# "Relations" Chapter 9

## Topic: 9.1
## "Relation & Their Properties"

### what is relation?

A realation b/w two sets is a collection of ordered pairs containing one object from each set. If the object x is from the set one & object y is from second set, then the objects are said to be related if the ordered pair (x,y) is in the relation. A function is a type of relation.

### What is Binary relation?

A binary relation over set X & Y is a subset of the cartesian product X×Y; that is, it is a set of ordered pairs (x,y) consisting of elements x in X & y in Y.
Binary relation of X & Y is a subset of X×Y.

What is meant by relation on set?

Relations from a set A to itself are of special interest.

e.g: let $A = \{1,2,3,4\}$, $A \times A = ?$

$R = \{(1,1),(1,2),(1,3),(1,4),(2,3),(2,2),(2,4),(3,3),(3,4)$

$(4,4)\}$

$R_1 = \{(a,b) \mid a = b\}$ $= \{(1,1), (2,2), (3,3)$ $(2,4),(3,3),(2,3),(3,4)$

$R_2 = \{(a,b) \mid a \leq b\}$ $= \{(1,1),(1,2),(1,3),(1,4),$

$R_3 = \{(a,b) \mid a \geq b\}$ $= \{(1,1),(2,2),(3,3),(4,4)$

$R_4 = \{(a,b) \mid a+1 \leq b\}$ $= \{(1,2),(1,3),(1,4),\cancel{(2,3)},(3,$

Describe Properties of Relation?

**1. Reflexive relation:**

A relation R on a set A is called reflexive if $(a,a) \in R$ for every element $a \in A$

e.g: $R = A \times A = \{(1,1),(1,2),(1,3),(1,4),(2,3),(2,4),(2,2),(3,3),(3,4)$

a ~~set~~ relation is reflexive if a relation

$a = b$ pair

e.g: from above $(1,1),(2,2),(3,3),(4,4)$

$R_1 = \{(1,2),(1,1),(1,3)\}$ X

$R_2 = \{(1,1),(1,2),(2,2),(2,3),(3,3),(4,4)\}$ ✓

**2. Symmetric relation:**

if $(b,a) \in R$ whenever $(a,b) \in R$, for all $a,b \in$

mean $(a,b) \in R \wedge (b,a) \in R$

$R_1$ is not symmetric bcz $(1,3)$

$(3,1)$ is not $\in R$, $R_2$ is also not symmetric

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### 3. transitive relation:

A relation R on a set A is called transitive if whenever $(a,b) \in R$ & $(b,c) \in R$, then $(a,c) \in R$, for all $a,b,c \in A$

(2,1) (3,2) (3,1) (4,2) (4,3)

e.g = $\{ \cancel{(1,2)}, (2,1), (3,1), (3,2), (4,1), (4,2), (4,3) \}$

as we know (2,1), (3,2), (3,1), (4,2) & (4,3) are all present in a set so its transitive.

### what is meant by Composite of relation.

let R be the relation from A to B, & S be the relation from B to C, then composition SoR or RoS will be $(x,y) \in SoR \Leftrightarrow \exists s \in B (x,z) \in R \wedge (z,y) \in S$

let $A = \{1,2,3,4\}$

$R = \{(1,2),(2,4),(3,2),(4,3)\}$

$S = \{(1,1),(2,4),(3,1),(4,4)\}$

Find SoR & RoS

SoR =

| Rx | Ry | Sx | Sy |
|---|---|---|---|
| 1 → | 2 | 1 | 1 |
| 2 → | 4 | 2 → | 4 |
| 3 → | 2 | 3 → | 1 |
| 4 → | 3 | 4 → | 4 |

SoR $= \{(1,4),(2,4),(3,4),(4,1)\}$

RoS =

| Sx | Sy | Rx | Ry |
|---|---|---|---|
| 1 → | 1 | 1 → | 2 |
| 2 → | 4 | 2 | 4 |
| 3 → | 1 | 3 | 2 |
| 4 → | 4 | 4 → | 3 |

SoR $= \{(1,2),(2,3),(3,2),(4,3)\}$

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Topic 4.2

# "n-ary Relations & Their Application"

## What is n-ary relation?

Relationship among elements ~~is called~~ from more than two sets is called n-ary relation.

e.g:

let $A_1, A_2, \ldots A_n$ be sets. An n-ary relation on these sets is a subset of $A_1 \times A_2 \times A_3 \ldots A_n$. The sets $A_1, A_2, \ldots A_n$ are called domain & n is called degree

$R = \{(a_1, a_2), (a_3, a_4), \ldots\} \subseteq A \times A$

$R_n = \{(a_{11}, a_{12}, a_{13} \ldots a_{1n}) \times (a_{21}, a_{22}, \ldots a_{2n}) \ldots\}$

## What is extension & intension?

The current collection of n-ary is called extension & the permanent part of database including name & attributes is called intension

## What is primary & composite key?

Domain of n-ary is called primary key

The cartesian product of the the values of set of domains is called composite key.

Topic: 9.3

# "Representing Relations"

**How to represent relation using metric?**

A relation b/w finite sets can be represented using zero-one matrices.

Suppose, $R$ is a relation from $A = \{a_1, a_2, \dots, a_m\}$ to $B = \{b_1, b_2, \dots, b_m\}$. The relation $R$ can be represented by the matrix $M_R = [m_{ij}]$, where

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R \end{cases}$$

e.g.:

$A = \{1, 2, 3\}$ & $B = \{1, 2\}$ $= \{(1,1)(1,2) | (2,1),(2,2) |, (3,1)(3,2)\}$

let $R$ be the relation from $A$ to $B$ containing $(a,b)$, if $a \in A$, $b \in B$ & $a > b$, in matrix, $R$ if $a_1 = 1$, $a_2 = 2$, & $a_3 = 3$ & $b_1 = 1$, $b_2 = 2$?

Ans = $R = \{(2,1), (3,1), (3,2)\}$ the matrix for $R$ is

$$M_R = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$



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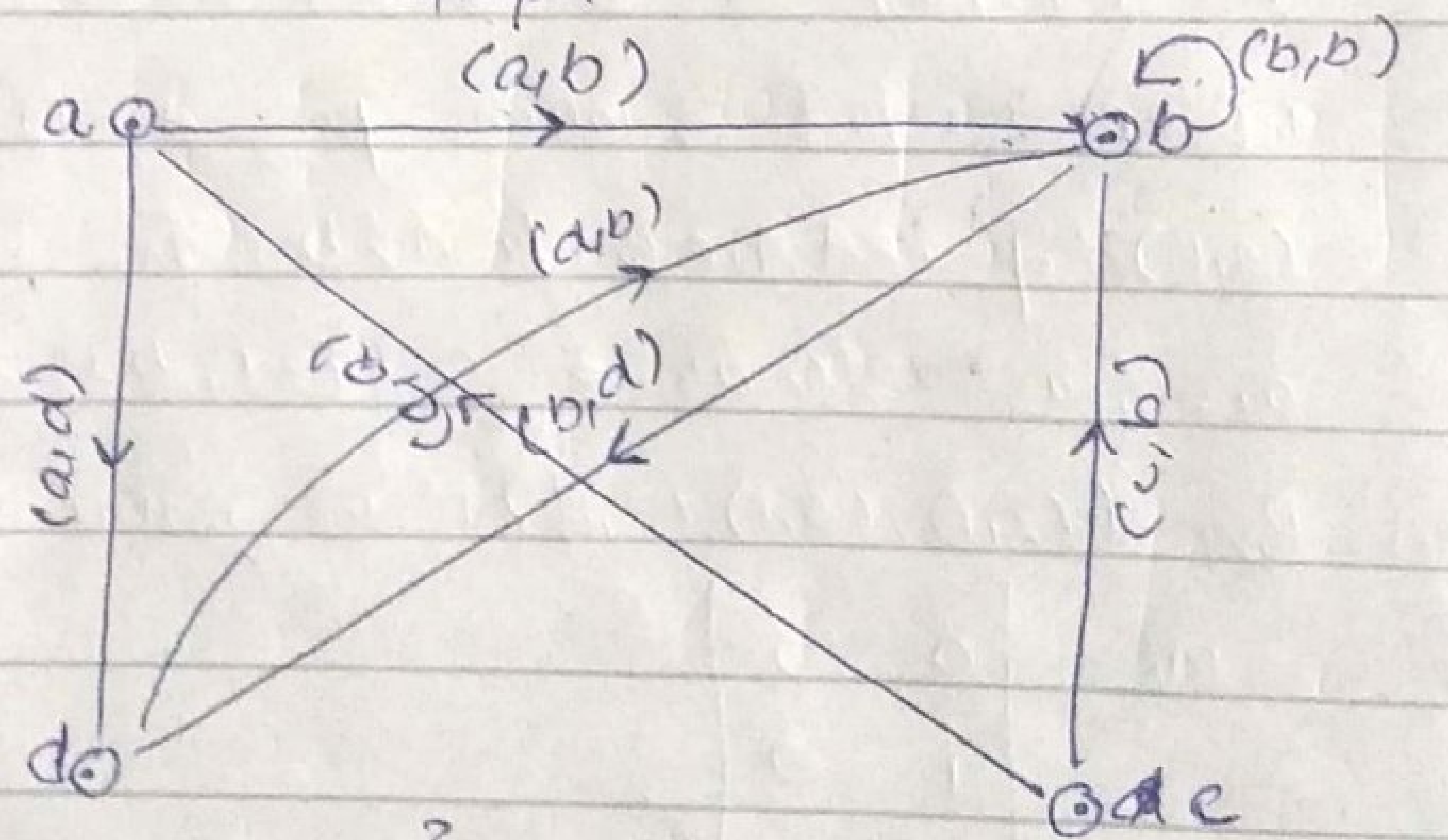
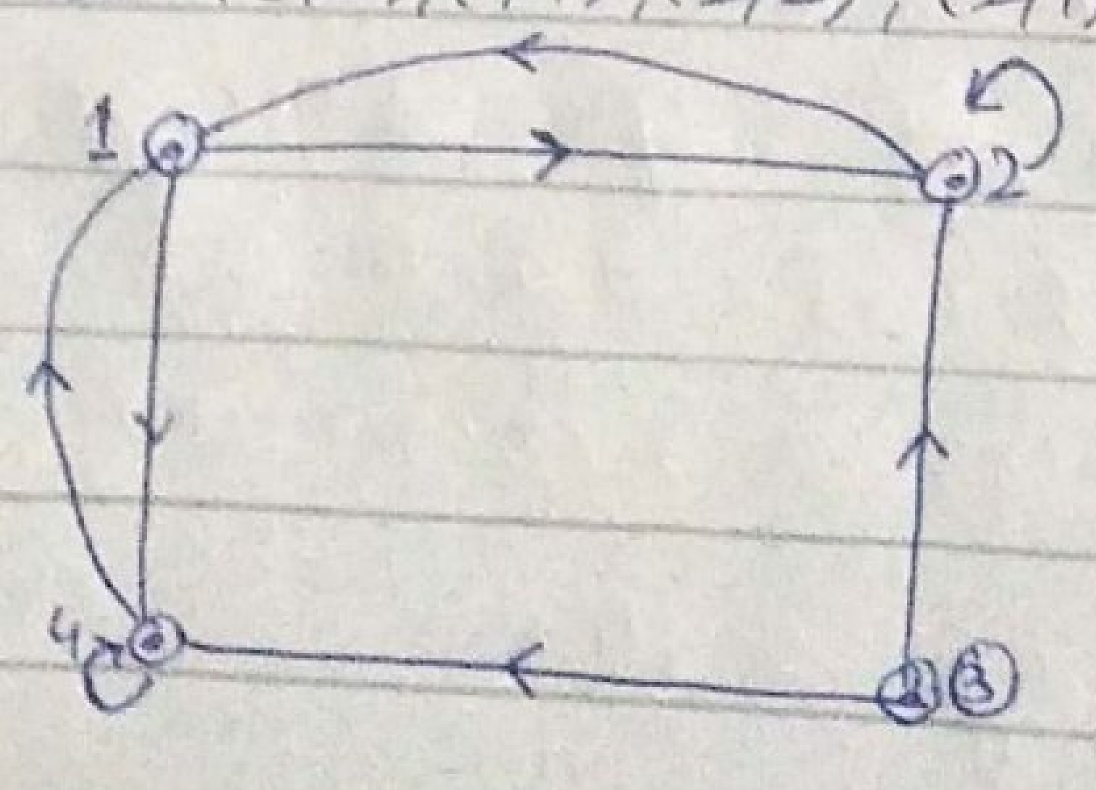
## How to represent relation using diagraphs?

A directed graph or direct graph, consist of a set V of vertices together with a set E of ordered pair of elements of V called edges.
The vertex a is called initial vertex of the edge (a, b), & vertex b is called the terminal vertex of edge (a, b)
Pictorial representation of a ~~graph~~ relation is called diagraphs or directed graphs.
e.g:
Directed graph with vertice a, b, c, & d & edges (a,b), (a,d), (b,b), (b,d), (c,a), (c,b), (d,b)

Graph




in case {1, 2, 3, 4}
Then R = {(1,2), (1,4), (2,2), (2,1), (3,2), (3,4), (4,4), (4,1)}



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Topic 9.4
# "Closures of Relations"

## Transitive closure: Relation:

let R & S is a relation, where S containg R such that S is a subset of every transitive relation containing R. S is the smallest relation that contain R. This relation is called transitive colosure of R.

let R = {(1,1),(1,2),(2,1),(3,2)} A = {1,2,3}

we have to add $\boxed{S=(2,2)}$ to make it transitive bcz it is not in R, by adding this new relation in R it becomes transitive closure of R.

⇒ It become reflexive closure by adding (2,2) & (3,3) into R

⇒ It become symmetric closure by adding (2,3) into R.

## Warshall's Algorithm:

An efficient method of finding the adjacency matrix of the transitive closure of relation R on finite set S from adjacency matrix of R.

let set S have = {a, b, c} or {1, 2, 3}

R = {(1,1), (1,2), (2,1), (3,2)

imaginary relation.

Warshall method. First make matrices of relation R

1 ⇒. $M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = W_0$

2 ⇒. Create more relation to make $W_1$, check pos

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 1 & & \\ 3 & 0 & & \end{array}$$

First Column : 1, 2

First Row : 1, 2

$R = \{(1,1), (1,2), (2,1), (2,2)\}$

$W_1 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

3 ⇒ Create relation again to make $W_2$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & & 1 & \\ 2 & 1 & 1 & 0 \\ 3 & & 1 & \end{array}$$

FC SC = 1, 2, 3

FR SR = 1, 2

$R = \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2)\}$

$W_2 = \begin{bmatrix} 1 & 1 & - \\ 1 & 1 & - \\ - & 1 & - \end{bmatrix} \Rightarrow W_2 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

4 ⇒ Create creation again $W_3$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & & & 0 \\ 2 & & & 0 \\ 3 & 1 & 1 & 0 \end{array}$$

TC = X

TR = 1, 2    R = { }

So $W_2 = W_3$

$W_3 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$W_3$ is called transitive closure



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