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Date ____ Discrete Structure Day M T W T F S

Short Questions

Q1: What is Absorption laws for sets?

Ans: Following two statements are the absorption laws for sets.

(i) $A \cup (A \cap B) = A$

(ii) $A \cap (A \cup B) = A$

Q2: Differentiate between mathematical induction and strong induction?

Ans: In mathematical induction we use "if $p(K)$ is true then $p(K+1)$ is true" While in strong induction you use "if $P(i)$ is true for all i less than or equal to K then $P(K+1)$ is true".

Q3: Determine the truth value of following Statements if the domain for all variables consists of all integers. $\forall n (n^2 \geq n)$

Ans: $\forall n (n^2 \geq n)$

$= n^2 \geq n$

$(1)^2 \geq 1$

$= 1 \geq 1$

$(2)^2 \geq 2$

$= 4 \geq 2$ So $n^2 \geq n$ is true statement

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Q4: Define Greedy algorithm with example?

Ans: Algoriths that make what seems to be the "best" choice at each step of solving an optimization problem.

Example:

Given a set of coins with several values, It is required to make a change using those coins for a particular amount of rupees using the minimum no of coins

Available coins = 1, 5, 10, 25

Amount to change = 36

Algorithm:

36 - 25 = 11 Coins Used = 25, 10, 1

11 - 10 = 1 Total no of coins = 3

1 - 1 = 0

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Q5: Define the term counterexample?

Ans: A counterexample is a special Kind of example that disproves a statement or proposition. An element for which P(x) is false is called a counter-example of ∀x P(x). For example the prime number 2 is a counterexample to the statement "All prime numbers are odd.

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Q6: What is the Cartesian product $A \times B \times C$, Where $A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{0, 1, 2\}$?

Ans:

The Cartesian Product is:

$$A \times B \times C = \left\{ \begin{array}{l} (0,1,0), (0,1,1), (0,1,2), (0,2,0), \\ (0,2,1) (0,2,2), (1,1,0), (1,1,1), \\ (1,1,2), (1,2,0), (1,2,1), (1,2,2) \end{array} \right\}$$

Q7: Let f be the function from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$ with $f(a) = 4$, $f(b) = 2$, $f(c) = 1$ and $f(d) = 3$ is f a bijection?

Ans: The function f is one-to-one and onto. It is one-to-one because no two values in the domain are assigned the same function value. It is onto because all four elements of the codomain are images of elements in the domain. Hence f is a bijection

Q8: What is the difference between Best Case and Worst Case Complexity of algorithm?

Ans: Worst Case:

By the worst case complexity of an algorithm, we mean the

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maximum number of operations needed to solve the given problem using this algorithm on input of specified size.

Best Case:

By the Best case complexity of an algorithm, we mean the minimum number of operations needed to solve the given problem using this algorithm on input of specified size.

Q9: Convert the hexadecimal expansion of $(80E)_{16}$ integers to a binary expansion?

Ans:

$(80E)_{16}$

$\downarrow$

$(100000001110)_2$

Hexa	Binary
8	1000
0	0000
E	1110

Q10: What are the two ways for the representation of Graph?

Ans: Following are the two ways for the representation of Graph.

(i) Adjacency lists

(ii) Adjacency Matrices

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Q11: What is the probability of getting a numbers greater than 4 when a dice is tossed?

Ans:

Total Outcome = 6 => {(1), (2), (3) (4), (5), (6)}

Favourable Outcome = (5, 6) = 2

$$\text{Probability} = \frac{\text{Favourable Outcome}}{\text{Total Outcome}}$$

$$P = \frac{2}{6}$$

Q12: Define the term Halting problem?

Ans: In computability theory, The halting problem is the decision problem that asks whether a Turing machine T eventually halts when given an input string x.

Q13: Define Binary Search Tree?

Ans: Binary Search Tree is a node-based binary tree data structure which has following properties.

(i) The left subtree of a node contains only nodes with Keys lesser than the node's Key.

(ii) The right subtree of a node contains

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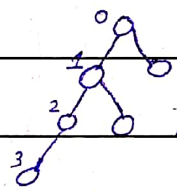
only nodes with keys greater than the node's key.

(iii) The left and right subtree each must also be a binary search tree.

Q14: How the Height of a tree is calculated?

Ans: The height of a tree is the length of the longest path from the root to any vertex. For calculating height start from main root now height is zero move downward and add 1 in height at each level



height = 3

Q15: How many rows appear in a truth table for each of these compound propositions? $(p \to r) \vee (\neg s \to \neg t) \vee (\neg u \to v)$

Ans: Total no of variables $= n = 6$

$$\text{rows} = 2^n$$
$$= 2^6$$
$$\text{rows} = 64$$

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Q16: Let $P(x)$ be the statement "x spends more than five hours every weekday in class" where the domain for x consists of all students. Express the quantifications in English $\exists x \neg P(x)$?

Ans: $P(x)$: x spends more than five hours every weekday in class

$\neg P(x)$: x does not spend more than five hours every weekday in class

$\exists x \neg P(x)$: There is some student in the class who does not spend more than five hours every weekday in class

Q17: Find the conjunction of the proposition p and q where p is the proposition "Today is Friday" and q is the proposition "It is raining today?

Ans: The conjunction of these propositions $p \wedge q$, is the proposition "Today is Friday, and it is raining today"

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Q.18: What are the negations of the statement "All goats are mammals"?

Ans: The negation of the statement "All goats are mammals" is "All goats are not mammals.

Q19: Define contradiction?

Ans: A compound proposition that is always false is called a contradiction for example $p \wedge \neg p$ is a contradiction

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Q20: What is Cartesian product of $A = \{a, b, c\}$ and $B = \{1, 2\}$?

Ans: $A = \{a, b, c\}$ $B = \{1, 2\}$

$$A \times B \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$$

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Q21: Define Commutative Law with the help of example?

Ans: For any two sets A and B

(i) $A \cup B = B \cup A$

(ii) $A \cap B = B \cap A$

Example:

$A = \{1, 2, 3, 4\}$ $B = \{3, 4, 5, 6\}$

(i) $A \cup B = \{1, 2, 3, 4, 5, 6\}$ $B \cup A = \{1, 2, 3, 4, 5, 6\}$

(ii) $A \cap B = \{3, 4\}$ $B \cap A = \{3, 4\}$

$\longleftrightarrow$

Q22: How many comparisons are needed for a binary search in a set of 64 elements?

Ans:

$$N = 64$$

$$\text{Comparisons} = \log_2 N$$

$$C = \log_2 64$$

$$C = 6$$

$\longleftrightarrow$

Q.23: Differentiate projection and join operator?

Ans: Projection:

The projection operation is denoted by the π symbol and is used to choose attributes from

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a relation. This operator shows the list of those attributes that we wish to appear in the result and rest attributes are eliminated from the table Π<attribute list>(relation)

Join:

Join operation is denoted by the $\bowtie$ symbol and is used to compound similar tuples from two relations into single longer tuple. $R \bowtie S$

$\longleftrightarrow$

Q24: What is pigeonhole principle?

Ans: Pigeonhole principle states that If K is a positive integer and K+1 or more objects are placed into K boxes, then there is at least one box containing two or more of the objects

$\longleftrightarrow$

Q25: How many bit strings of length eight either start with a 0 bit or end with the two bits 11?

Ans: No of bit strings of length 8 that start with 0: $2^7 = 128$

No of bit strings of length 8 that end with 11: $2^6 = 64$

No of bit strings of length 8 that

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start with 0 and end with 11: $2^5 = 32$

Applying the subtraction rule, the number is

$$128 + 64 - 32 = 160$$

$\longleftrightarrow$

Q26: Define reflexive closure and symmetric closure?

Ans: Reflexive Closure:

The reflexive closure of R is the relation that contains all ordered pairs of R and to which all ordered pairs of the form $(a,a) \in R (a \in A)$ were added

$$R \cup \Delta = R \{(a,a) \mid a \in A\}$$

Symmetric Closure:

The symmetric closure of R is the Union of the relation R with its inverse relation R^{-1} inverse relation R^{-1} is the set $\{(b,a) \mid (a,b) \in R\}$

$\longleftrightarrow$

Q27: Difference between tree and graph?

Ans:

Tree	Graph
(i) Only One path exist between two vertices	(i) More than one path allowed between two vertices
(ii) Root node is the	(ii) There is no

starting node of the tree	root node concept
(iii) Tree look like hierarchical	(iii) Graph looks like Network

Q28: Let P(x) be the statement "x spends more than 5 hours every work day in class" where the domain consists of all students. Express each of these quantification in English?

(a) $\forall x \sim P(x)$ (b) $\exists x\, P(x)$

Ans: P(x): x spends more than 5 hours every work day in class.

(a) $\forall x \neg P(x)$: It is not the case that every student in the class spends more than 5 hours every work day in class

(b) $\exists x\, P(x)$: There is some students who spends more than 5 hours every work day in class.

Q29: What is a recurrence relation?

Ans: A recurrence relation for the sequence $\{a_n\}$ is an equation that expresses a_n in terms of one or more of the previous terms of the sequence.

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Example: Fibonacci Sequence $f_0, f_1, f_2, f_3, \dots$ is defined by:

- Initial conditions: $f_0 = 0$; $f_1 = 1$
- Recurrence Relation: $f_n = f_{n-1} + f_{n-2}$

Q30: What is Full Adder?

Ans: A full adder is a digital circuit that performs addition. A full adder adds three one-bit binary numbers, two operands and a carry bit. The adder outputs two numbers, a sum and a carry bit.

Q31: Define Pascal's Triangle?

Ans: Pascal's triangle is a triangular pattern of numbers in which each number is equal to the sum of the two numbers immediately above it. Its a geometric arrangement of the binomial coefficients in a triangle.

Example:

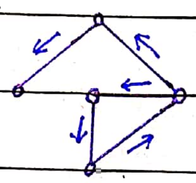
$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
1 & & 3 & & 3 & & 1 \\
 & & & \vdots & & &
\end{array}
$$

Q32: Define Euler path and Euler circuit?

Ans:

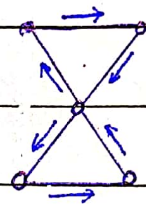
Euler Path

An Euler path is a path of edges that visits all the edges in a graph exactly once.



Euler Circuit

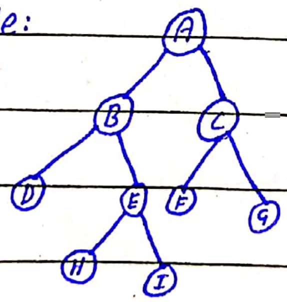
An Euler circuit is an Euler Path which starts and ends on the same vertex.



Q33: What is Leaf?

Ans: A vertex of a rooted tree is called a leaf if it has no children. A leaf is a node of vertex degree 1.

Example:



Here D, H, I, F and G are leaf nodes.

Q34: What is K-map?

Ans: A Karnaugh map (K-map) is a pictorial method used to minimize Boolian expressions without having to use Boolian algebra theorms and equation manipulations.

Example:

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

Truth table

B \ A	0	1
0	0	1
1	1	1

K-map

Q35: What is a Postulate?

Ans: A postulate is a statement that is accepted without proof. It's universally true and needs no proof.

Example:

A straight line segment can be drawn joining any two points.

Q36: Define Logical equivalence?

Ans: Two proposition are said to be logically equivalent if their truth tables are identical.

Example:

$\sim p \vee q$ is logically equivalent to $p \to q$

$\sim p \vee q \equiv p \to q$

p	q	$\sim p \vee q$	$p \to q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

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Q37: What is Pseudocode?

Ans: An informal high-level description of the operating principal of a Computer program or other algorithm is called Pseudocode.

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Q38: Define Recursive algorithm?

Ans: An algorithm is called recursive if it solves a problem by reducing it to an instance of the same problem with smaller input.

Example: Algorithm for Computing n!

Procedure factorial (n: nonnegative integer)

if n = 0 then return 1

else return n. factorial (n-1)

{output is n!}

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Q39: Define Permutation?

Ans: A permutation is a mathematical calculation of the number of ways a particular set can be arranged, where the order of the arrangement matters.

Example: {a, b, c, d} is a set of 4 distinct objects.

Permutations of this set are: abcd, dcba, abdc, etc

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Q40: What is Random Variable?

Ans: A random variable is a set of possible values from a random experiment.

Example: Tossing a coin: we could get Head or tails

Heads = 0 Tails = 1

Random variable = X = {0, 1}

Q41: What is Derangement?

Ans: A derangement is a permutation with no fixed points. A derangement of a set leaves no element in its orignal place.

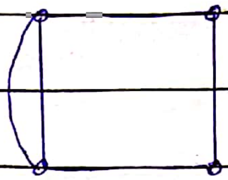
Example: The derangements of {1,2,3} are {2,3,1} and {3,1,2}, but {3,2,1} is not a derangement because 2 is a fixed point.

Q42: Define Multigraph and pseudograph?

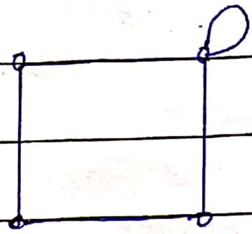
Ans:

Multigraph	Pseudograph
Graphs that may have multiple edges connecting the same vertices are called multigraphs. Self loops are not permitted in multigraphs.	Graphs that may include loops, and possibly multiple edges connecting the same pair of vertices or a vertex to itself are called pseudographs.

Example:



Example



Q43: What is Tree Traversal?

Ans: Traversing a tree means visiting every node in the tree. For example you might want to add all the values in the tree or find the largest one. For all these operations you will need to visit each node of the tree.

Q44: What is compound proposition? Give an example?

Ans: A compound proposition is a proposition formed from simpler propositions using logical connectors or some combination of logical connectors

Example:

$P \wedge q$: It is sunny and it is humid.

Q45: Give an indirect proof to the theorm "If $3n+2$ is odd, then n is odd"

Ans: Proof:- Let n be an integer,

Suppose $3n+2$ is odd, also n is not odd

$\Rightarrow$ n is an even number

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$\Rightarrow n = 2K$

Now $3n+2 = 3(2K)+2$

$= 6K+2$

$3n+2 = 2(3K+1)$

$\Rightarrow 3n+2$ is even this is a contradiction

$\Rightarrow n$ should be odd

Hence proved

$\longleftrightarrow$

Q46: What are the contrapositive, the converse, and the inverse of the conditional statement "The home team wins whenever it is raining"?

Ans: "q whenever p" means $p \to q$ so original statement is " If it is raining, then the home team wins.

Contrapositive: If the home team does not win, then it is not raining.

Converse: If the home team wins, then it is raining.

Inverse: If it is not raining, then the home team does not win.

$\longleftrightarrow$

Q47: What is bi-implication? State with example?

Ans: The bi-implication statement $p \leftrightarrow q$ is the proposition "p if and only if q".

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The bi-implication statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Bi-implications are also called Biconditional statements.

Example: "You can take the flight if and only if you buy a ticket."

Q48: Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent?

Ans:

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Q49: How many edges are there in a graph with ten vertices each of degree six?

Ans: According to Handshaking theorem the sum of the degrees of all vertices in a graph with m edges is 2m

Sum of degrees = $6 \cdot 10 = 60$

$2m = 60$

$m = \frac{60}{2}$

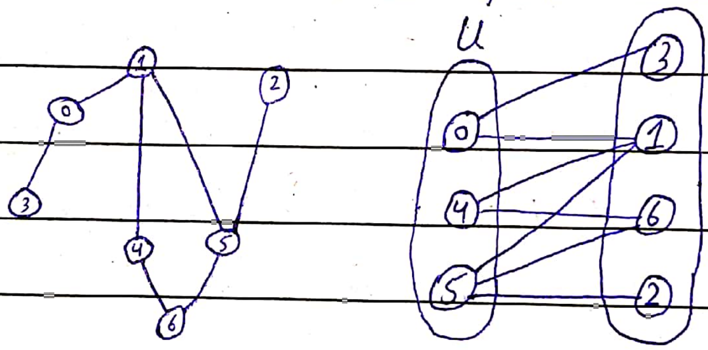
$m = 30$

$\longleftrightarrow$

Q50: What is a bipartite graph?

Ans: A graph whose vertices can be divided into two independent sets U and V such that there is no edge that connect vertices of same set.

Example:



$\longleftrightarrow$

Q51: What is the cardinality of each of these sets? $\{a, \{a\}, \{a, \{a\}\}\}$

Ans: $A = \{a, \{a\}, \{a, \{a\}\}\} = n(A) = 3$

$B = \{a\} = n(B) = 1$

$C = \{a, \{a\}\} = n(C) = 2$

$D = \{a\} = n(D) = 1$

$\longleftrightarrow$

Q52: State resolution rule?

Ans:

Name	Rule of inference	Tautology
Resolution	$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$

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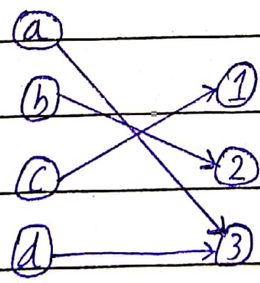
Q53: What is the difference between geometric progression and arithmetic progression?

Ans:

Geometric Progression	Arithmetic Progression
In geometric Progression Ratio of Consecutive terms is constant called the common ratio	In Arithmetic progression difference between consecutive terms is constant.
Example: 1, 3, 9, 27, 81 ..... ratio = 3	Example: 2, 6, 10, 14, 18 ..... diff = 4

Q54: Let f be the function from {a, b, c, d} to {1, 2, 3} defined by $f(a) = 3$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$ is f an onto function?

Ans:



Because all three elements of the codomain are images of elements in domain, we see that f is onto

Q55: State division algorithm?

Ans: Division algorithm states that given any two integers a and b with $a > 0$. there exists unique integers q and r such that $b = qa + r$ with $0 \leq r < a$. Where a is called divisor, b is called dividend, q is called quotient and r is called remainder.

Example:

Let $b = 49$, and $a = 9$

$b = q/a + r$

$49 = 5(9) + 4$

$49 = 49$

Q 56: What is the height of a rooted tree?

Ans: The height of a rooted tree is the length of the longest path from the root to any vertex. The height of a rooted tree is, the maximum of the levels of vertices.

Q 57: Define existential quantifications with suitable example?

Ans: The existential quantification of $P(x)$ is the proposition, "There exists an element x in the domain such that $P(x)$". We use the notation $\exists x P(x)$ for the existential quantification of $P(x)$. Here $\exists$ is called existential quantifier.

Example: Let $P(x)$ be the statement "x spends more than five hours every work day in class"

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then existential quantification of $P(x)$ $\exists x P(x)$ is "There is some student in the class who spend more than 5 hours every workday in class.

Q58: Show that $\neg(P \lor (\neg P \land q))$ and $\neg P \land \neg q$ are logically equivalent by developing a series of logical equivalences?

Ans:

P	q	$\neg P$	$\neg q$	$\neg P \land q$	$P \lor (\neg P \land q)$	$\neg(P \lor (\neg P \land q))$	$\neg P \land \neg q$
T	T	F	F	F	T	F	F
T	F	F	T	F	T	F	F
F	T	T	F	T	T	F	F
F	F	T	T	F	F	T	T

$$\neg(P \lor (\neg P \land q)) \equiv \neg P \land \neg q$$

Q59: By using the truth table show that it is tautology or not $(P \leftrightarrow Q) \leftrightarrow (P \rightarrow Q) \land (Q \rightarrow P)$?

Ans:

P	Q	$P \leftrightarrow Q$	$P \rightarrow Q$	$Q \rightarrow P$	$(P \leftrightarrow Q) \leftrightarrow (P \rightarrow Q)$	$(P \rightarrow Q) \land (Q \rightarrow P)$
T	T	T	T	T	T	T
T	F	F	F	T	T	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T

Q.60: State which rule of inference is basis of the following argument: "It is below freezing and raining now, therefore, it is below freezing now"?

Ans: Let P be the proposition "It is below freezing now" and q the proposition "It is raining now" This argument is of the form

$$\frac{p \wedge q}{\therefore p}$$

This argument uses the simplification rule.

Q61: If p = It is raining q = She will go to college "It is raining and she will not go to college" will be denoted by?

Ans: It is raining and she will not go to college will be denoted by "$p \wedge \neg q$".

Q62: Write down the name of the rule

$p \rightarrow q$

$q \rightarrow r$?

$\therefore p \rightarrow r$

Ans:

$p \rightarrow q$

$q \rightarrow r$ Hypothetical syllogism

$\therefore p \rightarrow r$

Q63: State HandShaking lemma?

Ans: Handshaking lemma states that the sum of the degrees of all vertices in an undirected graph with m edges is 2m. Let $G=(V,E)$ be an undirected graph with m edges then

$$2m = \sum_{v \in V} deg(v)$$

Q64: State inclusion-exclusion principle?

Ans: Inclusion-exclusion principle tells us how many elements are in the union of a finite numbers of finite sets. Let $A_1, A_2, \ldots A_n$ be finite sets then

$$|A_1 \cup A_2 \cup \cdots \cup A_n| = \sum |A_i| - \sum |A_i \cap A_j| + \sum |A_i \cap A_j \cap A_k| - \cdots + (-1)^{n+1} |A_1 \cap A_2 \cap \cdots \cap A_n|$$

Q65: What is isomorphism?

Ans: Two graphs G_1 and G_2 are said to be isomorphic, if there is one-to-one correspondence between their vertices and edges, and incidence relationship is preserved. Such one-to-one correspondence is called isomorphism

Q66: What are the basic Counting Principles?

Ans: Following two are the basic counting principles

(i) The Sum Rule

(ii) The Product Rule

Q67: From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the Committee. In how many ways it can be done?

Ans:

Selecting 3 men out of 7	Selecting 2 Women out of 6
$\frac{7!}{3! \cdot 4!}$	$\frac{6!}{2! \cdot 4!}$
$= 35$	$= 15$

No of ways $= 35 \times 15 = 525$

Q68: In how many different ways can the letters of the word "LEADING" be arranged in such a way that the vowels always come together?

Ans: L E A D I N G

$\boxed{E\ A\ I}$ L D N G

No of ways $= 5! \times 3! = 720.$

Q69: How many permutations of the letters "ASSESSINATION" contain the string "SES"?

Ans: A S $\boxed{S\ E\ S}$ S I N A T I O N

$$\text{Permutations} = \frac{11!}{2!\cdot 2!\cdot 2!\cdot 2!} \cdot \frac{3!}{2!}$$

$$= 7484400$$

Q70: Differentiate between Euler and Hamiltonian path?

Ans:

Euler path	Hamiltonian Path
A euler path is a path that passes through every edge exactly once	A Hamiltonian path is a path that passes through every vertex exactly once (Not every edge)

Q71: What is graph coloring problem?

Ans: Graph coloring problem is to assign colors to certain elements of a graph. The problem is, given m colors, find a way of coloring the vertices of a graph such that no two adjacent vertices are colored using same color.

Q72: Define tautology?

Ans: A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a tautology.

Example: $p \vee \neg p$

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Q73: Find the symmetric difference of $\{3,5,8\}$ and $\{2,5,6\}$?

Ans:

$$A = \{3, 5, 8\}$$
$$B = \{2, 5, 6\}$$
$$A \oplus B = \{3, 8, 2, 6\}$$

Q74: Let $A = \{eggs, milk, corn\}$ and $B = \{cows, goats, hens\}$ define a relation R from A to B by $(a, b) \in R$ iff a is produced by b.?

Ans: $A = \{eggs, milk, corn\}$ $B = \{cows, goats, hens\}$

$$A \times B = \begin{Bmatrix} (eggs, cows), (eggs, goats), (eggs, hens) \\ (milk, cows), (milk, goats), (milk, hens) \\ (corn, cows), (corn, goats), (corn, hens) \end{Bmatrix}$$

$$R' = \{(a, b) : a \text{ is Produced by } b, a \in A, b \in B\}$$

$R' = \{(eggs, hens), (milk, cows), (milk, goats)\}$

$\longleftrightarrow$

Q75: Write applications of minimum spanning tree?

Ans: Minimum spanning trees are used for network designs. They are also used to find approximate solutions for complex mathematical problems like Traveling Salesman problem other applications include Cluster Analysis and Entropy based image registration.

$\longleftrightarrow$

Q76: Define ceiling and floor function with example?

Ans:

Floor function	Ceiling function
Let x be a real number. The floor function rounds x down to the closest integer less than or equal to x.	Let x be a real number. The ceiling function rounds x upto the closest integer greater than or equal to x.
Example: $x = 2.5$ $f(2.5) = 2$	Example: $x = 2.5$ $f(2.5) = 3$

$\longleftrightarrow$

Q77: Define a function with example?

Ans: A function is a rule that assigns each input exactly one output. The set of all inputs for a function is called domain and set of all allowable outputs is called codomain. We would write $f: x \to y$ to describe a function with name f, domain x and codomain y.

Example: input $\rightarrow$ $\boxed{\text{Function}}$ $\rightarrow$ output

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is even} \\ x+5 & \text{if } x \text{ is odd} \end{cases}$$

$f(2) = 4 \qquad f(3) = 8$

$\longleftrightarrow$

Q78: What is modus ponen rule of inference?

Ans:

Name	Tautology	Rule of inference
Modus Ponens	$(p \wedge (p \to q)) \to q$	p $p \to q$ $\therefore q$

$\longleftrightarrow$

Q79: Write any two applications of Venn diagram?

Ans: Some applications of venn diagram is following.

(i) Sets can be represented graphically using venn diagrams.

(ii) Venn diagrams are used to Demonstrate De Morgan's Law.

(iii) Venn diagrams are useful in illustrating relationships in statistics and probability.

$\longleftrightarrow$

Q80: Differentiate between free variable and bound variable?

Ans: A variable, x is free iff the variable does not occur in the scope of a quantifier.
A variable, x is bound iff the variable occurs in the scope of a quantifier.

Example:

Bound $\exists x(Dx \land Px) \land Qx$ Free

Q81: Define Predicate?

Ans: A predicate is a property that some object has for example the statement "x is greater than 3" has two parts, The first part, the variable x, is the subject of the statement and the second part "is greater than 3" is the predicate which refers to the property that the subject of the statement can have.

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Q82: Evaluate this expression?

(11011 V 01010) ∧ (10001 V 11011)

Ans: Let A is 11011

B is 01010

C is 10001

A	B	C	A V B	C V A	(A V B) ∧ (C V A)
1	0	1	1	1	1
1	1	0	1	1	1
0	0	0	0	0	0
1	1	0	1	1	1
1	0	1	1	1	1

Q83: Differentiate between a function and a Relation?

Ans: The difference between function and relation is relation shows relationship between input and output whereas function derives one output for each input.

Q84: Verify whether $(P \wedge Q) \rightarrow (P \vee Q)$ is tautology or not?

P	Q	$P \wedge Q$	$P \vee Q$	$(P \wedge Q) \rightarrow (P \vee Q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

It's a tautology.

Q85: Let $Q(x)$ denote the statement "$x = x+1$". What is the truth value of the quantification $\exists x Q(x)$ where the domain consists of all real numbers?

Ans: Because $Q(x)$ is false for every real number x, the existential quantification of $Q(x)$ which is $\exists x Q(x)$, is false.

$\longleftrightarrow$

Q86: Write the names of an algorithm properties?

Ans: (i) Input (ii) Output
(iii) Definiteness (iv) Correctness
(V) Finiteness (Vi) Effectiveness
(vii) Generality.

$\longleftrightarrow$

Q87: What is the decimal expansion of the number with hexadecimal expansion $(2AE0B)_{16}$?

Ans:

$$(2AE0B)_{16} = 2\cdot 16^4 + 10\cdot 16^3 + 14\cdot 16^2 + 0\cdot 16^1 + 11\cdot 16^0$$
$$= 175627$$

$\longleftrightarrow$

Q88: Define partial ordering with example?

Ans: A relation R on a Set S is called a partial ordering if it is reflexive, antisymmetric, and transitive. A set S together with a partial ordering R is called a partially ordered set or poset.

Example:

The "greater than or equal" relation ($\geq$) is a partial ordering on the set of integers because:

Reflexivity: $a \geq a$ for every integer a

Antisymmetry: If $a \geq b$ and $b \geq a$, then $a = b$

Transitivity: If $a \geq b$ and $b \geq c$, then $a \geq c$

$\longleftrightarrow$

Q89: Encrypt the message "WATCH YOUR STEP" by translating the letters into numbers, applying the given encryption function, and then translating the numbers back into letters. $f(P) = (14P+21) \bmod 26$?

Ans:
W A T C H   Y O U R   S T E P
22 0 19 2 7   24 14 20 17   18 19 4 15

Lets apply the function $f(P) = (14P+21) \bmod 26$

By applying function we get 17, 21, 1, 23, 15   19, 9, 15, 25   18, 19, 4, 15   their tranlation is

RVBXP TJPZ NBZX

$\longleftrightarrow$

Q90: How many permutations of the letters ABCDEFG contain the string CFGA?

Ans: Because the letter CFGA must occur as a block, we can find the answer by finding the number of permutations of four objects, namely the block CFGA and individual letters B, D, E.

$$\text{Permutations} = 4! = 24$$

$\longleftrightarrow$

Q91: Determine whether the integers 10, 17 and 21 are pairwise relatively prime?

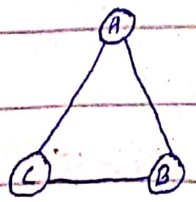
Ans: Because greatest common devisor $\gcd(10,17)=1$, $\gcd(10,21)=1$, and $\gcd(17,21)=1$, we conclude that 10, 17, and 21 are pairwise relatively prime.

$\longleftrightarrow$

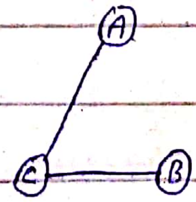
Q92: Define the spanning tree of a graph?

Ans: Let G be a simple graph. A spanning tree of G is a subgraph of G that is a tree containing every vertex of tree.

Example:



Graph G



Spanning tree

$\longleftrightarrow$

Q93: How can you produce the terms of a sequence if the first terms are 5, 11, 17, 23, 29, 35, 41, 47, 53, 59 ?

Ans: By applying following formula we can produce terms of sequence

$$a_n = a_1 + (n-1)d \qquad d = 6$$

$$a_{11} = 5 + (11-1)6$$

$$a_{11} = 5 + 60 = 65$$

Q94: What are the connected components of a graph?

Ans: A connected component of an undirected graph is a subgraph in which any two vertices are connected to each other by paths, and which is connected to no additional vertices in the supergraph

Q95: How many subsets with more than two elements does a set with 100 elements have?

Ans: STEP 1: Numbers of subsets with not more than two elements

$$n = 100 \qquad r = 0 \text{ or } 1 \text{ or } 2$$

$$C(100,2) = \frac{100!}{2!(100-2)!} = \frac{100!}{2!\,98!} = 4950$$

$$C(100,1) = \frac{100!}{1!(100-1)!} = \frac{100!}{1!\,99!} = 100$$

$$C=(100,0)=\frac{100!}{0!(100-0)!}=\frac{100!}{0!\,100!}=1$$

STEP2: Number of subsets with more than two elements

The given set has 100 elements which thus has 2^{100} subsets

Total subset − subsets not more than two elements

= subsets with more than 2 elements

$2^{100}-4950-100-1=1.2676506\times10^{30}$ subsets.

Q96: Define reflexive relation?

Ans: A relation R on a set A is called reflexive if $(a,a)\in R$ for every element $a\in A$.

Example: $A=\{1,2,3,4\}$

$R=\{(1,1),(1,2),(1,4),(2,1),(2,2),(3,3),(4,1),(4,4)\}$

Q97: Define Cardinality of a set?

Ans: Let S be a set, If there are exactly n distinct elements in S where n is a nonnegative integer, we say that S is a finite set and that n is the cardinality of S.

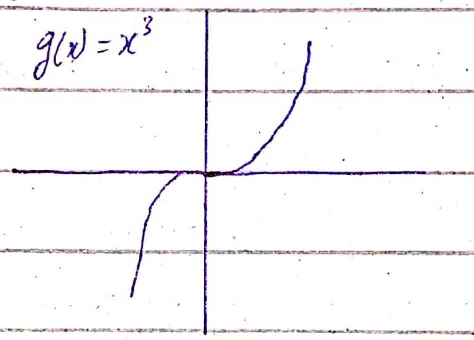
Example: $S=\{1,2,3,4\}$

$|S|=4$

Q98: Define monotonic function?

Ans: When a function is increasing on its entire domain or decreasing on its entire domain, we say that the function is strictly monotonic and we call it a monotonic function.

Example: Consider the function $g(x) = x^3$



The graph of g is increasing everywhere therefore g is a monotonic function.

Q99: Define Transitive closuer?

Ans: The transitive closure of a relation R is the smallest transitive relation containing R. Transitive closure of R is denoted by R^+. Let cardinality of A is n. Let R be the relation on A. Then

$$R^+ = R \cup R^2 \cup R^3 \cdots\cdots \cup R^n$$

Q100: Proof by contradiction with example?

Ans: Proof by contradiction is a proof that determines the truth of a statement by assuming the proposition is false, then working to show its falsity until the result of that assumption is a contradiction.

Example: Proof:- Let n be an integer.

Prove "If $3n+2$ is odd then n is odd"

Suppose $3n+2$ is odd also n is not odd

$\Rightarrow n$ is an even number

$\Rightarrow n = 2K$

Now $3n+2 = 3(2K)+2 = 6K+2 = 2(3K+1)$

$\Rightarrow 3n+2$ is even this is a contradiction

$\Rightarrow n$ should be odd

Q101: Construct truth table for $(p \vee q) \vee \sim p$

Ans:

p	q	$\neg p$	$p \vee q$	$(p \vee q) \vee \neg p$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

Q102: Prove that $p \to (q \to r)$ and $(p \wedge \neg r) \to \neg q$ are logically equivalant?

p	q	r	$\neg r$	$\neg q$	$q \to r$	$p \to (q \to r)$	$p \wedge \neg r$	$(p \wedge \neg r) \to \neg q$
T	T	T	F	F	T	T	F	T
T	T	F	T	F	F	F	T	F
T	F	T	F	T	T	T	F	T
T	F	F	T	T	T	T	T	T
F	T	T	F	F	T	T	F	T
F	T	F	T	F	F	T	F	T
F	F	T	F	T	T	T	F	T
F	F	F	T	T	T	T	F	T

$$p \to (q \to r) \equiv (p \wedge \neg r) \to \neg q$$

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Q103: What is worst case complexity of bubble sort?

Ans: The bubble sort uses $\frac{(n-1)n}{2}$ comparisons, so it has $O(n^2)$ worst-case complexity in terms of the number of comparisons used.

$\longleftrightarrow$

Q104: What is the space complexity of linear search algorithm?

Ans: We don't need any extra space to store anything. We just compare the given value with the elements in an array one by one so the space complexity of linear search is $O(1)$ which is constant.

$\longleftrightarrow$

Q105: What is a relation on a set?

Ans: A relation on a set A is a relation from A to A. In other words a relation on a set A is a subset of A X A.

Example: $A = \{1, 2, 3\ 4\}$

$R = \{(a,b) \mid a \text{ divides } b\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$.

$\longleftrightarrow$

Q106: How many ways are there to select five players from a 10 member tennis team to make a trip to a match at another school?

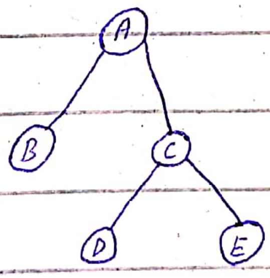
Ans: The answer is given by the numbers of 5-combinations of a set with 10 elements, The numbers of such combinations is

$$C(10,5) = \frac{10!}{5!5!} = 252.$$

Q107: Define a Tree?

Ans: A tree is a connected undirected graph with no simple circuits.

Example:



Q108: What is minimum spanning tree?

Ans: A minimum spanning tree in a connected weighted graph is a spanning tree that has the smallest possible sum of weights of its edges.

Q109: How many relations are there on a set with n elements?

Ans: A relation on a set A is subset of $A \times A$. Because $A \times A$ has n^2 elements when A has n elements, and a set with m elements has 2^m subsets, there are 2^{n^2} subsets of $A \times A$. Thus there are 2^{n^2} relations on a set with n elements.

Q110: How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits?

Ans: There are 26 letters, A-Z in uppercase alphabets and 10 digits 0 through 9. Assuming repetition is allowed. Each letter position has 26 possibilities and each digit position has 10 possibilities thus

$$26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17,576,000$$

Q111: How many permutations of the letters of the word PANAMA can be made, if P is too be the first letter in each argument?

Ans: No of letters = 6 $\quad 1 \times 5! = 5!$

P A N A M A $\quad 5! = 120$

Q112: Suppose that there are eight runners in a race first will get gold medal the second will get silver and third will get bronze. How many different ways are there to award these medals if all possible outcomes of race can occur and there is no tie?

Ans: The number of different ways to award the medals is the number of 3-permutations of a set of eight elements. Hence there are $P(8,3) = 8 \cdot 7 \cdot 6 = 336$ possible ways to award the medals.

$\longleftrightarrow$

Q113: Construct truth table for $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg p)$?

Ans:

p	q	$\neg p$	$p \leftrightarrow q$	$p \leftrightarrow \neg p$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg p)$
T	T	F	T	F	T
T	F	F	F	F	F
F	T	T	F	F	F
F	F	T	T	F	T

$\longleftrightarrow$

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Q114: Show that this conditional statement is tautology $\neg(p \to q) \to p$?

Ans:

p	q	$p \to q$	$\neg(p \to q)$	$\neg(p \to q) \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

Q115: Write the simplification rule of inference?

Ans:

Name	Tautology	Rule of inference
Simplification	$(p \land q) \to p$	$\dfrac{p \land q}{\therefore p}$

Q116: Define the fallacies?

Ans: An assumption or series of steps which is seemingly correct but contains a flawed argument is called a fallacy. In other words fallacies are common forms of incorrect reasoning which lead to invalid arguments.

Q117: Define disjoint set?

Ans: Two sets are called disjoint if their intersection is the empty set.

Example: $A=\{1,3,5\}$ $B=\{2,4,6\}$

$A \cap B = \{\ \}$

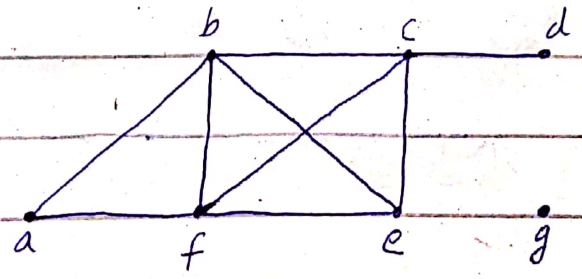
Q118: Is the function $f(x)=x^2$ from the set of integers to the set of integers onto?

Ans: The function f is not onto because there is no integer x with $x^2=-1$, for instance.

Q119: Define isolated vertex in a graph?

Ans: A vertex of degree zero is called isolated vertex. It follows that an isolated vertex is not adjacent to any vertex.

Example:



here g is isolated vertex

Q120: Define pre-order traversal in a tree?

Ans: Pre-order traversal is a traversal method in which the root node is visited first, then the left subtree and finally the right subtree this process goes on untill all nodes are visited.

Q121: What is average case complexity of linear search algorithm?

Ans: The linear search algorithm uses $\frac{n+1}{2}$ comparisons so it has O(n) average case complexity in terms of number of comparisons used.

Q122: List down the name of three graph models?

Ans: Following are the names of models.

(i) Social Networks (ii) Information Networks

(iii) Biological Networks (iv) Tournaments

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