Q- What is Discrete Mathematics?

Discrete Mathematics concerns processes that consists of a sequence of individual steps.
This distinguishes it from calculus, which studies continously changing processes. While the ideas of calculus were fundamental to the science & technology of the industrial revolution, the ideas of discrete mathematics underline the science & technology specific to the computer age.

Q- What is Logic?

It is the study of the principles & methods that distinguishes b/w a valid & an invalid argument.

1- logic / statement.
2- sets & functions
3- Algorithms & ~~Cate~~ complexity No:s. theory

4- Advance counting
5- Relations
6- graph
7. tree

* A statement is also reffered to as a **proposition**. so, when we use say a statement or proposition this must be clear to you that these are the same.

Discrete Structure / Math

→ Sequence (series / step by step) of individual steps (sets of rules)

→ based on principles

Example:

2 + 2 = 4 answer

value | value

operator

★ **Every sentence is not a statement.**

Logic

→ valid / invalid = difference

→ sequence of individual series / steps

CHAPTER: 1 . Statement / Proposition

1– wri

Statement:

1- In written form. 2- In comments form 3- No

3- No formula can be written (+, -, *, /, ≤, ≥)

CHAPTER: 1 Statement / Proposition

~~1- wri~~

Statement:

1- In written form. 2- In comments form ~~3- No~~

3- No formula can be written (+, -, *, /, ≤, ≥)

4- when statement is started it should be terminate.

A statement is a declarative sentence that is either true or false but not both.

S.Q ⇒ when statement should be valid or invalid

→ valid = Truth value (T)

→ invalid → Truth value (F)

→ ~~Statement wrong~~. ~~Sten~~ Sentence wrong = statement wrong

---

Example of Statement:

Q → given is statement or not ?

↳ i- There should be no question in it & no ? is there.

ii- There should be no command

4 + 2 = 6 — T
statement & logic is true

4 + 2 = 7 — F
statement is true, logic is false

what is your name? variable X Commands ✓ question ✓ } not a statement

Your name is Asad } statement
name / logic

Q→ What are basic logic operators?

Opreators used in discrete structure

i- → NOT → Negation $\sim$

ii- • → Conjunction → AND $\wedge$

iii- • → Dis-junction → OR $\vee$

A statement within a statement is called compound statement

→ (What is your name) & (4+2=6)

X

Propositions: is a declarative sentence (that is, a sentence that declares a fact) that is either true or fasle, but not both.

All the following declarative sentences are propositions;

i- Washington, D.C., is the capital of USA.

ii- Toronto is the capital of Canada.

iii- 1+1 = 2

iv- 2+2 = 3

Propositions 1 & 3 are true, whereas 2 & 4 are false.

→ The area of logic that deals with propositions is called the propositional calculus or propositional logic.

→ Many mathematical statements are constructed by combining one or more propositions. New propositions, called compound propositions, are formed from existing propositions using logical operators.

X	Y	$X \land Y$	$X \lor Y$	$\widetilde{X \land Y}$	$\widetilde{X \lor Y}$
T	T	T	T	F	F
T	F	F	T	T	F
F	T	F	T	T	F
F	F	F	F	T	T

Q- Define truth value?

True & False are called the truth values.

1- Close the door.

Q- Define truth value?

True & False are called the truth values.

1- Close the door.

False

2- $x+1=2$

False

3- He is very Rich person True

Compound Statement:

more than one

at least 2 sentence.

→ Simple statements could be used to build a compound ~~what are~~ statement.

Examples:

- $3+2=5$ $\boxed{\&}$ Lahore is a city in Pakistan

combining 2 statements by using &

- The grass is green or It is hot today.
- Discrete Mathematics is not difficult to me.

AND, OR, NOT are called LOGICAL CONNECTIVES.

SYMBOLIC REPRESENTATION

Statements are Symbolically represented by letters such as p, q, r

Examples:

p = Islamabad is the capitall of Pakistan

q = 17 is divisible by 3.

Logical Connectives

CONNECTIVE	MEANINGS	SYMBOL	CALLED
Negation	not	$\sim$	Tiled
Conjunction	and	$\wedge$	Hat
Disjunction	or	$\vee$	vel
Conditional	if... then...	$\rightarrow$	Arrow
Biconditional	if & only if	$\leftrightarrow$	Double arrow

Examples:

$p \vee q$ = Islamabad is the capital of Pakistan OR 17 is divisible by 3.

$p \wedge q$ = Islamabad is the capital of Pakistan AND 17 is divisible by 3

$\sim p$ = It is not the case that Islamabad is the capital of Pakistan

→ Islambad is not the capital of Pakistan.

What is conjunction? $\wedge$

Any 2 proposition can be combined by the word "and" to form a compound preposition called conjunction.

Truth Table:

$p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

∵ both values should be true.

What is disjunction? $\vee$

Any 2 propositions can be combined by the word "or" to form a compound proposition called disjunction

Truth Table:

$P \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

$\therefore$ both value ~~should~~ be false than F

Define Negation?

A single proposition ~~which~~ whose value can be changed by using negation

Truth Table:

P	$\sim p$
T	F
F	T

$\widetilde{p \vee q}$	$\widetilde{p \wedge q}$
F	F
F	T
F	T
T	T

What is conditional statement proposition? $\rightarrow$ IF

A statement that can be written in the form If P then Q where p & q are true sentences. P is called hypothesis Q is called what is conditional statement proposition? → IF

A statement that can be written in the form If P then Q where p & q are true sentences. P is called **hypothesis** & Q is called **conclusion**.

→ if "P then Q means" Q must be true whenever P is true. → we will use if statement

Example

P	q		$p \to q$
T	T		T
T	F		F
F	T		T
F	F		T

P q = True T

P False = T

P implies q

$p \to q$

The conditional statement is also called implication

Q - what is bi-conditional statement? ↔ IF & only IF

A bi-conditional statement is defined to be true whenever both parts have the same truth value. It is denoted by ↔

p	q	p↔q
T	T	T
F	F	F
F	T	F
F	F	T

$X = T$
$Y = T$ } → same = True

$X = F$
$Y = F$ → same = True

De-morgan's Law

$\sim(p \wedge q) = \sim p \vee \sim q$

same

p	q	$(p \wedge q)$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$(\sim p \vee \sim q)$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Contradiction = False

Tautology = T → always True

Define tautology?

A compound statement that is always true no matter true no matter what the value of a propositional variables is called tautology.

P	~P	Pv~P
T	F	T
F	T	T

~~contradiction~~

Tautology

$\rightarrow F$

Define contradiction?

A compound statement that is always False is called contraction

p	-p	p∧-p
T	F	F
F	T	F

} contradiction

p ~p

$(p \wedge q) \quad (p \vee q)$

$\sim[((p \wedge q) \wedge r)] = \sim p \vee \sim q \vee \sim r$

p	q	r	p∨q	(p∨q)∨r	p∧q	(p∧q)∧r
T	T	T	T	T	T	T
T	T	F	T	T	T	F
T	F	T	T	T	F	F
T	F	F	T	T	F	F
F	T	T	T	T	F	F
F	T	F	T	T	F	F
F	F	T	T	T	F	F
F	F	F	F	F	F	F

~p	~q	~r	~p∨~q	(~p∨~q)∨~r	~((p∧q)∧r)
F	F	F	F	F	F
F	F	T	F	T	T
F	T	F	T	T	T
F	T	T	T	T	T
T	F	F	T	T	T
T	F	T	T	T	T
T	T	F	T	T	T
T	T	T	T	T	T

Double negation laws

$\tilde{p}(\tilde{p}) = p$

p	q	$\tilde{p}$	$\tilde{q}$
T	T	F	F
T	F	F	T
F	T	T	F
F	F	T	T

p	$\tilde{p}$	$\tilde{p}(\tilde{p})$	$\tilde{p}(p)$	$p \vee p$	$p \vee \tilde{p}$
T	F	F	F	T	T
T	F	F	F	T	T
F	T	T	F	F	T
F	T	T	F	F	T

Domination Laws

$p \wedge F = F$

$p \vee T = T$

Identity Laws

$p \wedge T = p$

$\tilde{p} \vee F = p$

Associative Laws

$A + B = B + A$

$A + (B + C) = (A + B) + C$

p q r

Ass of the Law

$P \vee (q \vee r) = (P \vee q) \vee r$ equal

P	q	r	$q \vee r$	$P \vee (q \vee r)$	$P \vee q$	$(P \vee q) \vee r$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	T	F	T
F	F	F	F	F	F	F

$P \wedge (q \wedge r) = (P \wedge q) \wedge r$

P	q	r	$q \wedge r$	$P \wedge (q \wedge r)$	$P \wedge q$	$(P \wedge q) \wedge r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	F	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

2.3.5.Q.

→ **Predicate:** most powerful method/way to solve the statement Mathematically

or

$\boxed{\text{Propositional function } P(F)}$

Mathematically:

P(A) ⇒ P = predicates

P(B)

The statemement involving variables such as

$x > 3$

in this type of statements

'x' is subject

3 is predicate.

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Let P(x) denotes the statement $x>3$
P(x) is also said to be propositional function

Q. What are the truth values for P(4) & P(2)?

$P(4) = 4>3$ (True)

$P(2) = 2>3$ False

→ A predicate is a sentance that contains a finite no: of variables & become a statement ~~with~~ when specific values are substituted for the variables.

Example:

$x>3$, $x=y+3$, $x+y=z$

→ Predicate:

- Large/wide statement
- most powerfull method to express wide range of statements in mathematical form
- identify P → predicate
- also called propositional function
- Relational Propositon function.

→ Answer will be in mathematically form

Relational Operators

$>, <, ==, !=, \geq, \leq$

$x>6$

P(x)

P(8), P(2), P(12), P(3)

① P(8)

$8>6 = T$

② P(2)

$2>6 = F$

which of the Sentence are Proposition

A) $2+3=5$

No, this is not a proposition, Logic is true, but

B) $x+2=11$

c) $5+7=10$

d) Answer the question

e) Do not Pass go.

f) What time is it?

QUANTIFIER:

- Identify by capital Q
- it is used to summarize the statement OR limit the variable of a proposition.
- Two Types
  ↳ Universal: all cases true. → apply on all cases.
  → Universal mean for all values
  → identify by $\forall$

1- English / statement
2- $P(x)$ — $P(1,2)$ → another variable
3- True / False

Just for Real nos

Rules:

→ Universal quantification for all values of X / True
→ $P(x) = P(a) \& P(b)$ ⇒ we will not get confused if variable changes in the question.

Truth table
Negation
Conjunction/dis-junction
Conditional /Bi-conditional

3- P(x) = sentence.
If the value of x is replace with sentence

Existential quantifier:
Kisi 1 side pr value exist krti ho
1st side must be true.

Rules:
→ At least 1 value will True
→ P(x) = P(a) P(b) ⇒ if variable changes not be confused
→ P(x) = sentence
If the value of x is replace with sentence.

PROOFS:-

Proof/ methods It is a sequence of logical deductions based on accepted assumptions & previously proven statements

→ arguments / valid arguments or invalid

→ Types: / METHODS OF PROOFS:

1- direct proof; 2- indirect proof

It is a conditional statement $p \to q$ is constructed when the 1st

1- Direct proof:- step of assumption that is 'p' is true subsequent conditional $p \to q$ steps are constructed showing Bi-conditional that 'q' must also be true

2. Indirect proff:- An extremely useful type of having negation indirect proof is known as proof by contraposition.

$$\left.\begin{array}{l} p \to q \\ \tilde{q} \to \tilde{p} \end{array}\right\} \text{same answer}$$

proof by contraposition

Rules of Inference:

$p \to q$

2. Indirect proff:- An extremely useful type of having negation indirect proof is known as proof by contraposition.

$$\left.\begin{array}{l} p \to q \\ \sim q \to \sim p \end{array}\right\} \text{same answer}$$

proof by contraposition

Rules of Inference :

$p \to q$

q depends on p

~~METHODS OF PROOF~~:-

Q→ what is meant by Formal proof?

A finite sequence of sentences, each of which is an axiom, an assumptions, or follows from the preceding sentences in the sequence by a rote of inference

e.g $(a+b)^2 = a^2 + b^2 + 2ab$

Q→ Describe informal proof?

A proof where more than one rule of inference ~~may be~~ used in each step, where steps may be skipped, where the axioms being assumed & rule of inference used are not explicity stated.

Chapter: 2 Set & Functions

Set: A set is an unorderd collection of elements objects/elements

→ A collection of unorder no: / values
{ }

1- { 1, 2, 3, 4 } - Natural
2- { 0, 1, 2, 3, 4, ..9 } All
3- { 1, 2, 3, 4

Real - number (0-9) +

Union U ⇒ collection of all same ni ayeingy.... remaining ayeingy.

~~A = { 0, 1, 2, 3, 4 }~~

B = { 1, 2, 3 }

A U B = { 0, 1, 2, 3, 4 }

∩ intersection ⇒ same ⇒ 2 sets lazmi hony chayein

A·B, A×B, A×B cartesian product ⇒ All possible sample

$A = \{ a_1, a_2, a_3, a_4 \}$ / $B = \{ B_1, B_2, B_3 \}$

$\{a_1, B_1\}, \{a_1, B_2\}, \{a_1, B_3\}, \{a_2, B_1\}, \{a_2, B_2\}, \{a_2, B_3\},$
$\{a_3, B_1\}, \{a_3, B_2\}, \{a_3, B_3\}, \{a_4, B_1\}, \{a_4, B_2\}, \{a_4, B_3\},$ ~~{a4, B1}~~

Power of set ϕ - indication

All possible subset of this set

$S = \{0, 1, 2, 3\} = 4^4 = 16$ → sets

Power rule sample doesn't repeat

Subset all values availabe in another set. $\subseteq$

FUNCTION:

A → If element same then it is a function
B → element / Domain

Range/co-domain → wo set jis pr implementation kini ho. ha

A → B

A = domain B = co-domain

Conditions:

1- elements should be equal

2- A > B — no to no issue

B < A → mapping is not possible

A k hr element ko B pr apply krna hai → or phir skip

repeat ~~na~~ na hoka koi element

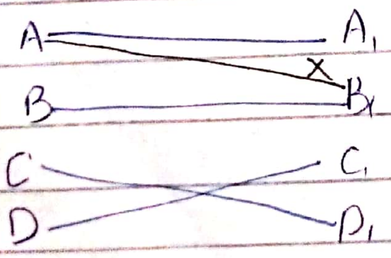
if B = 4 elements

A = 5 elements

A — B

One to one - function should be increase continously
or
decrease "

Domain co-domain



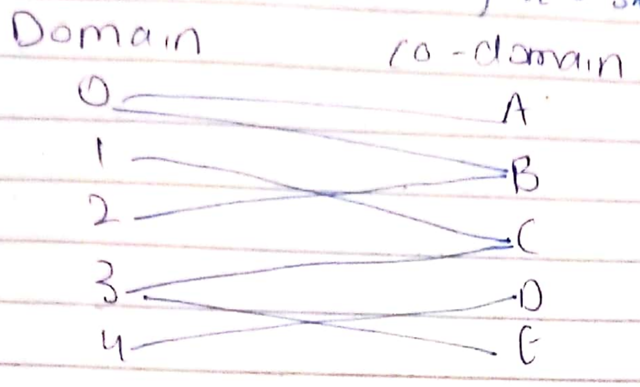
can't take 2 from any at a point

co-domain domain

can take 2 from any at a point.

On-to function

- for all
- all elements should be done, no one will be empty
- co-domain masy ke koi skip na hua ho



Floor function $= \lfloor \ \rfloor$

Celing function $= \lceil \ \rceil$

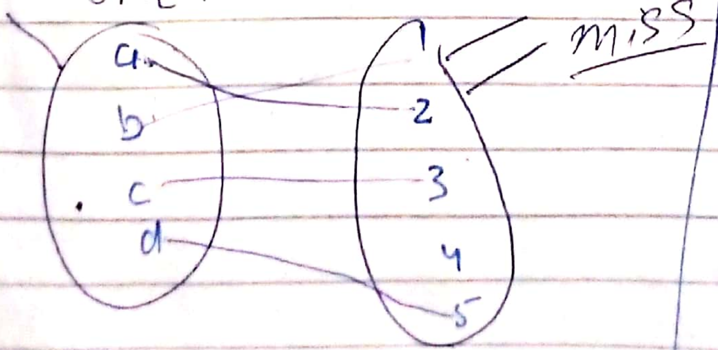
Its compoulsary k fractions waly no:s ayeingy.

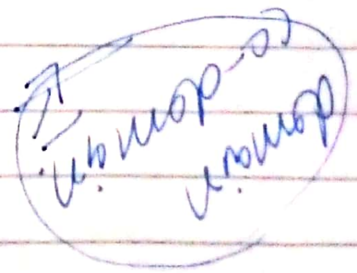
$\lceil \frac{1}{2} \rceil + 3$, $\lfloor \frac{2}{4} \rfloor$ to, ...etc

Inverse of function

- Pehly domain ho, phir co-domain, elements equal hon.
- 1 to one ho / onto function ho.

One to one - function





book chap: 8

Chap: 7

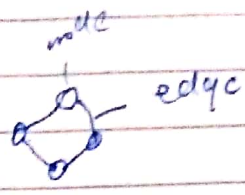
Graph Theory

1817 chap

Tree
3 - long

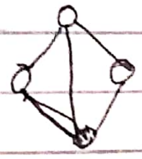
2 - long

80 - 85 marks



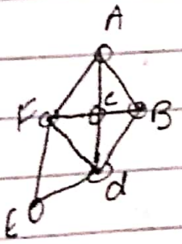
Graph:- It is a collection of verteces & edges.
'G

Vertex ⇒ node/path — V
Edge ⇒ line — E



G(v) = { a, b, c, d }
G(E) = { ab, ac, ad, ba, bc, ca, cb, cd, da, dc }

G(V) = { 4 }
G(E) = { 10 }



G(v) = { a, b, c, d, e, f)

G(E) = { ab, ae, af, ba, bc, bd, ca, cb, cd, cf, db, dc, df, de,
ed, ef, fa, fc, fd, fe }.

G(v) = { 6 }
G(E) = { 20 }

Types of Graphs:

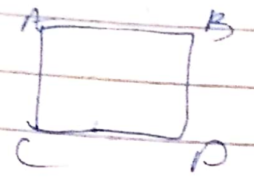
1. Simple graph

without direction/undirected

vertex

edge

The graph that donot include loop & each edge connectes two diff.. pair of vertices



2. Multigraph / Directed graph

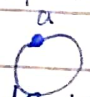
In which loop exist & tell direction

Special Type of graph.

3. Cylic Graph

→ A walk is closed if end points aresame. Such graph is called cyclic graph

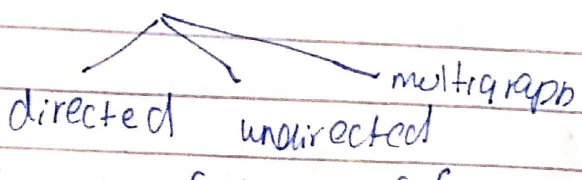
e.g



→ A graph include loop is called graph.

→ Directed OR endirected graph.

4- Webgraph / site / web page / link



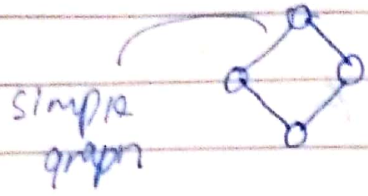
G = no. of V + no. of E

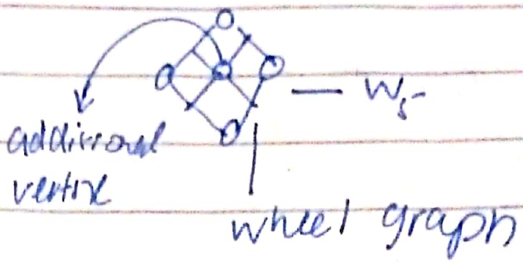
~~Cycle graph~~:-

5 - Wheel graph (W_n) no. of vertices

→ when we add an additional vertex in cycle graph is caled wheel graph.

→ Additional vertix are connecting with all vertices.



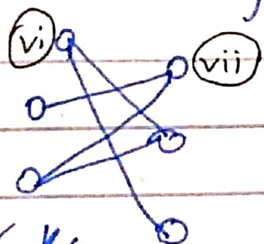


3- Complete graph
→ Denoted K_n.
→ One edge b/w each pair of vertices.
→ Directed or undirected graph.


K_2

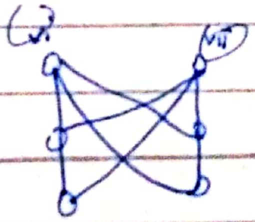
→ N mean no: of vertices.

→ 4- Bi- Partite Graph:-
→ Denoted by K_n
→ vertices are divided into 2 disjoint subset.
→ subset are connecting with each other

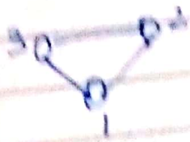


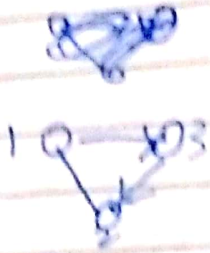
K_6

→ 5- Complete Bi- Partite graph
→ Denoted By K_n
→ Vertices are divided into 2 disjoint subset.
→ Vertices are not connected within a set.



6- Matrices in Graph





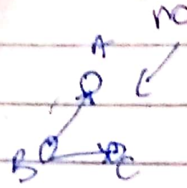
$$= \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

7- Graph Isomorphism

→ Isomorphism mean 2 graph in same in the term of no: of vertices & no: of edges.

→ Directed or undirected graph.

→ most common problem in graph theory



no: of vertices			List/subset
~~ϕ~~	1	A	0
1	2	B	2
~~2~~	3	C	0

8- Connectivity/path/edges

loop

→ with respect of vertix

→ with respect of edges.

Path:-

A path is sequence b/w 2 vertices.

Scanned with CamScanner

Hamilton Path/circuits→ledges

→ A simple path that passes through every vertices exactly one is called Hamilton Path.

→ A simple circuit/edge that passes through every vertices exactly one is called Hamilton circuits.

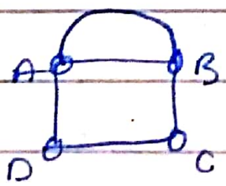
Weighted Graph

→ A graph may include cost or value /weight is called weighted graph

→ sometimes apply weight on directed or undirected.

Planner graph

→ A graph with no edges crossing is called planner graph.



A—B

A—B

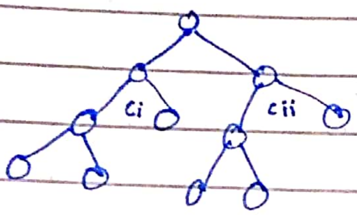
A—D—C—B

Chapter: 9 Tree

→ Some special type of graph.
→ A simple graph with no multiple edges, no loop

Types of Tree

Binary Tree



Tree ~~Traver~~ Traversal Pg# 650

1- Pre-order: Pg#651

visit Root (Root, left, Right)
subtree ~~(L—Root—R)~~
left to Right

2- In-order Pg#653

left most sub tree (L-R-Root)
visit Root
visit other subtree, left to right

3- Post-order Pg# 654

visit subtree left to right (Left-Right-Root)
visit root ~~Root—L—R)~~

Binary Search Tree (BST)

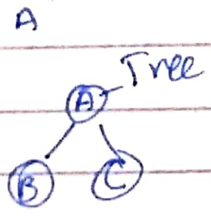
Root visit
i) Left side → --
ii) Right side → ++
iii) No duplication
iv) arranged value

item larger → right
item smaller → left

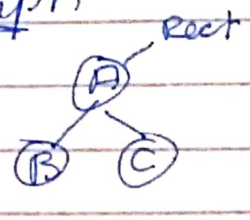
Spaining Tree

A simple Graph **G** in a subgraph of Tree, that containing all the vertices of graph G in a Tree

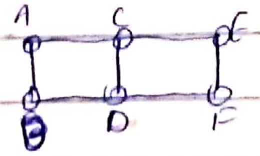
<u>graph</u>



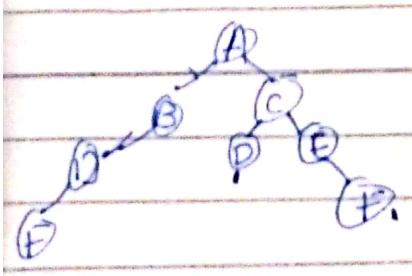
<u>Sub graph</u>

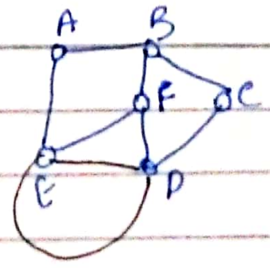


Example



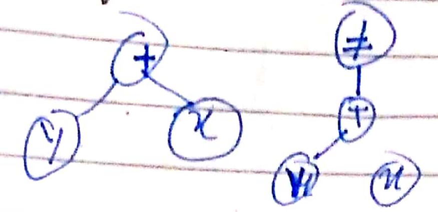
6 = V
7 = E





pg#no: 657, 658, 659

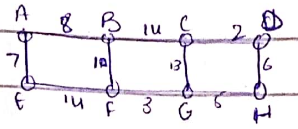
$(y+x) \uparrow 2) + (x+y)$



• weight edge k uper lgta ha.

Minimum Spanning Tree

- Connected graph
- weighted graph ⇒ maximum weight.
- all possible smallest weighted edges will be sumed up.



Value	No of edges	weight
1	AB	8
2	BC	14
3	CD	2
4	AE	7
5	BF	10
6	CG	13
7	DH	6
8	EF	14
9	FG	3
10	GH	5

Sum $\boxed{87}$

book

INSERTION SORT

Pg # 189, 190

Imp topic in graph

- → isoporphism
- → Degree In-out, degree
- → graph coloring problem

Imp topic in trees

- → Tree Traversal
- → Binary Search Tree
- → Searching / Insertion / deletion