# Past paper 2020

## Short Question.

① $\lim_{x \to 1} \frac{x^6 - 1}{x^{10} - 1}$

$= \lim_{x \to 1} \frac{6x^5 - 0}{10x^9 - 0}$

Apply Limit $\frac{6(1)^5}{10(1)^5} = \frac{6}{10} = \frac{3}{5}$ Ans.

② Two function which are contineous on their

Ans: $f(x) = \sin x$, $f(x) \cos x$, $f(x) \tan x$

$f(x) = e^x$, $f(x) = a^x$ $f(x) = \ln x$

③ Prove that limit doesnot exist

$\lim_{x \to 0} \frac{|x|}{x}$

LHS

$\lim_{x \to 0^-} \frac{-\cancel{x}}{\cancel{x}}$

$\lim_{x \to 0} (-1)$

R.H.S

$\lim_{x \to 0^+} \frac{x}{x}$

$\lim_{x \to 0} \frac{1}{1}$

Limit doesnot exist.

④ Eqvuation of tangent Line hyperbola $= \frac{3}{x}$ at the point $(3, 1)$

$y - y_1 = m(x - x_1)$

$m = \frac{3}{x}$ $x_1 = 3$ $y_1 = 1$

$y - 1 = \frac{3}{x}(x - 3)$

$y - 1 = \frac{3\cancel{x}}{\cancel{x}} - \frac{3 \cdot 3}{x}$

$y - 1 = 3 - \frac{9}{x}$

$y - 1 + 3 = -\frac{9}{x}$

$(y + 2)x = -9$

$2x + xy + 9 = 0$

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(5) $f'$ if $f(x) = \frac{1-x}{2+x}$

$f'(x) = \frac{d}{dx}\left(\frac{1-x}{2+x}\right)$

$f'(x) = \frac{(2+x)\,d/dx(1-x) - (1-x)\,d/dx(2+x)}{(2+x)^2}$

$f'(x) = \frac{(2+x)(0-1) - (1-x)(0+1)}{2+x^2}$

$f'(x) = \frac{(2+x)(-1) - (1-x)}{2+x^2} \rightarrow \frac{-2-\cancel{x}-1+\cancel{x}}{2+x^2} = \frac{-3}{2+x^2}$

(6)

(7) find $f'(4)$ if $f(x) = \sqrt{x}\ g(x)$ wher $g(4) = 2$ $g'(4) = 3$

$f(x) = \sqrt{x}.\ g(x)$

$f'(x) = \sqrt{x}\ \frac{d}{dx}\ g(x) + g(x)\ d/dx\ \sqrt{x}$

$= \sqrt{x}\ g'(x) + g(x) \cdot \frac{1}{2} x^{-1/2}$

$g'(4) = 3 \qquad g(4) = 2$

$x = 4$

$f'(4) = \sqrt{4}\ g'(4) + g(4)\ \frac{1}{2\sqrt{4}}$

$f'(4) = \sqrt{4} \cdot (3) + \cancel{2} \cdot \frac{1}{\cancel{2} \cdot 2}$

$= \sqrt{(2)^2} \cdot 3 + \frac{1}{2}$

$= 6 + \frac{1}{2}$

$= \frac{12+1}{2} = \frac{13}{2}$ Ans.

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8) derivative of function $y = 2^{3^{x^2}}$ w.r.t $x$

$y = 2^{3^{x^2}}$

Apply log on both side.

$\ln y = \ln 2^{3^{x^2}}$

$\ln y = 3^{x^2} \ln 2$

$\frac{d}{dx} a^x$

$a^x \cdot \ln a \cdot \frac{d}{dx}(x)$

Taking derivative

$\frac{1}{y} \cdot dy/dx = \ln 2 \cdot d/dx\, 3^{x^2}$

$= \ln 2 \cdot 3^{x^2} \ln 3 \cdot 2x$

$dy/dx = y \cdot \ln 2 \cdot \ln 3\, 3^{x^2} \cdot 2x$

$dy/dx = 2^{3^{x^2}} \cdot 3^{x^2} \cdot 2x \cdot \ln 3 \cdot \ln 2$ Ans.

⑨ $y'$ if $\sin(x+y) = y^2 \cos x$.

$\frac{d}{dx} \sin(x+y) = \frac{d}{dx} y^2 \cos x$.

$\cos(x+y) \cdot dy/dx = y^2 \cdot -\sin x + \cos x\, 2y\, dy/dx$

$\cos(x+y)\, dy/dx = -\sin x y^2 + 2y \cos x\, dy/dx$.

$\cos(x+y) \frac{dy}{dx} - 2y\cos x \frac{dy}{dx} = -\sin x y^2$

$dy/dx\,(\cos(x+y) - 2y\cos x) = -\sin x y^2$

$$\frac{dy}{dx} = \frac{-y^2 \sin x}{\cos(x+y) - 2y\cos x}$$ Ans.

⑩ $y' = ?$ $y = x^{1/x}$

$\ln y = \ln x^{1/x}$

$\ln y = \frac{1}{x} \ln x$

$\frac{1}{y} dy/dx = \frac{1}{x} \cdot \frac{1}{x} + \ln x \cdot (-1)x^{-1-1}$

$= \frac{1}{x^2} + \ln x\,(-1)x^{-2} \rightarrow \frac{1}{x^2} - \frac{\ln x}{x^2} = \frac{1 - \ln x}{x^2}$

$\frac{1}{y} \cdot dy/dx =$ ~~[illegible]~~ Ans.

$dy/dx = (1 - \ln x / x^2) \cdot x^{1/x}$.

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⑪ Linearization of function.

$f(x) = \sqrt{x+3}$ at $a=1$

$f(x) = f(a) + f'(a)(x-a)$

$f'(x) = \frac{1}{2}(x+3)^{1/2-1} \cdot (1)$

$= \frac{1}{2}(x+3)^{-1/2}$

$\underline{\underline{a=1}}$

* $f(a) = \sqrt{a+3} = \sqrt{1+3} = \sqrt{4} = 2$
* $f'(a) = \frac{1}{2}(a+3)^{-1/2} = \frac{1}{2}(1+3)^{-1/2} = \frac{1}{2}(4)^{-1/2}$

$\frac{1}{2} \cdot \frac{1}{\sqrt{4}} \rightarrow \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$= 2 + \frac{1}{4}(x-1)$ Ans.

⑫ Point of inflection.

If function is increasing before $x=a$ and after $x=0$ such a point is called point of inflection.

⑬ $\lim\limits_{x \to 1} \frac{x^a - 1}{x^b - 1}$ L hopital Rule.

$\lim\limits_{x \to 1} \frac{ax^{a-1} - 0}{bx^{b-1} - 0}$

Apply Limit.

$= \frac{a(1)^{a-1}}{b(1)^{b-1}}$

$= \frac{a}{b}$ Ans.

(14) Antiderivative of function.

$f(x) = 6\sqrt{x} - \sqrt[6]{x}$

$= 6\int \sqrt{x}\,dx - \int (x)^{1/6}\,dx$

$= 6 \cdot \frac{(x)^{1/2+1}}{3/2} - \frac{x^{1/6+1}}{1/6+1} + c$

$= 6 \cdot \frac{x^{3/2}}{3/2} - \frac{x^{7/6}}{7/6} + c$

$= \cancel{6}^{2} \cdot \frac{2}{\cancel{3}} x^{3/2} - \frac{6}{7} x^{7/6} + c$

$= 4x^{3/2} - \frac{6}{7} x^{7/6} + c$ Ans.

(15) given that $\int_4^9 \sqrt{x}\,dx = \frac{38}{3}$ what is value.

$\int_9^4 \sqrt{t}\,dt$

$= \left[\frac{t^{1/2+1}}{\frac{1}{2}+1}\right]_9^4$

$= \frac{2}{3}\left[t^{3/2}\right]_9^4 \rightarrow \frac{2}{3}\left[(4)^{3/2} - (9)^{3/2}\right]$

$= \frac{2}{3}\left[(2^2)^{3/2} - (3^2)^{3/2}\right]$

$= \frac{2}{3}\left[(16 - 27)\right]$

$= \frac{2}{3}(-11) = \frac{-22}{3}$ Ans.

(16) Parabola $y^2 = 12x$.

Focus = ? directrix.

$y^2 = 4ax$ $\quad 4ax = 12x \quad a = 12/4 = 3$

$F = (0, 3)$ $\quad D = y = -3$

$F = (0, a)$ $\quad D = (y = -a)$

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Long Question:-

Q2 $f(x) = \frac{x^2-1}{|x-1|}$

$\lim_{x \to 1^+} f(x)$ and $\lim_{x \to 1^-} f(x)$ exist?

$f(x) = \frac{x^2-1}{\pm(x-1)}$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} - \frac{x^2-1}{x-1}$

$\lim_{x \to 1^-} f(x) = -\lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$

Apply Limit

$= -(1+1)$

$= -2$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{x^2-1}{x-1}$

$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$

= apply Limit

$= (1+1)$

$= 2.$

Yes Limit exist's.

visit tShahab.blogspot.com for more.

Date: (7)

Q 3 (b) Find $\frac{d^9}{dx^9}(x^8 \ln x)$

$\frac{dy}{dx} = \frac{d}{dx}(x^8 \ln x)$

$= x^8 \cdot \frac{1}{x} + \ln x \cdot 8x^7$

$= x^7 + \ln x \, 8x^7 \rightarrow x^7(1 + 8\ln x)$ $1^{st.}$

$\frac{d^2y}{dx^2} = x^7 \cdot (0 + 8\,1/x) + (1 + 8\ln x)7x^6$

$= 8x^{7-1} + 7x^6 + 56\ln x \, x^6$

$= x^6(8 + 7 + 56\ln x)$

$= x^6(15 + 56\ln x)$ $2^{nd}$

$\frac{d^3y}{dx^3} = x^6 \cdot (0 + 56 \cdot \frac{1}{x}) + (15 + 56\ln x)6x^5$

$= x^{6-1}56 + 90x^5 + 336\ln x \, x^5$

$= x^5(56 + 90 + 336\ln x)$

$= x^5(146 + 336\ln x)$ $3^{rd}$

$\frac{d^4y}{dx^4} = x^5(0 + 336 \cdot \frac{1}{x}) + (146 + 336\ln x) \cdot 5x^4$

$= 336x^4 + 730x^4 + 1680x^4 \ln x$

$= x^4(336 + 730 + 1680\ln x)$

$= x^4(1066 + 1680\ln x)$ $4^{th}$

$\frac{d^5y}{dx^5} = x^4(0 + 1680\frac{1}{x}) + (1066 + 1680\ln x)4x^3$

$= 1680x^3 + 4264x^3 + 6720\ln x \cdot x^3$

$= x^3(1680 + 4264 + 6720\ln x)$

$= x^3(5944 + 6720\ln x)$ $5^{th}$

$d^6y/dx = x^3(0 + 6720\frac{1}{x}) + (5944 + 6720\ln x) \cdot 3x^2$

$= 6720x^2 + 17832x^2 + 20160 \ln x \cdot x^2$

$= x^2(6720 + 17832 + 20160 \ln x)$

$= x^2(24552 + 20160\ln x)$

$= x^2(24552 + 20160\ln x)$ $6^{th}$

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$$\frac{d^7y}{dx^7} = x^2(24552 + 20160\ln x)$$
$$= x^2\left(0 + 20160\frac{1}{x}\right) + (24552 + 20160\ln x).2x$$
$$= x^{2-1}\,20160 + 49104x + 40320\ln x.x$$
$$= x(20160 + 49104 + 40320\ln x)$$
$$= x(69{,}264 + 40320\ln x) \quad \underline{7^{th}}$$

$$\frac{d^8y}{dx^8} = x.\left(0 + 40320.\frac{1}{x}\right) + (69{,}264 + 40320\ln x).1$$
$$= 40320 + 69264 + 40320\ln x$$
$$= 109{,}584 + 40320\ln x \quad 8^{th}$$

$$\frac{d^9y}{dx^9} = 0 + 40320.\frac{1}{x}.$$
$$= 40320.\frac{1}{x} \quad \underline{9^{th}}$$

Ans

**Q3 (A part)**

**Let** $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ mx + b & \text{if } x > 2 \end{cases}$ **find m=? differe**

Continuity

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} mx + b$$
$$= 2m + b$$

$$\lim_{x\to 2^-} f(x) = \lim x^2$$
$$= (2)^2 = 4$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$$

$$4 = 2m + b \quad -(1)$$

put the value of m in eq/u (1)

$$4 = 2(4) + b$$
$$= 8 + b$$

Differentiability.

$$\lim_{x\to 2^-} f(x) = +x^2$$
$$\lim_{x\to 2^+} f(x) = mx + b.$$

$$\frac{d}{dx}\Big|x^2_{x=2} = \frac{d}{dx}\Big|mx+b_{x=2}$$

$$[2x]_{x=2} = m$$
$$2(2) = m$$
$$m = 4$$

$$4 - 8 = b$$

$$\boxed{b = -4}$$

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Q4 (a) Absolute minimum of function

$f'(x) = 12x^3 - 84x^2 + 12x + 24$ $\quad -1 \le x \le 7$

$x = -1 \quad x = 7$

$f(x) = 3x^4 - 28x^3 + 6x^2 + 24x$

$f(-1) = 3(-1)^4 - 28(-1)^3 + 6(-1)^2 + 24(-1)$

$= 3 + 28 + 6 - 24 = 13$

$f(7) = 3(7)^4 - 28(7)^3 + 6(7)^2 + 24(7)$

$= 7203 - 9604 + 294 + 168 = -1939$

$f(x) = 7 \quad y = -1939$ minimum.

(b) Find $f$ if $f''(x) = 2 + \cos x \quad f(0) = -1 \quad f(\pi/2) = 0$

$\int \frac{d^2y}{dx} = \int 2 + \cos x \, dx$

$\frac{dy}{dx} = 2x + \sin x$

again integrate

$\int dy/dx = \int 2x + \sin x$

$y = \cancel{2}\frac{x^2}{\cancel{2}} - \cos x$

$y = x^2 - \cos x$

$f(x) = x^2 - \cos x$

$f(0) = 0 - \cos 0 = -1$

$f(\pi/2) = (\pi/2)^2 - \cos \pi/2$

$= 0$

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Q5 (b) $\int x^a \sqrt{b+cx^{1+a}}\, dx$

$\int x^a \, (b+cx^{1+a})^{1/2}$ $\qquad c \cdot x^{1+a}$

$\int x^a \cdot (b+cx^{1+a})^{1/2}\, dx.$ $\qquad c \cdot (1+a) x^{1+a-1}$

multiply and divide $(c+ca)$ $\qquad (c+ca)\, x^a$

$$\frac{1}{c+ca} \int (b+cx^{1+a})^{1/2} \cdot x^a \cdot (c+ca)\, dx$$

$$\frac{1}{c+ca} \left[ \frac{(b+cx^{1+a})^{3/2}}{3/2} \right] + c$$

$$\frac{2}{3(ca+c)} \cdot (b+cx^{1+a})^{3/2} + c \quad \text{Ans.}$$

(a) derivative of function. $y = \int_{\cos x}^{5x} \cos(u^2)\, dx$

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Q6 (a) Find the slope of tanget of cordioal.

$r = 1 + \sin\theta$ $\theta = \pi/3$

$\frac{dr}{d\theta} = 0 + \cos\theta$

$\frac{dr}{d\theta} = \cos \pi/3.$

Slope of Tangent $= \frac{f'(\theta) \cdot \sin\theta}{f'(\theta) \cdot \cos\theta}$

$= \frac{\cos \pi/3 \cdot \sin \pi/3}{\cos \pi/3 \cdot \cos \pi/3}$

$= \tan \pi/3$

$= \tan 60^\circ =$

$4y^2 + 8y + (\;)$

$(2y)^2 + 2(2)(2)$

(b) Sketch the conic and find foci

$9x^2 - 4y^2 - 72x + 8y + 176 = 0$ $a = 9$ $b = -4$ $h = 0$ $g = -72$ $f = 8$

$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$

• $h^2 - ab \rightarrow 0 - (9)(-4) = +36 > 0$ parbola.

$a > b$

Now $9x^2 - 4y^2 - 72x + 8y \pm 176$

$(9x^2 - 72x + 144) - (4y^2 - 8y + 4) = -176 + 144 + 4$

$(3x - 12)^2 - (4y + 2)^2 = -28$. divided by $-28$.

$\frac{(3x-12)^2}{(-28)} - \frac{(3x-12)^2}{(-28)} = \frac{-28}{-28} = 1$

$\frac{(3x-12)^2}{(2\sqrt{7})^2} - \frac{(3x-12)^2}{(2\sqrt{7})^2} = 1$ At foci $(\pm c, 0)$

$c^2 = a^2 + b^2$

$c^2 = (2\sqrt{7})^2 + (2\sqrt{7})^2$

$c^2 = 28 + 28 = 56$

$\sqrt{c^2} = 2\sqrt{14}$

At foci $(\pm 2\sqrt{14}, 0)$