Q#1(a)

let $S = \{(a,0,0) : a \in R\} \subseteq R^3$

Take $u, v \in S$

$\Rightarrow u = (a,0,0)$

$v = (b,0,0)$

(i) $u+v = (a+b,0,0) \in S$

(ii) $ku = (ka,0,0) \in S$

$\Rightarrow S$ is a subspace

Q#1(b)

$S = \{(a,1,1) : a \in R\} \subseteq R^3$

Take $u, v \in S \Rightarrow u = (a,1,1), v = (b,1,1)$

(i) $u+v = (a,1,1) + (b,1,1)$

$= (a+b,2,2) \notin S$

S is not closed under addition. $\Rightarrow$ S is not subspace.

Q#1(c)

$S = \{(a,b,c) : a,b,c \in R, b = a+c\} \subseteq R^3$

Take $u, v \in S \Rightarrow$ $u = (a_1, b_1, c_1)$ s.t $b_1 = a_1 + c_1$ — (1)

$v = (a_2, b_2, c_2)$ s.t $b_2 = a_2 + c_2$ — (2)

(i) $u+v = (a_1+a_2, b_1+b_2, c_1+c_2)$

$b_1 + b_2 = a_1 + c_1 + a_2 + c_2$ (using (1) & (2))

$= (a_1+a_2) + (c_1+c_2)$

$\Rightarrow u+v \in R^3$

(ii) $ku = (ka_1, kb_1, kc_1)$

$kb_1 = k(a_1+c_1)$ using (1)

$= ka_1 + kc_1$

$\Rightarrow ku \in S \Rightarrow$ S is a subspace.

(d) $S = \{(a,b,c) : b = a+c+1,\ a,b,c \in \mathbb{R}\}$

Take $u, v \in S$

$\Rightarrow u = (a_1, b_1, c_1) : b_1 = a_1 + c_1 + 1$ — (1)

$v = (a_2, b_2, c_2) : b_2 = a_2 + c_2 + 1$ — (2)

Now

$u+v = (a_1+a_2, b_1+b_2, c_1+c_2)$

$b_1 + b_2 = (a_1+c_1+1) + (a_2+c_2+1)$ using (1) & (2)

$= (a_1+a_2) + (c_1+c_2) + 2$

$\neq (a_1+a_2) + (c_1+c_2) + 1$

$\Rightarrow u+v \notin S$

$\Rightarrow S$ is not subspace.

(e) $S = \{(a,b,0) : a,b \in \mathbb{R}\} \subseteq \mathbb{R}^3$

Now Take $u, v \in S$

$\Rightarrow u = (a_1, b_1, 0)$

$v = (a_2, b_2, 0)$

$\Rightarrow u+v = (a_1+a_2, b_1+b_2, 0+0) \in S$

$ku = k(a_1, b_1, 0)$

$= (ka_1, kb_1, 0) \in S$

$\Rightarrow S$ is a subspace.

Q#2 (a)

Sum of two diagonal matrices is diagonal.

(i) sum of two diagonal matrices is diagonal

(ii) Scalar multiple of diagonal matrix is diagonal

Hence set of all diagonal $n \times n$ matrices form a subspace of M_{nn}.

Q#2 (b)

No it is not a subspace.

Take $A = \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 2 & 0 \end{bmatrix}$ in M_{22}

$\det A = 0$; $\det B = 0$ but $A+B = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ and $\det(A+B) \neq 0$

i.e the set is not closed under addition.

Q#2 (c)

$S =$ Set of all $n \times n$ matrices A s.t $tr A = 0$.

Take $A, B \in S$ s.t $tr(A) = 0$, $tr(B) = 0$

(i) $A+B \in S$ $\quad (\because tr(A+B) = tr(A) + tr(B) = 0+0 = 0)$

(ii) $kA \in S$ $\quad (\because tr(kA) = k\,tr(A) = k(0) = 0$

Q#2(d)

$S =$ set of all symmetric matrices.

Let $A, B \in S$ s.t $A^t = A$; $B^t = B$

(i) $A+B \in S$ $\quad (\because (A+B)^t = A^t + B^t = A+B)$

(ii) $kA \in S$ $\quad (\because (kA)^t = kA^t = kA)$

Q#2(e)

$S =$ Set of All Antisymmetric matrices

Let $A, B \in S \Rightarrow A^t = -A$; $B^t = -B$

(i) $A+B \in S$ $(\because (A+B)^t = A^t + B^t = -A-B = -(A+B))$

(ii) $kA \in S$ $(\because (kA)^t = kA^t = k(-A) = -kA)$

Q#2(f)

$S =$ Set of all $n \times n$ matrices A for which $Ax = 0$ has only trivial solution.

(Remark: $Ax=0$ has only trivial solution iff A is invertible iff $\det A \neq 0$)

This is not a subspace.

since sum of two invertible matrices need not to be invertible. Take $A = \begin{bmatrix} 2 & 1 \\ 5 & 4 \end{bmatrix}$ $B = \begin{bmatrix} -2 & 1 \\ -5 & 4 \end{bmatrix}$ $\det A \neq 0$; $\det B \neq 0$

$\det(A+B) = 0$

(g) $S =$ Set of all matrices A s.t $AB = BA$, B is fixed.

Take $A_1, A_2 \in S$

$$\Rightarrow A_1B = BA_1 \quad \text{(i)} \quad \& \quad A_2B = BA_2 \quad \text{(ii)}$$

Now

(i) $A_1 + A_2 \in S$ $\left(\because (A_1+A_2)B = A_1B + A_2B = BA_1 + BA_2 = B(A_1+A_2)\right)$

(ii) $kA \in S$ $\left(\because (kA)B = k(AB) = k(BA) = B(kA)\right)$

Q#3 (a)

$$S = \{a_0 + a_1x + a_2x^2 + a_3x^3;\ a_0 = 0\}$$
$$= \{a_1x + a_2x^2 + a_3x^3;\ a_1, a_2, a_3 \in \mathbb{R}\}$$

Take $f, g \in S$

$$\Rightarrow f = a_1x + a_2x^2 + a_3x^3$$
$$g = b_1x + b_2x^2 + b_3x^3$$

(i) $f + g = (a_1+b_1)x + (a_2+b_2)x^2 + (a_3+b_3)x^3 \in S$

(ii) $kf = k(a_1x + a_2x^2 + a_3x^3) = ka_1x + ka_2x^2 + ka_3x^3 \in S$

$\Rightarrow S$ is a subspace of P_3.

Q#3(b)

$$S = \{a_0 + a_1x + a_2x^2 + a_3x^3 : a_0 + a_1 + a_2 + a_3 = 0\}$$

Take $f, g \in S$

$$\Rightarrow f = a_0 + a_1x + a_2x^2 + a_3x^3 \ ; \quad a_0 + a_1 + a_2 + a_3 = 0$$
$$g = b_0 + b_1x + b_2x^2 + b_3x^3 \ ; \quad b_0 + b_1 + b_2 + b_3 = 0$$

(i) $f + g = (a_0+b_0) + (a_1+b_1)x + (a_2+b_2)x^2 + (a_3+b_3)x^3 \in S$

because $(a_0+b_0) + (a_1+b_1) + (a_2+b_2) + (a_3+b_3)$
$= (a_0 + a_1 + a_2 + a_3) + (b_0 + b_1 + b_2 + b_3)$
$= 0 + 0$
$= 0$

(ii) $kf = k(a_0 + a_1x + a_2x^2 + a_3x^3)$
$= ka_0 + ka_1x + ka_2x^2 + ka_3x^3 \in S$

because

$ka_0 + ka_1 + ka_2 + ka_3 = k(a_0 + a_1 + a_2 + a_3) = k(0) = 0$

Q # 3(c)

This set is not closed under scalar multiplication.

Take $f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

s.t a_0, a_1, a_2, a_3 are rational numbers.

Now Take k = irration number (any)

Then $kf = ka_0 + ka_1 x + ka_2 x^2 + ka_3 x^3$

but ka_0, ka_1, ka_2, ka_3 are irrational numbers.

(Product of a rational and irrational number is irrational).

Q # 3(d)

S = given set

Here $f = a_0 + a_1 x \quad , a_0, a_1 \in \mathbb{R}$

$g = b_0 + b_1 x \quad , b_0, b_1 \in \mathbb{R}$

(i) $f + g = (a_0 + b_0) + (a_1 + b_1)x \in S$ (as $a_0 + b_0, a_1 + b_1 \in \mathbb{R}$)

$kf = ka_0 + ka_1 x \in S$ (as $ka_0, ka_1 \in \mathbb{R}$)

Q # 4(a)

S = All function f in $F(-\infty, \infty)$ for which $f(0) = 0$

Take $f, g \in S \Rightarrow f(0) = 0 , g(0) = 0$

(i) $(f+g)(0) = f(0) + g(0) = 0 + 0 = 0$

$\Rightarrow f + g \in S$

(ii) $(kf)(0) = k(f(0)) = k(0) = 0$

$\Rightarrow S$ is a subspace.

Q # 4(b)

Given set is not closed under addition.

Take $f, g \in S$ such that $f(0) = 1 , g(0) = 1$

(i) $(f+g)(0) = f(0) + g(0) = 1 + 1 = 2 \neq 1$

Q#5 check yourself.

only (b) is not a subspace.

Q#6

$L = \{x = at;\ y = bt,\ z = ct\ ,\ t \in \mathbb{R}\} \subseteq \mathbb{R}^3$

Take $v_1, v_2 \in L$

$\Rightarrow v_1 = (x_1, y_1, z_1)$ s.t $x_1 = at_1\ ;\ y_1 = bt_1\ ;\ z_1 = ct_1\ ;\ t_1 \in \mathbb{R}$

$v_2 = (x_2, y_2, z_2)$ s.t $x_2 = at_2\ ;\ y_2 = bt_2\ ;\ z_2 = ct_2\ ;\ t_2 \in \mathbb{R}$

(i) $v_1 + v_2 = (x_1 + x_2,\ y_1 + y_2,\ z_1 + z_2) \in L$

because $x_1 + x_2 = at_1 + at_2 = a(t_1 + t_2)$

$y_1 + y_2 = bt_1 + bt_2 = b(t_1 + t_2)$

$z_1 + z_2 = ct_1 + ct_2 = c(t_1 + t_2)$ where $t_1 + t_2 \in \mathbb{R}$

(ii) $kv_1 = (kx_1, ky_1, kz_1) \in L$

because $kx_1 = k(at_1) = (ka)t_1 = a(kt_1)$

$ky_1 = k(bt_1) = b(kt_1)$

$kz_1 = k(ct_1) = c(kt_1)$

Q#7 (a)

let $(2,2,2) = k_1 u + k_2 v$

$\Rightarrow (2,2,2) = k_1(0,-2,2) + k_2(1,3,-1)$

$\Rightarrow$ $0k_1 + k_2 = 2$

$-2k_1 + 3k_2 = 2$

$2k_1 - k_2 = 2$

$$\Rightarrow \begin{bmatrix} 0 & 1 \\ -2 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

7

$$\left[\begin{array}{cc|c} 0 & 1 & 2 \\ -2 & 3 & 2 \\ 2 & -1 & 2 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 2 & -1 & 2 \\ -2 & 3 & 2 \\ 0 & 1 & 2 \end{array}\right] \quad R_1 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{cc|c} 2 & -1 & 2 \\ 0 & 2 & 2 \\ 0 & 1 & 2 \end{array}\right] \quad R_2 + R_1$$

System is not consitent.
from last two rows, we have

$2K_2 = 2$; $\boxed{K_2 = 2}$
$\Rightarrow \boxed{K_2 = 1}$

which is not possible at same time.

Q # 7 (b) do yourself

Q # 7 (c)

$(0, 0, 0) = k_1 (0, -2, 2) + k_2 (1, 3, -1)$

$0K_1 + K_2 = 0$ ← ① $\Rightarrow \boxed{K_2 = 0}$

$-2K_1 + 3K_2 = 0$ — ② using ① $\Rightarrow \boxed{K_1 = 0}$

$2K_1 - K_2 = 0$ — ③ using ① $\Rightarrow \boxed{K_1 = 0}$

Thus $\boxed{K_1 = K_2 = 0}$

$\Rightarrow (0, 0, 0) = 0(0, -2, 2) + 0(1, 3, -1)$.

Q # 8 (b)

$(6, 11, 6) = k_1 u + k_2 v + k_3 w$

$(6, 11, 6) = k_1 (2, 1, 4) + k_2 (1, -1, 3) + k_3 (3, 2, 5)$

$(6, 11, 6) = (2k_1 + k_2 + 3k_3, k_1 - k_2 + 2k_3, 4k_1 + 3k_2 + 5k_3)$

(8)

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 2 \\ 4 & 3 & 5 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 6 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 6 \\ 1 & -1 & 2 & 11 \\ 4 & 3 & 5 & 6 \end{array}\right]$$

$$R \sim \left[\begin{array}{ccc|c} 1 & -1 & 2 & 11 \\ 2 & 1 & 3 & 6 \\ 4 & 3 & 5 & 6 \end{array}\right] \quad R_1 \Leftrightarrow R_2$$

$$R \sim \left[\begin{array}{ccc|c} 1 & -1 & 2 & 11 \\ 0 & 3 & -1 & -16 \\ 0 & 7 & -3 & -38 \end{array}\right] \quad \begin{array}{l} R_2 - 2R_1 \\ R_3 - 4R_1 \end{array}$$

$$R \sim \left[\begin{array}{ccc|c} 1 & -1 & 2 & 11 \\ 0 & 1 & -\frac{1}{3} & \frac{-16}{3} \\ 0 & 7 & -3 & -38 \end{array}\right] \quad \frac{1}{3}R_2$$

$$R \sim \left[\begin{array}{ccc|c} 1 & -1 & 2 & 11 \\ 0 & 1 & -\frac{1}{3} & -16/3 \\ 0 & 0 & -\frac{2}{3} & -\frac{2}{3} \end{array}\right]$$

$-3+\frac{7}{3}$; $-38+\frac{112}{3}$

$\frac{-9+7}{3}$; $\frac{-114+112}{3}$

By backward substitution

$$-\frac{2}{3}K_3 = -\frac{2}{3}$$

$$\Rightarrow \boxed{K_3 = 1}$$

$$K_2 - \frac{1}{3}K_3 = \frac{-16}{3}$$

$$\Rightarrow K_2 - \frac{1}{3} = -\frac{16}{3}$$

$$\Rightarrow K_2 = \frac{-16}{3} + \frac{1}{3} = \frac{-15}{3} = -5 \Rightarrow \boxed{K_2 = -5}$$

$$K_1 - K_2 + 2K_3 = 11$$

$$k_1 - (-5) + 2(1) = 11 \Rightarrow K_1 = 11 - 2 - 5 = 4$$

$$\Rightarrow \boxed{K_1 = 4}$$

Scanned with CamScanner

Q# 9 (a)

$$\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix} = k_1 A + k_2 B + k_3 C$$

$$\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix} = k_1 \begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix} + k_2 \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + k_3 \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix} = \begin{bmatrix} 4k_1 + k_2 & -k_2 + 2k_3 \\ -2k_1 + 2k_2 + k_3 & -2k_1 + 3k_2 + 4k_3 \end{bmatrix}$$

Comparing coefficients

$$4k_1 + k_2 = 6$$
$$-k_2 + 2k_3 = -8$$
$$-2k_1 + 2k_2 + k_3 = -1$$
$$-2k_1 + 3k_2 + 4k_3 = -8$$

$$\begin{bmatrix} 4 & 1 & 0 \\ 0 & -1 & 2 \\ -2 & 2 & 1 \\ -2 & 3 & 4 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ -1 \\ -8 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 4 & 1 & 0 & 6 \\ 0 & -1 & 2 & -8 \\ -2 & 2 & 1 & -1 \\ -2 & 3 & 4 & -8 \end{array}\right]$$

$$R \sim \left[\begin{array}{ccc|c} -2 & 2 & 1 & -1 \\ 0 & -1 & 2 & -8 \\ 4 & 1 & 0 & 6 \\ -2 & 3 & 4 & -8 \end{array}\right] \quad R_1 \leftrightarrow R_3$$

OR

$$R \sim \left[\begin{array}{ccc|c} -2 & 2 & 1 & -1 \\ 0 & -1 & 2 & -8 \\ 4 & 1 & 0 & 6 \\ -2 & 3 & 4 & -8 \end{array}\right] \quad \begin{array}{l} R_3 + 2R_1 \\ R_4 - R_1 \end{array}$$

$$R \sim \left[\begin{array}{ccc|c} -2 & 2 & 1 & -1 \\ 0 & -1 & 2 & -8 \\ 0 & 5 & 2 & 4 \\ 0 & 1 & 3 & -7 \end{array}\right]$$

$$R \sim \left[\begin{array}{ccc|c} -2 & 2 & 1 & -1 \\ 0 & 1 & -2 & +8 \\ 0 & 5 & 2 & 4 \\ 0 & 1 & 3 & -7 \end{array}\right] \quad -R_2$$

$$R \sim \left[\begin{array}{ccc|c} -2 & 2 & 1 & -1 \\ 0 & 1 & -2 & 8 \\ 0 & 0 & 12 & -36 \\ 0 & 0 & 5 & -15 \end{array}\right] \quad \begin{array}{l} R_3 - 5R_2 \\ R_4 - R_2 \end{array}$$

By backward substitution.

$5k_3 = -15$ or $12k_3 = -36$

$\Rightarrow \boxed{k_3 = -3}$ or $\boxed{k_3 = -3}$

$$R \sim \left[\begin{array}{ccc|c} 1 & -1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & -1 & 2 & -8 \\ 4 & 1 & 0 & 6 \\ -2 & 3 & 4 & -8 \end{array}\right] \quad -\frac{1}{2} R_1$$

proceed in usual way

$K_2 - 2K_3 = 8$

$K_2 - 2(-3) = 8$

$K_2 + 6 = 8$

$K_2 = 8 - 6$

$\boxed{K_2 = 2}$

$-2K_1 + 2K_2 + K_3 = -1$

$\Rightarrow -2K_1 + 2(2) + (-3) = -1$

$\Rightarrow -2K_1 + 4 - 3 = -1$

$\Rightarrow -2K_1 + 1 = -1$

$-2K_1 = -2$

$\boxed{K_1 = 1}$

Q#9(b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0A + 0B + 0C$

Q#9(c) Do yourself.

Q#10 (a)

$-9 - 7x - 15x^2 = K_1P_1 + K_2P_2 + K_3P_3$

$= K_1(2 + x + 4x^2) + K_2(1 - x + 3x^2) + K_3(3 + 2x + 5x^2)$

$-9 - 7x - 15x^2 = (2K_1 + K_2 + 3K_3) + (K_1 - K_2 + 2K_3)x + (4K_1 + 3K_2 + 5K_3)x^2$

$\Rightarrow 2K_1 + K_2 + 3K_3 = -9$

$K_1 - K_2 + 2K_3 = -7$

$4K_1 + \cancel{3}K_2 + 5K_3 = -15$

Solve $\begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 2 \\ 4 & 3 & 5 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} -9 \\ -7 \\ -15 \end{bmatrix}$

Hint You can use Crammer's rule too when the matrix is a square matrix.

Q#10 (b) & (d) Try yourself

Q # 10 (c) $0 = 0p_1 + 0p_2 + 0p_3$.

Q#11 (a)

let $v \in \mathbb{R}^3 \Rightarrow v = (a, b, c)$

Now

v_1, v_2, v_3 Spans $\mathbb{R}^3$

if each vector v in $\mathbb{R}^3$ can be written as linear Combination of v_1, v_2, v_3.

i.e $v = k_1v_1 + k_2v_2 + k_3v_3$ for some $k_1, k_2, k_3 \in \mathbb{R}$

$(a,b,c) = k_1(2,2,2) + k_2(0,0,3) + k_3(0,1,1)$

or

$$2k_1 + 0k_2 + 0k_3 = a$$
$$2k_1 + 0k_2 + k_3 = b$$
$$2k_1 + 3k_2 + k_3 = c$$

or
$$\begin{bmatrix} 2 & 0 & 0 \\ 2 & 0 & 1 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

or $AX = b$ has a solution for every b.

which is possible only if $\det A \neq 0$

$$\det(A) = \begin{vmatrix} 2 & 0 & 0 \\ 2 & 0 & 1 \\ 2 & 3 & 1 \end{vmatrix} = 2\begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix} - 0 + 0$$
$$= 2(0-3) = -6 \neq 0$$

Hence v_1, v_2, v_3 Spans $\mathbb{R}^3$.

Q#11 (b) do yourself.

Q# 12 (a)

vector $(2,3,-7,3) \in \text{Span}\{v_1, v_2, v_3\}$
if and only if
$(2,3,7,3) = k_1 v_1 + k_2 v_2 + k_3 v_3$
i.e problem is to find k_1, k_2, k_3
Solution follows same as problem 7, 8 !

Q# 12 (b), (c), (d) parts are same to solve.

Q #13

P_2 = Set of all polynomials with degree less or equal to 2.

$= \{a_0 + a_1 x + a_2 x^2 ;\ a_0, a_1, a_2 \in \mathbb{R}\}$

Take $p(x) \in P_2$

$\Rightarrow P(x) = a_0 + a_1 x + a_2 x^2$

The problem is to find k_1, k_2, k_3, k_4 s.t

$P(x) = k_1 P_1 + k_2 P_2 + k_3 P_3 + k_4 P_4$

$a_0 + a_1 x + a_2 x^2 = k_1(1 - x + 2x^2) + k_2(3 + x) + k_3(5 - x + 4x^2) + k_4(-2 - 2x + 2x^2)$

$$\Rightarrow \begin{aligned} k_1 + 3k_2 + 5k_3 - 2k_4 &= a_0 \\ -k_1 + k_2 - k_3 - 2k_4 &= a_1 \\ 2k_1 + 0k_2 + 4k_3 + 2k_4 &= a_2 \end{aligned} \quad \text{or} \quad \begin{bmatrix} 1 & 3 & 5 & -2 \\ -1 & 1 & -1 & -2 \\ 2 & 0 & 4 & 2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

Now Consider

$$\left[\begin{array}{cccc|c} 1 & 3 & 5 & -2 & a_0 \\ -1 & 1 & -1 & -2 & a_1 \\ 2 & 0 & 4 & 2 & a_2 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 3 & 5 & -2 & a_0 \\ 0 & 4 & 4 & -4 & a_0+a_1 \\ 0 & -6 & -6 & 6 & a_2-2a_0 \end{array}\right] \begin{array}{l} R_2+R_1 \\ R_3-2R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 3 & 5 & -2 & a_0 \\ 0 & 1 & 1 & -1 & \frac{1}{4}(a_0+a_1) \\ 0 & -6 & -6 & 6 & a_2-2a_0 \end{array}\right] \frac{1}{4}R_2$$

$$\sim \left[\begin{array}{cccc|c} 1 & 3 & 5 & -2 & a_0 \\ 0 & 1 & 1 & -1 & \frac{1}{4}(a_0+a_1) \\ 0 & 0 & 0 & 0 & (a_2-2a_0)+\frac{3}{2}(a_0+a_1) \end{array}\right] R_3+6R_2$$

which is inconsistent i.e
System has no Solution.
Hence given polynomials do not span P_2.

<u>Q#14</u>

$\cos 2x = \cos^2 x - \sin^2 x \in \text{Span}\{\cos^2 x, \sin^2 x\}$

$3+x^2 \neq k_1 \cos^2 x + k_2 \sin^2 x \notin \text{Span}\{\cos^2 x, \sin^2 x\}$

$1 = \cos^2 x + \sin^2 x \in \text{Span}\{\cos^2 x, \sin^2 x\}$

$\sin x \neq k_1 \cos^2 x + k_2 \sin^2 x \notin \text{Span}\{\cos^2 x, \sin^2 x\}$

$0 = 0\cos^2 x + 0\sin^2 x \in \text{Span}\{\cos^2 x, \sin^2 x\}$

Q#15 Hint

Reduce the matrix A ~~into~~ by Gauss elimination method.

(i) If one free variable occur answer is a line.
(ii) If two free variables occur answer is a plane.
(iii) If $x=0, y=0, z=0$ is only solution then origin is answer.

Q# 19 (a)

$\{T_A(u_1), T_A(u_2)\}$ spans R^2

if $\{Au_1, Au_2\}$ spans R^2.

$$Au_1 = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1-2 \\ 0+4 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix} = (-1, 4)$$

$$Au_2 = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1-1 \\ 0+2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix} = (-2, 2)$$

The problem is to find $\{(-1,4), (-2,2)\}$ spans R^2 or not. (like problem 11).

Q# 19(b), Q# 20 are the same.

Q#22 (Hint)

To show that
$\text{Span}\{v_1, v_2, v_3\} = \text{Span}\{w_1, w_2\}$
we need to show (1) each v_1, v_2, v_3 is a linear combination of w_1 & w_2.
(2) Each w_1, w_2 is a linear combination of v_1, v_2 and v_3.