BS-Chemistry-Semester (VI) Basic Statistics سوموار 27، دسمبر 2021ء

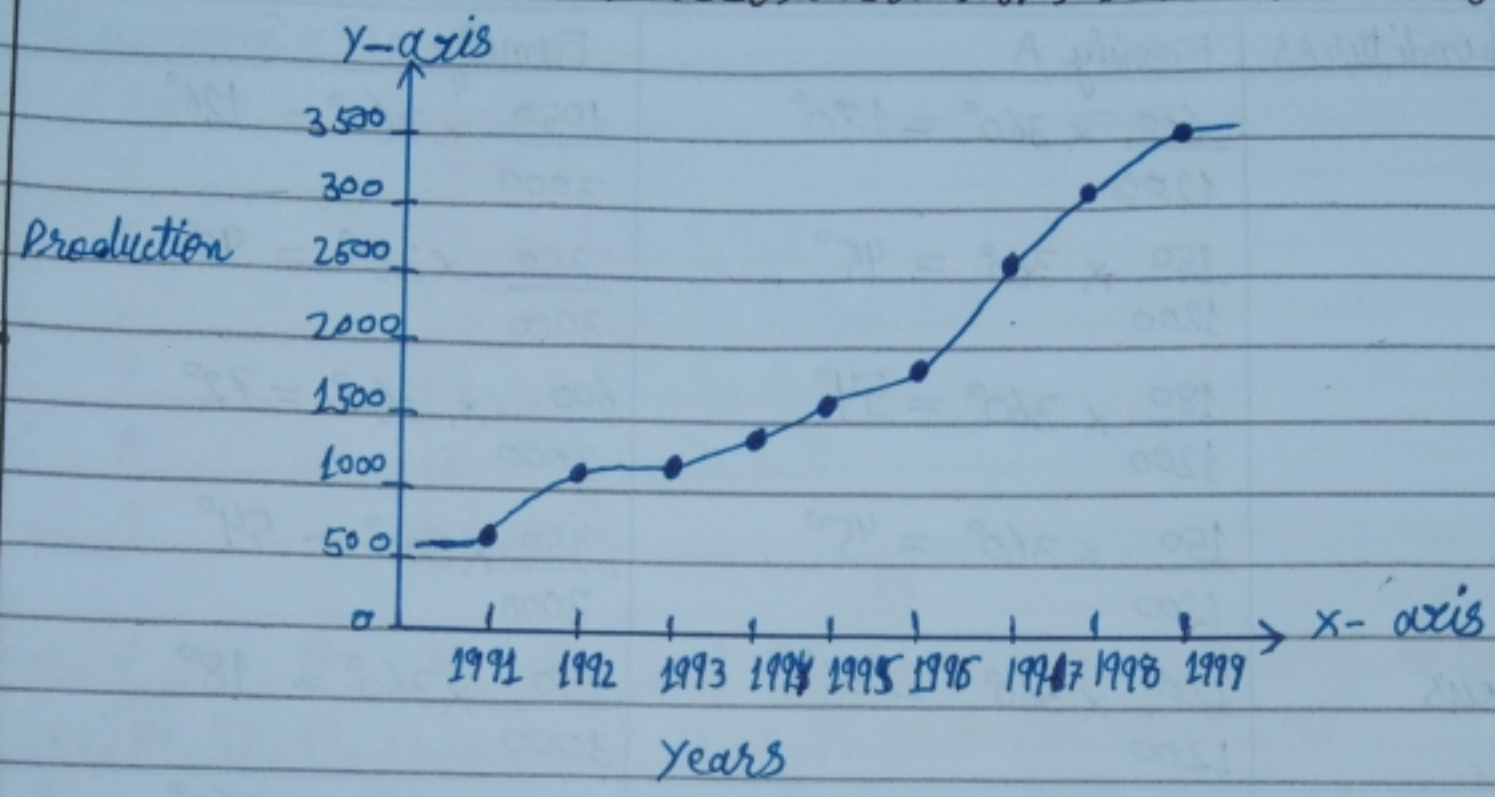
(Long Questions)

①

For the following data draw a line chart:

Years: 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1998, 1999

Production: 928, 1088, 1326, 1456, 1767, 1984, 2645, 3206, 3493

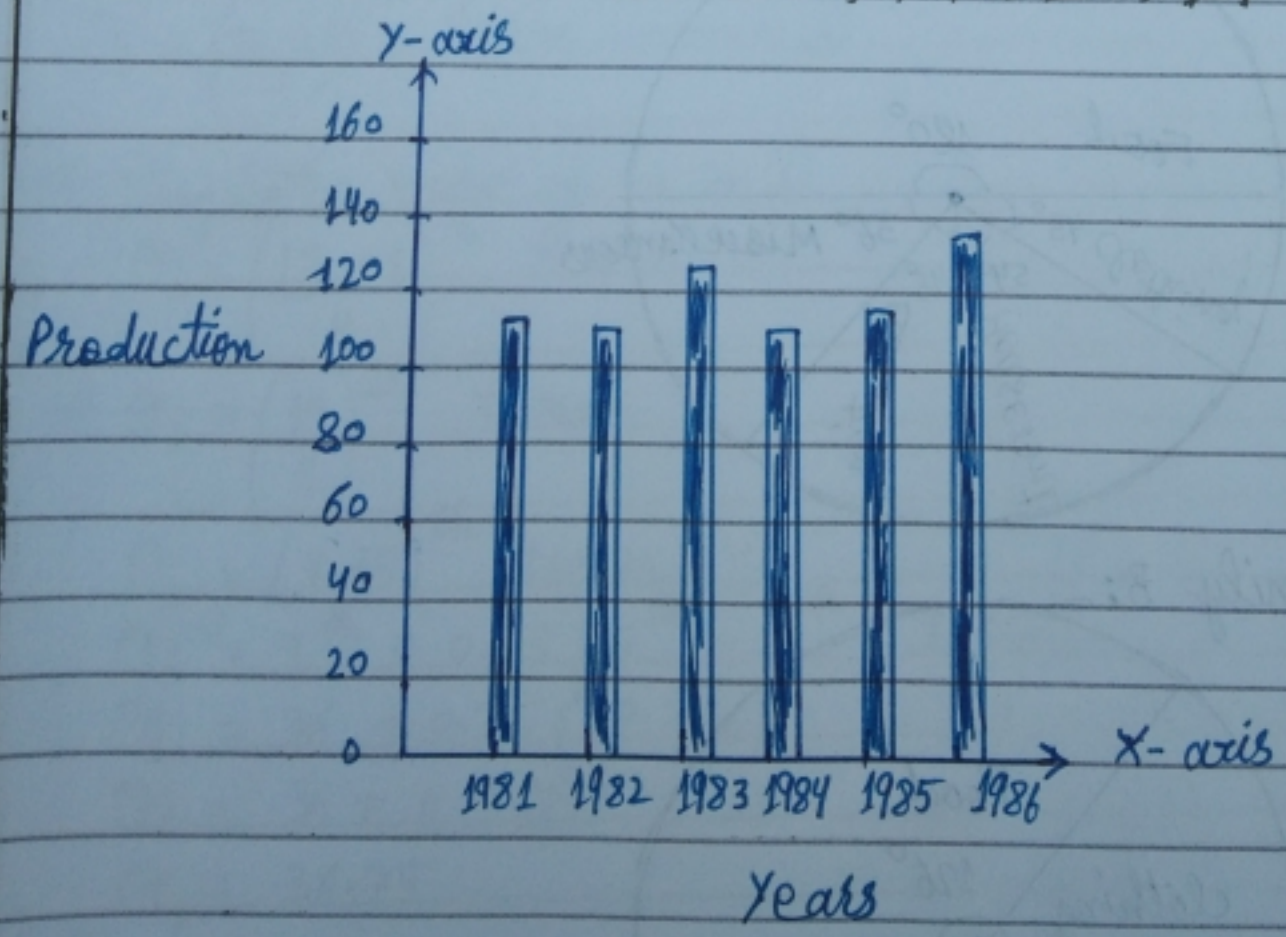


②

Draw a simple bar chart:

Year: 1981, 1982, 1983, 1984, 1985, 1986

Production: 115, 113, 124, 109, 117, 139



③

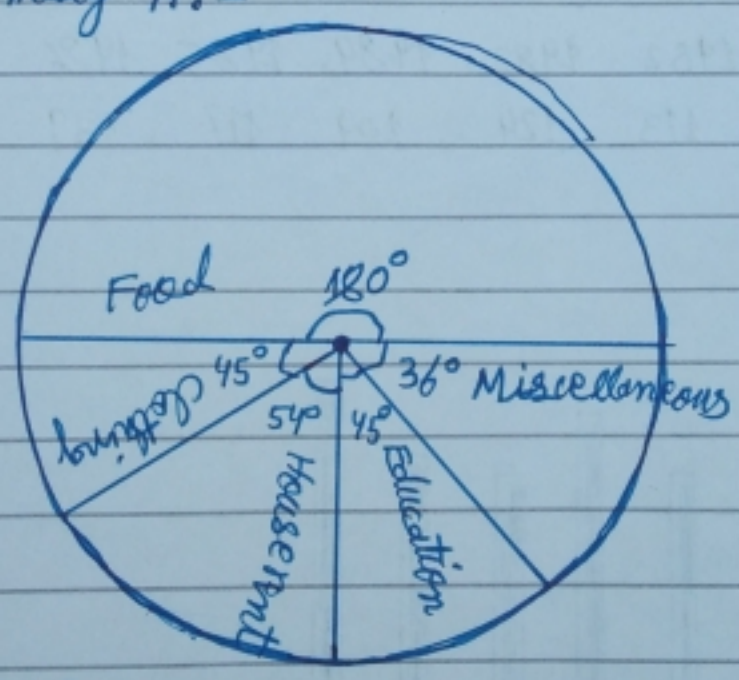
Construct a Pie chart to compare the budgets of two families A and B.

Items of expenditure:	Food	Clothing	House rent	Education	Miscellaneous	Total
Family A:	600	150	180	150	120	1200
Family B:	1050	750	600	450	150	3000

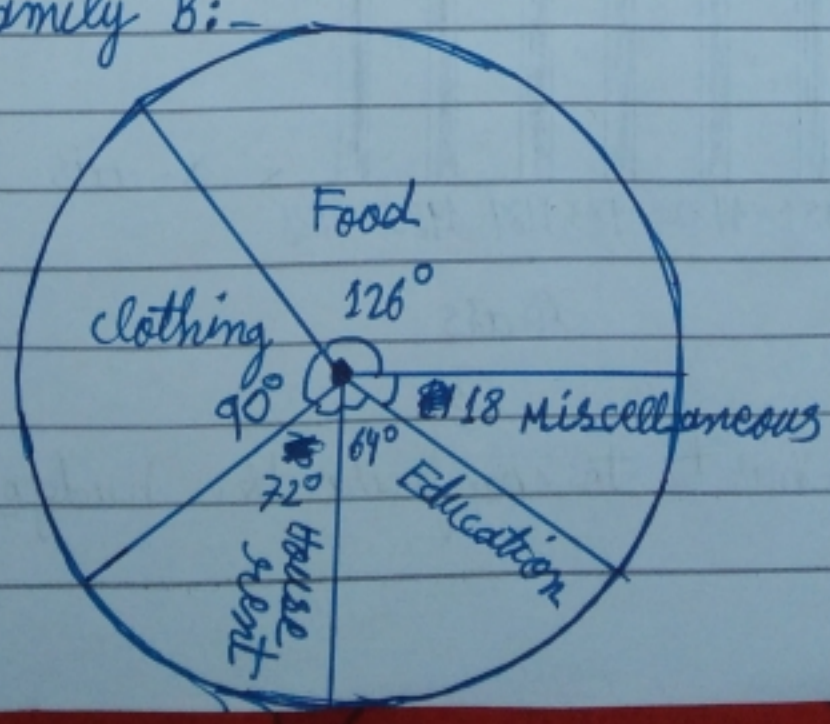
Pie - sector $= \frac{\text{Part}}{\text{Total}} \times 360^\circ$

Items of expenditures	Family A	Family B
Food	$\frac{600}{1200} \times 360^\circ = 180^\circ$	$\frac{1050}{3000} \times 360^\circ = 126^\circ$
Clothing	$\frac{150}{1200} \times 360^\circ = 45^\circ$	$\frac{750}{3000} \times 360^\circ = 90^\circ$
House rent	$\frac{180}{1200} \times 360^\circ = 54^\circ$	$\frac{600}{3000} \times 360^\circ = 72^\circ$
Education	$\frac{150}{1200} \times 360^\circ = 45^\circ$	$\frac{450}{3000} \times 360^\circ = 54^\circ$
Miscellaneous	$\frac{120}{1200} \times 360^\circ = 36^\circ$	$\frac{150}{3000} \times 360^\circ = 18^\circ$
Total	$= 360^\circ$	$= 360^\circ$

- Pie chart for family A:-



- Pie-chart for family B:-



④

For the following data construct stem and leaf display:
48, 53, 52, 49, 51, 52, 63, 60, 53, 64, 59, 58, 47, 49, 45, 64, 79, 65, 62, 60, 68, 65, 73, 88, 69, 83, 78, 81, 86, 92, 75, 85, 81, 77, 87, 76, 75, 73, 92.

Minimum = 48

Maximum = 92

Stem	Leaf
4	8, 9, 7, 9, 5
5	3, 2, 1, 2, 3, 9, 8
6	3, 0, 4, 4, 5, 2, 0, 8, 5, 9
7	9, 3, 8, 5, 7, 6, 5, 3
8	8, 3, 1, 6, 5, 1, 7
9	2, 1, 2

⑤

Find Q_1, Q_2, Q_3, D_7 and P_{64} from the following data:
53, 74, 82, 42, 39, 20, 81, 68, 28, 67, 54, 93, 70, 30, 55, 36, 37.

Arranging the observations in ascending order:
20, 28, 30, 36, 37, 39, 42, 53, 54, 55, 58, 67, 68, 70, 74, 81, 82, 93.

$n = 18$

- $Q_1 = ?$

$$Q_1 = \left(\frac{n+1}{4}\right)^{th} \text{ term} \quad \text{——— (1)}$$

Putting the value of n in (1)

$$Q_1 = \left(\frac{18+1}{4}\right)^{th} \text{ term}$$

$$Q_1 = \left(\frac{19}{4}\right)^{th} \text{ term}$$

$$Q_1 = (4.75)^{th} \text{ term}$$

$$Q_1 = 4^{th} + 0.75\left(5^{th} - 4^{th}\right) \text{ term}$$

$$Q_1 = 36 + 0.75(37 - 36)$$

$$Q_1 = 36 + 0.75(1)$$

$$Q_1 = 36.75$$

- $Q_2 = ?$

$$Q_2 = 2\left(\frac{n+1}{4}\right)^{th} \text{ term} \quad \text{——— (2)}$$

Putting the value of n in ②

$$Q_2 = 2\left(\frac{18+1}{4}\right)^{th} \text{ term}$$

$$Q_2 = 2\left(\frac{19}{4}\right)^{th} \text{ term}$$

$$Q_2 = 2(4.75)^{th} \text{ term}$$

$$Q_2 = (9.5)^{th} \text{ term}$$

$$Q_2 = 9^{th} + 0.5(10^{th} - 9^{th})$$

$$Q_2 = 54 + 0.5(55-54)$$

$$Q_2 = 54 + 0.5(1)$$

$$Q_2 = 54 + 0.5$$

$$Q_2 = 54.5$$

- $Q_3 = ?$

$$Q_3 = 3\left(\frac{n+1}{4}\right)^{th} \text{ term} \quad \text{——— ③}$$

Putting the value of n in ③

$$Q_3 = 3\left(\frac{18+1}{4}\right)^{th} \text{ term}$$

$$Q_3 = 3\left(\frac{19}{4}\right)^{th} \text{ term}$$

$$Q_3 = 14.25$$

$$Q_3 = 14^{th} + 0.25(15^{th} - 14^{th})$$

$$Q_3 = 70 + 0.25(74-70)$$

$$Q_3 = 70 + 0.25(4)$$

$$Q_3 = 70 + 1$$

$$Q_3 = 71$$

- $D_7 = ?$

$$D_7 = 7\left(\frac{n+1}{10}\right)^{th} \text{ term} \quad \text{——— ④}$$

Putting the value of n in ④

$$D_7 = 7\left(\frac{18+1}{10}\right)^{th} \text{ term}$$

$$D_7 = 7\left(\frac{19}{10}\right)^{th} \text{ term}$$

$$D_7 = (13.3)^{th} \text{ term}$$

$D_7 = 13^{th} + 0.3(14^{th} - 13^{th})$

$D_7 = 68 + 0.3(70 - 68)$

$D_7 = 68 + 0.3(2)$

$D_7 = 68 + 0.6$

$D_7 = 68.6$

- $P_{64} = ?$

$P_{64} = 64\left(\frac{n+1}{100}\right)^{th}$ term ——— (5)

Putting the value of n in (3)

$P_{64} = 64\left(\frac{19}{100}\right)^{th}$ term

$P_{64} = 64(0.19)^{th}$ term

$P_{64} = (12.16)^{th}$ term

$P_{64} = 12^{th} + 0.16(13^{th} - 12^{th})$

$P_{64} = 67 + 0.16(68 - 67)$

$P_{64} = 67 + 0.16(1)$

$P_{64} = 67 + 0.16$

$P_{64} = 67.16$

(6)

Find the probability that atleast one head turn off, toss twice.

S	H	T
H	HH	HT
T	TH	TT

$n(S) = 4$

Let A ~ atleast one head

$A = \{HH, HT, HT, TT\}$

$n(A) = 3$

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$

(7)

What is the probability of getting exactly two head when three balanced coins are tossed once.

S	HH	HT	TH	TT
H	HHH	HHT	HTH	HTT
T	THH	THT	TTH	TTT

$n(S) = 8$

Let B ~ exactly two heads,

$B = \{HHT, HTH, THH\}$

$n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{8}$$

(3.17) (8)

Find arithmetic mean for the following data:

25, 15, 18, 30, 20, 20, 12, 9, 16, 15.

$$\text{Mean}, \bar{x} = \frac{\text{sum of observations}}{\text{number of observations}}$$

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{180}{10} = 18$$

(3.18(b)) (9)

Find arithmatic mean for following data:

85, 70, 10, 75, 500, 8, 42, 250, 40, 36.

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{1116}{10} = 111.6$$

(3.19(a)) (10)

Find arithmetic mean for following data:

60, 80, 90, 96, 120, 150, 200, 360, 480, 520, 1060, 1200, 1450, 2500, 7200.

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{15366}{15} = 1037.73$$

(3.19(b)) (11)

Find geometric mean for following data:

60, 80, 90, 96, 120, 150, 200, 360, 480, 520, 1060, 1200, 1450, 2500, 7200

$$\text{Geometric mean} = \text{antilog}\left(\frac{\Sigma \log x}{n}\right)$$

(on next page)

x	$\log x$
60	$\log 60 = 1.7781$
80	$\log 80 = 1.9031$
90	$\log 90 = 1.9542$
96	$\log 96 = 1.9823$
120	$\log 120 = 2.0792$
150	$\log 150 = 2.176$
200	$\log 200 = 2.3010$
360	$\log 360 = 2.5563$
480	$\log 480 = 2.6812$
520	$\log 520 = 2.7160$
1060	$\log 1060 = 3.0253$
1200	$\log 1200 = 3.0792$
1450	$\log 1450 = 3.1614$
2500	$\log 2500 = 3.3979$
7200	$\log 7200 = 3.8573$
$n = 15$	$\Sigma \log x = 38.6486$

Geometric mean = antilog $\left(\frac{38.6486}{15}\right)$

Geometric mean = antilog (2.5766)

Geometric mean = 377.2

(3.27) - (12)

A man gets a rise of 10% in salary at the end of his first year of service and further rises 20% and 25% at the end of second and third year respectively. The rise in each case being calculated on on his salary at the begining of the year. To what annual % age increase is this equivalent.

Let the basic income salary = 100

Increase after one year = 10%

= 100 + 10 = 110

Increase after second year = 20%

= 100 + 20 = 120

Increase after third year = 25%

= 100 + 25 = 125

Geometric mean $= (x_1 \times x_2 \times x_3 \cdots x_n)^{1/n}$

Geometric mean $= (110 \times 120 \times 125)^{1/3} = 118.16$

Average % age increase = Geometric mean − basic income salary

Average % age increase $= 118.16 - 100$

Average % age increase $= 18.16\%$

(3.30) (13)

Find the harmonic mean of the following frequency distribution:

Weekly income(Rs): 35−39, 40−44, 45−49, 50−54, 55−59, 60−64, 65−69

No. of workers: 15 13 17 29 11 10 5

x	f	x	fx
35−39	15	37	555
40−44	13	42	546
45−49	17	47	799
50−59	29	52	1508
55−59	11	57	627
60−64	10	62	620
65−69	5	67	335
------	$\Sigma(f) = 100$	--------	$\Sigma f(x) = 4990$

$$\bar{x} = \frac{\Sigma(fx)}{\Sigma(f)}$$ ∵ for grouped data

$$\bar{x} = \frac{4990}{100} = 49.9$$

(3.31) (14)

Calculate the harmonic mean for the distribution given below:

Variables: 0−5, 5−10, 10−15, 15−20, 2025, 25−30, 30−35, 35−40

Frequency: 2 5 7 13 21 16 8 3

x	f	x	$f(x)$
0−5	2	2.5	5
5−10	5	7.5	37.5
10−15	7	12.5	87.5
15−20	13	17.5	227.5
20−25	21	22.5	472.5
25−30	16	27.5	440
30−35	8	32.5	260
35−40	3	37.5	112.6
	$\Sigma(f) = 75$		$\Sigma(fx) = 1642.5$

$$\bar{x} = \frac{\Sigma(fx)}{\Sigma(f)}$$

$$\bar{x} = \frac{1642.5}{75} = 21.9$$

(3.45 (b))

(15)

The weight of 40 male students at a university are given below, find the harmonic mean:

Weight: 118-126, 127-135, 136-144, 145-153, 154-162, 163-171, 172-180

frequency: 3 5 9 12 5 4 2

x	f	x	fx
118-126	3	122	366
127-135	5	131	655
136-144	9	140	1260
145-153	12	149	1788
154-162	5	158	790
163-171	4	167	668
172-180	2	176	352
	$\Sigma f = 40$		$\Sigma(fx) = 5879$

$$\bar{x} = \frac{\Sigma(fx)}{\Sigma(f)}$$

$$\bar{x} = \frac{5879}{40} = 146.97$$

(3.46)

(16)

The following table shows the distribution of the maximum loads in short tons supported by certain cable produced by a company. Find the harmonic mean.

Max. loads short tons: 9.8-10.2, 10.3-10.7, 10.8-11.2, 11.3-11.7, 11.8-12.2, 12.3-12.7

No. of cables 1 7 12 17 14 6 4

x	f	x	fx
9.8-10.2	7	10	70
10.3-10.7	12	10.5	126
10.8-11.2	17	11	187
11.3-11.7	14	11.5	161
11.8-12.2	6	12	72
12.3-12.7	4	12.5	50
	$\Sigma f = 60$		$\Sigma(fx) = 666$

$$\bar{x} = \frac{\Sigma(fx)}{\Sigma(f)}$$

$$\bar{x} = \frac{666}{60} = 11.1$$

(3.41) (17)

The following distribution shows Kilowatt-hours of electricity used in one by 75 residential consumers in a certain locality of 98 - Find harmonic mean.

Consumption in kilowatt hours: 5-25, 25-44, 45-64, 65-84, 85-104, 105-124, 125-144, 145-164

No. of consumers : 4 6 14 22 4 5 7 3

x	f	x	fx
5-24	4	14.5	58
25-44	6	34.5	207
45-64	14	54.5	763
65-84	22	74.5	1639
85-104	14	94.5	1323
105-124	5	114.5	572.5
125-144	7	134.5	941.5
145-164	3	154.5	463.5
	$\Sigma(f) = 75$		$\Sigma(fx) = 5967.5$

$$\bar{x} = \frac{\Sigma(fx)}{\Sigma(f)}$$

$$\bar{x} = \frac{5967.5}{75} = 79.57$$

(3.32) (18)

Find arithmetic mean, 64, 65, 66, 65, 66, 67.

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{64+65+66+65+66+67}{6} = \frac{393}{6} = 65.5$$

(19)

If a car is running at the rate of 15 km/hr during the first 30 km, at 20 km/hr during the second 30 km and at 25 km/hr during the third 30 km. Distance is constant but times are variable then find harmonic mean?

$$\bar{x} = \frac{n}{\Sigma\left(\frac{1}{x}\right)}$$ ∵ for ungrouped data.

$$\bar{x} = \text{Reciprocal of } \frac{1/15 + 1/20 + 1/25}{3}$$

$$\bar{x} = \frac{3}{0.06667 + 0.5000 + 0.04000}$$

$$\bar{x} = \frac{3}{0.15667} = 19.15 \text{ km/hr approximately}$$

(3.19) (20)

Calculate harmonic mean of the annual incomes of 15 families:
60, 80, 90, 96, 120, 150, 200, 360, 480, 520, 1060, 1200, 1450, 2500, 7200 : Rs

x	$1/x$
60	$1/60 = 0.0166$
80	$1/80 = 0.0125$
90	$1/90 = 0.0111$
96	$1/96 = 0.0104$
120	$1/120 = 0.0083$
150	$1/150 = 0.0066$
200	$1/200 = 0.005$
300	$1/300 = 0.0027$
480	$1/480 = 0.0021$
520	$1/520 = 0.0019$
1060	$1/1060 = 0.0009$
1200	$1/1200 = 0.0008$
1450	$1/1450 = 0.007$
2500	$1/2500 = 0.0004$
7200	$1/7200 = 0.0001$
$n = 15$	$\Sigma(1/x) = 0.0801$

$$\bar{x} = \frac{n}{\Sigma(1/x)}$$

$$x = \frac{15}{0.0801} = 187.7$$

(3-18 (b)) (21)

The monthly incomes of 10 families in a certain locality are given

below, find the harmonic mean.

Family:	A	B	C	D	E	F	G	H	I	J
Income:	85	70	10	75	500	8	42	250	40	36

x	$1/x$
85	$1/85 = 0.0118$
70	$1/70 = 0.0143$
10	$1/10 = 0.10$
75	$1/75 = 0.133$
500	$1/500 = 0.002$
8	$1/8 = 0.125$
42	$1/42 = 0.238$
250	$1/250 = 0.004$
40	$1/40 = 0.25$
30	$1/30 = 0.0333$
$n = 10$	$\Sigma(1/x) = 0.3525$

$$\bar{x} = \frac{n}{\Sigma(1/x)}$$

$$\bar{x} = \frac{10}{0.3525} = 28.36$$

(3.28) (22)

A man travels from A to B at average speed of 30 miles per hour and ~~there~~ returns from B to A along the same route at an average speed of 60 miles per hour, find average speed of entire journey.

x	$1/x$
30	$1/30 = 0.0333$
60	$1/60 = 0.0166$
$n = 2$	$\Sigma(1/x) = 0.05$

$$\bar{x} = \frac{n}{\Sigma(1/x)}$$

$$\bar{x} = \frac{2}{0.05} = 40 \text{ miles}$$

(3.28 c) (23)

Find out the average speed of person who rides the first mile at the rate of 8 miles an hour, the next mile at the rate of 7.5 miles an hour and the third mile at the rate of 5.5 miles

an hour.

x	$1/x$
8	$1/8 = 0.125$
7.5	$1/7.5 = 0.1333$
5.5	$1/5.5 = 0.1818$
$n = 3$	$\Sigma(1/x) = 0.4401$

$$\bar{x} = \frac{n}{\Sigma(1/x)}$$

$$\bar{x} = \frac{3}{0.4401} = 6.82 \text{ miles}$$

3.29 (b)

(24)

Find out the average rate of motion in the case of a person who rides the first mile at the rate of 10 miles an hour, the next mile at the rate of 8 miles per hour and the third mile at the rate of 6 miles per hour.

x	$1/x$
10	$1/10 = 0.1$
8	$1/8 = 0.125$
6	$1/6 = 0.1666$
$n = 3$	$\Sigma(1/x) = 0.3916$

$$\bar{x} = \frac{n}{\Sigma(1/x)}$$

$$\bar{x} = \frac{3}{0.3916} = 7.66$$

Example: 3.9

(25)

Find harmonic mean from the frequency distribution of weights as:

Weight (grams): 65-84, 85-104, 105-124, 125-144, 145-164, 165-184, 185-204

f : 9 10 17 10 5 4 5

Weight (grams)	x_i	f_i	$f_i(1/x_i)$
65-84	74.5	9	0.12081
85-104	94.5	10	0.10582
105-124	114.5	17	0.14847
125-144	134.5	10	0.07435
145-164	154.5	5	0.03236
165-184	174.5	4	0.02292

185-204	194.5	5	0.02571
Σ	----	60	0.53044

$$\bar{x} = \frac{n}{\Sigma f_i (1/x_i)}$$ ∵ for grouped data

$$\bar{x} = \frac{60}{0.53044} = 113.11 \text{ grams}$$

3.30

(26)

Find harmonic mean of frequency distribution:

Weekly income Rs: 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69

No. of workers: 15 13 17 29 11 10 5

x	f	x	f/x
35-39	15	37	0.4054
40-44	13	42	0.3095
45-49	17	47	0.3617
50-54	29	52	0.5577
55-59	11	57	0.1929
60-64	10	62	0.1613
65-69	5	67	0.0746
----	$\Sigma f = 100$		$\Sigma(f/x) = 2.0631$

$$\bar{x} = \frac{\Sigma f}{\Sigma (f/x)}$$

$$\bar{x} = \frac{100}{2.0631} = 48.47$$

(27)

Calculate harmonic mean for distribution given below:

Variable: 0-5, 5-10, 10-15, 15-20, 20-25, 25-30, 30-35, 35-40

Frequency: 2 5 7 13 21 16 8 3

x	f	x	f/x
0-5	2	2.5	0.8
5-10	5	7.5	0.6666
10-15	7	12.5	0.56
15-20	13	17.5	0.7428
20-25	21	22.5	0.9333
25-30	16	27.5	0.5818
30-35	8	32.5	0.2461

35-40	3	37.5	0.8
------	$\Sigma f = 75$	------	$\Sigma(f/x) = 4.6106$

$$\bar{x} = \frac{\Sigma f}{\Sigma(f/x)}$$

$$\bar{x} = \frac{75}{4.6106} = 16.27.$$

3.17(a)

(28)

The number of cars crossing a certain bridge in a big city in 10 intervals of five minutes each was recorded as: 25, 15, 18, 30, 20, 12, 9, 16, 15, 20 find geometric mean.

x	$\log x$
25	$\log 25 = 1.3979$
~~18~~ 15	$\log 15 = 1.1761$
~~30~~ 18	$\log 18 = 1.2552$
30	$\log 30 = 1.4771$
20	$\log 20 = 1.3010$
12	$\log 12 = 1.0791$
9	$\log 9 = 0.9542$
16	$\log 16 = 1.2041$
15	$\log 15 = 1.1761$
20	$\log 20 = 0.3010$
$n = 10$	$\Sigma \log x = 12.322$

Geometric mean $= \text{antilog}\left(\frac{\Sigma \log x}{n}\right)$ ∵ for ungrouped data

Geometric mean $= \text{antilog}\left(\frac{12.322}{10}\right)$

Geometric mean $= \text{antilog}(1.2322)$

Geometric mean $= 17.06$

3.18(b)

(29)

The monthly incomes of 10 families in a certain locality are given. calculate geometric mean of the incomes (Rs).

85, 70, 10, 75, 500, 8, 42, 250, 40, 36.

x	$\log x$
85	$\log 85 = 1.9294$
70	$\log 70 = 1.8451$

10	$\log 10 = 1$
75	$\log 75 = 1.8751$
500	$\log 500 = 2.6989$
8	$\log 8 = 0.9031$
42	$\log 42 = 1.6232$
250	$\log 250 = 2.3979$
40	$\log 40 = 1.6020$
36	$\log 36 = 1.5563$
$n = 10$	$\Sigma \log x = 17.431$

Geometric mean = antilog $\left(\frac{\Sigma \log x}{n}\right)$

Geometric mean = antilog $\left(\frac{17.431}{10}\right)$

Geometric mean = antilog (1.7431)

Geometric mean = 55.35

(2.31) (30)

Calculate geometric mean for the distribution given below:

Variable: 0-5, 5-10, 10-15, 15-20, 20-25, 25-30, 30-35, 35-40

Frequency: 2 5 7 13 21 16 8 3

x	f	x	$\log x$	$f \log x$
0-5	2	2.5	0.3979	0.7958
~~10-15~~ 5-10	5	7.5	0.8751	4.3755
~~15-20~~ 10-15	7	12.5	1.0969	7.6783
~~20-25~~ 15-20	13	17.5	1.2430	16.159
~~25-30~~ 20-25	21	22.5	1.3522	28.3962
~~30-35~~ 25-30	16	27.5	1.4393	23.0288
~~35-40~~ 30-35	8	32.5	1.5119	12.0952
35-40	3	37.5	1.5740	4.722
------	$\Sigma(f) = 75$	----	-----	$\Sigma f \log x = 97.2508$

Geometric mean = antilog $\left(\frac{\Sigma f \log x}{\Sigma f}\right)$

Geometric mean = antilog $\left(\frac{97.2508}{75}\right)$

Geometric mean = antilog (1.2967)

Geometric mean = 19.8

A sample of 40 children from New York state showed that mean time ~~by~~ they spend watching television is 28.50 hours per week with a standard deviation of 4 hours. Another sample of 35 children from California showed that the mean time spent by them watching television is 23.25 hours per week with standard deviation of 5 hours using a 2.5% level of significance, can you conclude that the mean time spent watching television by children in New-York is greater than that for in California. Assume that standard deviation of two population are equal.

$\bar{x}_1 = 28.50$ hours

$\bar{x}_2 = 23.25$ hours

$S_1 = 4$

$S_2 = 5$

$n_1 = 40$

$n_2 = 35$

- $H_0 : \mu_1 - \mu_2 = 0$

  $H_1 : \mu_1 - \mu_2 \neq 0$

- $\alpha = 2.5\%$

$$t = \frac{(\bar{x}_1 - \bar{x}_2)(\mu_1 - \mu_2)}{S_p \sqrt{1/n_1 + 1/n_2}}$$

with $d.f = n_1 + n_2 - 2$

$$S_p = \sqrt{\frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{(n_1 + n_2 - 2)}}$$

$$S_p = \sqrt{\frac{(40-1)16 + (35-1)25}{40 + 35 - 2}} = \sqrt{\frac{1474}{73}} = \sqrt{20.19} = 4.49$$

$$t = \frac{(28.5 - 23.25) - (0)}{4.49 \sqrt{1/40 + 1/35}}$$

$$t = \frac{5.25}{4.49(0.2302)} = \frac{5.25}{1.033598} = 5.0793$$

$d.f = 73$

- Critical region:

$t \geq t_{\frac{\alpha}{2}}$, V

$5.0793 \geq t_{\frac{0.025}{2}}, 73$

$5.0793 \geq 0.01, 73$

$5.0793 \geq 2.326$

- Conclusion:-

Reject, H_0

(32)

Calculate and analyze the correlation between the number of study hours and the number of sleeping hours of different students. Also interpret the result:

$x = 11, 12, 13, 14, 15$

$y = 15, 14, 13, 12, 16$

x	y	$(x-\bar{x})$	$(x-\bar{x})^2$	$(y-\bar{y})$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
11	15	-2	4	1	1	-2
12	14	-1	1	0	0	0
13	13	0	0	-1	1	0
14	12	1	1	-2	4	-2
15	16	2	4	2	4	4
$\bar{x}=13$	14	0	10	0	10	0

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2 \Sigma(y-\bar{y})^2}}$$

$r = 0$

So, no relation exists.

(33)

Arrange the following marks in a frequency table, taking classes as: 10 - 19, 20 - 29 ..... Also make a histogram.

3, 81, 58, 81, 85, 75, 61, 20, 84, 81, 87, 67, 65, 62, 61, 59, 58, 57, 91, 87, 43, 83, 40, 73, 43, 33, 95, 65, 29, 43, 42, 65, 64, 57, 95, 92, 49, 15, 27, 22, 78, 94, 48, 39, 62, 48, 77, 29, 36.

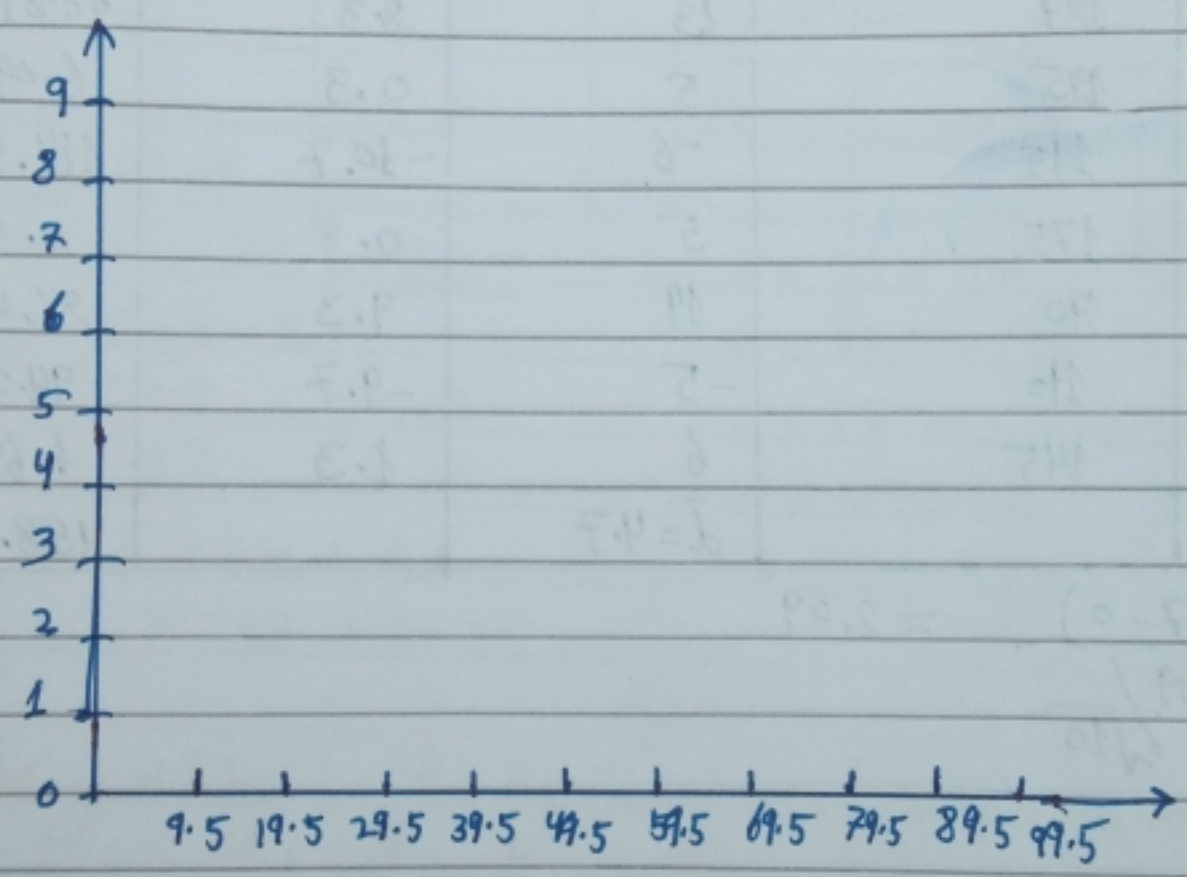
- Arranging in ascending order:-

3, 15, 22, 27, 29, 29, 33, 36, 39, 40, 42, 43, 43, 43, 48, 48, 49, 57, 57, 58, 58, 59, 61, 61, 62, 62, 64, 65, 65, 65, 67, 70, 73, 75, 77, 78, 81, 81, 81, 83, 84, 85, 87, 87, 91, 92, 94, 95, 95.

$n = 50$

(On next page)

Class interval	f	Commulative f	Relative f	Percentage f	Class boundry	Mid-points
0-9	1	1	1/50	1/50 x 100 = 2	-0.5-9.5	4.5
10-19	2	2	2/50	2/50 x 100 = 4	9.5-19.5	14.5
20-29	4	6	4/50	4/50 x 100 = 8	19.5-29.5	24.5
30-39	3	9	3/50	3/50 x 100 = 6	29.5-39.5	34.5
40-49	8	17	8/50	8/50 x 100 = 16	39.5-49.5	44.5
50-59	5	22	5/50	5/50 x 100 = 10	49.5-59.5	54.5
60-69	10	32	10/50	10/50 x 100 = 20	59.5-69.5	64.5
70-79	5	37	5/50	5/50 x 100 = 16	69.5-79.5	74.5
80-89	8	45	8/50	8/50 x 100 = 16	79.5-89.5	84.5
90-99	5	50	5/50	5/50 x 100 = 10	89.5-99.5	94.5
-------	50	------	------	-------	------	------



(34)

In a group of 20 adults 4 out of 7 women and 2 out of 13 women wear glasses. What is the probability that a person chosen at random from the group is a women or someone who wear glasses.

No.	Women	Men	Total
Wear glasses	4	2	6
Not wear glasses	3	11	14
Total	7	13	20

$$P(\text{women wear glasses}) = \frac{P(A \cap B)}{N}$$

$$P(\text{women wear glasses}) = \frac{4}{20} = \frac{1}{5}$$

Example (18.8)

10 individuals put through asterous physical training the army, their weights are recorded before and after the training. Using $\alpha = 0.05$, would you say that the programme affects the average weight of individuals? Assume the distribution of weights before and after to be approximately normal.

Weight before: 125, 195, 160, 171, 140, 201, 170, 176, 195, 139

Weight after: 136, 201, 158, 184, 145, 195, 175, 190, 190, 145

Weight before x_1	Weight after x_2	$d = x_2 - x_1$	$d - \bar{d}$	$(d - \bar{d})^2$
125	136	11	6.3	39.69
195	201	6	1.3	1.69
160	158	-2	-6.7	44.89
171	184	13	8.3	68.89
140	145	5	0.3	6.09
201	195	-6	-10.7	114.49
170	175	5	0.3	0.09
176	190	14	9.3	86.49
195	190	-5	-9.7	94.09
135	145	6	1.3	1.69
		$\bar{d} = 4.7$		458.1

$$t = \frac{(4.7 - 0)}{7.09/\sqrt{10}} = 2.09$$

$$r = 9$$

- Critical region:-

$$t \geq t_{\frac{\alpha}{2}, V}$$

$$2.09 \geq 0.025, 9$$

$$2.09 \geq 2.262$$

- Confidence Interval:-

$$\bar{d} \pm t_{\frac{\alpha}{2}, V} \frac{S_d}{\sqrt{n}}$$

- Conclusion:-

Do not reject H_0

Do not reject that $\mu_d = 0$

(36)

The following table presents shair strength (in kn/mm) and weld diameters (in mm) for a sample of spot welds. Compute from diameter and interpret the ~~diam~~ parameter.

x	y	$(x-\bar{x})$	$(x-\bar{x})^2$	$(y-\bar{y})$	$(x-\bar{x})(y-\bar{y})$
4.2	51	-0.8	0.64	-26.44	21.152
4.4	54	-0.6	0.36	-23.44	14.064
4.6	69	-0.4	0.16	-8.44	3.376
4.8	81	-0.2	0.04	3.56	-0.712
5.0	75	0	0	-2.44	0
5.2	79	0.2	0.04	1.56	0.312
5.4	89	0.4	0.16	11.56	4.624
5.6	101	0.6	0.36	23.56	14.136
5.8	98	0.8	0.64	20.56	16.448
$\bar{x}=5$	$=77.44$		2.4		73.4

$$b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

$$b = \frac{73.4}{2.4} = 30.58$$

$$a = \bar{y} - b\bar{x}$$

$$a = 77.44 - (30.58)(5)$$

$$a = 77.44 - 152.92$$

$$a = -75.48$$

$$y = a + bx + \varepsilon$$

$$y = -75.48 + 30.58x + \varepsilon$$

(37)

One of the question on the Business week subscriber study was in the past 12 months when travelling for business, what types of air line ticket did you purchase most often? The data obtained are shown in the table. Use 5% of significance and test for the independence of type of flight and type of ticket. give conclusion.

Type of flight	Type of flight	
	Domestic flight B_1	International flight B_2
First class A_1	29	22
Business/executive class A_2	95	121
Full fare economy A_3	518	135

(on next page)

Type of ticket	Type of flight		
	Domestic flight	International flight	Total
First class A_1	29 (A_1B_1)	22 (A_1B_2)	51 A_1
Business/executive class A_2	95 (A_2B_1)	121 (A_2B_2)	216 A_2
Full fare economy A_3	518 (A_3B_1)	135 (A_3B_2)	653 A_3
	642	278	920
	B_1	B_2	N

- H_0: the two variables/ ~~critical~~ criteria of classification are independent
  H_1: the two variables/ criteria of classification are dependent
- level of significance: 0.05
- Test statistics:

$$x^2 = \frac{\sum (o_{ij} - e_{ij})^2}{e_{ij}}$$

$$d.f = (r-1)(c-1)$$
$$d.f = (3-1)(2-1)$$
$$d.f = (2)(1)$$
$$d.f = 2$$

$$e = \frac{(A)(B)}{N}$$

$$e_{A_1B_1} = \frac{(51)(642)}{920}, \quad e_{A_1B_2} = \frac{(51)(278)}{920}$$

$$e_{A_1B_1} = 35.59, \quad e_{A_1B_2} = 15.41$$

$$e_{A_2B_1} = \frac{(216)(642)}{920}, \quad e_{A_2B_2} = \frac{(216)(278)}{920}$$

$$e_{A_2B_1} = 150.73, \quad e_{A_2B_2} = 65.27$$

$$e_{A_3B_1} = \frac{(653)(642)}{920}, \quad e_{A_3B_2} = \frac{(653)(278)}{920}$$

$$e_{A_3B_1} = 655.68, \quad e_{A_3B_2} = 192.32$$

O_{ij}	e_{ij}	$O_{ij} - e_{ij}$	$(O_i - e_{ij})^2$	$\frac{(O_{ij} - e_{ij})^2}{e_{ij}}$
29	35.59	-6.59	43.4281	1.2202
95	150.73	-55.73	3105.8329	20.6053
518	655.68	-137.68	18955.7824	28.9101
22	15.41	6.59	43.4281	2.8182
121	65.27	55.73	3105.8329	47.5844

135	192.32	-57.32	328.5824	17.08394
				$x^2 = 118.2221$

- Critical region :-

$$x^2 = x^2_{\alpha, d.f}$$

$$118.2221 \geq x^2_{(0.05)(2)}$$

$$118.2221 \geq 5.99$$

- Conclusion :

Reject, H_0

(38)

The Los angles times regularly reports the air quality index for various areas of Southern California. A sample quality index values for pomona (a fruit tree) provided the following data 28, 42, 58, 48, 45, 55, 60, 49, 50. Compute sample variance and standard deviation.

x	$(x-\bar{x})$	$(x-\bar{x})^2$
28	-20.33	413.3089
42	-6.33	40.0689
45	-3.33	11.0889
48	-0.33	0.1089
49	0.67	0.4489
50	1.67	2.7889
55	6.67	44.4889
58	9.6	93.5089
60	11.67	136.1889
$\bar{x} = 48.33$		$\Sigma = 742.0001$

- Variance : ∵ for sample data

$$S^2 = \frac{\Sigma(x-\bar{x})^2}{N-1}$$

$$S^2 = \frac{742.0001}{9-1} = \frac{742.0001}{8} = 92.7500$$

- Standard deviation : ∵ for sample data

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{N-1}}$$

$$\sigma = \sqrt{\frac{742.0001}{9-1}} = \sqrt{\frac{742.0001}{8}} = \sqrt{92.7500} = 9.6307$$

(39)

Three applicants are to be selected at random out of 4 boys and 6 girls. What is the probability of selectivity i) all girls ii) all boys iii) atleast one by.

Boys = 4

Girls = 6

Total = 10

$$n(S) = \binom{10}{3}$$

$$^{n}C_{r} = \frac{10!}{3!(10-3)!} = \frac{10!}{3!(7)!} = \frac{362800}{6(5040)} = \frac{3628800}{30240} = 120$$

- All girls:-

$$n(A) = \binom{4}{0} \cdot \binom{6}{3}$$

$$n(A) = \left(\frac{4!}{0!(4-0)!}\right)\left(\frac{6!}{3!(6-3)!}\right) = \left(\frac{24}{24}\right) \cdot \left(\frac{720}{3!(3)!}\right) = \frac{720}{6(6)} = \frac{720}{36} = 20$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{20}{120} = \frac{1}{6}$$

- All boys:

$$n(B) = \binom{6}{0}\binom{4}{3}$$

$$n(B) = \frac{6!}{0!(6-0)!}\left(\frac{4!}{3!(4-3)!}\right) = \left(\frac{720}{720}\right)\left(\frac{4!}{3!(1)!}\right) = \frac{24}{6} = 4$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{120} = \frac{1}{30}$$

- Atleast one boy:-

$$n(C) = \binom{4}{1}\binom{6}{2} + \binom{4}{2}\binom{6}{1} + \binom{4}{3}\binom{6}{0}$$

$$n(C) = \left(\frac{4!}{1!(4-1)!}\right)\left(\frac{6!}{2!(6-2)!}\right) + \left(\frac{4!}{2!(4-2)!}\right)\left(\frac{6!}{1!(6-1)!}\right) + \left(\frac{4!}{3!(4-3)!}\right)\left(\frac{6!}{0!(6-0)!}\right)$$

$$n(C) = \left(\frac{4!}{1! \cdot 3!}\right)\left(\frac{6!}{2! \cdot 4!}\right) + \left(\frac{4!}{2! \cdot 2!}\right)\left(\frac{6!}{1! \cdot 5!}\right) + \left(\frac{4!}{3! \cdot 1!}\right)\left(\frac{6!}{0! \cdot 6!}\right)$$

$$n(C) = \left(\frac{24}{6}\right)\left(\frac{720}{48}\right) + \left(\frac{24}{4}\right)\left(\frac{720}{120}\right) + \left(\frac{24}{6}\right)\left(\frac{720}{720}\right)$$

$$n(C) = (4)(15) + (6)(6) + (4) = 100$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{100}{120} = 1.2$$

2.8(b)

(40)

Tabulate the following marks in a grouped frequency distribution.
74, 49, 103, 95, 90, 118, 52, 88, 101, 96, 72, 56, 64, 110, 97, 59, 62, 62, 96, 82, 65, 85, 105, 116, 91, 83, 99, 52, 76, 84, 89, 77, 104, 96, 84, 62, 58, 66, 100, 80, 54, 75, 55, 99, 104, 78, 66, 96, 83, 57, 60, 51, 114, 120, 121, 92, 95, 88, 64, 63, 78.

The lowest marks = 49, heighest marks = 121, range = 121 − 49 = 72
Let us take 10 marks as class interval i.e $h = 10$ and place the lower class-limit of the lowest class or group at 40. Then the frequency distribution is constructed as follows:

Marks	Tally	Frequency
40 − 49	I	1
50 − 59	~~IIII~~ IIII	9
60 − 69	~~IIII~~ IIII	9
70 − 79	~~IIII~~ II	7
80 − 89	~~IIII~~ ~~IIII~~	10
90 − 99	~~IIII~~ ~~IIII~~ II	12
100 − 109	~~IIII~~ I	6
110 − 119	IIII	4
120 − 129	II	2
Total	----	60

(41)

The followings are the scores made by two batsman A and B in a series of innings; who is the better as a run getter?

A : 12, 15, 6, 73, 7, 19, 199, 36, 84, 29
B : 47 12 76 48 4 51 37 48 13 0

x_1	x_2	$(x_1-\bar{x}_1)$	$(x_2-\bar{x}_2)$	$(x_1-\bar{x}_1)^2$	$(x_2-\bar{x}_2)^2$
12	47	−36	13.4	1296	179.56
15	12	−33	−21.6	1089	466.56
6	76	−42	42.4	1764	1797.76
73	48	25	14.4	625	207.36
7	4	−41	−29.6	1681	876.16
19	51	−29	17.4	841	302.76
199	37	151	3.4	22801	11.56
36	48	−12	14.4	144	207.36
84	13	36	−20.6	1296	424.36
29	0	−19	−33.6	361	1128.96

$\bar{x}_1 = \frac{\Sigma x_1}{n_1}$, $\bar{x}_2 = \frac{\Sigma x_2}{n_2}$

$\bar{x}_1 = \frac{480}{10} = 48$, $\bar{x}_2 = \frac{336}{10} = 33.6$

$S.D = \sqrt{\frac{\Sigma (x_1 - \bar{x}_1)^2}{n_2}}$, $S.D = \sqrt{\frac{\Sigma (x_2 - \bar{x}_2)^2}{n_2}}$

$S.D = \sqrt{\frac{31898}{10}} = 56.48$, $S.D = \sqrt{\frac{56024}{10}} = 23.67$

For A, $COV = \frac{S.D}{\bar{x}_1} \times 100$, For B, $COV = \frac{S.D}{\bar{x}_2} \times 100$

$COV = \frac{56.48}{48} \times 100 = 117.67\%$, $COV = \frac{23.67}{33.6} \times 100 = 70.45\%$

Batsman B is more consistent player. (capable)

(42)

Calculate mean deviation from mean of the data set: 45, 32, 37, 46, 41, 48, 36, 36

- Arranging in ascending order:-

32, 36, 36, 37, 39, 41, 45, 46, 48.

$\bar{x} = \frac{\Sigma x}{n}$

- $\bar{x} = \frac{360}{9} = 40$

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
32	-8	8
36	-4	4
36	-4	4
37	-3	3
39	-1	1
41	1	1
45	5	5
46	6	6
48	8	8
		$\Sigma = 48$

Mean deviation $= \frac{\Sigma (x_i - \bar{x})}{n}$

Mean deviation $= \frac{40}{9} = 4.44$

Coefficient of mean deviation = $\frac{\text{mean deviation}}{\text{average}}$

Coefficient of mean deviation = $\frac{4.44}{40} = 0.111$

(43)

For a set of values, the following sums are found:
$n = 15$, $\Sigma x = 480$, $\Sigma x^2 = 15735$. Find mean and standard deviation.

$$\text{Mean} = \frac{\Sigma x}{n}$$

$$\text{Mean} = \frac{480}{15} = 32$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$\text{Standard deviation} = \sqrt{\frac{15735}{15} - \left(\frac{480}{15}\right)^2}$$

$$\text{Standard deviation} = \sqrt{\frac{15735}{15} - \frac{230400}{25}}$$

$$\text{Standard deviation} = \sqrt{1049 - 1024} = \sqrt{25} = 5$$

(44)

Given the following set of values:

x :	20	11	15	10	17	19
y :	5	15	14	7	8	9

(a) Determine the equation of least square regression line?

Regression line $= Y = a + bx$

$$a = Y - b\bar{x}$$

$$b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

• Calculations:-

$$\bar{x} = \frac{\Sigma x}{n}, \quad \bar{y} = \frac{\Sigma y}{n}$$

$$x = 15.33, \quad \bar{y} = 11.33$$

x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})^2$	$(x-\bar{x})(y-\bar{y})$
20	5	4.67	-6.33	21.81	-29.56
11	15	-4.33	3.67	18.75	-15.89
15	14	-0.33	2.67	0.11	-0.88

10	17	-5.33	5.67	28.41	-30.22
17	8	1.67	-3.33	2.79	-5.56
19	9	3.67	-2.33	13.47	-8.55
				85.34	-90.66

$a = \bar{y} - b\bar{x}$

$a = 11.33 - (-1.06)(15.33) = 11.33 + 16.24 = 27.58$

$b = \dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$

$b = \dfrac{-90.66}{85.34} = -1.06$

Fitted regression line is or equation of least square regression line:

$y = 27.58 - 1.06x$

(b) Find predicted values of y for X = 10, 11, 15, 17, 19, 20.

$X = 10$

$\hat{y} = 27.58 - 1.06(10) = 27.58 - 10.6 = 16.98$

$X = 11$

$\hat{y} = 27.58 - 1.06(11) = 27.58 - 11.66 = 15.92$

$X = 15$

$\hat{y} = 27.58 - 1.06(15) = 27.58 - \cancel{1.06(}15.9 = 11.68$

$X = 17$

$\hat{y} = 27.58 - 1.0(17) = 27.58 - 18.02 = 9.56$

$X = 19$

$\hat{y} = 27.58 - 1.06(19) = 27.58 - 20.14 = 7.44$

$X = 20$

$\hat{y} = 27.58 - 1.06(20) = 27.58 - 21.2 = 6.38$

45

The measurements of resistance of by electronic component measure by multimeter at 10 different times given as:

Time :

x-measurement:

Calculate standard deviation

Standard deviation $= \sqrt{\dfrac{\Sigma(x_i - \bar{x})^2}{N-1}}$

~~Standard deviation~~ $\bar{x} = \dfrac{857}{10} = 85.7$

x	$(x-\bar{x})$	$(x-\bar{x})^2$
85	-0.7	0.49
94	8.3	68.89
96	10.3	106.09
80	-5.7	32.49
77	-8.7	75.69
78	-7.7	59.29
91	5.3	28.09
95	9.3	86.49
77	8.7	75.69
84	-1.7	2.89
		536.1

Standard deviation $= \sqrt{\frac{536.1}{10-1}} = \sqrt{\frac{536.1}{9}} = \sqrt{59.567} = 7.718$

(46)

Consider the following pocket money of 10 students of a class.
$X = 100, 105, 108, 108, 110, 115, 101, 103, 104, 105$ - Calculate geometric mean.

x	$\log x$
100	$\log 100 = 2$
105	$\log 105 = 2.02$
108	$\log 108 = 2.03$
100	$\log 100 = 2$
110	$\log 110 = 2.04$
115	$\log 115 = 2.06$
101	$\log 101 = 2.00$
103	$\log 103 = 2.01$
104	$\log 104 = 2.02$
105	$\log 105 = 2.02$
	$\Sigma \log x = 20.2$

Geometric mean $= \text{antilog}\left(\frac{\Sigma \log x}{n}\right)$

Geometric mean $= \text{antilog}\left(\frac{20.2}{10}\right)$

Geometric mean $= \text{antilog } 2.02$

Geometric mean $= 104.7 = 105$

Consider the following measurement on energy per particle:
68, 49, 70, 80, 77, 70, 77, 59, 77, 80.

(a) Calculate mean of energy per particle:-

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{707}{10} = 70.7$$

(b) Calculate median per energy particle

- Arranging data in ascending order:
  49, 59, 68, 70, 77, 77, 77, 80, 80.
- No. of observations, $n = 10$

$$\text{Median} = \frac{1}{2}\left[\text{size of}\left(\frac{n^{th}}{2} + \frac{n+2^{th}}{2}\right)\text{obs}\right]$$

$$\bar{x} = \frac{1}{2}\left[\text{size of}\left(\frac{10^{th}}{2} + \frac{10+2^{th}}{2}\right)\text{obs}\right]$$

$$\bar{x} = \frac{1}{2}\left[5^{th} + 6^{th} \text{ observation}\right]$$

$$\bar{x} = \frac{1}{2}\left(70 + 77\right) = 73.5$$

(c) Mode:-

Mode $= 77$

(d) Standard deviation:-

x	$(x-\bar{x})$	$(x-\bar{x})^2$
68	-2.7	7.29
49	-21.7	48.98
70	-0.7	0.49
80	9.3	86.49
77	6.3	39.69
70	-0.7	0.49
77	6.3	39.69
59	-11.7	136.89
77	6.3	39.69
80	9.3	86.49
		908.1

Standard deviation $= \sqrt{\dfrac{\Sigma (x-\bar{x})^2}{N-1}}$

Standard deviation $= \sqrt{\dfrac{908.1}{10-1}} = \sqrt{100.9} = 10.04$

(48)

Consider the data on energy per particle (Y) and temperature (X):

Temperature (X) : 10, 12, 14, 16, 18, 20

Energy (Y) : 18, 16, 14, 12, 10, 8

(a) Find coefficient of linear correlation of energy per particle.

$$r = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma (x-\bar{x})^2 \, \Sigma (y-\bar{y})^2}} \qquad \because \bar{x} = 15, \bar{y} = 13$$

x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
10	18	−5	5	25	25	−25
12	16	−3	3	9	9	−9
14	14	−1	1	1	1	−1
16	12	1	−1	1	1	−1
18	10	3	−3	9	9	−9
20	8	5	−5	25	25	−25
	78				70	−70

$$r = \frac{-70}{\sqrt{70 \times 70}} = \frac{-70}{\sqrt{70^2}} = \frac{-70}{70} = -1$$, perfect negative correlation.

(b) Carry out the computations to obtain estimation, regression equation, $y = a + bx$

$$y = a + b\bar{x} \quad \text{—— (1)}$$

$$a = y - b\bar{x} \quad \text{—— (2)}$$

$$b = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\Sigma (x-\bar{x})^2 (y-\bar{y})^2} = -1$$

Putting the values of b in (2)

$$a = 13 - (-1)(15) = 13 + 15 = 28$$

Then, $y = 28 - x$, estimated regression line.

(49)

Given a random sample of 25 observations from a normal population for which population mean μ is unknown of population standard deviation σ is 5. Suppose the sample mean is found to be $\bar{x} = 45$. Find 95% confidence interval for μ.

- $H: \mu = 0$

  $H_1: \mu \neq 0$
- $\alpha = 0.05$
- Test statistics:-

$$t = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

with, $\nu = n - 1$

- Calculations:

$\bar{x} = 45$

$\mu_0 = 0$

$\sigma = 5$

$n = 25$

$$t = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

$$t = \frac{45 - 0}{5/\sqrt{25}} = \frac{45}{5/5} = 45$$

$\nu = n - 1$

$\nu = 25 - 1$

$\nu = 24$

- Critical region:

$H_1: \mu \neq 0$

$t \geq t_{\frac{\alpha}{2}, \nu}$

$45 \geq t_{\frac{0.05}{2}, 24}$

$45 \geq t_{0.025, 24}$

$45 \geq 2.064$

- Conclusion:

Reject H_0

95% confidence interval

$$\bar{x} \pm t_{\frac{\alpha}{2}, \nu} \frac{5}{\sqrt{n}}$$

$$45 \pm 2.064 \times \frac{5}{\sqrt{25}}$$

$$45 \pm 2.064 \times \frac{5}{5}$$

45 ± 2.064

$45 + 2.064$, $45 - 2.064$

47.064 , 42.936

(59)

Consider, $n = 50$, $\bar{x} = 1850$ and $\sigma = 100$. Test the null hypothesis $H_0 : \mu = 1800$ against alternative hypothesis $H_1 : \mu > 1800$ at 1% level of significance.

- $H_0 : \mu = 1800$

  $H_1 : \mu > 1800$
- Level of significance

  $\alpha = 0.01$
- Test statistic,

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$v = n - 1$$

- Calculation:

$n = 50$

$\bar{x} = 1850$

$\sigma = 100$

$\mu_0 = 1800$

$$t = \frac{1850 - 100}{100/\sqrt{50}} = \frac{50}{100/7.07} = \frac{50}{14.14}$$

$t = 3.54$

$v = n - 1$

$v = 50 - 1 = 49$

- Critical region:

  $H_1 : \mu > 1800$

  $t \geq t_{\alpha, v}$

  $3.54 \geq t_{0.01, 49}$

  $3.54 \geq 2.326$
- Conclusion:

  Reject H_0.

(51)

Following are the consective weeks earning of 3 salemen by a form.

A : 52, 75, 80, 67 = 274
B : 81, 71, 45, 85 = 282
C : 60, 47, 54, 65 = 226

Calculate F-test at 5% level of significance whether difference mean earning of Saim is significant.

- $H_0 : \mu_1 = \mu_2 = \mu_3$
  H_1 : all three means are not equal
- $\alpha = 0.05$
- Test statistic:-

$$F = \frac{S_b^2}{S_w^2}$$

$$V_1 = K-1$$
$$V_2 = n-k$$

- Calculations:
- correlation factor, $C.F = \frac{T^2}{n}$

$$\text{correlation factor} = \frac{(782)^2}{12} = \frac{611524}{12} = 50960.33$$

- total sum of square, $TSS = \Sigma_i \Sigma_j x^2_{ij} - C.F$

$$TSS = 53020 - 5096.33 = 2059.67$$

- between sum of square, $BSS = \frac{\Sigma_i T^2_j}{r} - C.F$

$$BSS = \frac{205676}{4} - 50960.33 = 458.67$$

- within sum of square, $WSS = TSS - BSS$

$$WSS = 2059.67 - 458.67 = 1601$$

- ANOVA table:-

Source of variation	degree of freedom	sum of square	mean of square
Between samples	$V_1 = K-1$ $V_1 = 3-1$ $V_1 = 2$	458.67	$S_b^2 = \frac{458.67}{2}$ $S_b^2 = 229.34$
Within samples	$V_2 = n-k$ $V_2 = 12-3$ $V_2 = 9$	1601	$S_w^2 = \frac{1601}{9}$ $S_w^2 = 177.89$

$$F = \frac{S_b^2}{S_w^2}$$

$$F = \frac{229.34}{177.89} = 1.289$$

- Critical region:

$F \geq F_{\alpha}, V_1, V_2$

$1.289 \geq 5_{0.05}, 2, 9$

$1.289 \geq 4.26$

- Conclusion:-

Do not reject H_0

18.7 (a)

(3) (52)

A random sample of 10 boys had the following I.Qs: 70, 120, 110, 101, 88, 107, 83, 95, 100, 98. Find 95% are normally distributed and the variance is not known.

- $H_0 : \mu = 0$

  $H_1 : \mu \neq 0$

- $\alpha = 0.05$

- Test statistics:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$\nu = n - 1$$

- Calculations:

$$\bar{x} = \frac{972}{10} = 97.2$$

$\mu = 0$

$n = 10$

$$s = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n-1}}$$

x	$(x - \bar{x})$	$(x - \bar{x})^2$
70	-27.2	739.84
120	22.8	519.84
110	12.8	163.84
101	3.8	14.44
88	-9.2	84.64
83	14.2	201.64
95	-2.2	4.84
10	9.8	96.04

100		
98	2.8	7.84
	0.8	0.64
		1833.6

$$S = \sqrt{\frac{1833.6}{10-1}} = \sqrt{\frac{1833.6}{9}} = \sqrt{203.73} = 14.27$$

then $t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$

$$t = \frac{97.2 - 0}{14.27/\sqrt{10}} = \frac{97.2}{14.27/3.16} = \frac{97.2 \times 3.16}{14.27} = \frac{307.37}{14.27} = 21.54$$

with $\nu = n - 1$

$\nu = 10 - 1$

$\nu = 9$

- Critical region:-

$H_1 : \mu \neq 0$

$t \geq t_{\frac{\alpha}{2}, \nu}$

$t \geq t_{0.025, 9}$

$21.54 \geq t_{0.025, 9}$

$21.54 \geq 2.262$

- Conclusion:-

Reject, H_0.

$\bar{x} \pm t_{\frac{\alpha}{2}, \nu} \frac{S}{\sqrt{n}}$

$97.2 \pm 2.262 \times \frac{14.27}{\sqrt{10}}$

$97.2 \pm 2.262 \times \frac{14.27}{3.16}$

97.2 ± 10.21

$97.2 + 10.21$, $97.2 - 10.21$

107.41 , 86.99

$86.99 - 107.41$

It means our parameter does not lie in this interval, so we reject it.

(18.16) (53)

A random sample of 16 values from a normal population

showed a mean of 14.5 inches and sum of squares of deviations from this mean equal to 135 (inches)^2. Show that the assumption of a mean of 13.5 inches for the population is not reasonable and that 95% confidence limits for this mean are 39.9 and 43.1 inches.

- $H_0 : \mu = 0$
  $H_1 : \mu \neq 0$
- $\alpha = 0.05$

$$t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$$

with, $\nu = n - 1$

$\bar{x} = 14.5$

$S = 5$

$n = 5$

$\mu_0 = 0$

$$t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$$

$$t = \frac{14.5 - 0}{5/\sqrt{16}} = \frac{14.5 \times 4}{5} = 11.6$$

with, $\nu = n - 1$

$\nu = 16 - 1 = 15$

$t \geq t_{\frac{\alpha}{2}, \nu}$

$11.6 \geq t_{\frac{0.05}{2}, \nu}$

$11.6 \geq 2.262$

- Conclusion:-
  Reject, H_0

18.8

54

The masses of 12 balls bearing taken at random from a batch are, 31.4, 33.1, 35.9, 34.7, 34.5, 33.4, 35.0, 32.5, 36.9, 36.4, 35.8, 33.2. Calculate a 90% confidence interval for the mean of the population, supposed normal, from which these masses were drawn.

- $H_0 : \mu = 0$
  $H_1 : \mu \neq 0$

- $\alpha = 0.05$

with, $\nu = n - 1$

$\bar{x} = 34.4$

$\mu_0 = 0$

$n = 12$

x	$(x-\bar{x})$	$(x-\bar{x})^2$
31.4	−3	9
33.1	−1.3	1.69
35.9	1.5	2.25
34.7	0.3	0.09
33.4	−1	1
~~33.4~~ ~~34.7~~ 34.5	0.1	0.01
~~38~~ 35.0	0.6	0.36
32.5	−1.9	3.61
36.9	2.5	6.25
36.4	2	4
35.8	1.4	1.96
33.2	−1.2	1.44
412.8		36.66

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

$$S = \sqrt{\frac{31.66}{12-1}} = \sqrt{2.88} = 1.697$$

Then, $t = \dfrac{34.4 - 0}{1.697/\sqrt{12}} = \dfrac{34.4}{1.697/3.46} = \dfrac{34.4 \times 3.46}{1.697} = 70.22$

with, $\nu = n - 1$

$\nu = 12 - 1 = 11$

- Critical region:-

$H_1 : \mu \neq 0$

$t \geq t_{\frac{\alpha}{2}, \nu}$

$70.22 \geq t_{\frac{0.05}{2}, 11}$

$70.22 \geq 2.201$

- Conclusion:-

Reject, H_0

$$\bar{x} \pm t_{\frac{\alpha}{2}, v} \frac{s}{\sqrt{n}}$$

$$34.4 \pm 2.201 \times \frac{1.697}{\sqrt{12}}$$

$$34.4 \pm 1.08$$

$34.4 + 1.08$, $34.4 - 1.08$

35.48 33.2

18.5(a) (55)

The nine items of a sample had the following values: 45, 47, 50, 48, 52, 47, 49, 53, 51. Does the mean of nine items differ significantly from an assumed normal population mean of 47.5 at a $\alpha = 0.05$?

- $H_0: \mu = 47.5$
  $H_1: \mu \neq 47.5$
- $\alpha = 0.05$
- Test statistics

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

with, $v = n - 1$

- Calculations:

$\bar{x} = 49.11$

$\mu_0 = 47.5$

$n = 9$

$$s = \sqrt{\frac{\Sigma(x - \bar{x})}{n-1}}$$

x	$(x-\bar{x})$	$(x-\bar{x})^2$
45	-4.11	16.89
47	-2.11	4.45
50	0.89	0.79
52	2.89	8.35
48	-1.11	1.23
47	-2.11	4.45
49	-0.11	0.01
53	3.89	15.13
51	1.89	3.57
		54.87

$$S = \sqrt{\frac{54.87}{9-1}} = \sqrt{6.85875} = 2.62$$

Then, $t = \frac{49.11 - 47.5}{2.62/\sqrt{9}} = \frac{1.61 \times 3}{2.62} = 1.84$

with, $\nu = n - 1$

$\nu = 9 - 1$

$\nu = 8$

- Critical region:-

$H_1 : \mu \geq t_{\frac{\alpha}{2}}, \nu$

$1.84 \geq t_{\frac{0.05}{2}}, 8$

$1.84 \geq 2.306$

- Conclusion:-

Do not reject, H_0.

18.15(a)

(56)

10 cartoons are taken at random on automatic filling machine. The mean net weight of the 10 cartoons is 15.90 oz, and the sum of squared deviations is 0.276. Does the sample mean differ significantly from the intended weight of 16 oz?

- $H_0 : \mu = 15.90$

  $H_1 : \mu \neq 15.90$

- $\alpha = 0.05$

- Test statistics:

$$t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$$

with, $\nu = n - 1$

- Calculations:-

$\bar{x} = 16$

$n = 10$

$S = 0.276$

$\mu_0 = 15.9$

$$t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$$

$$t = \frac{16 - 15.9}{0.276/\sqrt{10}} = \frac{0.1 \times 3.16}{0.276} = 1.15$$

$\nu = n - 1$

$\nu = 10 - 1 = 9$

- Critical region:

$H_1 : \mu \neq 15.9$

$t \geq t_{\frac{\alpha}{2}}, \nu$

$1.15 \geq t_{\frac{0.05}{2}}, 9$

$1.15 \geq 2.262$

- Conclusion:

Do not reject H_0

(1817)

(57)

In the past a machine has produced washers having a thickness of 0.050 inches. To determine whether machine is in proper worker order a sample of 10 washers is choosen for which the mean thickness is 0.053 inches of the standard deviation is 0.003 inches. Test the hypothesis that the machines is in proper working order using $\alpha = 0.05$ and 0.09.

- $H_0 : \mu = 0.050$
- $H_1 : \mu \neq 0.050$
- $\alpha = 0.050$
- $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

with, $\nu = n - 1$

- Calculations:

$\bar{x} = 0.053$

$s = 0.003$

$n = 10$

$\mu_0 = 0.050$

$t = \dfrac{0.053 - 0.050}{0.003/\sqrt{10}} = \dfrac{0.003 \times 3.16}{0.003} = 3.16$

$\nu = 10 - 1 = 9$

- Critical region:

$\alpha = 0.05$, $\alpha = 0.01$

$t \geq \frac{\alpha}{2}, \nu$, $t = t_{\frac{\alpha}{2}}, \nu$

$3.16 \geq t_{\frac{0.05}{2}, 9}$, $3.16 \geq t_{\frac{0.01}{2}, 9}$

$3.16 \geq t_{0.025, 9}$, $3.16 \geq t_{0.005, 9}$

$3.16 \geq 2.262$, $3.16 \geq 3.250$

- Conclusion:-

Reject , H_0

Exercise: 18.18 (58)

A manufacturer claims that his light bulb have an average life time of 1500 hours. A purchaser decides to check this claim and finds that for 6 bulbs the life times are: 14, 72, 1486, 1401, 1350, 1610, 1590 hours. Does this evidence support the manufacturer's claim. Assume that the lifetimes of bulbs are normally distributed.

- $H_0: \mu = 1500$

$H_1: \mu \neq 1500$

$$t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$$

with $v = n-1$

$\bar{x} = 1484.8$

$\mu_0 = 500$

$n = 6$

$S = 102.8$

x	$(x-\bar{x})$	$(x-\bar{x})^2$
1472	-12.8	163.84
1486	1.2	1.44
1401	-83.8	7022.44
1350	-134.8	18171.04
1610	125.2	15675.04
1590	102	11067.04
8909		52100.84

$$S = \sqrt{\frac{52100.84}{6-1}} = \sqrt{10420.168} = 102.08$$

~~$S = 102.08$~~ $t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$

$$t = \frac{1484.8 - 1500}{102.08/\sqrt{6}} = \frac{-15.2 \times 2.45}{102.08} = \frac{-37.23}{102.08} = -0.36$$

with, $\nu = n-1$

$\nu = 6-1 = 5$

- Critical region:

$H_1 : \mu = 1500$

$t \leq -t_{\frac{\alpha}{2}}, \nu$

$-0.36 \leq -2.571$

- Conclusion:

Reject, H_0

(18.19 b)

(59)

Two random samples taken independently from normal populations with an identical variance yield the results:

No.	n	$\bar{x}$	S^2
Sample I	12	10	1200
Sample II	~~18~~ 18	25	900

Test the hypothesis that the true difference between the population means is 10, that is, that $\mu_2 - \mu_1 = 10$ against the alternative that, $\mu_2 - \mu_1 > 10$ at the 5 percent level of significance.

- $H_0 : \mu_2 - \mu_1 \leq 10$

$H_1 : \mu_2 - \mu_1 >$

- $\alpha = 0.05$

- Test statistics:-

$$t = \frac{(x_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

with, $\nu = n_1 + n_2 - 2$

- Calculations:

$n_1 = 12$, $n_2 = 18$

$x_1 = 10$, $\bar{x}_2 = 25$

$S_1^2 = 1200$, $S_2^2 = 900$

- To calculate S_p

$$S_p = \sqrt{\frac{(n-1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}}$$

$$S_p = \sqrt{\frac{(12-1)1200 + (18-1)900}{12 + 18 - 2}} = \sqrt{\frac{11 \times 1200 + 17 \times 900}{28}}$$

$$S_p = \sqrt{\frac{13200 + 15300}{28}} = \sqrt{\frac{28500}{28}} = \sqrt{1017.85} = 32$$

Now, $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$$t = \frac{(10-25)-(10)}{32\sqrt{\frac{1}{12} - \frac{1}{18}}} = \frac{10-25-10}{32\sqrt{0.083 + 0.0556}} = \frac{-25}{32\sqrt{0.139}} = \frac{-25}{32 \times 0.073}$$

$$t = \frac{-25}{11.936} = -1.96$$

with, $v = n_1 + n_2 - 2$

$v = 12 + 18 - 2 = 30 - 2 = 28$

- Critical region:

$H_1 : \mu_2 - \mu_1 > 10$

then, $t \geq t_{\frac{\alpha}{2}, v}$

$-1.96 \geq t_{0.05, 28}$

$-2.0 \geq 1.701$

- Conclusion:

Do not reject H_0

(69)

Write the formula's of given terms.

- Arithmatic mean:-

$\bar{x} = \dfrac{\Sigma x}{n}$ ∵ for ungrouped data.

$\bar{x} = \dfrac{\Sigma(fx)}{\Sigma(f)}$ ∵ for grouped data

- Harmonic mean:-

$\bar{x} = \dfrac{n}{\Sigma(1/x)}$ ∵ for ungrouped data

$\bar{x} = \dfrac{\Sigma(fx)}{\Sigma(f/x)}$ ∵ for grouped data

- Geometric mean:-

$\bar{x} = \sqrt[n]{x_1, x_2 \cdots x_n} = (x_1, x_2 \cdots x_n)^{1/n}$

$\bar{x} = \text{antilog}\left(\dfrac{\Sigma \log x}{n}\right)$ ∵ for ungrouped data

$$\bar{x} = \text{antilog}\left(\frac{\Sigma f \log x}{\Sigma f}\right)$$ ∵ for grouped data

- **Median:-**

$$\bar{x} = \left(\frac{n+1}{2}\right)^{th}$$ ∵ for ungrouped data

$$\bar{x} = lb + \frac{h}{f}\left(\frac{n}{2} - c\right)$$ ∵ for grouped data

→ lb = lower class boundry of $n/2$ group.
→ h = common interval
→ f = frequency of $n/2$ group.
→ $n/2$ = indicating median group.
→ c = commulative frequency.

- **Mode:-**

$$\bar{x} = lb + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$ ∵ grouped data

→ lb = lower class boundry of f_m
→ f_m = maximum frequency
→ f_1 = frequency before f_m
→ f_2 = frequency after f_m
→ h = common interval

- **Geometric mean:-** (Prove that)

$$\text{Geometric mean} = \text{antilog}\left(\frac{\Sigma \log x_i}{n}\right)$$

$$\text{Geometric mean} = \frac{1}{n}\left(\log x_1 + \log x_2 \cdots\cdots \log x_n\right)$$

$$\because \log x_1 + \log x_2 \cdots \log x_n = \log x_i$$

$$\text{Geometric mean} = \frac{1}{n}\left(\log x_i\right)$$

$$\text{Geometric mean} = \text{antilog}\left(\frac{\Sigma \log x}{n}\right)$$

(Proved)