-: STATISTICS :-

Past Paper 2020

-: QUESTION 1 :-

(i)

Discuss mathematical expectation and its properties?

Ans The expected value of discrete random variable of "X" is equal to sum of the product of each value of "X" and the corresponding value "P(X)". That is :

$$E(x) = \sum X \cdot P(X)$$

Properties:

1) $E(c) = c$
2) $E(ax) = a\,E(x)$
3) $E(x+a) = E(x) + a$
4) $E(ax+b) = a\,E(x) + b$
5) $E(x+y) = E(x) + E(y)$
6) $E(x-y) = E(x) - E(y)$
7) $E(xy) = E(x) \cdot E(y)$
8) $E(x - E(x)) = 0$

(ii)

Discuss moment generating function?

Ans The moment generating function (m.g.f) usually denoted by $M_0(t)$ of a random variable "X" about the origin if it exists; is defined as an expected value of the random variable e^{tx}, where "t" is a real variable lying in a neighbourhood of zero.

That is:

$$M_0(t) = E(e^{tx})$$

$$= \sum e^{tx} f(x_i) \text{, if 'x' is discrete}$$

$$\text{and } M_0(t) = \int_{-\infty}^{+\infty} e^{tx} f(x)\, dx \text{, if 'x' is continuous.}$$

(iii)

Differentiate between discrete and continous random variable?

Discrete R.V	Continuous R.V.
A random variable "X" is defined to be discrete if it can assume values which are finit or countably infinite	A random variable "X" is defined to be continuous if it can assume every possible value in an interval $[a,b]$ $a<b$, where 'a' & 'b' may be $-\infty$ & ∞ respectively.

(iv)

What is role of third moment in skewness?

Not In Syllabus.

(v)

What do you meant by sampling without replacement?

Ans Sampling without replacement is performed when an object is not replaced in the population after it has been selected.

(vi)

Define Variance and its properties?

Ans The variance of a set of observation is defined as a mean of squares of deviation of all the observation from their mean.

Properties:

1) $Var(c) = 0$
2) $Var(X+a) = Var(X)$
3) $Var(aX) = a^2 Var(X)$
4) $Var(X+Y) = Var(X) + Var(Y)$
5) $Var(X-Y) = Var(X) + Var(Y)$

(vii)

Differentiate between population

and Sample?

Population	Sample
A population is a set of data that characterizes some phenomenon.	A sample is a set of data selected from a population.

(viii)

Explain type one error and type two error?

Type 1 error	Type 2 error
If we reject a true null hypothesis, the error is called a type-I error.	If we accept a false null hypothesis, the error is called a type-II error.

(ix)

Define Control Chart and make

its visual presetation?

Control Chart:

The control chart is one of the most useful tool for study variation, it is the graphic representation of the distribution of variation of statistic.

Visual Presentation:

Not In Syllabus.

(x)

Differentiate between the Null and Alternative Hypothesis?

Null Hypothesis	Alternative Hypothesis
The Null Hypothesis is a statement about the value of a population paraments.	The Alternative hypothesis is hypothesis for which the researcher wants to gather supporting evidence
⇒ Denoted by 'H_0'.	⇒ Denoted by 'H_1'.

(xi)

Explain Chi-Square distribution?

Ans Let $z_1, z_2, ---, z_n$ be normally and independently distributed variable with zero mean (0) and unit variance (1). Then a random variable express by the quantity $\chi^2 = \sum_{i=1}^{n} z_i^2$ is defined as a chi-square random variable with 'n' degree of freedom that is chi-square random variable is defined as a sum of squares of 'n' independent standard normal variable.

$$P.d.f = f(\chi^2) = \frac{1}{2^{n/2}\Gamma n/2} \chi^{2(\frac{n}{2}-1)} e^{-\chi^2/2}$$

$$, \quad 0 < \chi^2 < \infty$$

(xii)

Define Sampling distribution?

Ans A probability distribution consisting of all possible value of sample statistic is called sampling distribution.

(xiii)

Differentiate between acceptance and rejection region?

Acceptance Region	Rejection Rajion
The set of value for the test statistic that lead to accept H_0 called the acceptance region.	The rejection region is set of possible computed values of test statistic for which null hypothesis will be rejected.

(xiv)

For a certain set of data $E(X^2) = 100$ and $Var(X) = 50$, Then $E(X) = ?$

Solution.

$\because \quad Var(X) = E(X^2) - [E(X)]^2$

$50 = 100 - [E(X)]^2$

$[E(X)]^2 = 100 - 50$

$[E(X)]^2 = 50$

Taking square root both sides.

$E(X) = \sqrt{50}$

$$\boxed{E(X) \approx 7.071}$$

(XV)

Define estimation and its types?

Ans The process by which we get information about unknown value of population parameters by using sample values.

Types:

1) Point Estimation.
2) Interval Estimation.

(xvi)

Explain Poisson distribution?

Ans The limiting form of binomial distribution, when 'p', the probability of success is very small but 'n', the number of trials is so large. This is called poisson distribution.

P.d.f: $\lim\limits_{\substack{n \to \infty \\ p \to 0}} b(x; n, p) = \frac{\mu^x e^{-\mu}}{x!}$

-: QUESTION 2 :-

Find χ^2 dist/test whether two attributes are independent.

Attributes	A_1	A_2	A_3	Total
B_1	215	325	60	600
B_2	135	175	90	400
Total	350	500	150	1000

Solution:

★ Null Hypothesis

⇒ Two attributes are independent.

★ Alternative Hypothesis:

⇒ Two attributes are not independent

★ Level of Significance

⇒ $\alpha = 0.05$

★ Test Statistics:

$$\chi^2 = \Sigma \frac{(f_o - f_e)^2}{f_e} \quad \text{with } d.f = (r-1)(c-1)$$

Now,

The table is given as:

f_o	f_e	$(f_o-f_e)^2$	$(f_o-f_e)^2/f_e$
215	$600 \times 350/1000 = 210$	25	5/42
135	$400 \times 350/1000 = 140$	25	6/28
325	$600 \times 500/1000 = 300$	625	25/12
175	$400 \times 500/1000 = 200$	625	25/8
60	$600 \times 150/1000 = 90$	900	10
90	$400 \times 150/1000 = 60$	900	15

$$\because \chi^2 = \Sigma \left[\frac{(f_o - f_e)^2}{f_o} \right]$$

$$\chi^2_c = \frac{5125}{168} \Rightarrow \boxed{30.506}$$

$$d.f = (r-1)(c-1) \Rightarrow (2-1)(3-1)$$

$$\boxed{d.f = 2}$$

$$\chi^2_{\alpha(\chi)} = \chi^2_{(0.05)(2)} = 5.991$$

$$\boxed{\chi^2_{\alpha(\chi)} = 5.991}$$

★ Critical Region: Reject H_o if

$\Rightarrow$ if $\chi^2_c \geq \chi^2_{\alpha(\chi)}$ otherwise accept H_o

★ Conclusion.

Since our $\chi^2_c \geq \chi^2_{\alpha(\chi)}$, so reject H_o.

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-: QUESTION 3 :-

Assume that a population consist of 7 similar containers having the following weights, 9, 8, 7, 9, 6, 10, 7.

(a) Find the mean and standard deviation of the given population.

(b) Draw sample of 2 size without replacement and calculate the mean weight of each sample.

(c) Sampling distribution of sample mean?

(d) Verify $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \sqrt{\frac{\sigma(N-n)}{n(N-1)}}$.

Sol:

$X = 9, 8, 7, 9, 6, 10, 7$

$N = 7$, $n = 2$

Total possible Sample Outcomes

$$W.O.R.S = \binom{N}{n} = \binom{7}{2}$$

$$\Rightarrow \boxed{21}$$

No	Samples	Samples Mean ($\bar{x}$)
1	9, 8	$9+8/2 = 8.5$
2	9, 7	$9+7/2 = 8$
3	9, 9	$9+9/2 = 9$
4	9, 6	$9+6/2 = 7.5$
5	9, 10	$9+10/2 = 9.5$
6	9, 7	$9+7/2 = 8$
7	8, 7	$8+7/2 = 7.5$
8	8, 9	$8+9/2 = 8.5$
9	8, 6	$8+6/2 = 7$
10	8, 10	$8+10/2 = 9$
11	8, 7	$8+7/2 = 7.5$
12	7, 9	$7+9/2 = 8$
13	7, 6	$7+6/2 = 6.5$
14	7, 10	$7+10/2 = 8.5$
15	7, 7	$7+7/2 = 7$
16	9, 6	$9+6/2 = 7.5$
17	9, 10	$9+10/2 = 9.5$
18	9, 7	$9+7/2 = 8$
19	6, 10	$6+10/2 = 8$
20	6, 7	$6+7/2 = 6.5$
21	10, 7	$10+7/2 = 8.5$

Now,

Sampling dist of Sample Mean:

$\bar{x}$	Tally	f	$f(\bar{x})$	$\bar{x}^2$	$\bar{x}^2 f(\bar{x})$
6.5	$\vert\vert$	2	2/21	42.25	84.5/21
7	$\vert\vert$	2	2/21	49	98/21
7.5	$\vert\vert\vert\vert$	4	4/21	56.25	225/21
8	$\cancel{\vert\vert\vert\vert}$	5	5/21	64	320/21
8.5	$\vert\vert\vert\vert$	4	4/21	72.25	289/21
9	$\vert\vert$	2	2/21	81	162/21
9.5	$\vert\vert$	2	2/21	90.25	180.5/21

$\because E(\bar{x}) = \mu_{\bar{x}} = \sum \bar{x} f(\bar{x})$

$\mu_{\bar{x}} = 168/21 \Rightarrow \boxed{\mu_{\bar{x}} = 8}$ — ①

Also, $\mu = \sum x/N \Rightarrow 56/7$

$\boxed{\mu = 8}$ — ②

From ① & ②

$$\boxed{\mu_{\bar{x}} = \mu}$$

Hence proved.

Now,

$$S.E(\bar{x}) = \sqrt{\Sigma(\bar{x}^2 \cdot f(x)) - \Sigma(\bar{x} \cdot f(\bar{x}))^2}$$

$$\sigma_{\bar{x}} = \sqrt{1359/21 - (8)^2}$$

$$\boxed{\sigma_{\bar{x}} = 0.845} \quad — (3)$$

And,

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$$

$$\sigma = \sqrt{\frac{460}{7} - (8)^2} = \boxed{1.309}$$

Also, $$\sigma_{\bar{x}} = \sqrt{\frac{\sigma(N-n)}{n(N-1)}}$$

$$\sigma_{\bar{x}} = \sqrt{\frac{(1.309)(7-2)}{2(7-1)}}$$

$$\boxed{\sigma_{\bar{x}} = 0.8} \quad — (4)$$

From (3) & (4)

$$\boxed{\sigma_{\bar{x}} = \sigma}$$

Hence proved.

QUESTION 4

A process is in control when the average amount of instant coffee that is packed in a jar is 6 oz. The standard deviation is 0.2 oz. A sample of 100 jars is selected at random and sample average is found to be 6.1 oz. Is the process out of control?

(b) Show that total area under the curve for Normal dist. is unity?

(a)

Not In Syllabus

(b)

Sol

$$\because \int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$\because f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\Rightarrow \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

Let $\frac{x-\mu}{\sigma} = z \Rightarrow x-\mu = \sigma z$

$dx = \sigma\, dz$

$$\Rightarrow \int_{-\infty}^{+\infty} \frac{1}{\cancel{\sigma}\sqrt{2\pi}} e^{-\frac{z^2}{2}} \cancel{\sigma}\, dz$$

$$\Rightarrow \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-\frac{z^2}{2}} dz$$

Let $y = z^2/2$

$dy = z\,dz \Rightarrow z = \sqrt{2y}$

$$\Rightarrow \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} e^{(\sqrt{2y}/2)^2} \cdot 1/z\, dy$$

$$\Rightarrow \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} \frac{e^{-y}}{\sqrt{2y}} dy$$

$$\Rightarrow \frac{2}{\sqrt{2\pi}\cdot\sqrt{2}} \int_{0}^{\infty} e^{-y} y^{-1/2} dy$$

$$\Rightarrow \frac{2}{2\sqrt{\pi}}\left(\frac{\boxed{1}}{\sqrt{2}}\right)$$

$$\Rightarrow \frac{1}{\sqrt{\pi}}(\sqrt{\pi})$$

$$\Rightarrow \boxed{1} \text{ (unity).}$$

Hence proved.

QUESTION 5

(a)

If $f(x) = 6 - |7-x|/36$, for $X = 2, 3, 4, ---, 12$. Find mean and variance of X.

Sol.

$$X = 2, 3, 4, ---, 12$$

&

$$f(x) = \frac{6 - |7-x|}{36}$$

So,

X	f(x)	x·f(x)	x^2	$x^2 \cdot f(x)$
2	1/36	2/36	4	4/36
3	2/36	6/36	9	18/36
4	3/36	12/36	16	48/36
5	4/36	20/36	25	100/36
6	5/36	30/36	36	180/36
7	6/36	42/36	49	294/36
8	5/36	40/36	64	320/36
9	4/36	36/36	81	324/36
10	3/36	30/36	100	300/36
11	2/36	22/36	121	242/36
12	1/36	12/36	144	144/36

$\because$ Mean $= E(x) = \sum x \cdot f(x)$

$$= \frac{252}{36}$$

$$\boxed{\text{Mean} = 7}$$

$$Var(x) = \sum x^2 \cdot f(x) - (\sum (x) \cdot f(x))^2$$

$$\Rightarrow \frac{1974}{36} - (7)^2 = \frac{35}{6}$$

$$\boxed{Var(x) = 5.83}$$

Also,

(b)

If X is binomially distribute with mean 3.49 and variance 1.783, find the complete binomial probability distribution.

Sol.

Mean $= np = 3.49$ — ①

Var $= npq = 1.783$ — ②

Dividing ① & ②

$$\frac{\cancel{np}q}{\cancel{np}} = \frac{1.783}{3.49} \Rightarrow \boxed{q = 0.51}$$

$p = 1 - q \Rightarrow 1 - 0.51 \Rightarrow \boxed{p = 0.49}$

Also,

$np = 3.49 \Rightarrow n(0.49) = 3.49$

$n = \dfrac{3.49}{0.49} \Rightarrow \boxed{n = 7}$

So,

$p = 0.49, \; q = 0.51, \; n = 7$

Then,

X	$P(x=x) = \binom{n}{x} p^x q^{n-x}$
0	$\binom{7}{0}(0.49)^0(0.51)^7 = 0.00897$
1	$\binom{7}{1}(0.49)^1(0.51)^6 = 0.0603$
2	$\binom{7}{2}(0.49)^2(0.51)^5 = 0.1739$
3	$\binom{7}{3}(0.49)^3(0.51)^4 = 0.2785$
4	$\binom{7}{4}(0.49)^4(0.51)^3 = 0.2676$
5	$\binom{7}{5}(0.49)^5(0.51)^2 = 0.1542$
6	$\binom{7}{6}(0.49)^6(0.51)^1 = 0.0494$
7	$\binom{7}{8}(0.49)^7(0.51)^0 = 0.00678$

QUESTION 6

(a)

The probability that a patient recovers from delicate heart operation is 0.90. What is the probability that exactly five of next seven patients having this operation survive?

Sol.

$P = 0.9 \Rightarrow q = 1 - p = 1 - 0.9$

$q = 0.1$

$n = 7$

$x = 5$

So,

$$P(x = n) = \binom{n}{x} p^x q^{n-x}$$

$$P(x = 5) = \binom{7}{5} p(0.9)^5 (0.1)^{7-5}$$

$$\boxed{P(x = 5) = 0.1240}$$

(b)

A bag contains 4 red and 6 black balls. A sample of 4 balls is selected from the bag without replacement. Let X be the number of red balls. Find the probability distribution for X also compute the mean and variance.

Sol.

$K = 4$, $N = 4+6$

$N = 10$

$N-K = 6$

$n = 4$

X	$P(x=x) = \binom{K}{x}\binom{N-K}{n-x} / \binom{N}{n}$
0	$\binom{4}{0}\binom{6}{4} / \binom{10}{4} = 15/210$
1	$\binom{4}{1}\binom{6}{3} / \binom{10}{4} = 80/210$
2	$\binom{4}{2}\binom{6}{2} / \binom{10}{4} = 90/210$
3	$\binom{4}{3}\binom{6}{1} / \binom{10}{4} = 24/210$
4	$\binom{4}{4}\binom{6}{0} / \binom{10}{4} = 1/210$

$$\text{Mean} = \frac{nK}{N} = \frac{(4)(4)}{10} = \boxed{1.6}$$

$$Var(X) = npq \frac{N-n}{N-1} \Rightarrow n\left(\frac{K}{N}\right)\left(\frac{N-K}{N}\right)\left(\frac{N-n}{N-1}\right)$$

$$\Rightarrow 4\left(\frac{4}{10}\right)\left(\frac{6}{10}\right)\left(\frac{6}{9}\right)$$

$$\boxed{Var(x) = 0.64}$$