χ^2

Ch#17 Chi-square distribution

→ imp

1) Explain chi-square distribution:-

Let $z_1, z_2, \ldots z_n$ be normally and independently distributed variable with zero means and unit variance. Then a random variable expressed by the quantity

$$\chi^2 = \sum_{i=1}^{n} z_i^2 \quad \text{Range } (0 \text{ to } \infty)$$

2) uses of chi-square dist:-

i) goodness of fit.
ii) test of idependence.
iii) test of homogency.

3) parameter of chi-squar?

The chi-square distribution contains only ondy parameter called number of degree of freedom.

$df = n-1$

4) Properties of chi-square dist?

i) The chi-square is a continous distribution range from zero to infinity. i.e $0 < \chi^2 < \infty$

ii) The mean of chi-square distribution equal to number of degree of freedom and its variance equal to twice number of degree of freedom.

i.e $E(\chi^2) = n$

$Var(\chi^2) = 2n$

iii) Moments of χ^2 distribution.

$M_0(t) = (1-2t)^{-n/2}$

First four moments about mean

$\mu_1' = E(\chi^2) = n$

$\mu_2 = Var(\chi^2) = 2n$

$\mu_3 = 8n$

$\mu_4' = 12n^2 + 48n$

$\beta_1 = \frac{8}{n}$, $\beta_2 = 3 + \frac{12}{n}$

iv) when n increase skewness decreases.

Scanned with CamScanner

i) Testing hypothesis about variance of a normal:-

$$\chi_{n-1} = \frac{ns^2}{\sigma^2} = \frac{\Sigma(x-\bar{x})^2}{\sigma^2}$$

Test Based on chi-square distr.

i) Hypothesis:-

$H_0: \sigma^2 = \sigma_0^2$, $H_1: \sigma^2 \neq \sigma_0^2$

$H_0: \sigma^2 \leq \sigma_0^2$, $H_1: \sigma^2 > \sigma_0^2$

$H_0: \sigma^2 \geq \sigma_0^2$, $H_1: \sigma^2 < \sigma_0^2$

ii) Level of significance:-

$\alpha = 5\%$, $\alpha = 1\%$ etc.

iii) Test statistic:-

$$\chi^2_{n-1} = \frac{ns^2}{\sigma_0^2} = \frac{\Sigma(x-\bar{x})^2}{\sigma_0^2}$$

iv) Critical region:-

v) calculation:-

vi) conclusion:-

= F

Critical region

i) $H_1 : \sigma^2 \neq \sigma^2$

$\chi^2 < \chi^2_{(1-\alpha/2)(n-1)}$, $\chi^2 > \chi^2_{(\alpha/2)(n-1)}$

ii) $H_1 : \sigma^2 > \sigma^2$

$\chi^2 > \chi^2_{\alpha(n-1)}$

iii) $H_1 : \sigma^2 < \sigma^2$

$\chi^2 < \chi^2_{1-\alpha(n-1)}$

Scanned with CamScanner

Formula

① $\chi^2_{n-1} = \frac{ns^2}{\sigma^2}$ ungroup Data

② $\chi^2_{n-1} = \sum\frac{(o-e)^2}{e}$ [group Data or Table Form]