CH # 2

Ex = 2.1

Same 1-2-3-4-

Find all minors and co-factors of A

I) ⇒ $A = \begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$

Sol=

Minors = M_{ij} ∴ i = no. of row

j = no. of columns

First of all we find M_{11}.

$M_{11} = \begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$

$M_{11} = \begin{vmatrix} 7 & -1 \\ 1 & 4 \end{vmatrix}$

$M_{11} = 28 - (-1) \Rightarrow \boxed{M_{11} = 29}$

Similarly for $M_{12}, M_{13}, M_{21}, M_{22}, M_{23}, M_{31}, M_{32}, M_{33}$

Co-Factors

$C_{ij} = (-1)^{i+j} M_{ij}$

$C_{11} = (-1)^{1+1} M_{11}$

$C_{11} = (-1)^2 (29)$

$C_{11} = 29$

Similarly, for $C_{12}, C_{13}, C_{21}, C_{22}, C_{23}, C_{31}, C_{32}, C_{33}$.

In exercise 5-8, evaluate the determinant. If matrix is invertible, use Eq (2) to find inverse.

(7): $\begin{bmatrix} -5 & 7 \\ -7 & -2 \end{bmatrix}$

Sol:

let $A = \begin{bmatrix} -5 & 7 \\ -7 & -2 \end{bmatrix}$

A matrix is invertible if $|A| \neq 0$ so,

$|A| = \begin{vmatrix} -5 & 7 \\ -7 & -2 \end{vmatrix}$

$= 10 - (-49)$

$= 10 + 49$

$|A| = 59$

Now inverse $\Rightarrow A^{-1} = \dfrac{\text{adj of } A}{|A|}$

adj of $A = \begin{bmatrix} -2 & -7 \\ 7 & -5 \end{bmatrix}$

$A^{-1} = \dfrac{1}{59} \begin{bmatrix} -2 & -7 \\ 7 & -5 \end{bmatrix}$ Ans.

In exercise 9-14 use the arrow technique to evaluate the determinent.

11):
$$\begin{vmatrix} -2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{vmatrix}$$

Sol:

let

$$|A| = \begin{vmatrix} -2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{vmatrix}$$

$$= -2\begin{vmatrix} 5 & -7 \\ 6 & 2 \end{vmatrix} - 1\begin{vmatrix} 3 & -7 \\ 1 & 2 \end{vmatrix} + 4\begin{vmatrix} 3 & 5 \\ 1 & 6 \end{vmatrix}$$

$$= -2(10+42) - 1(6+7) + 4(18-5)$$

$$= -104 - 13 + 52$$

$$= -65 \quad \text{Ans.}$$

In exercise 15-18 find all values of λ for which $\det(A) = 0$.

(17) $A = \begin{bmatrix} \lambda-1 & 0 \\ 2 & \lambda+1 \end{bmatrix}$

Sol:

$$|A| = \begin{bmatrix} \lambda-1 & 0 \\ 2 & \lambda+1 \end{bmatrix} = 0$$

$$(\lambda-1)(\lambda+1) - 0 = 0$$

$$\lambda^2 - 1 = 0$$

$$\lambda^2 = 1$$

$$\lambda = \pm 1 \quad \text{Ans}$$

In exercise 21-26 evaluate det(A) by a cofactor expansion along a row or column of your choice.

21) ⇒ $A = \begin{bmatrix} -3 & 0 & 7 \\ 2 & 5 & 1 \\ -1 & 0 & 5 \end{bmatrix}$

Sol:-

Expand by row 1 -

$$|A| = -3\begin{vmatrix} 5 & 1 \\ 0 & 5 \end{vmatrix} - 0\begin{vmatrix} 2 & 1 \\ -1 & 5 \end{vmatrix} + 7\begin{vmatrix} 2 & 5 \\ -1 & 0 \end{vmatrix}$$

$$= -3(25-0) - 0(10+1) + 7(0-(-5))$$

$$= -75 - 0 + 35$$

$$= -40 \quad \underline{\text{Ans}}$$

Ex 2.2

In exercise 1-4, verify that $\det(A) = \det(A^T)$

3): $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 5 & -3 & 6 \end{bmatrix}$

Sol=

We have to proof that.

$\det(A) = \det(A^T)$

L.H.S => $\det(A) = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 5 & -3 & 6 \end{vmatrix}$

$= 2\begin{vmatrix} 2 & 4 \\ -3 & 6 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 4 \\ 5 & 6 \end{vmatrix} + 3\begin{vmatrix} 1 & 2 \\ 5 & -3 \end{vmatrix}$

$= 2(12+12) + 1(+6-20) + 3(-3-10)$

$= 48 - 14 - 39$

$= -5$

R.H.S => $\det(A^T)$.

$A^T = \begin{bmatrix} 2 & 1 & 5 \\ -1 & 2 & -3 \\ 3 & 4 & 6 \end{bmatrix}$

$\det(A^T) = 2\begin{vmatrix} 2 & -3 \\ 4 & 6 \end{vmatrix} - 1\begin{vmatrix} -1 & -3 \\ 3 & 6 \end{vmatrix} + 5\begin{vmatrix} -1 & 2 \\ 3 & 4 \end{vmatrix}$

$= 2(12+12) - 1(-6+9) + 5(-4-6)$

$= 48 - 3 - 50$

$= -5$

Hence proved.

In exercise 9-14. evaluate the determinant of matrix by first reducing the matrix to row echelon form and then using some combination of row operations.

9): $\begin{bmatrix} 3 & -6 & 9 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix}$

Sol:

let $A = \begin{bmatrix} 3 & -6 & 9 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix}$

Now, for converting into row echelon form:

$R_1 + R_2 \Rightarrow R_1$

$A = \begin{bmatrix} 3-2 & -6+7 & 9-2 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & 7 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix}$

$R_2 + 2R_1 \Rightarrow R_2$

$A = \begin{bmatrix} 1 & 1 & 7 \\ 0 & 9 & 12 \\ 0 & 1 & 5 \end{bmatrix}$

$R_2 + 8R_1 \Rightarrow R_2$

$A = \begin{bmatrix} 1 & 1 & 7 \\ 0 & 9-8 & 12-40 \\ 0 & 1 & 5 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 & 7 \\ 0 & 1 & -28 \\ 0 & 1 & 5 \end{bmatrix}$$

$R_3 - R_2 \Rightarrow R_3$

$$A = \begin{bmatrix} 1 & 1 & 7 \\ 0 & 1 & -28 \\ 0 & 0 & 33 \end{bmatrix}$$

$R_3 * \frac{1}{33}$

$$A = \begin{bmatrix} 1 & 1 & 7 \\ 0 & 1 & -28 \\ 0 & 0 & 1 \end{bmatrix}$$

Now

$$|A| = 1\begin{vmatrix} 1 & -28 \\ 0 & 1 \end{vmatrix} - 1\begin{vmatrix} 0 & -28 \\ 0 & 1 \end{vmatrix} + 7\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 7 \\ 0 & 9 & 12 \\ 0 & 1 & 5 \end{bmatrix}$$

Exchanging R_2 by R_3 ∴ when exchange write (-1)

$$A = (-1)\begin{bmatrix} 1 & 1 & 7 \\ 0 & 1 & 5 \\ 0 & 9 & 12 \end{bmatrix}$$

Taking 3 common from R_3

$$A = -3\begin{bmatrix} 1 & 1 & 7 \\ 0 & 1 & 5 \\ 0 & 3 & 4 \end{bmatrix}$$

$R_3 - 2R_2 \Rightarrow R_3$

$$= -3 \begin{bmatrix} 1 & 1 & 7 \\ 0 & 1 & 5 \\ 0 & 0 & -11 \end{bmatrix}$$

$\therefore$ Now taking (-11) common from R_3.

$$= (-3)(-11) \begin{bmatrix} 1 & 1 & 7 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= 33 \begin{bmatrix} 1 & 1 & 7 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Now expanding A as

$$|A| = 33\left(1\begin{vmatrix} 1 & 5 \\ 0 & 1 \end{vmatrix} - 1\begin{vmatrix} 0 & 5 \\ 0 & 1 \end{vmatrix} + 7\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}\right)$$

$|A| = 33\,[\,1 - 0 + 0\,]$

$|A| = 33$ Ans-

24) (a): Prove that

$$\det \begin{bmatrix} 0 & 0 & a_{13} \\ 0 & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = -a_{13}a_{22}a_{31}$$

Sol:

$$\Rightarrow 0\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - 0\begin{vmatrix} 0 & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} 0 & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = -a_{13}a_{22}a_{31}$$

$0 - 0 + a_{13}[0 - a_{22}a_{31}] = -a_{13}a_{22}a_{31}$

$-a_{13}a_{22}a_{31} = -a_{13}a_{22}a_{31}$

Hence proved.

Ex = 2.3

In Exercise 1-4 verify that $\det(kA) = k^n \det(A)$:

(1). $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$; $k = 2$

Sol:

we have to show that

$\det(kA) = k^n \det(A)$

LHS $\Rightarrow$ $\det(kA) = ?$

$kA = 2\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} -2 & 4 \\ 6 & 8 \end{bmatrix}$

$\det(kA) = (-2)(8) - (6)(4)$

$= -16 - 24$

$= -40.$

Now RHS

$k^n \det(A)$

$\det(A) = \begin{vmatrix} -1 & 2 \\ 3 & 4 \end{vmatrix}$

$= -4 - 6$

$= -10$

$k^n = 2^2 = 4$

$k^n \det(A) = 4(-10) \Rightarrow k^n \det(A) = -40$

Hence proved.

In Exercise 5-6 verify that $\det(AB) = \det(BA)$.

(5) :> $A = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -1 & 3 \\ 7 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix}$

Sol:

L.H.S => $\det(AB)$ =?

$$AB = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 & 3 \\ 7 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+7+0 & -2+1+0 & 6-2+0 \\ 3+28+0 & -3+4+0 & 9+8+0 \\ 0+0+10 & 0+0+0 & 0+0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -1 & 4 \\ 31 & 1 & 17 \\ 10 & 0 & 2 \end{bmatrix}$$

$$\det(AB) = 9\begin{vmatrix} 1 & 17 \\ 0 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 31 & 17 \\ 10 & 2 \end{vmatrix} + 4\begin{vmatrix} 31 & 1 \\ 10 & 0 \end{vmatrix}$$

$= 18 + 1(-108) + 4(-10)$

$= 18 - 108 - 40$

$= -130$

Now R.H.S

$\det(BA)$ =?

$$BA = \begin{bmatrix} 1 & -1 & 3 \\ 7 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2-3+0 & 1-4+0 & 0+0+6 \\ 14+3+0 & 7+4+0 & 0+0+4 \\ 10+0+0 & 5+0+0 & 0+0+2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -3 & 6 \\ 17 & 11 & 4 \\ 10 & 5 & 2 \end{bmatrix}$$

$$\det(BA) = -1\begin{vmatrix} 11 & 4 \\ 5 & 2 \end{vmatrix} -(-3)\begin{vmatrix} 17 & 4 \\ 10 & 2 \end{vmatrix} +6\begin{vmatrix} 17 & 11 \\ 10 & 5 \end{vmatrix}$$

$= -1(-2) + 3(+6) + 6(-25)$

$= +2 + 18 - 150$

$= -130$

Hence proved.

In Exercise 7-14 Use determinants to decide weather the matrix is invertible.

7). $A = \begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$

Sol:

matrix is invertible if $|A| \neq 0$

$$|A| = 2\begin{vmatrix} -1 & 0 \\ 4 & 3 \end{vmatrix} - 5\begin{vmatrix} -1 & 0 \\ 2 & 3 \end{vmatrix} + 5\begin{vmatrix} -1 & -1 \\ 2 & 4 \end{vmatrix}$$

$= 2(-3-0) - 5(-3-0) + 5(-4+2)$

$= -6 + 15 - 10$

$= -1$ Ans.

Matrix is invertible.

In Exercise 15-18 find values of k for which matrix A is invertible.

15). $A = \begin{bmatrix} k-3 & -2 \\ -2 & k-2 \end{bmatrix}$

Sol:

$\therefore$ since matrix is invertible so $|A| \neq 0$

$|A| = \begin{vmatrix} k-3 & -2 \\ -2 & k-2 \end{vmatrix}$

$(k-3)(k-2) - (-2)(-2) \neq 0$

$k^2 - 5k + 6 - 4 \neq 0$

$k^2 - 5k + 2 \neq 0$

Using quadratic formula:

$k \neq \frac{5-\sqrt{17}}{2}$; $k \neq \frac{5+\sqrt{17}}{2}$

In Exercise 24-29 solve by cramer rule.

24) $7x_1 - 2x_2 = 3$

$3x_1 + x_2 = 5$

Sol:

$\begin{bmatrix} 7 & -2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$A \quad x = B$

Now, we have to find x_1 and x_2 so,

$x_1 = \frac{|A_1|}{|A|}$ and $x_2 = \frac{|A_2|}{|A|}$

firstly, $|A| = \begin{vmatrix} 7 & -2 \\ 3 & 1 \end{vmatrix}$

$= 7 - (-6)$

$= 13$

and for $A_1 = \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix}$

So, $|A_1| = \begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix}$

$= 3 - (-10)$

$= 13$

$x_1 = \frac{|A_1|}{|A|}$, $x_1 = \frac{13}{13}$, $\boxed{x_1 = 1}$

and for x_2 ; $A_2 = \begin{bmatrix} 7 & 3 \\ 3 & 5 \end{bmatrix}$; $|A_2| = \begin{vmatrix} 7 & 3 \\ 3 & 5 \end{vmatrix}$

$|A_2| = 35 - 9$

$|A_2| = 26$

$x_2 = \frac{|A_2|}{|A|}$

$x_2 = \frac{26}{13}$; $\boxed{x_2 = 2}$

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