(iii) Generating Functions:

→ Let $\{A\}$ be any sequence with terms $a_0, a_1, a_2, a_3, \ldots$ then, G.F. $G(A,z)$ of a sequence $\{A\}$ is infinite series.

$$G(A,z) = \sum_{n=0}^{\infty} a_n \cdot z^n.$$

$$= a_0 + a_1 z + a_2 z^2 + \cdots + \infty.$$

Examples:

1- $a_n = c \;, n \geq 0$

$$G(A,z) = \sum_{n=0}^{\infty} a_n \cdot z^n.$$

$$= \sum_{n=0}^{\infty} c \cdot z^n.$$

$$= c \sum_{n=0}^{\infty} z^n.$$

$$= c(1+z+z^2+\cdots)$$

in this;

$a=1 \;, r=z.$

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{1}{1-z}.$$

$$= c \cdot \left(\frac{1}{1-z}\right)$$

$$= \boxed{\frac{c}{1-z}}$$

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2- $a_n = b^n \; , \; n \geq 0.$

$$G(A,z) = \sum_{n=0}^{\infty} a_n \cdot z^n.$$

$$= \sum_{n=0}^{\infty} b^n \cdot z^n.$$

$$= \sum_{n=0}^{\infty} (b \cdot z)^n.$$

$$= 1 + bz + (bz)^2 + \cdots$$

in this;

$a = 1 \; , \; r = bz.$

$$S_\infty = \frac{a}{1-r}.$$

$$S_\infty = \frac{1}{1-bz}$$

$$= \boxed{\frac{1}{1-bz}}$$

3- $a_n = c \cdot b^n \; , \; n \geq 0.$

$$G(A,z) = \sum_{n=0}^{\infty} a_n \cdot z^n.$$

$$= \sum_{n=0}^{\infty} c \cdot b^n \cdot z^n.$$

$$= c \sum_{n=0}^{\infty} (b \cdot z)^n.$$

$$= c(1 + bz + (bz)^2 + \cdots).$$

in this;

$a = 1 \; , \; r = bz.$

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{1}{1-bz}$$

$$= \boxed{\frac{c}{1-bz}}$$

4- $a_n = n \;, n \geq 0$

$$G(A, z) = \sum_{n=0}^{\infty} a_n \cdot z^n.$$

$$= \sum_{n=0}^{\infty} n \cdot z^n.$$

$$= 0 + z + 2z^2 + 3z^3 + \cdots$$

$$= z(1 + 2z + 3z^2 + \cdots).$$

$= z(1-z)^{-2}$ By using bionomial theorm:

$$= \boxed{\frac{z}{(1-z)^2}}.$$