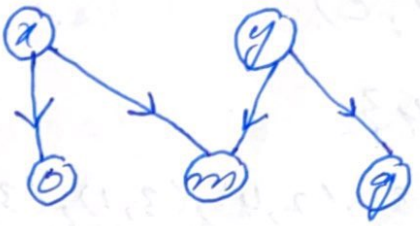
(iii) Graph of relation representation:



→ Closure of Relations:

⇒ The closure of relation R with respect to property P is the relation obtained by adding minimum number of ordered pairs to R to obtain property P.

(i) Reflexive closure.
(ii) Symmetric closure.
(iii) Transitive closure.

(i) Reflexive closure:

$A = \{1, 2, 3, 4\}$.

$R = \{(1,1), (1,3), (2,4), (3,1), (3,3), (4,3)\}$

∴ Reflexive closure should have:

$(1,1), (2,2), (3,3), (4,4)$.
✓ × ✓ ×.

So, $R \cup \{(2,2), (4,4)\}$.

(ii) Symmetric closure:

∴ In symmetric closure relation should have:

$(a,b) \times (b,a)$ } $(c,d) \times (d,c)$

e.g: $A = \{1,2,3,4\}$.

$R = \{(1,1),(1,3),(2,4),(3,1),(3,3),(4,3)\}$.

$(1,3) \rightarrow (3,1)$ ✓

$(2,4) \rightarrow \underline{(4,2)}$ ×.

$(4,3) \rightarrow \underline{(3,4)}$ ×

So, $R \cup \{(4,2),(3,4)\}$.

(iii) Transitive Closure.

e.g: $A = \{1,2,3\}$.

$R = \{(1,2),(2,3),(3,3)\}$.

→ For transitive closure we will use following formula:

$(R \cup R^2 \cup R^3)$ Where;

$R^2 = \underbrace{R \circ R}_{\text{composition}}$ And $R^3 = R^2 \circ R$.

Solve:

$R^2 = R \circ R$.

$R^2 = \{(1,2),(2,3),(3,3)\} \circ \{(1,2),(2,3),(3,3)\}$

$R^2 = \{(1,3),(2,3),(3,3)\}$.

$R^3 = R^2 \circ R$.

$R^3 = \{(1,\underline{3}), (2,\underline{3}), (3,\underline{3})\} \circ$

$\{(1,2), (2,3), (\underline{3},3)\}$.

$R^3 = \{(1,3), (2,3), (3,3)\}$.

Now, $R \cup R^2 \cup R^3$.

$= \{(1,2), (2,3), (3,3)\} \cup$
$\{(1,3), (2,3), (3,3)\} \cup$
$\{(1,3), (2,3), (3,3)\}$

$R \cup R^2 \cup R^3 = \{(1,2), (2,3), (3,3), (1,3)\}$.

Transitive closure.

$\therefore$ Transitivity: (a,b) (b,c) $\Downarrow$ (a,c) $(1,2)$ $(2,3)$ $\Downarrow$ $(1,3)$ ✓ $(2,3)$ $(3,3)$ $\Downarrow$ $(2,3)$ ✓

→ Equivalence Relations:

→ A relation "R" on a set "A" is said to be equivalence if it is reflexive, symmetric and transitive.

$\therefore R_4 = \{\ \}$.

e.g: $A = \{1, 2, 3\}$.

$R_1 = \{(1,1), (2,2), (3,3)\}$ R ✓ S, T ✓

$R_2 = \{(1,1), (2,2), (3,3), (2,1), (1,2)\}$ R, S, T ✓

$R_3 = \{(1,1), (2,2), (3,3), (3,2), (1,3)\}$ R ✓, S ✗.