Popoulation

→ Finite popoulation:- A popoulation is said to be finite if it is possible to count its element.

For Example:-

→ Popoulation of words in a book.
→ popoulation of student in a college.

INFINITE Popoulation:- A popoulation is said to be infinite if it is impossible to count its elements.

Example -

→ Popoulation of jesms in the body of patient.
→ Popoulation of stase in the sky.

Sampling Unit:-

Sampling unit are those basic unit of popoulation of which the sample

is selected.
These units must be distinct and should not be overlaping) → common.

Example:-

The sampling unit may be student light ~~bubble~~ bulb, a house hold, a farm a city e.t.c

Census:-

The collection of information from all the unit of population is called making a census.

Basic aims of Sampling:-

To get maximum information about a population without examing each and every unit of population.

To find reliability of estimates derived from the sample.

Advantages Of Sampling:-

It is cheaper than complete count.
The data are collected and analyzed more quickly.

Sampling saves the time.

A small & fraction of population gives sometimes comprehensive & detail result.

Target popoulation:-

A population about which we want to get some information is called target population.

Real popoulation:-

A population consisting of the items which are all present physically is called real population.

Hypothetical popoulation:-

A population consist of the results of repeated trails is called hypothetical population

QUESTION 11.1

Given Data:-

$n = 2$

$N = 5$

$\binom{N}{n} = \binom{5}{2} = 10$

Because of without replacement-

Sno	Sample value	$\bar{x} = \Sigma x / n$
1	3,6	4.5
2	3,9	6
3	3,12	7.5
4	3,15	9
5	6,9	7.5
6	6,12	9.0
7	6,15	10.5
8	9,12	10.5
9	9,15	12.0
10	12,15	13.5

The Sampling Distribution of Sample mean $(\bar{X})$ & its mean & varience are.

$\bar{X}$	f	$f(\bar{X})$	$\bar{X} f(\bar{X})$	$\bar{X}^2$	$\bar{X}^2 f(\bar{X})$
4.5	1	$f/\Sigma f = 1/10$	4.5/10	20.25	20.25/10
6	1	1/10	6/10	36	36/10
7.5	2	2/10	15/10	56.25	112.5/10
9	2	2/10	18/10	81	162/10
10.5	2	2/10	21/10	110.25	220.5/10
12	1	1/10	12/10	144	144/10
13.5	1	1/10	13.5/10	182.25	182.25/10
	$\Sigma f = 10$		$\Sigma \bar{X} f(\bar{X}) = \frac{90}{10}$		87.75/10

relative frequency $(\bar{X}) = \frac{f}{\Sigma f}$

Mean $E(\bar{X}) = \Sigma \bar{X} f(\bar{X})$

(Mean of sampling distribution of $(\bar{X})$)

$= \frac{90}{10} \rightarrow 9$

$E(\bar{X}) = 9$

$Var(X) = \Sigma \bar{X}^2 f(\bar{X}) - (\Sigma \bar{X} f(\bar{X}))^2$

$= \frac{877.5}{10} - \left(\frac{90}{10}\right)$

$= 87.75 - 81$

$= 6.75$

⇒ Sampling without Replacemet ^NC_n

Sample mean $\mu_{\bar{x}} = \sum \bar{x} f(\bar{x})$

Sample Varience $\sigma^2_{\bar{x}} = \sum \bar{x}^2 f(\bar{x}) - [\sum \bar{x} f(\bar{x})]^2$

Sample S.D $\sigma_{\bar{x}} = \sqrt{\quad}$ "

population mean $\mu = \frac{\sum x}{N}$

population varience

papoulation S.D $\sigma^2 = \frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2$

$\sigma = \sqrt{\quad}$ "

Verification:-

(i) $\mu_{\bar{x}} = \mu$

$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} \times \frac{N-n}{N-1}$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}}$

Example 11.2

Data:-

$n = 3$

$N = 4$

population = 4, 5, 5, 7

Number of sample that can be drawn without replacement $^NC_n = {}^4C_3$

$= 4$

Possible Sample (x_1, x_2, x_3)	Sample mean $\left(\bar{x} = \frac{\sum x}{n}\right)$
4, 5, 5	14/3
4, 5, 7	16/3
4, 5, 7	16/3
5, 5, 7	17/3

Sampling Distribution of Sample mean

$\bar{x}$	f	$f(\bar{x})$	$\bar{x}f(\bar{x})$	$\bar{x}^2$	$\bar{x}^2 f(\bar{x})$
14/3	1	1/4	14/12	196/9	196/36
16/3	2	2/4	32/12	256/9	512/36
17/3	1	1/4	17/12	289/9	289/36
	$\sum f = 4$		$\sum = \bar{x}f(\bar{x}) = \frac{63}{12}$		$\sum \bar{x}^2 f(\bar{x}) = \frac{997}{36}$

For mean of Sampling Distribution-

$$\mu_{\bar{x}} = \sum \bar{x} f(\bar{x})$$

$$\mu_{\bar{x}} = \frac{63}{12}$$

$$\mu_{\bar{x}} = 5.25$$

For S.D of Sampling Distribution-

$$S.D(\bar{x}) = \sqrt{\sum \bar{x}^2 f(\bar{x}) - [\sum \bar{x} f(\bar{x})]^2}$$

$$= \sqrt{\frac{997}{36} - \left(\frac{63}{12}\right)^2}$$

$= \sqrt{27.694 - 27.562}$

$S.D(\bar{X}) = \sqrt{0.132}$

$= 0.363$

Population:-

X	X^2
4	16
5	25
5	25
7	49
$\Sigma X = 21$	$\Sigma X^2 = 115$

population mean $= \mu = \frac{\Sigma X}{N}$

$= \frac{21}{4}$

$\mu = 5.25$

population S.D $= \sigma = \sqrt{\frac{\Sigma X^2}{N} - \left(\frac{\Sigma X}{N}\right)^2}$

$= \sqrt{\frac{115}{4} - \left(\frac{21}{4}\right)^2}$

$= \sqrt{28.75 - 27.56} \Rightarrow \sqrt{1.19}$

$S.D = \sigma = 1.091$

Verification:-

$$\mu_{\bar{x}} = \mu$$

$$5.25 = 5.25$$

L.H.S = R.H.S

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}}$$

$$0.363 = \frac{1.091}{\sqrt{3}} \times \sqrt{\frac{4-3}{4-1}}$$

$$0.363 = \frac{1.091}{7.732} \times \sqrt{\frac{1}{3}}$$

$$0.363 = \frac{1.091}{1.732} \times 0.577$$

$$0.363 = 0.6299 \times 0.577$$

$$0.363 = 0.363$$

L.H.S = R.H.S

Exercise

Data

$n = 2$

$N = 5$

Population $= 3, 7, 11, 15, 19$

Number of Samples that can be drawn without replacement $= {}^{N}C_{n} = {}^{5}C_{2}$

$= 10$

Possible Samples	$\bar{X}$
3, 7	5
3, 11	7
3, 15	9
3, 19	11
7, 11	9
7, 15	11
7, 19	13
11, 15	13
11, 19	15
15, 19	17

Sampling Distribution of Sample mean

$\bar{X}$	f	$f(\bar{X})$	$\bar{X} f(\bar{X})$	$\bar{X}^2$	$\bar{X}^2 f(\bar{X})$
5	1	1/10	5/10	25	25/10
7	1	1/10	7/10	49	49/10
9	2	2/10	18/10	81	162/10
11	2	2/10	22/10	121	242/10
13	2	2/10	26/10	169	338/10
15	1	1/10	15/10	225	225/10
17	1	1/10	17/10	289	289/10
	$\Sigma f = 10$		110/10		$\Sigma = 1330/10$

$$\mu_{\bar{x}} = \bar{x} f(\bar{x})$$

$$= \frac{110}{10} \Rightarrow 11$$

$$S.D = \sqrt{\bar{x}^2 f(\bar{x}) - [\bar{x} f(\bar{x})]^2}$$

$$= \sqrt{\frac{1330}{10} - \left[\frac{110}{10}\right]^2}$$

$$= \sqrt{133 - 121}$$

$$= \sqrt{12} \Rightarrow S.D \rightarrow 3.46$$

QUESTION #3

Data

$n = 2$
$N = 4$
Population $= 16, 18, 20, 22$

No of Sample that can be drawn without replacement $^{N}C_{n} = {}^{4}C_{2} = 6$

Possible Samples (x_1, x_2)	Sample mean $\bar{x} = \frac{\Sigma x}{n}$
16, 18	17
16, 20	18
16, 22	19
18, 20	19
18, 22	20
20, 22	21

Sampling Distribution of Sample mean

$\bar{x}$	f	$f(\bar{x})$	$\bar{x}f(\bar{x})$	$\bar{x}^2$	$\bar{x}^2 f(\bar{x})$
17	1	1/6	17/6	289	289/6
18	1	1/6	18/6	324	326/6
19	2	2/6	38/6	361	722/6
20	1	1/6	20/6	400	400/6
21	1	1/6	21/6	441	441/6
	$\Sigma = 6$		$= 114/6$		$\frac{2176}{6}$

$$\mu_{\bar{x}} = \sum \bar{x} f(\bar{x})$$

$$= \frac{114}{6} \Rightarrow 19$$

$$Var(\bar{x}) = \sum \bar{x}^2 f(\bar{x}) - [\sum \bar{x} f(\bar{x})]^2$$

$$= \frac{2176}{6} - \left[\frac{114}{6}\right]^2 \Rightarrow 362.67 - 361$$

$$= 1.67$$

Population:-

X	X^2
16	256
18	324
20	400
22	484
$\Sigma = 76$	$\Sigma = 1464$

$$\mu = \frac{\sum x}{N} = \frac{76}{4}$$

$$\mu = 19$$

$$\sigma^2 = \frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2$$

$$= \frac{1464}{4} - \left(\frac{76}{4}\right)^2$$

$$\sigma^2 = 366 - 361$$

$$= 5$$

Verification:-

$$\mu_{\bar{x}} = \mu$$

$$19 = 19$$

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7^2

$7-361$

$$Var(\bar{X}) = \frac{\sigma^2}{n} \times \frac{N-n}{N-1}$$

$$1.67 = \frac{5}{2} \times \frac{4-2}{4-1}$$

$$1.67 = \frac{5}{2} \times \frac{2}{3}$$

$$1.67 = 2.5 \times 0.6$$

$$1.67 = 1.67$$

$$L.H.S = R.H.S$$

Question # 07

$n = 2$

$N = 3$

Population = 20, 40, 60

Number of sample that can be drawn with replacement $(N)^n = (3)^2 = 9$

Possible Sample (X_1, X_2)	Sample mean $\bar{X} = \Sigma X / n$
20, 420	20
20, 40	30
20, 60	40
40, 20	30
40, 40	40
40, 60	50
60, 20	40
60, 40	50
60, 60	60

Sampling Distribution of Sample mean

$\bar{X}$	f	$f(\bar{X})$	$\bar{X}f(\bar{X})$	$\bar{X}^2$	$\bar{X}^2 f(\bar{X})$
20	1	1/9	20/9	400	400/9
30	2	2/9	60/9	900	1800/9
40	3	3/9	120/9	1600	4800/9
50	2	2/9	100/9	2500	5000/9
60	1	1/9	60/9	3600	3600/9
	$\Sigma = 9$		$\Sigma = \frac{360}{9}$		$= \frac{15600}{9}$

$$Var(\bar{x}) = \Sigma \bar{x}^2 f(\bar{x}) - [\Sigma \bar{x} f(\bar{x})]^2$$

$$= \frac{1500}{9} - \left[\frac{3600}{9}\right]^2$$

$$= 1733.33 - 1600$$

$$= 133.33$$

Population :-

X	X^2
20	400
40	1600
60	3600
$\Sigma = 120$	$\Sigma = 5600$

$$\sigma^2 = \frac{\Sigma X^2}{N} - \left(\frac{\Sigma X}{N}\right)^2$$

$$= \frac{5600}{3} - \left(\frac{120}{3}\right)^2$$

$$= 1866.67 - 1600$$

$$\sigma^2 = 266.67$$

QUESTION #2

Data :-

$n = 3$

$N = 5$

Population :- 2, 6, 8, 12, 14

Number of samples that can be drawn without replacement

$^{N}C_n = {}^5C_3 = 10$

Possible Sample (X_1, X_2, X_3)	Sample mean $\bar{X} = \frac{\Sigma X}{n}$
2, 6, 8	16/3
2, 6, 12	20/3
2, 6, 14	22/3
2, 8, 12	22/3
2, 8, 14	24/3
2, 12, 14	28/3
6, 8, 12	26/3
6, 8, 14	28/3
6, 12, 14	32/3
8, 12, 14	34/3

Sampling Distribution Of Sample Mean -

$\bar{X}$	f	$f(\bar{X})$	$\bar{X} f(\bar{X})$	$\bar{X}^2$	$\bar{X}^2 f(\bar{X})$
16/3	1	1/10	16/30	259/9	256/90
20/3	1	1/10	20/30	400/9	400/90
22/3	2	2/10	44/30	484/9	968/90
24/3	1	1/10	24/30	576/9	576/90
26/3	1	1/10	26/30	676/9	676/90
28/3	2	2/10	56/30	784/9	1568/90
32/3	1	1/10	32/30	1024/9	1024/90
34/3	1	1/10	34/30	1156/9	1156/90
	$\Sigma f = 10$		$\Sigma = \frac{252}{30}$		$\Sigma = \frac{6624}{90}$

$$\mu_{\bar{x}} = \Sigma \bar{x} f(\bar{x})$$

$$\mu_{\bar{x}} = \frac{252}{30}$$

$$\mu_{\bar{x}} = 8.4$$

$$\sigma^2_{\bar{x}} = \Sigma \bar{x}^2 f(\bar{x}) - [\Sigma \bar{x} f(\bar{x})]^2$$

$$\sigma^2_{\bar{x}} = \frac{6624}{90} - \left(\frac{252}{30}\right)^2$$

$$\sigma^2_{\bar{x}} = 73.6 - (8.4)^2$$

$$= 73.6 - 70.56$$

$$\sigma^2_{\bar{x}} = 3.04$$

Population:-

x	x^2
2	4
6	36
8	64
12	144
14	196
$\Sigma = 42$	$\Sigma = 444$

$(\bar{x})$
/90
0/90
8/90
/90
/90
8/90
4/90
/90
624
90

$$\mu = \frac{\Sigma x}{N}$$

$$\mu = \frac{42}{5}$$

$$\mu = 8.4$$

$$\sigma^2_{\bar{x}} = \frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2$$

$$= \frac{444}{5} - \left(\frac{42}{5}\right)^2$$

$$= 88.8 - (8.4)^2$$

$$= 88.8 - 70.56$$

$$= 18.24$$

Verification:-

$$\mu_{\bar{x}} = \mu$$

$$8.4 = 8.4$$

$$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} \times \frac{N-n}{N-1}$$

$$3.04 = \frac{18.24}{3} \times \frac{5-3}{5-1}$$

$$= \frac{18.24}{3} \times \frac{2}{4}$$

$$3.04 = 3.04$$

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QUESTION # 05

Data

$n = 3$

$N = 6$

Population = 4, 5, 5, 7, 9, 10

Number of sample that can be drawn without replacement $^{N}C_{n} = {}^{6}C_{3} = 20$

Possible Sample	Sample mean
4, 5, 5	14/3
4, 5, 7	16/3
4, 5, 9	18/3
4, 5, 10	19/3
4, 5, 7	16/3
4, 5 9	18/3
4, 5, 10	19/3
4, 7 9	20/3
4, 7, 10	21/3
4 9, 10	23/3
5, 5, 7	17/3
5, 5, 9	19/3
5, 5, 10	20/3
5, 7, 9	21/3
5, 7, 10	22/3
5, 9, 10	24/3
5, 7, 9	21/3
5, 7, 10	22/3
5, 9, 10	24/3
7, 9, 10	26/3

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Sampling Distribution of Sample mean.

$\bar{x}$	$f($	$f(\bar{x})$	$\bar{x}f(\bar{x})$	$\bar{x}^2$	$\bar{x}^2 f(\bar{x})$
14/3	1	1/20	14/60	196/9	196/180
16/3	2	2/20	32/60	256/9	512/180
17/3	1	1/20	17/60	289/9	289/180
18/3	2	2/20	36/60	324/9	648/180
19/3	3	3/20	57/60	361/9	1083/180
20/3	2	2/20	40/60	400/9	800/180
21/3	3	3/20	63/60	441/9	1323/180
22/3	2	2/20	44/60	484/9	968/180
23/3	1	1/20	23/60	529/9	529/180
24/3	2	2/20	48/60	570/9	1152/180
26/3	1	1/20	26/60	676/9	676/180
	$\Sigma = 20$		$\Sigma = \frac{400}{60}$		$\Sigma = \frac{8176}{180}$

$$\mu_{\bar{x}} = \Sigma \bar{x} f(\bar{x})$$

$$= \frac{400}{60} \Rightarrow \frac{20}{3} \Rightarrow \mu_{\bar{x}} = 6.667$$

$$\sigma^2_{\bar{x}} = \Sigma \bar{x}^2 f(\bar{x}) - [\Sigma \bar{x} f(\bar{x})]^2$$

$$= \frac{8176}{180} - \left(\frac{400}{60}\right)^2$$

$$= 45.4222 - (6.667)^2$$

$$= 45.422 - 44.44$$

$$\sigma^2_{\bar{x}} = 0.98$$

Papoulation:-

x	x^2
4	16
5	25
5	25
7	49
9	81
10	100

$$\mu = \frac{\Sigma X}{N}$$

$$\mu = \frac{40}{6} \Rightarrow 6.667$$

$$\sigma^2 = \frac{\Sigma X^2}{N} - \left(\frac{\Sigma X}{N}\right)^2$$

$$= \frac{296}{6} - \left(\frac{40}{6}\right)^2$$

$$= 48 \quad 49.33 - 44.44$$

$$\sigma^2 = 4.89$$

Verification:-

$$\mu_{\bar{X}} = \mu$$

$$6.667 = 6.667$$

$$\sigma^2_{\bar{X}} = \frac{\sigma^2}{n} \times \frac{N-n}{N-1}$$

$$0.98 = \frac{4.89}{3} \times \frac{6-3}{6-1}$$

$$0.98 = 0.98$$

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QUESTION #6

Data:-

$n = 2$

$N = 2$

Population = 24, 35

No of Sample that can be drawn with replacement $(N)^n = (2)^2 = 4$

Possible Samples (X_1, X_2)	Sample mean $\bar{X} = \frac{\Sigma X}{n}$
24, 24	24
24, 35	29.5
35, 24	29.5
35, 35	35

Sampling Distribution Of Sample mean-

$\bar{X}$	f	$f(\bar{X})$	$\bar{X} f(\bar{X})$	$\bar{X}^2$	$\bar{X}^2 f(\bar{X})$
24	1	1/4	24/4	576	576/4
29.5	2	2/4	59/4	870.25	1740.5/4
35	1	1/4	35/4	1225	1225/4
	$\Sigma f = 4$		$\Sigma = \frac{118}{4}$		$\Sigma = \frac{3541.5}{4}$

$\mu_{\bar{X}} = \Sigma \bar{X} f(\bar{X})$

$\mu_{\bar{X}} = \frac{118}{4}$

$\mu_{\bar{X}} = 29.5$

$$\sigma_{\bar{x}} = \sqrt{\Sigma \bar{x}^2 f(\bar{x}) - [\Sigma \bar{x} f(\bar{x})]^2}$$

$$\sigma_{\bar{x}} = \sqrt{\frac{3541.5}{4} - \left(\frac{118}{4}\right)^2}$$

$$= \sqrt{885.375 - (29.5)^2}$$

$$= \sqrt{885.375 - 870.25}$$

$$\sigma_{\bar{x}} = 3.89$$

Population:-

X	X^2
24	576
35	1225
$\Sigma X = 59$	$\Sigma X^2 = 1801$

$$\mu = \frac{\Sigma X}{N} \Rightarrow \mu = \frac{59}{2} \Rightarrow 29.5$$

$$\sigma = \sqrt{\frac{\Sigma X^2}{N} - \left(\frac{\Sigma X}{N}\right)^2}$$

$$\sigma = \sqrt{\frac{1801}{2} - \left(\frac{59}{2}\right)^2}$$

$$\sigma = \sqrt{900.5 - (29.5)^2}$$

$$\sigma = \sqrt{900.5 - 870.25}$$

$$\sigma = \sqrt{30.25}$$

$$\sigma = 5.5$$

Verification:-

$$\mu_{\bar{x}} = \mu$$

$$29.5 = 29.5$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$3.89 = \frac{5.5}{\sqrt{2}}$$

$$3.89 = \frac{5.5}{1.4142}$$

$$3.89 = 3.89$$

Sampling with Replcement $(N)^n$

↳ If the Sampling unit is drawn from the population & is returned to the population before the next Selection then this procedure is called **Sampling with replacement**

↳ In this case the size of population doesnot change and the probability of selection for every unit remains same

↳ In This case unit may be selected more then once

Sampling without Replacement

$^N C_n$

↳ If the selected sample unit is not returned to the population before the next section then this procedure is called **Sampling without replacement**

↳ In this case the size of population will be reduced after every selection, So the probability of selection for each unit will be different.

↳ In this case a unit is selected only once

↳ Sampling Distribution:-

The probability distribution of any sample statistic called sampling distribution.

↳ Probability Sampling / Random Sampling-

The process in which the selection of units from population depends upon the chance is called **probability Sampling**.

↳ The probability of selection for every unit may be the equal or unequal but it should be taken & not equal to zero

Types Of probability Sampling

- Simple Random Sampling
- Systematic Sampling
- Stratified Random Sampling

↳ Non-Probability Sampling:-

In non-probability Sampling, the selection of Sample is not based upon chance. Some body may use his experience for the selection of the sample.

It is also called non Random Sampling

Types:-

- Purposive Sampling
- Quota Sampling

Bias :-

Bias is the difference b/w the **expected value** of statistic and the true value of **unknown parameter**.

It is defined as Bias = $E(T) - \theta$

$E(T) - \theta$ → Parameter

↓ expected

↓ Statistics

> If $E(T) > \theta$ then biasness will be **positive**.

> $E(T) < \theta$ business will be negative.

Sampling Error :-

The difference between the Sample statistics & Corresponding population parameter is called Sampling error

Let "T" be the sample statistics used to estimate the population u parameter θ then sampling error is defined as

$$E = T - \theta$$

↳ Non-Sampling Error:-

The non Sampling error are caused by human error lik faulty interviewing incomplete or inaccurated responses e.t.c

Example:-

These errors are common for both sample survey and census

↳ Standard Error:-

Standard error is the standard deviation of sampling distribution of any statistics.

↳ Central Limit Theorm:-

If parent population is normal the Sampling distribution of $\bar{X}$ is exactly normal with mean $\mu_{\bar{x}}$ & SD $\sigma_{\bar{x}}$ regardless of the sample size if parent population is not good normal Then for large Sample size ($n > 30$) sampling distribution of $\bar{x}$ is approximatly normal and $\bar{x}$ can be converted into z

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \Rightarrow z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$