Ch#7 Random Variable

Random variable :-

A variable whose value can be determined by the outcomes of random experiment is known as random variable.
It is commonly denoted by capital letters X, Y, Z etc

Probability distribution :-

If a random variable 'x' as value $x_1, x_2, \ldots x_n$ with corresponding probability $P(x_1), P(x_2), \ldots P(x_n)$
$\{x_1, P(x_1)\}, \{x_2, P(x_2)\}, \ldots \{x_n, P(x_n)\}$
in tabular form it can be written as.

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x	P(x)
x_1	$P(x_1)$
x_2	$P(x_2)$
⋮	⋮
x_n	$P(x_n)$

Properties of P.d

i) $P(x) \geq 0$
ii) $\sum P(x) = 1$

Discrete random variable:-

A variable is said to be discrete if it can assume finite or countable infinite value called discrete random variable.

Example:-

i) The occurence of "No. of heads" when two coins are to tossed.

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ii) The occurence of "sum of dots" when a pair of dice rolled.

P.d.F :-

Let x be a discrete random variable taking on distinct values $x_1, x_2, \ldots x_n$. Then function denoted by
$$F(x) = P(x) = \Sigma f(x)$$

Continous random variable :-

A random variable is said to be continous if it can assume every possible value in an interval $[a, b]$, $a < b$.

Example :-

i) The life time of light bulb.

ii) The temperature of room.

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P.D.F :-

i) $f(x) \geq 0$

ii) $f(x) = \int_{-\infty}^{\infty} f(x)\,dx$, for all x

Properties :-

i) $f(x) \geq 0$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

iii) The probability x takes on value in interval $[c,d]$, $c<d$ given as

$$P(c < x \leq d) = F(d) - F(c)$$

$$= \int_{-\infty}^{d} f(x)\,dx - \int_{-\infty}^{c} f(x)\,dx$$

$$= \int_{c}^{d} f(x)\,dx$$

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Mathematical Expectation :-

Let a random variable x takes the value $x_1, x_2, \ldots x_n$ with corresponding probability $f(x_1), f(x_2) \ldots f(x_n)$. then mathematical expectation or expected value of x denoted by $E(x)$ is define as

$$E(x) = x_1 f(x_1) + x_2 f(x_2) + \ldots x_n f(x_n)$$

$$E(x) = \text{Mean} = \Sigma x f(x)$$

Mean :-

$$E(x) = \Sigma x f(x)$$

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx \text{ (continouse)}$$

Variance :-

$$\begin{aligned} \text{Var}(x) &= E(x^2) - [E(x)]^2 \\ &= \Sigma x^2 f(x) - [\Sigma x f(x)]^2 \end{aligned}$$

Standard deviation :-

$$S.D(x) = \sqrt{E(x^2) - [E(x)]^2}$$

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Properties Expected values:-

$E(c) = c$, If c is a constant

$E(ax+b) = aE(x)+b$

$E(x+y) = E(x) + E(y)$

$E(x-y) = E(x) - E(y)$

$E(xy) = E(x) \cdot E(y)$

If x and y are independent

Chebyshev's Inequality

If x is a random variable having mean μ and variance $\sigma^2 > 0$ and K is any postive constant, then the probability that a value of x falls within K standard deviation of mean is at least $\left(1 - \frac{1}{K^2}\right)$ that is

$$P(\mu - K\sigma < x < \mu + K\sigma) \geq 1 - \frac{1}{K^2}$$

Moment Generating Function:-

The moment generating function (m.g.f) usually denoted by $M_0(t)$ of a random variable x about the origin if it exists is defined as the expected value of the r.v e^{tx} where t is a real variable lying in a neighbourhood of zero.

$$M_0(t) = E(e^{tx}) = \sum_{i=1}^{\infty} e^{tx} f(x) \quad (x \text{ discrete})$$

$$M_0(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx \quad , (x \text{ continous})$$

Cumulant Generating Function:-

A cumulant generating function (CGF) takes the moment of a probability density function and generates the cumulant. A cumulant of a probability distribution is a sequence of numbers that describes the distribution

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in a useful, compact way.

$$K(t) = \log_e M_o(t)$$

$$= K_1 t + K_2 \frac{t^2}{2!} + K_3 \frac{t^3}{3!} + \cdots + K_r \frac{t^r}{r!} + \cdots$$

The rth comulant given as

$$K_r = \left[\frac{d^r}{dt^r} \log_e M_o(t) \right]_{t=0}$$

Coefficient of cumulant :-

$$K_1 = \mu_1' = \mu_1$$

$$K_2 = \mu_2' - \mu_1'^2 = \mu_2$$

$$K_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3 = \mu_3$$

$$K_4 = \mu_4' - 4\mu_3'\mu_1' - 3\mu_2'^2 + 12\mu_2'\mu_1'^2 - 6\mu_1'$$

$$K_4 = \mu_4 - 3\mu_2^2$$

Characteristic Function :-

The m.g.f does not exist for many probability distribution. We then use another function called the characteristic function (c.f). The character function of a r.v x

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denoted by $\phi(t)$ is defined as expected value of the r.v e^{itx}

$\phi(t) = E(e^{itx})$

$\phi(t) = \sum e^{itx} f(x)$ Discrete

$\phi(t) = \int_{-\infty}^{\infty} e^{itx} f(x)\,dx$ continouse

Joint distribution:-

The distribution of two or more random variables which are observed simultaneously when an experiment is performed, is called their Joint distribution

Bivariate Distribution Function:-

Let X and Y be two random variable defined on the same sample S. Then the function $F(x,y)$ defined by

$F(x,y) = P(X \leq x \text{ and } Y \leq y)$

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Where $F(x,y)$ gives the probability that X will take on a value less than or equal to x and at the same time, Y will take on a value less than or equal to y, is called a bivariate or Joint distribution of X and Y.

Properties :-

i) $F(x,-\infty) = F(-\infty,y) = 0$

$F(-\infty,+\infty) = 1$

ii) $F(x,y)$ non-decreasing function of x and y, and is continuous on the right

iii) If $x_1 < x_2$ and $y_1 < y_2$ then

$P(x_1 \le X < x_2 ; y_1 \le Y < y_2) =$

$P(X < x_2, Y < y_2) - P(X < x_2, Y < y_1)$

$- P(X < x_1, Y < y_2) + P(X < x_1, Y < y_1)$

$= F(x_2,y_2) - F(x_2,y_1) - F(x_1,y_2) + F(x_1,y_1) \geq 0$

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Marginal Probability Function:-

From the Joint probability function for (x, y) we can obtain the individual probability function of x and y. Such individual probability function are called marginal probability function.

Conditional Probability Function:-

Let x and y be two discrete random variable with Joint probability function $f(x,y)$. Then the conditional probability function for x given $Y=y$, denoted as $f(x|y)$ is defined by.

$$f(x_i|y_j) = P(X=x_i \mid Y=y_j)$$

$$= \frac{P(X=x_i \text{ and } Y=y_j)}{P(Y=y_j)}$$

$$= \frac{f(x_i, y_j)}{h(y_j)}, \quad i=1,2,\dots, j=1,2,\dots$$

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Independence :-

Two discrete random variable x and y are said to be satistical independence, if and only if for all possible pairs of value (x_i, y_j) the Joint probability function $f(x,y)$ can be expressed as the product of the two merginal probability functions. that is, X and Y are independent, if

$$f(x,y) = P(X = x_i \text{ and } Y = y_j)$$
$$= p(X = x_i), p(Y = y_j) \text{ for all } i \text{ and}$$
$$= g(x)\, h(y)$$

Continuous Bivariate distributon:

The bivariate probability density function of contnuous random variable X and Y is an integrable function $f(x,y)$ satisfying following property.

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i) $f(x$

ii) $\int_{-\infty}^{\infty}\int^{\infty}$

iii) $P(a$

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i) $f(x,y) \geq 0$, for all (x,y)

ii) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$

iii) $P(a \leq x \leq b, c \leq y \leq d) = \int_{a}^{b}\int_{c}^{d} f(x,y)\,dy\,dx$

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