Some Important Derivative

- $\frac{d}{dx}(c)=0$, where c is constant
- $\frac{d}{dx}(x^n)=nx^{n-1}$
- $\frac{d}{dx}\left[f(x)\right]^n=n\left[f(x)\right]^{n-1}\frac{d}{dx}\left[f(x)\right]$

- $\frac{d}{dx}\sin x=\cos x$
- $\frac{d}{dx}\cos x=-\sin x$
- $\frac{d}{dx}\tan x=\sec^2 x$
- $\frac{d}{dx}\cot x=-\csc^2 x$
- $\frac{d}{dx}\csc x=-\csc x\cot x$
- $\frac{d}{dx}\sec x=\sec x\tan x$

- $\frac{d}{dx}Sin^{-1}x=\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}Cos^{-1}x=\frac{-1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}Tan^{-1}x=\frac{1}{1+x^2}$
- $\frac{d}{dx}Cot^{-1}x=\frac{-1}{1+x^2}$
- $\frac{d}{dx}Sec^{-1}x=\frac{1}{x\sqrt{x^2-1}}$
- $\frac{d}{dx}Csc^{-1}x=\frac{-1}{x\sqrt{x^2-1}}$

- $\frac{d}{dx}a^x=a^x\ln a$
- $\frac{d}{dx}e^x=e^x$
- $\frac{d}{dx}\log_a x=\frac{1}{x\ln a}$
- $\frac{d}{dx}\ln x=\frac{1}{x}$

- $\frac{d}{dx}\sinh x=\cosh x$
- $\frac{d}{dx}\cosh x=\sinh x$
- $\frac{d}{dx}\tanh x=\operatorname{sech}^2 x$
- $\frac{d}{dx}\coth x=-\operatorname{csch}^2 x$
- $\frac{d}{dx}\operatorname{sech} x=-\operatorname{sech} x\tanh x$
- $\frac{d}{dx}\operatorname{csch} x=-\operatorname{csch} x\coth x$

- $\frac{d}{dx}Sinh^{-1}x=\frac{1}{\sqrt{x^2+1}}$
- $\frac{d}{dx}Cosh^{-1}x=\frac{1}{\sqrt{x^2-1}}$
- $\frac{d}{dx}Tanh^{-1}x=\frac{1}{1-x^2}$
- $\frac{d}{dx}Coth^{-1}x=\frac{1}{1-x^2}$
- $\frac{d}{dx}Sech^{-1}x=\frac{-1}{x\sqrt{1-x^2}}$
- $\frac{d}{dx}Csch^{-1}x=\frac{-1}{x\sqrt{1+x^2}}$

Some Standard nth Derivative

- $\frac{d^n}{dx^n}(ax+b)^m=\frac{m!}{(m-n)!}a^n(ax+b)^{m-n}\ \ if\ m\geq n$
- $\frac{d^n}{dx^n}\left(\frac{1}{ax+b}\right)=\frac{(-1)^n n!a^n}{(ax+b)^{n+1}}$
- $\frac{d^n}{dx^n}\left[\ln(ax+b)\right]=\frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$
- $\frac{d^n}{dx^n}e^{ax}=a^ne^{ax}$
- $\frac{d^n}{dx^n}\sin(ax+b)=a^n\sin\left(ax+b+n.\frac{\pi}{2}\right)$
- $\frac{d^n}{dx^n}\cos(ax+b)=a^n\cos\left(ax+b+n.\frac{\pi}{2}\right)$
- $\frac{d^n}{dx^n}e^{ax}\sin(bx+c)=(a^2+b^2)^{\frac{n}{2}}e^{ax}\sin\left(bx+c+n\tan^{-1}\frac{b}{a}\right)$
- $\frac{d^n}{dx^n}e^{ax}\cos(bx+c)=(a^2+b^2)^{\frac{n}{2}}e^{ax}\cos\left(bx+c+n\tan^{-1}\frac{b}{a}\right)$

Leibniz's Theorem

- $\frac{d^n}{dx^n}(uv)={}^nC_0u^{(n)}v+{}^nC_1u^{(n-1)}v'+{}^nC_2u^{(n-2)}v''+\ldots+{}^nC_{n-1}u'v^{n-1}+{}^nC_nuv^n$

Some Important Integrals

- $\int c\,dx = cx$, where c is constant
- $\int x^n dx = \dfrac{x^{n+1}}{n+1}, \quad n \neq -1$,
- $\int e^x dx = e^x$
- $\int a^x dx = \dfrac{a^x}{\ln a}$
- $\int \dfrac{dx}{x} = \ln|x|$
- $\int \sin x\,dx = -\cos x$
- $\int \sec^2 x\,dx = \tan x$
- $\int \sec x \tan x\,dx = \sec x$
- $\int \cos x\,dx = \sin x$
- $\int \csc^2 x\,dx = -\cot x$
- $\int \csc x \cot x\,dx = -\csc x$
- $\int \tan x\,dx = \ln|\sec x| = -\ln|\cos x|$
- $\int \sec x\,dx = \ln|\sec x + \tan x|$
- $\int \cot x\,dx = \ln|\sin x| = -\ln|\csc x|$
- $\int \csc x\,dx = \ln|\csc x - \cot x| = -\ln|\csc x + \cot x|$
- $\int \dfrac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\dfrac{x}{a}$ or $-\cos^{-1}\dfrac{x}{a}$
- $\int \sinh x\,dx = \cosh x$
- $\int \dfrac{dx}{a^2+x^2} = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a}$ or $-\dfrac{1}{a}\cot^{-1}\dfrac{x}{a}$
- $\int \cosh x\,dx = \sinh x$
- $\int \dfrac{dx}{x\sqrt{x^2-a^2}} = \dfrac{1}{a}\sec^{-1}\dfrac{x}{a}$ or $-\dfrac{1}{a}\csc^{-1}\dfrac{x}{a}$
- $\int \operatorname{sech}^2 x\,dx = \tanh x$
- $\int \dfrac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1}\dfrac{x}{a} = \ln(x+\sqrt{x^2+a^2})$
- $\int \operatorname{csch}^2 x\,dx = -\coth x$
- $\int \dfrac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\dfrac{x}{a} = \ln(x+\sqrt{x^2-a^2})$
- $\int \operatorname{sech} x \tanh x\; dx = -\operatorname{sech} x$
- $\int \dfrac{dx}{x\sqrt{a^2-x^2}} = -\dfrac{1}{a}\operatorname{sech}^{-1}\dfrac{x}{a} = -\dfrac{1}{a}\ln\left|\dfrac{a+\sqrt{a^2-x^2}}{x}\right|$
- $\int \operatorname{csch} x \coth x\; dx = -\coth x$
- $\int \dfrac{dx}{x\sqrt{a^2+x^2}} = -\dfrac{1}{a}\operatorname{csch}^{-1}\dfrac{x}{a} = -\dfrac{1}{a}\ln\left|\dfrac{a+\sqrt{a^2+x^2}}{x}\right|$
- $\int \tanh x\; dx = \ln|\cosh x|$
- $\int \dfrac{dx}{a^2-x^2} = \dfrac{1}{2a}\ln\left|\dfrac{a+x}{a-x}\right| = \dfrac{1}{a}\tanh^{-1}\dfrac{x}{a}$ *if* $x^2 < a^2$
- $\int \coth x\; dx = \ln|\sinh x|$
- $\int \dfrac{dx}{x^2-a^2} = \dfrac{1}{2a}\ln\left|\dfrac{x-a}{x+a}\right| = -\dfrac{1}{a}\coth^{-1}\dfrac{x}{a}$ *if* $x^2 > a^2$
- $\int \sqrt{a^2-x^2}\,dx = \dfrac{x\sqrt{a^2-x^2}}{2} + \dfrac{a^2}{2}\sin^{-1}\left(\dfrac{x}{a}\right)$
- $\int \sqrt{x^2+a^2}\,dx = \dfrac{x\sqrt{x^2+a^2}}{2} + \dfrac{a^2}{2}\ln\left|\dfrac{x+\sqrt{x^2+a^2}}{a}\right|$
- $\int \sqrt{x^2-a^2}\,dx = \dfrac{x\sqrt{x^2-a^2}}{2} - \dfrac{a^2}{2}\ln\left|\dfrac{x+\sqrt{x^2-a^2}}{a}\right|$

Differentiation of Trigonometric Functions

1) $\frac{d}{dx}(Sinx) = Cosx\ \frac{d}{dx}(x)$

2) $\frac{d}{dx}(Cosx) = -Sinx\ \frac{d}{dx}(x)$

3) $\frac{d}{dx}(tanx) = Sec^2x\ \frac{d}{dx}(x)$

4) $\frac{d}{dx}(Cotx) = -Cosec^2x\ \frac{d}{dx}(x)$

5) $\frac{d}{dx}(Secx) = Secx.tanx\ \frac{d}{dx}(x)$

6) $\frac{d}{dx}(Cesecx) = -Cosecx.Cotx\ \frac{d}{dx}(x)$

Differentiation of Inverse Trigonometric Functions

1) $\frac{d}{dx}(Sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}\ \frac{d}{dx}(x)$

2) $\frac{d}{dx}(Cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}\ \frac{d}{dx}(x)$

3) $\frac{d}{dx}(tan^{-1}x) = \frac{1}{1+x^2}\ \frac{d}{dx}(x)$

4) $\frac{d}{dx}(Cot^{-1}x) = \frac{-1}{1+x^2}\ \frac{d}{dx}(x)$

5) $\frac{d}{dx}(Sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}\ \frac{d}{dx}(x)$

6) $\frac{d}{dx}(Cosec^{-1}x) = \frac{-1}{|x|\sqrt{x^2-1}}\ \frac{d}{dx}(x)$

Differentiation of Hyperbolic Functions

1) $\frac{d}{dx}(Sinhx) = Coshx\ \frac{d}{dx}(x)$

2) $\frac{d}{dx}(Coshx) = Sinhx\ \frac{d}{dx}(x)$

3) $\frac{d}{dx}(tanhx) = Sech^2x\ \frac{d}{dx}(x)$

4) $\frac{d}{dx}(Cothx) = -Cosech^2x\ \frac{d}{dx}(x)$

5) $\frac{d}{dx}(Sechx) = -Sechx.tanhx\ \frac{d}{dx}(x)$

6) $\frac{d}{dx}(Cosechx) = -Cosechx.cothx\ \frac{d}{dx}(x)$

Differentiation of Inverse Hyperbolic Functions

1) $\frac{d}{dx}(Sinh^{-1}x) = \frac{1}{\sqrt{1+x^2}}\ \frac{d}{dx}(x)$

2) $\frac{d}{dx}(Cosh^{-1}x) = \frac{1}{\sqrt{x^2-1}}\ \frac{d}{dx}(x)$

3) $\frac{d}{dx}(tanh^{-1}x) = \frac{1}{1-x^2}\ \frac{d}{dx}(x)$

4) $\frac{d}{dx}(Coth^{-1}x) = \frac{-1}{x^2-1}\ \frac{d}{dx}(x)$

5) $\frac{d}{dx}(Sech^{-1}x) = \frac{-1}{|x|\sqrt{1-x^2}}\ \frac{d}{dx}(x)$

6) $\frac{d}{dx}(Cosech^{-1}x) = \frac{-1}{|x|\sqrt{x^2+1}}\ \frac{d}{dx}(x)$

Integration Formulas

1 $\int \sin(ax+b)\, dx = -\frac{1}{a}\cos(ax+b) + C$

2 $\int \cos(ax+b)\, dx = \frac{1}{a}\sin(ax+b) + C$

3 $\int \tan(ax+b)\, dx = -\frac{1}{a}\ln\left|\cos(ax+b)\right| + C = \frac{1}{a}\ln\left|\sec(ax+b)\right| + C$

4 $\int \cot(ax+b)\, dx = \frac{1}{a}\ln\left|\sin(ax+b)\right| + C$

5 $\int \operatorname{cosec} x\, dx = \ln\left|\operatorname{cosec} x - \cot x\right| + C = ln\left|\tan\frac{x}{2}\right| + C$

6 $\int \sec x\, dx = \ln\left|\sec x + \tan x\right| + C = ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$

7 $\int e^{ax+b}\, dx = \frac{1}{a} \times e^{ax+b} + C$

8 $\int a^{bx+c}\, dx = \frac{1}{b\ln a} \times a^{bx+c} + C$

9 $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right)$

10 $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right)$

11 $\int \sqrt{a^2-\boldsymbol{x^2}}\, dx = \frac{x}{2}\sqrt{a^2-\boldsymbol{x^2}} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

12 $\int \frac{1}{\sqrt{x^2+a^2}}\, dx = \sinh^{-1}\left(\frac{x}{a}\right) + C = ln\left(\frac{x+\sqrt{x^2+a^2}}{a}\right) + C$

13 $\int \frac{1}{\sqrt{x^2-a^2}}\, dx = \cosh^{-1}\left(\frac{x}{a}\right) + C = ln\left(\frac{x+\sqrt{x^2-a^2}}{a}\right) + C$

14 $\int \frac{1}{x^2+a^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

15 $\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$

16 $\int \frac{1}{a^2-x^2}\, dx = \frac{1}{2a}\, ln\left|\frac{a+x}{a-x}\right| + C$

17 $\int \frac{1}{x^2-a^2}\, dx = \frac{1}{2a}\, ln\left|\frac{x-a}{x+a}\right| + C$

Hyperbolic Formulas

① $cosh^2x - sinh^2x = 1$

② $cosh^2x + sinh^2x = cosh2x$

Adding ① and ②

③ $2cosh^2x = 1 + cosh2x$

Subtracting ② from ①

④ $2sinh^2x = cosh2x - 1$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \; .... = \frac{\pi^2}{12}$$

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \; .... = \frac{\pi^2}{6}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - ....$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - ....$$

$$\text{Cos x} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - ....$$

$$\text{Sin x} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - ....$$

nth order derivatives

$$\frac{d^n}{dx^n}\sin(ax+b) = a^n \sin\left[n\frac{\pi}{2} + ax + b\right]$$

$$\frac{d^n}{dx^n}\cos(ax+b) = a^n \cos\left[n\frac{\pi}{2} + ax + b\right]$$

$$\frac{d^n}{dx^n}a^x = a^x\,(lna)^n$$

$$\frac{d^n}{dx^n}e^{ax} = a^n\,e^{ax}$$

$$\frac{d^n}{dx^n}\left[\frac{1}{ax+b}\right] = \frac{a^n\,(-1)^n\,n!}{(ax+b)^{n+1}}$$

$$\frac{d^n}{dx^n}e^{ax}\sin(bx+c) = [a^2+b^2]^{\frac{n}{2}}\,e^{ax}\sin(bx+c+n\tan^{-1}\frac{b}{a})$$

$$\frac{d^n}{dx^n}e^{ax}\cos(bx+c) = [a^2+b^2]^{\frac{n}{2}}\,e^{ax}\cos(bx+c+n\tan^{-1}\frac{b}{a})$$

$\tan^{-1}z = \frac{1}{2}\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right)$	$\tanh^{-1}z = \frac{1}{2}\cosh^{-1}\left(\frac{1+z^2}{1-z^2}\right)$

$$\int\frac{dx}{a+bcosx} = \frac{1}{\sqrt{a^2-b^2}}\cos^{-1}\left(\frac{b+acosx}{a+bcosx}\right)+c \qquad ,a > b$$

$$\int\frac{dx}{a+bcosx} = \frac{1}{\sqrt{b^2-a^2}}\cosh^{-1}\left(\frac{b+acosx}{a+bcosx}\right)+c \qquad ,a < b$$

$$\int\frac{dx}{a+bcosx} = \frac{1}{a}\tan\frac{x}{2}+c \qquad ,a = b$$

Fundamental Identities	***Triple Angle Identities***
i $\sin^2\theta + \cos^2\theta = 1$ ii $1 + \tan^2\theta = \sec^2\theta$ iii $1 + \cot^2\theta = \operatorname{cosec}^2\theta$	$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Sum or Difference into Product

$$\sin P + \sin Q = 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$$

$$\sin P - \sin Q = 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$$

$$\text{Cos } P + \text{Cos } Q = 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$$

$$\text{Cos } P - \text{Cos } Q = -2\sin\frac{P+Q}{2}\sin\frac{P-Q}{2}$$

Product into Sum or Difference

$$2\sin\alpha\cos\beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$$

$$2\cos\alpha\sin\beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$$

$$2\cos\alpha\cos\beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$$

$$-2\sin\alpha\sin\beta = \cos(\alpha + \beta) - \cos(\alpha - \beta)$$

Double Angle Identities

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\cos 2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

Half Angle Identities

$$1 + \text{Cos }\theta = 2\cos^2\frac{\theta}{2}$$

$$1 - \text{Cos }\theta = 2\sin^2\frac{\theta}{2}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$