definitions of linear Algebra

1) **Vectors in R^n :-**

The set of all n-tuples of real numbers, denoted by R^n is called n-space. A particular n-tuple in R^n, say $u = (a_1, a_2, \dots a_n)$ is called a point or vector. The number a_i is called co-ordinates. When discussing the space R^n, we use term scalar for elements of R.

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2) **Dot (inner) PRODUCT :-**

Consider arbitrary vectors 'u' and 'v' in R^n; say,

$$u = (a_1, a_2, \dots a_n) \text{ and } v = (b_1, b_2, \dots b_n)$$

The dot product or inner product or scalar product of 'u' and 'v' is denoted and defined by

$$u \cdot v = a_1 b_1 + a_2 b_2 + \dots + a_n b_n.$$

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3) **Harmitian Matrices :-**

A complex matrix A is said to be Harmitian or skew symmetric according as

$$A^H = A \quad \text{or} \quad A^H = -A$$

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4) **Symmetric Matrix :-**

A matrix A is symmetric if $A^T = A$

The symmetric elements in A are equal, or $A^T = A$.
Thus A is symmetric.

5) **Triangular Matrices :-**
A square matrix $A = [a_{ij}]$ is upper triangular or simply triangular if all entries below the (main) diagonal are = to 0 that is $a_{ij} = 0$ for $i > j$.
e.g;

$$\begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix}$$

6) **Invertible Non-Singular Matrices :-**
A square matrix A is said to be invertible or non-singular if there exists a matrix B such that

$$AB = BA = I$$

I is identity Matrix.

7) **Identity Matrix, Scalar Matrix :-**
The n-square identity or unit matrix denoted by I_n or simply I, is the n-square matrix with 1's on the diagonal and 0's elsewhere. It is similar to scalar

$$AI = IA = A$$

8) Diagonal and Trace :-

Let $A = [a_{ij}]$ be an n-square matrix. The diagonal or main diagonal of A consists of the elements with the same subscripts that is

$$a_{11}, a_{22}, a_{33}, \dots a_{nn}$$

The trace of A, written tr (A) is the sum of diagonal elements.

$$tr(A) = a_{11} + a_{22} + a_{33} + \dots a_{nn}.$$

9) Transpose of Matrices :-

The transpose of Matrix is the interchange of rows into columns and columns as rows.

It is denoted by A^T.

e.g;

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

10) Square Matrices :-

A Matrix which consists of same number of rows and columns

e.g;

$$\begin{bmatrix} 1 & 1 \\ 2 & 5 \end{bmatrix}$$

11) Matrix Multiplication :-

The product of Matrix A and B is written as AB.

Product AB of a row matrix $A = [a_i]$ and a column Matrix $B = [b_i]$ with the ~~the~~ same no. of elements is defined to be a scalar matrix (1×1), multiplying corresponding entries and adding.

$$AB = [a_1, a_2, \cdots a_n] \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = a_1 b_1 + a_2 b_2 + \cdots \cdots a_n b_n = \sum_{k=1}^{n} a_k b_k.$$

12) Matrices :-

A Matrix A over a field K or, simply a matrix A is a rectangular array of scalars usually presented in.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

The rows of such a matrix A are the m horizontal lists of scalars.

$$(a_{11}, a_{12} \cdots a_{1n}), (a_{21}, a_{22}, \cdots a_{2n}), (a_{m1}, a_{m2}, \cdots a_{mn})$$

13) Complex Numbers :-

It is an ordered pair (a, b) of real numbers where equality, addition and multiplication is defined.

$(a,b) = (c,d)$ if and only if
$a = c$, $b = d$.

14) **Cross Product :-**

$u \times v$ is a vector
it is also called vector product, cross product or outer product of u and v.

e.g:

$i \times j = k$, $j \times k = i$, $k \times i = j$

$j \times i = -k$, $k \times j = -i$, $i \times k = -j$

15) **Vectors in R^3 (spatial vectors) :-**

Vectors in R^3, called spatial vectors. A special notation used for such vectors are

$i = [1, 0, 0]$ denotes unit vector in x direction

$j = [0, 1, 0]$ " " " " y "

$k = [0, 0, 1]$ " " " " z "

16) **Curves in R^n :-**

Let D be an interval on the real line R. A continuous function $F : D \to R^n$ is a curve in R^n Thus, to each point $t \in D$ there is assigned to the following point in R^n

$$F(t) = [F_1(t), F_2(t), \ldots F_n(t)]$$

- derivatives :

$$V(t) = \frac{dF(t)}{dt} = \left[\frac{dF_1(t)}{dt}, \frac{dF_2(t)}{dt}, \dots, \frac{dF_n(t)}{dt}\right]$$

which is tangent to the curve

$$T(t) = \frac{V(t)}{\| V(t) \|}.$$

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17) Distance, Angle, Projection :-

- distance :-

$$d(u,v) = (u-v) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \dots + (a_n - b_n)^2}$$

- Angle :-

Angle b/w non-zero vectors.

$$\cos \theta = \frac{u \cdot v}{|u||v|}$$

by Schwarz inequality

$$-1 \leq \frac{u \cdot v}{|u||v|} \leq 1$$

- Projection :-

Projection onto non-zero vectors.

$$\text{proj}(u,v) = \frac{u \cdot v}{|v|^2} v = \frac{u \cdot v}{v \cdot v} v$$

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18) Norm (length of a vector) :-

Length of a vector in R^n denoted by $\|u\|$, defined to be the non. negative sq. root.

$$\|u\| = \sqrt{u \cdot u} = \sqrt{a_1^2 + a_2^2 + \dots a_n^2}$$

sq. root of the sum of the sq. of the vectors.

In unit vector.

$$\hat{v} = \frac{1}{\|v\|} v = \frac{v}{\|v\|}$$

19) Orthogonal Matrix :-

It is a real sq. matrix whose columns and rows are orthogonal / orthonormal vectors.

The determinant of orthogonal matrix is either 1 or -1

20) Rank of a Matrix :-

The number of non-zero rows is called of a matrix.

e.g:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 2 & 3 \end{bmatrix} \rightarrow \text{non-zero.}$$

R_2 is the Rank of. Matrix.

21) **Diagonalization of Matrix :-**

Diagonalization of Matrix is In similar Matrix $B = P^{-1}AP$ are in the diagonal matrix in which all the upper values are icon and lower are zero.

e.g;

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \Rightarrow D = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

22) **Linear Algebra :-**

A branch of mathe--matics that is concerned with mathematical operations / structures closed under the operation of scalar ~~and~~ multiplication and addition.

23) **Null Space :-**

A subspace of a vector space consisting of vectors that under a given linear transformation are mapped onto zero.

24) **SubSpace :-**

Subspace are all vectors, they have the same dimensions.

25) **Vector Space :-**

A vector space is any set of objects with a notion of addition and scalar multiplication that behaves like vectors in Rn.

26) **Linear Combination :-**

If one vector is equal to the sum of scalar multiples of other vectors, it is said to be linear combination.

e.g;

$$a = 2b + 3c.$$

27) **Echelon form :-**

- All non zero rows are above any rows of all zeros.
- Each leading entry of a row is in a column to the right of the leading entry of row above it.
- All entries of a column below a leading entry are zeros.

28) **Gauss - Jordan Elimination :-**

It is also known as row reduction, is an algorithm for solving systems of linear equations. It consists of a sequence of operations.

29) **Cauchy-Schwarz Inequality :-** The theorem that the square of the integral of the product of two function is less than or equal to the product of the integrals of the square of each function.

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30) **Eigen value and vector :-**

- Eigen vectors are the directions along with a particular linear transformation acts by compressing.
- Eigen value can be referred as the strength of the transfor- -mation in the direction of eigen vector.

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31) **Linear dependent :-** The property of one set having at least one linear combination of its elements equal to zero when the coefficients are taken from another given set. and at least one of its coefficients is not equal to zero.

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32) **Linear Independent :-** The property of a set having no linear combination

of all its elements equal to zero when coefficients are taken from a given set unless the coefficient of each element is zero.

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33) **Row Operation :-**

Row operation are the simple operations that allow us to transform a system of linear equation into equivalent system, interchanging two equations, add, multiply ....

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34) **Linear Space :-**

A linear space is a space consisting of a collection of points and a set of lines subject to the.

- Any line of has atleast two points
- Any two distint points of ~~beyond~~ belong to exactly one line.

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35) **Co-factor :-**

The signed minor of a matrix, Minor ... an alternative name for the determinant of a smaller matrix.
