Ch#6 Counting

Sum Rule :-

The Sum rule states that if an event A can occure in m ways and event B can occure in n disjoint ways, then the event A or B can occure in m+n ways.

Product Rule :-

Suppose that a procedure can be broken down into a sequence of two task. if there are m Ways to do the first task ~~and for each of these ways of doing the first task~~ and there are n ways to do the second task, then there are m×n ways to do the procedure.

Sum Rule with sets :-

Given two set A and B, if $A \cap B = \emptyset$
Then

$$|A \cup B| = |A| + |B|$$

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Cartesian Product:-

Cartesian Product of two set A and B and defined as.

$A \times B = \{(a, b) ; a \in A, b \in B\}$

Product Rule With set:-

Given two set A and B, we have

$|A \times B| = |A| . |B|$

imp

Inclusion- Exclusion Principle:-

For any finite set A and B, we have

$|A \cup B| = |A| + |B| - |A \cap B|$

Division Rule:-

There are 'n/d' ways to do a task if it can ~~to~~ be done using a procedure that can be carried out in 'n' ways, and for every ways 'w', exactly 'd' of the 'n' ways correspond to way 'w'.

P **Permutation :-**

The arrangment of object with some order is called permutation.
It is denoted by ^nP_r.

$$^nP_r = \frac{n!}{(n-r)!}$$

n = total object
r = selected object

P **Combination :-**

The Selection of object without any ~~arrangment~~ order is called Combination.
It is denoted by ^nC_r.

$$^nC_r = \frac{n!}{r!(n-r)!}$$

n = total object
r = Selected object

Permutation With Repetition:-

An r-permutation with repetition of a set S is the number of ways to choose r element from S with repetition allowed S where orders matters.

(Theoram)

1. Prove that $P(n,r) = \dfrac{n!}{(n-r)!}$?

Proof :-
For postive integer n and r with $1 \leq r \leq n$, we have

$P(n,r) = n(n-1)(n-2) \cdots\cdots (n-r+1)$
$\times$ and $\div$ $(n-r)(n-r-1) \cdots\cdots 3.2.1$

$$P(n,r) = \frac{n(n-1)(n-2)\cdots\cdots(n-r+1)(n-r)(n-r-\cdots 3.}{(n-r)(n-r-1)\cdots\cdots 3.2.1}$$

$$P(n,r) = \frac{n!}{(n-r)!}$$ proved

Henced proved.

2. Prove that $\dfrac{n!}{r!(n-r)!}$

Proof :-
Let x be the set of n elements. Let A be set of r-Permutation of x and Let B be set of r-combination of x, We find $|B|$

Let $f: A \longrightarrow B$ be the function mapping r-permulation to r-Combination defined by

$$f(x_1, x_2, x_3, ----, x_r) = (x_1, x_2, x_3, --- x_r)$$

(permulation) (combination)

There f is $r!$ - to - 1 function.

In particular

$$f^{-1}(x_1, x_2, x_3, ----, x_r) = \{(x_1, x_2, x_3, --- x_r), (x_2, x_1, x_3, -- x_r)$$

------- all $r!$ permutation}

By the division rule

$$|A| = r! |B|$$

$$\Rightarrow |B| = \frac{|A|}{r!}$$

$$|B| = \frac{P(n,r)}{r!}$$

$$|B| = \frac{n!}{(n-r)! \, r!}$$ - proved

Combination With repetition:-

$$\binom{n+r-1}{r}$$ Formula

6.2)

State Pigeonhole Principal:-

If K is a postive integer and $K+1$ or more object are placed into K boxes, then there is at least one box containing two or more of the objects.

State Generalized Pigeonhole Princi

If N objects are placed into K boxes, then there is atleast one box containing at least $\lceil N/K \rceil$ objects.

Proof:-

Suppose that none of the boxes contains more than $\lceil N/K \rceil - 1$ objects.
The total number of object is

$$K\left(\left\lceil \frac{N}{K} \right\rceil - 1\right) < K\left(\left(\frac{N}{K} + 1\right) - 1\right) = N$$

where the inequality $\lceil N/K \rceil < N/K + 1$ has been used.

Proof inclusion - Exclusion Principal

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Proof :-

compute

$$|A \cup B| = |A \cap B| + |A \setminus B| + |B \setminus A|$$

$$= |A \cap B| + (|A| - |A \cap B|) + (|B| - |A \cap B|)$$

$$= \cancel{|A \cap B|} + |A| - |A \cap B| + |B| - \cancel{|A \cap B|}$$

$\therefore |A \setminus B| = |A| - |A \cap B|$

$\therefore |B / A| = |B| - |A \cap B|$

$$\boxed{|A \cup B| = |A| + |B| - |A \cap B|}$$

This is a special case of inclusion-Exclusion Principal.

Ch#9 Relations

Set :-

Well - defined collection of distinct object is called set.

Well - defined :-

About any object you may be able to decide whether this object belongs to the collection or not.

Cartesian product :-

Cartesian product of two set A and B denoted by $A \times B$ and define as

$$A \times B = \{(a,b);\ a \in A,\ b \in B\}$$

Relation :-

All Subset of Cartesian product $A \times B$ are called relation from A to B.

Example :-

$A = \{1, 2\}$, $B = \{a, b\}$

Cartesian product $= A \times B = \{(1,a)(1,b),(2,a),(2,b)\}$

Total Relation $= 2^4 = 16$

$A \times B = \{(1,a),(1,b)(2,a),(2,b)\}$

$R_1 = \{ \ \} = \phi$

$R_2 = \{(1,a)\}$

$R_3 = \{(1,b)\}$, $R_4 = \{(2,a)\}$, $R_5 = \{(2,$

$R_6 = \{(1,a),(1,b)\}$, $R_7 = \{(1,a),(2,a)\}$

$R_8 = \{(1,a),(2,b)\}$, $R_9 = \{(1,b),(2,a)\}$

$R_{10} = \{(1,b),(2,b)\}$, $R_{11} = \{(2,a),(2,b)\}$

$R_{12} = \{(1,a),(1,b),(2,a)\}$

$R_{13} = \{(1,a),(1,b),(2,b)\}$

$R_{14} = \{(1,a),(2,a),(2,b)\}$

$R_{15} = \{(1,b),(2,a),(2,b)\}$

$R_{16} = \{(1,a),(1,b),(2,a),(2,b)\}$

Domain = First element each relation

Range = Second element of each relation.

Function :-

Suppose that to each element of a set A we assign a unique element of a set B, the collection of such assignments is called a function from A to B.

The set A is called the domain of the function and the set B is called target se

$f : A \rightarrow B$

Types of

i) one-to-one

ii) onto function

iii) Into function

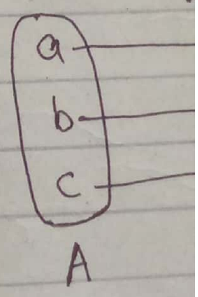
iv) Bijective fun

v) Injective fu

vi) Surjective f

One-to-on

function f:
be one-
different el
domain A



onto f

$f : A \rightarrow B$

an on

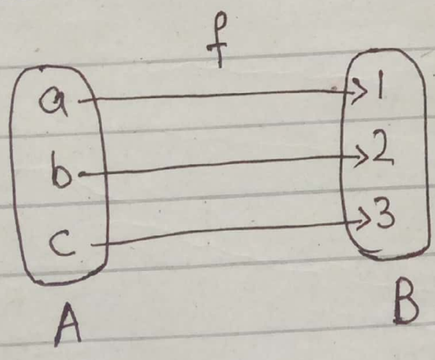
$f : A \to B$

Types of function :-

i) one-to-one function
ii) onto function
iii) Into function
iv) Bijective function
v) Injective function
vi) Surjective function.

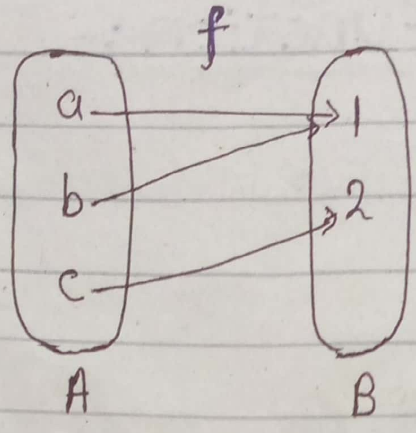
One-to-one function :- (injective)

A function $f : A \to B$ is said to be one-to-one function if different elements in the domain A have distinct images.
واضح



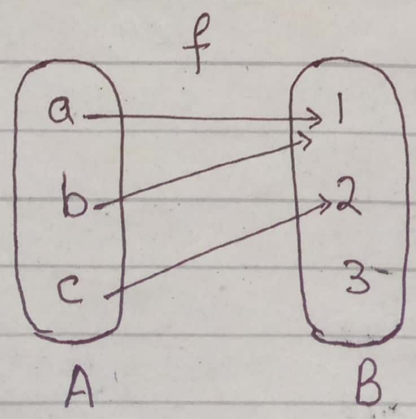
onto function :- (Surjective)

A function $f : A \to B$ is said to be an onto function if every element of B is the image of some element in A.



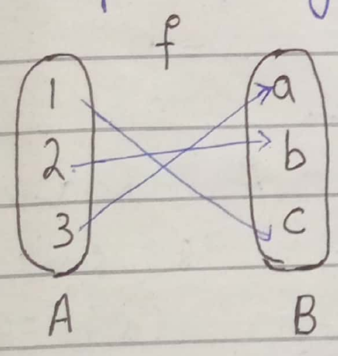
Into function :-

A function $f : A \rightarrow B$ is said to be into function if every element of set B has atleast one element which is not associated with any element of set A.



Bijective function:-

A function $f: A \rightarrow B$ is said to be a bijective if f is both one-to-one and onto, that is every element in A has a unique element in B and every element of B has a pre-image in set A.



Properties of Relation:-

Reflexive Relation:-

A Relation R on a set A is called reflexive if $(a,a) \in R$ for every element $a \in A$.

ii) **Symmetric Relation :-** A relation R on a set A is called symmetric if $(a,b) \in R$ whenever $(b,a) \in R$ for all $a, b \in A$.

iii) **Antisymmetric Relation :-** A Relation R on a set A is called antisymmetric if $(a,b) \in R$ and $(b,a) \in R$ then $a = b$.

iv) **Transitive Relation :-** A Relation R on a set A is called transitive if whenever $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ for all $a, b, c \in A$.

Binary relation :- Let A and B be sets. A binary relation from A to B is a subset of $A \times B$.

um#3 $A=\{0,1,2\}$, $B=\{a,b\}$

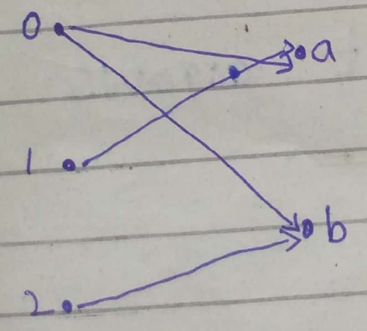
cartesian product = ?

cartesian product = $A\times B=\{(0,a),(0,b),$
$(1,a),(1,b),(2,a),(2,b)\}$

Let $A=\{1,2\}$, $B=\{a,b\}$
cartesian product = $A\times B=\{(1,a),(1,b),(2,a),(2,b)\}$
Relation = $2^4=16$

$R=\{\{\emptyset\},\{(1,a)\},\{(1,b)\},\{(2,a)\},\{(2,b)\},$
$\{(1,a),(1,b)\},\{(1,a),(2,a)\},\{(1,a),(2,b)\}$
$,\{(1,b),(2,a)\},\{(1,b),(2,b)\},\{(2,a),(2,b)\}$
$,\{(1,a),(1,b),(2,a)\},\{(1,a),(1,b),(2,b)\},$
$\{(1,a),(2,a),(2,b)\},\{(1,b),(2,a),(2,b)\}$
$,\{(1,a),(1,b),(2,a),(2,b)\}\}$

Relation represent graphically
$A=\{0,1,2\}$, $B=\{a,b\}$, $R=\{(0,a),(0,b),(1,a),(2,b)\}$



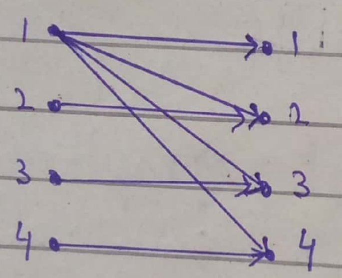
Relation represent arrows

R	a	b
0	1	1
1	1	0
2	0	1

4) $A = \{1, 2, 3, 4\}$, Which ordered Pairs are in relation $R = \{(a,b) \mid a \text{ divides } b\}$?

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)$

Relation represent graphicall



Relation represent arrows

R	1	2	3	4
1	1	1	1	1
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1

Relation and Properties of Relation

Relation :-

#7 Relation = $\{1, 2, 3, 4\}$

$R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)\}$
$R_2 = \{(1,1), (1,2), (2,1)\}$
$R_3 = \{(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)\}$
$R_4 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$
$R_5 = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$?
$R_6 = \{(3,4)\}$

Reflexive relation :-

$\therefore (a,a) \in R \quad \forall\, a \in A$

$R_3 = \{(1,1), (2,2), (3,3), (4,4)\}$ Reflexive
$R_5 = \{(1,1), (2,2), (3,3), (4,4)\}$ Reflexive

Symmetric relation :-

$\therefore (a,b) \in R$ whenever $(b,a) \in R$
$\forall\, a, b \in A$

R_2 symmetric
$(1,2) \in R$, $(2,1) \in R$

R_2, R_3 symmetric

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R_3 symmetric relation

$(1,2) \in R$, $(2,1) \in R$

iii) **Anti-Symmetric relation :-**

$$\left[\begin{array}{l} (a,b) \in R \quad \text{and} \quad (b,a) \in R \\ \text{then} \quad a = b \end{array}\right].$$

R_4, R_5, R_6 anti-symmetric

iv) **Transitive relation :-**

$$\left[\begin{array}{l} (a,b) \in R \quad \text{and} \quad (b,a) \in R \\ \text{then} \quad (a,c) \in R \end{array}\right]$$

R_4, R_5, R_6 transitive

$\therefore$

Exam#5 Relation Set of integer

i) **Reflexive relation :-**

R_1, R_3, R_4 reflexive relation

$R_1 = \{(a,b) \mid a \leq b\}$

$R_1 = \{(1,1), (2,2), (3,3)\}$

ii) **Symmetric relation :-**

R_3, R_4, R_6

Anti-symmetric relation :-

R_1, R_2, R_4, R_5

Transitive relation :-

R_1, R_2, R_3, R_4

Combining/Composite Relation :-

Let R be a relation from a set A to a set B and S a relation from B to a set C. The composite of R and S is the relation consisting of ordered pairs (a,c) where $a \in A$, $c \in C$ and for which there exists an element $b \in B$ such that $(a,b) \in R$ and $(b,c) \in S$. We denote the composite of R and S by $S \circ R$.

17) $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4\}$, Relation
$R_1 = \{(1,1), (2,2), (3,3)\}$, $R_2 = \{(1,1), (1,2), (1,3), (1,$

$R_1 \cup R_2 = ?$, $R_1 \cap R_2 = ?$, $R_1 - R_2 = ?$
$R_2 - R_1 = ?$

Ans. $R_1 \cup R_2 = ?$

$R_1 \cup R_2 = \{(1,1), (2,2), (3,3)\} \cup \{(1,1), (1,2), (1,3), (1,4$

$R_1 \cup R_2 = \{(1,1), (1,2), (1,3), (1,4), (2,2), (3,3)\}$ Ans

$R_1 \cap R_2 = ?$

$R_1 \cap R_2 = \{(1,1), (2,2), (3,3)\} \cap \{(1,1), (1,2), (1,3), ($

$R_1 \cap R_2 = \{(1,1)\}$ Ans

$R_1 - R_2 = ?$

$R_1 - R_2 = \{(1,1), (2,2), (3,3)\} - \{(1,1), (1,2), (1,3), (1$

$R_1 - R_2 = \{(2,2), (3,3)\}$ Ans

$R_2 - R_1 = ?$

$R_2 - R_1 = \{(1,1), (1,2), (1,3), (1,4)\} - \{(1,1), (2,2), (3,3)\}$

$R_2 - R_1 = \{(1,2), (1,3), (1,4)\}$ Ans

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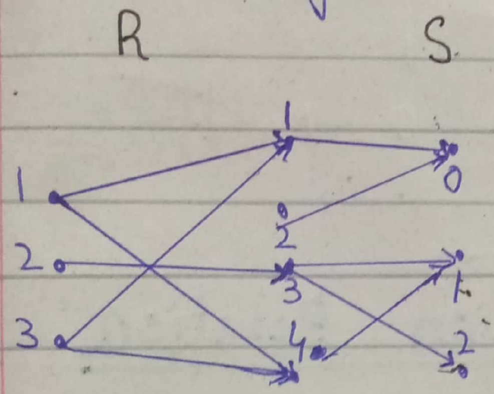
Q.10) What is Composite relation R and S, R relation {1,2,3} to {1,2,3,4}, R = {(1,1),(1,4),(2,3),(3,1),(3,4)} and S from {1,2,3,4} to {0,1,2} S = {(1,0),(2,0),(3,1),(3,2),(4,1)} ?

Sol. $S \circ R = \{(1,0),(1,1),(2,1),(2,2),(3,0),(3,1)\}$

Constructing $S \circ R$



$1 \to 1 \to 0$ (1,0)
$1 \to 4 \to 1$ (1,1)
$2 \to 3 \to 1$ (2,1)
$2 \to 3 \to 2$ (2,2)
$3 \to 1 \to 0$ (3,0)
$3 \to 4 \to 1$ (3,1)

Power relation:-

Let R be a relation on the set A. The power R^n, $n = 1, 2, 3, \ldots\ldots$ are defined recursively by

$R^1 = R$ and $R^{n+1} = R^n \circ R$

Remark

$R^2 = R \circ R$

$R^3 = R^2 \circ R = (R \circ R) \circ R$ so on

22) $R = \{(1,1),(2,1),(3,2),(4,3)\}$ Find Power R^n, $n = 2, 3, 4, \ldots$

Sol

9.3) Representation Relation

1) $A = \{1,2,3\}$, $B = \{1,2\}$, $R = a > b$, What is ~~composite~~ ~~relati~~ matrix represen R?

Sol $A \times B = \{(1,1),(1,2),(2,1),(2,2),(3,1),(3,2)\}$

$R = a > b = \{(2,1),(3,1),(3,2)\}$

$$M_R = \begin{array}{c|c} & B \\ & \begin{array}{cc} 1 & 2 \end{array} \\ A \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \end{array}$$

2) $A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2, b_3, b_4\}$, matrix representing?

Sol

$A \times B = \{(a_1, b_1), (a_1, b_2), (a_1, b_3), (a_1, b_4), (a_2, b_1), (a_2, b_2)$
$(a_2, b_3), (a_2, b_4), (a_3, b_1), (a_3, b_2), (a_3, b_3), (a_3, b_4)\}$

$R = \{(a_1, b_2), (a_2, b_1), (a_2, b_3), (a_2, b_4), (a_3, b_1), (a_3, b_3), (a_3, b_4)\}$

$$M_R = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{cccc} b_1 & b_2 & b_3 & b_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

4) $M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

What are matrix representing $R_1 \cup R_2$, $R_1 \cap R_2$?

Sol

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

5) Find matrix representing $S \circ R$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}, \quad M_S = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

Sol

$R = \{(1,1),(1,3),(2,1),(2,2)\}$

$S = \{(1,2),(2,3),(3,1),(3,3)\}$

$S \circ R = \{(1,1),(2,2),(1,2),(2,3),(1,3)\}$

R → A to B
S → B to c
SoR = A to c

$$M_{S \circ R} = M_R \odot M_S = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(1,3) (3,1)
a b b c
(a,c) = (1,1)
(2,1), (1,2)
(1,2), (

6) Find matrix representing R^2

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Sol

$M_{R^2} =$

Directed graph/diagraph :- (Page# 625 (Book)

A directed graph consist of a set V of vertices together with a set E of ordered pairs of elements of V called edge. The vertex called initial vertex of the edge (a,b) and vertex B called terminal.

Closures of Relation

Closures of Relation:-

If R is a relation on a set A, then the closure of R with respect to P, if it exists, is the relation of S on A with property P that contains R and subset of every subset $A \times A$ containing R with property P.

Types of closure:-

Reflexive closure:-

The reflexive closure of a relation R on A is obtained by adding (a,a) to R for each $a \in A$.

Symmetric closure:-

The symmetric closure of R is obtained by adding (b,a) to R for each $(a,b) \in R$.

Transitive closure:-

Let R be a relation on a set A. The connectivity relation R^* consist of the pair (a, b) such that there is a path of length at least one from a to b in R.

In other words,

$$R^* = \bigcup_{n=1}^{\infty} R^n$$

Note The transitive closure of a relation R equal the connectivity relation R^*.

9.5) Equivalance Relation:-

A relation on a set A is called equivalance relation if it is reflexive, symmetric and transitive.

Equivalance Classes:-

Let R be an equivalance relation on a set A. The set of all element that are related to an element a of A is called equivalence class of a. It is denoted by $[a]_R$.

8) What equivalance class of integer for equivalance relation?

Sol Because integer equivalent itself and its negtive equivalence relation

$[a] = \{-a, a\}$, unless $a = 0$

$[7] = \{-7, 7\}$

$[-5] = \{5, -5\}$, $[0] = \{0\}$

Partial ordering :-

A relation R on a set S is called partial order if it is reflexive, antisymmetric and transitive.

Poset/Partially ordered set :-

A set S together with a partial ordering R is called partially ordered set or poset.

It is denoted by (S, R)

∴ S = element of Poset.

Incomparable :-

The element a and b of a poset $(S, \leq)$ are called comparable if either $a \leq b$ or $b \leq a$. When a and b are element of S such that neighter $a \leq b$ or $b \leq a$, a and b are called incomparable.

Totally ordered/Chain :-

If $(S, \leq)$ is a poset and every two element of S are comparable, S is called a totally ordered or linearly ordered set. A totally ordered set is also called a chain.

$\leq$ total order or linear order.

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Maximal Elements:-

An element of a ~~set~~ poset is called maximal if it is not less than any element of the Pos[e]

Minimal Element:-

An element of a poset is called minimal if it is not greater than any element of the poset.

greatest element:-

An element in a poset that is greater than every other element, such an element is called greatest element.

least element:-

An element is called the least element if it is less than all the other element in the poset.

Lattices:-

A partially ordered set in which every pair of elements has both a least upper bound and a greatest lower bou[nd] is called a lattice.

Theoram

Let R be a relation on set A. There is path length n, n postive integer from a to b if and only if $(a,b) \in R^n$

Proof:-

Basic step By use mathematical induction, there is a path a to b length if and only if $(a,b) \in R$, true for $n=1$
~~Assume that theoram is true for $n=1$~~
This is inductive hypothesis,
Path of length $n+1$ from a to b if and only if there are element $c \in A$ such that length from a to c.
So,
$(a,c) \in R$
and path of length n from c to b
that is,
$(c,b) \in R^n$
Conversely,
by inductive hypothesis
Path of length $n+1$ from a to b if and only if an element c with
$(a,c) \in R$ and $(c,b) \in R^n$

But such an element if and only if $(a,b) \in R^{n+1}$

Henced proved.

2) **The relation R on set A is transitive if and only if $R^n \subseteq R$ for $n = 1, 2, 3, \ldots\ldots$?**

Proof:-

Suppose that $R^n \subseteq R$ for $n = 1, 2, 3, \ldots$

In particular,

$R^2 \subseteq R$

We have to prove that R is transitive

Let $(a,b), (b,c) \in R$ Then

$(a,c) \in R^2$

But $R^2 \subseteq R$ so that

$(a,c) \in R$

This prove that R is transitive

Conversery,

Suppose that R is transitive

We have to prove that

$R^n \subseteq R$ for $n = 1, 2, 3, \ldots$

Assume that $R^n \subseteq R$ for some integer n, $n = 1$ result true

We have to prove that

$R^{n+1} \subseteq R$.

Let $(a,b) \in R^{n+1} = R^n \circ R$ there exist $x \in A$ such that

$(a,x) \in R$ and $(x,b) \in R^n \subseteq R$

Since R is transitive

$(a,x), (x,b) \in R$

$\Rightarrow (a,b) \in R$

Hence

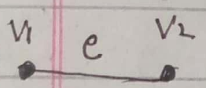
$R^{n+1} \subseteq R$.

Ch#10 Graphs

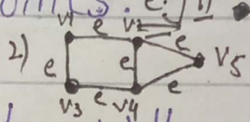
10-1 Graphs and Graph Models

Graph :-

A graph $G=(V,E)$ consists of V, a nonempty set of vertics (nodes) and E a set of edges. Each edge has either one of two vertics associated with it called its endpoints. An edge is said to connect its endpoints. e.g 1)

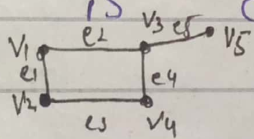


2)



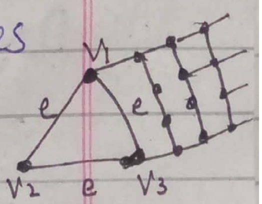
Finite graph :-

A graph with a finite vertex set and a finite edge set is called finite graph. e.g



Infinite graph :-

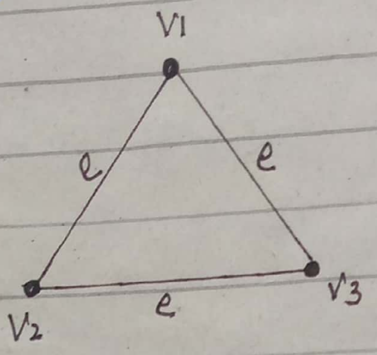
A graph with an infinite vertex set or an infinite number of edges is called infinite group. e.g



Simple graph :-

A graph in which each edge connect two different vertices and where no two edges connect the same pair of vertices is called simple graph.

Example :-



(10.2) Graph Terminology

(Example 1, 3, 4)

Adjacent :-

Two vertices **U** and **V** in an undirected graph **G** are called adjacent in **G** if **U** and **V** are endpoints of an edge **e** of **G**. Such an edge **e** is called incident with the vertics **U** and **V** and **e** said to connect **U** and **V**.

imp ✓ **Degree of a vertex :-**

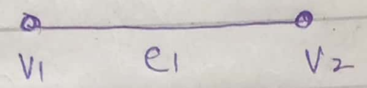
The degree of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degre

of that vertex.
The degree of the vertex V is denoted by $deg(V)$.

Pendex Vertex

A vertex with degree (edge) 1 is called pendex vertex.

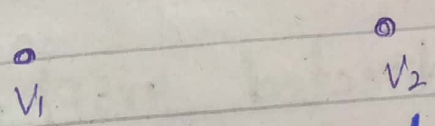
e.g



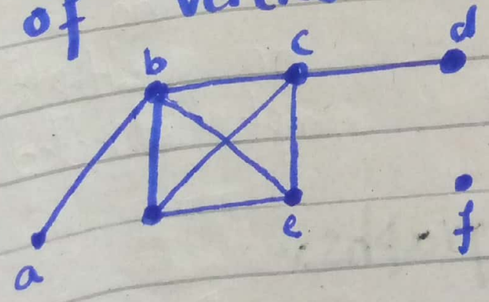
Isolated Vertex:-

A vertex with zero degree is called isolated vertex.

e.g



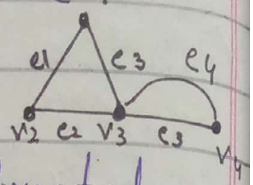
ple What degree and neighborhoods of vertics in graph.



$\left[\begin{array}{l} V = \text{vertex} \\ e = \text{edge} \end{array}\right]$

Multigraphs:-

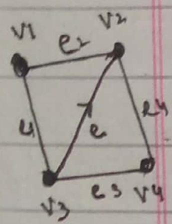
Graphs that may have multiple edge connecting the same vertices are called multiple graph. e.g



Directed graph:-

A directed graph (V, E) consist of a non-empty set of vertices V and a set of directed edge E. Each directed edge is associated with an ordered pair of vertices. The directed edge associated with the ordered pair (U, V) is said to start at U and end at V.

e.g



Types

Simple directed graph:-

When a directed graph has no loops and has no multiple directed edge is called simple directed graph.

Directed multigraphs:-

Directed graphs that may have multiple directed edge from a vertex to a second vertex are used to model such network called D.M

The Handshaking theoram:-

Let $G = (V, E)$ be an undirected graph with m edge. then.

$$2m = \sum_{v \in V} \deg(v)$$

-(Example)-

How many edge there in a graph with 10 vertices each of degree six?

Sum of degree of vertics $6 \cdot 10 = 60$

$$2m = 60$$

$\boxed{m = 30}$, m is the number of edge.

In-degree vertex:-

~~In a graph with directed edges~~ the in-degree of a vertiex v denoted by $\deg^-(v)$, is the number of edges with v as their terminal vertex.

out-degree Vertex:-

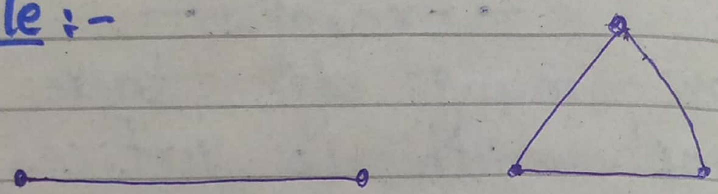
The out-degree of v, denoted by $\deg^+(v)$, is the ... of edges with v as their initial vertex.

Scanned with CamScanner

Complete Graph :-

A complete graph on **n** vertices denoted by **Kn**, is a simple graph that contains exactly one edge b/w each pair of distinct vertices.

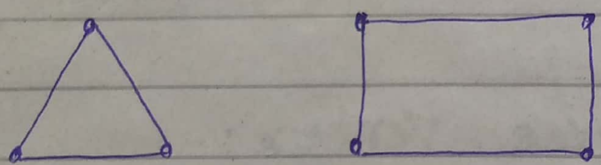
Example :-



Cycle :-

A closed path is called cycle.

Example :-



Bipartile graph :-

A Simple graph G is called bipartile if its vertex set V can be partitioned into two distinct set V_1 and V_2 such that every edge in Graph Connect a vertex V_1 and a vertex in V_2. When this condition hold, we call the pair (V_1, V_2) a bipartition of a vertex V of G.

Complete bipartile graph :-

A complete bipartile graph denoted by $K_{m,n}$ is a simple graph whose vertex set can be partitioned into two mutually disjoint non-empty subset A and B containing m and n vertices, such that each vertex set A is connected to every vertex in set B, but the vertices within the set not connected.

HALL'S MARRIAGE Theoram :-

The bipartile graph $G=(V, E)$ with bipartition (V_1, V_2) has a complete matching from V_1 to V_2 if and only if $|N(A)| \geq |A|$ for all subsets of A of V_1.

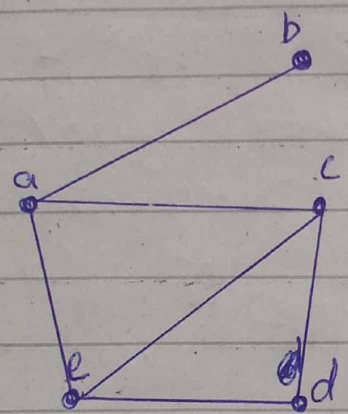
Example 1, 2, 3, 4, 5, 6, 7, 8, 9, 1[illegible]

10.3) Representing graph.

Exam#1

Vertex	adjacent vertex
a	b, c, e
b	a
c	a, d, e
d	c, e
e	a, c, d

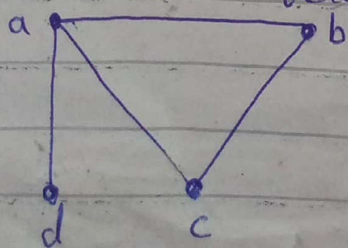
Sol



Incidence Matrices:-

In mathema[tics] an incidence matrix is a logical matrix that shows the relationships b/w two classes of object usually called an incidence relation.

e.g



Discrete Mathematics Past Paper 20

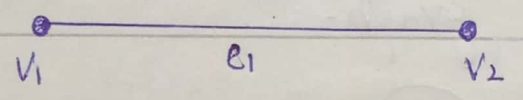
(Short Q.)

1) What pendex vertex graph and example?

A vertex with degree 1 is called pendex vertex.

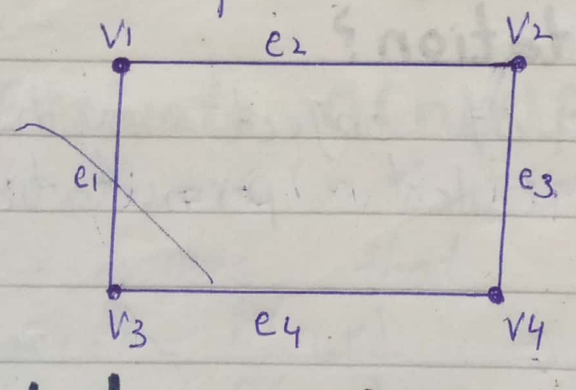
Example:-

Vertex 'V_1' and 'V_2' have a connected edge V_1V_2. So with respect to V_1 only one edge toward V_2. so degree 1.



2) Give example of finite group?

This graph consists of finite number of vertices and edges.

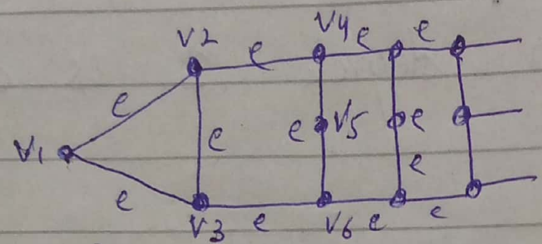


3) What is Pigeon hole principal?

If K is a postive integer and K+1 or more object are placed into K boxes, then there is at least one box containing two or more of the object.

4) Give example of infinite grou

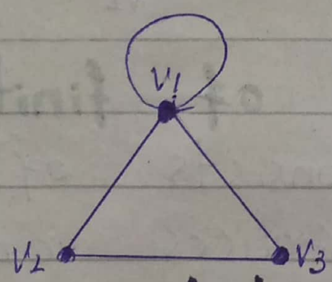
A graph with an infinite vert set and infinite number of edg



5) What is pseudograph and exam

A graph having no paraller edges but having self loop in it is called pseudo graph.

Example:-



6) Define permutation?

The arrangment of object with some order is called permutation.

$$P(n,r) = \frac{n!}{(n-r)!}$$

7) Define Combination?

The ~~ar~~ selection of object without any arrangment is called Combinati

group
vertex
edge

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

1) Evaluate C(100, 50) ?

Sol $n = 100, \quad r = 3$

we know that

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

$$C(100,50) = \frac{100!}{50!(100-50)!} \Rightarrow \frac{100!}{50!\,50!} \Rightarrow 10089134$$

Examp

) Evaluate P(10, 3) ?

Sol $n = 10, \quad r = 3$

we know that

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(10,3) = \frac{10!}{(10-3)!} \Rightarrow \frac{10\times9\times8\times\cancel{7!}}{\cancel{7!}} = \underline{\underline{720}}$$

10) Evaluate C(n, r) P(r, r) ?

$$P(n,r) = C(n,r) \cdot P(r,r)$$

This implies that

$$C(n,r) = \frac{P(n,r)}{P(r,r)} \Rightarrow \frac{n!/(n-r)!}{r!/(r-r)!}$$

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

11) Write inclusion- ~~and~~ exclusion Principal?

For any finite set A and B.
We have

$|A \cup B| = |A| + |B| - |A \cap B|$

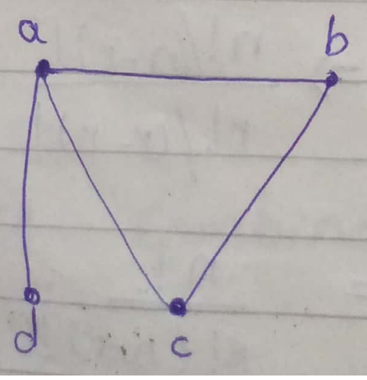
12) Explain degree of graph?

The degree of a ~~matrix~~ vertex in an undirected graph is the number of edge incident with it, except that a loop at a vertex contribute twice to the degree.

13) Incidence matrix?

In mathematic an incidence matrix is a logical matrix that shows the relationsh b/w two classes of object, usually called an incidence relatio

e.g



Give example of local area network?

3. a) The network in our Computar lab, home wifi network and small bussiness network are common example of lan.

What is ring topology?

In this topology, each computer is connected to the next computar with the last one connected to the first. Thus, a ring of computar is formed.

What is mesh network?

In a mesh topology, every device in the network is physically connected to every other devices in the network. A message can be sent on different possible paths from source to destination. Mesh provide improved performace and relability.

State Sum Rule?

The sum rule states that if an event A can occure in m ways and event B can occure in n disjoint ways, then the event A or B can occure in m+n ways.

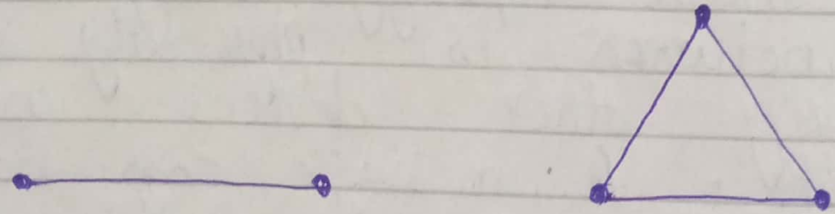
11 How many different - three letters initial can people hav

There are 26 letter

1: Define complete graph and giv example?

A complete graph on n vertices denoted by K_n, is a simple graph that contains exactly one edge b/w each pair of distinct vertices.

e.g



Draw graph if deg(a) = 2, deg(b) = deg(c) = 1 and deg(d) = 1 ?