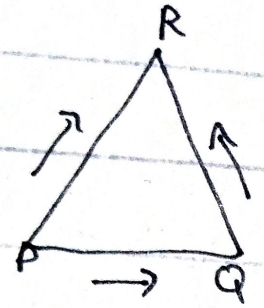
xv) Find area of triangle $P(1,-1,0)$ $Q(2,1,-2)$ $R(-1,1,2)$



$= \frac{1}{2}\left[\vec{PQ} \times \vec{PR}\right]$

$\Rightarrow \vec{PQ} = \vec{OQ} - \vec{OP}$

$\Rightarrow 2i + j - 2k - i + j - 0k$

$= i + 2j - 2k$

$\vec{PR} = \vec{OR} - \vec{OP}$

$= -i + j + 2k - i + j - 0k$

$= -2i + 2j + 2k$

$$\left|\vec{PQ} \times \vec{PR}\right| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -2 \\ -2 & 2 & 2 \end{vmatrix}$$

$= i(4+4) - j(2-4) + k(2+4)$

$= |8i + 2j + 6k|$

$= \sqrt{(8)^2 + (2)^2 + (6)^2}$

$= \sqrt{64 + 36 + 4}$

$= \sqrt{104}$

$= \left(\cancel{8}(26)\right)^{1/\cancel{2}} \Rightarrow \sqrt{26}$ Sq. units.