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Real Vector space let v be a arbitrary non empty set with two operation addition(+) and multiplication showed be satisfied.

i)

if $u, v \in V$

$u + v \in V$

ii)

$u + v = v + u \quad \forall \; u, v \in V$

iii)

$u + (v + w) = (u + v) + w$

iv)

$\exists \, 0 \in V$

$u + 0 = 0 + u = u$

matrix of the zero or indentify $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

v)

$u \in V \quad \exists \, -u$

$u - u = 0 \quad -u + u = 0$

This is the inverse.

vi)

k be any scalar and

$u \in V \quad ku \in V$

V = Vector space

vii)

$k(u+v) = ku + kv$
↳ scalar

viii)

$(k+m)u = ku + mu$
↳ vector

ix)

$k(mu) = (km)u$

x)

$1 \cdot u = u \cdot 1 = u$

Example

$V = \{x, y, z \in R^3 : 2x + 3y^2 - 4z = 0\}$

is vector space or not?

Sol:
Taking two point in the vector space
Let
$u = (2,0,1)$ $v = (3,0,2)$
first step
Add the two point

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$U+V = (10,0,3)$

second step

put the value of two point in the equation.

$$2x+3y^3-4z=0$$
$$2(10)+3(0)-4(12)=0$$
$$20+0-36\neq 0$$
$$-16\neq 0$$

so the L.H.S $\neq$ R.H.S

so this is not the vector space.

Example 2

$V = \{x,y,z \in R^3 : (a-b)c=0\}$

Find is vector space or not?

Sol.

Taking two point

Let

$u=(4,3,0)$ $v=(0,1,5)$

First step

$U+V=(4,4,5)$

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The set $R^3 = \{(x, y, z);\ x, y, z \in R\}$ is vector space over field R.

Under addition and scalar multiplication defined by

i) $u + u' = (x, y, z) + (x', y', z')$
$= (x + x', y + y', z + z')$

ii) $au = (ax, ay, az)$
$a \in R^3$

we check Axioms

i) Abilans group under addition

(i)

Closure property

Taking two vector from the vector space.

Let $u, v \in R^3$
such that
Let

$u = (x_1, y_1, z_1)$
$v = (x_2, y_2, z_2)$

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u+v

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R.

$$u+v=(x_1,y_1,z_1)+(x_2,y_2,z_2)$$

$$=(x_1+x_2,\ y_1+y_2,\ z_1+z_2)$$

Such that closure property holds.

ii)

Commutative property

Let $u=(x_1,y_1,z_1)$
$v=(x_2,y_2,z_2)$

$$u+v=(x_1,y_1,z_1)+(x_2,y_2,z_2)$$

$$=(x_1+x_2,\ y_1+y_2,\ z_1+z_2)$$

$$=(x_2+x_1,\ y_2+y_1,\ z_2+z_1)$$

$$=(x_2,y_2,z_2)+(x_1,y_1,z_1)$$

We know that
$v=(x_2,y_2,z_2)$
$u=(x_1,y_1,z_1)$
put the value

$$u+v=(x_2,y_2,z_2)+(x_1,y_1,z_1)$$
$$u+v=v+u$$

So prove that Commutative property holds.

(iv) Identity property

$0 \in (0,0,0)$

so the $0 \in R^3$

$\forall u \in R^3$

so the

$$0+u = u+0 = u$$

So the identity property holds.

(v) Inverse property

Let

$u \in R^3 \quad \exists \; -u \in R^3$

The reverse present in the vector space

$u+(-u) = 0 \qquad -(u)+u = 0$

$u-u = 0 \qquad -u+u = 0$

(vi) Associated property

~~Let~~ $u, v, w \in R^3$ ~~so~~

~~the~~

Let

$u = (x_1, y_1, z_1)$

$V = (x_2, y_2, z_2)$

$Z = (x_3, y_3, z_3)$

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$u+(v+w) = (x_1, y_1, z_1) + \big((x_2, y_2, z_2) + (x_3, y_3, z_3)\big)$

Given the condition to add

$= (x_1, y_1, z_1) + (x_2+x_3, y_2+y_3, z_2+z_3)$

$= (x_1+x_2+x_3, y_1+y_2+y_3, z_1+z_2+z_3)$

$= \big((x_1+x_2, y_1+y_2, z_1+z_2)\big) + (x_3, y_3, z_3)$

We know that

$\because u+v = (x_1+x_2, y_1+y_2, z_1+z_2)$

$\because w = (x_3, y_3, z_3)$

put in the value

$= \big((x_1+x_2, y_1+y_2, z_1+z_2)\big) + (x_3, y_3, z_3)$

$u+(v+w) = (u+v)+w$

So the associated property holds.

$\longleftrightarrow$

vi)

k be any scalar and

$u \in R \quad \exists \; ku \in R$

let $u = (x_1, y_1, z_1)$

$ku = k(x_1, y_1, z_1)$

$= (kx_1, ky_1, kz_1)$

$= (k(x_1), ky_1, kz_1) \in R$

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So the Axiom 6 holds.

vii)

Take Two vector in the vector space and take 1 scalar.

$\forall\ u, v \in R \quad \exists\ k \in R$

$$k(u+v) = ku + kv$$

Let

$u = (x_1, y_1, z_1)$

$v = (x_2, y_2, z_2)$

$= k(x_1 + x_2, y_1 + y_2 + z_1 + z_2)$

$= k((x_1, y_1, z_1) + (x_2, y_2, z_2))$

we know that

$x_1, y_1, z_1 = u$

$x_2, y_2, z_2 = v$

$= k((u) + (v))$

$= k(u) + k(v)$

$= ku + kv$

Hence prove that

$k(u+v) = ku + kv$

The Axiom vii) Holds

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e Vector

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viii)

Taking Two Scalar And one vector

$k, m \in R$ & $u \in$ Vector space

$(k+m)u = ku + mu$

prove

Let $u = (x, y, z)$

$= (k+m)(x, y, z)$

$kx, ky, kz + mx + my + mz$

Taking "k" common and "m"

$= k(x, y, z) + m(x, y, z)$

we know that

$u = (x, y, z)$

$= ku + m(u)$

$(k+m)u = ku + mu$

Hence prove that.

ix)

Taking Two vector and One scalar.

$k(mu) = (km)u$

let

$u = x_1, y_1, z_1$

$v = x_2, y_2, z_2$

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$k((x_1, y_1, z_1)(x_2, y_2, z_2))$

$= k(x_1x_2, y_1y_2, z_1z_2)$

$= (kx_1x_2, ky_1y_2, kz_1z_2)$

$= (kx_2, ky_2, kz_2)(x_1, y_1, z_1)$

~~we~~ Taking the k common

$(k(x_2, y_2, z_2))(x_1, y_1, z_1)$

we know that

$m = x_2, y_2, z_2$

$u = x_1, y_1, z_1)$

$(k(m))u = km(u)$

$= (km)u$

Hence prove that

$k(mu) = (km)u$

So The Axiom is holds

(x)

So the 1 is any Scalar value taken from the vector space and u is a vector taken from the vector space.

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$1 \in R^3$

let

$1 \cdot u$

$1 \cdot ($

$(1 \cdot$

$($

The

$1 \in R^3$ And $u \in R^3$

let

$u = (x, y, z)$

$1 \cdot u = u \cdot 1 = u$

$1 \cdot u = u \cdot 1$

$1 \cdot (x, y, z) = (x, y, z)(1)$

$(1 \cdot x, 1 \cdot y, 1 \cdot z) = (x \cdot 1, y \cdot 1, z \cdot 1)$

$(x, y, z) = (x, y, z)$

Hence prove that

The Axioms (10) holds.