Date:

Short Question

Find what the value of K the vector $(1,-2,K)$ in $\mathbb{R}^3$ be a linear combination of vectors $(3,0,-2)$ & $(2,-1,-5)$.

(3-Dimensional Space)

L.C

Linear combination

Expression of L.C

$V = \alpha V_1 + \beta V_2 + \gamma V_3$

L.C. vector   Scalar α,γ Vectors

Sol

Since vector $(1,-2,k)$ is L.C of V

So,

$K = (1,1,1)$
$V = (1,2,-3)$
$W = (1,-4,3)$

Then which vectors

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17-06-21

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Q Let $S = \{u = (1,2,1), v = (2,9,0), w = (3,3,4)\}$ form basis of $\mathbb{R}^3$. Find the vector in $\mathbb{R}^3$ whose co-ordinate Vector relative to basis S is $(-1, 3, 2)$. (388)

$\longleftrightarrow$

Q Write the bases for the vector space $M_{2\times 2}$ of 2x2 matrices. (390)

$\longleftrightarrow$

Q Use Wronskian method to show that $F_1 = 1$, $F_2 = e^x$, $F_3 = e^{2x}$ are linearly independent. (377)

$\longleftrightarrow$

$A \ni n$

Q Let V be Vector Space & U belongs to V ($U \in V$) then show that $(-1)u = -u$

$\longleftrightarrow$

Q If A is in invertible matric & n is non-negative Int than show that $(A^n)^{-1} = (A^{-1})^n$

$\longleftrightarrow$

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Q If A is invertible matrics then A^t is also invertible & $(A^t)^{-1} = (A^{-1})^t$

$\longleftrightarrow$

Q If B & C are both inverses of matrix A than $B = C$

$\longleftrightarrow$

Q Define Bases. (392)

$\longleftrightarrow$

" null space (405)

$\longleftrightarrow$

" ~~Rang~~ & Rank & nullity of homomorphism (422)

$\longleftrightarrow$

Q Show that matrix P is orthognol iff ~~and~~ (if an only if), P^t is orthognal (464)

$\longleftrightarrow$

Define Similar Matrices (640)

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Define ~~chariter~~ crachtristic equation

Q " eigen value? (643)

State Calay's Milton Theorem (574)

Define Trace of Matrix (51)

Write the bases of $P_n(x)$.

Normalize the vector (1, 2, 4, 5)

Q Conside the Vector $u = (1, -5, 3)$
Find $\|u\|_\infty$, $\|u\|_1$, $\|u\|_2$

Q vector space K is scalar and

Date:

Q inverse of $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

Q Matrix $\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$

is zero of $g(x) = x^2 + 3x - 10$

Q find x, y, z, t if

$$\begin{bmatrix} x+y & 2z+t \\ x-y & z-t \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 5 \end{bmatrix}$$

Q if $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$

Show that $A^4 = I_2$

Q if $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$ & $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

find a ~~and~~ and b

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Linear Algebra

S.Q.

Norm (Length)

$$\|x\|_1 = \sum_{i=1}^{m} |x_i| \rightarrow \text{(Norm 1)}$$

$$\|x\|_2 = \left(\sum_{i=1}^{n} |x_i|^2\right)^{\frac{1}{2}} \rightarrow \text{(Simple (Norm}_2\text{))}$$

$$\|x\|_\infty = \max_{1 \le i \le m} |x_i| \rightarrow \text{(Norm infinity)}$$

Example $x = \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix}$

$$\|x_1\| = |2| + |5| + |-3| = 10$$

$$\|x_2\| = \sqrt{2^2 + 5^2 + (-3)^2} = \sqrt{4+25+9} = \sqrt{41}$$

$$\|x\|_\infty = \max\{|2|, |5|, |-3|\} = 5$$

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① ~~Find Eigen value and~~

~~Bases for Eigen Space~~ (5.2) (7.1.4)

~~of $A=\begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$~~

② ~~diagonalize $\begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix}$~~

③ ~~Matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ Satisfies~~

~~its Characteristic Eq.~~

④ ~~Eigen values and Eigen Vectors~~

of $A=\begin{bmatrix} 1 & 2 \\ -3 & 2 \end{bmatrix}$

⑤ $X=\begin{bmatrix} 1 \\ 2 \end{bmatrix} \rightarrow$ Eigen vector

$A=\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$ Eigen value

$$\underset{A}{\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}} \underset{X}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}} = \begin{bmatrix} 3 \\ 6 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

λX

Maths

Date: 01-07-21

CH # 3

CH #4: Vector Space

~~3/-7~~

Let V be non-empty set & F be a field

$V \downarrow$ Vector eg $\vec{v}, \vec{u}$

$F \downarrow$ Scalars eg 2, a, c

Linear Combination

Let $v_1, v_2, v_3, \dots, v_n$ be the vectors in V

& $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ be the Scalars in Field F

Then the expression of this form

$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \dots + \alpha_n v_n$ is L.C of the vector $v_1, v_2, \dots, v_n$

$\therefore \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \dots + \alpha_n v_n = v \rightarrow \vec{v}$ is Linear Combination of $v_1, v_2, v_3, \dots, v_n$,

Linear Space

Let $v_1 v_2, v_3 \dots v_n$ be the vectors in V then set of all L.C of $v_1, \dots, v_n$ is called Linear Space.

Linear Independent (w/p Lord, Shiv) (page#202)

$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n = 0$ if

$\forall \alpha_i = 0$ then vectors $v_1 v_2, \dots v_n$ are called linearly independent & if any $\alpha_i \neq 0$

.... L. independent.)

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eg $v_1 = (1, 2, 2, -1)$
$v_2 = (4, 9, 9, -4)$
$v_3 = (5, 8, 9, -5)$

$$\alpha v_1 + \beta v_2 + \gamma v_3 = 0$$

$\alpha(1$

$(\alpha, 2\alpha, 2\alpha, -1\alpha) + (4\beta, 9\beta, 9\beta, -4\beta) + (5\gamma, 8\gamma, 9\gamma, -5\gamma)$

$(\alpha, 2\alpha, 2\alpha, -\alpha) + (4\beta, 9\beta, 9\beta, -4\beta) + (5\gamma, 8\gamma, 9\gamma, -5\gamma) = 0$

$(\alpha + 4\beta + 5\gamma, 2\alpha + 9\beta + 8\gamma, 2\alpha + 9\beta + 9\gamma, -\alpha - 4\beta - 5\gamma) = 0$

$\alpha v_1 = \cancel{\alpha + 2\alpha + 2\alpha - \alpha}$
$= \alpha +$

$= (\alpha + 4\beta + 5\gamma) + (2\alpha + 9\beta + 8\gamma) + (2\alpha + 9\beta + 9\gamma) + (-\alpha - 4\beta - 5\gamma) = 0$

$\alpha + 4\beta + 5\gamma = 0$ — eq (i)
$2\alpha + 9\beta + 8\gamma = 0$ — (ii)
$2\alpha + 9\beta + 9\gamma = 0$ — (iii)
$-\alpha - 4\beta - 5\gamma = 0$ — (iv)

$2\alpha + 9\beta + 8\gamma = 0$
$2\alpha + 9\beta + 9\gamma = 0$
$- \quad - \quad -$

$-\gamma = 0$
$\boxed{\gamma = 0}$

put γ in eq (i)
$\alpha + 4\beta = 0$ — (A)
put γ in eq (ii)
$2\alpha + 9\beta = 0$ — (B)

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$2\alpha + 9\beta = 0$
$\alpha + 4\beta = 0$

$2\alpha + 9\beta = 0$
$2\alpha + 8\beta = 0$
$-\quad -$

$\boxed{\beta = 0}$

Put α, & β in eq(i)

$\alpha + 4\beta + 5\gamma = 0$
$\boxed{\alpha = 0}$

α, β, γ scalars are 0 so these vectors V are linearly independent.

=> Page 191
Ex # 2

=> in square matrix A
if det of A = 0
then vectors are linearly dependent

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Q Determine whether the vectors $v = (3,3,-4)$ is a linear combination of $x = (1,2,3)$ $y = (2,3,7)$, $z = (3,5,6)$.

$$\alpha x + \beta y + \gamma z = v$$

$$\alpha(1,2,3) + \beta(2,3,7) + \gamma(3,5,6) = (3,3,-4)$$

$$(\alpha + 2\alpha, 3\alpha) + (2\beta + 3\beta + 7\beta) + (3\gamma + 5\gamma + 6\gamma) = (3,3,-4)$$

$$\alpha + 2\beta + 3\gamma = 3 \quad -(i)$$
$$2\alpha + 3\beta + 5\gamma = 3 \quad -(ii)$$
$$3\alpha + 7\beta + 6\gamma = -4 \quad -(iii)$$

$2 \times$ eq(i) $-$ eq(ii)

$$2\alpha + 4\beta + 6\gamma = 6$$
$$2\alpha + 3\beta + 5\gamma = 3$$
$$- \quad - \quad - \quad -$$
$$\beta + \gamma = 3 \quad -(A)$$

$3 \times$ eq(i) $-$ eq(iii)

$$3\alpha + 6\beta + 9\gamma = 9$$
$$3\alpha + 7\beta + 6\gamma = -4$$
$$- \quad - \quad - \quad +$$
$$-\beta + 3\gamma = 13 \quad -(B)$$

Solving eq(A) & (B)

$$\beta + \gamma = 3$$
$$-\beta + 3\gamma = 13$$
$$4\gamma = 16$$
$$\boxed{\gamma = 4}$$

put in eq(B)

$$-\beta + 3(4) = 13$$
$$-\beta + 12 = 13$$
$$\boxed{\beta = -1}$$

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Date:

put γ & β in eq(1)

$\alpha + 2\beta + 3\gamma = 3$

$\alpha + 2(-1) + 3(4) = 3$

$\alpha - 2 + 12 = 3$

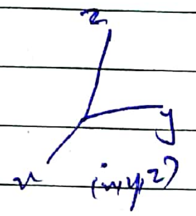
$\alpha + 10 = 3$

$\boxed{\alpha = -7}$, $\boxed{\beta = -1}$, $\boxed{\gamma = 4}$

Linear Span

$\mathbb{R}^3$

3-D



$e_1 = (1,0,0)$

$e_2 = (0,1,0)$

$e_3 = (0,0,1)$

e.g $(4,5,6)$

$4(1,0,0) + 5(0,1,0) + 6(0,0,1) = (4,0,0) + (0,5,0) + (0,0,6)$

$= (4,5,6)$

e_1, e_2, e_3 are spaning whole $\mathbb{R}^3$

$Sp(e_1, e_2, e_3) = \mathbb{R}^3$

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(independent)

Date:

$\Rightarrow e_1, e_2$ & e_3 are independent

$$\alpha(1,0,0) + \beta(0,1,0) + \gamma(0,0,1) = (0,0,0)$$

$$(\alpha, \beta, \gamma) = (0,0,0)$$

$$\gamma = 0,\ \beta = 0,\ \alpha = 0$$

$\Rightarrow$ Therefore e_1, e_2, e_3 are basis of $\mathbb{R}^3$.

$\Rightarrow$ So $e_1, e_2,$ & e_3 are linearly independent.

Basis

Vectors $v_1, v_2, \dots, v_n$ form basis of V.S (vector space) if

(i) $v_1, v_2, \dots v_n$ are linearly independent

(ii) $\text{Span}(v_1, v_2, \dots, v_n) = V.S$ $(\mathbb{R}^3)$

Dimensions

No of vectors in basis bases

e.g

$$M_{2\times2} = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix},\ a,b,c,d \in \mathbb{R}^3 \right\}$$

$$\text{Basis} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

~~det~~ $\begin{bmatrix} 4 & 5 \\ 7 & 8 \end{bmatrix}$

$$4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 5\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 7\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + 8\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

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Dim of $M_{2\times 2} = 4$

$\rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$\longleftrightarrow$

#210

Q Let w be the subspace of $\mathbb{R}^4$ spand by the vectors

$u_1 = [1, 2, -1, 3]$
$u_2 = [2, 4, 1, -2]$
$u_3 = [3, 6, 3, -7]$
$u_4 = [1, 2, -4, 11]$
$u_5 = [2, 4, -5, 14]$

Q Find the bases & dimension of w

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 2 \\ 2 & 4 & 6 & 2 & 4 \\ -1 & 1 & 3 & -4 & -5 \\ 3 & -2 & -7 & 11 & 14 \end{bmatrix}$$

~~[illegible]~~ $2R_1 - R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 3 & -4 & -5 \\ 3 & -2 & -7 & 11 & 14 \end{bmatrix}$$

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Interchange R_2 & R_4

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 2 \\ 3 & -2 & -7 & 11 & 14 \\ -1 & 1 & 3 & -4 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

~~$R_2 + 2R_3 \rightarrow R_2$~~

$R_1 + R_3 \rightarrow R_3$

$3R_1 - R_2 \rightarrow R_2$

6-(-2)
6+2= 8

9-(-7)
16

3-11
6-14

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 2 \\ 0 & 8 & 16 & -8 & -8 \\ 0 & 3 & 6 & -3 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\frac{1}{8} \times R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 2 \\ 0 & 1 & 2 & -1 & -1 \\ 0 & 1 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 - R_3 \rightarrow R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 2 \\ 0 & 1 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 2 & 3 & 1 & 2 \\ 0 & 1 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

pivot elements occurs in column 1 & 2
So $w_1 = (1,0,0,0)$ & $w_2 = (2,1,0,0)$
form basis of subspace of R^4

Consequently vectors $v_1 = (1,-2,-1,3)$
& $v_2 = (2,4,1,-2)$
form basis of subspace of R^4
whose dimension is 2.

Q # 210

Use Wronskian method &. . . . .

Solution

The wronskian is

$$W(x) = \begin{vmatrix} 1 & e^x & e^{2x} \\ 0 & e^x & 2e^{2x} \\ 0 & e^x & 4e^{2x} \end{vmatrix} \begin{matrix} \to f_1, f_2, f_3 \\ \to \text{derivative of first row} \\ \to \text{'' '' 2nd row} \end{matrix}$$

$= 1(e^x . 4e^{2x} - 2e^{2x} . e^x) - 0$

$= 4e^{3x} - 2e^{3x}$

$= 2e^{3x}$

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This function is obviously not identically zero on $(-\infty, \infty)$, so f_1, f_2 & f_3 form a linearly independent set

e

$\Rightarrow$ put any value from $(-\infty, \infty)$

in $2e^{3x}$ the

$e^x \neq 0$

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