Principal of Emclusion & Exclusion

Formula

$|A \cup B| = |A| + |B| - |A \cap B|$

Examples:

In a class the no of students having Computer as a major subject is 25, the no of students having Maths as a major subject is 13 and the no of students having both as major subject is 8.
How many students are in classs

$|A \cup B| = |A| + |B| - |A \cap B|$
$= 25 + 13 - 8$
$= 30$

Relation & Their Properties

$R = \{x / x \quad x < 10\}$

Relation:

A binary relation from A to B is a subset of $A \times B$.

Example: Here is a set who have elements
$A = \{1, 2, 3\}$

$A \times A$ = cartesian prod = $\{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

Relation is denoted by R

$R_1 = \{((a,b)) / a \leq b\}$

$R_1 = \{(1,1)(1,2)(1,3)(2,2)(2,3)(3,3)\}$

$R_2 = \{ (a,b) \mid a<b \}$

$= (1,2), (1,3) (2,3)$

Properties

Example: Consider these relations on the set of integers

$R_1 = \{(a,b) \mid a \leq b\}$

$R_2 = \{(a,b) \mid a > b\}$

$R_3 = \{ (a,b) \mid a = b \text{ or } a = -b \}$

$R_4 = \{ (a,b) \mid a = b \}$

$R_5 = \{ (a,b) \mid a = b+1 \}$

Which of these relations contains each pair $(1,1)$, $(1,2)$, $(2,1)$, $(1,-1)$, $(2,2)$?

Solution

$R_1 = (1,1), (1,2) (2,2)$

$R_2 = (2,1), (1,-1)$

$R_3 = (1,1), (1,-1), (2,2)$

$R_4 = (1,1), (2,2)$

$R_5 = (2,1)$

So

Pair $(1,1)$ is in relation R_1, R_3, R_4

Pair $(1,2)$ " " " R_1

Pair $(2,1)$ " " " R_2, R_5

Pair $(1,-1)$ " " " R_2, R_3

Pair $(2,2)$ " " " R_1, R_3, R_4

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Properities

(i) Reflexive Relation :

~~For~~ all the elements present in a set, then the relation should have all the members of set in (a,a) form.

Example $A = \{1, 2, 3\}$
Then its relation should have,
$R = \{(1,1), (2,2), (3,3)\}$

(ii) Symmetric Relation :

A relation is a symmetric relation if $(b,a) \in R$ (relation) whenever $(a,b) \in R$, for all $(a,b) \in A$ (set).

Example : $A \{1, 2, 3\}$
Then Symmetric Relation would be as $R = \{(1,2), (2,1)\}$

* A relation on a set A, if $(a,b) \in R$ and $(b,a) \in R$, for all $(a,b) \in A$ then $a=b$ is called antisymmetric

(iii) Transitive Relation :

If we have two pairs (a,b) & (b,c) then we should have one other pair in ~~relation~~ set which have (a,c) pair in relation

Example = $\{(\overset{a}{1}, \overset{b}{2}), (\overset{b}{2}, \overset{c}{3}), (\overset{a}{1}, \overset{c}{3})\}$

④ Equivalance Relation

The relation which have all the three above relations i.e transitive symmetric & reflexive, then it will be equivalance relation

Example: $R = \{(1,1), (2,2), (1,2), (2,1), (1,3), (2,3)\}$

n-ary Relation:-

Relation having more than one set

Examples: $A_1, A_2, A_3, \ldots, A_n$ are sets then

for n-ary relation we have to find product of all of them.

If we have 3 sets suppose

$A = \{1,2\}$, $B = \{3,4\}$, $C = \{5,6\}$

Combining Relations.

Relations are combined using union & intersection procedures.

Example: $R_1 = \{(1,1)\ (1,2)\ (1,3)\}$

$R_2 = \{(2,1)\ (1,1)\ (2,3)\}$

$R_1 \cup R_2 = \{(1,1), (1,2), (1,3), (2,1) (2,3)\}$

$R_1 \cap R_2 = \{(1,1)\}$

$R_1 - R_2 = \{(1,2), (1,3)\}$

$R_2 - R_1 = \{(2,1), (2,3)\}$

These ~~processe~~ procedures are called combining or composition of positions of relations.

Representation of Relation

Relations can be represent by using matrices.

Q: Represent the relation $R = \{(2,1), (3,1), (3,2)\}$ by using matrix when $A = \{(1,2,3)\}$ $B = \{1,2\}$

Solution:

$$\begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \end{array}$$

Q: Represent the relation on $\{1,2,3\}$ with a matrix (a) $(1,1), (1,2), (1,3)$

Solution:

If there is only one set is given then the same set will become row & column.

$$\begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Q: Represent the order pair of following

$\{1,2,3\}$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}$$

Solution: $\{(1,1), (1,3), (2,2), (3,1), (3,3)\}$

Q:

$$M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(i) $M_{\bar{R}} = ?$

Sol:

$$M_{\bar{R}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(ii) $M_{R^2} = ?$

$$M_{R^2} = M_R \odot M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+1 & 0+1+0 & 0+0+1 \\ 0+1+0 & 1+1+0 & 1+0+0 \\ 0+0+1 & 1+0+0 & 1+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

Q: $M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(a) $M_{R_1} \cup M_{R_2} = ?$

$$M_{R_1} \cup M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

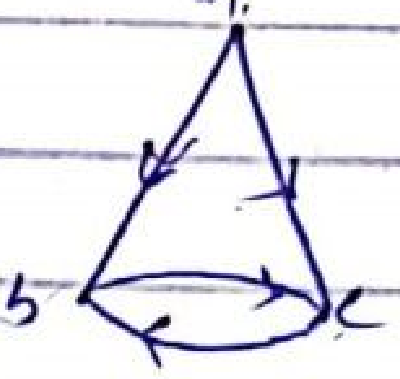
It is like 'OR' operation

(b) $M_{R_1} \cap M_{R_2}$

$$M_{R_1} \cap M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

It is like AND Operation

Q: List the order pairs in the relation by directed graph



Solution

$R = \{(a,b), (a,c), (b,c)(c,b)\}$

Partial Ordering

The relation in which reflexive relation, anti symmetric and transitive relations are present is called partial ordering

Example:

$S = \{0,1,2\}$

$R = \{(0,0), (1,1), (1,2), (2,2), (3,3)\}$

Solu:

This relation is partial ordering because it support all reflexive (i.e (0,0), (1,1)), antisymmetric (i.e (0,1), (1,1), (2,2), (3,3)) and Transitive relation (i.e (1,1), (1,2)).

$\underset{a\,b}{(1,1)}$ $\underset{b\,c}{(1,2)}$

so $\underset{a\,c}{(1,2)}$. which already exists