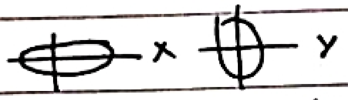
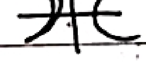
"Parabola"

Date:..............................

Conic Sections:

Ellipse 

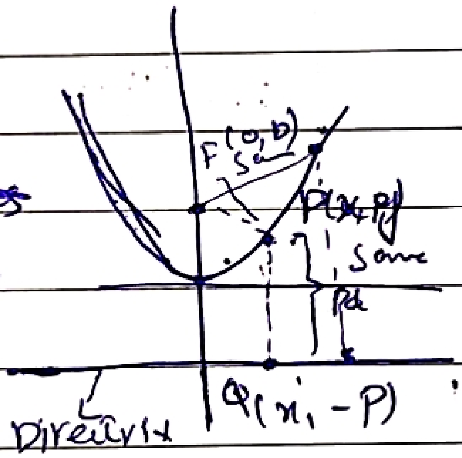
parabola  ; hyperbola 

Parabola (defination)

A set that consist of all the points in a plane equidistant from a given fixed point and a fixed line in ~~the~~ the plane in a parabola.
The fixed point is called focus
The fixed line is called directrix.

$|FP| = |PQ|$

~~fixed point~~ focus
focus کو directrix اور directrix کو focus
سے فاصلہ same ہوتا ہے



equation of parabola.

$\boxed{x^2 = 4py}$ or $x^2 = 4ay$ slower

p = ~~constant number~~ (distance of focus point from origin)
4 = constant number

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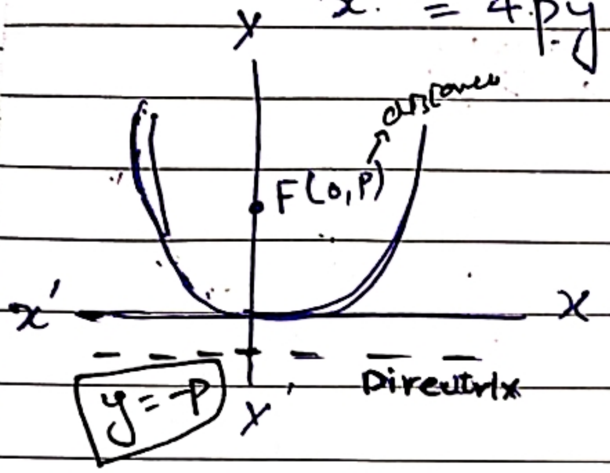
focus point ہمیشہ parabola کے درمیان میں ہوتا مطلب parabola کے اندر ہوتا ہے

1st **equation**:-

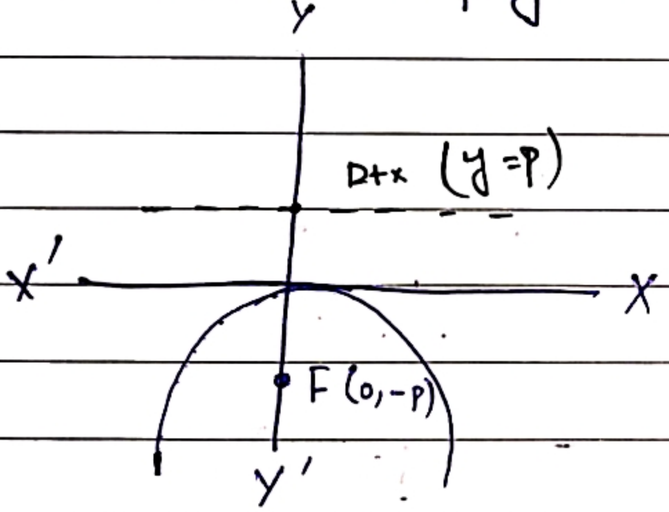
$x^2 = 4py$ -



اس equation میں x دو بار آیا ہے x^2 کی وجہ سے اور "y" صرف ایک بار تو ہم x کے دونوں positive اور -ve پارٹ کو لیں گے اور y کا صرف +ve والا part.

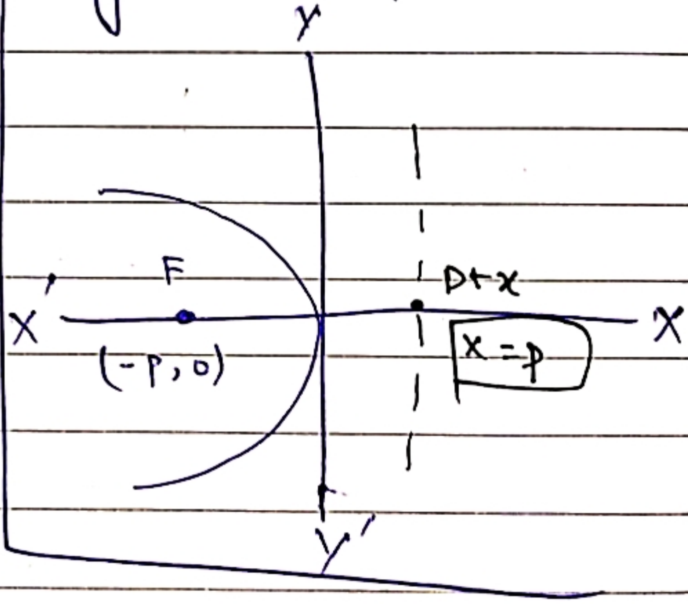
2nd equation.

$x^2 = -4py$



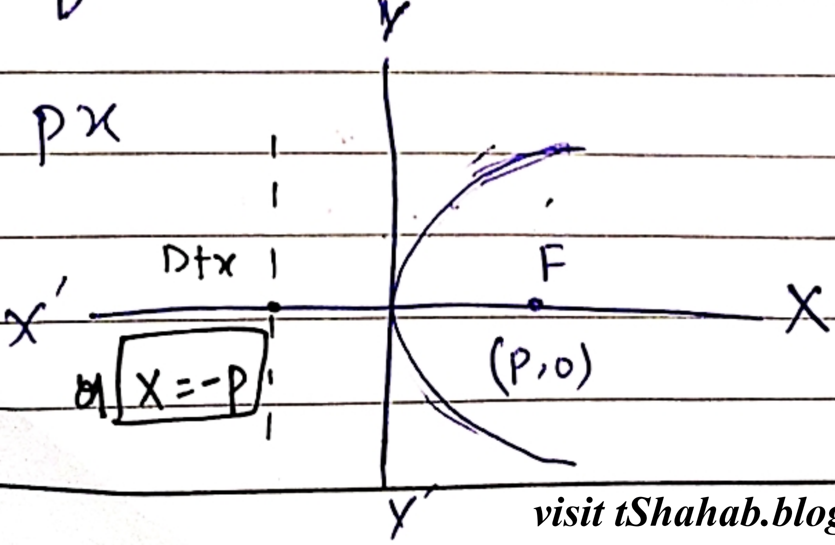
4th equation

$y^2 = -4px$



3r equation

$y^2 = 4px$



Date:................................

Axis of symmetry: / axis-

وہ لائن axis ہے جس کے دونوں طرف parabola کا equal part موجود ہے۔

for $x^2 = 4py$

x - axis

for $x^2 = -4py$

y - axis

for $y^2 = 4p\cancel{y}x$

x - axis

for $y^2 = -4px$

x - axis.

Find focus & Directrix of parabola:

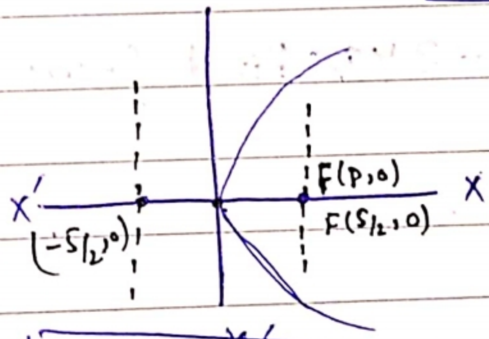
1) $y^2 = 10x$

Sol: general equation

$y^2 = 4p.x$ (compare)

$4 \cdot p = 10$

$p_y = \frac{10}{4}$; $\boxed{p = \frac{5}{2}}$



Dtx = $\boxed{x = -5/2}$; $\boxed{\text{Focus} = \frac{5}{2}}$

$\boxed{\text{axis} = x\text{-axis}}$

Parabola Questions

Date:.................................

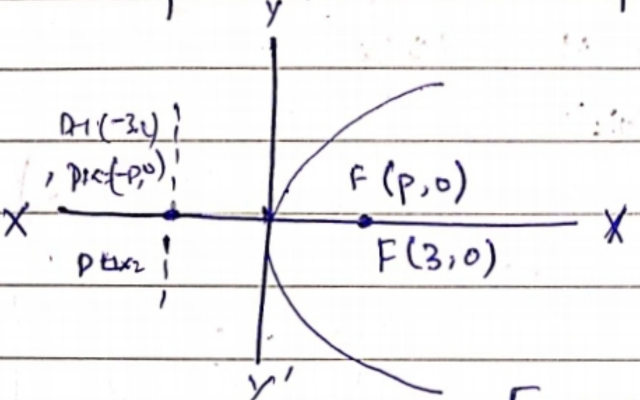
10.1 (Q 9 − 16) Q 39, 40

9) = $y^2 = 12x$ find focus, directrix

compare it with general equation:

~~$x^2 = 4py$~~ $y^2 = 4px$

$4p = 12 \Rightarrow p = \frac{12}{4} \Rightarrow \boxed{p = 3}$



Focus = (3,0), Dtx = (-3,0)

15) $x = -3y^2$ Find focus & Dtx

~~Compare with general equation~~

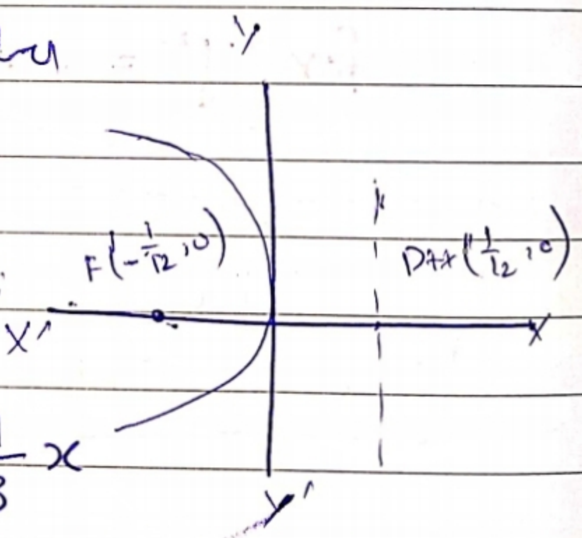
~~$x^2 = 4py$~~

Rearrange it

$-3y^2 = x$ then compare:

$y^2 = -4p\cancel{y}$

$y^2 = \frac{x}{-3}$ or $y^2 = -\frac{1}{3}x$



$\cancel{y^2}\ -4p\cancel{y} = -\frac{1}{3} \Rightarrow p = -\frac{1}{3 \times -4}$

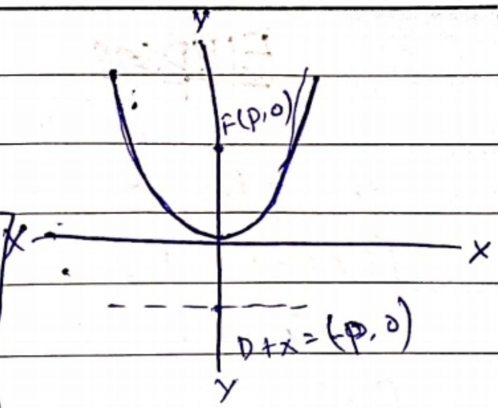
$p = \frac{-1}{-12} \Rightarrow \boxed{p = 12}$

$x^2 = 6y$

compare with general equation

$x^2 = 4py$

$4p = 6 \Rightarrow p = \frac{\cancel{6}^{3}}{\cancel{4}_{2}} \Rightarrow \boxed{p = \frac{3}{2}}$



Focus $= \left(\frac{3}{2}, 0\right)$, Directrix $= \left(-\frac{3}{2}, 0\right)$.

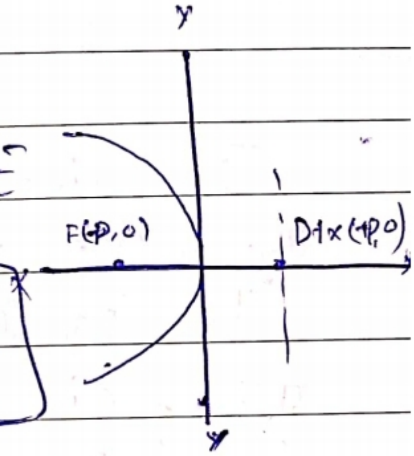
$x = 2y^2$

rearrange

$-2y^2 = x$ compare it gen G.E

$y^2 = -4px$

$-4p = -2 \Rightarrow p = \frac{-2}{-4} \Rightarrow \boxed{p = \frac{1}{2}}$



Focus $= \left(-\frac{1}{2}, 0\right)$, D+x $= \left(\frac{1}{2}, 0\right)$

"ELLIPSES" Date:..............................

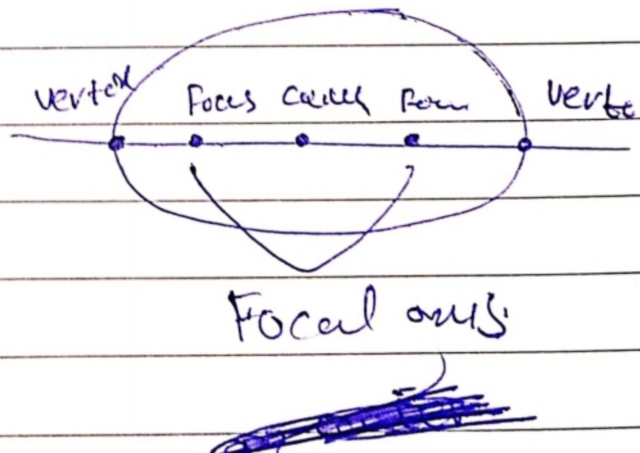
Ellipses

An ellipse is the set of points in a plane whose distances from two fixed points in the Plane have a constant sum. The Two fixed points are foci of the ellipses

The line through the foci of an ellipse is the ellipse's focal axis.
The point on the axis halfway between the foci is the center
The points where the focal axis and ellipse cross are the ellipse's vertices.

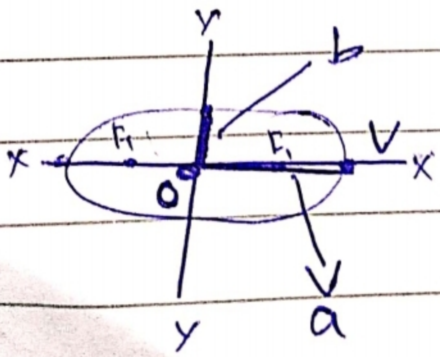
e.g.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



a = Semi-major axis

b = semi-minor axis.



what is c?

$$c = \sqrt{a^2 - b^2}$$

by finding c we can find focal points.

جب a کی value b سے بڑی آ جائے تو اسکا graph x axis-along بنے گا۔

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$F_1 = (-c, 0)$, $F_2 = (c, 0)$

$V = (-a, 0)$, $V_2 = (a, 0)$

if ellipses is shifted from the origin then

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Q# 22 $9x^2 + 10y^2 = 90$. | Find focus points vertices, sketch.

S.E

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Sol:

$\div$ by 90.

$$= \frac{9x^2}{90} + \frac{10y^2}{90} = \frac{90}{90}$$

$$= \frac{1}{10}x^2 + \frac{1}{9}y^2 = 1 \quad \text{or} \quad \frac{x^2}{10} + \frac{y^2}{9} = 1$$

Now compare with S.E.

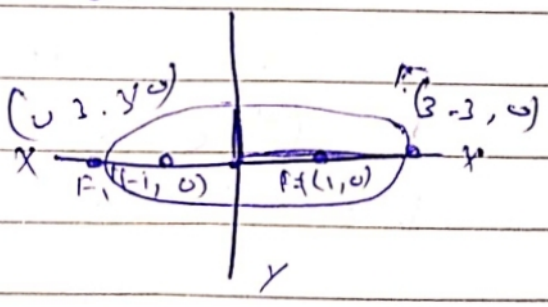
$$\frac{x^2}{\sqrt{(10)}^2} + \frac{y^2}{\sqrt{(9)}^2} = 1 \quad \text{or} \quad \frac{x^2}{(\sqrt{10})^2} + \frac{y^2}{3^2} = 1$$

Now

$a = \sqrt{10}$, $b = 3$

$a = 3.3$, $b = 3$.

ellipse on x-axis



C, (0, 0)

$c = \sqrt{a^2 - b^2} = \sqrt{10 - 9} = \sqrt{1}$

$F = (\pm c, 0)$, $V = (\pm a, 0) = (\sqrt{10}, 0)$

$F = (\pm 1, 0)$ $V =$

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$$6x^2 + 9y^2 = 54$$

$\div$ by 54.

$$\frac{6x^2}{54} + \frac{9}{54}y^2 = 1$$

$$\frac{1}{9}x^2 + \frac{1}{6}y^2 = 1 \quad \text{or} \quad \frac{x^2}{9} + \frac{y^2}{6} = 1$$

compare with G.E

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a^2 = 9 \quad , \quad b^2 = 6$$

$$\frac{x^2}{(3)^2} + \frac{y^2}{(\sqrt{6})^2} = 1$$

$a = 3 \quad , \quad b = \sqrt{6}$ SmA

SMA $\therefore a = 3 \quad , \quad b = 2.44$

$\therefore a > 2.44$

so ellipse on x-axis → for x-axis

$F = (\pm c, 0) \quad , \quad c = \sqrt{a^2 - b^2}$

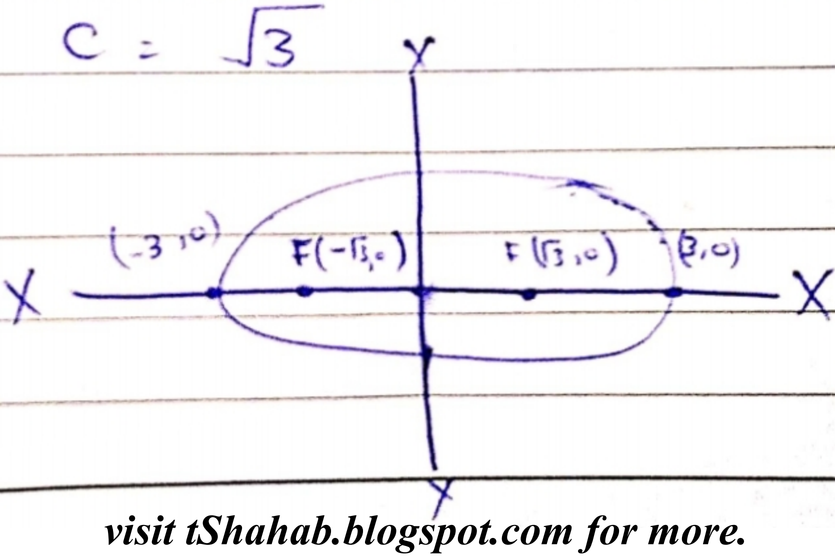
$V = (\pm a, 0)$

$c = \sqrt{9 - 6}$

$c = \sqrt{3}$

$$F = (\pm\sqrt{3}, 0)$$

$$V = (\pm 3, 0)$$



Date:

$$169x^2 + 25y^2 = 4225$$

$\div$ B.S by 4225

$$\frac{169}{4225}x^2 + \frac{25}{4225}y^2 = \frac{4225}{4225}$$

$$\frac{1}{25}x^2 + \frac{1}{169}y^2 = 1 \text{ or } \frac{x^2}{25} + \frac{y^2}{169} = 1$$

compare with G.E

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a^2 = 5^2$, $b^2 = 13^2$

$$\frac{x^2}{5^2} + \frac{y^2}{13^2} = 1$$

$$\frac{x^2}{5^2} + \frac{y^2}{13^2} = 1$$

$a = 5$, $b = 13$

$b > a$

So, ellipse on y-axis

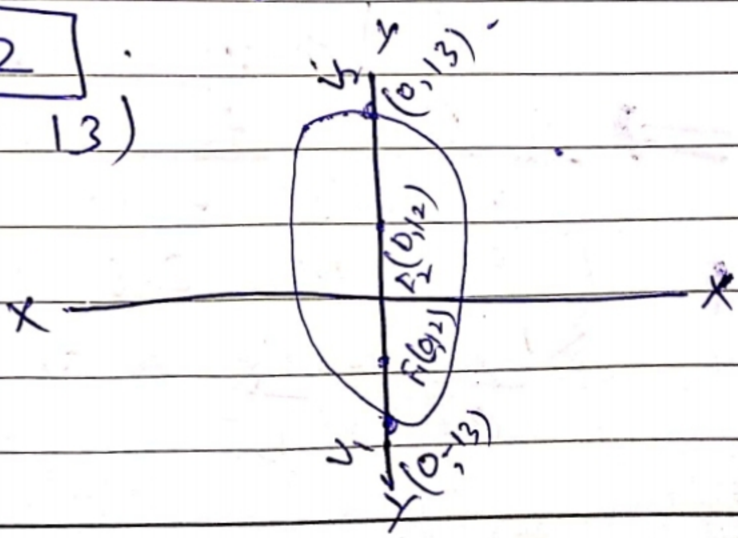
$F = (0, \pm c)$, $V = (0, \pm b)$

$c = \sqrt{b^2 - a^2}$

$c = \sqrt{13^2 - 5^2} \Rightarrow c = \sqrt{169 - 25}$

$c = \sqrt{144} = \boxed{c = 12}$

$F = (0 \pm 12)$, $V = (0 \pm 13)$



"HYPERBOLAS"

Date:...............................

Def Hyperbolas:-

" A hyperbola is the set of points in a plane whose distances from two fixed points in the plane have a constant difference. The two fixed points are the foci of the hyperbola. "

1) ~~Standard equation~~

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad , \text{ Focus points} = O(\pm c, 0)$$

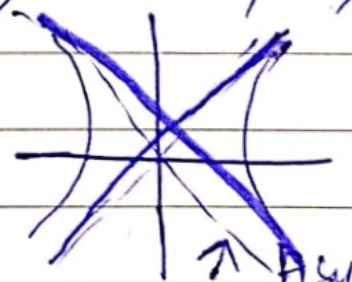
& ~~$c = \sqrt{a^2 + b^2}$~~

$$c^2 = a^2 + b^2 \quad \text{or}$$

$$c = \sqrt{a^2 + b^2}$$

2) Asymptotes:-

ایسی دو لائنز جو hyperbola کی دونوں برانچز کو cut کئے بغیر گزرتی ہیں۔



Asympotoes

=> method to calculate Asympotes

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$ we replace "1" in Q.E by "0"

$$\frac{x^2}{a^2} = \frac{y^2}{b^2} \quad \text{or} \quad \frac{y^2}{b^2} = \frac{x^2}{b^2}$$

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here $y^2 = \frac{b^2}{a^2} x^2$ by takesquareroot on B.S

$y = \pm \frac{b}{a} x$ are The Two asympototes of hyperbola-

Exercise:

Q31:- $8x^2 - 2y^2 = 16$

Standard equation =? Foci =?

asympototes =? vertices =?

÷ equation by 16

$\frac{8x^2}{16} - \frac{2y^2}{16} = \frac{16}{16}$

$\frac{1}{2}x^2 - \frac{1}{8}y^2 = 1 \Rightarrow \frac{x^2}{2} - \frac{y^2}{8} = 1$

Now compare with - S.E

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{\sqrt{2}^2} - \frac{y^2}{\sqrt{8}^2} = 1$

$c = \sqrt{b^2 + a^2} \Rightarrow c = \sqrt{2+8} \Rightarrow \sqrt{10}$

$a = \sqrt{2}$, $b = \sqrt{8}$, $c = \sqrt{10}$

$F = (\pm c, 0)$, $V = (\pm a, 0)$

$F = (\pm\sqrt{10}, 0)$, $V = (\pm\sqrt{2}, 0)$

asymptotes =?

$\cancel{\frac{8x^2}{2}}$ $y = \pm \frac{b}{a} x$

$y = \pm \frac{\sqrt{8}}{\sqrt{2}} x \Rightarrow y = \pm \frac{\sqrt{2}\times\cancel{\sqrt{2}}\times\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} x$

$\boxed{y = \cancel{\pm} \pm 2x \text{ A}}$

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$9x^2 - 16y^2 = 144$ -

Standard equation

Foci. Vertices, asymptotes = ?

$\div$ bs by 144.

$$\frac{9}{144}x^2 - \frac{16}{144}y^2 = \frac{144}{144}$$

$$\frac{1}{16}x^2 - \frac{1}{9}y^2 = 1$$

Now compare with S.E

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{\sqrt{4}^2} - \frac{1}{\sqrt{3}^2} = 1$$

$a = \sqrt{4}$, $b = \sqrt{3}$

$F = (\pm c, 0)$, $V = (\pm a, 0)$

$c = \sqrt{b^2 + a^2}$

F: $c = \sqrt{(\sqrt{3})^2 + \sqrt{(4)^2}} \Rightarrow c = \sqrt{3 + 4}$

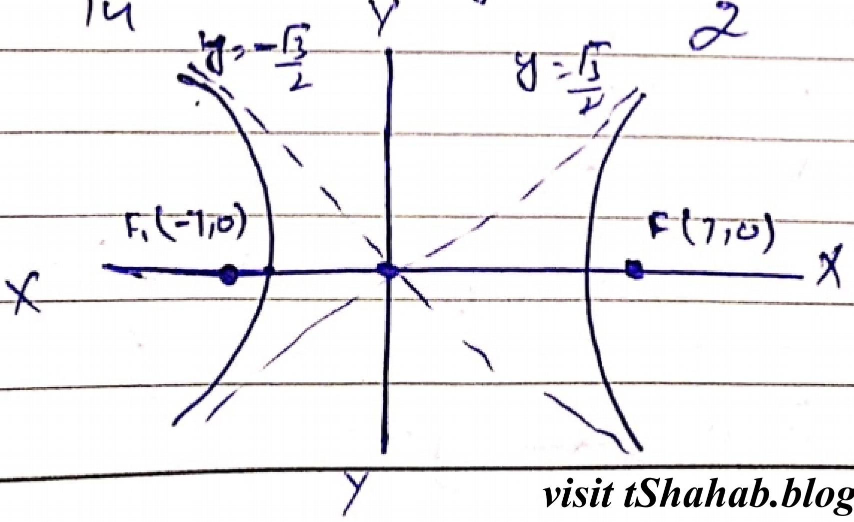
$c = \sqrt{7}$

$F = (\pm\sqrt{7}, 0)$, $V = (\pm\sqrt{4}, 0)$

asymptotes = ?

$$y = \pm \frac{b}{a} x$$

$$y = \pm \frac{\sqrt{3}}{\sqrt{4}} x \quad \text{or} \quad y = \pm \frac{\sqrt{3}}{2} x$$



$\sqrt{2} \cdot \sqrt{2}$ $\sqrt{2\times 2}$ $\sqrt{4}$ $= 2.\sqrt{2}$

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$y^2 - x^2 = 8$

$\div$ by 8 on B.S

$\frac{y^2}{8} - \frac{x^2}{8} = 1$ Now compare with S.E

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow \frac{y^2}{\sqrt{8}^2} - \frac{x^2}{\sqrt{8}^2} = 1$

$a = \sqrt{8}$, $b \sqrt{8}$

$F = (\pm c, 0)$, $V = (\pm a, 0)$

$c = \sqrt{b^2 - a^2} \Rightarrow c = \sqrt{(\sqrt{8})^2 - (\sqrt{8})^2}$