

QUESTION: Perform AES and give output of round 2 by using following data ?

$R_0 =$ 54 68 61 74 73 20 6D 79
20 4B 75 6E 67 20 46 75

$R_1 =$ E2 32 FC F1 91 12 91 88 B1
59 E4 E6 D6 79 A2 93

$R_2 =$ 56 08 20 07 C7 1A B1 8E 76
43 55 69 A0 3A F7

State Matrix = $\begin{bmatrix} 54 & 4F & 4E & 20 \\ 77 & 6E & 69 & 54 \\ 6F & 65 & 6E & 77 \\ 20 & 20 & 65 & 6F \end{bmatrix}$

Sol:-

Step 1 Add Round Key

$$= \begin{bmatrix} 54 & 4F & 4E & 20 \\ 77 & 6E & 69 & 54 \\ 6F & 65 & 6E & 77 \\ 20 & 20 & 65 & 6F \end{bmatrix} \oplus \begin{bmatrix} 54 & 73 & 20 & 67 \\ 68 & 20 & 4B & 90 \\ 61 & 6D & 75 & 46 \\ 74 & 79 & 6E & 75 \end{bmatrix}$$

$$54 = 01010100$$

$$4F = 01001111$$

$$54 = 01010100$$

$$73 = 01110011$$

$$00000000 = 00$$

$$00111100 = 3C$$

$$4E = 01001110$$

$$20 = 00100000$$

$$20 = 00100000$$

$$67 = 01100111$$

$$01101110 = 6E$$

$$01000111 = 47$$

$$77 = 01110111$$

$$6E = 01101110$$

$$68 = 01101000$$

$$20 = 00100000$$

$$00011111 = 1F$$

$$01001110 = 4E$$

$$69 = 01101001$$

$$54 = 01010100$$

$$4B = 01001011$$

$$20 = 00100000$$

$$00100010 = 22$$

$$01101000 = 74$$

$$6F = 01101111$$

$$65 = 01100101$$

$$61 = 01100001$$

$$6D = 01101101$$

$$00001110 = 0E$$

$$00001000 = 08$$

$$6E = 01101110$$

$$77 = 01110111$$

$$75 = 01110101$$

$$46 = 01000110$$

$$00011011 = 1B$$

$$00110001 = 31$$

$$20 = 00100000$$

$$20 = 00100000$$

$$74 = 01110100$$

$$79 = 01111001$$

$$01010100 = 54$$

$$01011001 = 59$$

$$65 = 01100101$$

$$6F = 01101111$$

$$6E = 01101110$$

$$75 = 01110101$$

$$00001011 = 0B$$

$$00011010 = 1A$$

=	00	3C	6E	47
	1F	4E	22	74
	0E	08	1B	31
	54	59	0B	1A

Step 2:-

Substitution Method

=	63	EB	9F	A0
	C0	2F	93	92
	AB	30	AF	C7
	20	CB	2B	A2

Step 3:-

Shifting Rows

$$= \begin{bmatrix} 63 & EB & 9F & A0 \\ 2F & 93 & 92 & C0 \\ AF & C7 & AB & 30 \\ A2 & 20 & CB & 2B \end{bmatrix}$$

Step 4:-

Mixed Columns

$$= \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} 63 & EB & 9F & A0 \\ 2F & 93 & 92 & C0 \\ AF & C7 & AB & 30 \\ A2 & 20 & CB & 2B \end{bmatrix}$$

$$= 02 \cdot 63 \oplus 03 \cdot 2F \oplus 01 \cdot AF \oplus 01 \cdot A2$$

$$= (x)(01100011) \oplus (x+1)(00101111) \oplus 01$$

$$(10101111) \oplus 01 \cdot (10100010)$$

$$= (x)(x^6 + x^5 + x^1 + 1) \oplus (x+1)(x^5 + x^3 + x^2 + x^1 + 1)$$

$$= (x^7 + x^6 + x^2 + x) \oplus (x^6 + x^4 + x^3 + x^2 + x + x^5 + x^1 + x^0 + 1)$$

$$= (11000110) \oplus (0110001)$$

$$\begin{array}{r}
 11000110 \\
 01110001 \\
 10101111 \\
 + 10100010 \\
 \hline
 10111010 = \text{BA}
 \end{array}$$

$$= 02 \cdot EB \oplus 03 \cdot 93 \oplus 01 \cdot C7 \oplus 01 \cdot 20$$

$$= x(11101011) \oplus (x+1)(10010011) \oplus 01(11000111) \oplus 01(00100000)$$

$$= x(x^7 + x^6 + x^5 + x^3 + x' + 1) \oplus (x+1)(x^7 + x^4 + x' + 1)$$

$$= (x^8 + x^7 + x^6 + x^4 + x^2 + x) \oplus (x^8 + x^5 + x^2 + x^3 + x^7 + x^4 + x' + 1)$$

$$= (x^1 + x^3 + 1 + x^7 + x^6 + x' + x^2 + x') \oplus (x^4 + x^3 + 1 + x^5 + x^2 + x' + x^7 + x^4 + x' + 1)$$

$$= (11001111) \oplus (10100100) \oplus (11000111) \oplus (00100000)$$

$$11001111$$

$$10101100$$

$$11000111$$

$$+ 00100000$$

$$10000100 = 84$$

$$= 02 \cdot 9F \oplus 03 \cdot 92 \oplus 01 \cdot AB \oplus 01 \cdot CB$$

$$= (x)(00111111) \oplus (x+1)(10010010) \oplus 01(10101011) \oplus 01(11001011)$$

$$= (x)(x^7 + x^4 + x^3 + x^2 + x + 1) \oplus (x+1)$$

$$(x^7 + x^4 + x)$$

$$= (x^8 + x^5 + x^4 + x^3 + x^2 + x) \oplus (x^8 + x^5 + x^3 + x^2 + x + 1)$$

$$= (x^4 + x^3 + 1 + x^5 + x^2 + x^3 + x^2 + x) \oplus$$

$$(x^4 + x^3 + 1 + x^5 + x^2 + x^7 + x + x)$$

$$= (00100111) \oplus (10101111) \oplus (10101011) \oplus$$

$$(11001011)$$

$$00100111$$

$$10101111$$

$$10101011$$

$$+ 11001011$$

$$11101000 = \mathbf{E8}$$

$$= 02 \cdot A0 \oplus 03 \cdot C0 \oplus 01 \cdot 30 \oplus 01 \cdot 2B$$

$$= (x)(10100000) \oplus (x+1)(11000000) \oplus$$

$$01(00110000) \oplus 01(00101011)$$

$$= x(x^7 + x^5) \oplus (x+1)(x^7 + x^5)$$

$$= (x^8 + x^6) \oplus (x^8 + x^7 + x^7 + x^6)$$

$$= (x^4 + x^3 + 1 + x^6) \oplus (x^4 + x^3 + 1 + x^6)$$

$$= (01011001) \oplus (01011001) \oplus (00110000) \oplus$$

$$(00101011)$$

$$\begin{array}{r}
 01011001 \\
 01011001 \\
 00110000 \\
 + \underline{00101011}
 \end{array}$$

$$00011011 = 1B$$

$$= 01 \cdot 63 \oplus 02 \cdot 2F \oplus 03 \cdot AF \oplus 01 \cdot A2$$

$$= 01(01100011) \oplus (\alpha)(00101111) \oplus (\alpha+1)(10101111) \oplus 01(10100010)$$

$$= \alpha(00101111) \oplus (\alpha+1)(10101111)$$

$$= \alpha(\alpha^5 + \alpha^3 + \alpha^2 + \alpha^1 + 1) \oplus (\alpha+1)(\alpha^7 + \alpha^5 + \alpha^4 + \alpha^3 + \alpha^2 + \alpha^1 + 1)$$

$$= (\alpha^6 + \alpha^4 + \alpha^3 + \alpha^2 + \alpha^1) \oplus (\alpha^8 + \alpha^6 + \alpha^5 + \alpha^4 + \alpha^3 + \alpha^2 + \alpha^1 + 1)$$

$$= (\alpha^6 + \alpha^4 + \alpha^3 + \alpha^2 + \alpha^1) \oplus (\alpha^1 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6 + \alpha^7 + \alpha^2 + \alpha^3 + \alpha^2 + \alpha^1 + 1)$$

$$= (01011110) \oplus (11011000) \oplus (01100011) \oplus (10100010)$$

$$01100011$$

$$01011110$$

$$11011000$$

$$+ \underline{10100010}$$

$$01011011 = 575$$

$$= 01 \cdot EB \oplus 02 \cdot 93 \oplus 03 \cdot C7 \oplus 01 \cdot 20$$

$$= 01(11101011) \oplus (\alpha)(10010011) \oplus (\alpha+1)(11000111) \oplus 01(00100000)$$

$$\begin{aligned}
 &= x(x^4 + x^3 + x^2 + 1) \oplus (x+1)(x^4 + x^3 + x^2 + 1) \\
 &= (x^8 + x^7 + x^6 + x^5) \oplus (x^5 + x^4 + x^3 + x^2 + 1) \oplus (x^4 + x^3 + x^2 + x + 1) \\
 &= (x^8 + x^7 + x^6 + x^5) \oplus (x^5 + x^4 + x^3 + x^2 + 1) \oplus (x^4 + x^3 + x^2 + x + 1) \\
 &= (11101011) \oplus (00111111) \oplus (01010000) \oplus (00100000)
 \end{aligned}$$

$$11101011$$

$$00111111$$

$$01010000$$

$$00010000$$

$$10100100 = A4$$

$$= 01 \cdot 9F \oplus 02 \cdot 92 \oplus 03 \cdot AB \oplus 01 \cdot CB$$

$$= 01(10011111) \oplus (x)(10010010) \oplus (x+1)$$

$$(10101011) \oplus 01(11001011)$$

$$= (x)(x^7 + x^6 + x^5) \oplus (x+1)(x^7 + x^6 + x^5 + x^4 + 1)$$

$$= (x^8 + x^7 + x^6) \oplus (x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$$

$$= (x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1) \oplus (x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$$

$$= (10011111) \oplus (00111111) \oplus (11100110) \oplus (11001011)$$

$$\begin{aligned}
 &= x(x^7 + x^6 + x^5 + x^4 + 1) \oplus (x+1)(x^7 + x^6 + x^5 + x^4 + 1) \\
 &= (x^8 + x^7 + x^6 + x^5 + x^4) \oplus (x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1) \\
 &= (x^8 + x^7 + x^6 + x^5 + x^4) \oplus (x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1) \\
 &= (1001111) \oplus (10010010) \oplus (01001101) \oplus (01000100)
 \end{aligned}$$

$$\begin{array}{r}
 1001111 \\
 10010010 \\
 01001101 \\
 + 01000100 \\
 \hline
 00000110 = 06
 \end{array}$$

$$= 01.A0 \oplus 01.C0 \oplus 02.30 \oplus 03.2B$$

$$\begin{aligned}
 &= 1(10100000) \oplus 1(11000000) \oplus (x)(00110000) \oplus \\
 &\quad (x+1)(00101011)
 \end{aligned}$$

$$\begin{aligned}
 &= x(x^8 + x^7) \oplus (x+1)(x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1) \\
 &= (x^9 + x^8) \oplus (x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1) \\
 &= (10100000) \oplus (11000000) \oplus (01100000) \oplus (0111101)
 \end{aligned}$$

$$\begin{array}{r}
 10100000 \\
 11000000 \\
 01100000 \\
 + 0111101 \\
 \hline
 0111101 = 7D
 \end{array}$$

00100110

10010011

11000111

+ 01000000

00110010 = **32**

$$= 03 \cdot 9F \oplus 01 \cdot 92 \oplus 01 \cdot AB \oplus 02 \cdot CB$$

$$= (x+1)(10011111) \oplus 1(10010010) \oplus 1(10101011) \oplus$$

$$(x)(11001011)$$

$$= (x+1)(x^7+x^6+x^5+x^4+x^3+x^2+x^1+1) \oplus x(x^7+x^6+x^5+x^4+x^3+x^2+x^1+1)$$

$$= (x^8+x^7+x^6+x^5+x^4+x^3+x^2+x^1+1) \oplus (x^8+x^7+x^6+x^5+x^4+x^3+x^2+x^1+1)$$

$$\oplus (x^8+x^7+x^6+x^5+x^4+x^3+x^2+x^1)$$

$$= (x^4+x^3+1+x^7+x^5+1) \oplus (x^4+x^3+1+x^7+x^5+x^4+x^3+x^2+x^1)$$

$$= (10111000) \oplus (10001111)$$

$$= (10111000) \oplus (10010010) \oplus (10101011) \oplus$$

$$(10001111)$$

10111000

10010010

10101011

+ 10001111

00001110 = **0E**

$$= 03 \cdot A0 \oplus 01 \cdot C0 \oplus 01 \cdot 30 \oplus 02 \cdot 2B$$

$$= (x+1)(10100000) \oplus 1(11000000) \oplus$$

$$1(00110000) \oplus (x)(00101011)$$

$$= (x+1)(x^7+x^6) \oplus (x)(x^5+x^3+x^1+1)$$

$$= 03 \cdot 63 \oplus 01 \cdot 2F \oplus 01 \cdot AF \oplus 02 \cdot A2$$

$$= (x+1)(01100011) \oplus 1(00101111) \oplus 1(10101111) \oplus$$

$$x(10100010)$$

$$= (x+1)(x^6+x^5+x^1+1) \oplus x(x^7+x^5+x^1)$$

$$= (x^7+x^6+x^2+x^1+x^6+x^5+x^1+1) \oplus$$

$$(x^8+x^1+x^2)$$

$$= (x^7+x^5+x^2+1) \oplus (x^8+x^1+x^2+x^6+x^5+x^1+1)$$

$$= (10100101) \oplus (00101111) \oplus (10101111) \oplus$$

$$(01011101)$$

$$10100101$$

$$00101111$$

$$10101111$$

$$+ 01011101$$

$$0111010 = 7A$$

$$= 03 \cdot EB \oplus 01 \cdot 93 \oplus 01 \cdot C7 \oplus 02 \cdot 20$$

$$= (x+1)(11101011) \oplus 1(10010011) \oplus 1(11000111) \oplus$$

$$x(00100000)$$

$$= (x+1)(x^7+x^6+x^5+x^3+x^1+1) \oplus x(x^5)$$

$$= (x^8+x^7+x^6+x^5+x^4+x^3+x^2+x^1+x^6+x^5+x^3+x^1+1)$$

$$\oplus (x^6)$$

$$= (x^8+x^7+x^6+x^5+x^4+x^3+x^2+x^1) \oplus (x^6)$$

$$= (00100100) \oplus (10010011) \oplus (11000111) \oplus$$

$$(01000000)$$

M T W T F S

$$= (x^7 + x^6 + 1 + x^6 + x^5 + x^4 + x^3 + x^2 + x) \oplus$$

$$(x^4 + x^3 + 1 + x^7 + x^6 + x^5 + x^2 + x)$$

$$= (01100011) \oplus (00101111) \oplus (01000101) \oplus$$

$$(11111101)$$

$$01100011$$

$$00101111$$

$$01000101$$

$$+ 11111101$$

$$11110000 = F4$$

$$= 01.EB \oplus 01.93 \oplus 02.C7 \oplus 03.20$$

$$= 1(11101011) \oplus 1(10010011) \oplus (x)(11000111) \oplus$$

$$(x+1)(00100000)$$

$$= x(x^7 + x^6 + x^2 + x^1 + 1) \oplus (x+1)(x^5)$$

$$= (x^8 + x^7 + x^3 + x^2 + x^1) \oplus (x^6 + x^5)$$

$$= (x^4 + x^3 + 1 + x^7 + x^6 + x^2 + x^1) \oplus (x^6 + x^5)$$

$$= (11101011) \oplus (10010011) \oplus (10010101) \oplus$$

$$(01100000)$$

$$11101011$$

$$10010011$$

$$10010101$$

$$+ 01100000$$

$$10001101 = 8D$$

$$= 01.9F \oplus 01.92 \oplus 02.AB \oplus 03.CB$$

$$= 01(10011111) \oplus 1(10010010) \oplus x(10101011) \oplus (x+1)$$

$$\begin{array}{r}
 1001111 \\
 0011111 \\
 1110010 \\
 + 1100101 \\
 \hline
 \end{array}$$

$$10001101 = 8D$$

$$= 01.A0 \oplus 02.C0 \oplus 03.30 \oplus 01.2B$$

$$= 01(10100000) \oplus (x)(11000000) \oplus (x+1)$$

$$(00110000) \oplus 01(00101011)$$

$$= x(x^7 + x^6) \oplus (x+1)(x^5 + x^4)$$

$$= (x^8 + x^7) \oplus (x^6 + x^5 + x^4 + x^3)$$

$$= (x^4 + x^3 + 1 + x^7) \oplus (x^6 + x^4)$$

$$= (10100000) \oplus (10011011) \oplus (01010000) \oplus (00101011)$$

$$10100000$$

$$10011011$$

$$01010000$$

$$+ 00101011$$

$$01000000 = 40$$

$$= 01.63 \oplus 01.2F \oplus 02.AF \oplus 03.A2$$

$$= 1(01100011) \oplus 1(00101111) \oplus (x)$$

$$(10100011) \oplus (x+1)(10100010)$$

$$= x(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1) + (x+1)$$

$$(x^7 + x^6 + x^5)$$

$$= (x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1) + (x^8 + x^7 + x^6 + x^5 + x^4)$$

$$\begin{aligned}
 &= (x^0 + x^4 + x^7 + x^8) \oplus (x^4 + x^7 + x^8 + x^9) \\
 &= (x^0 + x^4 + x^7 + x^8 + x^9) \oplus (x^4 + x^7 + x^8 + x^9) \\
 &= (1111011) \oplus (11000000) \oplus (00110000) \oplus \\
 &\quad (01010100)
 \end{aligned}$$

$$= 1111011$$

$$11000000$$

$$00110000$$

$$+ 01010100$$

$$01011100 = 5D$$

$$= 03 \cdot A0 \oplus 01 \cdot C0 \oplus 01 \cdot 30 \oplus 02 \cdot 2B$$

$$\begin{aligned}
 &= (x+1)(10100000) \oplus 1(11000000) \oplus \\
 &\quad 1(00110000) \oplus
 \end{aligned}$$

$$= \begin{bmatrix} BA & 84 & E8 & 1B \\ 75 & A4 & 8D & 40 \\ F4 & 8D & 06 & 7D \\ 7A & 32 & 0E & 5D \end{bmatrix}$$

Add Round-key 1

$$\begin{bmatrix} BA & 84 & E8 & 1B \\ 75 & A4 & 8D & 40 \\ F4 & 8D & 06 & 7D \\ 7A & 32 & 0E & 5D \end{bmatrix} \oplus \begin{bmatrix} E2 & 91 & B1 & D6 \\ 32 & 12 & 59 & 79 \\ FC & 91 & E4 & A2 \\ F1 & 88 & E6 & 93 \end{bmatrix}$$

$$= \begin{bmatrix} 58 & 159 & 39 & CD \\ 47 & B6 & D4 & 39 \\ 08 & 1C & E2 & DF \\ 8B & BA & E8 & CE \end{bmatrix}$$

Substitution Method

$$= \begin{bmatrix} 6A & 59 & CB & BD \\ A0 & 4E & 48 & 12 \\ 30 & 9C & 98 & 9E \\ 3D & F4 & 9B & 8B \end{bmatrix}$$

Shifting Rows

$$= \begin{bmatrix} 6A & 59 & CB & BD \\ 4E & 48 & 12 & A0 \\ 98 & 9E & 30 & 9C \\ 8B & 3D & F4 & 9B \end{bmatrix}$$

Mixed Columns:-

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} 6A & 59 & CB & BD \\ 4E & 48 & 12 & A0 \\ 98 & 9E & 30 & 9C \\ 8B & 3D & F4 & 9B \end{bmatrix}$$

$$= 01.6A \oplus 02.4E \oplus 03.98 \oplus 01.8B$$

$$= (01101010) \oplus x(01001110) \oplus (x+1)(10011000) \oplus (10001011)$$

$$= x(x^6 + x^3 + x^2 + x^1) \oplus (x+1)(x^7 + x^4 + x^3)$$

$$= (x^7 + x^4 + x^3 + x^2) \oplus (x^8 + x^5 + x^4 + 1 + x^5 + x^4 + x^7 + x^4 + x^3)$$

$$= (01101010) \oplus (10011100) \oplus (10110011) \oplus (10001011)$$

$$01101010$$

$$10011100$$

$$10110011$$

$$+ 10001011$$

$$11001110 = CE$$

$$= 01.59 \oplus 02.48 \oplus 03.9E \oplus 01.3D$$

$$= (01011001) \oplus (2x)(01001000) \oplus (x+1)(10011110) \oplus (00111101)$$

$$= x(x^6 + x^3) \oplus (x+1)(x^7 + x^4 + x^3 + x^2 + x^1)$$

$$= (x^7 + x^4) \oplus (x^8 + x^5 + x^4 + 1 + x^5 + x^4 + x^7 + x^4 + x^3 + x^2 + x^1)$$

$$= (01011001) \oplus (10010000) \oplus (10111001) \oplus (00111101)$$

$$01011001$$

$$10010000$$

$$10111001$$

$$+ 00111101$$

$$01001101 = 4D$$

$$= 01.6A \oplus 01.4E \oplus 02.98 \oplus 03.5B$$

$$= (01101010) \oplus (01001110) \oplus x(10011000) \oplus (x+1)(10001011)$$

$$= x(x^7 + x^6 + x^5) \oplus (x+1)(x^7 + x^6 + x^5 + 1)$$

$$= (x^7 + x^6 + x^5 + 1 + x^5 + x^6) \oplus (x^7 + x^6 + x^5 + 1 + x^7 + x^6 + x^5 + 1 + x^7 + x^6 + x^5 + 1)$$

$$= (01101010) \oplus (01001110) \oplus (00101001) \oplus (10000110)$$

$$01101010$$

$$01001110$$

$$00101001$$

$$+ 10000110$$

$$10001001 = 89$$

$$= 01.59 \oplus 01.48 \oplus 02.9E \oplus 03.3D$$

$$= (01011001) \oplus (01001000) \oplus x(10011110) \oplus (x+1)(00111101)$$

$$= x(x^7 + x^6 + x^5 + x^4 + x^3) \oplus (x+1)(x^5 + x^4 + x^3 + x^2 + 1)$$

$$= (x^7 + x^6 + x^5 + 1 + x^5 + x^4 + x^3 + x^2) \oplus (x^5 + x^4 + x^3 + x^2 + 1 + x^5 + x^4 + x^3 + x^2 + 1)$$

$$= (01011001) \oplus (01001000) \oplus (00100111) \oplus (01000111)$$

$$01011001$$

$$01001000$$

$$00100111$$

$$+ 01000111$$

$$01110001 = 71$$

$$= 01 \cdot CB \oplus 02 \cdot 12 \oplus 03 \cdot 30 \oplus 01 \cdot F4$$

$$= (11001011) \oplus (x)(00010010) \oplus (x+1)(00110000) \oplus (11110100)$$

$$= x(x^7+x^6) \oplus (x+1)(x^5+x^4)$$

$$= (x^5+x^4) \oplus (x^4+x^3+x^2+x)$$

$$= (11001011) \oplus (00100100) \oplus (01010000) \oplus (11110100)$$

$$11001011$$

$$00100100$$

$$01010000$$

$$+ 11110100$$

$$01001011 = 4B$$

$$= 01 \cdot BD \oplus 02 \cdot A0 \oplus 03 \cdot 9C \oplus 01 \cdot 9B$$

$$= (10111101) \oplus x(10100000) \oplus (x+1)(10011100) \oplus (10011011)$$

$$= x(x^7+x^6) \oplus (x+1)(x^7+x^6+x^5+x^4)$$

$$= (x^4+x^3+x^2+1+x^6) \oplus (x^7+x^6+x^5+1+x^5+x^4+x^3+x^2+x^7+x^6+x^5+x^4)$$

$$= (10111101) \oplus (01011011) \oplus (10111111) \oplus (10011011)$$

$$10111101$$

$$01011011$$

$$10111111$$

$$+ 10011011$$

$$11000010 = C2$$

$$= 02 \cdot CB \oplus 03 \cdot 12 \oplus 01 \cdot 30 \oplus 01 \cdot F4$$

$$= x(11001011) \oplus (x+1)(00010010) \oplus (00110000) \oplus (11110100)$$

$$= x(x^7 + x^6 + x^3 + x' + 1) \oplus (x+1)(x^4 + x')$$

$$= (x^8 + x^7 + x^4 + x^2 + x') \oplus (x^5 + x^2 + x^4 + x')$$

$$= (x^8 + x^3 + x^1 + 1 + x^7 + x^4 + x^2 + x') \oplus (x^5 + x^2 + x^4 + x')$$

$$= (10001101) \oplus (00110110) \oplus (00110000) \oplus (11110100)$$

$$10001101$$

$$00110110$$

$$00110000$$

$$+ 11110100$$

$$01111111 = 7F$$

$$= 02 \cdot BD \oplus 03 \cdot A0 \oplus 01 \cdot 9C \oplus 01 \cdot 9B$$

$$= x(10111101) \oplus (x+1)(10100000) \oplus (10011100) \oplus (10011011)$$

$$= x(x^7 + x^5 + x^4 + x^3 + x^2 + 1) \oplus (x+1)(x^7 + x^5)$$

$$= (x^8 + x^6 + x^4 + 1 + x^6 + x^5 + x^4 + x^3 + x^2) \oplus$$

$$(x^4 + x^3 + x' + 1 + x^6 + x^7 + x^5)$$

$$= (01100001) \oplus (11111011) \oplus (10011100) \oplus (10011011)$$

$$01100001$$

$$11111011$$

$$10011100$$

$$+ 10011011$$

$$10011101 = 9D$$

$$= 02.6A \oplus 03.4E \oplus 01.98 \oplus 01.8B$$

$$= x(01101010) \oplus (x+1)(01001110) \oplus (10011000) \oplus (10001011)$$

$$= x(x^6 + x^5 + x^3 + x^1) \oplus (x+1)(x^6 + x^5 + x^3 + x^1)$$

$$= (x^7 + x^6 + x^4 + x^2) \oplus (x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x^1)$$

$$= (11010100) \oplus (11010010) \oplus (10011000) \oplus (10001011)$$

$$11010100$$

$$11010010$$

$$10011000$$

$$+ 10001011$$

$$00010101 = 15$$

$$= 02.59 \oplus 03.48 \oplus 01.9E \oplus 01.3D$$

$$= x(01011001) \oplus (x+1)(01001000) \oplus (10011110) \oplus (00111101)$$

$$= x(x^6 + x^4 + x^3 + 1) \oplus (x+1)(x^6 + x^3)$$

$$= (x^7 + x^5 + x^4 + x^1) \oplus (x^7 + x^4 + x^3)$$

$$= (10110010) \oplus (11011000) \oplus (10011110) \oplus (00111101)$$

$$10110010$$

$$11011000$$

$$10011110$$

$$+ 00111101$$

$$11001001 = C9$$

$$BA = 10111010$$

$$E2 = \frac{11100010}{01011000} = 58$$

$$E8 = 11101000$$

$$B1 = \frac{10110001}{01011001} = 59$$

$$75 = 01110101$$

$$32 = \frac{00110010}{01000111} = 47$$

$$8D = 10001101$$

$$59 = \frac{01011001}{11010100} = 44$$

$$F4 = 11110100$$

$$FC = \frac{11111100}{00001000} = 08$$

$$06 = 00000110$$

$$E4 = \frac{11100100}{11100010} = E2$$

$$7A = 01111010$$

$$F1 = \frac{11110001}{10001011} = 8B$$

$$0E = 00001110$$

$$E6 = \frac{11100110}{11101000} = E8$$

$$84 = 100001000$$

$$91 = \frac{10000001}{00010101} = 15$$

$$1B = 00011011$$

$$D6 = \frac{11010110}{11001101} = CD$$

$$A4 = 10100100$$

$$12 = \frac{00010010}{10110110} = B6$$

$$40 = 01000000$$

$$79 = \frac{01111001}{00111001} = 39$$

$$8D = 10001101$$

$$91 = \frac{10010001}{00011100} = 1C$$

$$7D = 01111101$$

$$A2 = \frac{10100010}{11011111} = DF$$

$$32 = 00110010$$

$$88 = \frac{10001000}{10111010} = BA$$

$$5D = 10101110$$

$$93 = \frac{10010011}{11001110} = CE$$

$$= 03.64 + 01.4E \oplus 01.98 \oplus 02.8B$$

$$= (x+1)(01101010) \oplus (01001110) \oplus (10011000) \oplus (10001011)$$

$$= (x+1)(x^6 + x^5 + x^4 + x^3) \oplus x(x^7 + x^6 + x^5 + x^4 + 1)$$

$$= (x^7 + x^6 + x^5 + x^4 + x^6 + x^5 + x^4 + x^3) \oplus (x^8 + x^7 + x^6 + 1 + x^7 + x^6 + x^5)$$

$$= (1011110) \oplus (01001110) \oplus (10011000) \oplus (00001101)$$

$$1011110$$

$$01001110$$

$$10011000$$

$$+ 00001101$$

$$01100101 = 65$$

$$= 03.59 \oplus 01.48 \oplus 01.9E \oplus 02.3D$$

$$= (x+1)(01011001) \oplus (01001000) \oplus (10011110) \oplus$$

$$x(00111101)$$

$$= (x+1)(x^6 + x^5 + x^4 + 1) \oplus (x)(x^6 + x^5 + x^4 + x^3 + 1)$$

$$= (x^7 + x^6 + x^5 + x^4 + x^6 + x^5 + x^4 + x^3 + 1) \oplus (x^7 + x^6 + x^5 + x^4 + x^3)$$

$$= (11101011) \oplus (01001000) \oplus (10011110) \oplus (01111010)$$

$$11101011$$

$$01001000$$

$$10011110$$

$$+ 01111010$$

$$01000111 = 47$$

$$= 01 \cdot CB \oplus 01 \cdot 12 \oplus 02 \cdot 30 \oplus 03 \cdot F4$$

$$= (11001011) \oplus (00010010) \oplus (21)(00110000) \oplus$$

$$(21+1)(11110100)$$

$$= x(x^5 + x^7) \oplus (x+1)(x^7 + x^6 + x^5 + x^4 + x^2)$$

$$= (x^6 + x^5) \oplus (x^7 + x^6 + x^5 + 1 + x^4 + x^3 + x^2 + x + x^0 + x^{12})$$

$$= (11001011) \oplus (00010010) \oplus (01100000) \oplus (00000111)$$

$$11001011$$

$$00010010$$

$$01100000$$

$$+ 00000111$$

$$10111110 = BE$$

$$= 01 \cdot BD \oplus 01 \cdot A0 \oplus 02 \cdot 9C \oplus 03 \cdot 9B$$

$$= (10111101) \oplus (10100000) \oplus x(10011100) \oplus$$

$$(x+1)(10011011)$$

$$= x(x^7 + x^4 + x^3 + x^2) \oplus (x+1)(x^7 + x^6 + x^5 + x^4 + 1)$$

$$= (x^8 + x^6 + x^5 + 1 + x^5 + x^4 + x^3) \oplus (x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$$

$$= (10111101) \oplus (10100000) \oplus (00100011) \oplus (10110110)$$

$$10111101$$

$$10100000$$

$$00100011$$

$$+ 10110110$$

$$10001000 = 88$$