Chapter # "1"

"Logic & Proofs"

Define Proposition?

A proposition is a declarative sentence or statement which is either true or false.

e.g:

1⇒ $1+1=2$ true ✓

2⇒ $9<6=6$ False ✗

3⇒ where are you going? (Not a proposition)

4⇒ sit down (Not a proposition)

Define truth value?

True & False is called the truth value.

Compound Proposition?

when one or more proposition are connected Through various connectivities is called compound ~~con~~ proposition.

e.g:

"$\underbrace{\text{Roses are red}}_{\text{Proposition 1}}$ & $\underbrace{\text{violets are blue}}_{\text{Proposition 2}}$."

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Define primitive proposition?

A proposition is said to be primitive, if it cannot be broken down into simpler proposition.

e.g:

Roses are red. (Unbreakable)

what are basic logic operators?

3 types

1. Conjunction ⇒ AND ⇒ $\wedge$
2. Disjunction ⇒ OR ⇒ $\vee$
3. Negation ⇒ NOT ⇒ $\sim$

Define truth table?

A mathematical table used in logic to get all possible value/combinations of input for the output.

" Conjunction truth table.

$P \wedge q$

P	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

∴ both value should be true

Disjunction truth table

$P \vee q$

P	q	$P \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

∴ One value should be

" Negation truth Table

p	~p
T	F
F	T

what is conjunction?

Any two proposition can be combined by the word "and" to form a compound proposition called conjuction.

what is disjunction?

Any two proposition can be combined by the word "Or" to form a compound proposition called disjunction.

Define negation?

A single proposition which value can be changed by using negation.

what is exclusive OR proposition? (XOR)

let p & q be two propositions. The exclusive OR of p & q (denoted by $p \oplus q$) is a proposition that simply means exactly one of p & q will be true but both cannot be true.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

what is conditional statement proposition?

A statement that can be written in the form " If P Then Q " where p & q are two senteces. P is called hypothesis & Q is called conclusion.

if " P Then Q means " Q must be true whenever P is true.

e.g: $P \rightarrow q$

P	q	$P \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

A conditional statement is also called implication.

what is bi-conditional statement?

A bi-conditional statement is defined to be true whenever both parts have the same truth value. Denoted by " $\longleftrightarrow$ "

P	q	$P \longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Define Precedence? write the precedence of logical operators?

An ordering of logical operators designed to allow the dropping of paratheses in logical expression.

Operator	Precedence
$\sim$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Define bit strings?

A sequence of bits is called bit string. A bit string used to represent sets or to manipulate binary data. The elements of bit string are numbered from zero up to the no. of bits in the string less one, in right to left order.

e.g:

01001100

"Truth table"

T	1
F	0

"Exercise 1.1"

Q1:

Sentence	Proposition	truth value
b. Miami is the capital of Florida	Yes	False
d. $5+7=10$	Yes	False
7. Answer This Question	NO	—

Q3:

a. Mei does not have an MP3 player

c. $2+1 \neq 3$

Q6:

a. True

c. False

e. True

Q8:

a. I didnot bought a lottery ticket This wee

c. If i bought a lottery ticket then win million jackpot

e. I bought a lottery ticket & won million dollar jackpo

g. I didnot bought a lottery ticket & not won million dollar jackpot.

Q11:

b. $P \wedge -q$

d. $P \vee q$

7. $(P \vee q) \wedge (P \rightarrow -q)$

Q12:

a. If you have flu then you miss the final examination
miss final examination
c. if you ~~have flu~~ then you can not pass the course
you miss the final exam, you can't pass the course.
e. (If you have flu then you can't pass the course & if
f. If you have flu & miss the final exam Or either you
cannot miss the final exam & pass the course.

Q16:

b. False

d. False

Q17:

a. False

c. True

Q19: Exclusive, inclusive or intened.

a. ~~Inclusive~~ exclusive or

b. ~~inclusive~~ ~~Exclusive~~ inclusive or

c. intened

d. inclusive

Q23: statement written if p then q

a. If the wind blows from northeast the its snow.

b. If it is warm week then apple trees will blomed.

c. If the lakers were beaten then pistons win championship.

d. If to get the top of long peak then it necessary to walk 8 miles

e. If you are world famous then you get tenure as professor.

f If you want to drive more than 400 miles then you would buy gasoline.

g. If you bought CD player less than 90 days then its guarantee is good.

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Q25:

a. you can buy an icecream if & only if it is hot outside.

b. you will win the contest if & only if you hold the winning ticket.

c. you will get permotion if & only if you have connections.

d. your mind will decay, & conversely if & only if you watch television.

e. The train run late only if you take it.

$(P \to Q)$ $(Q \to P)$ $(Q \to \neg P)$ $(\neg P \to \neg$

Q27: State the converse, contrapositive & inve

a: Converse: I will ski tomorrow only if it snows today.

Contrapositive: I will not ski tomorrow if it is not snow today.

Converse: If it is not snow today then i will not ski tomorrow.

Q29: Row in truth table

a. 2

b. 16

c. 64

d. 16

Q31: Construcling truth table?

a:

P	$\neg P$	$P \wedge \neg P$
T	F	F
F	T	F

c: $(P \vee \neg q) \to q$

P	q	$\neg q$	$(P \vee \neg q)$	$(P \vee \neg q) \to q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

e: $(P \to q) \leftrightarrow (\neg q \to \neg P)$

P	q	$\neg P$	$\neg q$	$P \to q$	$\neg q \to \neg P$	$(P \to q) \leftrightarrow (\neg q \to \neg P)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Q33: Construct truth table

a: $(P \vee q) \to (P \oplus q)$

P	q	$P \vee q$	$P \oplus q$	$(P \vee q) \to (P \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

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7: $(p \oplus q) \to (p \oplus \neg q)$

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \to (p \oplus$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

Q43: Bit Strings:

a: 101 1110, 010 0001

101 1110
010 0001

111 1111 OR

000 0000 AND

111 1111 XOR

Q44: Evaluate each expression.

0 1 0 1 0
1 1 0 1 1 $\oplus$

1 0 0 0 1
0 1 0 0 0 $\oplus$

0 0 0 0 1

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Application Of Propositional Logic

Define Boolean Searches?

logical connectives are used extensively in searches of large collections of information such as indexes of web pages. Because these searches employ techniques from propositional logic, they are called boolean searches.

In boolean searches, connective AND, connective OR connective NOT or sometime AND NOT is used.

e.g: Mexico & italy
Mexico or italy
Mexico not italy.

Define logic puzzles?

The puzzles that can be solved using logical reasoning are known as logic puzzle.

e.g: Suodoku puzzle
Minesweaper etc.

Define logic circuits?

An electronic circuit having one or more inputs & only one output. The relationship b/w the input & output based on a certain logic.

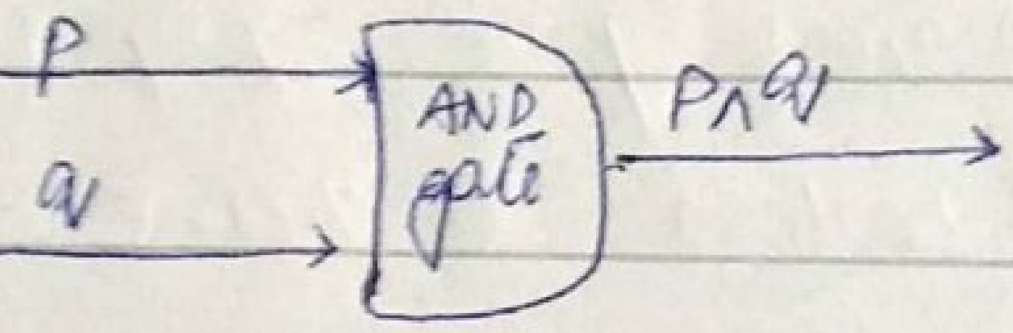
Describe basic logic gates?

There are 3 basic logic gates.

- AND gate
- OR gate
- NOT gate or inverter

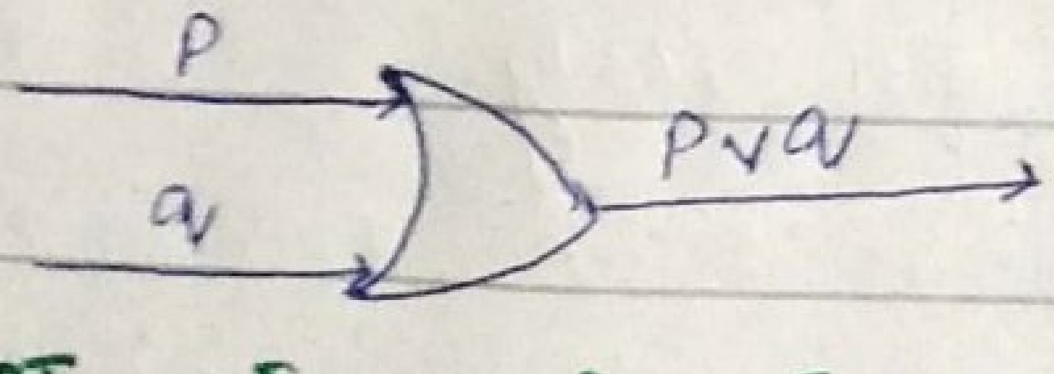
AND gate:

A gate that gives the value one if and only if all operands are one.



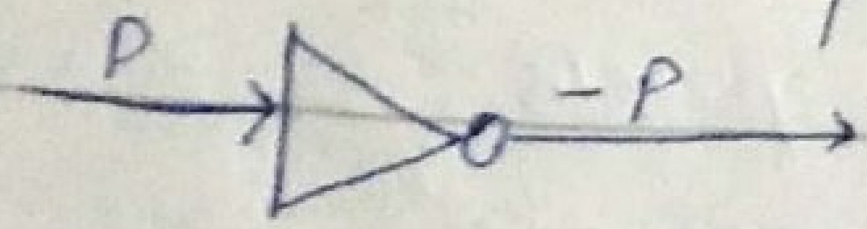
OR gate:

A gates that give the value one if any of one operand is one.



NOT gate or inverter:

The inverse value of a operand. (single)



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: PROPOSITIONAL EQUIVALANCE:

what is propositional equivalance?

The replacement of a statement with another statement with the same truth value is called propositional equivalance.

Define tautology?

A compound statement that is always true, no matter what the value of a propositional variables, is called tautology.

P	$-P$	$P \vee -P$
T	F	T
F	T	T

Tautology

Define Contradiction?

A compound statement that is always False is called contradiction.

P	$-P$	$P \wedge -P$
T	F	F
F	T	F

Contradiction

Define contingency?

A compound statement that is neither tautology nor a contradiction is called contingency.

Define logical Equivalance?

Compound proposition that have the same truth value in all possible cases are called logical equivalance.

e.g:

The compound proposition p & q are called logically equivalent if $p \leftrightarrow q$ is a tautology.

Denoted by $p \equiv q$ or $p \Leftrightarrow q$

De-morgan Law (Logic)?

$\neg(p \lor q) \equiv -p \land -q$

$\neg(p \land q) \equiv \neg p \lor \neg q$

Logical Equivalance for conditional statement

$p \to q \equiv \neg p \lor q$

$p \to q \equiv -q \to -p$

$p \lor q \equiv -p \to q$

$p \land q \equiv \neg(+p \to -q)$

$\neg(p \to q) \equiv p \land -q$

$(p \to q) \land (p \to r) \equiv p \to (q$

$(p \to r) \land (q \to r) \equiv (p \lor q) \to$

$(p \to q) \lor (p \to r) \equiv p \to (q$

$(p \to r) \lor (q \to r) \equiv (p \land q) \to$

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logical Equivilance bi-conditional statements

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$p \leftrightarrow q \equiv \neg p \leftrightarrow q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Sk Sudoku Puzzle:

A puzzle represented by 9×9 grid made up of nine 3×3 subgrids, known as blocks.

" Exercise 1.3 "

Q1:

P	$P \wedge T = F$	$P \wedge F = F$	$P \vee P = P$
T	T = T	F = F	$T \vee T = T = P$
F	T = F	F = F	$F \vee F = F = P$

Q2: Show $\neg(\neg p)$ & p are logically equivalent.

$\neg(\neg p) \equiv p$

$\neg(\neg p) \equiv p$ by double negation law

Q3: Use truth table to verify commutative law.

Commutative law $p \vee q \equiv q \vee p$ or $p \wedge q \equiv q \wedge p$

p	q	$p \vee q$ =	$q \vee p$	$p \wedge q$ =	$q \wedge p$
T	T	T	T	T	T
T	F	T	T	F	F
F	T	T	T	F	F
F	F	F	F	F	F

Q4: Associative law

$(P \vee q) \vee r \equiv P \vee (q \vee r)$

$(P \wedge q) \wedge r \equiv P \wedge (q \wedge r)$

P	q	r	$(P \vee q) \vee r$	$\equiv$	$P \vee (q \vee r)$	$(P \wedge q) \wedge r$	$\equiv$	$P \wedge (q \wedge r)$
T	T	T	$(T \vee T) = T$	$\equiv$	T	T	$\equiv$	T
T	T	F	$(T \vee F) = T$	$\equiv$	T	F	$\equiv$	F
T	F	T	$(T \vee T) = T$	$\equiv$	T	F	$\equiv$	F
T	F	F	$(T \vee F) = T$	$\equiv$	T	F	$\equiv$	F
F	T	T	$(T \vee T) = T$	$\equiv$	T	F	$\equiv$	F
F	T	F	$(T \vee F) = T$	$\equiv$	T	F	$\equiv$	F
F	F	T	$(F \vee T) = T$	$\equiv$	T	F	$\equiv$	F
F	F	F	$(F \vee F) = F$	$\equiv$	F	F	$\equiv$	F

Q5: Distributive law:

$P \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge r)$

P	q	r	$P \wedge (q \vee r)$	$\equiv$	$(P \wedge q) \vee (P \wedge r)$
T	T	T	T	$\equiv$	T
T	T	F	T	$\equiv$	T
T	F	T	T	$\equiv$	T
T	F	F	F	$\equiv$	F
F	T	T	F	$\equiv$	F
F	T	F	F	$\equiv$	F
F	F	T	F	$\equiv$	F
F	F	F	F	$\equiv$	F

Q6: De morgan law:

$$\neg(P \vee q) \equiv \neg P \wedge \neg q$$

P	q	$-(P \vee q)$	$\equiv$	$-P \wedge -q$
T	T	F	$\equiv$	F
T	F	F	$\equiv$	F
F	T	F	$\equiv$	F
F	F	T	$\equiv$	T

Q7: Use de morgan law find negation of Statements:

a) $\underbrace{\text{Jan is rich}}_{P}$ $\underbrace{\&}_{\wedge}$ $\underbrace{\text{happy}}_{q}$

$$\neg(P \wedge q) = -P \vee -q$$

Ans: Jan is not rich or not happy

Q9: Show tautology using truth table

a) $(P \wedge q) \rightarrow P$

P	q	$P \wedge q$	$(P \wedge q) \rightarrow P$
T	T	T	T
T	F	F	T
F	T	F	$\cancel{F}$ T
F	F	F	$\cancel{F}$ T

c) $\neg P \rightarrow (P \rightarrow q)$

P	q	$\neg P$	$(P \rightarrow q)$	$\neg P \rightarrow (P \rightarrow q)$
T	T	F	$\cancel{F}$ T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

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c) $\neg(P \to q) \to P$

P	q	$(P \to q)$	$-(P \to q)$	$-(P \to q) \to P$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

Q11: Without using truth table

a) $(P \wedge q) \to P$

$-(P \wedge q) \vee P$

$\neg(F) \vee P$

$T \vee P$

~~P~~ T Ans.

b) $\neg P \to (P \to q)$

$\neg P \to (-P \vee q)$

$-(-P) \vee (-P \vee q)$

$P \vee (T)$

T Ans.

c) $\neg(P \to q) \to P$

$-(\neg(P \to q)) \vee P)$

$(-P \vee q) \vee P$

$T \vee P$

T

Q13: Absorption law

$P \vee (P \wedge q) \equiv P \qquad P \wedge (P \vee q) \equiv P$

P	q	$P \vee (P \wedge q) \equiv P$	$P \wedge (P \vee q) \equiv P$
T	T	T = P	T = P
T	F	T = P	T = P
F	T	F = P	F = P
F	F	F = P	F = P

Q14: Show whether tautology

$(-P \wedge (P \rightarrow q)) \rightarrow -q$

$(-P \wedge (-p \vee q)) \rightarrow -q$

$(-P \wedge T) \rightarrow -q$

$(F) \rightarrow -q$

$-(F) \vee q$

$-F \vee -(T)$

$T \vee F$

T

Q16: Show that $P \leftrightarrow q$ & $(P \wedge q) \vee (\neg P \wedge -q)$

are logically equivalent

$P \leftrightarrow q \equiv (P \rightarrow q) \wedge (q \rightarrow P)$

$\equiv (-p \vee q) \wedge (-q \vee p)$ or

$(-q \rightarrow P) \wedge (-p \rightarrow q)$

$= (-(-q) \vee p) \wedge (-(p) \vee q))$

$= (q \vee p) \wedge (p \vee q)$

$= -((q \vee p) \rightarrow -(p \vee q)$

$= -((q \vee p) \vee (-(p \vee q))$

$= (q \wedge p) \wedge (p \wedge -q)$

Logically equivalent

Q16: $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

p	q	$\neg p$	$\neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Q18: $p \rightarrow q$ $\quad \neg q \rightarrow \neg p$

$p \rightarrow q$	$\neg q \rightarrow \neg p$
T	T
F	F
T	T
T	T

Q20: $\neg(p \oplus q)$ $\quad p \leftrightarrow q$

$\neg(p \oplus q)$	$p \leftrightarrow q$
F = T	T
T = F	F
T = F	F
F = T	T

Q22 $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

p	q	r	$(p \rightarrow q) \wedge (p \rightarrow r)$	$p \rightarrow (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

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Q29: Show Tautology

$$(p \to q) \wedge (q \to r) \to (p \to r)$$

p	q	r	$p \to q$	$q \to r$	$p \to r$	All.
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

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"Predicates And Quantifiers"

Define predicate?

A predicate is a sentence that contains a finite number of variables & becomes a statement when specific values are substituted for the variables.

For example:

$x > 3$, $x = y + 3$, $x + y = 2$

Define Quantifiers?

Quantifiers are words that refer to quantities such as 'some' or 'all' & tell for how many elements a given predicate is true.

"$\forall$" for all

'$\exists$' atleast one

What is n-place predicate?

A statement to form $P(x_1, x_2, x_3, \ldots, x_n)$ is the value of propositional function P at the n-tuple $(x_1, x_2, x_3, \ldots x_n)$ & P is also called n-place predicate or an ary predicate

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What is Quantification?

To create proposition from propositional function is called Quantification.

$$\underbrace{x + y = 17}$$

∴ propositional function

either they are equal or may not

There two type of quantification

- Universal Quantification
- Existetial Quantification.

Define universal Quantification?

Universal Quantification states that the statements within its scope is true for every value of the specific variable.

denoted by "$\forall$" symbol.

e.g :

$\forall x \, P(x)$

means

For all value of x in the domain of $P(x)$ is true.

e.g : $P(x) = x + 1 > x$

it is true for every real Number

but if $P(x) = x + 1 < x$

it becomes false every real number.

Define Existential Quantifier?

It state that the statement within it se are true for some values of the specif variable.

denoted by "$\exists$" symbol.

e.g $\exists x P(x)$

for specific values of x domain P(x) true.

e.g: if $P(x) = \underset{\text{positive}}{x+1 \geq 7}$

true for all real number greater than and equal to 6

What is the difference b/w pre-conditio & post condition?

The pre-condition statement indicates wha must be true before the function is called

$x = 3;$

$y = 4;$

$x = y - 1$ (statement)

The post condition indicates that what will be true when the function finishes its work

$3 = 4 - 1$

$3 = 3$

true.

Describe counter-example?

A special kind of example that disaproves a statement or proposition.

e.g: P(x) = "every x is a prime number" for every integer x

it becomes false at many points because every integer is not a prime number.

Define uniqueness Quantifier?

The property of being the one & only one in a proposition is called uniqueness quantifier denoted by "$\exists!$" symbol.

e.g: $\exists! x\ P(x)$

$P(x) = "x - 1 = 0"$

only true for 1

What is meant by Quantifier with restricted domain?

An abbreviated notation used to restrict the domain of a quantifier is know as Quantifier restricted domain.

e.g:

$\forall x < 0\ (x^2 > 0)$

stricted to all negative real numbers.

What is meant by Precedence of Quantifi

The quantifien "∀" & "∃" have higher precedenc then logical operaton.

e.g:

$$\forall x\, P(x) \vee Q(x)$$

means $((\forall x\, P(x)) \vee Q(x))$

What is binding variables?

A bound variable is a variable that w previously free, but has been bound to a specific value or set of values.

e.g: x become bound variable when we write;

$$\forall x, (x+1)^2 = x^2 + 2x + 1$$

How logical equivalance involve Quantifier

Statements involving predicates & quantifiers are logically equivalent if they have the same truth value for all applications & for all domains of discourse.

e.g: $\forall x(P(x) \wedge Q(x)) = \forall x(P(x)) \wedge \forall x(Q(x))$

What is negating Quantified Expression?

Reverse of a Quantified expression is known as negating quantified expression.

e.g:

$\forall x(Px)$

Negating: $\neg \forall x(Px)$

Translation of expression from English to logical Equivalance & from logical Equivalence to english.

S: "Every student in the class has studied Calculus"

include variable, "x"

S(x) "For every student x in the class studied Calculus."

$\forall x S(x)$. translated

$\exists x S(x)$

From a class or In a class there is atleast one student x who studied Calculus. (translated).

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Define lewiss Carroll Example?

let consider three proposition expression

1. $A(x)$ = All lions are fierce.
2. $B(x)$ = Some lions do not drink coffe
3. $C(x)$ = Some fierce creature do not coffee.

$A(x)$ & $B(x)$ is called premises & $C(x)$ is called conculsion. The enitre set is called argument. x is lions, x is fierce, x i

We can expres these statements as.

1. $\forall x(A(x) \longrightarrow B(x))$
2. $\exists x(\cancel{B}) A(x) \wedge \neg B(x))$
3. $\exists x(B(x) \wedge \neg C(x))$

2^{nd} Example.

$P(x)$ 1. All humming birds are richly colored

$Q(x)$ 2. No large birds live on honey

$R(x)$ 3. Birds that donot live on honey are dull in color.

$S(x)$ 4. Humming birds are small.

"x is humming birds", "x is richly colored"

"x is live on honey" "x is dull in color"

"~~x is birds small~~" "~~x is large~~"

1. $\forall x (P(x) \longrightarrow S(x))$
2. $\neg \exists x (Q(x) \wedge R(x))$
3. $\forall x (\neg (Rx) \longrightarrow \neg S(x))$
4. $\forall x (B(x) \wedge \neg Q(x))$

Exercise 1.4

Q1: $P(x) =$ "$x \leq 4$" Find truth value

a: $P(0)$ = true $\because 0 \leq 4$ $0 < 4$ ✓ or $0 = 4$ ✗

c: $P(6)$ = False $\because 6 \leq 4$ $6 < 4$ ✗ or $6 = 4$ ✗

Q2: Statement "the word x contains the letter a"

Find truth value?

b: P(lemon) = False

d: P(false) = True

Q4: Statement "if $P(x)$ then $x := 1$ is executed"

$P(x) = x > 1$, find where it executed.

a: $x = 0$ Not executed

b: $x = 1$ Not executed

c: $x = 2$ Executed.

Q5: Statement "x has spends more than five hours every weekday in class" x consist of all students. Find Quantification in English.

b: $\forall x \, P(x)$

For all value of x has spends more than five hours every weekday.

d: $\exists x \, \neg P(x)$

There is atleast one or some values for x that has not spend more then 5 hours in a class.

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Q7: Translate statement in English

$C(x)$ = "x is a comedian"

$F(x)$ = "x is Funny"

x is all people

a: $\forall x(C(x) \rightarrow F(x))$

For all people or every comedian is funny

c: $\exists x(C(x) \rightarrow F(x))$

one or some comedian are funny

Q9: Translate into Quantification. logically

$P(x)$ = " x can speak Russian "

$Q(x)$ = " x knows the computer lang C++ "

a: There is a student....

$\exists x(P(x) \wedge Q(x))$

c: Every student at.....

$\forall x(P(x) \vee (Q(x))$

Q12: Statement " $x+1>2x$ "

Find truth values.

b: $Q(-1)$ = true $\therefore -1+1>2(-1) \therefore 0>-2$

d: $\exists x Q(x)$ = true $\therefore 0+1>2(0) = 1>0$

f: $\exists x -Q(x)$ = true $\therefore 2+1>2(2) = 3 \ngtr 4$

g: $\forall x -Q(x)$ = False $\therefore -1+1 \ngtr 2(-1)$ $0 \ngtr -2$

Q13: Determine the truth value. domain is all integers

a: $\forall n(n+1>n)$ true

c: $\exists n(n=-n)$ true $\because$ $0=-(0)$ ✓

Q16: Determine the truth value, domain is all real Numbers:

b: $\exists x(x^2=-1)$ False $\because$ square is always +ive

d: $\forall x(x^2 \neq x)$ False $\because$ $(0)^2=0$ ✓

Q17 & 18 & 19 & 20:

17: a: $\exists x\, P(x)$ $P(0) \vee P(1) \vee (P(2) \vee P(3) \vee P(4)$

c: $\exists x \neg P(x)$ $\neg P(0) \vee \neg(P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$

f: $\neg \forall x\, P(x)$ $\neg(P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4))$

18: b: $\forall x\, P(x)$ $(P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2))$

d: $\forall x \neg P(x)$ $(\neg P(-2) \wedge \neg P(-1) \wedge \neg P(0) \wedge \neg P(1) \wedge \neg P(2))$

f: $\neg \forall x\, P(x)$ $\neg(P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2)$

19: a: $\exists x(Px)$ $P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5)$

c: $\neg \exists x\, P(x)$ $\neg(P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5))$

e: $\forall x((x \neq 3) \rightarrow P(x)) \vee \exists x \rightarrow P(x)$

$(P(1) \wedge P(2) \wedge P(4) \wedge P(5))) \vee (P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5))$

20: b: $\forall x\, P(x)$ $(P(-5) \wedge P(-3) \wedge P(-1) \wedge P(1) \wedge P(3) \wedge P(5))$

d: $\exists x((x \geq 0) \wedge P(x))$

$((1 \geq 0) \wedge P(1)) \vee ((3 \geq 0) \wedge P(3)) \vee ((5 \geq 0) \wedge P(5))$

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Q ~~23~~ 35: Counter example

a: $\forall x(x^2 \geq x)$ ∴ counter example for every negati integer

b: $\forall x(x > 0 \vee x < 0)$ ∴ counter example for 0

c: $\forall x(x = 1)$ ∴ for every except "1" integer.

Q36: Counter example

a: $\forall x(x^2 \neq x)$ ∴ for ~~every under root ve~~ 0

b: $\forall x(x^2 \neq 2)$ ∴ for $\sqrt{2}$

c: $\forall x(|x| > 0)$ ∴

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"Nested Quantifier"

Nested Quantifier:

The quantifier that occur within the scope of other quantifiers is called nested Quantifier.

For example:

$$\forall x \exists y \, P(x,y)$$

"Exercise 1.5"

Q1: Translate statements into English

a: $\forall x \exists y (x<y)$

For all real number x there is a real number y where x is less than y.

c: $\forall x \forall y \exists z (xy = z)$

For all real no. x & for all real no y there exist a real number "z" where product of x & y is equal to z.

Q2:

b: $\forall x \forall y ((x \geq 0) \wedge (x<0)) \longrightarrow (x-y>0))$

For all real no: of x & y if x greater than & or equal to zero & x is less than 0 then the subtraction of y from x is greater then 0 or zero.

Q3 & Q4: translation

a: There is some students in your class that has sent message to some student in your class.

c: All students in your class has sent email to some student in your class.

e: Some students in your class has sent email message to all students in your class

b: Some students of your class has attened all computer courses.

d: Some computer course class have been attened by all students of your school.

f: All students of your school have taken all computer science courses.

Q5 & Q6:

a: Sarah Smith has visited website "www.att.com".

c: Some websites has been visited by Jose Orez.

e: Some website has been visited by ~~all students~~ a person beside a david oacher visited.

b: There is some students x that have been given math class at your school.

d: There is a some students that been give math 222 & CS 212 classes at your school.

f: There is a student beside from your school who has also ~~take~~ classes ~~from~~ being given classes at your school.

Q8: translate Eng to Quantifiers.

b: $\neg\forall x\, Q(x,y)$

d: $\forall y \exists x\, Q(x,y)$

Q9:

a: $\forall x\, L(x, \text{Jerry})$

c: $\exists y \forall x\, L(x,y)$

e: $\exists y \neg\forall x\, L(\text{lydia}, y)$

f: $\neg\forall x\, L(x,y)$

h: $\exists y\, L(\text{lynn}, y)$

j: $\exists z \forall x \forall y\, ((x \neq y) \wedge (L(x$ ... $\rightarrow (y, z))$

Q21:

(A) Every positiver integer is the sum of

x

square of four integers.

y

~~$\forall x \exists a \exists b \exists c \exists d (x > (x+y)^2)$~~

$\forall x \exists a \exists b \exists c \exists d ((x>0) \to x = a^2+b^2+c^2+d^2)$

Q22:

There is a positive integer that is the sum of three squares.

$\forall$ $\exists x \exists a \exists b \exists c ((x>0) \to x \neq a^2+b^2+c^2)$

Q23:

$\forall x \forall y R((x<0) \wedge (y<0) \to (xy>0))$

a: ~~$\forall x \forall y ((x<0) \to x$~~ ~~$\forall x \exists a \exists b \forall y ((x<0) \to ab = y$~~$)$

e: ~~$\forall x \forall y ((x>0) \wedge (y>0)$~~ $\Leftrightarrow \forall x \exists a \exists b (a \neq b) \wedge \forall c (c^2 = x \Leftrightarrow (c = a \vee c = b)))$

Q24:

b: There is a number x for all nuber of y that the product of both x & y is equal to y.

d: There is a number x & y whose is not equal to zero & the product of the no. is also not zero.

Q26: "x+y = x−y", truth values

b: $Q(2,0)$ T ✓ e: $\exists x \exists y Q(x,y)$ T ✓ h: $\forall y \exists x Q(x,y)$ T ✓

d: $\exists x Q(x,2)$ F ✗ g: $\exists y \forall x Q(x,y)$ T ✓

Q45 & 46: Determine truth values

a: True

c: true

b: False

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"PROOFS"

Define Proof?

A proof is a sequence of logical deductions based on accepted assumptions & previously proven statements.

What is meant by Formal proof?

A finite sequence of sentences, each of which is an axiom, an assumptions, or follows from the preceeding sentences in ~~which~~ the sequence by a rule of inference.

e.g $(a+b)^2 = a^2+b^2+2ab$

Describe informal proof?

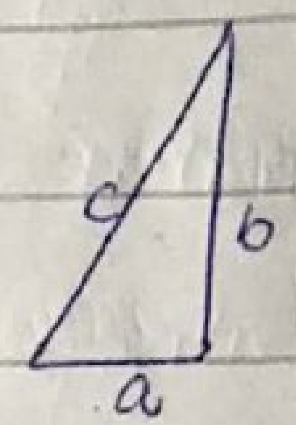
A proof where more than one rule of inference may be used in each step, where steps may be skipped, where the axioms being assumed & rule of inference used are not explicity stated.

e.g: simple english logical statements.

What is Theorem?

A non-self evident statement that has be proven to be true on the basis of generally accepted statements.

For example:



"Pythagorass Theorem."

What is lemma?

lemma is generally minor, proven proposition which is used as a stepping stone to a larger result.

Also known as helping Theorem.

Forexample:

$\triangle$ a, b & c have equal length means also have equal angles.

What is corollarry & Conjecture?

A corollary is a Theorem that follows on from another Theorem.

A conjecture is a conclusion or a proposition which is suspected to be true due to preliminary supporting evidence.

What is meant by direct proof?

"A way of showing the truth & falsehood of a given statement by a straight forward combination of established facts, usually axioms, existing lemmas & theorems, without making any further assumptions.

Direct proof means,

$P \rightarrow Q$

The integer n is even if there exists an integer k such that $n = 2k$, & n is odd if there exists an integer k such that $n = 2k+1$

e.g: Prove that if a is odd & b is even, then $a+b$ is odd

$a = 2k+1$

$b = 2k$ $\quad\quad \therefore k$ is integer (Z^+)

$a+b = 2k+1+2k$

$= \underline{4k}+1$

$= a+b$ $\quad$ Proved

$4k$ $\therefore$ Answer will be also an even integer by adding 1 it becoms odd.

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Explain Indirect proof?

A method of proof that do not start with premises & end with conclusion are called indirect proof.

Mainly two type

- Contradiction $p \rightarrow q$
- Contrapositive $\neg q \rightarrow \neg p$

Contradiction:

1. Assume p is true & q is false.
2. Show that $\neg p$ is also true.
3. Then we have $p \wedge (\neg p)$ is true.
4. But it is impossible bcz $p \wedge (\neg p)$ always false. It's a contradiction
5. So q cannot be false, therefore it is true.

⇒ if a is odd no. & b is even then a+b is odd.

$P(a, b)$ $Q(a+b)$

1. $a = 2k+1$ (true)

   $b = 2k$ (true) Assume

   $a+b = \text{odd}$ (False) Assume

2. $a = -(2k+1) = -2k-1$ (true)

   $b = -(2k) = -2k$ (true)

3. $P \wedge -(p)$ is true

   $\underset{T}{2k+1} \wedge \underset{F}{-2k \mp 1}$

4. impossible
5. q is true $a+b = \text{odd}$

Contrapositive $(\neg q) \rightarrow (\neg p)$

if $\underbrace{3n+2 \text{ is odd}}_{p}$ Then $\underbrace{n \text{ is odd}}_{q}$

to prove $(\neg q) \rightarrow (\neg p)$

n is even, Then $n = 2k$

$3(2k)+2$ $\quad\therefore k$ is a $\mathbb{Z}$

$6k+2$

$2(\underline{3k+1})$ become even its mean $(\neg q)$ proved

Thus, $3n+2$ is even, $\neg q \rightarrow \neg p \equiv p \rightarrow q$.

Describe vacuous proof?

A proof of statement of the form $p \rightarrow q$ which prove p ~~without using p~~ is false.

e.g: Prove that if $\underbrace{1+1=1}_{p}$, Then "$\underbrace{\text{I am true}}_{q}$"

" The premises is false $1+1 \neq 1$, Then I am true " is true.

Describe trivial proof?

A proof of statement of the form $p \rightarrow q$ which proves q is true without using p.

e.g: Prove that if $\underbrace{x>0}_{p}$ Then $\underbrace{(x+1)^2 - 2x > x^2}_{q}$

$p \longrightarrow q$

$\therefore$ by using q

$$(x+1)^2 - 2x > x^2$$

$$x^2+1+\cancel{2x}-\cancel{2x} > x^2$$

$$x^2+1 > x^2 \quad \text{Proved}$$

"Exercise 1.7"

Q1: Sum of two integers is even:

$P \rightarrow$ two integers $a+b$ odd integers

$q \rightarrow$ sum is even $(a+b)$

$a = 2k+1$ odd no

$b = 2l+1$ " "

$(a+b) = (2k+1)+(2l+1)$

$= 2k+2l+2$

$= 2(k+l+1)$

$\therefore k$ & l are integers so it proved that sum of two odd integers is even

Q2: Sum of two even integer is even:

$P \rightarrow$ two even integers a & b

$q \rightarrow$ sum is even c

$c = a+b$

$\therefore a = 2k, \; b = 2l$

$c = 2k + 2l$

$c = 2(k+l)$

Proved.

Q3: Square of an even number is an even number:

$p \rightarrow$ Square of an even number $a^2 = (a \times a)$

$q \rightarrow$ Square is also even number

$a^2 = a \times a$

$\therefore a = 2k$

$a^2 = (2k \times 2k)$

$= 4k$ $\therefore k$ is an integer

proved, The square of a number is even.

Q5: Prove That if $m+n$ & $n+p$ are even integers, where m, n & p are integers, Then $m+p$ is even. what kind of proof did u we?

$\therefore m+n = 2k,$

$n+p = 2l$

$m+n+n+p = 2k+2l$

$m+2n+p = 2k+2l$

$m+p = 2k+2l+2n$

$m+p = 2(k+l+n)$

$m+p = 2(k+l+n)$ Proved.

Q7: Every odd integer is The difference of two squares

$P \rightarrow$ every odd integers, a odd

$q \rightarrow$ Difference of two squares.

$\therefore a = 2k+1$

$a = (2k+1)^2 - k^2 = 4k^2+1+4k - k^2 = 3k^2+1+4k$

Q8: if n is a perfect square, then $n+2$ is even not a perfect square!

p : $n^2 = n \times n$ perfect square. a

q : $n+2$ is not a perfect square.

$n \times n = n+2$

$\therefore$ n is a integer

$n = a+2$

$= (a \times a)+2$

= Proved.

Q10: Product of two rational no. is rational

p = 2 number (rational number)

let r & s are two rational numbers

q = Product is also rational q

$p \to q$

$q = rs$

$\therefore r = \frac{p}{q}$, $s = \frac{t}{u}$

$q = rs$

$= \left(\frac{p}{q}\right)\left(\frac{t}{u}\right)$

$= \frac{pt}{qu}$

Proved.

Q12: Product of two non-zero rational no. & an irrational no is irrational

$P \to$ two number r & s

r is rational & s is irrational

$r = \frac{P}{q}$, $s = \sqrt{t}$

$q \to$ Product is also irrational

$q = rs$

$= \frac{P}{q} \cdot \sqrt{t}$

$\therefore \frac{P}{q}$ & t & are non-zero rational & irrational

Proved irrational

Q14: if x rational & $x \neq 0$ then $1/x$ is rational.

$P \to x$ is a rational no $\frac{P}{q}$

then $q \to 1/x$ is rational $1/P/q = \frac{q}{P}$

$P \to q$

$x = \frac{1}{x}$

$\frac{P}{q} = \frac{1}{P/q}$

$\frac{P}{q} = \frac{q}{P}$ Proved its a rational.

Q16: if m & n are integers & mn is even, Then m is even or n is even.

$m = 2k$

$n = 2l$

$mn = 2k(2l)$

$mn = 4kl$

$m = \frac{4kl}{n}$ or $n = \frac{4kl}{m}$

where m, k, l & n are integer,

Q17: if n is an integer & n^3+5 is odd, Then n is even using

(a) Contraposition. $(\neg q) \rightarrow (\neg p)$

$P = n^3 + 5$ is odd s.n is integer

$P = 2k + 1$

$= 2(n^3 + 5) + 1$

$= 2n^3 + 10 + 1 = 2n^3 + 11$

$q = n$ is even

$(\neg p) = n^3 + 5$ is not odd, mean it even

$\neg p = 2k$

$= 2n^3 + 10$

$\neg q = n$ is not even, its odd

$(\neg q) \rightarrow (\neg p)$

Proved, if n is not even Then n^3+5 is not odd.

Contradiction: $P \to Q$

$P = n^3+5$ is odd

$P = 2k + \cancel{0}1$

$= 2(n^3+5)+1$

$Q = n$ is even means $n = 2k$

$= 2(n^3+5)+1$

$= (2((2k)^3+5)+1)$

$= (2(8k^3+5)+1)$

$= 16k^3+10+1$

$= 16k^3+11$ where k is an integer.

$P = n^3+5$ so it is odd

$n^3+5 = 2k+1$

Q n is even

$P \to Q$

$n^3+5 = 2k+1$

$n^3 = 2k+1-5$

$n^3 = 2k-4$

$n^3 = 2(k-2)$

$n \cdot n^2 = 2(k-2)$

Proved n is even, where is any integer for example $-1, 1$

$n^3 = 2(-1-2)$, $n^3 = 2(1-2)$

$n^3 = 2(-3)$ $n^3 = 2(-1)$

$n^3 = -6$ $n^3 = -2$

n is even.

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"Proof Methods & Strategy"

Q: What is meant by proof by cases?

To prove a conditional statement of the form

$$(p_1 \vee p_2 \vee p_3 \cdots p_n) \rightarrow q$$

the tautology

$$[((p_1 \vee p_2 \vee p_3 \cdots p_n) \rightarrow q] \leftrightarrow [(p_1 \rightarrow q) \wedge (p_2 \rightarrow q) \wedge \cdots (p_n \rightarrow q)$$

can be use as a rule of interference. This shows that the original conditional statement with the hypothesis made up of disjunction of the proposition $p_1, p_2, \ldots p_n$ can be proved by providing each of the n conditional statements $p_i \rightarrow q$, $i = 1, 2, 3, \cdots n$ individually. This argument is called proof by cases.

Q: Describe exhaustive proof?

Some theorem can be proven by examining a relatively small no. of examples. Such proofs are called exhaustive proof or proofs by exhaustion.

For example;

if n is a natural no.

$n \leq 3$ & $n \geq 2$

then is prime