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MAth 3215

CS 2017

Long

Q.2

$$\begin{bmatrix} 1 & -4 \\ 4 & 2 \end{bmatrix}$$

Find characteristic equation and eigen values also find eigen vector.

Sol.

Eigen values

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & -4 \\ 4 & 2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(2-\lambda) + 16 = 0$$

$$2 - \lambda - 2\lambda + \lambda^2 + 16 = 0$$

$$\lambda^2 - 3\lambda + 18 = 0$$

So the characteristic equation ~~values~~ is

$\lambda^2 - 3\lambda + 18 = 0$

Now find the eigen values

$\cancel{\lambda^2 - 6\lambda + 3\lambda + 18 = 0}$

$\cancel{\lambda(\lambda - 6) + 3}$

$\cancel{\lambda^2 + 3\lambda -}$

$$\begin{array}{c|cc|c} & \lambda^2 & -3\lambda & +18 = 0 \\ 6 & & 6 & -18 \\ \hline & 1 & -3 & 0 \end{array}$$

$\lambda - 3 = 0$

$\boxed{\lambda = 3}$

So the eigen values is

$\lambda = 6, 3$

Eigenvector when
$\lambda = 6$

$$\begin{bmatrix} 1-6 & -4 \\ 4 & 2-6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -5 & -4 \\ 4 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-5x_1 - 4x_2 = 0$$
$$-4x_1 \mp 4x_2 = 0$$
$$= 0$$

$$-5x_1 - 4x_2 = 0$$
$$-5x_1 = 4x_2$$
$$x_1 = \frac{4}{5}x_2$$

So the eigenvector is

$$= \begin{bmatrix} \frac{4}{5}x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4/5 \\ 1 \end{bmatrix}$$

so the eigenvector
$$\begin{bmatrix} 4/5 \\ 1 \end{bmatrix}$$

when λ 3

$$\begin{bmatrix} 1-3 & -4 \\ 4 & 2-3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -4 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x_1 - 4x_2 = 0$$

$$4x_1 - x_2 = 0$$

$$-2(x_1 + x_2) = 0$$

$$x_1 + x_2 = 0$$

$$x_1 = -x_2$$

So the eigen~~values~~ vector

$$= \begin{bmatrix} -x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

So the eigenvector =

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$ Ans.

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Q.3

solve for x

$$\begin{vmatrix} 1 & 2+x & 3 \\ 2 & 1 & 3+x \\ 3 & 2+x & 1 \end{vmatrix} = 0$$

Now we take the det

$$\begin{vmatrix} 1 & 2+x & 3 \\ 2 & 1 & 3+x \\ 3 & 2+x & 1 \end{vmatrix} = 0$$

$$1\begin{vmatrix} 1 & 3+x \\ 2+x & 1 \end{vmatrix} - (2+x)\begin{vmatrix} 2 & 3+x \\ 3 & 1 \end{vmatrix}$$

$$+3\begin{vmatrix} 2 & 1 \\ 3 & 2+x \end{vmatrix} = 0$$

$$1-(3+x)(2+x) \quad -(2+x)(2-9-3x)$$

$$+3(4+2x-3)=0$$

$$1-(6+3x+2x+x^2)-((2+x)(-7-3x)$$

$$+3(1+2x)=0$$

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$=0$

$+x$

$9-3x)$

$-3x)$

$=0$

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$1-(6+5x+x^2)-(-14-6x-7x-3x^2)$

$+3+6x$

$+3, +6x$

$1-6-5x-x^2+14+13x+3x^2=0$

$2x^2+18x+9=0$

$2x^2+14x+12=0$

$2(x^2+7x+6)=0$

$x^2+7x+6=0$

$x^2+6x+x+6=0$

$x(x+6)+1(x+6)=0$

$(x+1)(x+6)=0$

$x+1=0 \qquad x+6=0$

$\boxed{x=-1} \qquad \boxed{x=-6}$

So the values of the x is

$x=-1,-6$

with proof

$2(-1)^2+14(-1)+12=0$

$2+(-14)+12=0$

$14-14=0$

$0=0$

Hence prove that

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Q. 4 (a)

find the rank of the matrix

$$\begin{bmatrix} 2 & 1 & 3 & 5 \\ 15 & 8 & 1 & 12 \\ 11 & 5 & 8 & 6 \\ 12 & 8 & 7 & 10 \end{bmatrix}$$

Now we find the rank.

First we make the matrix echelon form.

$$= \begin{bmatrix} 2 & 1 & 3 & 5 \\ 30-30 & 16-15 & 2-15 & 24-75 \\ 11 & 5 & 8 & 6 \\ 12 & 8 & 7 & 10 \end{bmatrix} R_2 \to 2R_2 - 15R_1$$

$$\begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 1 & -13 & -49 \\ 11 & 5 & 8 & 6 \\ 12 & 8 & 7 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 1 & -13 & -49 \\ 22-22 & 10-11 & 14-33 & 12-55 \\ 12 & 8 & 7 & 10 \end{bmatrix} R_3 \to 2R_3 - 11R_1$$

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$$= \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 1 & -13 & -49 \\ 0 & -1 & -19 & -43 \\ 12 & 8 & 7 & 10 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$= \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 1 & -13 & -49 \\ 0 & 0 & -19-13 & -43-49 \\ 12 & 8 & 7 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 1 & -13 & -49 \\ 0 & 0 & -32 & -92 \\ 12 & 8 & 7 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 1 & -13 & -49 \\ 0 & 0 & -32 & -92 \\ 12-12 & 8-6 & 7-18 & 10-30 \end{bmatrix}$$ $R_4 \to R_4 - 6R_1$

$$= \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 1 & -13 & -49 \\ 0 & 0 & -32 & -92 \\ 0 & 2 & -11 & -20 \end{bmatrix}$$

$R_4 \to R_4 - 2R_2$

$\to 2R_2 - 15R_1$

$\to 2R_3 - 11R_1$

$$\begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 1 & -13 & -49 \\ 0 & 0 & -32 & -92 \\ 0 & 2-2 & -11+26 & -20+98 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 1 & -13 & -49 \\ 0 & 0 & -32 & -92 \\ 0 & 0 & 15 & 78 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 1 & -13 & -49 \\ 0 & 0 & -8 & -23 \\ 0 & 0 & 15 & 78 \end{bmatrix}$$

$$R_4 \to 8R_4 + 15R_3$$

$$= \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 1 & -13 & -49 \\ 0 & 0 & -8 & -23 \\ 0 & 0 & -120+120 & 624-345 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 1 & -13 & -49 \\ 0 & 0 & -8 & -23 \\ 0 & 0 & 0 & 279 \end{bmatrix}$$

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So the rank of this matrix is 4 Because the 4 row are non-zero.

4. (b)

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$ Find A^{-1} with cofactors

$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 3 \\ 0 & 2 \end{vmatrix} = -2$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 3 \\ 1 & 2 \end{vmatrix} = -(-3) = 3$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = +1$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} = -4$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} = 2+1 = 3$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = +2$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6-1 = 5$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -1 \\ 0 & 3 \end{vmatrix} = -3$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} = -1$

So the inverse of the matrix is

$|A| = 3$

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$$A^{-1} = \frac{1}{3}\begin{bmatrix} -2 & 3 & +1 \\ -4 & 3 & 2 \\ 5 & -3 & -1 \end{bmatrix}^{t}$$

Ans-

$$A^{-1} = \frac{1}{3}\begin{bmatrix} -2 & -4 & 5 \\ 3 & 3 & -3 \\ 1 & 2 & -1 \end{bmatrix}$$

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Q. 6

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 0 \\ 3 & 1 & 5 \end{bmatrix}$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 0 \\ 1 & 5 \end{vmatrix} = 10$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = -5$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix} = -11$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix} = -2$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = \cancel{8}\ 7$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 3 & 4 \\ 2 & 0 \end{vmatrix} = -8$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 4 \\ 1 & 0 \end{vmatrix} = 4$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 3 \\ 1 & \cancel{8}\ 2 \end{vmatrix} = 1$$

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$$= \begin{bmatrix} 10 & -5 & -5 \\ -11 & -2 & 7 \\ -8 & 4 & 1 \end{bmatrix}$$

So the det

$$|A| = 2\begin{vmatrix} 2 & 0 \\ 1 & 5 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} + 4\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$$

$$= 2(10) - 3(5) + 4(1-6)$$

$$= 20 - 15 - 20$$

$$= -15$$

$$= \begin{bmatrix} 10/-15 & -5/-15 & -8/-15 \\ -11/-15 & -2/-15 & 7/-15 \\ -8/-15 & 4/-15 & 1/-15 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2/3 & 1/3 & 1/3 \\ 11/15 & 2/15 & -7/15 \\ -8/-15 & 4/-15 & 1/-5 \end{bmatrix}$$

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So

$= -2/3$

$= -2/3$

$= -2$

$=$

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So the det(A^{-1})

$$\begin{vmatrix} -2/3 & 1/3 & 1/3 \\ 11/15 & 2/15 & -7/15 \\ 8/15 & 4/-15 & 1/-15 \end{vmatrix}$$

$$= -2/3 \begin{vmatrix} 2/15 & -7/15 \\ 4/-15 & 1/-15 \end{vmatrix} - \frac{1}{3} \begin{vmatrix} 11/15 & -7/15 \\ 8/15 & 1/-15 \end{vmatrix}$$
$$+ \frac{1}{3} \begin{vmatrix} 11/15 & 2/15 \\ 8/15 & 4/-15 \end{vmatrix}$$

$$= -2/3\left(\frac{-2}{225} - \frac{28}{225}\right) - \frac{1}{3}\left(\frac{-11}{225} + \frac{56}{225}\right)$$
$$+ \frac{1}{3}\left(\frac{-44}{225} - \frac{16}{225}\right)$$

$$= -2/3\left(\frac{-2-28}{225}\right) - \frac{1}{3}\left(\frac{-11+56}{225}\right)$$
$$+ \frac{1}{3}\left(\frac{-44-16}{225}\right)$$

$$= -2/\cancel{3}\left(-\frac{\overset{10}{\cancel{30}}}{225}\right) - \frac{1}{3}\left(\frac{45}{225}\right) + \frac{1}{\cancel{3}}\left(-\frac{\overset{20}{\cancel{60}}}{225}\right)$$

$$= \frac{20}{225} - \frac{45}{675} - \frac{20}{225}$$

$$\frac{60 - 45 - 60}{675} = \frac{-45}{675}$$

t is missing but answer is correct

det $(A^{-1})^t$

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$$= \frac{60-45-60}{675}$$

$$= \frac{-45}{675}$$

$$|A^{-1}| = -\frac{1}{15}$$

~~or~~

Check that answer is correct or not

$$AA^{-1} = I$$

$$(-\cancel{15})\left(\frac{-1}{\cancel{15}}\right) = 1$$

$$(-1)(-1) = 1$$

$$1 = 1$$

Hence prove that

L.H.S = R.H.S