BS 3rd Term Examination 2020

Subject: I.T **Paper: Linear Algebra (MATH-3215)**

Time Allowed: 2:30 Hours **Maximum Marks: 80**

Note: Objective part is compulsory. Attempt any three questions from subjective part.

Objective Part (Compulsory)

Q.1. Write short answers of the following in 2-3 lines each on your answer sheet. **(16*2)**

i. Determine whether equation $x + y^2 = 0$ is linear in x and y.

ii. Find the inverse $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, if exists.

iii. Determine whether the given matrix is elementary or not $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

iv. Compute the indicated quantity $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}^{30}$.

v. Write any two properties of determinant of matrix.

vi. Find the value of k for which the matrix is invertible $\begin{bmatrix} k-2 & -2 \\ -2 & k-2 \end{bmatrix}$.

vii. Let $v=(0,5)$ and $w = (-1,4)$, Find the components of $v+w$.

viii. State Cauchy Schwarz Inequality.

ix. Let $v = (-2, 3, 0, 6)$, find all scalars k such that $||kv|| = 5$.

x. Determine whether u and v are orthogonal vectors or not $u = (1,2,-1)$, $v = (0,1,1)$.

xi. Is the vector $v = (2,1,1)$ a linear combination of $u = (1,1,1)$ and $v = (1,0,1)$.

xii. Define basis for vector space.

xiii. Show that the given set form a basis for R^2, $\{(2,1),(3,0)\}$.

xiv. Define Hermitian and Skew Hermitian matrix.

xv. Find $\bar{u}$, $Re(u), Im(u)$ and $||u||$ if $u = (2 - \iota, 4\iota, 1 + \iota)$.

xvi. Define Vector space.

Subjective Part (3*16)

Q.2. **a)**. Solve the linear system by Guassian elimination

$$2x_1 + 2x_2 + 2x_3 = 0;$$
$$-2x_1 + 5x_2 + 2x_3 = 1;$$
$$8x_1 + x_2 + 4x_3 = -1.$$

b) Use row operations to find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$.

Q.3. **a)** Find the eigenvalues of $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$

b) Determine whether the set of all vectors of the form (a, 1, 1) is subspaces of R^3 or not?

Q.4. **a)** Express the vector $6 + 11x + 6x^2$ as a linear combination

$$p_1 = 2 + x + 4x^2, p_2 = 1 - x + 3x^2, p_3 = 3 + 2x + 5x^2.$$

b) Show that the set of vectors (1, 2, 1); (2, 9, 0) and (3, 3, 4) form a basis for R^3.

Q.5. **a)** Find a matrix P that diagonalize A, where $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

b) Apply the Gram Schmidt process to transform the basis vectors $u_1 = (1,1,1), u_2 = (0,1,1), u_2 = (0,0,1)$ into an orthogonal basis.

Q.6. Let V be the set of all order pairs of real numbers. Check whether V is a vector space over R under the given operation. If not, state the axioms which fail to hold

$$(a, b) + (c, d) = (a + c, b + d) \text{ and } k(a, b) = (k^2a, k^2b).$$