Elementry Linear Algebra

Chapter 1
System of Linier Equation and matrices

Section 1.1
Introduction to systems of Linear Equation:

Linear Algebra:

The study of matrices and related topics that forms the mathematical field is called Linear Algebra.

Linear Equation:

A general linear equation in the 'n' variables $x_1, x_2, \dots x_n$ can be expressed as

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = b$$

where $a_1, a_2, \dots a_n$ and b are constents and a's are not all zero. In the special cases when $n=2$ or $n=3$

$$a_1x + a_2y = b \quad (a_1, a_2 \text{ not both zero})$$

$$a_1x + a_2y + a_3z = b \quad (a_1, a_2, a_3 \text{ not all zero})$$

Homogeneous Linear Equation:

A case in which $b=0$

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = 0$$ called homogeneous linier equation in 'n' variable.

NOTE:

In Linear equation no term in product of variable and no variable is present other then one power and logrithmic and trignometric function.

Examples:

$x + 3y = 7$ $\quad x_1 - 2x_2 - 3x_3 + x_4 = 0$

$\frac{1}{2}x - y + 3z = -1$

are linear equations.

$x + 3y^2 = 4$ $\quad 3x + 2y - xy = 5$

$\sin x + y = 0$

are non linear equations.

System of linear equation:

A finit set of linear equation is called a system of linear equation or simply linear system. The variable are called unknowns. For example

$5x + 7y = 3$

$2x - y = 4$ is linear system has x, y unknowns.

$4x_1 - x_2 + 3x_3 = -1$

$3x_1 + x_2 + 9x_3 = -4$ is linear system has x_1, x_2, x_3 unknows.

A general linear system of m equation and n unknown can be writen as

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$
$$\vdots \qquad\qquad \vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots \; a_{mn}x_n = b_m$$

NOTE.

اگر دو یا دو سے زیادہ Linear equation ہوں تو اکٹھی آ جائیں تو

تو ان کا Linear system بن جاتا ہے۔

اور جتنے Variable ہوں گے Linear system میں اتنے ہی unknown ہوں گے

Solution of Linear system:

حل ہیں

A solution of liner system in 'n' unknown is a sequence of 'n' numbers for which make a true statement. For example

$$x - y = 1$$
$$x + y = 0$$
$$x = \frac{1}{2}, \quad y = -\frac{1}{2} \quad \text{make order pair}$$
$$\left(\frac{1}{2}, -\frac{1}{2}\right) \quad \text{which give true statement}$$

NOTE: (x, y)

اگر دو Variable ہوں تو Answer

Answer ordered pair میں آئے گا۔ اگر تین Variable ہوں گے تو Answer order triple میں آئے گا اور اگر n-Variable ہوں گے تو order n tuple میں آئے گا

Consistent System:

A linear system is called consistent if there exist at least one solution.

Inconsistent System:

A linear system is called inconsistent if there is no solution.

Linear system in two and three Variable:

Linear system with two variable or three variable have three possibilities.

i) no solution ii) one solution iii) Infinit sol.

i) The lines may intersect at only one point in this case have one solution.

ii) if lines are parallel then no solution

iii) if one line coinside to the other line then infinit many solution.