Discrete Structure (CMP-2111)

Class: BS (CS, SE, IT), M.sc IT

Short and Long Questions

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Answer the following Questions

1) **Differentiate between Euler and Hamiltonian path.**
2) **What is compound preposition?**
3) **Give an indirect proof to the theorem "if 3n + 2 is odd, then n is odd".**
4) **What is the contrapositive, the converse ad inverse of the conditional statement? "the football team wins whenever it is raining"**
5) **What is bi-implication? State with an example.**
6) **State resolution rule?**
7) **Show that ⌐ (P V Q) and ⌐P ^ ⌐Q are logically equivalent.**
8) **How many edges are there in a graph with ten vertices each of degree six?**
9) **What is the cardinality of each of these sets {a, {a}, {a, {a}}}?**
10) **Height of tree is 7. Find sum of height?**
11) **What is the difference between geometric progression and arithmetic progression?**
12) **State division algorithm?**
13) **Define the chromatic number of a graph.**
14) **What is the height of rooted tree?**
15) **Let f be the function from {a, b, c, d} to {1,2, 3} defined by f(a) = 3, f(b) = 2, f(c) = 1, f(d)=3 is f an onto function.**
16) **What is graph coloring problem?**
17) **Define tautology.**

18) Find the symmetric difference of {3,5 ,8}and {2,5,6}.

19) Let A = {eggs, milk, corn} and B = {cows, goats, hens}

Define a relation R from A to B by (a, b) ∈R *iff* a is produced by b.

20) Among 200 people, 150 either swim or jog or both. If 85 swim and 60 swims and jog, how many jogs?

21) Write applications of minimum spanning tree.

22) Define ceiling and floor function with example.

23) State pigeonhole principle?

24) Define a function with example.

25) What is modus ponen rule of inference?

26) Write any two applications of Venn diagram.

27) Differentiate between free variable and bound variable.

28) Define predicate?

29) Evaluate of this expression

$$(1\ 1011 \vee 0\ 1010) \wedge (1\ 0001 \vee 1\ 1011)$$

30) Differentiate between a function and a Relation.

31) What is the expansion of $(x + y)^4$? By using Pascal's Triangle.

32) What is the minimum number of students in a class to be sure that two of them are born in the same month?

33) Define permutation and combination.

34) What is optimal solution?

35) What is divide and conquer approach?

36) What is the value of this prefix expression?

$$* + 3 + 3 \uparrow 3 + 3\ 3\ 3$$

37) What is closure of relations? Define reflexive closure and transitive closure.

38) Is set theory deals continuous values? Explain to motivate your answer.

39) Define Bipartite Graph with example.

40) What is graph coloring problem? Explain with example.

41) How many one-to-one functions are there from a set with three elements to a set with four elements.

42) Give any two applications of Mathematical induction and Recursion.

43) Write any four applications of Venn diagram.

44) Differentiate between a Graph and Tree

45) Determine the truth value of each of these statements if the domain consists of all real numbers.

a) ∃x(x3 = −1) **b) ∃x(x4 < x2)**

c) ∀x((−x)2 = x2) **d) ∀x(2x > x)**

46) Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

a) **No one is perfect.**
b) **Not everyone is perfect.**
c) **All your friends are perfect.**
d) **At least one of your friends is perfect.**
e) **Not everybody is your friend or someone is not perfect.**

47) Use a Venn diagram to illustrate the set of all months of the year whose names do not contain the letter R in the set of all months of the year.

48) Suppose that a connected planar simple graph has 30 edges. If a plane drawing of this graph has 20 faces, how many vertices does this graph have?

49) Find the next three terms of the sequence 2 43, –81, 27, . . .

50) Find S = $\sum_{k=48}^{100} 3.(2)^k$

51) Define Binary search tree?

52) How the height of tree is calculated?

53) Define the term halting problem?

54) What is the probability of getting a number greater than 4 when a disc is tossed?

55) What are the two ways for the representation of graph?

56) Convert the Hexadecimal expansion of $(80E)_{16}$ **to a binary expansion?**

57) What is the difference between Best Case, and Worse Case complexity of an algorithm?

58) Define the term counterexample.

59) Define greedy algorithm with example.

60) Differentiate between mathematical induction and strong induction?

61) What is the Absorption Law for sets?

62) How many rows appear in a truth table for teach of these compound preposition?

(p →r) v (¬s →¬t) v (¬u →v)

63) let P(x) be the statement, "x spends more than five hours every weekday in class" where the domain for x consists of all students. Express the quantification in English.

$\exists x \neg P(x)$.

64) determine the truth value of following statement if the domain for all variables consist of all integers. $\forall n\ (n^2 \geq n)$

65) what is the Cartesian product of A x B x C, where A = {0,1}, B= {1,2} and C= {0,1,2}?

66) Let f be the function from {a, b, c, d} to {1,2,3,4} with f(a)=4, f(b)=2, f(c)=1, and f(d)=3, is f a bijection?

67) find the conjunction of the propositions p and q where p is the proposition "Today is Friday" and q is the proposition "it is raining today".

68) Define Contradiction.

69) What are the negations of the statements, "All goats are mammals"?

70) what is pigeonhole principle?

71) Difference between projection and join operator?

72) How many bit strings of length 8 either start with a 0 bit or end with the two bits 11?

73) Define Reflexive closure and symmetric closure?

74) What is a Recurrence relation?

75) Define commutative law with the help of example?

76) How many permutations of the letter ASSESSINATION contain the string SES?

77) How many comparisons are needed for a binary search in a set of 64 elements?

78) What is the Cartesian product of A= {a, b, c} and B= {1,2}?

79) Define this function $f(x)= (x+1)/(x^2+2)$ onto or one-to-one. Domain consists of all integers?

80) What is a relation on a set?

81) How can you produce the terms of a sequence if the first 10 terms are, 5, 11,17,23,29,35,41,47,53,59?

82) Determine whether each of these functions is a bijection from R to R (a) $f(x) = x^2+1$ (b) $f(x)=x^3$

83) How many ways are there to select five players from a 10 members tennis team to make a trip to a match at another school?

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84) Define Euler Path?

85) What is the Worst-case complexity of Bubble sort?

86) State the division algorithm?

87) What is the space complexity of linear search algorithm?

88) What is minimum spanning tree?

89) How many different license plates can be made if each plate contains a sequence of three uppercase three letters followed by three digits?

90) Define Existential Quantifier?

91) How many relations are there on a set with n elements?

92) What is simplification rule of inference?

93) Define the fallacies?

94) Define disjoint set?

95) State principle of inclusion exclusion?

96) Define isolated vertex in a graph?

97) Define preorder traversal in a tree?

98) What is the average case complexity of linear search algorithm?

99) List down the name of three graph models?

100) Define isomorphism of graphs?

101) How many permutations of the letter of the word PANAMA can be made, if p is to be the first letter in each argument?

102) How many permutations of the letters ABCDEFGH contain the string ABC?

103) Suppose that there are eight runners in a race first will get gold medal, the second will get silver and third will get bronze. How many different ways are there to award these medals if all possible outcomes of race can occur and there is no tie?

104) State which rule of inference is basis of the following argument.: "it is below freezing and raining now, therefore, it is below freezing now".

105) if p= it is raining q= she will go to college "it is raining and she will not go to college" will be denoted by?

106) Determine the truth value of following statement if the domain consist of all real numbers.

$$\exists x\ (x^4 < x^2)$$

107) Define Idempotent law?

108) Differentiate between proposition and predicate with example.

109) How many different elements does A x B x C have if A has m elements, B has n elements and C has p elements?

110) Let A= {1,2,3,4}, and R is a relation defined by “a divides b”. write R as a set of ordered pair, draw directed graph.

111) Define the term Halting problem.

112) A bag contains 6 white, 5 black and 4 red balls. Find the probability of getting a white or a black ball in a single draw.

113) What is cyclic graph? Give an example of cyclic graph?

114) State handshaking lemma?

115) What is the basic counting principle?

116) Write the names of an algorithm properties?

117) Define Partial ordering with example?

118) Determine whether the integers 10,17 and 21 are pairwise relatively prime?

119) Encrypt the message WATCH YOUR STEP by translating the letters into numbers, applying the given encryption function, and then translating the numbers back into letters. f(p)= (14p+21) mod26.

120) How can you produce the terms of a sequence if the first 10 terms are 5,11,17,23,29,35,41,47,53,59?

121) How many permutations of the letters ABCDEFG contain the string CFGA?

122) Find recurrence relation of the sequence S(n) = 5^n

123) How many subsets with more than two elements does a set with 100 elements have?

124) What are connected components of a graph?

125) What is height of rooted tree?

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Long Questions

1)

a) State which rule of inference is used in the argument:

If it rains today, then we will not have a barbecue today. If we do not have a barbecue today, then we will have a barbecue tomorrow. Therefore, if it rains today, then we will have a barbecue tomorrow.

b) Prove that 21 divides $4^{n+1} + 5^{2n-1}$ whenever *n* is a positive integer.

c) Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?

If George does not have eight legs, then he is not an insect.
George is an insect.

Therefore: George has eight legs.

2)

a) Find S = $\sum_{k=50}^{100}(k)^2$

b) Proof that √2 is irrational by using contradiction method

3)

a) Define the following graphs with example.

Planar graph, Bipartite graph, Regular graph, complete graph

b) Define the following relations with example.

I. Reflexive relation
II. Symmetric relation
III. Transitive relation

4)

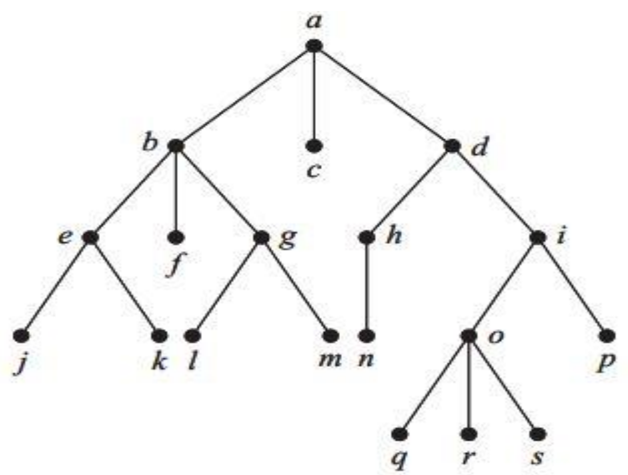
a) How many bit strings of length four do not have two consecutive 1's by using tree diagram?

b) Draw a 3-ary tree with height 4?

5)

a) How many license plates can be made using either two letters followed by four digits or two digits followed by four letters?

b) Consider the tree shown below.



I. **What is the level of *n*?**

II. **What is the height of this rooted tree?**

III. **What are the children of *b*?**

IV. **What is the parent of *g*?**

V. **What are the siblings of *j*?**

VI. **What are the descendants of *f*?**

6) **Translate these statements into English, where *C(x)* is "*x* is a comedian" and *F(x)* is "*x* is funny" and the domain consists of all people.**
   a) $\forall x(C(x) \rightarrow F(x))$ **b)** $\forall x(C(x) \wedge F(x))$
   c) $\exists x(C(x) \rightarrow F(x))$ **d)** $\exists x(C(x) \wedge F(x))$
7) **How many bit strings of length four do not have two consecutive 1's using Tree diagram?**
8) **Prove that 21 divides $4^{n+1} + 5^{2n-1}$ whenever *n* is a positive integer.**
9) **Give a proof by contradiction of the theorem "If 3*n* + 2 is odd, then *n* is odd**
10) **Prove that if *m* + *n* and *n* + *p* are even integers, where m, n, and p are integers, then m + p is even. What kind of proof did you use?**
11) **Prove that $2^{1/2}$ is irrational number by using contradiction method.**
12) **Let A = {1,2,3,4,5} and we define functions f:A →A and then g:A→A :**
    f(1)=3, f(2)=5, f(3)=3, f(4)=1, f(5)=2
    g(1)=4, g(2)=1, g(3)=1, g(4)=2, g(5)=3
    Find the composition functions fog and gof.
13) **Which term of the arithmetic sequence**
    4, 1, -2, ……., is -77?
14) **Prove the following using Venn Diagrams:**
    (i) $A - (A - B) = A \cap B$
    (ii) $(A \cap B)^c = A^c \cup B^c$
    (iii) $A - B = A \cap B^c$

15) **Define a sequence b0, b1, b2, . . . by the formula**

$b_n = 5^n$, for all integers $n \geq 0$.

Show that this sequence satisfies the recurrence relation $b_k = 5b_{k-1}$, for all integers $k \geq 1$.

16)

Use mathematical induction to prove that

$$\sum_{i=1}^{n+1} i2^i = n \cdot 2^{n+2} + 2, \qquad \text{for all integers } n \geq 0$$

17) **Two cards are drawn at random from an ordinary pack of 52 cards. Find the probability p that (i) both are spades, (ii) one is a spade and one is a heart.**

18) **A man visits a family who has two children. One of the children, a boy, comes into the room. Find the probability that the other child is also a boy if**

(i) The other child is known to be elder,

(ii) Nothing is known about the other child

19) **Use mathematical induction to show that 2 divides $n^2 + n$, whenever n is a positive integer.**

20) **A coin is flipped 10 times where each flip come up either heads or tails. How many possible outcomes.**

a) **Are there in total?**
b) **Contain exactly two heads?**
c) **Contain at most three tails?**
d) **Contain the same number of heads and tails?**

21) **Prove that if m + n and n + p are even integers, where m, n and p are integers, m+ p is even. What kind of proof did you use?**

22) **Suppose that a password for a computer system must have at least 8, but no more than 12, characters, where each character in the password is a lower case English letter, an uppercase English letter, a digit, or one of the six special characters *, >, <, !, +and =**

a) **How many different passwords are available for this computer system?**
b) **How many of these passwords contain at least one occurrence of at least one of the six special characters?**

23) **Find each of these values.**

a) **(992 mod 32)3 mod 15**
b) **(34 mod 17)2 mod11**
c) **(193mod23)2mod31**
d) **(893mod 79)4 mod 26**

Find the formula for the sequence with the following four five

I. **1, -1/2, 1/3, -1/4**
II. **0, 1/2, 2/3, 3/4**
III. **-1, 8, -27, 64,**

24) **Let** $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **and** $M_S = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **Find** M_{SoR}

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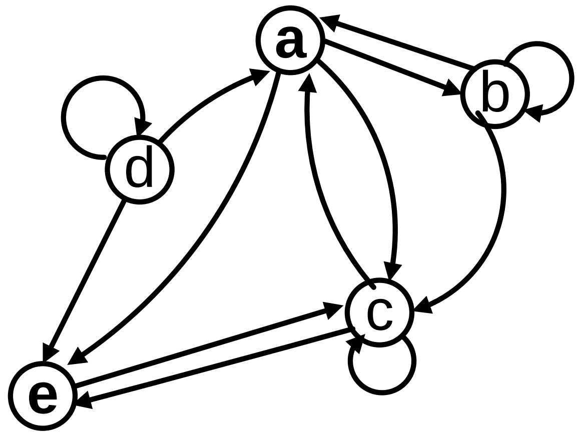
a) **A digraph of the set A = {a, b, c, d, e} is given.**

i. **Find the Matrix M_R on the digraph?**

ii. **Find that the digraph symmetric, reflexive, transitive or antisymmetric?**



25) **Let R be the relation {(1,2), (1,3), (2,3), (2,4), (3,1)}, and let S be the relation {(2,1), (3,1), (3,2), (4,2)}. Find (S o R) and (R S).**

26) **Out of 5 mathematicians and 7 engineers, a committee consisting of 2 mathematicians and 3 engineers is to be formed. In how many ways can this be done if**

a. **Any mathematician and any engineer can be included?**

b. **One particular engineer must be in the committee?**

27) **Serial numbers for a product are to be made using four letters (using any letter of the alphabet) followed by 2 single-digit numbers. For example, JGRB297 is one such serial number. How many such serial numbers are possible if neither letters nor numbers can be repeated?**

28) **Use a tree diagram to find the number of bit strings of length seven with no three consecutive 0's and no three consecutive 1's**

29) **From a group of 7 men and 6 women, 5 persons are to be selected to form a committee so that at least 3 are there on the committee. In how many ways can it be done?**

30) **In how many different ways can the letters of the word "LEADING" be arranged in such a way that the vowels always come together?**

31) **A school has scheduled three volleyball games, two soccer games, and four basketball games. You have a ticket allowing you to attend three of the games. In how many ways can you go to two basketball games and one of the other events?**

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32) **use rule of inference to show that the hypotheses "Randy works hard, if Randy works hard, when he is a dull boy, and if Randy is a dull boy then he will not get the job ""imply the conclusion Randy will not get the job"**

33) **what is the wrong with this statement? Let H (x) be "x is happy". Given the premise** $\exists x H(x)$, **we conclude that H(John), therefore john is happy.**

34) **Encrypt the message UPLOAD using the RSA system with n=53.61 and e=17.**

35) **What is the proof by mathematical induction? use mathematical induction to prove that $K^3 - K$ is divisible by 3. Whenever K is a positive integer.**

36) **Using alphabetical order, construct binary search tree for the words in the sentence "the quick brown fox jumps over the lazy dog".**

37) **How many different spanning trees does each of these simple graphs have?**
   a) **K3**
   b) **K4**
   c) **K2,2**
   d) **C5**

38) **Prove that following are logically equivalent developing a series of logically equivalences.**
   a) **$(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is tautology?**
   b) **$(p \rightarrow q) \rightarrow (r \rightarrow s)$ and $(p \rightarrow r) \rightarrow (q \rightarrow s)$**

39) **Show that the premises, "A students in this class has not read the book", "and everyone in this class passed the first exam" imply the conclusion, someone who passed the first exam has not read the book.**

40) **Use mathematical induction to prove that $1+3+5+\ldots\ldots \ldots. + (2n-1) = n^2$ for all integers $n \geq 1$.**

41) **Use the Insertion sort to sort 3,1,5,7,4 in decreasing order showing the list obtained at each step.**

42) **Draw a Binary search tree by inserting the following numbers from left to right.**

    19,10,8,17,4,10,6,13,43,47,33

    Determine the order, in which the vertices of the following binary trees will be visited under.
   a) **Preorder Traversal**
   b) **Inorder Traversal**
   c) **Postorder Traversal**

43) **Let f: $R \rightarrow R$ be defined by the rule $f(x) = x^3$. show that f is bijective.**

44) **Find a counterexample by, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.**
   a) $\forall x \forall y\ (x^2 = y^2 \rightarrow x = y)$
   b) $\forall x \exists y\ (y^2 = x)$
   c) $\forall x \forall y\ (xy \geq x)$

45) **Prove that following are logically equivalent by developing a series of logically equivalences.**
   a) **$\neg (p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$**
   b) **$(\neg p \leftrightarrow q) \leftrightarrow (p \leftrightarrow \neg q)$**

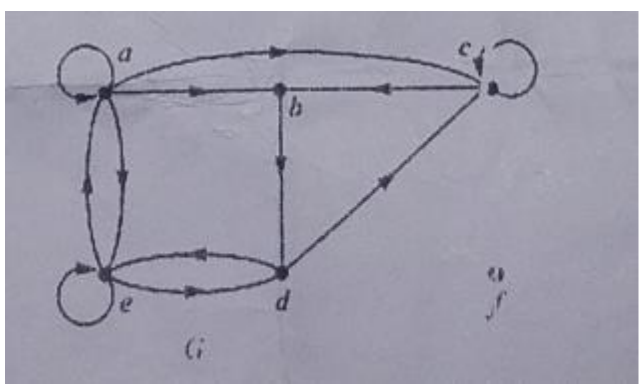
46) **Use the divide and conquer algorithm to put 6,1,2,5, -7,23,11,12,4,3 into decreasing order.**

47) **Mention whether the following problems are permutation or combination problem.**
   a) **How many ways can 6 tosses of coin yields 2 heads and 4 tails?**
   b) **How many lines can you draw using 3 noncolinear (not in a single line) points A, B and C on a plane?**
   c) **How many triangles can you make using 6 noncollinear points on a plane?**
   d) **In a certain country, the car number plate is formed by 4 digits, from the digits 1,2,3,4,5,6,7,8 and 9 followed by 3 letters from the alphabet. How many number plates can be formed if neither the digits nor the letters are repeated?**

48) **Write down base and recursive case for sum of array elements. Also solve for array of length 5 via tree convention.**

49) **Determine whether each of these functions is a bijection from R to R.**
   I. **$f(x)=-5x/2+4$**
   II. **$f(x)=-3x^2+7$**
   III. **$f(x)=(x+1)/(x^3+2)$**
   IV. **$f(x)=x^5+1$**

50) **find the in-degree and out-degree of the given graph. Also find whether the given graph has Hamiltonian or Euler tour. Show visually, also represent the graph into Adjacency matrix.**



51) **Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a Discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from computer science department?**

52) **Represent the following algebraic expressions in tree.**
(x+(y/x)) +3 and x+ (y/(x+3))

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53) For each of these relations on the set {1,2,3,4}, decide whether it is reflexive, symmetric, asymmetric, antisymmetric and transitive.

a) {(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)}
b) {(1,1), (2,2), (3,3), (4,4)}
c) {(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)}
d) {(1,3), (1,4), (2,3), (2,4) (3,1), (3,4)}

54) Describe an algorithm based on the linear search for determining the correct position in which to insert a new element in an already sorted list.

55) Define f: R→ R by the rule f(x)= 4x – 1 for all x € R is f onto? Prove or give a counter example.

Solution

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#01:- What is the absorption law. for sets.

→ $A \cup (A \cap B) \equiv A$ $\quad$ $P \vee (P \wedge q) \equiv P$

→ $A \cap (A \cup B) \equiv A$ $\quad$ $P \wedge (p \vee q) \equiv P$

Q#02:- How many Rows appear in a truthtable for each of the following preposition?

$$(P \to r) \vee (\neg s \to \neg t) \vee (\neg u \to v)$$

There are 6 variables in the preposition.

So $2^6 = 64$ rows will be appear in truth table.

Q#03: Let $P(x)$ be the Statement "x spends more than 5 hours every weekday in Class". Where the domain for x consist of all students. Express the quantification in English. $\exists x \neg P(x)$

1→ $\forall x P(x > 5)$ $\quad$ $\forall x (P(x) > 5)$

2→ There is at least no x such that $P(x)$

For some x $P(x)$

Q#04:- Determine the truth table of following statement if the domain for all variables consists of all integer. $\forall n (n^2 \geq n)$

Q#05: Find the conjunction of the proposition P & Q where
P = Today is Friday.
Q = It is rainy Day.

$P \wedge Q$ = Today is friday and it is rainy day.

Q#06:- Define contradiction?

A compound proposition that is always false is called contradiction ∴ $P \wedge \sim P$ is always false

Q#07:- What are negations of Statement.
"All goats are mammals"

¬ ("All goats are mamals")

→ All goats are not mammals.

Q#08:- Defin Reflexive and Symmetric closure?

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Q#09:- Define commutative law with example?

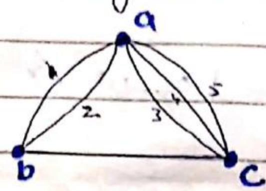
~~$P \vee P = P$~~

$$p \vee q = q \vee p$$

$$p \wedge q = q \wedge p$$

Q#10:- Define Euler-Path?

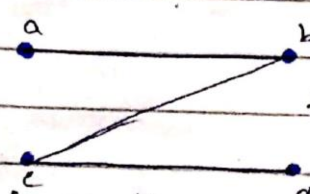
It is path that visit every edge exactly once and only once.



Q#11:- What is minimum Spanning Tree?

A tree that include all vertices but no cycle.

SPSWE



A minimum spanning tree is a connected weighted graph is a spanning tree that has the smallest possible sum of weights of its edges.

Q#12:- Define Existential Quantifier.

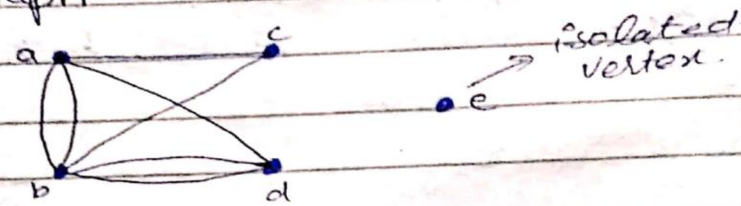
The existential quantifier of P(x) is the proposition "There exist an element x in the domain such that P(x)"

notation ⟶ $\exists x \, P(x)$

$\exists$ ⟶ is existential Quantifier

Q#13: Define isolated vertex in a graph.

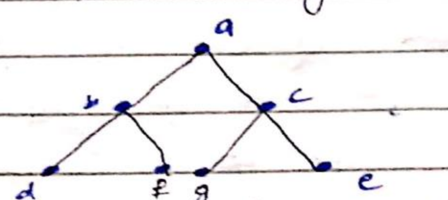
A vertex that has zero degree is called isolated vertex of graph.



Q#14:- Define Pre-order traversal in a tree?

A tree in which we have:-

1- Visit root
2- visit left
3- visit right



→ abdfcge

Q#15:- List down the three name of graph models?

1- Influence Graph
2- The Hollywood Graph
3- Round-Robin Tournaments
4- Call Graph
5- Web Graph.

Q#16:- Define isomorphism of the graph?

A graph in which all the vertices has the same degree and no of vertices is called isomorphism of graph.

Q#17: If $P =$ "It is Raining"
$Q =$ "She will go to college"
"It is Raining and she will go to college" will be denoted by?

$P \wedge Q$.

Q#18: Determine the truth value of following Statement if the domain consist of all real numbers.

$\exists x \, (x^4 < x^2)$

Q#19: Define idempotent Law?

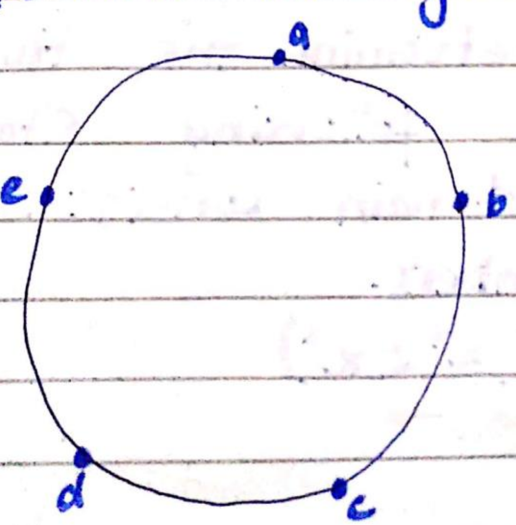
$$P \vee P = P$$

$$P \wedge P = P$$

Q#20: Differentiate between proposition and Predicate.

Proposition	Predicate
• A proposition is a declarative sentence that is either true or false but not both.	• is a sentensa that contain a infinite no. of variable and becomes a Statement when specific values are Substitud for the Variable => P(x) " and x has domain

Q#21:- What is cyclic Graph?



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Q What is Recurrence Relation?
A recurrence relation of the sequence {an} is an equation that expresses an in terms of one or more of previous terms of the sequence.
A sequence is called a solution of a recurrence relation if its terms satisfy the recurrence relation.
Exp
a{1, 2, 3, 5, 8, 13, 21}

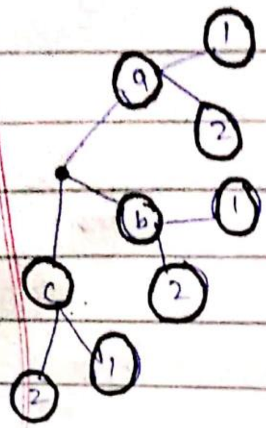
Q What is permutation of the letters ASSESSINATION contain the string SES?
Letters are 3A's, 4S's, 2I's, 2N's, 1T's and 1O's
Total number of ways these letter can be arranged $= n(S) = \frac{13!}{3!4!2!2!}$
a) if for S's come consecutively! in the word then we considers these 4S's as 1 group.

Q How many comparisons are needed for a binary search in set of 64 elements?
$\because N/2^K = 1$
$2^K = N$
Applying log (base 2) on both side
$\log(2^K) = \log N = K \log N$ $\therefore N = 64$
$K \log 64 \Rightarrow K = \log(2^6) \Rightarrow K = 6 \log 2 = 6$
K is the average worst case number of comparison needed.

Q What is the cartesian product of $A = \{a, b, c\}$, $B = \{1, 2\}$

$A \times B = \{a, b, c\} \{1, 2\}$

$= \{(a,1)(a,2)(b,1)(b,2)(c,1)(c,2)\}$



Q Define this function $f(x) = \dfrac{x+1}{x^2+2}$ onto or one-to-one.

Domain $\in \mathbb{R}$

$f(0) = \dfrac{0+1}{0+2} = 1/2$

$f(1) = \dfrac{1+1}{1+2} = 2/3$, $f(2) = \dfrac{2+1}{4+2} = 3/6 = 1/2$

$f(-1) = \dfrac{-1+1}{-1+2} = 0$, $f(-2) = \dfrac{-2+1}{4+2} = -1/6$

As $f(0) = 1/2$ and $f(2) = 1/2$ so the function is not ~~onto~~ one two one function and onto function

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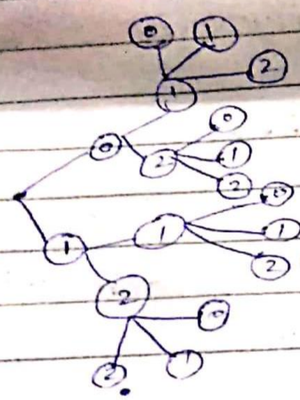
What is the cartesian product of $A \times B \times C$, wher $A = \{0,1\}$, $B = \{1,2\}$, $C = \{0,1,2\}$

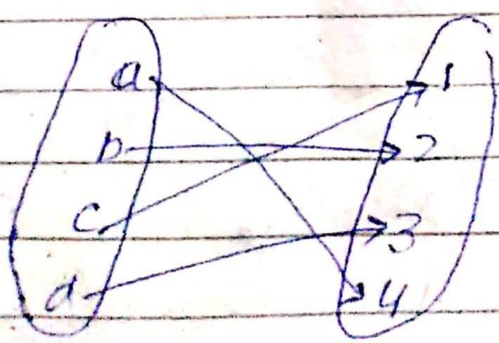
$A \times B = \{0,1\} \times \{1,2\}$
$\{(0,1)\ (0,2)\ (1,1)\ (1,2)\}$

$A \times B \times C = \{(0,1)\ (0,2)\ (1,1)\ (1,2)\} \times \{0,1,2\}$

$= \{(0,1,0)\ (0,1,1)\ (0,1,2)\ (0,2,0)\ (0,2,1)\ (0,2,2)\ (1,1,0)\ (1,1,1)\ (1,1,2)\ (1,2,0)\ (1,2,1)\ (1,2,2)\}$



let f be the function from $\{a,b,c,d\}$ with $f(a) = 4$, $f(b) = 2$, $f(c) = 1$, $f(d) = 3$, is f bijective?



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How many bit strings of length 8 either start with a 0 bit or end with the two bits 11?

0 _ _ _ _ _ 1 1

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$= 120$

Q Define Reflexive and symmetric closure?

- **Reflexive closure**

The Reflexive closure of a binary relation R on a set X is the smallest reflexive relation on X that contains R.

For example: if X is a set of distinct number and "xRy" means "x is less than y". then the reflexive closure of R is the relation "x is less or equal to y.

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Symmetric Closure:

The Symmetric closure S of a relation R on a set X is given by. in other words the Symmetric closure of R is Union of R with its Converse relation R^T

Difference b/w projection & Join operator.

Join Operator

An SQL join clause Corresponding to a join operation in relational algebra combines Columns from one or more tables in a relational database. it Creates a set that Can be saved as a table or use as it is. A join is a means for combining Columns from one or more tables by using values Common to each.

Projection operator

projection operator (π) is a Unary operator in relational algebra that performs a projection operation. it displays Columns of a relation or table ~~was~~ based on the specified attributes

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What is a relation on a set?

Let A and B be sets. A binary relation from A to B is a subset of A×B.

Exp

$A = \{1,2\}$, $B = \{0,1\}$

$A \times B = \{(1,0)(1,1)(2,0)(2,1)\}$

$R = \{(1,0)(2,0)\}$

Q How can you produce the terms of a sequence if the first 10 terms are
5, 11, 17, 23, 29, 35, 41, 47, 53, 59?

$$\{a_n\} = n+6\}_{n=-1}^{\infty}$$

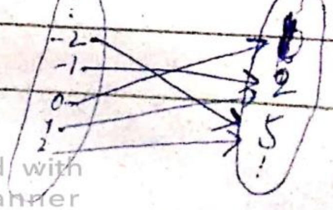
Q Determine whether each of these function is bijection from R to R

$f(x) = x^2+1$

$f(-2) = 4+1 = 5$, $f(2) = 4+1 = 5$

$f(-1) = 1+1 = 2$, $f(1) = 1+1 = 2$

$f(0) = 0+1 = 1$



So the function bijective

Q How many ways are there to Select five players from a 10 members tennis team to make a trip to a match at another School?

$$^{10}C_5 = \frac{10!}{5!\,(10-5)!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!}\ 5!}$$

$$= \frac{30240}{120} = 252$$

Q What is the worst-case complexity of linear search algorithm?

The worst case complexity of leaner search is O(n)
The complexity of binary search is O(log n)
The worst case complexity for merge sort is O(n log n). The worst case complexity for bubble sort is O(n²) and best case is O(n)

Q Define Existential Quantifier?

The symbol of Existential quantifier is ∃. Pronounced as "there is".

Exp

∃x student

There exist a student.

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Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

Truth tables..

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Because the truth values of the compound propositions $\neg(p \vee q)$ and $\neg p \wedge \neg q$ agree for all possible combinations of the truth values of p and q it follows that $\neg(p \vee q) \leftrightarrow (\neg p \wedge \neg q)$ is a tautology and that these compound propositions are logically equivalent.

Q Define tautology?

A compound proposition that is always true, no matter what the truth values of the propositions that occur in it, is called tautology.

08

Example $(p \wedge q) \rightarrow (p \vee q)$

$= (p \wedge q) \rightarrow (p \vee q).$

$= \sim(p \wedge q) \vee (p \vee q).$

$= \sim p \vee \sim q \vee p \vee q.$

$= \sim p \vee p \vee \sim q \vee q.$

$= T \vee T$

$= T$

Q What is bi-implications? State with an example?

Way to combine propositions that expresses that two propositions have the same truth value.

Let p and q be prepositions. The biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q". The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called bi-implications. There are some other common ways to express $p \leftrightarrow q$:

"p is necessary and sufficient for q".

"if p then q, and conversely".

"p iff q".

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P	q	$P \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Q What are the contrapositive, the converse, and inverse of the conditional statement.
"The ~~home~~ football Team wins whenever it is raining". ? q whenever p

Because "q whenever p" is one of the ways to express the conditional statement $p \to q$, the original statement can be rewritten as
"If it is raining, then the football team wins".
Consequently the contrapositive of this conditional statement is.
$\sim q \to \sim p$ "If the ~~foot~~ball does not win, then it is not raining".

The converse is
$q \to p$ "If the football Team wins, then it is raining".

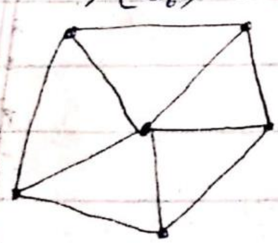
The inverse is
$\sim P \to \sim q$ "If it is not raining, then the football Team does not win".

Only the contrapositive is equivalent To the original statement.

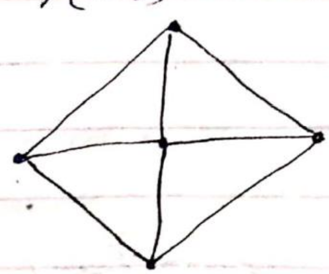
2 Define Chromatic number of a graph.

The chromatic number of a graph is the smallest number of colors needed To color the vertices of so that no Two adjacent vertices share the same color i.e., the smallest value of possible To obtain a k-coloring. Minimal colorings and chromatic numbers for a sample of graphs are illustrated below

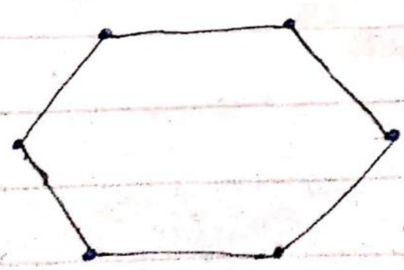
$\gamma(S_6) = 2$



$\gamma(W_5) = 3$



$\gamma(C_6) = 2$

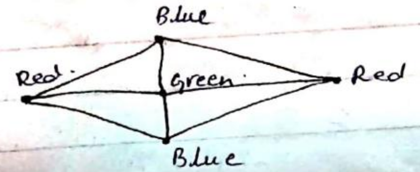


(11)

What is graph coloring problem? Explain with example.

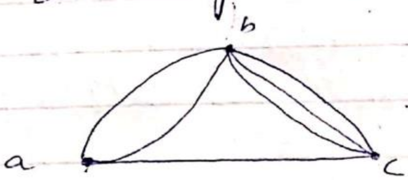
A coloring of simple graph is the assignment of a color to each vertex of the graph so the problem is that no two adjacent vertices are assigned the same color.

Example



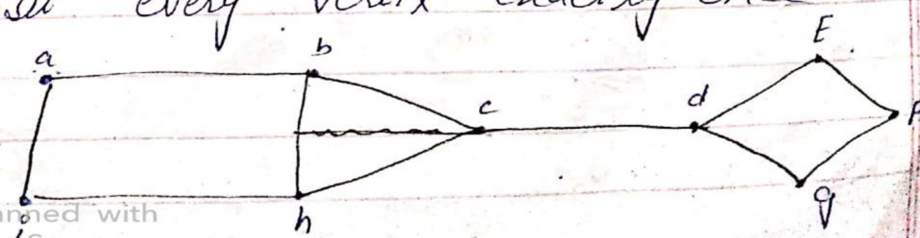
Q Different b/w Euler path & Hamitonian path?

Euler path:- It is a path that visit every edge exactly once and only once. Maximum two vertices have odd degree.



Hamitonian path:-

It is a path that visit every vertix exactly once.



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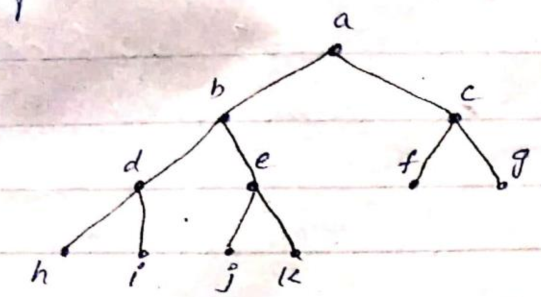
What is height of rooted Tree?

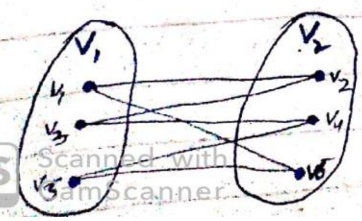
The height of a rooted tree is the maximum of the levels of vertices.



$\boxed{\text{Height} = 3}$

Define Bipartite Graph with partitions example?

A simple graph G is called bipartite if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2. When this condition holds, we call the pair (V_1, V_2) a bipartition of the vertex set V of G



Showing That C_6 is Bipartite.

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Q What are the Two ways for the representation of graph?

If a graph comprises 2 n

- Nodes: These are the most important components in any graph. Nodes are entities whose relationships are expressed using edges
- Edges: Edges are the components that are used to represent the relationship between various nodes in graph.

Q Difference b/w mathematical induction & Strong induction.

Mathematical induction	Strong induction:
Mathematical induction is a mathematical proof technique. It is essentially used to prove that a property P(n) holds for every natural number n, i.e. for n=0,1,2,3, and so on	Strong induction is a type of proof closely related to simple induction. As in simple induction, we have a statement P(n) about the

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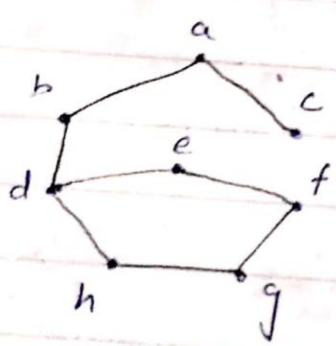
whole numbers n, and we want to prove that p(n) is True for every value of n.

Difference b/w Graph & Tree.

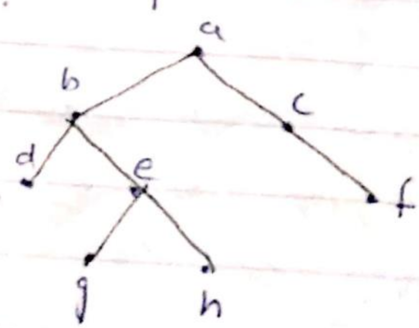
Graph

A Graph is a group of vertices and edges where an edge connects a pair of vertices



Tree

A Tree is considered as a minimally connected graph which must be connected and free from loops



Graph and Tree are the non-linear data structure which is used to solve various complex problems.

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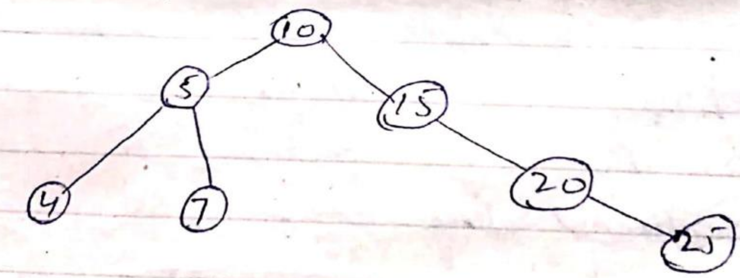
Define Binary search Tree?

A Binary search tree also known as an ordered binary Tree, is a node-based data structure in which each node has no more than two child nodes. The left sub-Tree contains only nodes with keys less than the parent node, the right sub-Tree contains only nodes with keys greater than the parent node.

Left $\leq$ parent
Right $\geq$ parent



Q How many edges are there in a graph with 10 vertices, each having degree six?

The sum of the degree of the vertices is $6 \times 10 = 60$

The handshaking theorem says $2m = 60$ So the number of edges is $m = 30$

$$\frac{\cancel{2}m}{\cancel{2}} = \frac{\cancel{60}^{30}}{\cancel{2}} = 30.$$

Evaluate of this expression.
(11011 ∨ 01010) ∧ (10001 ∨ 11011)

```
  11011          10001
∨ 01010        ∨ 11011
  11011          11011

∧   11011
    11011
    11011
```

Q Let A = {eggs, milk, corn}
B = {cows, goats, hens}
Define a relation R from A to B by (a, b) ∈ R iff a produced by b.

A × B = {(eggs cows), (eggs goats), (eggs hens)
(milk cows), (milk goats), (milk hens)
(corn cows), (corn goats), (corn hens)}

According to this Condition;
{(eggs, hens), (milk, cows), (milk, goats)}

Q what is closure of relations?
Define reflexive closure and Transitive closure?

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with the respect to property P is the relation obtained by adding the minimum number of orders pair to R. To Obtain ~~the property~~

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Closure of relations:-

The relation R = {(1,1), (1,2), (2,1), (3,2)} on the set A = {1,2,} is not reflexive.

Reflexive closure:- (add loops)

Any reflexive relation that contains R must also contain (2,2) and (3,3). Because this relation contain R, is reflexive and is contained within every reflexive relation that contains R, it is called the reflexive closure of R.

Transitive closure:-

Let R be a relation on a set A. The connectivity relation R* consists of pairs (a,b) such that there is a path of length at least one from a to b in R.

Determine the truth value of each of these statements if the domain consists of all real numbers.

a) $\exists x(x3=-1)$

False

b) $\exists x(x4<x2)$

False

c) $\forall x(-x)2=x2)$

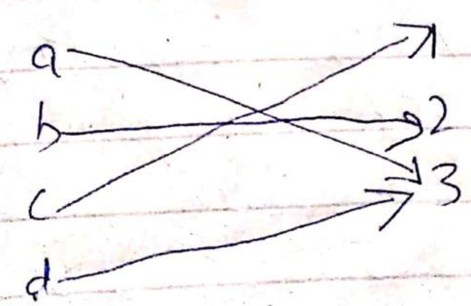
True

d) $\forall x(2x>x)$

True

---

let f be the function from $\{a,b,c,d\}$ to $\{1,2,3\}$ defined by $f(a)=3$ $f(b)=2$ $f(c)=1$ $f(d)=3$ is f an onto function



onto function.

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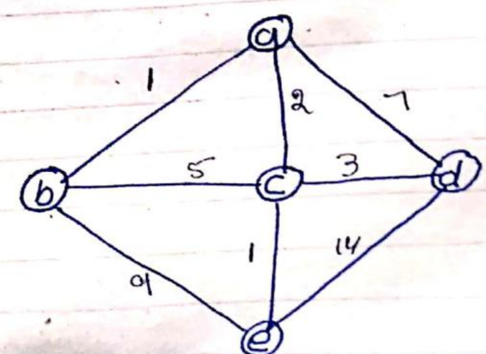
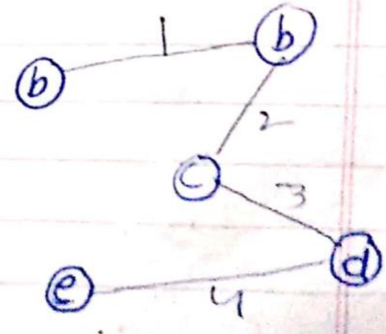
Minimum Spaning Tree

(19)

Minimum spaning tree is used to solve the problems of traveling salesmen.

"Minimum spaning tree or minimum weight spaning tree is a subset of the edges of a connected, edge-weighted undirected graph that connect all the vertices together without any cycle."





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Q Height of Tree is 7. Find sum of height?

Q How the height of Tree is calculated?

Suppose that a connected planar simple graph has 30 edges. If a plane drawing of this graph has 20 faces, how many vertices.

Q How many Permutation of the letters ABCDEFG contain the String CFGA?

$$4! = 4 \times 3 \times 2 \times 1$$
$$= 24$$

D	B	CFGA	E

= 4

Then we take 4 factorial

Q. How many Subsets with more than two elements does a set with 100 elements have?

In general, if a set has n elements, then the set han 2^n ~~elements~~ Subsets.

The given set has 100 elements which has 2^{100} elements.

Thus the number of subsets with more than 2 elements is then the number of subsets decreased by the number of subsets with at most 2 elements

$2^{100} - 4950 - 100 - 1 = 1,267,650,600,228,229,401,496,703,200,325$

Q find ~~the difference~~

$\therefore \quad C(100, 2) = \dfrac{100!}{2!(100-2)!} = 4950$

$C(100, 1) = \dfrac{100!}{1!(100-1)!} \quad 100$

$C(100, 0) = \dfrac{100!}{0!(100-0)!} = 1$

Is set theory deals continous value? Explain to motive your answer?

Set theory, branch of mathematics that deals with the properties of well-defined collection of objects, which may or may not be of a mathematical nature such as numbers or functions. The theory is less valuable in direct application ordinary experience than as a basis for precise and adaptable terminology for the definition of complex and sophisticated mathematical concepts.

Question#2

What is the value of prefix expression.

$* 3 + 3 \uparrow 3 + 333.$

$* + 3 + 3 \wedge 3 (3+3) 3$

 $* + 3 + 3 \wedge 3 6 3$

Yy

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$* + 3 + 3 (3^6) 3$

$* + 3 + 3\ 729\ 3$

$* + 3 (3 + 729) 3$

$* + 3\ 732\ 3$

$* (3 + 732) 3$

$* 735\ 3$

$(735) * 3$

$\boxed{2205}$ Ans

Question # 3

What is divide & conquer approach?

In computer science **divide and conquer** is an algorithm design paradigm based on multi-branched recussion.

⇒ A divide-and-conquer algorithm works by recursively breaking down a problem into two or more sub-proplems.

Give any two applications of mathematical induction and recursion?

⇒ Recursion occurs when a thing is defined in terms itself of its type

⇒ The most common applications of recursion is in **mathematics** and **computer science**, where a function being defined is applied within its own definition.

Question #5

How many one-to-one and functions are there from a set with three-element of a set with four elements?

There are 4 possible values for f(1). But since f is to be one-to-one you cannot pick just any of the four 4 to be f(2) so * for

value you assign to f(1).*
You have only 3 choices for f(2); similarly once you assign values to f(1) and f(2) only two choices remain for f(3).

So, your total is
$4 * 3 * 2 = 24$.

abc
abd
acd

Q Among 200 people, 150 either swim OR jog OR both. if 85 swim and 60 swim and jog how many jog?

$n(S \cup J) = 150$
$n(S) = 85$
$n(S \cap J) = 60$
$n(J) = ?$

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According To second ~~pt~~ rule of Inclusion – Exclusion principle:

$n(S \cup J) = n(S) + n(J) - n(S \cap J)$

$150 = 85 + n(J) - 60$

$150 = 25 + n(J)$

$150 - 25 = n(J)$

$n(J) = 125$

Q# what are the minimum number of students in a class to be sure that two of them are born in the same month?

According To pigeons hole principle:

Students = $\underline{\underline{13}}$

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State division algorithm?

Division algorithm state for any integer a and any positive integer b there exist unique integer q and r.

$$a = bq + r$$

b = divisor
q = quotient
r = reminder

Geometric and arithmetic progression?

Geometric Progression.

is also known as Geometric sequence is as sequence of number where each term after the first is found by multiply the pervious.

Example 2, 6, 18 ....
common is 3

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Arithmetic progression:

is also called Arithmetic Sequence is a sequence of number which differ from each other by a common difference

Example :- 2, 4, 6, 8 ....

State the resolution rule & Modus Ponens & tollens rule?

- Modus Ponens

$$\begin{array}{c} p \to q \\ \underline{p} \\ q \end{array}$$

- Modus Tollens

$$\begin{array}{c} \sim q \\ \underline{p \to q} \\ \sim p \end{array}$$

- **Resolution rule**

$$\begin{array}{l} p \vee q \\ \underline{\sim p \vee r} \\ \therefore q \vee r \end{array}$$

Tautology form

$$(p \vee q) \wedge (\sim p \vee r) \to (q \vee r)$$

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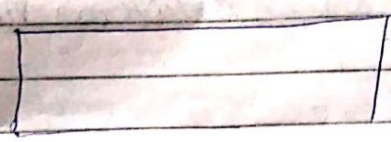
Q What is bi-implication state

The biimplication of p and q is true if and only both p and q ~~is true if~~ are true or both p and q are false

Example



The polygon has only four side, if and only if the polygones is quadrilateral.

Find the symmetric difference of $\{3,5,8\}$ and $\{2,5,6\}$

Symmetric difference $=$ $\underbrace{\{3,5,8\} - \{2,5,6\}}_{A-B}$

$= \underbrace{\{3,8\}}_{A-B} \quad \underbrace{\{2,6\}}_{B-A}$

$(A-B) \cup (B-A) = \{3,8\} \cup \{2,6\}$

$= \{2,3,6,8\}$

Diffrence between mathematical induction and strong induction.

Mathematical	Strong
if P(K) is true then p(K+1) is true.	if p(i) is true for all i less than or equal to K then p(K+1) is true, where p(K) is some statement depending on the positive integer K. They are not "identical" but they are equivalent.

Define greedy algorithm with example.

Greedy Algorithm :-

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A greedy algorithm is an algorithmic strategy that makes the best optimal choice at each small stage with the goal of this eventually leading to a globally optimum solution. This means that the algorithm picks the best solution at the moment without regard for consequences.

36 - 20 = 16   20

16 - 10 = 6   20 10

6 - 5 = 1   20 10 5

1 - 1 = 0   20 10 5 1

Diffrence between Best case and worst case complexity of an algorithm:-

Worst-case	Best case
~~Is~~ the algorithm is the function define by the	The algorithm is the function

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maximum number of steps taken on any instance of size n.

define by the minimum number of steps taken on any instance of size n.

Q. What is the probability of getting a number greater than 4. when a dise is tossed.

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5) (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6).

~~(1,3), (2,2), (3~~

$P(A) = \frac{n(A)}{n(S)} = \frac{30}{36}$

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Q What is a Optimal solution?

An optimal solution is a feasible solution where the objective function reaches its maximum (or minimum) value for example, the most profit or the least cost. A globally optimal solution is one where there are no other feasible solutions with better objective function values

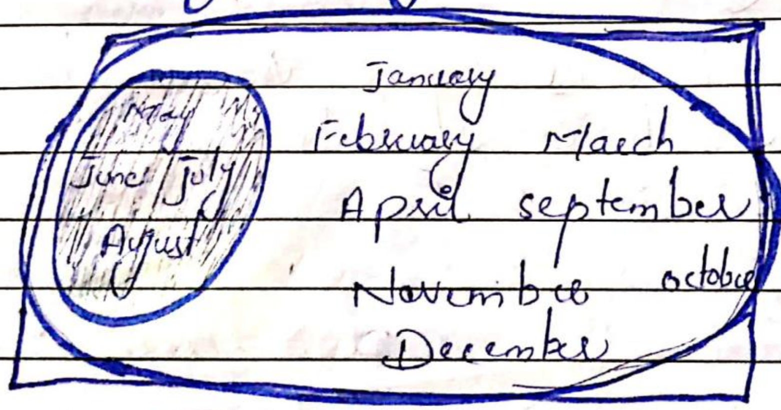
Q Define predicate?

A predicate is a sentence that contains a finite number of variables and becomes a statement when specific values are substituted for the variables. The domain of a predicate variable is the set of all values that may be substituted in place of the variable.

Q Define the term halting problem

Halting problem takes as input a computer program and input to the program and determin weather the program will eventually stop when run with this input.

Q Use the venndiagram to illustrate the sets of all months of the year whose name does not contain the letter R in the set of months of the year.



Q write any four applications of venn diagram

Math

Venn diagram are commonly

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used in Shools to teach basic math concept such as set union & intersection.

Computer science

program can use venndiagram to visualize computer language and heierarchies.

Buisness

Venndiagram can be use to compare and contrast product services / process or pretty much anything.

Technical reading comprehension

Teachers can use it to improve their students reading comprehension.

Find the next three terms of sequence 243, -81, 27.....

243 = 3^5
81 = 3^4
27 = 3^3
9 = 3^2

The next terms are

3^1 & 3^0 so, 0, & 1

Q Define functions with example?
A special relationship where each input has a single output. It is often written as "f(x)" where x is the input value.

Example:-
f(x) = x/2 ("f of x equals x divided by 2")
It is a function because each input "x" has a single output "x/2".
• f(2) = 1.

Q State Pigeonhole principle?
The pigeonhole principle states that if items are put into containers with, then at least one containes must contain more than one item. This theorem is exemplified in real life by truisms like "in any group of three gloves there must be at least two left gloves or at

least two right gloves".

Q Difference b/w free variable and bound variable.

Variables in the scope of some of quantifies are called bound variables. All other variables in the expression are called free variables. A propositional function that does not contain any free variables is a proposition and has a Truth Value.

Q What is the expansion of $(x+y)^4$? By using Pascal's Triangle.

The Triangle can be used to calculate the coefficients of the expansion of (a+b)n (a+b)n by Taking the exponent n and adding 1. The coefficients will correspond with line n+1

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of the Triangle. For $(x+y)^4$
$(x+y)^4$, $n=4$ so the coefficients of the expansion will correspond with line 5.

Q Define permutation & Combination?

Combinations and Permutations
For instance, both permutations and combinations are collections of objects. But while a combination is a collection of the objects where the order doesn't matter, a permutation is an arrangement of a group of objects where the order does matter.

Q What is modous ponen rule of inference?

The rule of inference called modus ponens takes two premises, one in the form "If p then q" and another in the form "p", and returns the conclusion "q".

Q Find $S = \sum_{k=48}^{100} 3.(4)^k$

Q Define the term counterexample?

An example that contradicts a statement.
(Show a statement to be false).

Example:-

The universal statement $\forall x\, P(x)$ is false $\exists x \in D$ such that $P(x)$ is false

Q What is the cardinality of each of these sets
$\{a, \{a\}, \{a\{a\}\}\}$?

Its cardinality is 3.

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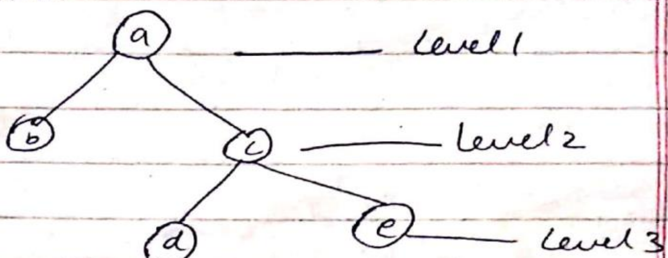
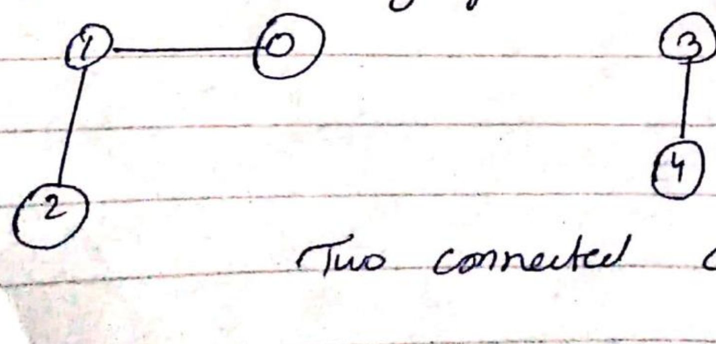
What is height of rooted tree?

Ans The level of node is one greater than the level of its parent.
The level of the root node 1
The height of rooted tree is the maximum level of any node in the tree



What are the connected components of a graph?

In graph theory, a connected components of an undirected graph is a subgraph in which any two vertices are connected to each other by path, and which is connected to no additional vertices in the subgraph.



Two connected components.

Q How many different license plates can be made if each plate contains a sequence of three uppercase three letters followed by three digits?

Product rule

26 26 26 10 10 10

$26 \times 26 \times 26 \times 10 \times 10 \times 10$

$= 17576\,00$ possible Plates

Q How many relations are there on a set with n elements

If irreflexive relation $\rightarrow 2^{n(n-1)}$

If symmetric relation $\rightarrow 2^{\frac{n(n+1)}{2}}$

anti-symmetric relation $\rightarrow 2^{\left(\frac{n(n+1)}{2}\right)}$

asymmetric$^{(n-1)}$ relation $\rightarrow 2^{\binom{n}{2}}$

Both reflexive and symmetric $\rightarrow 2^{\binom{n}{2}}$

Q Define Fallacies?

An invalid argument form often used incorrectly as a rule of inference

(or sometimes, more generally, an incorrect argument)

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Q Define disjoint set?

Two sets are called disjoint if their intersection is the empty set. Exp

$A = \{1,2,3\}$, $B = \{4,5,6\}$, $A \cap B = \phi$

Q State principle of inclusion Exclusion?

$n(A \cup B) = n(A) + n(B)$

↑ for not common elements

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

↑ for common elements

Q How many permutation → 07 wood DANAMA can be made, if P is to be th first letter in each argument?

P _ _ _ _ _

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$= 20 \times 6$

$= 120$

Q How many Permutation of the letters ABCDEFGH contain the string ABC?

A B C _ _ _ _ _

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$= 20 \times 6 = 120$

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Q Suppose that there are eight runners in a race. First will get ~~silver~~ gold medal, the second will get silver and third will get bronze. How many different ways are there to award these medals if all possible outcomes of race can accur and there is no tie?

$$P(8,3)$$

$$= \frac{8!}{5!}$$

$$= \frac{40320}{120} = 336$$

Q Determine the truth value of following.

$\exists x \; (x^4 < x^2)$ Domain $\rightarrow \mathbb{R}$

$\rightarrow$ False.

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Q How many different elements does AXBXC have if A has m element, B have n elements and C has p elements

∴ elements = m×n×p

Suppose = ~~24~~

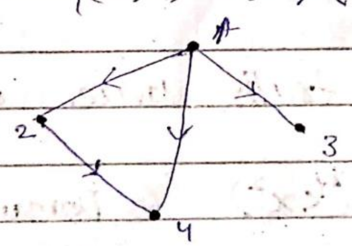
m = 4, n = 2, p = 3

→ m×n×p = 4×2×3 = 24

Q Let A = {1,2,3,4}, and R is a Relation defined by "a devides b". write R as a set of ordered pairs, ~~draw~~ Graph

A = {1,2,3,4}

R = {(2,4)(1,2)(1,3)(1,4)}



Q Define the term Halting problem?

The halting problem is an unsolvable decision problem. That ~~is, no turing machine exists that~~ asks whether a turing machine T eventually halts when given an input string x.

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A bag contains 6 white, 5 black and 4 red balls. Find the probability of getting a white or a black ball in a single draw?

white	Black	Red	Total
6	5	4	15

let A indicate the ~~number~~ ball ~~of~~ ~~wh~~ will be white

$n(A) = {}^6C_1 \cdot {}^5C_0 \cdot {}^4C_0$
$= 6$

Let B indicate the ball will be black.

$n(B) = {}^4C_1 \cdot {}^6C_0 \cdot {}^5C_0 = 4$

$n(S) = {}^{15}C_1 = 15$

Let D indicate that the ball will be white or black.

$n(D) = n(A) + n(B)$

$P(D) = P(A) + P(B)$

$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)}$

$= \frac{6}{15} + \frac{4}{15} = \frac{10}{15} = \frac{2}{3}$

∴ Handshaking lemma is a consequence of **degree sum formula**

[illegible]

Q State handshaking lemma?

In graph theory, a branch of mathematics, the handshaking lemma is the statement that every finite undirected graph has an even number of vertices with odd degree.

$$\sum_{v \in V} \deg v = 2|E|$$

Q What is the basic counting principle?

There are two basic counting principles

- Sum Rule
- product Rule.

Q Write the names of an algorithm properties?

Input: from a specified set

Output: from a specified set (Solution)

Definiteness of every step in the computation

Correctness of number of calculation steps

Effectiveness of each calculation step

Q Define Partial ordering with example?

A relation that is reflexive, antisymmetric and transitive is called partial ordering.

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Q Determine 10, 17, 21 are pairwise relatively prime?

Step I

Dividend	Divisor
10	1, 2, 5, 10
17	1, 17
21	1, 3, 7, 21

Step II

pair	gcd	
10, 17	1	relatively prime
10 21	1	" "
17 21	1	" "

Q Encrypt the message WATCH YOUR STEP by translating the letter into numbers, applying the given encryption function, and then translating the numbers back into letters. $f(P) = (14p+21) \bmod 26$

let us apply the function

22 0 19 2 7   24 14 20 17   18 19 4 15
W A T C H

17 21 1 23 15   19 9 15 25   13 12 5 23 $\therefore 14(22)+21 \bmod 26$

$= 329 \bmod 26$

$= 17$

then

RVBXP TJPZ NBZX   we encode it "22" with

$= 17$

Q Find the recursive relation of the sequence $S(n) = 5^n$

23

put $k = k+1$

$= 4^{k+1+1} + 5^{2(k+1)-1}$

$= 4^{k+2} + 5^{2k+2-1}$

$= 4^{k+2} + 5^{2k+1}$

$= 4^{k+1}.4 + 5^{2k-1+1+1}$

$= 4.4^{k+1} + 5^{2k-1}.5^{2}$

$= 4.4^{k+1} + 25.5^{2k-1}$

$= 4.4^{k+1} + (21+4).5^{2k-1}$

$= 4.4^{k+1} + 21.5^{2k-1} + 4.5^{2k-1}$

$= 4(4^{k+1} + 5^{2k-1}) + 21.5^{2k-1}$

$= 4(21p) + 21.5^{2k-1}$

$= 21(4p + 5^{2k-1})$

$= 21y$

Any number divisible by 21 if multiply by 21.

c) Find the argument from the following argument and determine whether it is valid, can we conclude the the conclusion is true. If the premises are true?

- If George does not have eight legs, then he is not an insect.

George is ~~not~~ an insect

Therefore, George has eight legs.

let p = George has eight legs

q = George is an insect

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$\sim p \to \sim q$

q

$\therefore p$

p	q	$\sim p$	$\sim q$	$\sim p \to \sim q$	q	p
T	T	F	F	T	T	T ✓
T	F	F	T	T	F	T
F	T	T	F	F	T	F
F	F	T	T	T	F	F

$\Rightarrow$ Valid Argument

2) b Prove that $\sqrt{2}$ is irrational by using contradicution method.

let $\sqrt{2}$ is rational

$\sqrt{2} = p/q$ (p/q is lowest term)

$(\sqrt{2})^2 = \left(\frac{p^2}{q^2}\right)$

$2 = p^2/q^2$

$2q^2 = p^2$ (i)

If p^2 is even then p is also even.

$p = 2k$

$(p)^2 = (2k)^2$

$p^2 = 4k^2$ (ii)

By comparing (i) & (ii)

$2q^2 = 4k^2$

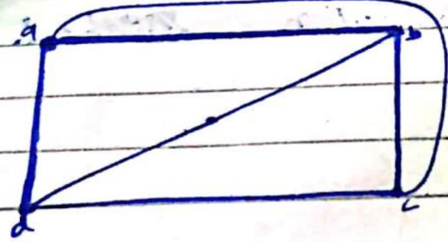
$$2k^2 = q^2$$

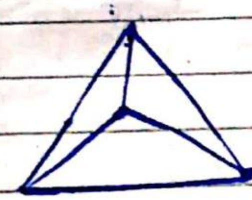
If q^2 is even then q is also even.

The given theorem is true $\Rightarrow \sqrt{2}$ is irrational.

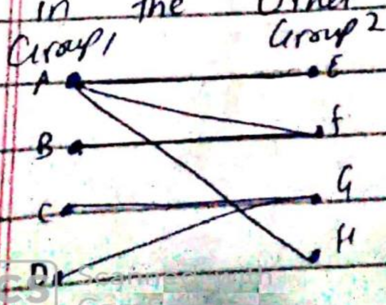
Q3 Define the following graphs with example:

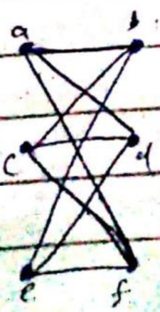
⇒ **Planar graph:-** A planar graph is a graph that can be drawn in such a way that no edges cross each other.

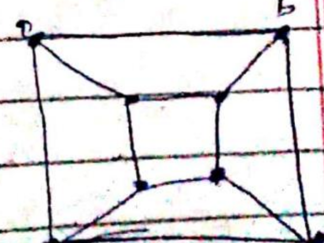




⇒ **Bipartite Graph:-** A graph whose vertex set can be divided into two ~~pool~~ disjoint subsets such that each edge connects a vertex in one of these subsets to a vertex in the other subsets.







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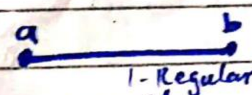
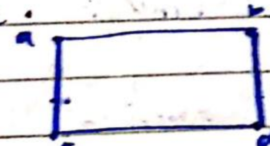
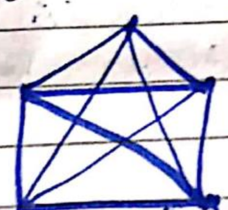
⇒ **Regular graph:** A Regular graph is a graph where each vertex has the same number of neighbours i.e every vertex has the same degree.





Complete graph:- A complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge.



b) Define the following relation with example:-

i) Reflexive relation- A relation R on set A is called reflexive if $(a,a) \in A$ for every elements $a \in A$.

⇒ For example: $R = \{1,2,3,4\}$;

$R_1 = \{(1,1),(1,2),(1,4),(2,1),(2,2),(3,3),(4,1)(4,4)\}$ are the reflexive relation because it contain all pairs of the form (a,a) namely, $(1,1)$, $(2,2)$, $(3,3)$ and $(4,4)$.

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ii Symmteric Relation:- A relation R on a set A is called **symmetric** if $(b,a) \in R$ whenever $(a,b) \in R$ for all $a, b \in A$.

⇒ For example:-

An example is the relation "**is equal to**" because if $a=b$ then $b=a$ is also true.

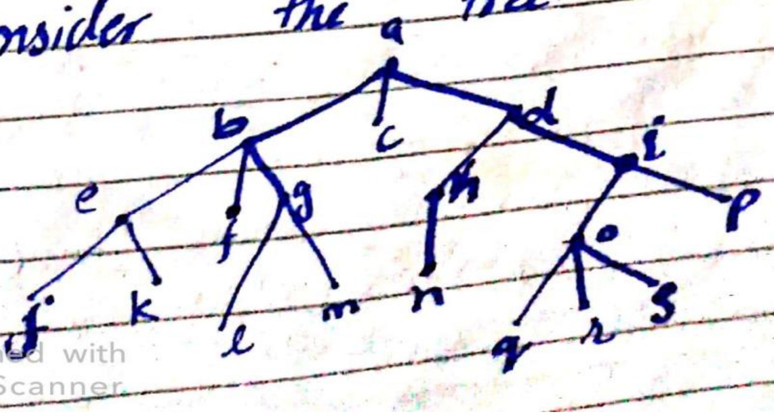
(iii) Transitive Relation:- A Relation R on a set A is called transitive if whenever $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ for all $a, b, c \in A$

For example:-

"is greater than", "is at least as great as" and "is equal to" are transitive relation.

Whenever $x>y$ and $y>z$ then also $x>z$

Ex b Consider the tree Shown below



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What is the level of n? 3

What is the height of this rooted tree? 5

What are the children of b? e, f, g, m

What is the parent of g? b

What are the sibling of j? K

What are the descendants of f? No

Q6 Translate these into english.

Where $C(x)$ = "x is comedian"

$F(x)$ = "x is funny"

and domain consist of all People:

a) $\forall x \, (C(x) \rightarrow F(x))$

Every comedian is funny

b) $\forall x \, (C(x) \wedge F(x))$

Every person is a funny comedian

c) $\exists x \, (C(x) \rightarrow F(x))$

There exist a person such that if She or he is a comedian then She or he is funny

d) $\exists x \, (C(x) \wedge F(x))$.

Some comedian are funny.

(29)

Q9 Give a proof by contradiction of the theorem "if 3n+2 is odd then n is odd.

In Contradiction $\therefore p \to q \Rightarrow p \to \sim q$

Contradiction:-

if 3n+2 is odd the n is even.

let $n = 2k$ —①

put eq ① in $3n+2$

$$= 3(2k)+2$$
$$= 6k+2$$
$$= 2(3k+1)$$

let $p = 3k+1$ then

$= 2p$ is even

then it is prove that the given Statement is contradiction.

Q10 Prove that if m+n and n+p is even int therefore m, n, p are integer, then m+p is even? What kind of proof did you use?

$$m+n = 2k \quad —①$$
$$n+p = 2x \quad —②$$

Add both Statement

$$m+n+n+p = 2k+2x$$
$$m+2n+p = 2k+2x$$

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$$m+p = 2k+2n-2n$$
$$m+p = 2(k+n-n)$$
let $y = k+n-n$ then
$m+p = 2y$ that is even.

Q25 Let R be the relation {(1,2), (1,3), (2,3), (2,4), (3,1)} and let S be the relation {(2,1), (3,1), (4,2)} find SoR and RoS.

R = {(1,2) (1,3) (2,3) (2,4) (3,1)}
S = {(2,1) (3,1) (4,2)}

SoR = {(1,1), (2,1) (2,2)}

RoS = {(2,2) (3,2), (2,3), (3,3), (4,3) (4,4), }

Q32 Use rule of inference to show that "the hypothesis Randy works hard, if Randy work hard, when he is a dull boy and if Randy is a dull boy then he will not get the job" "imply the conclusion Randy will not get the job."

let w be Randy work hard.
let d be Randy is a dull boy.
let j be Randy will get job.

w

is , $w \rightarrow d$

and

$d \rightarrow \sim j$ then

modus ponens first two hypothesis follow d

and, using modus ponens and the last conclusion / hypothesis, $\sim j$ which is desired conclusion.

$$\begin{array}{l} w, w \rightarrow d \\ d \rightarrow \sim j \\ \hline \therefore \sim j \end{array} \qquad \begin{array}{l} p \\ p \rightarrow q \\ \hline q \end{array}$$

Q3: What is wrong with this statement?
let $H(x)$ be "x is happy"
Given the premises $\exists x H(x)$, we conclude that $H(\text{john})$, therefore john is happy.

We know that some x exist that make $H(x)$ true, but we cannot conclude that john is one such x.

(32)

Q35 What is proof by Mathmetical Induction. Use M.I to prove that $k^3 - k$ is divisible by 3, where k is Positive integer

Base case:- put $k = 1$

$(k)^3 - k$ is divisible by 3

$= (1)^3 - (1)$

$= 1 - 1 = 0$ is also divisible by 3

put $K = k+1$

$(k)^3 - k = (k+1)^3 - (k+1)$

$= k^3 + 1 + 3k(k+1) - k - 1$

$= k^3 - k + 3k(k+1)$

$k^3 - k = 3p$ because it is divisible by 3

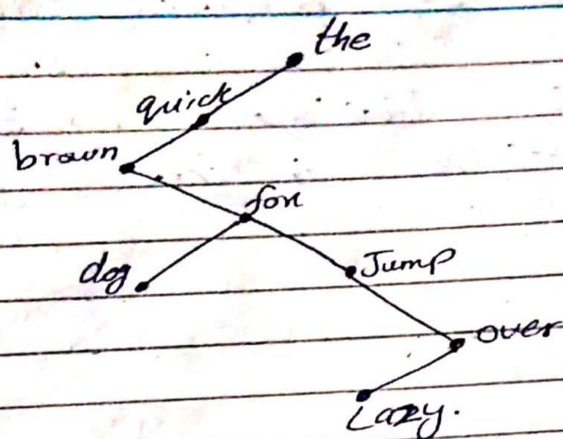
$= 3p + 3k(k+1)$

$= 3(p + k(k+1))$

Any number that is multiple of 3 also divisible by 3.

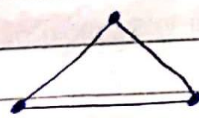
Q36 Using alphabetic order, construct a binary search tree for the words in the sentence: "The quick brown fox jumps over the lazy dog."

Binary Search Tree:- 33

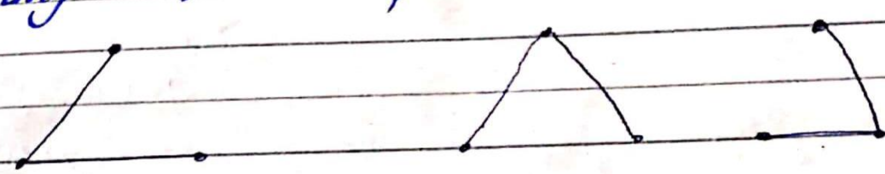


Q31 How many different spanningtree does each of these simple graph.

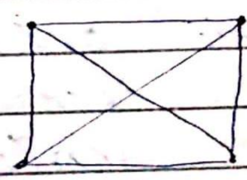
⇒ K_3



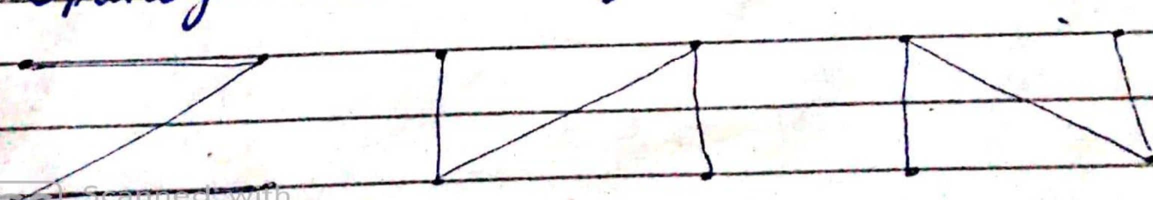
Spanning tree of K_3 = 3



K_4



Spaning - tree of = 16



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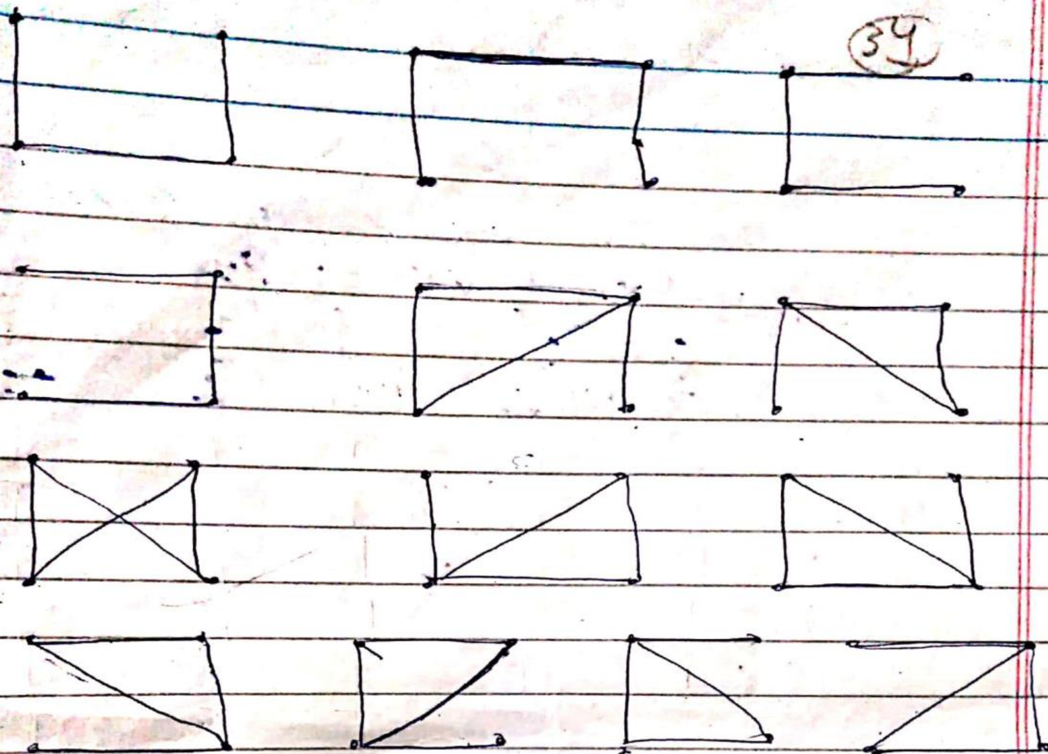
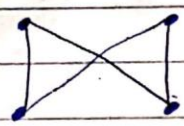
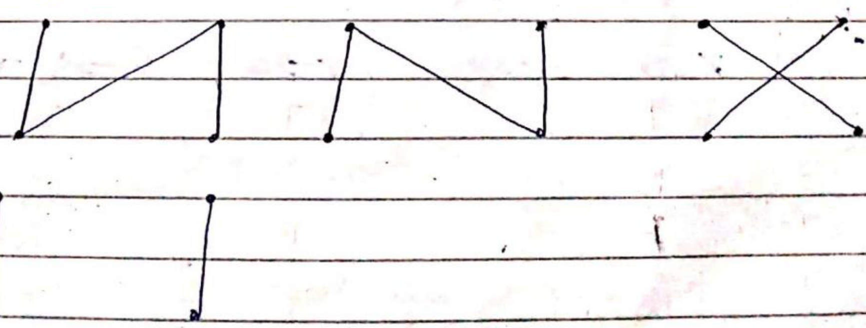


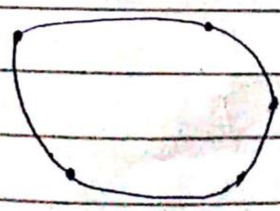
c) $K_{2,2}$



Spanning tree of $K_{2,2} = 4$



C_5



Spanning tree of C_5 = 5

Q38 Prove that are logically equivalent:-

a) $(p \vee q) \wedge (\sim p \vee r) \rightarrow (q \vee r)$ is tautology?

$\sim p$	p	q	r	$p \vee q$	$\sim p \vee r$	$q \vee r$	$\wedge$	$\rightarrow$
F	T	T	T	T	T	T	T	T
F	T	T	F	T	F	T	F	T
F	T	F	T	T	T	T	T	T
F	T	F	F	T	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	T	F	T	T	T	T
T	F	F	F	F	T	F	F	T

Hence Proved.

b) $(p \rightarrow q) \rightarrow (r \rightarrow s)$ and (2) $(p \rightarrow r) \rightarrow (q \rightarrow s)$

1) $(p \rightarrow q) \rightarrow (r \rightarrow s)$

p	q	r	s	$p \rightarrow q$	$r \rightarrow s$	$(p \rightarrow q) \rightarrow (r \rightarrow s)$
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	T	T	T
T	T	F	F	T	T	T
T	F	T	T	F	T	T
T	F	T	F	F	F	T
T	F	F	T	F	T	T
T	F	F	F	F	T	T

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F	T	T	T	T	T	T
F	T	T	F	T	F	F
F	T	F	T	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	T	T
F	F	T	F	T	F	F
F	F	F	T	T	T	T
F	F	F	F	T	T	T

Also same as above construct table for (2).

Q39 Show that the premises "A student in this class has not read the book" and everyone who passed the first ~~exam~~ exam" imply the conclusion someone who passed the first exam has not ~~yet~~ read the book.

$\forall \neg \exists \sim P$

Let $C(x)$ be "x is in the class"

let $B(x)$ be "x has read the book"

let $P(x)$ be "x passed the first exam"

$\exists x(C(x) \wedge \sim B(x))$

$\forall x(C(x) \longrightarrow P(x))$

has $\wedge$

$\therefore \exists x(P(x) \wedge \sim B(x))$.

Q40. Use M.I to prove $1+3+5+\ldots(2n-1)=n^2$ for all integers $n\geq 1$

put $n=1$

$1+3+5+ \ldots + (2(1)-1) = (1)^2$

$1 = 1$

put $n=k$

$1+3+5+\ldots+(2k-1) = k^2$

put $k=k+1$

$1+3+5+\ldots+(2(k+1)-1)=(k+1)^2$

$1+3+5+\ldots+(2k+1)=(k+1)^2$

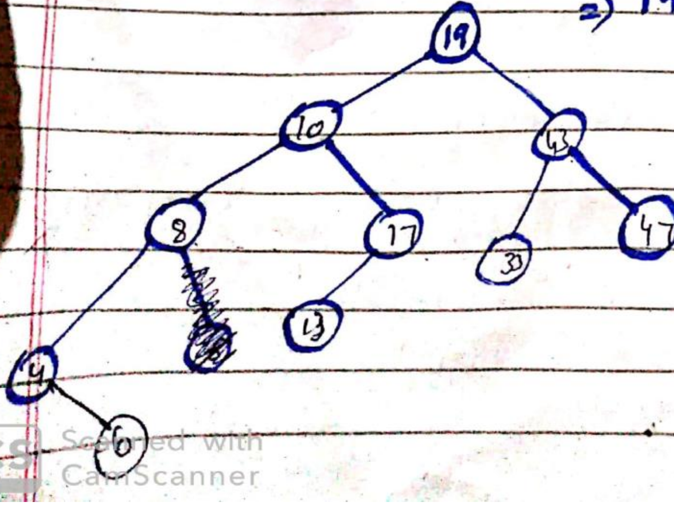
$k^2+2k+1=(k+1)^2$

$(k+1)^2=(k+1)^2$

L.H.S = R.H.S

Q42 Draw a binary search tree by inserting the number from left to right

=) 19, 10, 8, 17, 4, 10, 6, 13, 43, 47, 33



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(38)

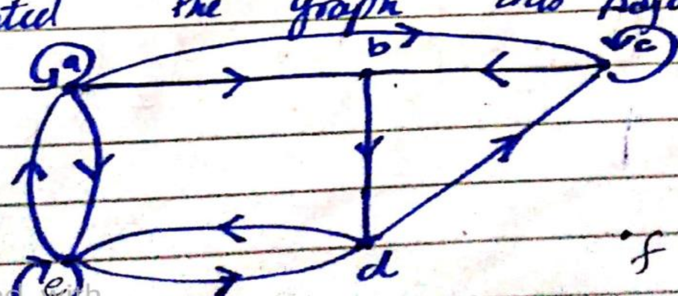
Determine the order in which vertices of the following binary trees will be visited under
1) preorder Traversal
2) Inorder Traversal
3) Postorder Traversal

1) Preorder Traversal:-
19, 10, 8, 4, 6, 17, 13, 43, 33, 47

=) Inorder Traversal.-
4, 6, 8, 10, 13, 17, 19, 33, 43, 47

=) Post order:-
6, 4, 8, 13, 17, 10, 33, 47, 43, 19

Q6 Find the in-degree and out-degree of the given graph. Also find whether the given graph has Hamiltonian or Euler tour. Show visually also represented the graph into Adjacency Matrix



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(54)

out

$deg(^{+}a) = 4$

$deg(^{-}a) = 2$

$deg(^{+}b) = 1$

$deg(^{-}b) = 2$

$deg(^{+}c) = 2$

$deg(^{-}c) = 3$

$deg(^{+}d) = 2$

$deg(^{-}d) = 2$

$deg(^{-}e) = 3$

$deg(^{+}e) = 3$

$deg(^{-}f) = 0$

$deg(^{+}f) = 0$

2) Find graph is Euler tour or Harmiltonian

~~Not Euler path.~~

~~Not Euler or Harmiltonian path.~~

Not Euler or Harmiltanian Tour

3) Adjancy Matrix :-

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

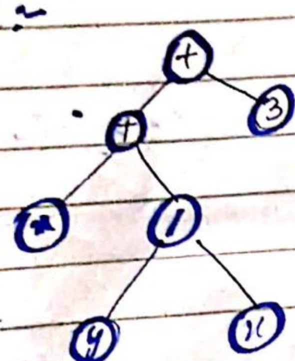
40

Q52) Represent the following algebric expression in tree

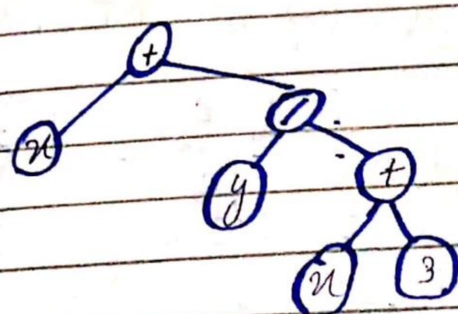
$(x+(y/x))+3$ and $x+(y/(x+3))$

$\Rightarrow (x+(y/x))+3$

→ In fix:-



→ Prefix:-



Q53 For each of these relation on set $A=\{1,2,3,4\}$ decide whether it is reflexive, symmetric, asymmetric, antisymmetric and transitive.

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(41)

a) $\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$

~~Symmetric~~, Transitive,

b) $\{(1,1),(2,2),(3,3),(4,4)\}$

Reflexive, Symmetric, transitive and antisymmetric

c) $\{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$

Reflexive, Symmetric, transitive

d) $\{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$

None of these Properties.

Q24 Let $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $M_S = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

find M_{SoR}

$$= \begin{bmatrix} 1+0+0 & 1+0+0 & 1+0+0 \\ 1+0+1 & 1+0+1 & 1+0+0 \\ 0+1+0 & 0+0+0 & 0+1+0 \end{bmatrix}$$

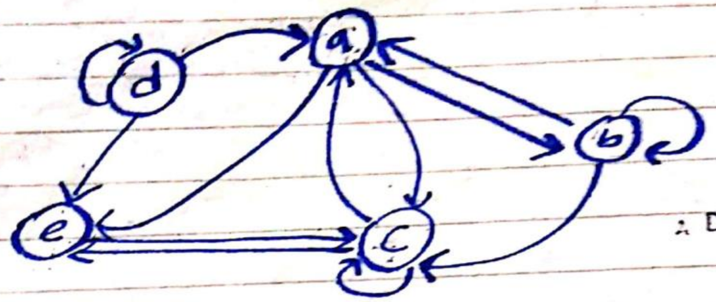
$$M_{SoR} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(42)

b) A digraph of the set $A = \{a, b, c, d, e\}$

i) Matrix M_R on the digraph?

ii) Find that the digraph symmetric, reflexive, antisymmetric or transitive?



∴ a to b
∴ b to c
∴ Transitive

∴ Directed graph
Loop =

i)

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

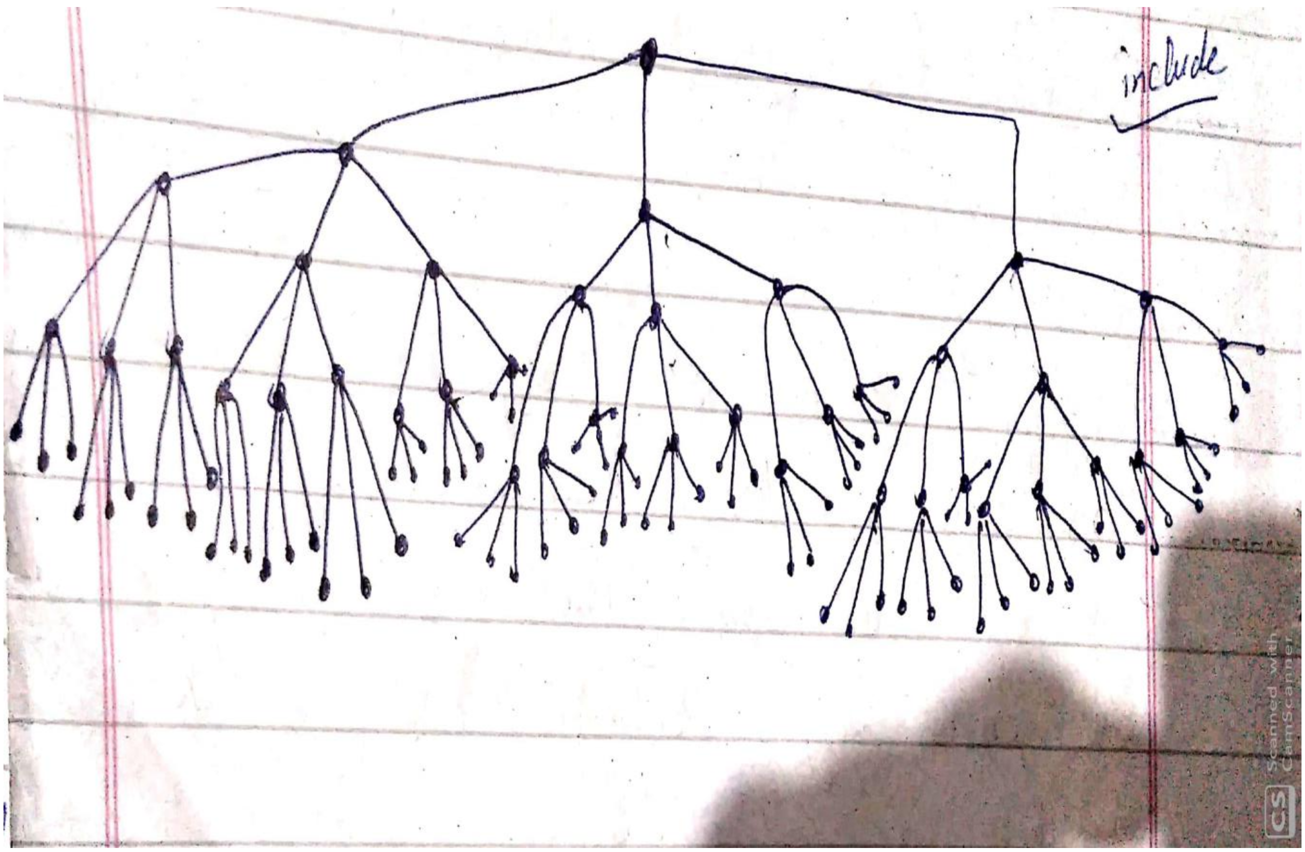
→ digraph is not reflexive because on every vertex there is no loop. Loop exist on some vertex. If loop exist on every vertex then it called reflexive

→ This graph has no degree same then its not symmetric

→ It is antisymmetric.

→ digraph is ~~not~~ transitive because there is an edge from a - b and b-c but also from a-c.

Draw a 3-ary tree with height 4?



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2(a) Find $S = \sum_{k=50}^{100} k^2$

$$\sum_{k=1}^{100} k^2 = \sum_{k=1}^{49} k^2 + \sum_{k=50}^{100} k^2$$

$$\sum_{k=50}^{100} k^2 = \sum_{k=1}^{100} k^2 - \sum_{k=1}^{49} k^2$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{100(100+1)(2(100)+1)}{6} - \frac{(49(49+1))(2(49)+1)}{6}$$

$$= \frac{100(101)(201)}{6} - \frac{49(50)(99)}{6}$$

$$= \frac{2030101}{6} - \frac{2450(99)}{6}$$

$$= 338350 - 40425$$

$$= 297925$$

4(a) How many bit Strings of length four do not have two consecutive 1's by using tree diagram?

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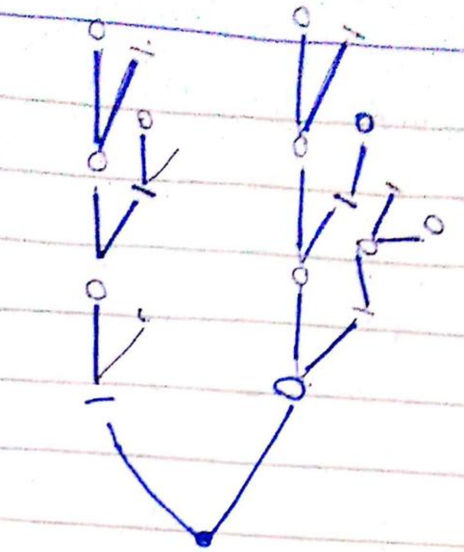

How many license plates can be made using either two letters followed by four digit or two digits followed by four letters.

$\underline{26}\ \underline{26}\ \underline{10}\ \underline{9}\ \underline{8}\ \underline{7}$

$26 \times 26 \times 10 \times 9 \times 8 \times 7 = 3407040$

$\underline{10}\ \underline{9}\ \underline{26}\ \underline{26}\ 26\ 26$

$10 \times 9 \times 26 \times 26 \times 26 \times 26 = 41127840$

$3407040 + \underline{41127840}\ \underline{44534880}$

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Q Suppose that there are 9 faculty members in the mathmetics department and 11 in the computer Science department. How many ways are there to select a comittee to develop a Discrete Math course at a school if the committee is to consist of three faculty members from the mathmatics department and four from computer science departments.

$$= {}^{9}C_3 * {}^{11}C_4$$

$$= 84 * 330$$

$$= 27720.$$

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) Serial number for a Product are to be made using four letters followed by 2 single-digit numbers. For example JGRB297 is one such serial number. How many such serial number are Possible if neither letters nor number can be reapted?

$\underline{26}\ \underline{25}\ \underline{24}\ \underline{23}\ \underline{10}\ \underline{9}\ \underline{8}$

$= 258336000$

30) In how many different ways can the letter of the word "LEADING" be arranged in such a way that the vowel always come together?

$$\frac{7!}{1!1!1!1!1!1!} = \frac{7!}{1!} = \frac{7\times6\times5\times4\times3\times2\times1}{1}$$

$= 5040$

Q Two cards are drawn at random from an ordinary pack of 52 cards. Find the probability p that

(i) both are spade.

Let A both are spade

$$P(A) = \frac{^{13}C_2 \cdot {}^{39}C_0}{^{52}C_2}$$

$$= \frac{78 \times 1}{1326}$$

$$= 0.05882$$

(ii) one is a spade and one is heart.

Let B be one is spade and one is heart

$$P(B) = \frac{^{13}C_1 \cdot {}^{13}C_1 \cdot {}^{26}C_0}{^{52}C_2} \qquad \because P(B) = \frac{n(A)}{n(S)}$$

$$= \frac{13 \cdot 13 \cdot 1}{1326}$$

$$= 0.12745$$

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let $A = \{1, 2, 3, 4, 5\}$ we define f
$f: A \to A$ and then $g: A \to A$

$f(1) = 3$, $f(2) = 5$, $f(3) = 3$, $f(4) = 1$
$f(5) = 2$

$g(1) = 4$, $g(2) = 1$, $g(3) = 1$, $g(4) = 2$
$g(5) = 3$.

find the fog and gof.

$\Rightarrow$ <u>fog</u>

$fog(1) = f(g(1)) = f(4)$ $\quad \because g(1) = 4$
$\qquad f(4) = 1$

$= \underline{1}$

$fog(2) = f(g(2)) = f(1)$

$= 3$

$fog(3) = f(g(3)) = f(1)$

$= 3$

$fog(4) = f(g(4)) = f(2)$

$= 5$

$fog(5) = f(g(5)) = f(3)$

$= 3$

$\Rightarrow$ <u>gof</u>

$gof(1) = g(f(1)) = g(3)$

$= 1$

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$gof(2) = g(f(2)) = g(5)$

$= 3$

$gof(3) = g(f(3)) = g(3)$

$= 1$

$gof(4) = g(f(4)) = g(1)$

$= 4$

$gof(5) = g(f(5)) = g(2)$

$= 2$

Which term the arthematic sequen

$4, +1, -2, \ldots$ is -77?

$4 - 3n$ is the term where

$n = 0, 1, 2, \ldots 27.$

$4 - 3(0) = 4$

$4 - 3(1) = 1$

$4 - 3(2) = -2$

$\vdots$

$4 - 3(27) = 4 - 81$

$= -77.$

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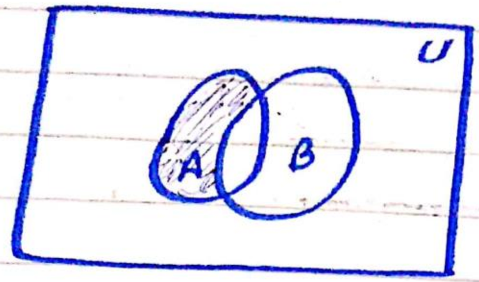
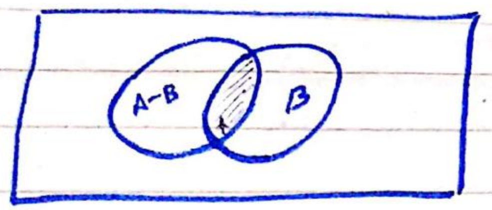
Prove the following using venn diagram.

$$A - (A-B) = A \cap B$$

L.H.S

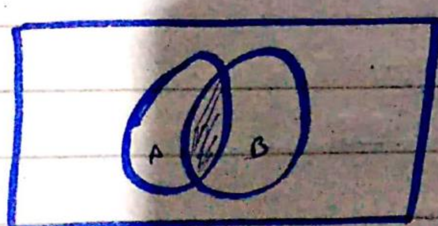


$A-B$



$A-(A-B)$

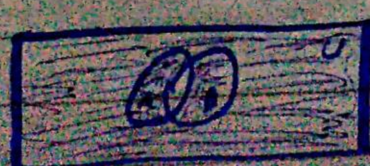
R.H.S



$\Rightarrow A \cap B$

L.H.S = R.H.S

$$(A \cap B)^c = A^c \cup B^c$$



$(A \cap B)^c$

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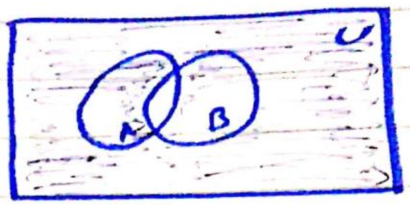
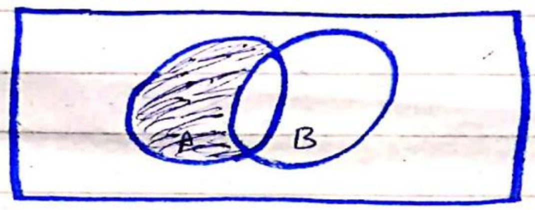
R.H.S



$A^c \cup B^c$

c) $A - B = A \cap B^c$

L.H.S:-

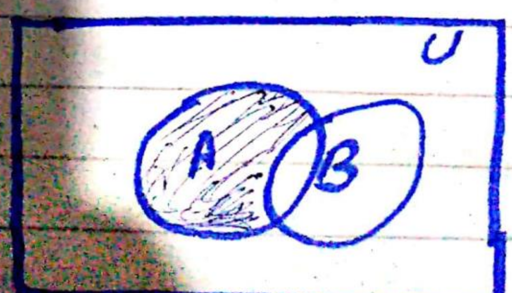


$A - B$

R.H.S:-

B'

$\Downarrow$



$A \cap \overline{B}$

Define the sequence b0, b1, b2, ...
$b_n = 5^n$ for all integer $n \geq 0$
Show that a sequence Satisfy the recurrence relation $b_K = 5b_{K-1}$ for all integer $K \geq 1$.

Sequence :-

$b_0, b_1, b_2, \ldots\ldots$

$b_n = 5^n$

Put $n = 0, 1, 2$ ———

$n=0 \Rightarrow b_0 = 5^0$

$b_0 = 1$

$n=1 \Rightarrow b_1 = 5^1$

$b_1 = 5$

$n=2 \Rightarrow b_2 = 5^2$

$= 25$

$n=3 \Rightarrow b_3 = 5^3$

$= 125$

$1, 5, 25, 125, \ldots\ldots$

Show that

$b_K = 5b_{K-1}$ for $K \geq 1$

Put $n=0$ and $K=1$

$b_1 = 5b_{1-1}$

$= 5b_0$

$b_1 = 5$

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$\Rightarrow$ put $n=1$, $k=2$

$b_2 = 5 b_{2-1}$

$\Rightarrow$ $b_2 = 5 b_1$ $\quad \therefore b_1 = 5$

$b_2 = 25$.

This show that sequense satisfy the recurrence relation.

Use mathematical induction $n \geq 0$

$$\sum_{i=1}^{n+1} i 2^i = n \cdot 2^{n+2} + 2$$

Base Case:

$$\sum_{i=1}^{1} i 2^i = 2.$$

$$1(2^1) = 2$$

$$2 = 2.$$

inductiv Case

$k = n \longrightarrow \sum_{i=1}^{k+1} i 2^i = k 2^{k+2} + 2.$

$n = k+1 \longrightarrow \sum_{i=1}^{k+2} i 2^i = (k+1) 2^{k+3} + 2.$

R.H.S.

$$\sum_{i=1}^{k+1} i 2^i + \sum_{i=1}^{k+2} i 2^i$$

$$k 2^{k+2} + 2 + (k+2) 2^{k+2}$$

$$2^{k+2}(k + k+2) + 2.$$

$$2^{k+2}(2k+2) + 2 \Rightarrow 2^{k+2} 2 (k+1) + 2.$$

$$2^{k+3}(k+1) + 2 \Rightarrow (k+1) 2^{k+3} + 2$$

Suppose that a password for a computer system must have at least 8, but no more than 12 characters, where each character in the password is a lower case English letter, an upper case English letter, a digit or one of the six special characters *, <, >, !, + and =.

a) How many different passwords are available for this computer system?

121

How many of these password contain at least one ~~occurrence~~ occurence of at least one of the six special characters?

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Q Prove that following logically equivalent developing a series of logically equivalence.

$(p \vee q) \wedge (\sim p \vee r) \rightarrow (q \vee r)$ is tautology

Use logical equivalence (2)

$(p \vee q) \wedge (\sim p \vee r) \rightarrow (q \vee r) \equiv \sim[(p \vee q) \wedge (\sim p \vee r)] \vee (q \vee r)$

$p \rightarrow q \equiv \sim p \vee q$

$\equiv \sim(p \vee q) \vee \sim(\sim p \vee r) \vee (q \vee r)$

$\equiv \sim p \wedge \sim q \vee (p \wedge \sim r) \vee (q \vee r)$

Use distributive law

$\equiv [(\sim p \vee \sim q) \vee p) \wedge (\sim p \wedge \sim q \vee \sim r)] \vee (q \vee r)$

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$\equiv [((\sim p \vee p) \wedge (\sim q \vee p)) \wedge ((\sim p \vee \sim r) \wedge (\sim q \vee \sim r))] \vee (q \vee r)$

Use negation law

$\equiv [T \wedge (\sim q \vee p) \wedge (\sim p \vee \sim r) \wedge (\sim q \vee \sim r)] \vee (q \vee r)$

Identity law

$\equiv [(\sim q \vee p) \wedge (\sim p \vee \sim r) \wedge (\sim q \vee \sim r)] \vee (q \vee r)$

Use associative law

$\equiv [(\sim q \vee p) \wedge (\sim p \vee \sim r) \wedge (\sim q \vee \sim r) \vee q] \vee r$

Use distributive law

$\equiv [((\sim q \vee p) \vee q) \wedge ((\sim p \vee \sim r) \vee q) \wedge ((\sim q \vee \sim r) \vee q)] \vee r$

Use associative law

$\equiv [((\sim q \vee q) \vee p) \wedge ((\sim p \vee \sim r) \vee q) \wedge ((\sim q \vee q) \vee \sim r)] \vee r$

Negation law

$\equiv [(T \vee p) \wedge ((\sim p \vee \sim r) \vee q) \wedge (T \vee \sim r)] \vee r$

Use dominant law

$\equiv [T \wedge ((\sim p \vee \sim r) \vee q) \wedge T] \vee r$

Identity law :-

$\equiv [T \wedge ((\sim p \vee \sim r) \vee q)] \vee r$

$\equiv ((\sim p \vee \sim r) \vee q) \vee r$

Use associative law

$\equiv (\sim p \vee \sim r) \vee (q \vee r)$

Commutative law

$\equiv (\sim p \vee \sim r) \vee (r \vee q)$

Use assosiative law

$\equiv ((\sim p \vee \sim r) \vee r) \vee q$

$\equiv \sim p \vee (\sim r \vee r) \vee q$

Negation law

$\equiv (\sim p \vee T) \vee q$

dominat law $\equiv T \vee q$

$\equiv T$ Hence Proved.

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b) Show that $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to r)$ are not logically equivalent.

p	q	r	s	$p \to q$	$r \to s$	$(p \to q) \to (r \to s)$	$p \to r$	$q \to r$	$(p \to r) \to (q \to r)$
T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	F	T	T	T
T	T	F	T	T	T	T	F	F	T
T	T	F	F	T	T	T	F	F	T
T	F	T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	T	T	T
T	F	F	T	F	T	T	F	T	T
T	F	F	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T	T	T
F	T	T	F	T	F	F	T	T	T
F	T	F	T	T	T	T	T	F	F
F	T	F	F	T	T	T	T	F	F
F	F	T	T	T	T	T	T	T	T
F	F	T	F	T	F	F	T	T	T
F	F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T

Hence Proved these are not logically equivalent

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Q Use a insertion sort 3, 1, 5, 7, 4 in descending order.

In descending order–

- Take 1 that is smaller then 3 then not switch.
- Take 5 then compare it with 3, 1 if greater then insert

3	1	5	7	4

After insertion

5	3	1	7	4

Take 7 and compare it with previous

5	3	1	7	4

then insert 7 at first

7	5	3	1	4

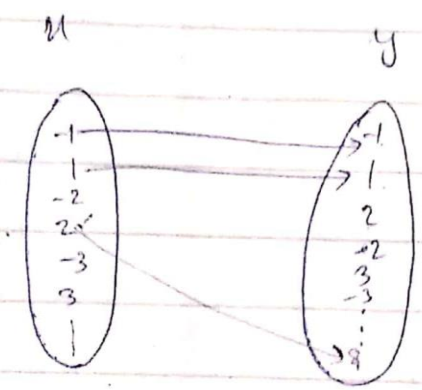
Take 4 and compare it with previous

7	5	3	1	4

0	1	2	3	4
7	5	4	3	1

Q. let $f: R \to R$ be defined
$f(x) = x^3$. Show that
f is bijective.

$\therefore f(x) = y$
$f(x) = x^3$



is not bijective

48 Write down base and recursive
case for sum of array elements.
Also solve for length 5 via tree
Convention:-

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(8) How many license plates can be made using either two letters followed by four digits or two digits followed by four letters?

Two letter followed by four digits.

$\underline{26}\ \underline{26}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10} = 6760000$

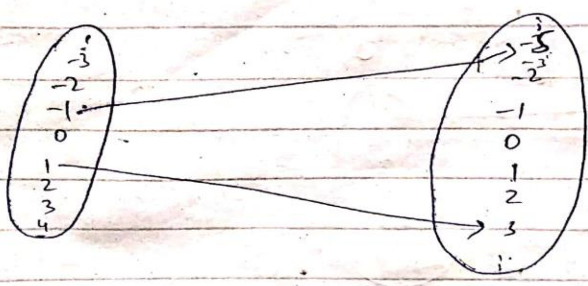
Two digit followed by four letters.

$\underline{10}\ \underline{10}\ \underline{26}\ \underline{26}\ \underline{26}\ \underline{26}$

$= 45697,600$

Define $f: R \to R$ by the rule $f(x) = 4x - 1$ for all $x \in R$

f is onto?



Not onto

$f(x) = 4x - 1$

put $x = 1$

$f(1) = 4(1) - 1$

$= 3$

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Also prove this function and give counter example?

Find the formula for the sequence with the following four fire

i) $1, -1/2, 1/3, -1/4$

ii) $0, 1/2, 2/3, 3/4$

iii) $+, 8, -27, 64.$

$n = 1$

$a_n = \frac{(-1)^{n+1}}{n}$

$\frac{(-1)^{1+1}}{1}, \frac{(-1)^{2+1}}{2}, \frac{(-1)^{3+1}}{3}$

$+1, -1/2, 1/3$

ii) $0, 1/2, 2/3, 3/4$

$a_n = \frac{n}{n+1}$

iii) $-1, 8, -27, 64$

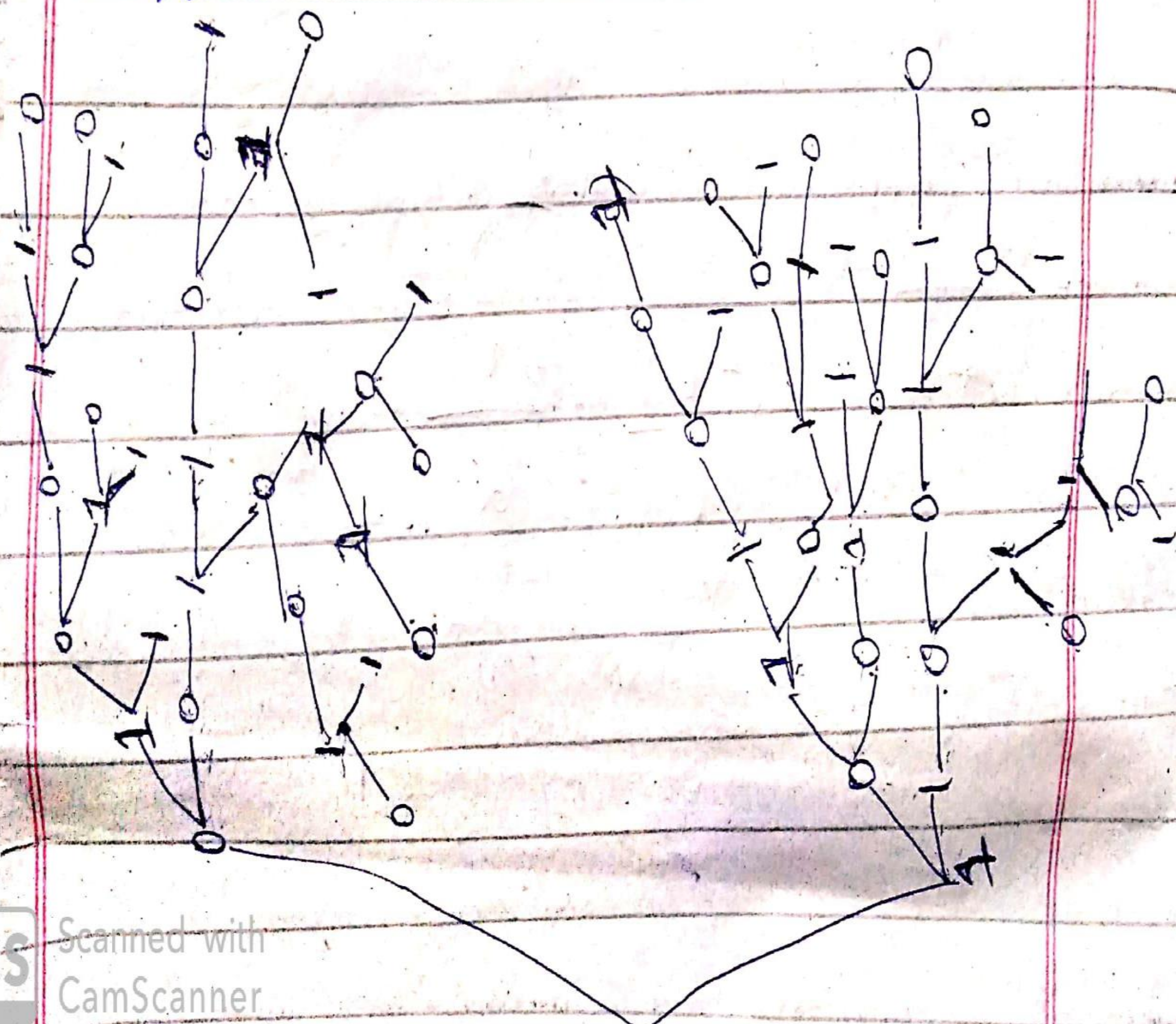
$\cancel{\left(\frac{+1}{n}\right)^3}$ $a_n = (-1)^n (n)^3$

where $n = 1$

$(-1)^1 (1)^3, (-1)^2 (2)^3, (-1)^3 (3)^3, (-1)^4 (4)^3$

$-1, 8, -27, 64$

28) Use a tree diagram to find the number of bit string of length seven with no three consecutive 0's and no three consecutive 1's.



Use a divide and Conquerer algorithm to put 6, 1, 2, 5, -7, 23, 11, 12, 4, 3 . into ~~dep~~ descending order.

Merge Sort:-

0	1	2	3	4	5	6	7	8	9
6	1	2	5	-7	23	11	12	4	3

0	1	2	3	4
6	1	2	5	-7

5	6	7	8	9
23	11	12	4	3

[6 | 1]   [2 | 5 | -7]   [23 | 11]   [12 | 4 | 3]

[6] [1]   [2 | 5] [-7]   [23] [11]   [12] [4 | 3]

[6] [1] [2] [5] [-7] [23] [11] [12] [4] [3]

0	1
6	1

0	1	2
6	2	1

0	1	2	3
6	5	2	1

0	1	2	3	4
6	5	2	1	-7

0	1	2	3	4	5
23	6	5	2	1	-7

0	1	2	3	4	5	6
23	11	6	5	2	1	-7

0	1	2	3	4	5	6	7
23	12	11	6	5	2	1	-7

0	1	2	3	4	5	6	7	8
23	12	11	6	5	4	2	1	-7

0	1	2	3	4	5	6	7	8	9
23	12	11	6	5	4	3	2	1	-7

23) Find each of these value.

a) $(99^2 \bmod 32)^3 \bmod 15$

Use theorem 5:

$$((99 \bmod 32)^2 \bmod 32)^3 \bmod 15$$

$9801 \bmod 32 = 9$

then

$= \cancel{(3^2 \bmod 32)^3 \bmod 15}$

$= \cancel{(9 \bmod 32)^3 \bmod 15}$ $\therefore 81 \bmod 32 = 9$

$= 9^3 \bmod 15$

$= 729 \bmod 15$

$= 9$

b) $(3^4 \bmod 17)^2 \bmod 11$

First evaluate the 4 power

$(81 \bmod 17)^2 \bmod 11$

$\therefore 81 \bmod 17 = 13$

$= 13^2 \bmod 11$

Use theorem 5.

$= (13 \bmod 11)^2 \bmod 11$ $\therefore 13 \bmod 11 = 2$

$= 2^2 \bmod 11$

$= 4 \bmod 11$

$= 4$

c) $(19^3 \bmod 23)^2 \bmod 31$

First evalue $19^3 = 6859$

$(6859 \bmod 23)^2 \bmod 31$

$= 5^2 \bmod 31$

$= 25 \bmod 31$

$= 25$

d) $(89^3 \bmod 79)^4 \bmod 26$

Use theorum

$((89 \bmod 79)^3 \bmod 79)^4 \bmod 26$

$\because 89 \bmod 79 = 10$

$= (10^3 \bmod 79)^4 \bmod 26$

$= (1000 \bmod 79)^4 \bmod 26$

$= 52^4 \bmod 26$

Use theorum

$= (52 \bmod 26)^4 \bmod 26$

$= 0 \bmod 26$

$= 0$

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Describe an algorithm based on linear search for determining the correct position for in which to insert a new element in an already sorted list?

Procedure Linear search (x : integer ; $a_1, a_2, \ldots$

A man visit a family who has two children, one of the children, a boy comes into the room. Find the probability that other child is also a boy if

i) The other child is known to be elder.

ii) Nothing is known about the other child

41) Determine whether each of these f is bijective from R to R.

i) $f(x) = -5x/2+4$

is a bijective from $R \to R$

ii) ~~$f(x) = 3x^2+7$~~ $f(x) = -3x^2+7$

$-3x^2 \leq 0$ for all x, $f(x) \leq 7$ for all x.
Thus f is not surjective, so it is not bijective. e.g $f(1) = f(-1)$

iii) $f(x) = (x+1)/(x^2+2)$

That is not bijective. Function

iv) $f(x) = x^5+1$

$f(x) = x^5+1$ is bijective.

$x = -2$

$f(2) = (2)^5 + 1 = -32+1$

$f(x) = -31$

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From a group of 7 men and 6 woman, 5 person are to be selected from a comittee so that at least 3 are threre on the committee. In how many ways can it be done?

Ans

Now select 3 from 7 and select rest 2 from remaining 6 females or both from male or one each.

And solution as mentioned above will be.

$= 7C_5 + 7C_4 * 6C_1 + {}^7C_3 * {}^6C_2$

$= 21 + 35 * 6 + 35 * 15 = 756$

OR

2) Another way total ways ~~ways~~ where 3 males are not there

All females $6C_5 = 6$

All but one ${}^6C_4 * 7C_1 = 15 * 7 = 105$

All but two $= {}^6C_3 * 7C_2 = 20 * 21 = 420$

So ways where 3 males are not there

$6 + 105 + 420 = 531$

Total $13C_5 = 1287$

So remaining way $= 1287 - 531 = 756$

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Out of 5 mathmatics and 7 engineers a Committee consisting of 2 mathematics and 3 engineers is to be formed, in how many ways can this be done if.
a) Any mathematics and any engineer can be included?
b) One Particular engineer must be in the committee?

$= {}^5C_2 * {}^7C_3$

$= 10 * 35$

$= 350$

b) ${}^5C_2 * {}^6C_1 * 1$, because the specific physicist take 1 from each part.

$= 150$

c) 2 particular mathematics can not be on the comittee.

$= {}^3C_2 * {}^7C_3$

$= 3 * 35$

$= 105$

31 A school has schedule three volleyball games, two soccer games, and four backet ball games you have a ticket / allowing you to attend three of the games. In how many ways can you go to two basketball games and one of the other events?

Pick 2 basket ball games

$^4C_2 = 4*3 / 1*2$

$= 6$ ways

Pick 1 other event : 5 ways

Total # of ways : $6*5 = 30$ways

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Q A coin is flipped 10 times where each flip come up either heads or Tail. How many possible outcome.

a) Are there in Total?

$2^{10} = 1024$

b) Contain exactly two head?

Let S is the total outcome

$n(S) = {}^{10}C_2$

$= 45$

c) Contain almost three tails?

$C(10,0) = \frac{10!}{0!(10-0)!}$ $\quad \because {}^nP_r = \frac{n!}{r!(n-r)!}$

$= 1$

$C(10,1) = \frac{10!}{1!(10-1)!} = \frac{10}{19!} = 10$

$C(10,2) = \frac{10!}{2!(10-2)!} = 45$

$C(10,3) = \frac{10!}{3!(10-3)!} = 120$

$= 1 + 10 + 45 + 120 = 176$

d) contain the same number of head and tails

$C(10,5) = \frac{10!}{5!(10-5)!} = \frac{10!}{5!5!} = 252$

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Encrypt a message UPLOAD using RSA system $n = 53 \cdot 61$ and $e = 17$.

$p = 53$

$q = 61$

UPLOAD

20 15 11 14 0 3

$= \gcd(e, (p-1)(q-1))$

$\therefore \gcd(e, (p-1)(q-1))$

$= \gcd(17, (53-1)(61-1))$

$= \gcd(17, (52)(60)) = 1$

~~$= \gcd(17, 52 \cdot 60$~~

$=$

2015 — 1114 — 0003

$M_1 = 2015$

$M_2 = 1114$

$M_3 = 0003$

Encrypt each block using the mapping

$C_x = M_x^e \pmod{n}$

$C_1 = 2015^{17} \pmod{3233} = 2545$

$C_2 = 1114^{17} \pmod{3233} = 2757$

$C_3 = 0003^{17} \pmod{3233} = 1211$

The encryption is then

2545 2757 1211

For any query contact at: asim.rana63@gmail.com

……………………………………………Good Luck………………………………………

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