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Linear equations

It is defined as It is an algebric equation in which each term has an exponent of one and graphing of equation results in a straight line.

$$y = mx + b$$

$$x + y = 2$$

System of linear equation

It is defined as "The system in which more than one linear equation involved"

Example:-

$$\begin{cases} x + y = 2 & \text{---} ① \\ 2x + 4y = 5 & \text{---} ② \end{cases}$$

Here more than one linear equation involves.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \longrightarrow \text{Matrix}$$

⇒ Matrix ki enteries always real numbers.

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$$\begin{bmatrix} a_{11}x + a_{12}y + a_{13}z = 0 \\ a_{21}x + a_{22}y + a_{23}z = 0 \\ a_{31}x + a_{32}y + a_{33}z = 0 \end{bmatrix} \rightarrow$$ system of linear equation

⇒ In system of linear equation variable k coefficient real numbers hon gy.

Homogeneous Linear equation

⇒ Right Hand side of linear equations should be zero (0)

⇒ The power of variable is equal to 1. (1)

Example:-

$$\begin{bmatrix} a_{11}x + a_{12}y + a_{13}z = 0 \\ a_{21}x + a_{21}y + a_{23}z = 0 \end{bmatrix} \rightarrow$$

Non-homogenous:-

$$\begin{bmatrix} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32} + a_{33} = b_3 \end{bmatrix}$$

→ Right hand side not equal to zero

Consistent

A linear equation system is consistent if it has **at least** one solution.

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Inconsistent

A linear system is inconsistent if it has no solution.

Imp points

⇒ Some system have more than one solution.

⇒ Some system have no solution.

⇒ linear equation ⟶ one equation involves

⇒ system of linear equation ⟶ more than one equation

⇒ **solve the linear equation**

$$x - y = 1$$
$$2x + y = 6$$

$$x - \cancel{y} = 1$$
$$2x + \cancel{y} = 6$$
$$3x = 7$$
$$x = 7/3$$

$$2\left(\frac{7}{3}\right) + y = 6$$
$$\frac{14}{3} + y = 6$$
$$y = 6 - \frac{14}{3}$$
$$y = \frac{18 - 14}{3}$$
$$y = 4/3$$

$\left(\frac{7}{3}, \frac{4}{3}\right)$

consistent

Inconsistent

$\Rightarrow$ $x + y = 4$

$3x + 3y = 6$

$$\cancel{3x} + \cancel{3y} = 12$$
$$-\cancel{3x} + \cancel{3y} = 6$$
$$\boxed{0 = 6}$$
$\rightarrow$ it contradicts the basic rule of maths.

$\Rightarrow$ $4x - 2y = 1$ —①

$16x - 8y = 4$ —②

Multiply by 4 eq(1)

$$\cancel{16x} - \cancel{8y} = \cancel{4}$$
$$-\cancel{16x} + \cancel{8y} = \cancel{4}$$
$$\boxed{0 = 0}$$
$\longrightarrow$ consistent

:- Parametric method:-

In this we express x in terms of y

∵ Here y is parameter

$\Rightarrow$ t ki value $0, 1, -1$

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$$4x - 2y = 1$$

$$4x = 1 + 2y$$

$$x = \frac{1+2y}{4}$$

$$x = \frac{1}{4} + \frac{1}{2}y$$

put $y = t$

$$x = \frac{1}{4} + \frac{1}{2}t$$

put value of t which is $0, -1, 1$

⇒ when $t = 0$ then

$$x = \frac{1}{4} + \frac{1}{2}(0)$$

$$x = \frac{1}{4}$$ ⇒ solution is $\left(\frac{1}{4}, 0\right)$

⇒ when $t = 1$ then

$$x = \frac{1}{4} + \frac{1}{2}(1)$$

$$x = \frac{1}{4} + \frac{1}{2}$$

$$x = \frac{1+2}{4}$$

$$x = 3/4$$ ⇒ $(3/4, 1)$

⇒ when $t = -1$

$$x = \frac{1}{4} + \frac{1}{2}(-1)$$

$$x = \frac{1}{4} - \frac{1}{2}$$

$$x = \frac{1-2}{4}$$ $x = \frac{-1}{4}$ ⇒ $\left(\frac{-1}{4}, -1\right)$

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⇒ if 3 variables involved

$$x - y + 2z = 5$$
$$2x - 2y + 4z = 10$$
$$3x - 3y + 6z = 15$$

$x - y + 2z = 5 \longrightarrow$ original equation

⇒ Hmesha 1st variable lain gy

$x = 5 + y - 2z \longrightarrow (*)$

put $y = r$, $z = s$ in $(*)$

$x = 5 + r - 2(s) \longrightarrow (**)$

put any values ⇒ it may or may not be same

⇒ (r, s) $\xrightarrow{\text{put}}$ $(**)$
$(1, 0)$

$x = 5 + 1 - 2(0)$
$x = 5 + 1 - 0$
$x = 6$

solution is $(6, 1, 0)$

⇒ (r, s)
$1, -1$ then

$x = 5 + 1 - 2(-1)$
$= 5 + 1 + 2$
$x = 8$

solution $(8, 1, -1)$

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Augmented matrix

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Q

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

ANS

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right]$$

5 (a)

$$\begin{bmatrix} 2 & 0 & 0 \\ 3 & -4 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

ANS

$$\left[\begin{array}{cc:c} 2 & 0 & 0 \\ 3 & -4 & 0 \\ 0 & 1 & 1 \end{array}\right] = \begin{bmatrix} 2x_1 + 0x_2 = 0 \\ 3x_1 - 4x_2 = 0 \\ 0x_1 + 1x_2 = 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 3 & 0 & -2 & 5 \\ 7 & 1 & 4 & -3 \\ 0 & -2 & 1 & 7 \end{bmatrix}$$

ANS

$$\left[\begin{array}{ccc:c} 3 & 0 & 2 & 5 \\ 7 & 1 & 4 & -3 \\ 0 & -2 & 1 & 7 \end{array}\right] = \begin{bmatrix} 3x_1 \overset{0x_2}{} - 2x_3 = 5 \\ 7x_1 + 1x_2 + 4x_3 = -3 \\ 0x_1 - 2x_2 + 1x_3 = 7 \end{bmatrix}$$

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Solve the linear System

$$x + y + 2z = 9 \longrightarrow 1$$
$$2x + 4y - 3z = 1 \longrightarrow 2$$
$$3x + 6y - 5z = 0 \longrightarrow 3$$

$-2 \times$ eq (1) and Add eq 2.

$$\begin{aligned} -\cancel{2x} - 2y - 4z &= -18 \\ \cancel{2x} + 4y - 3z &= 1 \\ \hline 2y - 7z &= -17 \end{aligned}$$

$$\begin{aligned} x + y + 2z &= 9 \\ 2y - 7z &= -17 \\ 3x + 6y - 5z &= 0. \end{aligned}$$

$-3 \times$ eq 1 and add eq 3.

$$\begin{aligned} -\cancel{3x} - 3y - 6z &= -27 \\ \cancel{3x} + 6y - 5z &= 0 \\ \hline 3y - 11z &= -27 \end{aligned}$$

$$\begin{aligned} x + y + 2z &= 9 \\ 2y - 7z &= -17 \longrightarrow 2' \\ 3y - 11z &= -27 \end{aligned}$$

$1/2 \times$ eq $(2)'$

$$y - 7/2\, z = -17/2.$$
$$x + y + 2z = 9$$

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$$y - \tfrac{7}{2} z = -17/2 \longrightarrow 2''$$
$$3y - 11z = -27 \longrightarrow 3'$$

$(-3) \times$ eq $2''$ and Add eq $3'$

$$-3y + \tfrac{21}{2} z = 51/2$$
$$3y - 11z = -27$$
$$-\tfrac{1}{2} z = -\tfrac{3}{2}$$

$$\boxed{z = 3}$$

$y = 2$, $x = 1$

system $(1, 2, 3)$

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Solve the linear System of linear equation by elementry row operation.

$$x + y + 2z = 9$$
$$2x + 4y - 3z = 1$$
$$3x + 6y - 5z = 0.$$

5th

1st $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 4th

2nd

3rd

Argumented Matrix.

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{array}\right]$$

$R_2 - 2R_1 \longrightarrow R_2$ Jis per operation kar operate karwana hota h.

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 3 & 6 & -5 & 0 \end{array}\right]$$

$R_3 - 3R_1 \longrightarrow R_3.$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 3 & 11 & -27 \end{array}\right]$$

$\frac{1}{2} \times R_2 \longrightarrow R_2.$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & -7/2 & -17/2 \\ 0 & 3 & -11 & -27 \end{array}\right]$$

$R_3 - 3 \times R_2 \longrightarrow R_3.$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & -1/2 & -3/2 \end{array}\right]$$

$-2 \times R_3 \longrightarrow R_3.$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$R_2 + 7/2 \times R_3 \longrightarrow R_2.$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$R_1 (-2) \times R_3 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$R_1 - R_2 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\left.\begin{array}{l} x = 1 \\ y = 2 \\ z = 3 \end{array}\right\}$$

For equations / for System of linear equation.

$\Rightarrow$ Multiply an equation through by a non-zero Constant.

$\Rightarrow$ Interchange two equation.

$\Rightarrow$ Add a Constant time one equation to another.

For Argumented Matrix.

$\Rightarrow$ Multiply a row through by a non-zero Constant.

$\Rightarrow$ Interchange two rows.

$\Rightarrow$ Add a Constant time one row to another.

These are called elementary row operations on a matrix.

There are following the methods to solve the linear equations.

1st

⇒ Gassian Elimination method

⇒ Row echlon method

2nd

⇒ Gass's Jorden method

⇒ Reduced row echlon method

<u>1st</u> Row echlon / Gassian elimination

Diagonal line کے نیچے سارے zero (0) ہوں.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

All are zero

Diagonal line

<u>2nd</u> Reduced row echlon / Gass's Jorden elimination

Diagonal line کے اوپر اور نیچے سارے zero ہوں

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

سب zero ہوں

Diagonal line

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$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$x = 0$ — 1

$y + 2z = 0$ — 2

$\boxed{0 = 1}$ — 3

From 3 it contradicts the basics of maths. This is called inconsistent solution.

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$0x + 0y + 0z = 0$ ⟶ It is valid

$x + 3z = -1$

$x = -1 - 3z$

$x = -1 - 3t$

$y - 4z = 2$

$y = 2 + 4z$

$y = 2 + 4t$

$t = 0$

$x = -1$

$y = 2$

$\Rightarrow (-1, 2, 0)$ ⟶ solution

:- Note :-

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

پہلی equation میں سے پہلی رو variable کو چھوڑ کر باقی سارے free variables ہوں گے

(c)

$$\left[\begin{array}{ccc|c} 1 & -5 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Degenerate equation

$$0x + 0y + 0z = 0$$
$$0x + 0y + 0z = 0$$

$$x - 5y + z = 4$$
$$x = 4 + 5y - z$$

$$y = s$$
$$z = t$$

$$x = 4 + 5s - t \qquad \begin{pmatrix} s & t \\ 1, & 0 \end{pmatrix}$$
$$x = 4 + 5(1) - 0$$
$$x = 4 + 5$$
$$x = 9 \qquad (9, 1, 0)$$

$$x = 4 + 5(-1) - 1$$
$$= 4 + (-5) - 1$$
$$x = 4 - 6$$
$$x = -2 \qquad (-2, -1, 1)$$

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Note:-

⇒ Left most column daikhna hota ha.

⇒ After solving parametric equation we get solution is called general solution.

Rectangular matrix

$$\left[\begin{array}{ccccc|c} 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -10 & 6 & 12 & 28 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{array}\right]$$

(interchange)

$\longleftrightarrow$ R_1 in to R_2

$$\left[\begin{array}{ccccc|c} 2 & 4 & -10 & 6 & 12 & 28 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{array}\right]$$

$\frac{1}{2} R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{array}\right]$$

جب جب reduced matrix ہوتا ہے تب تب وہ submatrix بن جاتا ہے

$-2R_1 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 0 & 0 & 5 & 0 & -17 & -29 \end{array}\right]$$

$-\frac{1}{2} R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & 5 & 0 & -17 & -29 \end{array}\right]$$

$R_3 - 5R_2 \longrightarrow R_3$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 1 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & 0 & 0 & 1/2 & 1 \end{array}\right]$$

$2R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

Row echlon form

$-1/2 R_3 + R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$R_1 - 6R_3 \longrightarrow R_1$

$$\left[\begin{array}{cccccc} 1 & 2 & -5 & 3 & 0 & 10 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 12 \end{array}\right]$$

$R_1 + 5R_2 \longrightarrow R_1$

$$\left[\begin{array}{cccccc} 1 & 2 & 0 & 3 & 0 & 7 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

Q Solve the following system

$$x_1 + 3x_2 - 2x_3 + 5x_4 = 4$$
$$2x_1 + 8x_2 - x_3 + 9x_4 = 9$$
$$3x_1 + 5x_2 - 12x_3 + 7x_4 = 7$$

This is Homogeneous linear eq.

Agumented matrix

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & 5 & 4 \\ 2 & 8 & -1 & 9 & 9 \\ 3 & 5 & -12 & 17 & 7 \end{array}\right]$$

$$R_2 - 2R_1 \longrightarrow R_2$$
$$R_3 - 3R_1 \longrightarrow R_3$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & 5 & 4 \\ 0 & 2 & 3 & -1 & 1 \\ 0 & -4 & 2 & -5 & -5 \end{array}\right]$$

$$\tfrac{1}{2} R_2 \longrightarrow R_2$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & 5 & 4 \\ 0 & 1 & 3/2 & -1/2 & 1/2 \\ 0 & -4 & -6 & 2 & -5 \end{array}\right]$$

$$4R_2 + R_3 \longrightarrow R_3$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & 5 & 4 \\ 0 & 1 & 3/2 & -1/2 & 1/2 \\ 0 & 0 & 0 & 0 & -3 \end{array}\right]$$

$0x_1 + 0x_2 + 0x_3 = -3$

Degenerate equation so this equation has no solution.

Solve the linear system

Non-homo

$x + 2y + 3z = 9$
$2x - y + z = 8$
$3x - z = 3$

⇒ system سے سے سے echlon کو reduced row form میں لے کر back substitution use کی گئی۔

By Gauss Jorden method

First make agumentary matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 2 & -1 & 1 & 8 \\ 3 & 0 & -1 & 3 \end{array}\right]$$

$R_2 (-2R_1) \longrightarrow R_2$
$R_3 + (-3R_1) \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & -5 & -5 & -10 \\ 0 & -6 & -10 & -24 \end{array}\right]$$

$(-1/5) R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & 1 & 2 \\ 0 & -6 & -10 & -24 \end{array}\right]$$

$R_3 + 6R_2 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -4 & -12 \end{array}\right]$$

$(-1/4) R_3 \longrightarrow R_3$ ⇒ This is for both square as well as rectangular matrix.

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Back substitution

(Always start from last row)

$\boxed{z = 3}$

$y + z = 2$

$y = 2 - z$

$y = 2 - 3$

$\boxed{y = -1}$

$x + 2y + 3z = 9$

$x + 2(-1) + 3(3) = 9$

$x - 2 + 3(3) = 9$

$x = \cancel{9} - \cancel{9} + 2$

$\boxed{x = 2}$

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$R_2 - R_3 \rightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$+ (-3) R_3 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$R_1 + (-R_2) \longrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \Rightarrow$$ Reduced row echlon form or Gass Jorden method.

$$\left.\begin{array}{l} x = 2 \\ y = -1 \\ z = 3 \end{array}\right] \longleftrightarrow$$ unique solution.

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Q Use Gass Jorden or Gass elimination to solve the system of linear system

$$x + y - z = 9$$
$$y + 3z = 3$$
$$-x - 2z = 2$$

Write Agumented matrix.

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & 1 & 3 & 3 \\ -1 & 0 & -2 & 2 \end{array}\right]$$

$R_3 + R_1 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & 1 & 3 & 3 \\ 0 & 1 & -3 & 11 \end{array}\right]$$

$R_3 - R_2 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & -4/3 \end{array}\right]$$

$z = -4/3$

$y + 3z = 3$

$y + 3\left(\frac{-4}{3}\right) = 3$

$y - 4 = 3$

$y = 3 + 4$

$\boxed{y = 7}$

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$x + y - 2 = 9$

$x + 7 + \frac{4}{3} = 9$

$x = 9 - \frac{4}{3} - 7$

$x = \frac{27 - 4 - 21}{3}$

$x = \frac{6 - 4}{3}$

$x = 2/3$

Q

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Q#06

$$2x_1 + 2x_2 - 2x_3 = 0$$
$$-2x_1 + 5x_2 + 2x_3 = 1$$
$$8x_1 + x_2 + 4x_3 = -1$$

$$\left[\begin{array}{ccc|c} 2 & 2 & -2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right]$$

$\frac{1}{2}R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right]$$

$R_2 + 2R_1 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 7 & 0 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right]$$

$\frac{1}{7}R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1/7 \\ 8 & 1 & 4 & -1 \end{array}\right]$$

$R_3 - 8R_1 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1/7 \\ 0 & -7 & 12 & -1 \end{array}\right]$$

$R_3 + 7R_2 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1/7 \\ 0 & 0 & 12 & 0 \end{array}\right]$$

By using Back

$12x_3 = 0$

$x_3 = 0$

$x_2 = 1/7$

$x_1 + x_2 - x_3 = 0$

$x_1 + \left(\frac{1}{7}\right) - 0 = 0$

$x_1 + \frac{1}{7} = 0$

$x_1 = -1/7$

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Q#8 $-2b + 3c = 1$

$3a + 6b - 3c = -2$

$6a + 6b + 3c = 5$

$\Rightarrow$ first convert this in to agumented matrix

$$\left[\begin{array}{ccc|c} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 5 \end{array}\right]$$

$\Rightarrow$ Interchange Row 1 & Row 2

$$\left[\begin{array}{ccc|c} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 6 & 6 & 3 & 5 \end{array}\right]$$

$\Rightarrow$ $\frac{1}{3} R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2/3 \\ 0 & -2 & 3 & 1 \\ 6 & 6 & 3 & 5 \end{array}\right]$$

$\Rightarrow$ $\frac{1}{2} R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2/3 \\ 0 & 1 & -3/2 & -1/2 \\ 6 & 6 & 3 & 5 \end{array}\right]$$

$\Rightarrow$ $R_3 - 6R_1 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2/3 \\ 0 & 1 & -3/2 & -1/2 \\ 0 & -6 & 9 & 10 \end{array}\right]$$

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$\Rightarrow R_3 + 6R_2 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2/3 \\ 0 & 1 & -3/2 & -1/2 \\ 0 & 0 & 0 & 13 \end{array}\right]$$

$\Leftarrow$ **Inconsistent** $\Rightarrow$

Question # 09

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10 \end{aligned}$$

$\Rightarrow$ first convert this into augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right]$$

$R_2 + R_1 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 3 & -7 & 4 & 10 \end{array}\right]$$

$-1R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 3 & -7 & 4 & 10 \end{array}\right]$$

$R_3 - 3R_1 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & -10 & -2 & -14 \end{array}\right]$$

$R_3 + 10R_2 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -104 \end{array}\right]$$

$-1/52\, R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\Rightarrow$ Row echlon form

$R_2 + 5R_3 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$R_1 - 2R_3 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$R_1 - R_2 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\Rightarrow$ **Reduced echlon form**

$$\begin{vmatrix} x_1 = 3 \\ x_2 = 1 \\ x_3 = 2 \end{vmatrix}$$

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Examples

Ex#3

unique solution

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 5 \end{array}\right]$$

This is basically reduced row echlon form

$$x_1 = 3$$
$$x_2 = -1$$
$$x_3 = 0$$
$$x_4 = 5$$

This system has unique solution

$$(x_1, x_2, x_3, x_4) \rightarrow (3, -1, 0, 5)$$

Ex#4

(a) $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

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The equation that corresponds to the last row of augumented matrix

$$0x + 0y + 0z = 1$$

This system is inconsistent.

(b)

$$\begin{bmatrix} 1 & 0 & 3 & -1 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$0x + 0y + 0z = 0$$

$$x_1 + 3\cancel{y}_3 = -1$$

$$\cancel{y}\ y - 4z = 2$$

x and y are leading 1's in the augmented

$$x + 3z = -1$$

$$x = -1 - 3z$$

$$y = 2 + 4z$$

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By parametrization

put $z = t$

$x = -1 - 3t$

$y = 2 + 4t$

put $t = 0$ in bothequations

$t = z = 0$

$x = -1 - 3(0)$

$x = -1$

$y = 2 + (4)(0)$

$= 2$ $(-1, 2, 0)$

for $t = 1$ in both equation

$x = -1 - 3(1)$

$= -1 - 3$

$= -4$

$y = 2 + (4)(1)$

$= 6$

$$\begin{array}{ccc} x & y & z \\ (-4, & 6, & 1) \end{array}$$

(c) $\begin{bmatrix} 1 & -5 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

we can omit the equations

$x - 5y + z = 4$

Thats x is the leading element

$x - 5y + z = 4$ —— (*)

put $y = t$

$z = s$ —— in (*)

$x - 5t + s = 4 \rightarrow (1)$

put $t = 0$ and $s = 1$ then eq (1) becomes

$x - 5(0) + 1 = 4$

$x + 1 = 4$

$x + 1 = 4$

$x = 4 - 1$

$x = 3$ $\Rightarrow (3, 0, 1)$

General solution

If a lineae system has infinitely many solutions then a set of parametric equations from which all solutions can be obtained by assigning numericals values to the parameters.

Elimination methods

$$\begin{bmatrix} 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -10 & 6 & 12 & 28 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{bmatrix}$$

step 1

locate the left most coloumn that does not consist entirely of zeros

$$\left[\begin{array}{ccccc|c} 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -10 & 6 & 12 & 28 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{array}\right]$$

↳ Left most non-zero coloumn

step 2

Interchange the top row with another row if necessary to bring entery nonzero entery to top of coloumn.

$R_1 \longrightarrow R_2$

$$\left[\begin{array}{ccccc|c} 2 & 4 & -10 & 6 & 12 & 28 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{array}\right]$$

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step 3/ If the entery that is now at the top of coloumn found in step (a) multiply the first row with $(\frac{1}{a})$ in order to leading 1

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{array}\right]$$

step 4/ $-2R_1 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 0 & 0 & 5 & 0 & -17 & -29 \end{array}\right]$$

step 5 Make echlon form

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 0 & 0 & 5 & 0 & -17 & -29 \end{array}\right]$$

left most non zero coloumn

$-\frac{1}{2}(R_2) \longrightarrow R_2$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & 5 & 0 & -17 & -29 \end{array}\right]$$

$-5R_1 + R_2 \longrightarrow R_3$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & 0 & 0 & 1/2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & 0 & 0 & 1/2 & +1 \end{array}\right]$$

$2(R_3) \longrightarrow R_3$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & -7/2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

Now that is echlon form

⇒ We make reduced row echlon form $\frac{7}{2} R_3 + R_2 \longrightarrow R_3$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$-6R_3 + R_1 \longrightarrow R_3$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -5 & 3 & 6 & 7 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{array}\right]$$

⇒ Reduced echlon form

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Example # 5

Gauss- Jorden elimination

Solve by Gauss- Jorden elimination

$$\begin{aligned} x_1 + 3x_2 - 2x_3 \qquad\qquad + 2x_5 &= 0 \\ 2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= -1 \\ 5x_3 + 10x_4 \qquad + 15x_6 &= 5 \\ 2x_1 + 6x_2 \qquad + 8x_4 + 4x_5 + 18x_6 &= 6 \end{aligned}$$

⇒ Augumented matrix

$$\left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{array}\right]$$

$R_2 - 2R_1 \longrightarrow R_2$

$R_4 - 2R_1 \longrightarrow R_4$

$$\left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 0 & 0 & 4 & 8 & 0 & 18 & 6 \end{array}\right]$$

$-R_2 \longrightarrow R_2$

$R_3 - 5R_2 \longrightarrow R_3$

$R_4 - 4R_2 \longrightarrow R_4$

$$\left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 6 & 2 \end{array}\right]$$

Interchange

$R_3 \longrightarrow R_4$

$\frac{1}{6} R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccccc|c} 1 & 3 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2 - 3R_3 \longrightarrow R_2$

$R_1 + 2R_2 \longrightarrow R_1$

$$\left[\begin{array}{cccccc|c} 1 & 3 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Corresponding system of eq

$$x_1 + 3x_2 + 4x_4 + 2x_5 = 0$$
$$x_3 + 2x_4 = 0$$
$$x_6 = 1/3$$

Solving for leading variables,

$$x_1 = -3x_2 - 4x_4 - 2x_5$$
$$x_3 = -2x_4$$
$$x_6 = 1/3$$

Solve parametically (r, s, t)

$x_1 = -3r - 4s - 2t$

$x_2 = r, \quad x_3 = -2s, \quad x_4 = s$

$x_5 = t \quad x_6 = 1/3$

Homogeneous linear system

A system of linear equation is said to be the homogeneous if the constant terms all are zero

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots a_{1n}x_n = 0$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = 0$$

⇒ Every homogeneous system of linear equations is consistent because all such systems have $x_1 = 0, x_2 = 0, \cdots x_n = 0$, as a solution

This solution is called trivial solutions

if there are other solutions they are called non-trivial solutions.

A homogenous linear system always has the trivial solution.

Two possibilities for its solution

The system has only the trivial solution.

it has infinitely many solutions in addition to trivial solutions

⇒ Elementary row operation donot alter coloumns of zeros in a matrix. so the reduced row echlon form of the augmented matrix of a homogenous linear system a final coloumn of zeros. This implies that the linear system to reduced row echlon form is homogeneous just like original system.

⇒ When we constructed the homogenous linear system corresponding to augmented matrix

⇒ We ignored the row of zeros because the corresponding equation

$$0x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 + 0x_6 = 0$$

Free variable Theorem for Homogeneous systems

If a homogenous linear system has n unknowns and if the reduced row echlon form of its augmented matrix has r non-zero rows then system has $n-r$ free variables.

Statement

A homogenous linear system with more unknowns than equations has infinitely many solution.

Gassian elimination and Back substitution

For large linear system that require a computer solution. It is generally more efficient to use Gassian elimination (reduction to row echlon form) followed by a unique technique known as Back substitution.

Example #7

$$\left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

To solve the corresponding system of equations

$$x_1 + 3x_2 - 2x_3 + 2x_5 = 0$$

$$x_3 + 2x_4 + 3x_6 = 1$$

$$x_6 = \frac{1}{3}$$

step

Solve the equation for the leading variables

$$x_1 = -3x_2 + 2x_3 - 2x_5$$

$$x_3 = 1 - 2x_4 - 3x_6$$

$$x_6 = 1/3$$

Begining with the bottom equation and working upward successively substitute each equation into all equations above it

substitute $x_6 = 1/3$ into second equation

$$x_1 = -3x_2 + 2x_3 - 2x_5$$
$$x_3 = -2x_4$$
$$x_6 = 1/3$$

substituting $x_3 = -2x_4$ in first eq

$$x_1 = -3x_2 + 2(-2x_4) - 2x_5$$
$$x_1 = -3x_2 - 4x_4 - 2x_5$$

$$x_3 = -2x_4$$
$$x_6 = 1/3$$

Assign arbitrarily values r, s, t

$\Rightarrow x_1 = -3r - 4s - 2t$

$\Rightarrow x_2 = r$

$\Rightarrow x_3 = -2s$

$\Rightarrow x_4 = s$

$\Rightarrow x_5 = t$

$\Rightarrow x_6 = 1/3$

Example 8

$$\left[\begin{array}{cccc|c} 1 & -3 & 7 & 2 & 5 \\ 0 & 1 & 2 & -4 & 1 \\ 0 & 0 & 1 & 6 & 9 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$0x_1 + 0x_2 + 0x_3 + 0x_4 = 1$$

Inconsistent

(b) $\left[\begin{array}{cccc|c} 1 & -3 & 7 & 2 & 5 \\ 0 & 1 & 2 & -4 & 1 \\ 0 & 0 & 1 & 6 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$0x_1 + 0x_2 + 0x_3 + 0x_4 = 0$

which has no effect on solution

4) $\left[\begin{array}{cccc|c} 1 & -3 & 7 & 2 & 5 \\ 0 & 1 & 2 & -4 & 1 \\ 0 & 0 & 1 & 6 & 9 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$

The last row corresponds to the equation

$x_4 = 0$

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Q No 10

$$2x_1 + 2x_2 + 2x_3 = 0$$
$$-2x_1 + 5x_2 + 2x_3 = 1$$
$$8x_1 + x_2 + 4x_3 = -1$$

⇒ Augmented matrix

$$\left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right]$$

$\frac{1}{2} R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right]$$

$(2) R_1 + R_2 \longrightarrow R_2$

$-8 R_1 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & 4 & -1 \end{array}\right]$$

$\frac{1}{7} R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & -7 & 4 & -1 \end{array}\right]$$

$7R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$0 = 0 \longrightarrow$ Ignore

$$x_1 + x_2 + x_3 = 0$$

$$x_2 + \frac{4}{7} x_3 = \frac{1}{7}$$

let $x_3 = t$

$$x_2 = \frac{1}{7} - \frac{4}{7} t$$

$$x_1 = \frac{-1}{7} - \frac{3}{7} t$$

In parametric form.

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Homogenous linear equation:-

Question # 15

$$2x_1 + x_2 + 3x_3 = 0$$
$$x_1 + 2x_2 = 0$$
$$x_2 + x_3 = 0$$

Augmented matrix

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$\Rightarrow \frac{1}{2}R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$\Rightarrow R_2 - R_1 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 0 \\ 0 & -3/2 & 3/2 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$\Rightarrow (-2/3)R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$(-1) R_2 + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$x_1 + \frac{1}{2} x_2 + 3x_3 = 0$$

$$x_2 - x_3 = 0$$

$$x_3 = 0$$

put in ①

$$x_2 - 0 = 0$$

$$x_2 = 0$$

$$x_1 + 0 + 0 = 0$$

$$x_1 = 0$$

$(0, 0, 0)$ is the solution of this system

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Q no16

$$2x_1 - x_2 - 3x_3 = 0$$
$$-x_1 + 2x_2 - 3x_3 = 0$$
$$x_1 + x_2 + 4x_3 = 0$$

Augmented matrix

$$\left[\begin{array}{ccc|c} 2 & -1 & -3 & 0 \\ -1 & 2 & -3 & 0 \\ 1 & 1 & 4 & 0 \end{array}\right]$$

$(1/2) R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & -1/2 & -3/2 & 0 \\ -1 & 2 & -3 & 0 \\ 1 & 1 & 4 & 0 \end{array}\right]$$

$R_1 + R_2 \rightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & -1/2 & -3/2 & 0 \\ 0 & 3/2 & 3/2 & 0 \\ 1 & 1 & 4 & 0 \end{array}\right]$$

$R_1 - R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & -1/2 & -3/2 & 0 \\ 0 & 3/2 & 3/2 & 0 \\ 0 & -3/2 & 4 & 0 \end{array}\right]$$

$(2/3) R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & -3/2 & -3/2 & 0 \\ 0 & 1 & 3/2 & 0 \\ 0 & 3/2 & 4 & 0 \end{array}\right]$$

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$(-3/2) R_2$

Q18)

$(-3/2)R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & -3/2 & -3/2 & 0 \\ 0 & 1 & 3/2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$x_1 - \frac{3}{2}x_2 - \frac{3}{2}x_3 = 0$$

$$x_2 + \frac{3}{2}x_3 = 0$$

$$x_3 = 0$$

$$x_2 = 0 \qquad x_1 = 0$$

$$(0, 0, 0)$$

Q18)

$$2x + 2y + 4z = 0$$
$$w - y - 3z = 0$$
$$2w + 3x + y + z = 0$$
$$-2w + x + 3y - 2z = 0$$

Agumented matrix

$$\left[\begin{array}{cccc|c} 0 & 2 & 2 & 4 & 0 \\ 1 & 0 & -1 & -3 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 0 \end{array}\right]$$

$R_2 \longrightarrow R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 0 \end{array}\right]$$

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$-2R_1 + R_3 \longrightarrow R_3$

$2R_1 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 0 & 3 & 3 & 7 & 0 \\ 0 & 1 & 1 & -8 & 0 \end{array}\right]$$

$(1/2) R_2 \longrightarrow R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 3 & 3 & 7 & 0 \\ 0 & 1 & 1 & -8 & 0 \end{array}\right]$$

$(-3)R_2 + R_3 \longrightarrow R_3$

$(-1)R_2 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -10 & 0 \end{array}\right]$$

$10R_3 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$w - y - 3z = 0$$

$$x + y + 2z = 0$$

$$z = 0$$

put z in (i) and (2)

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$w - y = 0$

$x + y = 0$

$x = 0$

By parametric method

$w = y$

$x = -y$

let $y = t$

$w = t$

$x = -t$

$z = 0$

solution to this

$(t, -t, t, 0)$

Q22

$$2I_1 - I_2 + 3I_3 + 4I_4 = 9$$
$$I_1 - 2I_3 + 7I_4 = 11$$
$$3I_1 - 3I_2 + I_3 + 5I_4 = 8$$
$$2I_1 + I_2 + 4I_3 + 4I_4 = 10$$

Agumented matrix

$$\left[\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 9 \\ 1 & 0 & -2 & 7 & 11 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{array}\right]$$

$R_2 \longrightarrow R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 7 & 11 \\ 2 & -1 & -2 & 4 & 9 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{array}\right]$$

$R_4 \overset{\text{minus}}{-} R_2 \longrightarrow R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 7 & 11 \\ 0 & 2 & 1 & 0 & 1 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{array}\right]$$

$(-3)R_1 + R_3 \longrightarrow R_3$

$(-2)R_1 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 7 & 11 \\ 0 & 2 & 1 & 0 & 1 \\ 0 & -3 & 7 & -16 & -25 \\ 0 & 1 & 8 & -10 & -12 \end{array}\right]$$

$R_4 \longrightarrow R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & 8 & -10 & -12 \\ 0 & 0 & 1 & -16 & -25 \\ 0 & 0 & -3 & 0 & -12 \end{array}\right]$$

$3R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & 8 & -10 & -12 \\ 0 & 0 & 31 & -46 & -61 \\ 0 & 2 & 1 & 0 & 1 \end{array}\right]$$

$-2R_2 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & 8 & -10 & -12 \\ 0 & 0 & 31 & -46 & -61 \\ 0 & 0 & -15 & 20 & 25 \end{array}\right]$$

$1/31 \; R_2 \longrightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & 8 & -10 & -12 \\ 0 & 0 & 1 & -46/31 & -61/31 \\ 0 & 0 & -15 & 20 & 25 \end{array}\right]$$

$(1/5)\; R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & 8 & -10 & -12 \\ 0 & 0 & 1 & -46/31 & -61/31 \\ 0 & 0 & -3 & 4 & 5 \end{array}\right]$$

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~~$R_3 + R_4 \longrightarrow R_4$~~

$2R_3 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 7 & 11 \\ 0 & 1 & 8 & -10 & -12 \\ 0 & 0 & 1 & -46/31 & -61/31 \\ 0 & 0 & 0 & -14/31 & -28/31 \end{array}\right]$$

$\left(-31/14\right) R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & 8 & -10 & -12 \\ 0 & 0 & 1 & -4 & -61/31 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

(1) $I_1 - 2I_1 + 7I_4 = 11$

(2) $I_2 + 8I_3 - 10I_4 = -12$

(3) $I_3 - 46/31\, I_4 = -61/31$

$$\boxed{I_4 = 2}$$

put I_4 in (3)

$$I_3 - 46/31\,(2) = \frac{-61}{31}$$

$$I_3 = \frac{-61}{31} + \frac{92}{31}$$

$$I_3 = \frac{-61+92}{31} = 31/31$$

$$\boxed{I_3 = 1}$$

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put both in 2

$I_2 + 8(1) - 10(2) = -12$

$I_2 = -12 - 8 - 20$

$\boxed{I_2 = 0}$

put I_2, I_4 in 1

$I_1 - 2(0) + 7(2) = 11$

$I_1 = 11 - 14$

$\boxed{I_1 = -3}$

Solution for this system

$(-3, 0, 1, 2)$.

Q 22

$$z_3 + z_4 + z_5 = 0$$
$$-z_1 - z_2 + 2z_3 - 3z_4 + z_5 = 0$$
$$z_1 + z_2 - 2z_3 \qquad - z_5 = 0$$
$$2z_1 + 2z_2 - z_3 \qquad + z_5 = 0$$

Augmented matrix

$$\left[\begin{array}{ccccc|c} 0 & 0 & 1 & 1 & 1 & 0 \\ -1 & -1 & 2 & -3 & 1 & 0 \\ 1 & 1 & -2 & 0 & -1 & 0 \\ 2 & 2 & -1 & 0 & 1 & 0 \end{array}\right]$$

$-R_2 \longrightarrow R_1$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & -2 & 0 & -1 & 0 \\ 2 & 2 & -1 & 0 & 1 & 0 \end{array}\right]$$

$R_1 - R_3 \longleftrightarrow R_3$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & -1 & -1 & 1 & 0 \end{array}\right]$$

$2R_1 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & -6 & 3 & 0 \end{array}\right]$$

$-3R_2 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & -9 & 0 & 0 \end{array}\right]$$

$\frac{1}{3} R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -9 & 0 & 0 \end{array}\right]$$

$9R_3 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Back substitution

$z_1 + z_2 - 2z_3 + 3z_4 - z_5 = 0$

$z_3 + z_4 + z_5 = 0$

$z_4 = 0$

$z_1 = -z_2 + 2z_3 - 3z_4 + z_5$

$z_1 = -z_2 + 2z_3 - 0 - z_5$

$z_1 = -z_2 + 2z_3 - z_5$

So

$$z_1 = -z_2 - 2z_3 + z_5$$
$$z_1 = -z_2 - z_5$$
$$z_1 = -z_2 - z_5$$

$$z_3 = -z_5$$

If we assign the arbitrary values s & t respectively to the z_2 & z_5 general solution is given by

$$z_1 = -s - t$$
$$z_3 = -t$$

So,

$$z_1 = -s - t$$
$$z_2 = s$$
$$z_3 = t$$
$$z_4 = 0$$
$$z_5 = t$$

Question #17

$$3x_1 + x_2 + x_3 + x_4 = 0$$
$$5x_1 - x_2 + x_3 - x_4 = 0$$

Augmented matrix

$$\left[\begin{array}{cccc|c} 3 & 1 & 1 & 1 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{array}\right]$$

$\frac{1}{3} R_1 \longrightarrow R_1$

$$\left[\begin{array}{cccc|c} 1 & 1/3 & 1/3 & 1/3 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{array}\right]$$

$-5R_3 + R_2 \longrightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 1/3 & 1/3 & 1/3 & 0 \\ 0 & -8/3 & -2/3 & -8/3 & 0 \end{array}\right]$$

$-3/8 \, R_2 \longrightarrow R_2$

$$\left[\begin{array}{cccc|c} 1 & 1/3 & 1/3 & 1/3 & 0 \\ 0 & 1 & 1/4 & 1 & 0 \end{array}\right]$$

$-1/3 \, R_2 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1/4 & 0 & 0 \\ 0 & 1 & 1/4 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1/4 & 0 & 0 \\ 0 & 1 & 1/4 & 1 & 0 \end{array}\right]$$

$$x_1 + 1/4 \, x_3 = 0$$
$$x_2 + x_3 \frac{1}{4} + x_4 = 0$$

so coresponding linear system

$$x_1 = -\tfrac{1}{4} x_3$$
$$x_2 = -\frac{1}{4} x_3 - x_4$$

if we assign x_3 & x_4 the arbitrary values s & t respectively, then genral solution is given by

$$x_1 = -\frac{1}{4} \cdot s$$
$$x_2 = \frac{-1}{4} s - t$$
$$x_3 = s$$
$$x_4 = t$$

Q # 19

Augmented matrix

$$\left[\begin{array}{cccc|c} 0 & 2 & 2 & 4 & 0 \\ 1 & 0 & -1 & -3 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ 3 & 1 & 3 & -2 & 0 \end{array}\right]$$

$R_1 \longrightarrow R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ 3 & 1 & 3 & -2 & 0 \end{array}\right]$$

$R_3 + R_4 \longrightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 0 & 4 & 4 & -1 & 0 \\ -2 & 1 & 3 & -2 & 0 \end{array}\right]$$

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$2R_1 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 0 & 4 & 4 & -1 & 0 \\ 0 & 1 & 1 & 4 & 0 \end{array}\right]$$

$R_2 \longleftrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 4 & 4 & -1 & 0 \\ 0 & 2 & 2 & 4 & 0 \end{array}\right]$$

$-4R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 0 & -17 & 0 \\ 0 & 2 & 2 & 4 & 0 \end{array}\right]$$

$-2R_2 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 0 & -17 & 0 \\ 0 & 0 & 0 & -4 & 0 \end{array}\right]$$

$-1/17\, R_3 \longrightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 0 & -17/-17 & 0 \\ 0 & 0 & 0 & -4 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -4 & 0 \end{array}\right]$$

$4R_3 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

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$-4R_3 + R_2 \longrightarrow R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$3R_3 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

corespondence linear system

$w - y = 0$

$x + y = 0$

$z = 0$

parametric method

$w = y$

$x = -y$

$z = 0$

we assign abitalry value t.

$w = t$

$x = t$

$z = 0$

$(t, -t, t, 0)$

$w = t, \quad x = -t$

$y = t \quad z = 0$

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$$x_1 + 3x_2 + x_4 = 0$$
$$x_1 + 4x_2 + 2x_3 = 0$$
$$-2x_2 - 2x_3 - x_4 = 0$$
$$2x_1 - 4x_2 + x_3 + x_4 = 0$$
$$x_1 - 2x_2 - x_3 + x_4 = 0$$

Augmented matrix

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 1 & 4 & 2 & 0 & 0 \\ 0 & -2 & 2 & 1 & 0 \\ 2 & -4 & 1 & 1 & 0 \\ 1 & -2 & -1 & 1 & 0 \end{array}\right]$$

$-R_2 + R_1 \longrightarrow R_2$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 2 & -4 & 1 & 1 & 0 \\ 1 & -2 & 1 & 1 & 0 \end{array}\right]$$

$-2R_1 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 0 & -10 & 1 & -1 & 0 \\ 1 & -2 & 1 & 1 & 0 \end{array}\right]$$

$R_5 - R_1 \longrightarrow R_5$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 0 & -10 & 1 & -1 & 0 \\ 0 & -5 & 1 & 0 & 0 \end{array}\right]$$

$2R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 2 & -3 & 0 \\ 0 & -10 & 1 & -1 & 0 \\ 0 & -5 & 1 & 0 & 0 \end{array}\right]$$

$-2R_5 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 2 & -3 & 0 \\ 0 & 0 & -1 & -1 & 0 \\ 0 & -5 & 1 & 0 & 0 \end{array}\right]$$

$5R_2 + R_5 \longrightarrow R_5$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 2 & -3 & 0 \\ 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 11 & -5 & 0 \end{array}\right]$$

$R_3 \longleftrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & -3 & 0 \\ 0 & 0 & 11 & -5 & 0 \end{array}\right]$$

$-R_3 \longrightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & +1 & +1 & 0 \\ 0 & 0 & 2 & -3 & 0 \\ 0 & 0 & 11 & -5 & 0 \end{array}\right]$$

$-2R_2 - R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -5 & 0 \\ 0 & 0 & 11 & -5 & 0 \end{array}\right]$$

$$-11R_3 + R_5 \longrightarrow R_5$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & & 0 \\ 0 & 0 & 0 & -16 & 0 \end{array}\right]$$

$$\tfrac{1}{2} R_4 \longrightarrow R_4$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -16 & 0 \end{array}\right]$$

$$16R_4 + R_5 \longrightarrow R_5$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$R_2 + R_4 \longrightarrow R_2$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$-2R_3 + R_2 \longrightarrow R_2$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

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$R_1 - R_4 \longrightarrow R_1$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$-3R_2 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{aligned} x_1 \qquad\qquad\qquad &= 0 \\ x_2 \qquad\qquad &= 0 \\ x_3 \qquad &= 0 \\ x_4 &= 0 \end{aligned}$$

so solution is trivial

$x_1 = 0$
$x_2 = 0$
$x_3 = 0$
$x_4 = 0$

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Question #7

$$x - y + 2z - w = -1$$
$$2x + y - 2z - 2w = -2$$
$$-x + 2y - 4z + w = 1$$
$$3x - 3w = -3$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{array}\right]$$

$-2R_1 + R_2 \longrightarrow R_2$

$R_1 + R_3 \longrightarrow R_3$

$-3R_1 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array}\right]$$

$1/3 R_2 \longrightarrow R_2$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & 6 & 0 & 0 \end{array}\right]$$

$-R_2 + R_3 \longrightarrow R_3$

$-3R_2 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

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$R_1 + R_2 \longrightarrow R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow x - w = -1$

$\Rightarrow y - 2z = 0$

put $w = s$

$z = \gamma$

$\Rightarrow x = s - 1$

$\Rightarrow y = 2\gamma.$

Matrices and Matrix Operations

A matrix is a rectangular array of numbers. The numbers in the array are called enteries in the matrix

Example

$$\begin{bmatrix} 1 & 2 \\ 3 & 0 \\ -1 & 4 \end{bmatrix}, \quad [\,2,\ 1\ \ 0,\ -3\,]$$

-:Rows:- The horizonal lines are called rows.

-:Coloumn:- The vertical lines are called coloumn.

-: Row matrix:- A matrix only one row is called row matrix or row vector

-: Coloumn matrix:- A matrix only one coloumn is called coloumn matrix or coloumn vector.

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Square matrix

A matrix 'A' with n rows and n columns is called square matrix

$$A = \begin{bmatrix} 2 & -3 \\ 7 & 0 \end{bmatrix}$$

⇒ number of rows is equal to number of columns.

-: Diagonal entries

$a_{11}, a_{22}, a_{33}, \cdots a_{nn}$

⇒ Equal matrices

Two matrices are defined to be equal if they have the same size and their corresponding entries are equal.

Equality of matrices

$A = \begin{bmatrix} 2 & 1 \\ 3 & x \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$

$C = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \end{bmatrix}$

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if $x = 5$ then A
values of
and B ar
all of th
entries

Example #4

Additi

$A = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

$B = \begin{bmatrix} -4 \\ 2 \\ 3 \end{bmatrix}$

$C = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$A + B = \begin{bmatrix} - \end{bmatrix}$

$A - B =$

The ex

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if $x = 5$ then $A = B$ but for all other values of x the matris A and B are not equal since not all of their corresponding entries are equal.

Example #4

Addition of and subtraction

$$A = \begin{bmatrix} 2 & 1 & 0 & 3 \\ -1 & 0 & 2 & 4 \\ 4 & -2 & 7 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -4 & 3 & 5 & 1 \\ 2 & 2 & 0 & -1 \\ 3 & 2 & -4 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$A+B = \begin{bmatrix} -2 & 4 & 5 & 4 \\ 1 & 2 & 2 & 3 \\ 7 & 0 & 3 & 5 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 6 & -2 & -5 & 2 \\ -3 & -2 & 2 & 5 \\ 1 & -4 & 11 & -5 \end{bmatrix}$$

The expressions $A+C$, $B+C$, $A-C$, $B-C$ are undefined

CHAPTER # 03

Euclidean vector spaces

Example
Finding the components of vector

$v = \overrightarrow{P_1P_2}$

$P_1(2,-1,4)$, $P_2(7,5,-8)$

$v = (7-2, 5-(-1), (-8)-4)$

$v = (5, 6, -12)$

ordered n-tuple

If n is a positive integer, then an ordered n-tuple is a sequence of a real numbers $(v_1, v_2, \ldots v_n)$. The set of all ordered n-tuples is called n-space and is denoted by R^n.

Equal vectors

Vectors $v = (v_1, v_2, \ldots v_n)$ and $w = (w_1, w_2, \ldots w_n)$ in R^n is said to be equivalent if

$v_1 = w_1, \; v_2 = w_2 \; \ldots \; v_n = w_n$

Equality of vectors

$(a, b, c, d) = (1, -4, 2, 7)$

$a = 1$

$b = -4$

$c = 2$

$d = 7$

Algebric operations using components

$v = (1, -3, 2)$

$w = (4, 2, 1)$

$v + w = (5, -1, 3)$

$2v = (2, -6, 4)$

$-w = (-4, -2, -1)$

$v - w = v + (-w)$

$= (-3, -5, 1)$

Linear combination

If w is a vector in R^n then w is said to be a linear combination of the vectors $v_1, v_2, \ldots v_r$ in R^n it can be expressed

$$W = k_1 v_1 + k_2 v_2 + \cdots + k_n v_n$$

where $k_1, k_2 \ldots k_r$ are scalars.
These scalars are called coefficients of linear combination.

Norm, Dot Product and Distance

If $v = (v_1, v_2 \ldots v_n)$ is a vector in R^n
Then the norm of v is called norm / length or magnitude of v by $|v|$ is defined by formula

$$|v| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$$

Example

$V = (-3, 2, 1)$ in R^3

$|V| = \sqrt{(-3)^2 + (2)^2 + (1)^2}$

$= \sqrt{9+4+1}$

$= \sqrt{14}$

$V = (2, -1, 3, -5)$ in R^4

$|V| = \sqrt{(2)^2 + (-1)^2 + (3)^2 + (-5)^2}$

$= \sqrt{4+1+9+25}$

$= \sqrt{39}$

Normalizing of a vector

$V = (2, 2, -1)$

$|V| = \sqrt{(2)^2 + (2)^2 + (-1)^2}$

$= \sqrt{4+4+1}$

$= \sqrt{9}$

$= 3.$

$$d = |\overrightarrow{P_1P_2}|$$

$$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Calculating Distance in R^n

$$u = (1, 3, -2, 7)$$
$$v = (0, 7, 2, 2)$$

$$d(u,v) = \sqrt{(1-0)^2 + (3-7)^2 + (-2-2)^2 + (7-2)^2}$$

$$= \sqrt{1 + 16 + 14 + 25}$$

$$= \sqrt{17 + 39}$$

$$= \sqrt{58}$$

If u and v are non zero vectors in R^2 or R^3 and if θ is the angle b/w u and v dot product of u and v is denoted by $u.v$

$$u \cdot v = |u|\,|v| \cos\theta$$

$u = 0$

$v = 0$

$u.v$ to be 0.

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Dot product

$|u| = 1 \quad |v| = \sqrt{8} = 2\sqrt{2}$

$\cos(45^\circ) = 1/\sqrt{2}$

$u \cdot v = |u|\,|v| \cos\theta$

$= |1|\,|2\sqrt{2}| \cos 45^\circ$

$= |1|\,|\cancel{2\sqrt{2}}| \cdot \frac{1}{\cancel{\sqrt{2}}}$

$= 2 \cdot 1$

$= 2$ <u>Ans</u>

Verify that

$$Au \cdot v = u \cdot A^T v$$

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$u = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$$

$$v = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$$

$$Au = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} \begin{bmatrix} 7 \\ 10 \\ 5 \end{bmatrix}$$

$$A^T v = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 4 & 0 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} -7 \\ 4 \\ -1 \end{bmatrix}$$

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$Au \cdot v =$

$u \cdot A^T v$

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$Au \cdot v = 7(-2) + 10(0) + 5(5) = 11$

$u \cdot A^T v = (-1)(-7) + 2(4) + 4(-1)$

$= 11$

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CHAPTER #04

General vector spaces

.: Example:-

Zero vector space

Let v consist of a single object, which we denote by 0

$0+0=0$ and

$k(0)=0$

Linear combination

$u=(1,2,-1)$

$v=(6,4,2)$

$w=(9,2,7)$

$w'=(4,-1,8)$

Solution

$w=k_1u+k_2v$ —— (A)

$(9,2,7)=k_1(1,2,-1)+k_2(6,4,2)$

$(9,2,7)=(k_1,2k_1,-k_1)+(6k_2,4k_2,2k_2)$

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$(9,2,7) = (K_1+6K_2,\ 2K_1+4K_2,\ -K_1+2K_2)$

$K_1+6K_2 = 9$ —— (1)

$2K_1+4K_2=2$ —— (2)

$-K_1+2K_2 = 7$ —— (3)

$2\times(1) \Rightarrow$

$$\begin{aligned} \cancel{2K_1} + 12K_2 &= 18 \\ -\cancel{2K_1} + 4K_2 &= +2 \\ \hline 8K_2 &= 16 \\ \boxed{K_2 = 2} \end{aligned}$$

$1) \Rightarrow \quad K_1 + 6(2) = 9$

$K_1 + 12 = 9$

$K_1 = 9-12$

$K_1 = -3$

$A \Rightarrow$

$W = -3u + 2v$

Check

$(9,2,7) = (-3(1,2,-1) + 2(6,4,2)$

$(9,2,7) = (-3,-6,3) + (12,8,4)$

$(9,2,7) = (9,2,7)$

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$$W' = K_1 U + K_2 U \quad \text{---} \; 2$$

$(4,-1,8) = K_1 (1,2,-1) + K_2 (6,4,2)$

$(4,-1,8) = K_1, 2K_1, -K_1 + 6K_2 + 4K_2 + 2K_2$

$(4,-1,8) = 1K_1 + 6K_2, 2K_1 + 4K_2, -K_1 + 2K_2)$

$K_1 + 6K_2 = 4 \quad \text{---} \; 1$

$2K_1 + 4K_2 = -1 \quad \text{---} \; 2$

$-K_1 + 2K_2 = 8 \quad \text{---} \; 3$

Add 1 and 3

$\cancel{K_1} + 6K_2 = 4$

$\cancel{-K_1} + 2K_2 = 8$

$8K_2 = 12$

$K_2 = 12/8$

$K_2 = 6/4$

$$\boxed{K_2 = 3/2}$$

$K_1 + 6(3/2) = 4$

$K_1 + 9 = 4$

$K_1 = 4 - 9$

$$\boxed{K_1 = -5}$$

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$w' = -5u + 3/2\, v$

Check

$(4,-1,8) = -5(1,2,-1) + 3/2\,(6,4,2)$

$= (-5,-10,5) + \frac{1}{2}(18,12,6)$

$= (-5,-10,5) + (9,6,3)$

$(4,-1,8) = (4,-4,8)$

w' is not L.C of u and v.

Testing for spanning

$v_1 = (1,1,2)$
$v_2 = (1,0,1)$
$v_3 = (2,1,3)$

$b = (b_1, b_2, b_3)$

$b = k_1v_1 + k_2v_2 + k_3v_3$

$(b_1, b_2, b_3) = k_1(1,1,2) + k_2(1,0,1) + k_3(2,1,3)$

$(b_1, b_2, b_3) = (k_1 + k_2 + 2k_3,\ k_1 + k_3,\ 2k_1 + k_2 + 3k_3)$

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$$K_1 + K_2 + 2K_3 = b_1$$
$$K_1 + K_3 = b_2$$
$$2K_1 + K_2 + 3K_3 = b_3$$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$|A| = 1\begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + 2\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}$$

$$= 1(0-1) - 1(3-2) + 2(1-0)$$

$$= -1 - 1 + 2$$

$$= -2 + 2 = 0$$

$$\boxed{|A| = 0}$$

Statement

The solution set of a homogenous linear system $Ax=0$ of m equations in n unknowns is a subspace of R^n.

Solution spaces of Homogenous system.

(a) = $\begin{bmatrix} 1 & -2 & 3 \\ 2 & -4 & 6 \\ 3 & -6 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

The solutions are

$\Rightarrow x = 2s - 3t$

$y = s$

$z = t$

$X = 2y - 3z$

or $x - 2y + 3z = 0$

This is the equation of a plane through the origin that is parallel to $v = (5,-1,1)$

Example :-

Linear independence of standard unit vectors

$e_1 = (1, 0, 0, 0, \ldots 0)$
$e_2 = (0, 1, 0 0 \ldots 0)$
$e_n = (0, 0, 0, \ldots 1)$

$i = (1, 0, 0)$
$j = (0, 1, 0)$
$K = (0, 0, 1)$

$$K_1 \hat{i} + K_2 \hat{j} + K_3 \hat{k} = 0$$

$K_1 = 0$
$K_2 = 0$
$K_3 = 0$

$$(K_1, K_2, K_3) = (0, 0, 0)$$

ih

Linear Independence in R^3

$$v_1 = (1, -2, 3)$$
$$v_2 = (5, 6, -1)$$
$$v_3 = (3, 2, 1)$$

$$k_1 v_1 + k_2 v_2 + k_3 v_3 = 0$$

$$k_1(1, -2, 3) + k_2(5, 6, -1) + k_3(3, 2, 1) = (0, 0, 0)$$
$$(k_1, -2k_1, 3k_1) + (5k_2, 6k_2, -1k_2) + (3k_3, 2k_3, k_3) = (0, 0, 0)$$

$$k_1 + 5k_2 + 3k_3 = 0 \quad \text{——— } (1)$$
$$-2k_1 + 6k_2 + 2k_3 = 0 \quad \text{——— } (2)$$
$$3k_1 - k_2 + k_3 = 0 \quad \text{——— } (3)$$

x (1) by (3) and ~~Add~~ subtract in (3)

$$\Rightarrow \quad \cancel{3k_1} + 15k_2 + 9k_3 = 0$$
$$\underline{-\cancel{3k_1} \mp k_2 \pm k_3 = 0}$$
$$16k_2 - 8k_3 = 0 \quad \text{——— } (A)$$

From (3) x by (3) and add (2)

$$18k_1 - \cancel{6k_2} + 6k_3 = 0$$
$$\underline{-2k_1 + \cancel{6k_2} + 2k_3 = 0}$$
$$16k_1 + 8k_3 = 0$$

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$16K_2 - 8K_3 = 0$

$16K_1 + 8K_3 = 0$

$16K_1 + 16K_2 = 0$

$16(K_1 + K_2) = 0$

$K_1 + K_2 = 0$

$\boxed{K_1 = -K_2}$ put in 1

$-K_2 + 5K_2 + 3K_3 = 0$

$4K_2 + 3K_3 = 0$

Multiply by 4 and subsr $4(4K_2 + 3K_3) = 0$

$16K_2 - 8K_3 = 0$

$-16K_2 + 12K_3 = 0$

$4K_3 = 0$

$\boxed{K_3 = 0}$

$4(K_2) + 3(0) = 0$

$4K_2 = 0$

$\boxed{K_2 = 0}$

$\boxed{K_1 = 0}$

Question #01

(a)

$u_1 = (-1, 2, 4)$
$u_2 = (5, -10, -20)$ in R^3

<u>Solution</u>

Consider

$a_1 u_1 + a_2 u_2 = 0$

$a_1(-1, 2, 4) + a_2(5, -10, -20) = (0, 0)$

$(-a_1, 2a_1, 4a_1) + (5a_2, -10a_2, -20a_2) = 0, 0$

$(-a_1 + 5a_2), (2a_1 + (-10a_2)), (4a_1 - 20a_2) = 0, 0$

$-a_1 + 5a_2 = 0$ —— 1

$2a_1 + (-10a_2 = 0$ —— 2

$4a_1 - 20a_2 = 0$

Multiply 4 to eq 1 and add in 3

$$\begin{aligned} -\cancel{4a_1} + \cancel{20a_2} &= 0 \\ \cancel{4a_1} - \cancel{20a_2} &= 0 \\ \hline 0 &= 0 \end{aligned}$$

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Exercise # 4.2

Question # 07

Which of the following are linear combination of

$u = (0, -2, 2)$ and $v = (1, 3, -1)$

(a)

$w = (2, 2, 2)$

$w = k_1 u + k_2 v$ —— (A)

$(2,2,2) = k_1(0,-2,2) + k_2(1,3,-1)$

$(2,2,2) = (0k_1, -2k_1, 2k_1) + (k_2, 3k_2, -k_2)$

$(2,2,2) = (0k_1 + k_2), (-2k_1 + 3k_2), (2k_1 - k_2)$

$2 = 0k_1 + k_2$

$\boxed{k_2 = 2}$

$-2k_1 + 3k_2 = 2$

$-2(k_1) + 3(2) = 2$

$-2k_1 + 6 = 2$

$-2k_1 = 2 - 6$

$-2k_1 = -4$

$k_1 = +4/+2$

$\boxed{k_1 = 2}$

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Hence $K_1 = 2$ and $K_2 = 2$ there is solution $(2,2)$ exis for a system of equation.

Linear combination is

$(2,2,2) = 2(0,-2,2) + 2(1,3,-1)$

$= (0,-4,4) + (2,6,-2)$

$= (2,2,2)$

(b) $W = (0,4,5)$

$W = K_1 u + K_2 v$ —— (A)

$(0,4,5) = K_1(0,-2,2) + K_2(1,3,-1)$

$= (0K_1 + K_2), (-2K_1 + 3K_2), (2K_1 - K_2)$

$0K_1 + K_2 = 0$

$\boxed{K_2 = 0}$

$-2K_1 + 3(0) = 4$

$-2K_1 = 4$

$K_1 = 4/-2$

$\boxed{K_1 = -2}$

Hence

$K_1 = -2$ and $K_2 = 0$

To check put in (A)

$(0,4,5) = -2(0,-2,2)$

$(0,4,5) \neq (0,4,-4)$

Hence it is not linear combination.

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(c) $(0,0,0)$

$W = 0,0,0$

$W = K_1 U + K_2 V$

$(0,0,0) = K_1(0,-2,2) + K_2(1,3,-1)$

$0,0,0 = K_2, -2K_1+3K_2, 2K_1-K_2$

$\boxed{K_2 = 0}$

$-2K_1 + 3(0) = 0$

$2K_1 = 0$

$\boxed{K_1 = 0}$

$(0,0,0) = 0(0,-2,2) + 0(1,3,-1)$

$0,0,0 = 0,0,0$

Hence it is linear combination

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Q/8/

Express the following

$u=(2,1,-4)$, $V=(1,-1,3)$

$W=(3,2,5)$

(a)

$(-9,-7,-15)$

$(-9,-7,-15) = K_1(2,1,-4) + K_2(1,-1,3) + K_3(3,2,5)$

$(-9,-7,-15) = (2K_1, K_1, -4K_1) + (K_2, -K_2, 3K_2) + (3K_3, 2K_3, 5K_3)$

$(-9,-7,-15) = (2K_1+K_2+3K_3), (K_1-K_2+2K_3), (-4K_1+3K_2+5K_3)$

$2K_1+K_2+3K_3=-9$

$K_1-K_2+2K_3=-7$

$-4K_1+3K_2+5K_3=-15$

Augumented matrix

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & -9 \\ 1 & -1 & 2 & -7 \\ 4 & 3 & 5 & -15 \end{array}\right]$$

$R_1 \longleftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -7 \\ 2 & 1 & 3 & -9 \\ 4 & 3 & 5 & -15 \end{array}\right]$$

$R_2 \longrightarrow -2R_1+R_2$, $R_3 \longrightarrow -4R_1+R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -7 \\ 0 & 3 & -1 & -5 \\ 0 & 7 & -3 & -13 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & -1 & 2 & -7 \\ 0 & 3 & -1 & 5 \\ 0 & 7 & -3 & 13 \end{array}\right]$$

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$\frac{1}{3}R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -7 \\ 0 & 1 & -11/3 & 5/3 \\ 0 & 7 & -3 & 11 \end{array}\right]$$

$R_3 \longrightarrow -7R_2 + R_3$

$R_1 \longrightarrow R_2 + R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5/3 & -16/3 \\ 0 & 1 & -11/3 & 5/3 \\ 0 & 0 & -2/3 & 4/3 \end{array}\right]$$

$-\frac{3}{2}R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5/3 & -16/3 \\ 0 & 1 & -11/3 & 5/3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$-\frac{5}{3}R_3 + R_1 \longrightarrow R_1$

$\frac{11}{3}R_3 + R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$K_1 = -2$

$K_2 = 1$

$K_3 = -2$

$(-9, -7, -15) = -2U + 1V - 2W$

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(b)

$(6, 11, 6)$

$$(6,11,6) = K_1(2,1,4) + K_2(1,-1,3) + K_3(3,2,5)$$

$$6 = 2K_1 + K_2 + 3K_3$$
$$11 = K_1 - K_2 + 2K_3$$
$$6 = 4K_1 + 3K_2 + 5K_3$$

Augumented matrix

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 6 \\ 1 & -1 & 2 & 11 \\ 4 & 3 & 5 & 6 \end{array}\right]$$

$\Rightarrow$ $R_1 \longleftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 11 \\ 2 & 1 & 3 & 6 \\ 4 & 3 & 5 & 6 \end{array}\right]$$

$\Rightarrow$ $-2R_1 + R_2 \longrightarrow R_2$
$-4R_1 + R_3 \longrightarrow R_3$
$1/3 R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 11 \\ 0 & 1 & -1/3 & -16/3 \\ 0 & 7 & -3 & -38 \end{array}\right]$$

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$-7R_2 + R_3 \longrightarrow R_3$

$R_2 + R_1 \longrightarrow R_1$

$-3/2 R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5/3 & 17/3 \\ 0 & 1 & -1/3 & -16/3 \\ 0 & 0 & 1 & -2/3 \end{array}\right]$$

$R_1 \longrightarrow -5/3 R_3 + R_1$

$R_2 \longrightarrow 1/3 R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$K_1 = 4$

$K_2 = -5$

$K_3 = 1$

$(6, 11, 6) = 4u - 5v + w$

(c)

$(0, 0, 0) = K_1(2, 1, 4) + K_2(1, -1, 3) + K_3(3, 2, 5)$

$K_1 = 0, \; K_2 = 0, \; K_3 = 0$

Q#9

which of the following are linear combination

$$A = \begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix}$$

(a) $\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix}$

$$W = \begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix} = K_1 \begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix} + K_2 \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + K_3 \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix} = \begin{bmatrix} 4K_1 & 0 \\ -2K_1 & -2K_1 \end{bmatrix} + \begin{bmatrix} K_2 & -K_2 \\ 2K_2 & 3K_2 \end{bmatrix} + \begin{bmatrix} 0 & 2K_3 \\ K_3 & 4K_3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix} = \begin{bmatrix} 4K_1 + K_2 & -K_2 + 2K_3 \\ -2K_1 + 2K_2 + K_3 & -2K_1 + 3K_2 + 4K_3 \end{bmatrix}$$

$4K_1 + K_2 = 6$

$-K_2 + 2K_3 = -8$

$-2K_1 + 2K_2 + K_3 = -1$

$-2K_1 + 3K_2 + 4K_3 = -8$

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Augumented matrix

$$\left[\begin{array}{ccc|c} 4 & 1 & 0 & 6 \\ -1 & 0 & 2 & -8 \\ -2 & 2 & 1 & -1 \\ -2 & 3 & 4 & -8 \end{array}\right]$$

$R_3 + 1/2 R_1 \longrightarrow R_3$

$R_4 + 1/2 R_1 \longrightarrow R_4$

$R_2 \longleftrightarrow R_4$

$R_3 - 5/7 R_2 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 4 & 1 & 0 & 6 \\ 0 & 7/2 & 4 & -5 \\ 0 & 0 & -13/7 & 39/7 \\ 0 & -1 & 2 & -8 \end{array}\right]$$

$R_4 + 2/7 R_2 \longrightarrow R_4$

$R_3 \longrightarrow R_4$

$R_4 + 13/22 R_3 \longrightarrow R_4$

$$\left[\begin{array}{ccc|c} 4 & 1 & 0 & 6 \\ 0 & 7/2 & 4 & -5 \\ 0 & 6 & 22/7 & -66/7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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$$4K_1 + K_2 = 6 \quad \text{——} (1)$$
$$\tfrac{7}{2}K_2 + 4K_3 = -5 \quad \text{——} (2)$$
$$\tfrac{22}{7}K_3 = -\frac{66}{7} \quad \text{——} (3)$$

Equation (3) becomes

$$K_3 = -3$$

Eq 2 $\Rightarrow$ $\tfrac{7}{2}K_2 + 4(-3) = -5$

$$\tfrac{7}{2}K_2 - 12 = -5$$

$$K_2 = 7 \cdot 2/7 = 2$$

$K_2 = 2$ put in (1)

$$4K_1 + 2 = 6$$

$$4K_1 = 4$$

$$\boxed{K_1 = 1}$$

$K_1 = 1$

$K_2 = 2$

$K_3 = -3$

w is linear combination of A, B, C.

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(b)

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$w = k_1 u_1 + k_2 v_2 + k_3 v_3$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = k_1 \begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix} + k_2 \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + k_3 \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix}$$

$4k_1 + k_2 = 0$ —— (1)

$-k_2 + 2k_3 = 0$ —— (2)

$-2k_1 + 2k_2 + k_3 = 0$ —— (3)

$-2k_1 + 3k_2 + 4k_3 = 0$ —— (4)

Us: Gauss Jorden Method

$$\left[\begin{array}{ccc|c} 4 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ -2 & 2 & 1 & 0 \\ -2 & 3 & 4 & 0 \end{array}\right]$$

$R_3 + \frac{1}{2} R_1 \longrightarrow R_3$

$R_4 + 1/2\, R_1 \longrightarrow R_4$

$$\left[\begin{array}{ccc|c} 4 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 5/2 & 1 & 0 \\ 0 & 7/2 & 4 & 0 \end{array}\right]$$

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$$R_2 \longleftrightarrow R_4$$

$$R_3 - 5/7 R_2 \longrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 4 & 1 & 0 & 0 \\ 0 & 7/2 & 4 & 0 \\ 0 & 0 & -13/7 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right]$$

$$R_4 - 12/7 R_2 \longrightarrow R_4$$

$$R_3 \xleftrightarrow{5} R_4$$

$$R_4 + \frac{13}{22} R_3 \longrightarrow R_4$$

$$\left[\begin{array}{ccc|c} 4 & 1 & 0 & 0 \\ 0 & 7/2 & 4 & 0 \\ 0 & 0 & 22/7 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$4K_1 + K_2 = 6 \longrightarrow 1$$

$$7/2\, K_2 + 4K_3 = -5 \longrightarrow 2$$

$$\frac{22}{7} K_3 = 0 \longrightarrow 3$$

$$\boxed{K_3 = 0}$$

$$Eq\ 2 \Longrightarrow 7/2\, K_2 = -5$$

$$K_2 = -10/7$$

put K_2 in eq 1

$$4K_1 - \frac{10}{7} = 6$$

$$4K_1 = 6 + \frac{10}{7}$$

$$K_1 = 13/7$$

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Q10/ $P_1 = 2 + x + 4x^2$
$P_2 = 1 - x + 3x^2$
$P_3 = 3 + 2x + 5x^2$

(a)

$-9, -7x - 15x^2$

$P = K_1P_1 + K_2P_2 + K_3P_3$

$(-9 - 7x, -15x^2) = K_1(2 + x + 4x^2) + K_2(1 - x + 3x^2)$
$+ K_3(3 + 2x + 5x^2)$

$(-9 - 7x - 15x^2) = (2K_1 + K_2 + 3K_3) + (K_1x + 2K_3x - K_2x)$

$-9 = 2K_1 + K_2 + 3K_3$ —— (1)
$-7 = K_1 - K_2 + 2K_3$ —— (2)
$-15 = 4K_1 + 3K_2 + 5K_3$ —— (3)

use Gaus Jorden

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & -9 \\ 1 & -1 & 2 & -7 \\ 4 & 3 & 5 & -15 \end{array}\right]$$

$R_1 \longleftrightarrow R_3$

$R_2 - 1/4\, R_1 \longrightarrow R_2$

$R_2 - 1/2\, R_1 \longrightarrow R_3$

$R_3 - 2/7\, R_2 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 4 & 3 & 5/2 & -15 \\ 0 & -7/4 & 3/4 & -13/4 \\ 0 & 0 & 2/7 & -4/7 \end{array}\right]$$

$\frac{7}{2} R_3 \longrightarrow R_3$

$R_2 - \frac{3}{4} R_3 \longrightarrow R_2$

$R_1 - 5 R_3 \longrightarrow R_1$

$-\frac{4}{7} R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 4 & 3 & 0 & -5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$R_1 - 2R_2 \longrightarrow R_1$

$\frac{1}{4} R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$K_1 = 2$

$K_2 = 1$

$K_3 = -2$

Hence

$-9 - 7x - 15x^2 = -2(2 + x + 4x^2) + 1(1 - x + 3x^2) - 2(3 + 2x + 5x^2)$

Q11
In each part determine wether the vectors span in R^3

$v_1 = (2,2,2)$
$v_2 = (0,0,3)$
$v_3 = (0,1,1)$

Vectors v_1, v_2, v_3 span in R^3 if they are independent

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$|A| = 2 \begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix}$$

$= 2(0-3)$

$= -6$

So v_1, v_2, v_3 are independent

They span in R^3.

(b) $V_1 = (2, -1, 3)$

$V_2 = (4, 1, 2)$

$V_3 = (3, -1, 8)$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 1 & 2 \\ 8 & -1 & 8 \end{bmatrix}$$

$$|A| = 2 \begin{vmatrix} 1 & 2 \\ -1 & 8 \end{vmatrix} + 1 \begin{vmatrix} 4 & 2 \\ 8 & 8 \end{vmatrix} + 3 \begin{vmatrix} 4 & 1 \\ 8 & -1 \end{vmatrix}$$

$$= 2(8+2) + 1(32-16) + 3(-4-8)$$

$$= 2(10) + 1(16) + 3(-12)$$

$$= 20 + 16 - 36$$

$$= 36 - 36 = 0$$

Vectors are dependent

So vector are not spaning in R^3.

Exercise #4.3

(a) $u_1 = \{-1, 2, 4\}$ $u_2 = \{5, -10, -20\}$ in R^3

u_2 is a scalar multiple of u_1.

(b) $u_1 = (3, -1)$, $u_2 = (4, 5)$
$u_3 = (-4, 7)$ in R^2

$4, 84 = 17u_2 - 19u_3$ so it is linearly dependent.

(d) $A = \begin{bmatrix} -3 & 4 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -4 \\ -2 & 0 \end{bmatrix}$ in M_{22}

Because B is a scalar multiple of A.

Q2/ Find
Vectors are linearly independent or independent in R^3

a $(-3, 0, 4)$, $(5, -1, 2)$, $(1, 1, 3)$

$a(-3, 0, 4) + b(5, -1, 2) + (1, 1, 3) = 0$

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$$-3a + 5b + c = 0 \quad \text{---} (1)$$
$$-b + c = 0 \quad \text{---} (2)$$
$$4a + 2b + 3c = 0 \quad \text{---} (3)$$

$$\begin{bmatrix} -3 & 5 & 1 \\ 0 & -1 & 1 \\ 4 & 2 & 3 \end{bmatrix} \left| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right.$$

$R_1 \longleftrightarrow R_3$

$R_3 + \frac{3}{4} R_1 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 4 & 2 & 3 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 13/2 & 13/4 & 0 \end{array}\right]$$

$R_2 \longleftrightarrow R_3$

$\frac{2}{13} R_2 \longrightarrow R_2$

$\frac{2}{3} R_3 \longrightarrow R_3$

$R_2 - \frac{13}{4} R_3 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 4 & 2 & 3 & 0 \\ 0 & 13 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$R_1 - 3R_3 \longrightarrow R_1$

$\frac{2}{13} R_2 \longrightarrow R_2$

$R_1 - 2R_2 \longrightarrow R_1$

$\frac{1}{4} R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$a = 0, \; b = 0, \; c = 0$

linearly independent.

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(b)

$(-2,0,1), (3,2,5), (6,-1,1), (7,0,-2)$

$a(-2,0,1) + b(3,2,5) + c(6,-1,1) + d(7,0,-2) = 0$

$-2a + 3b + 6c + 7d = 0$ — (1)

$2b - c = 0$ (2)

$a + 5b + c - 2d = 0$ (3)

$$\left[\begin{array}{cccc|c} -2 & 3 & 6 & 7 & 0 \\ 0 & 2 & -1 & 0 & 0 \\ 1 & 5 & 1 & -2 & 0 \end{array}\right]$$

Gauss Jorden

$a - 79/29\, d = 0$

$a = 79/29\, d$

$b + 3/29\, d = 0$

$b = 3/29\, d$

$c + 6/29\, d = 0$

$c = -6/29\, d$

Linearly dependent

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Q3/(a)

$(3, 8, 7, -3), (1, 5, 3, -1)\ (2, -1, 2, 6), (4, 2, 6, 4)$

Same solution (Method as of Q2

Q#4

(b) $1+3x+3x^2,\ x+4x^2,\ 5+6x+3x^2$
$7+2x-x^2$

$a(1+3x+3x^2,\ x+4x^2,\ 5+6x+3x^2,\ 7+2x-x^2)$

$a+3ax+3a^2x+bx+4bx^2+5c+6cx+3cx^2$
$+7d+2dx-dx^2=0$

$(a+5c+7d)+x(3a+b+6c+2d)$
$+x^2(3a+4b+3c-d)=0$

$a+5c+7d=0$ — (1)
$3a+b+6c+2d=0$ — (2)
$3a+4b+3c-d=0$

The corresponding matrix

$$\left[\begin{array}{cccc|c} 1 & 0 & 5 & 7 & 0 \\ 3 & 1 & 6 & 2 & 0 \\ 3 & 4 & 3 & -1 & 0 \end{array}\right]$$

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Gauss Jorden

$a - 17/4\ d = 0$

$a = 17/4$

$b + 5/4\ d = 0$

$b = -5/4\ d$

$c + 9/4\ d = 0$

$c = -9/4\ d$ linearly dependent

Q#5

$a = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$

in M_{22}

$aA + bB + cC = 0$

$$a\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} + b\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + c\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a & 0 \\ a & 2a \end{bmatrix} + \begin{bmatrix} b & 2b \\ 2b & b \end{bmatrix} + \begin{bmatrix} 0 & c \\ 2c & c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a+b & 2b+c \\ a+2b+2c & 2a+b+c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$a+b=0$

$2b+c=0$

$a+2b+2c=0$

$2a+b+c$ — (4)

Eq 3 − 2 { 4

$a+2b+2c-2(2a+b+c)=0$

$a+2b+2c-4a-2b-2c=0$

$-3a=0$

$\boxed{a=0}$

$\boxed{b=0}$, $\boxed{c=0}$

$a, b, c = 0$

Hence A, B, C are linearly Independent.

Q#6

$$\begin{bmatrix}1 & 0\\ 1 & k\end{bmatrix}, \begin{bmatrix}-1 & 0\\ k & 1\end{bmatrix}, \begin{bmatrix}2 & 0\\ 1 & 3\end{bmatrix}$$

$$a\begin{bmatrix}1 & 0\\ 1 & k\end{bmatrix} + b\begin{bmatrix}-1 & 0\\ k & 1\end{bmatrix} + c\begin{bmatrix}2 & 0\\ 1 & 3\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}a-b+2c & 0\\ a+bk+c & ak+b+3c\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$a-b+2c=0$ — (1)

$a+bk+c=0$ — (2)

$ak+b+3c=0$ — (3)

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$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & k & 1 \\ k & 1 & 3 \end{bmatrix} \neq 0$$

$$1\begin{bmatrix} k & 1 \\ 1 & 3 \end{bmatrix} + 1\begin{bmatrix} 1 & 1 \\ k & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & k \\ k & 1 \end{bmatrix}$$

$3k - 1 + 3 - k + 2 - 2k^2 = 0$

$2k^2 + 4 - 2k = 0$

$2(k^2 - k - 2) = 0$

$k^2 - 2k + k - 2 = 0$

$k(k-2) + 1(k-2) = 0$

$k - 2 = 0$ $\qquad$ $k + 1 = 0$

$\boxed{k = 2}$ $\qquad$ $\boxed{k = -1}$

Q#7

$V_1 = (2, -2, 0)$
$V_2 = (6, 1, 4)$
$V_3 = (2, 0, -4)$

$a(2,-2,0) + b(6,1,4) + c(2,0,-4) = 0$

$2a + 6b + 2c = 0$
$-2a + b = 0$
$4b - 4c = 0$

$$\left[\begin{array}{ccc|c} 2 & 6 & 2 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 4 & -4 & 0 \end{array}\right]$$

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Gauss Jordan
$a = 0$, b
Linearly

So They don
plan

Q#8

$V_1 = (-1, 2, 3)$,
$V_3 = (-3$
$V_2 = -2V_1$

Vectors lie

V_3 is n
V_3 dono

Q#9

Vectors
indep

$K_1(0, 3, 1, -$

6

3

Gauss Jorden

$a=0$, $b=0$, $c=0$

Linearly independent.

so They donot lie is same plane.

Q#8

$V_1=(-1,2,3)$, $V_2=(2,-4,-6)$

$V_3=(-3,6,0)$

$V_2=-2V_1$

Vectors lies in same line

V_3 is not multiple of V_1 or V_2

V_3 donot lies wit V_1, V_2.

Q#9

Vectors V_1, V_2 and V_3 are linearly independent in R^4.

$$K_1(0,3,1,-1)+K_2(6,0,5,1)+K_3(4,-7,1,3)=(0,0,0,0)$$

$$6K_2+4K_3=0$$

$$3K_1 \quad -7K_3=0$$

$$K_1+5K_2+K_3=0$$

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$$\left[\begin{array}{ccc|c} 0 & 6 & 4 & 0 \\ 3 & 0 & -7 & 0 \\ 1 & 5 & 1 & 0 \end{array}\right]$$

$$3K_2 + 2K_3 = 0$$

$$K_2 = -2/3 \, K_3$$

$$K_1 + 5K_2 + K_3 = 0$$

$$K_1 - \frac{10}{3} K_3 + K_3 = 0$$

$$K_1 = 7/3 \, K_3$$

$\Rightarrow$ Linear system has non-trivial solution.

$\Rightarrow$ Vector equation has non-trivial.

$\Rightarrow$ v_1, v_2, v_3 form a linearly dependent

b) $K_1 V_1 = -K_2 V_2 - K_3 V_3$

$V_1 = -\frac{K_2}{K_1} V_2 - \frac{K_3}{K_1} V_3$

$\Rightarrow$

$V_1 = 2/7\, V_2 - 3/7\, V_3$

$\Rightarrow K_2 V_2 = -K_1 V_1 - K_3 V_3$

$V_2 = 7/2\, V_1 + 3/2\, V_3$

$\Rightarrow K_3 V_3 = -K_1 V_1 - K_2 V_2$

$V_3 = -7/3\, V_1 + 2/3\, V_2$

Q#10

$K_1 V_1 + K_2 V_2 + K_3 V_3 = 0$

$K_1(1,2,3,4) + K_2(0,1,0,-1) + K_3(1,3,3,3)$

$K_1 + K_3 = 0$

$2K_1 + K_2 + 3K_3 = 0$

$3K_1 + 3K_2 = 0$

$4K_1 - K_2 + 3K_2 = 0$

Gauss Jorden

$K_1 + K_3 = 0$

$K_1 = -K_3$

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Linear system has non-trivial solution vector by

Vectors v_1, v_2, v_3 is form a linearly dependent set in $\mathbb{R}^3$.
Same as Q9(b)

Q11/

$$V_1 = (\lambda, -1/2, -1/2)$$

$$V_2 = (-1/2, \lambda, -1/2)$$

$$V_3 = (-1/2, -1/2, \lambda)$$

$$a(\lambda, -1/2, -1/2) + b(-1/2, \lambda, -1/2) + c(-1/2, -1/2, \lambda) = 0$$

$$a\lambda - 1/2\, b - 1/2\, c = 0 \quad \text{—} (1)$$

$$-1/2\, a + \lambda b - 1/2\, c = 0 \quad \text{—} (2)$$

$$-1/2\, a - 1/2\, b + \lambda c = 0 \quad \text{—} (3)$$

$$\begin{bmatrix} \lambda & -1/2 & -1/2 \\ -1/2 & \lambda & -1/2 \\ -1/2 & -1/2 & \lambda \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\det(A) = \lambda\begin{vmatrix} \lambda & -1/2 \\ -1/2 & \lambda \end{vmatrix} + 1/2\begin{vmatrix} -1/2 & -1/2 \\ -1/2 & \lambda \end{vmatrix} - \frac{1}{2}\begin{vmatrix} -1/2 & 2 \\ -1/2 & -1/4 \end{vmatrix}$$

$$= \lambda(\lambda^2 - 1/4) + 1/2(-2/\lambda - 1/4) - 1/2(1/4 + \lambda/2)$$

$$= \lambda^3 - \lambda/4 - 1/2 - 1/8 - 1/8 - \lambda$$

$$= \lambda^3 - \frac{5\lambda}{4} - \frac{1}{\lambda} - 1/4$$

$$= \lambda^3 - 3/4\,\lambda - 1/4$$

$$= \lambda^3 - 3/4\,\lambda - 1/4 = 0$$

$$4\lambda^3 - 3\lambda - 1 = 0$$

$$\lambda + 1 = 0 \qquad 2\lambda - 1 = 0$$

$$\boxed{\lambda = -1} \qquad \lambda = 1/2$$

Here given vectors ar linearly dependent.

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Q12/ The set with one vector is linearly dependat under condition that vector v is not zero

if $v = 0$ $\Rightarrow$ $K \cdot v = 0$ for all $K \in R$

$\Rightarrow$ It has non trivial solution

$\Rightarrow$ set is dependent if $v \neq 0$
$K \cdot V = 0$

$\Rightarrow$ Has only trivial solution $K = 0$ Hence solution is independent

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Exercise # 4.4

Question #1

Use the following of vectors form of a basis for R

$\{(2,1),(3,0)\}$

Solution

We have to show linearly independent and span R^2.

$a(2,1)+b(3,0)=0$ —— (1)

$a(2,1)+b(3,0)=x$ —— (2)

$(2a,a)+(3b)=x$

$2a+3b, a = x$

$2a+3b=x, \quad a=x$

$2a+3b=4, \quad a=2$ —— (3)

Thus we have show that (3) system has trivial solution

But co-efficient matrix of system is

$$A=\begin{bmatrix} 1 & 3 \\ 1 & 0 \end{bmatrix}$$

$\det(A) = 3$

which proves that the vectors v_1 and v_2 form a basis for R^2.

Q#02

$$\{(3,1,-4),(2,5,6),(1,4,8)\}$$

$$a(3,1,-4)+b(2,5,6)+c(1,4,8)$$

$$(3a,a,-4a)+(2b,5b,6b)+(c,4c,8c)$$

$$3a+2b+c=y$$

$$a+5b-14c=z$$

$$-4a+6b+8c=q$$

Augumented matrix

$$A=\begin{bmatrix}3 & 2 & 1\\ 1 & 5 & 4\\ -4 & 6 & 8\end{bmatrix}$$

$$\det(A)=3(40-24)-2(8+16)+1(6+20)$$

$$=3(16)-2(24)+1(26)$$

$$=48-48+26$$

$$26\neq 0$$

which proves that V_1, V_2, V_3 form a basis for R^3.

Zibara

c, 8c)

Q3/

$x^2+1,\ x^2-1,\ 2x-1$

$P_1(x) = x^2+1, \quad P_2(x) = x^2-1, \quad P_3(x) = 2x-1$

$a(x^2+1) + b(x^2-1) + c(2x-1) = 0$

$(ax^2+a) + (bx^2-b) + (2cx-c) = 0$

$ax^2 + bx^2 + 2cx - b + a - c = 0$

$x^2(a+b) + 2cx + (a-c-b) = 0$

$a+b = 0$ —— (1)

$2c = 0$ —— (2)

$a-b-c = 0$ —— (3)

$\boxed{c=0}$

$a-b-c=0$

$a-b=c$

$a-b=0$

$\boxed{a=b}$

$b+b=0$

$2b=0$

$\boxed{b=0}$

$a+b=0$

$a+0=0$

$\boxed{a=0}$

Hence, the given polynomials are linearly independent.

$\Rightarrow$ Polynomials form a basis for P^2.

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Q#4

Show that the following

$1+x, \quad 1-x, \quad 1-x^2, \quad 1-x^3$

$a(1+x)+b(1-x)+c(1-x^2)+d(1-x^3)=0$

$a+ax+b-bx+c-cx^2+d-dx^3=0$

$x^3(-d)+x^2(-c)+(a-b)x+a+c+d+b=0$

$-d=0$

$\boxed{d=0}$

$\boxed{c=0}$

$a-b=0$ —— (1)

$a+b+c+d=0$

$a+b+c+d=0$

$a+b+0=0$

$a+b=0$ —— (2)

Eq (1) and (2)

$a-\cancel{b}+a+\cancel{b}=0$

$2a=0$

$\boxed{a=0}$

$a-b=0$

$a=b$

$\boxed{b=0}$

$\Rightarrow$ The given polynomials form a basis for P^3.

Q#5

Show that the following matrices form a basis for M_{22}!

$$\begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$$c_1 \begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix} + c_2 \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix} + c_4 \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \in M_{22}$$

$$\begin{bmatrix} 3c_1 & 6c_1 \\ 3c_1 & -6c_1 \end{bmatrix} + \begin{bmatrix} 0 & -c_2 \\ -c_2 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -8c_3 \\ -12c_3 & -4c_3 \end{bmatrix} + \begin{bmatrix} c_4 & 0 \\ -c_4 & 2c_4 \end{bmatrix} = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$$

$$3c_1 + 0 + 0 + c_4 = b_1$$
$$6c_1 - c_2 - 8c_3 + 0 = b_2$$
$$3c_1 - c_2 - 12c_3 - c_4 = b_3$$
$$-6c_1 + 0 - 4c_3 + 2c_4 = b_4$$

$$3c_1 + c_4 = b_1$$

$$|A| \begin{bmatrix} 3 & 0 & 0 & 1 \\ 6 & -1 & -8 & 0 \\ 3 & -1 & -12 & -1 \\ -6 & 0 & -4 & 2 \end{bmatrix}$$

$$\det(A) = 3 \begin{vmatrix} -1 & -8 & 0 \\ -1 & -12 & -1 \\ 0 & -4 & 2 \end{vmatrix} + 0 + 0 + 1 \begin{bmatrix} 6 & -1 & -8 \\ 3 & -1 & -12 \\ -6 & 0 & -4 \end{bmatrix}$$

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$= 3[-1(-24)-4] + 8(-2)] - 1[6(4) + 1(-12+72) - 8(-6)]$

$= 3[-1(-28)-16] - 1[24+60+48]$

$= 3[28-16] - 1(132)$

$= 3[12] - 132$

$= 36 - 132$

$= -96 \neq 0$

Hence S is a for M_{22}.

Q # 06

Show that the following matrices form a basis for M_{22}.

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$c_1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + c_2\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} + c_3\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} + c_4\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \in M_{22}$$

$$\begin{bmatrix} c_1 & c_1 \\ c_1 & c_1 \end{bmatrix} + \begin{bmatrix} c_2 & -c_2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -c_3 \\ c_3 & 0 \end{bmatrix} + \begin{bmatrix} c_4 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$$

$c_1 + c_2 + c_4 = b_1$ —— (1)

$c_1 - c_2 - c_3 = b_2$ —— (2)

$c_1 + c_3 = b_3$ —— (3)

$\boxed{c_1 = b_4}$ —— (4)

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$\cancel{a_4 + a_3 = c}$

$\cancel{a_3 = c - d}$

$\cancel{d = a_2 - (c - d) = b}$

$c_1 + c_3 = b_3$

$c_3 = b_3 - c_1$

$\cancel{c_1} - c_2 - b_3 - \cancel{c_1} = b_2$

$-c_2 - b_3 = b_2$

$-(c_2 + b_3) = b_2$

$c_2 + b_3 = b_2$

$c_2 = b_2 - b_3$

$\cancel{c_1} - b_2 - b_3 - \cancel{b_3} - \cancel{c_1} = b_2$

$-\cancel{b_2} - 2b_3 = \cancel{b_2}$

$\boxed{b_3 = 0}$

$= \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$ can be written as linear combination of vector form c.

→ S spans M_{22}.

⇒ S Linearly independent

$$\alpha_1 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \alpha_2 \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} + \alpha_3 \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} + \alpha_4 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\alpha_1 + \alpha_2 + \alpha_4 = 0$$
$$\alpha_1 - \alpha_2 - \alpha_3 = 0$$
$$\alpha_1 + \alpha_3 = 0$$
$$\boxed{\alpha_1 = 0}$$
$$\boxed{\alpha_2 = 0}, \boxed{\alpha_3 = 0}$$

→ Vector eq has only trivial Solution

→ set S is linearly independent

→ S forms of a basis for M_{22}

Q7/

$\{(2,-3,1), (4,1,1), (0,-7,1)\}$

$a(2,-3,1) + b(4,1,1) + c(0,-7,1) = 0$

$(2a,-3a,a) + (4b,b,b) + (0,-7c,c) = 0$

$2a + 4b = 0$ — (1)

$-3a + b - 7c = 0$ (2)

$a + b + c = 0$ — (3)

$$\begin{bmatrix} 2 & 4 & 0 \\ -3 & 1 & -7 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

uses guass jorden method

$$\begin{bmatrix} 2 & 4 & 0 \\ -3 & 1 & -7 \\ 1 & 1 & 1 \end{bmatrix}$$

$R_3 \longleftrightarrow R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ -3 & 1 & -7 \\ 2 & 4 & 0 \end{bmatrix}$$

$R_1 + R_2 + R_3 \longrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 6 & -6 \\ 2 & 4 & 0 \end{bmatrix}$$

$2R_1 - R_3 \longrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 6 & -6 \\ 0 & -2 & 2 \end{bmatrix}$$

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$R_1 \longleftrightarrow R_2$

$R_2 - (-2/3) R_1 \longrightarrow R_2$

$R_3 + 1/3 R_1 \longrightarrow R_3$

$R_3 - 2/7 R_2 \longrightarrow R_3$

$3/14 R_2 \longrightarrow R_2$

$R_1 - R_2 \longrightarrow R_2$

$1/35 R_1 \longrightarrow R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 6 & -6 \\ 0 & -2 & 2 \end{bmatrix}$$

$2R_1 + R_3 \longrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 6 & -6 \\ 0 & 0 & 4 \end{bmatrix}$$

$1/4 R_3 \longrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 6 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$6R_1 + R_2 \longrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$a + 2c = 0$

$\boxed{a = -2c}$

$b - c \cancel{0} = 0$

$b = c$

$\boxed{c = c}$

This is no trivial solution

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Q1

$(1,6,4), (2,4,-1), (-1,2,5)$

$a(1,6,4) + b(2,4,-1) + c(-1,2,5) = 0$

$a + 2b - c = 0$

$6a + 4b + 2c = 0$

$4a - b - 5 = 0$

$$\begin{bmatrix} 1 & 2 & -1 \\ 6 & 4 & 2 \\ 4 & -1 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$a + c = 0$

$\boxed{a = -c}$

$\boxed{b = c}$

$c = c$

→ No trivial solution

→ vectors are linearly depend

→ set of vectors is not a basis for R^3

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Q#8

Show that the following vectors donot form a basis for

$1-3x+2x^2, \quad 1+x+4x^2, \quad 1-7x$

Three vectors in polynomial form that must be shown to not form a basis for R_2.

$$\begin{bmatrix} 1 & 1 & 1 \\ -3 & 1 & -7 \\ 2 & 4 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$Ax = b$

$x = A^{-1}b$

$$\det(A) = \begin{vmatrix} 1 & 1 & 1 \\ -3 & 1 & -7 \\ 2 & 4 & 0 \end{vmatrix}$$

$$1\begin{vmatrix} 4 & -7 \\ 4 & 0 \end{vmatrix} - 1\begin{vmatrix} -3 & -7 \\ 2 & 0 \end{vmatrix} + 1\begin{vmatrix} -3 & 1 \\ 2 & 4 \end{vmatrix}$$

$1(0+28) - 1(-12+0) + 1(-12-2)$

$= 28 - 12 - 14$

$= 28 - 28$

$= 0$

$|A| = 0$

They donot span.

Q#9

Q#9

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$$

$$a\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} + c\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} + d\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$$

$$a + 2b + c = 0 \quad (1)$$

$$-2b - c - d = 0 \quad (2)$$

$$a + 3b + c + d = 0 \quad (3)$$

$$a + 2b + d = \quad (4)$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ & -2 & -1 & -1 \\ 1 & 3 & 1 & 1 \\ 1 & 2 & & \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Gauss jorden

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$a - d = 0$$

$$a = d$$

$$b + d = 0$$

$$b = -d$$

$$c - d = 0$$

$$c = d$$

$$d = d$$

→ Hence, given matrices are linearly dependent then every 2×2 matrix cannot be expressed

$$a\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix} + b\begin{bmatrix}2 & -2\\ 3 & 2\end{bmatrix} + c\begin{bmatrix}1 & -1\\ 1 & 0\end{bmatrix} + d\begin{bmatrix}0 & -1\\ 1 & 1\end{bmatrix}$$

→ The matrices is not a basis for M_{22}

Q10/ $v_1 = \cos^2 x$
$V_2 = \sin^2 x$, $V_3 = \cos 2x$

Sol/

Let V be a space spanned by

$$\left\{\begin{array}{l} v_1 = \cos^2 x \\ V_2 = \sin^2 x \\ V_3 = \cos 2x \end{array}\right\}$$

Chap # 05 Ex # 5.1

Q1 Find eigen values and associative eigen vector of a matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

suppose

let $V = [x \; y]^t = \begin{bmatrix} x \\ y \end{bmatrix}$

$\therefore AV = dV$

$$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = d \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x + 2y \\ 3x + 2y \end{bmatrix} = \begin{bmatrix} dx \\ dy \end{bmatrix}$$

$x + 2y = dx$ — (*)

$3x + 2y = dy$ — (**)

from (*)

$x + 2y = dx$

$x - dx + 2y = 0$

$x(1-d) + 2y = 0$

from (**)

$3x - dy + 2y = 0$

$3x - y(d - 2) = 0$

$3x + (2-d)y = 0$

$$\begin{vmatrix} 1-d & 2 \\ 3 & 2-d \end{vmatrix} = 0$$

$(1-d)(2-d) - 6 = 0$

$2 - d - 2d + d^2 - 6 = 0$

$d^2 - 3d - 4 = 0$

$(d-4)(d+1) = 0$

$\boxed{d = 4, \; d = -1}$

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Hence $\lambda = 4, -1$ are eigen values

$\lambda = -1$ in eq (i) $\Rightarrow$

$2x + 2y = 0$

or $x + y = 0$

or $\boxed{y = -x}$

Thus

$$V = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -x \end{bmatrix} = x \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

eigen vector is $[1 \ -1]^t$

and $\lambda = 4$ in eq (i)

$-3x + 2y = 0$

$2y = 3x$

$y = \frac{3}{2} x$

$$V = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ \frac{3}{2} x \end{bmatrix} = \frac{x}{2} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

eigen vectors is $[2 \ \ 3]^t$

OR $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Homo Sapiens

Q2/ $A = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$, $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Let

$$V = [x \ \ y]^t = \begin{bmatrix} x \\ y \end{bmatrix}$$

$AV = \lambda V$

$$\begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 5x - y \\ x - 3y \end{bmatrix} = \begin{bmatrix} \lambda x \\ \lambda y \end{bmatrix}$$

$$5x - y = \lambda x \quad \text{--- (1)}$$

$$x - 3y = \lambda y \quad \text{--- (2)}$$

From (1)

$$5x - y - \lambda x = 0$$

$$5x - \lambda x - y = 0$$

$$x(5 - \lambda) - y = 0$$

From (2)

$$x - 3y - \lambda y = 0$$

$$x - y(3 + \lambda) = 0$$

For non-trivial solution

$$\begin{vmatrix} 5 - \lambda & -1 \\ 1 & -(3 + \lambda) \end{vmatrix} = 0$$

$$(5 - \lambda)(-3 - \lambda) + 1 = 0$$

$$-15 - 5\lambda + 3\lambda + \lambda^2 + 1 = 0$$

$$\lambda^2 - 2\lambda - 14 = 0$$

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Question # 05

Find the characteristics equation the eigenvalues and basis for the eigenspaces of the matrix.

Q05 (a)

(a) $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

characteristic equation $= tI - A$

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$= t\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} t & 0 \\ 0 & t \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} t-1 & 0-4 \\ 0-2 & t-3 \end{bmatrix}$$

$$= \begin{bmatrix} t-1 & -4 \\ -2 & t-3 \end{bmatrix}$$

$$\begin{vmatrix} t-1 & -4 \\ -2 & t-3 \end{vmatrix}$$

$$(t-1)(t-3) - (-4)(-2)$$

$$t^2 - 3t - t + 3 - (-8)$$

$$t^2 - 4t - 5$$

$$t^2 - 5t + 1t - 5 = 0$$

$$t(t-5) + 1(t-5) = 0$$

$$t+1 = 0 \qquad t-5 = 0$$

$$\boxed{t = -1} \qquad \boxed{t = 5}$$

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$(t-5)(t+1)=0$ raaf ra

eigen value 5

eigen value -1

b) $\begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix}$

$tI - A = 0$

$t\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -7 \\ 4 & 2 \end{bmatrix}$

$\begin{bmatrix} t & 0 \\ 0 & t \end{bmatrix} - \begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} t+2 & 0+7 \\ 0-1 & t-2 \end{bmatrix}$

$= \begin{bmatrix} t+2 & 7 \\ -1 & t-2 \end{bmatrix}$

$= (t+2)(t-2) - (7)(-1)$

$= t^2 - \cancel{2t} + \cancel{2t} - 4 + 7$

$= t^2 + 3$

$t^2 + 3 = 0$

No real eigen values

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Q1/

Characteristic equation :-

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\lambda I - A = 0$$

$$\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{bmatrix} = 0$$

$$= [(\lambda-1)(\lambda-1) - 0] - [2(\lambda-1) - 0] + [0 - 2(\lambda-1)]$$

$$= [(\lambda^2 - \lambda - \lambda + 1)] - [2\lambda - 2] + [-2\lambda + 2]$$

$$\lambda^2 - 2\lambda + 1 - 2\lambda + \cancel{2} - 2\lambda \cancel{+2} = 0$$

$$\lambda^2 - 6\lambda + 1 = 0$$

$$\lambda\left(\lambda - 6 + \frac{1}{\lambda}\right) = 0$$

$$\lambda - 6 + \frac{1}{\lambda} = 0$$

$$\frac{1}{\lambda}\left[\lambda - 6 + 1\right] = 0$$

$$\frac{1}{\lambda}\left[\lambda - 5\right] = 0$$

$$(\lambda-2)(\lambda-3)(\lambda-1) = 0$$

Question # 03

$A=\begin{bmatrix}3 & 0\\-1 & 2\\1 & 1\end{bmatrix}$, $B=\begin{bmatrix}4 & -1\\0 & 2\end{bmatrix}$

$C=\begin{bmatrix}1 & 4 & 2\\3 & 1 & 5\end{bmatrix}$

$D=\begin{bmatrix}1 & 5 & 2\\-1 & 0 & 1\\3 & 2 & 4\end{bmatrix}$ $E=\begin{bmatrix}6 & 1 & 3\\-1 & 1 & 2\\4 & 1 & 3\end{bmatrix}$

(a)

D+E

$$\begin{bmatrix}1 & 5 & 2\\-1 & 0 & 1\\3 & 2 & 4\end{bmatrix}+\begin{bmatrix}6 & 1 & 3\\-1 & 1 & 2\\4 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}7 & 6 & 5\\-2 & 1 & 3\\7 & 3 & 7\end{bmatrix}$$

(b)

D-E

$$\begin{bmatrix}1 & 5 & 2\\-1 & 0 & 1\\3 & 2 & 4\end{bmatrix}-\begin{bmatrix}6 & 1 & 3\\-1 & 1 & 2\\4 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-5 & 4 & -1\\0 & -1 & -1\\-1 & 1 & 1\end{bmatrix}$$

(c)

5A

$$=5\begin{bmatrix}3 & 0\\-1 & 2\\1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}15 & 0\\-5 & 10\\5 & 5\end{bmatrix}$$

(d)

-7C

$$=-7\begin{bmatrix}1 & 4 & 2\\3 & 1 & 5\end{bmatrix} = \begin{bmatrix}-7 & -28 & -14\\-21 & -7 & -35\end{bmatrix}$$

⇒ tr A

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$$

tr A = [trace is only defined for square matrix and A is not a square matrix.

Question #04

(a) $2A^T + C$

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$2A^T = \begin{bmatrix} 6 & -2 & 2 \\ 0 & 4 & 2 \end{bmatrix}$$

$$2A^T + C = \begin{bmatrix} 6 & -2 & 2 \\ 0 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 2 & 4 \\ 3 & 5 & 7 \end{bmatrix}$$

b) $D^T - E^T$

$$D^T - E^T = \begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

(c) $(D-E)^T$

$$= \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

$$(D-E) = \begin{bmatrix} -5 & 4 & -1 \\ 0 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$(D-E)^T$

$$\begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}.$$

(d) $B^T + 5C^T$

$$\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix} + 5\begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix}$$

= undefined

(f) $B - B^T$

$$= \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

(f) $\frac{1}{2}C^T - \frac{1}{4}A$

$$= \frac{1}{2}\begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1/2 & 3/2 \\ 4/2 & 1/2 \\ 2/2 & 5/2 \end{bmatrix}$$

$$\frac{1}{4}A = \frac{1}{4}\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3/4 & 0/4 \\ -1/4 & 2/4 \\ 1/4 & 1/4 \end{bmatrix}$$

$$\frac{1}{2}C^T - \frac{1}{4}A = \begin{bmatrix} 1/2 & 3/2 \\ 4/2 & 1/2 \\ 2/2 & 5/2 \end{bmatrix} - \begin{bmatrix} 3/4 & 0/4 \\ -1/4 & 2/4 \\ 1/4 & 1/4 \end{bmatrix}$$

(g) $2E^T - 3D^T$

$$2E^T = 2\begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 12 & -2 & 8 \\ 2 & 2 & 2 \\ 6 & 4 & 6 \end{bmatrix}$$

$$3D^T = 3\begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -3 & 9 \\ 15 & 0 & 6 \\ 6 & 3 & 12 \end{bmatrix}$$

$$2E^T - 3D^T = \begin{bmatrix} 12 & -2 & 8 \\ 2 & 2 & 2 \\ 6 & 4 & 6 \end{bmatrix} - \begin{bmatrix} 3 & -3 & 9 \\ 15 & 0 & 6 \\ 6 & 3 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 1 & -1 \\ -13 & 2 & -4 \\ 0 & 1 & -6 \end{bmatrix}$$

(h) $(2E^T - 3D^T)^T$

$$= \begin{bmatrix} 9 & 1 & -1 \\ -13 & 2 & -4 \\ 0 & 1 & -6 \end{bmatrix}$$

Transponse

$$= \begin{bmatrix} 9 & -13 & 0 \\ 1 & 2 & 1 \\ -1 & -4 & -6 \end{bmatrix}$$

(i) $(CE)E$

$$CD = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 9 & 14 \\ 17 & 25 & 27 \end{bmatrix}$$

$$(CD)E = \begin{bmatrix} 3 & 9 & 14 \\ 17 & 25 & 27 \end{bmatrix} \cdot \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 65 & 26 & 69 \\ 185 & 69 & 182 \end{bmatrix}$$

(j) (LBA)

undefined

k) $tr(DE^T)$

$$E^T = \begin{bmatrix} 6 & 1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix}$$

$$D \cdot E^T = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 8 \\ -1 & 0 & 1 \\ 9 & 4 & 12 \end{bmatrix}$$

$$tr(D \cdot E^T) = \begin{bmatrix} 6+0+12 \end{bmatrix} = 18$$

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(e) tr(BC)

$$BC = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

= undefined.

Question # 05

(a) AB

$$\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+0 & -3+0 \\ -4+0 & 1+4 \\ 4+0 & -1+2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix}$$

(b) (BA)

BA is undefined because B has 2x2 matrix and A has 3 x 2. matrix.

(c) 3E(D)

$$= 3 \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 18 & 3 & 9 \\ -3 & 3 & 6 \\ 12 & 3 & 9 \end{bmatrix}$$

$$3E(D) = \begin{bmatrix} 18 & 3 & 9 \\ -3 & 3 & 6 \\ 12 & 3 & 9 \end{bmatrix} \cdot \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 15 & 18 \\ 3 & 0 & 6 \\ 36 & 6 & 36 \end{bmatrix}.$$

Question #06

(a)

$(2D^T - E)A$

$$2D^T = 2\begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 6 \\ 10 & 0 & 4 \\ 4 & 2 & 8 \end{bmatrix}$$

$$2D^T - E = \begin{bmatrix} 2 & -2 & 6 \\ 10 & 0 & 4 \\ 4 & 2 & 8 \end{bmatrix} - \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -3 & 3 \\ 11 & -1 & 2 \\ 0 & 1 & 5 \end{bmatrix}$$

$$(2D^T - E)A = \begin{bmatrix} -4 & -3 & 3 \\ 11 & -1 & 2 \\ 0 & 1 & 5 \end{bmatrix} \cdot \Big[$$

(b) $(4B)C + 2B$

$$4B = 4\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 16 & -4 \\ 0 & 8 \end{bmatrix}$$

$$(4B)C = \begin{bmatrix} 16 & -4 \\ 0 & 8 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

$$=$$

Question #11

(a)

$$2x_1 - 3x_2 + 5x_3 = 7$$
$$9x_1 - x_2 + x_3 = -1$$
$$x_1 + 5x_2 + 4x_3 = 0$$

$Ax = b$

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 9 & -1 & 1 \\ 1 & 5 & 4 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$b = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 5 \\ 9 & -1 & 1 \\ 1 & 5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix}$$

(b)

$$4x_1 \quad - 3x_3 + x_4 = 1$$
$$5x_1 + x_2 = 3$$
$$2x_1 - 5x_2 + 9x_3 - x_4 = 0$$
$$3x_2 - x_3 + 7x_4 = 2$$

$Ax = b$

$$A = \begin{bmatrix} 4 & 0 & -3 & 1 \\ 5 & 1 & 0 & -9 \\ 2 & -5 & 9 & -1 \\ 0 & 3 & -1 & 7 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & -3 & 1 \\ 5 & 1 & 0 & -8 \\ 2 & -5 & 9 & -1 \\ 0 & 3 & -1 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \end{bmatrix}$$

Question # 12

(a)

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= -3 \\ 2x_1 + x_2 \qquad &= 0 \\ -3x_2 + 4x_3 &= 1 \\ x_1 \qquad + x_3 &= 5 \end{aligned}$$

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 0 \\ 0 & -3 & 4 \\ 1 & 0 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$b = \begin{bmatrix} -3 \\ 0 \\ 1 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 0 \\ 0 & -3 & 4 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 1 \\ 5 \end{bmatrix}$$

(b)

$$\begin{aligned} 3x_1 + 3x_2 + 3x_3 &= -3 \\ -x_1 - 5x_2 - 2x_3 &= 3 \\ -4x_2 + x_3 &= 0 \end{aligned}$$

$$A = \begin{bmatrix} 3 & 3 & 3 \\ -1 & -5 & -2 \\ 0 & -4 & 1 \end{bmatrix}$$

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$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$b = \begin{bmatrix} -3 \\ 3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 3 & 3 \\ -1 & -5 & -2 \\ 0 & -4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 0 \end{bmatrix}$$

Question (13)

change Matrix equation into linear equations

(a)

$$\begin{bmatrix} 5 & 6 & -7 \\ -1 & -2 & 3 \\ 0 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$

$$5x_1 + 6x_2 - 7x_3 = 2$$
$$-x_1 - 2x_2 + 3x_3 = 0$$
$$4x_2 - x_3 = 3$$

(b)

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 5 & -3 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -9 \end{bmatrix}$$

$$x_1 + x_2 + x_3 = 2$$
$$2x_1 + 3x_2 = 2$$
$$5x_1 - 3x_2 - 6x_3 = -9$$

Question # 14

(a)

$$\begin{bmatrix} 3 & -1 & 2 \\ 4 & 3 & 7 \\ -2 & 1 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$3x_1 - x_2 + 2x_3 = 2$$

$$4x_1 + 3x_2 + 7x_3 = -1$$

$$-2x_1 + x_2 + 5x_3 = 4$$

(b)

$$\begin{bmatrix} 3 & -2 & 0 & 1 \\ 5 & 0 & 2 & -2 \\ 3 & 1 & 4 & 7 \\ -2 & 5 & 1 & 6 \end{bmatrix}\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3w - 2x + z = 0$$

$$5w + 2y - 2z = 0$$

$$3w + x + 4y + 7z = 0$$

$$-2w + 5x + y + 6z = 0$$

Question # 15

Find value of k

$$\{k, \ 1 \ \ 1\} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix}\begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix} = 0$$

Applying matrices multiplication

$$\begin{bmatrix} k+1 & k+2 & -1 \end{bmatrix}\begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix} = 0$$

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Question # 14

(a)

$$\begin{bmatrix} 3 & -1 & 2 \\ 4 & 3 & 7 \\ -2 & 1 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$3x_1 - x_2 + 2x_3 = 2$$

$$4x_1 + 3x_2 + 7x_3 = -1$$

$$-2x_1 + x_2 + 5x_3 = 4$$

(b)

$$\begin{bmatrix} 3 & -2 & 0 & 1 \\ 5 & 0 & 2 & -2 \\ 3 & 1 & 4 & 7 \\ -2 & 5 & 1 & 6 \end{bmatrix}\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3w - 2x + z = 0$$

$$5w + 2y - 2z = 0$$

$$3w + x + 4y + 7z = 0$$

$$-2w + 5x + y + 6z = 0$$

Question # 15

Find value of k

$$\{k, 1, 1\}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix}\begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix} = 0$$

Applying matrices multiplication

$$\begin{bmatrix} k+1 & k+2 & -1 \end{bmatrix}\begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix} = 0$$

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$(k+1)k + k + 2 - 1 = 0$

$k^2 + k + k + 2 - 1 = 0$

$k^2 + 2k + 1 = 0$

$k^2 + 1k + 1k + 1 = 0$

$k(k+1) + 1(k+1) = 0$

$(k+1)(k+1) = 0$

$\boxed{k = -1}$

Question #16

$$\begin{bmatrix} 2 & 2 & k \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ k \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 4+3k & 6+k \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ k \end{bmatrix} = 0$$

$12 + 8 + 6k + 6k + k^2 = 0$

$12 + 8 + 12k + k^2 = 0$

$k^2 + 12k + 20 = 0$

$$k = \frac{-12 \pm \sqrt{12^2 - (4)(1)(20)}}{2}$$

$$k = \frac{-12 \pm \sqrt{64}}{2}$$

$k = -2, \; k = -10$

$k = -2$

$k = -10$

Question #17

Use coloumn row expansion of AB to express this product as a sum of matrices

$A=\begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$ $B=\begin{bmatrix} 0 & 1 & 2 \\ -2 & 3 & 1 \end{bmatrix}$

$c_1=\begin{bmatrix} 4 \\ 2 \end{bmatrix}$, $c_2\begin{bmatrix} -3 \\ -1 \end{bmatrix}$ $r_1\begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$

$r_2\begin{bmatrix} -2 & 3 & 1 \end{bmatrix}$

$$\begin{bmatrix} 4 \\ 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \end{bmatrix}+\begin{bmatrix} -3 \\ -1 \end{bmatrix}\begin{bmatrix} -2 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 4 & 8 \\ 0 & 2 & 4 \end{bmatrix}+\begin{bmatrix} 6 & -9 & -3 \\ 2 & -3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0+6 & 4-9 & 8-3 \\ 0+2 & 2-3 & 4-1 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -5 & 5 \\ 2 & -1 & 3 \end{bmatrix}$$

Q19

$A=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $B\begin{bmatrix} 1 & 3 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

$c_1=\begin{bmatrix} 1 \\ 4 \end{bmatrix}$, $c_2\begin{bmatrix} 2 \\ 5 \end{bmatrix}$, $c_3\begin{bmatrix} 3 \\ 6 \end{bmatrix}$ $r_1=\begin{bmatrix} 1 & 3 \end{bmatrix}$

$r_2=\begin{bmatrix} 3 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \end{bmatrix}+\begin{bmatrix} 2 \\ 5 \end{bmatrix}\begin{bmatrix} 3 & 4 \end{bmatrix}+\begin{bmatrix} 3 \\ 6 \end{bmatrix}\begin{bmatrix} 5 & 6 \end{bmatrix}$ $r_3=\begin{bmatrix} 5 & 6 \end{bmatrix}$

$$\begin{bmatrix} 1 & 3 \\ 4 & 12 \end{bmatrix}+\begin{bmatrix} 6 & 8 \\ 15 & 20 \end{bmatrix}+\begin{bmatrix} 15 & 18 \\ 30 & 36 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 11 \\ 19 & 32 \end{bmatrix}+\begin{bmatrix} 15 & 18 \\ 30 & 36 \end{bmatrix}$$

$$\begin{bmatrix} 22 & 29 \\ 49 & 68 \end{bmatrix}$$

Question # 23

$$\begin{bmatrix} a & 3 \\ -1 & a+b \end{bmatrix} = \begin{bmatrix} 4 & d-2c \\ d+2c & -2 \end{bmatrix}$$

Two matrices are equal iff all of the corresponding elements of two matrices are equal

$$\boxed{a = 4}$$

$d-2c = 3$ —— (1)

$d+2c = -1$ —— (2)

$-2 = a+b$ —— (3)

$-2 = 4+b$

$-2-4 = b$

$$\boxed{b = -6}$$

$d-2c = 3$ —— *

$d+2c = -1$ —— **

Adding * and **

$d - \cancel{2c} = 3$

$d + \cancel{2c} = -1$

$2d = 2$

$$\boxed{d = 1}$$

$d-2c = 3$

$1-2c = 3$

$-2c = 3-1$

$-c = 2/2$ $\boxed{c = -1}$

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Question # 24

$$\begin{bmatrix} a-b & b+a \\ 3d+c & 2d-c \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 7 & 6 \end{bmatrix}$$

$a-b=8$

$b+a=1$

$3d+c=7$

$2d-c=6$

$$\left.\begin{aligned} a-\cancel{b} &= 8 \\ \cancel{b}+a &= 1 \end{aligned}\right\}$$

Adding these

$2a=9$

$\boxed{a=9/2}$

$a-b=8$

$\frac{9}{2}-b=8$

$-b=8-\frac{9}{2}$

$=\frac{16-9}{2}$

$-b=\frac{7}{2}$

$\boxed{b=-7/2}$

Add

$3d+c=7$

$2d-c=6$

$5d=13$

$\boxed{d=13/5}$

$3\left(\frac{13}{5}\right)+c=7$

$\frac{39}{5}+c=7$

$c=7-\frac{39}{5}$

$=\frac{35-39}{5}$

$c=-4/5$

Exercise # 1.4

Q (1) (a)

Associative law of matrix

$A+(B+C) = (A+B)+C$

L.H.S

$B+C = \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$

$\begin{bmatrix} 0+4 & 2+1 \\ +1-3 & -4-2 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -2 & -6 \end{bmatrix}$

$(A+B)+C = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -2 & -6 \end{bmatrix}$

$= \begin{bmatrix} 3+4 & -1+3 \\ 2-2 & 4-6 \end{bmatrix}$

$= \begin{bmatrix} 7 & 2 \\ 0 & -2 \end{bmatrix}$

R.H.S = $(A+B)+C$

$A+B = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix}$

$\begin{bmatrix} 3+0 & -1+2 \\ 2+1 & 4-4 \end{bmatrix} \quad \begin{bmatrix} 3 & 1 \\ 3 & 0 \end{bmatrix}$

$(A+B)+C = \begin{bmatrix} 3 & 1 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$

$= \begin{bmatrix} 7 & 2 \\ 0 & -2 \end{bmatrix}$

$A(BC) = (AB)C$

$BC = \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$

$\begin{bmatrix} 0+6 & 0-4 \\ 4+12 & 1+8 \end{bmatrix} = \begin{bmatrix} 6 & -4 \\ 16 & 9 \end{bmatrix}$

$$\begin{bmatrix} -18-16 & -12\cdot 9 \\ \cdot 12+64 & -8+36 \end{bmatrix}$$

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$$A(BC) = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -6 & -4 \\ -16 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} -34 & 21 \\ 52 & 28 \end{bmatrix}$$

R.H.S $(AB)C$

$$AB = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix} \Rightarrow \begin{bmatrix} 0-1 & 6+4 \\ 0+4 & 4-16 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 10 \\ 4 & -12 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -1 & 10 \\ 4 & -12 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} -4-30 & -1-20 \\ 16+36 & 4+24 \end{bmatrix}$$

$$= \begin{bmatrix} -34 & 21 \\ 52 & 28 \end{bmatrix}.$$

(c) $A(B+C) = AB+AC$

L.H.S $A(B+C)$

$$B+C = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ -2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 12+2 & 9+6 \\ 8-18 & 6+24 \end{bmatrix} = \begin{bmatrix} 14 & 15 \\ -0 & -18 \end{bmatrix}$$

R.H.S

$AB+AC$

$$AB = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 0-1 & 6+4 \\ 0+4 & 4-16 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 10 \\ 4 & -12 \end{bmatrix}$$

$$AC = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+3 & 3+2 \\ 8-12 & 2-8 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -4 & -6 \end{bmatrix}$$

$$AB+BC = \begin{bmatrix} -1 & 10 \\ 4 & -12 \end{bmatrix} + \begin{bmatrix} 15 & 5 \\ -4 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 15 \\ 0 & -18 \end{bmatrix}.$$

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(d) $(a+b)c = ac+bc$

L.H.S

$(a+b)c$

$$(4-7)\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$

$$-3\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} -12 & -3 \\ 9 & 6 \end{bmatrix}$$

R.H.S

$ac+bc$

$$ac = 4\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 16 & 4 \\ -12 & -8 \end{bmatrix}$$

$$bc = -7\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} -28 & -7 \\ 21 & 14 \end{bmatrix}$$

$$ac+bc = \begin{bmatrix} 16 & 4 \\ -12 & -8 \end{bmatrix} + \begin{bmatrix} -28 & -7 \\ 21 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 16-28 & 4-7 \\ -12+21 & -8+14 \end{bmatrix}$$

$$= \begin{bmatrix} -12 & -3 \\ 9 & 6 \end{bmatrix}$$

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Exercise 1.4

Question #02

(a)

$$a(BC) = (aB)C = B(aC)$$

$$a(BC) = 4\left(\begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}\right)$$

$$= 4\begin{bmatrix} 0 & 2 \\ -3 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ -12 & 32 \end{bmatrix}$$

$(aB)C$

$$= \left(4\begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix}\right)\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 8 \\ 4 & -16 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 8 \\ -12 & 32 \end{bmatrix}$$

$B(aC)$

$$= \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix}\left(4\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} 16 & 4 \\ -12 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 8 \\ -12 & 32 \end{bmatrix}$$

Hence proved

(b) $A(B-C) = AB - AC$

$$A(B-C) = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \left(\begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} -4 & 1 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -12 & -1 \\ 4 & -8 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 2 & -16 \end{bmatrix}$$

$$AC = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -1 \\ -6 & -8 \end{bmatrix}$$

$$AB - AC = \begin{bmatrix} 0 & -2 \\ 2 & -16 \end{bmatrix} - \begin{bmatrix} 12 & -1 \\ -6 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} -12 & -1 \\ 4 & -8 \end{bmatrix}$$

Hence proved

(c) $(B+C)A = BA+CA$

$$(B+C)A = \left(\begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}\right)\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 \\ -2 & -6 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 12 & -3 \\ -4 & -24 \end{bmatrix}$$

$$BA+CA = \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 2 & -16 \end{bmatrix} + \begin{bmatrix} 12 & -1 \\ -6 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -3 \\ -4 & -24 \end{bmatrix}$$ Hence proved.

d) $a(bC) = (ab)C$

$$= 4\left(-7\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}\right)$$

$$= 4\begin{bmatrix} -28 & -7 \\ 21 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 112 & -28 \\ 84 & 56 \end{bmatrix}$$

$$= (4)(-7)\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$

$$= -28\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 112 & -28 \\ 84 & 56 \end{bmatrix}$$ Hence proved.

Question # 3

(a) $(A^T)^T = A$

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$$

$$(A^T)^T = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$$ Hence proved.

(b) $(AB)^T = B^T A^T$

L.H.S

$$(AB)^T$$

$$= \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 0 & -2 \\ 2 & -16 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 0 & 2 \\ -2 & -16 \end{bmatrix}$$

R.H.S =

$$B^T = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \\ -2 & -16 \end{bmatrix}$$ Hence

L.H.S = R.H.S

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Question # 4

(a)

$$(A+B)^T = A^T + B^T$$

L.H.S

$$A+B = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 \\ 3 & 0 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 3 & 3 \\ 1 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 \\ 1 & 0 \end{bmatrix}$$

L.H.S = R.H.S

(b) $(aC)^T = aC^T$

$$aC = 4\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 16 & 4 \\ -12 & -8 \end{bmatrix}$$

$$(aC)^T = \begin{bmatrix} 16 & -12 \\ 4 & -8 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 4 & -3 \\ 1 & -2 \end{bmatrix}$$

$$= 4\begin{bmatrix} 4 & -3 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 16 & -12 \\ 4 & -8 \end{bmatrix}$$

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Question # 05

$$A = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}$$

Find A^{-1}

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$a = 2$, $b = -3$, $c = 4$, $d = 4$

$$A^{-1} = \frac{1}{(2)(4) - (-3)(4)} \begin{bmatrix} 4 & 3 \\ -4 & 2 \end{bmatrix}$$

$$= \frac{1}{8 + 12} \begin{bmatrix} 4 & 3 \\ -4 & 2 \end{bmatrix}$$

$$= \frac{1}{20} \begin{bmatrix} 4 & 3 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1/5 & 3/20 \\ -1/5 & 1/10 \end{bmatrix}$$ **Ans**

$$B = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

B^{-1}

$$B^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$a = 3$, $b = 1$, $c = 5$, $d = 2$

$$= \frac{1}{(3)(2) - (1)(5)} \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

$$= \frac{1}{6-5}\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

Question #7

$$C = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$C^{-1} = \frac{1}{6-0}\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 0 \\ 0 & 1/3 \end{bmatrix}$$

Question #8

$$D = \begin{bmatrix} 6 & 4 \\ -2 & -1 \end{bmatrix}$$

$$D^{-1} = \frac{1}{-6+8}\begin{bmatrix} -1 & -4 \\ 2 & 6 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -1 & -4 \\ 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 & -2 \\ 1 & 3 \end{bmatrix} \quad \underline{\text{Ans}}$$

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Q# 9

$$A = \begin{bmatrix} \frac{1}{2}(e^{x}+e^{-x}) & \frac{1}{2}(e^{x}-e^{-x}) \\ \frac{1}{2}(e^{x}-e^{-x}) & \frac{1}{2}(e^{x}+e^{-x}) \end{bmatrix}$$

determinant $A = ad - bc$

$$\frac{1}{2}(e^{x}+e^{-x})\cdot\frac{1}{2}(e^{x}+e^{-x}) - \frac{1}{2}(e^{x}-e^{x})\frac{1}{2}(e^{x}-e^{-x})$$

$$= \frac{1}{2}(e^{x}+e^{-x})^2 - \frac{1}{2}(e^{x}-e^{-x})^2$$

= Apply Difference of squares

$$\frac{1}{2}(e^{x}+e^{-x})+\frac{1}{2}(e^{x}-e^{-x})\left(\frac{1}{2}e^{x}+e^{-x}+\frac{1}{2}e^{x}-e^{-x}\right)$$

$$\left(\frac{1}{2}e^{x}+\cancel{\frac{1}{2}e^{-x}}+\frac{1}{2}e^{x}\cancel{-\frac{1}{2}e^{-x}}\right)\left(\cancel{\frac{1}{2}e^{x}}+\cancel{\frac{1}{2}e^{-x}}-\cancel{\frac{1}{2}e^{x}}+\cancel{\frac{1}{2}e^{-x}}\right)$$

cancel opposite term.

$$\left(\frac{1}{2}e^{x}+\frac{1}{2}e^{x}\right)\left(\frac{1}{2}e^{-x}+\frac{1}{2}e^{-x}\right)$$

$$(e^{x})(e^{-x}) = 1$$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} \frac{1}{2}(e^{x}+e^{-x}) & \frac{1}{2}(e^{x}-e^{-x}) \\ \frac{1}{2}(e^{x}-e^{-x}) & \frac{1}{2}(e^{x}+e^{-x}) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}(e^{x}+e^{-x}) & \frac{1}{2}(e^{x}-e^{-x}) \\ \frac{1}{2}(e^{x}-e^{-x}) & \frac{1}{2}(e^{x}+e^{-x}) \end{bmatrix}$$

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Question # 10

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$A^{-1} = \frac{1}{\cos^2\theta - (-\sin^2\theta)} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

$$A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$ Ans

Question # 11

$$(A^T)^{-1} = (A^{-1})^T$$

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$$

$$(A^T)^{-1} = \frac{1}{12+2} \begin{bmatrix} 4 & -2 \\ 1 & 3 \end{bmatrix}$$

$$= \frac{1}{14} \begin{bmatrix} 4 & -2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4/14 & -2/14 \\ 1/14 & 3/14 \end{bmatrix}$$

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$$(ABC)^T = C^T B^T A^T$$

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$

$$ABC = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ -6 & 32 \end{bmatrix}$$

$$(ABC)^T = \begin{bmatrix} 0 & -6 \\ -2 & 32 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 4 & -3 \\ 1 & -2 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}$$

$$C^T B^T = \begin{bmatrix} 4 & -3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -3 \\ 2 & 8 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -3 \\ 2 & 8 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -6 \\ -2 & 32 \end{bmatrix} \quad \text{Ans}$$

Question 16

$$(5A^T)^{-1} = \begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}$$

$$(5A^T)^{-1} = B$$

$$B = \begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}$$

$$5A^T = B^{-1}$$

$$A^T = \frac{1}{5} B^{-1}$$

$$(A^T)^T = \left(\frac{1}{5} B^{-1}\right)^T$$

$$A = \left(\frac{1}{5} B^{-1}\right)^T \qquad \because (A^T)^T = A$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}^{-1} = \frac{1}{(-3)(2)-(-1)(5)} \begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix}$$

$$= \frac{1}{-6+5} \begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix}$$

$$= -1 \begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -1 \\ 5 & 3 \end{bmatrix}$$

$$A = \left(\frac{1}{5} B^{-1}\right)^T = \left(\frac{1}{5} \begin{bmatrix} -2 & -1 \\ 5 & 3 \end{bmatrix}\right)^T$$

$$= \begin{bmatrix} -2/5 & -1/5 \\ 1 & 3/5 \end{bmatrix}^T = \begin{bmatrix} -2/5 & 1 \\ -1/5 & 3/5 \end{bmatrix}$$

Question # 19

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \quad \text{(a) } A^3$$

$$A^3 = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (3\times 3)+(1\times 2) & (3\times 1)+(1\times 1) \\ (2\times 3)+(1\times 2) & (2\times 1)+(1\times 1) \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (11\times 3)+(4\times 2) & (11\times 1)+(4\times 1) \\ (8\times 3)+(3\times 2) & (8\times 1)+(3\times 1) \end{bmatrix}$$

$$= \begin{bmatrix} 41 & 15 \\ 30 & 11 \end{bmatrix}$$

(b) A^{-3} $\qquad A^3 = \begin{bmatrix} 41 & 15 \\ 30 & 11 \end{bmatrix}$

$$(A^3)^{-1} = \frac{1}{(41\times 11) - 15\times 30} \begin{bmatrix} 11 & -15 \\ -30 & 41 \end{bmatrix}$$

$$= 1 \times \begin{bmatrix} 11 & -15 \\ -30 & 41 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & -15 \\ -30 & 41 \end{bmatrix}$$

$$A^2 - 2A + I$$

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$A \cdot A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (9) + (1 \times 2) & (3 \times 1) + (1 \times 1) \\ (2 \times 3) + (1 \times 2) & (2 \times 1) + (1 \times 1) \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix} - 2\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix} - \begin{bmatrix} 6 & 2 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 2 \\ 4 & 2 \end{bmatrix}$$

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Question # 20

$$A = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$A^3 = A \cdot A \cdot A$$

$$= \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 0+1 \\ 8+4 & 0+1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 \\ 12 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8+4 & 1\times 0 + 1\times 1 \\ 12\times 2+4 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 1 \\ 28 & 1 \end{bmatrix} \quad \text{Ans}$$

$$(A^3)^{-1} = \frac{1}{12-28} \begin{bmatrix} 1 & -1 \\ -28 & 12 \end{bmatrix}$$

$$= \frac{1}{-16} \begin{bmatrix} 1 & -1 \\ -28 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} -1/16 & 1/16 \\ 28/16 & -12/16 \end{bmatrix} \quad \text{Ans}$$

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$A^2 - 2A + I$

$= \begin{bmatrix} 4 & 1 \\ 12 & 1 \end{bmatrix} - 2\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 1 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 8 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 \\ 4 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 4 & 0 \end{bmatrix}$ Ans

Question # 23

23/ $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

Find all values of a, b, c, d

$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$

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Question # 25

$$3x_1 - 2x_2 = -1$$
$$4x_1 + 5x_2 = 3$$

$$Ax = b$$
$$x = A^{-1}b$$

$$A^{-1} = \frac{\begin{bmatrix} 5 & 2 \\ -4 & 3 \end{bmatrix}}{23} \Rightarrow \frac{\begin{bmatrix} 5 & 2 \\ -4 & 3 \end{bmatrix}}{23}$$

$$A^{-1} = \begin{bmatrix} 5/23 & 2/23 \\ -4/23 & 3/23 \end{bmatrix}$$

$$x = A^{-1}b$$
$$x = \begin{bmatrix} 5/23 & 2/23 \\ -4/23 & 3/23 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$x = \begin{bmatrix} -5/23 & -6/23 \\ 4/23 & +9/23 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -5-6/23 \\ 4+9/23 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -11/23 \\ 13/23 \end{bmatrix}$$

Question # 21

$$P(x) = x - 2$$

$$A = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$P(A) = A - 2I$$

$$= \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 4 & -1 \end{bmatrix} \quad \text{Ans}$$

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Exercise # 1.5

Question # 01

(a) $\begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$

The given matrix is elementary.
We can obtain it from the identity matrix by adding -5 times the first row to second row.

$$= \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix} \rightarrow -5R_1 + R_2 \longrightarrow R_2$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

(b) $\begin{bmatrix} -5 & 1 \\ 1 & 0 \end{bmatrix}$

The given matrix is not elementary.

$$\begin{bmatrix} -5 & 1 \\ 1 & 0 \end{bmatrix}$$

(c) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

This matrix is not elementary because matrix has a zero row.
We cannot obtain a matrix with a zero row from identity matrix.

$$d = \begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⇒ Not elementary

Question # 02

$$a = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{3} \end{bmatrix}$$

⇒ Elementary matrix

(b) $$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

⇒ Elementary matrix

c) $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix}$$

⇒ Elementary matrix

d $$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

⇒ This is not elementary matrix

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Question # 03

Find a row operation and corresponding elementary matrix that will restore the given elementary matrix is identity matrix

(a)

$$\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$$

The row operation to get the identity matrix

$$R_1 = 3 \times R_2 + R_1$$

$$\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \xrightarrow{3R_2 + R_1 \rightarrow R_1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} -7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The row operation to get identity matrix

$$R_1 = \frac{-1}{7} \times R_1$$

$$\begin{bmatrix} -7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\frac{-1}{7}R_1 \rightarrow R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix}$$

The row operation to get identity matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix} \xrightarrow{5R_1 + R_3 \longrightarrow R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The row operation to get the identity matrix.

$R_1 \longleftrightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question # 04

(a)

$$\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

The row operation to get identity matrix

$$\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \xrightarrow{3R_1+R_2 \to R_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

The row operation to get identity matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Interchange $R_1 \longrightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question # 05

Elementary matrix E and a matrix A are given. Identify row operation to E and verify that the product EA from operation to A.

(a)

$$E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & -6 & -6 \end{bmatrix}$$

Interchange R_1 and R_2

So Resulting matrix is

$$EA = \begin{bmatrix} 3 & -6 & -6 & -6 \\ -1 & -2 & 5 & -1 \end{bmatrix}$$

Multiplying matrix E and A

$$E \cdot A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & -6 & -6 & -6 \\ -1 & -2 & 5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -6 & -6 & -6 \\ -1 & -2 & 5 & -1 \end{bmatrix}$$

$R_3 - \frac{1}{5}R_1 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 1 & -1/2 & 0 & 1 & -2 \\ 0 & -1 & 1/2 & -1 & 0 & 1 \end{array}\right]$$

$R_3 + R_2 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 1 & -1/2 & 0 & 1 & -2 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{array}\right]$$

The matrix has no inverse.

Find inverse

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$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_3 - R_1 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right]$$

$R_3 - R_2 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -2 & -1 & -1 & 1 \end{array}\right]$$

$-1/2\, R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right]$$

$R_2 - R_3 \longrightarrow R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right]$$

$-1/2\, R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right]$$

$R_2 - R_3 \longrightarrow R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right]$$

$R_1 - R_3 \longrightarrow R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 1/2 \\ 0 & 1 & 0 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right]$$

$R_2 - 2R_1 \longrightarrow R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1/2 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & -1 \end{array}\right]$$

$-R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1/2 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & +1 \end{array}\right]$$

$R_2 - R_3 \longrightarrow R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$R_1 - 3R_3 \longrightarrow R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1/2 & 3 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$R_1 - 3R_2 \longrightarrow R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7/2 & 0 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 7/2 & 0 & -3 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Question # 16

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 5 & 0 \\ 1 & 3 & 5 & 7 \end{bmatrix}$$

$A^{-1} = ?$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 3 & 5 & 0 & 0 & 0 & 1 & 0 \\ 1 & 3 & 5 & 7 & 0 & 0 & 0 & 1 \end{array}\right]$$

$-1R_1 + R_2, R_3, R_4 \longrightarrow R_2, R_3, R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 3 & 5 & 0 & -1 & 0 & 1 & 0 \\ 0 & 3 & 5 & 7 & -1 & 0 & 0 & 1 \end{array}\right]$$

$-R_2 + R_3 \,\&\, R_4 \longrightarrow R_3, R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 5 & 7 & 0 & -1 & 0 & 1 \end{array}\right]$$

$-R_3 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 & -1 & 1 \end{array}\right]$$

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$\Rightarrow \frac{1}{3} R_2 \longrightarrow R_2$

$\frac{1}{5} R_3 \longrightarrow R_3$

$\frac{1}{7} R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/3 & 1/3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/5 & 1/5 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1/7 & 1/7 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/3 & 1/3 & 0 & 0 \\ 0 & -1/5 & 1/5 & 0 \\ 0 & 0 & -1/5 & 1/7 \end{bmatrix}$$

Question # 17

$$A = \begin{bmatrix} 2 & -4 & 0 & 0 \\ 1 & 2 & 12 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & -1 & -4 & -5 \end{bmatrix}$$

$A^{-1} = ?$

$$\left[\begin{array}{cccc|cccc} 2 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \longleftrightarrow R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 2 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right]$$

$-2R_1 + R_2 \longrightarrow R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & -8 & -24 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \longleftrightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -8 & -24 & 0 & 1 & -2 & 0 & 0 \end{array}\right]$$

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$\Rightarrow -1 R_2 \longrightarrow R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 5 & 0 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -8 & -24 & 0 & +1 & -2 & 0 & 0 \end{array}\right]$$

$\Rightarrow 8 R_2 + R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 5 & 0 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 8 & 40 & 1 & -2 & 0 & -8 \end{array}\right]$$

$1/2 R_3 \longrightarrow R_3$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 5 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 8 & 40 & 1 & -2 & 0 & -8 \end{array}\right]$$

$-8R_3 + R_4 \longleftrightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 5 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 40 & 1 & -2 & -4 & -8 \end{array}\right]$$

$1/40 R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 5 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1/40 & -2/40 & -4/40 & -8/40 \end{array}\right]$$

$-5R_4 + R_2 \longrightarrow R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & -1/8 & 1/4 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 & 1/40 & -1/20 & -1/10 & -1/5 \end{array}\right]$$

$-4R_3 + R_2 \longrightarrow R_2$

$-12R_3 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 0 & 1 & -6 & 0 \\ 0 & 1 & 0 & 0 & -1/8 & 1/4 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 & 1/40 & -1/20 & -1/10 & -1/5 \end{array}\right]$$

$-2R_2 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/4 & 1/2 & -3 & 0 \\ 0 & 1 & 0 & 0 & -1/8 & 1/4 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 & 1/40 & -1/20 & -1/10 & -1/5 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1/4 & 1/2 & -3 & 0 \\ -1/8 & 1/4 & -3/2 & 0 \\ 0 & 0 & 1/2 & 0 \\ 1/40 & -1/20 & -1/10 & -1/5 \end{bmatrix}$$

$\Rightarrow -1R_2 \longrightarrow R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 5 & 0 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -8 & -24 & 0 & +1 & -2 & 0 & 0 \end{array}\right]$$

$\Rightarrow 8R_2 + R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 5 & 0 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 8 & 40 & 1 & -2 & 0 & -8 \end{array}\right]$$

$1/2\, R_3 \longrightarrow R_3$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 5 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 8 & 40 & 1 & -2 & 0 & -8 \end{array}\right]$$

$-8R_3 + R_4 \longleftrightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 5 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 40 & 1 & -2 & -4 & -8 \end{array}\right]$$

$1/40\, R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 5 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1/40 & -2/40 & -4/40 & -8/40 \end{array}\right]$$

$-5R_4 +$

$-4R_3$

$-12R_3$

$-2R$

$-5R_4 + R_2 \longrightarrow R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & -1/8 & 1/4 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 & 1/40 & -1/20 & -1/10 & -1/5 \end{array}\right]$$

$-4R_3 + R_2 \longrightarrow R_2$

$-12R_3 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 0 & 1 & -6 & 0 \\ 0 & 1 & 0 & 0 & -1/8 & 1/4 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 & 1/40 & -1/20 & -1/10 & -1/5 \end{array}\right]$$

$-2R_2 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/4 & 1/2 & -3 & 0 \\ 0 & 1 & 0 & 0 & -1/8 & 1/4 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 & 1/40 & -1/20 & -1/10 & -1/5 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1/4 & 1/2 & -3 & 0 \\ -1/8 & 1/4 & -3/2 & 0 \\ 0 & 0 & 1/2 & 0 \\ 1/40 & -1/20 & -1/10 & -1/5 \end{bmatrix}$$

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Q#18

$$A = \left[\begin{array}{cccc|cccc} 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 & 0 & 1 & 0 \\ 2 & 1 & 5 & -3 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \longleftrightarrow R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 & 0 & 1 & 0 \\ 2 & 1 & 5 & -3 & 0 & 0 & 0 & 1 \end{array}\right]$$

$-2R_1 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ & -1 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 5 & -5 & 0 & -2 & -0 & 1 \end{array}\right]$$

$R_2 \longleftrightarrow R_3$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -5 & 0 & -2 & 0 & 1 \end{array}\right]$$

$-R_2 \longrightarrow R_2$

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$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -5 & 0 & -2 & 0 & 1 \end{array}\right]$$

$$-R_2 + R_4 \longrightarrow R_4$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 0 & -2 & 1 & 1 \end{array}\right]$$

$$-4R_3 + R_4 \longrightarrow R_4$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & -4 & -2 & 1 & 1 \end{array}\right]$$

$$1/2\, R_3 \longrightarrow R_3$$

$$-1/5\, R_4 \longrightarrow R_4$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 4/5 & 2/5 & -1/5 & -1/5 \end{array}\right]$$

$-R_4 + R_1 \longrightarrow R_1$

$3R_3 + R_2 \longrightarrow R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -4/5 & 3/5 & 1/5 & 1/5 \\ 0 & 1 & 0 & 0 & 3/2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 4/5 & 2/5 & -1/5 & -1/5 \end{array}\right]$$

Q19

$$\begin{bmatrix} K_1 & 0 & 0 & 0 \\ 0 & K_2 & 0 & 0 \\ 0 & 0 & K_3 & 0 \\ 0 & 0 & 0 & K_4 \end{bmatrix}$$

$$= \left[\begin{array}{cccc|cccc} K_1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & K_2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & K_3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & K_4 & 0 & 0 & 0 & 1 \end{array}\right]$$

$1/K_1 \, R_1 \longrightarrow R_1$

$1/K_2 \, R_2 \longrightarrow R_2$

$1/K_3 \, R_3 \longrightarrow R_3$

$1/K_4 \, R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/K_1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1/K_2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/K_3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1/K_4 \end{array}\right]$$

(b)

$$\begin{bmatrix} K & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & K & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{cccc|cccc} K & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & K & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$1/K\ R_1 \longrightarrow R_1$

$1/K\ R_3 \longrightarrow R_3$

$$\left[\begin{array}{cccc|cccc} 1 & 1/K & 0 & 0 & 1/K & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1/K & 0 & 0 & 1/K & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$1/K\ R_4 + R_3 \longrightarrow R_3$

$-1/K\ R_2 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/K & -1/K & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/K & -1/K \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Q2. $\begin{bmatrix} 0 & 0 & 0 & K_1 \\ 0 & 0 & K_2 & 0 \\ 0 & K_3 & 0 & 0 \\ K_4 & 0 & 0 & 0 \end{bmatrix}$

$$= \left[\begin{array}{cccc|cccc} 0 & 0 & 0 & K_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & K_2 & 0 & 0 & 1 & 0 & 0 \\ 0 & K_3 & 0 & 0 & 0 & 0 & 1 & 0 \\ K_4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \longleftrightarrow R_4$

$R_2 \longleftrightarrow R_3$

$$\left[\begin{array}{cccc|cccc} K_4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & K_3 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & K_2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & K & 1 & 0 & 0 & 0 \end{array}\right]$$

$1/K_4 \; R_1 \longrightarrow R_1$

$1/K_3 \; R_2 \longrightarrow R_2$

$1/K_2 \; R_3 \longrightarrow R_3$

$1/K_1 \; R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1/K_4 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1/K_3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/K_2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1/K_1 & 0 & 0 & 0 \end{array}\right]$$

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(b)

$$A = \begin{bmatrix} k & 0 & 0 & 0 \\ 1 & k & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & k \end{bmatrix} \qquad A^{-1} = ?$$

$$= \left[\begin{array}{cccc|cccc} k & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & k & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & k & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & k & 0 & 0 & 0 & 1 \end{array}\right]$$

Each row multiply by $1/k$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/k & 0 & 0 & 0 \\ 1/k & 1 & 0 & 0 & 0 & 1/k & 0 & 0 \\ 0 & 1/k & 1 & 0 & 0 & 0 & 1/k & 0 \\ 0 & 0 & 1/k & 1 & 0 & 0 & 0 & 1/k \end{array}\right]$$

$-1/k \, R_1 + R_2 \longrightarrow R_2$

$-1/k \, R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/k & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/k^2 & 1/k & 0 & 0 \\ 0 & 0 & 1 & 0 & 1/k^3 & -1/k^2 & 1/k & 0 \\ 0 & 0 & 1/k & 1 & 0 & 0 & 0 & 1/k \end{array}\right]$$

$-1/k \, R_3 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/k & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/k^2 & 1/k & 0 & 0 \\ 0 & 0 & 1 & 0 & 1/k^3 & -1/k^2 & 1/k & 0 \\ 0 & 0 & 0 & 1 & -1/k^4 & 1/k^3 & -1/k^2 & 1/k \end{array}\right]$$

Q#21

$$\begin{bmatrix} c & c & c \\ 1 & c & c \\ 1 & 1 & c \end{bmatrix}$$

$R_1 \longleftrightarrow R_3$

$$= \begin{bmatrix} 1 & 1 & c \\ 1 & c & c \\ c & c & c \end{bmatrix}$$

$-1R_2 + R_2 \longrightarrow R_2$

$-cR_1 + R_3 \longrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & c \\ 0 & -1+c & 0 \\ 0 & 0 & c-c^2 \end{bmatrix}$$

$c - c^2 = c(1-c)$

$$\boxed{c=0,\ c=+1}$$

Q#22

$$\begin{bmatrix} c & 1 & 0 \\ 1 & c & 1 \\ 0 & 1 & c \end{bmatrix}$$

$R_1 \longleftrightarrow R_2$

$$\begin{bmatrix} 1 & c & 1 \\ c & 1 & 0 \\ 0 & 1 & c \end{bmatrix}$$

$R_2 \longleftrightarrow R_3$

$$\begin{bmatrix} 1 & c & 1 \\ 0 & 1 & c \\ c & 1 & 0 \end{bmatrix}$$

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$-cR_1 + R_3 \longrightarrow R_3$

$-c^2-1 \; R_2 + R_3 \longrightarrow R_3$

$$\begin{bmatrix} 1 & c & 1 \\ 0 & 1 & c \\ 0 & 0 & c^3-2c \end{bmatrix}$$

$c^3 - 2c = 0$

$c(c^2-2) = 0$

$c = 0 \; , \quad c^2 - 2 = 0$

$c^2 = 2$

$c = \sqrt{2}$

Q # 23 /

$$A = \begin{bmatrix} -3 & 1 \\ 2 & 2 \end{bmatrix}$$

$2R_2 + R_1 \longrightarrow R_1$

$$E_1 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$-2R_1 + R_2$

$$\begin{bmatrix} 1 & 5 \\ 0 & -8 \end{bmatrix} \longleftarrow \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$-1/8 \, R_2$

$$\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \longleftarrow E_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1/8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ -5 times $R_2 + R_1 \longrightarrow R_1$

$$E_4 = \begin{bmatrix} 1 & -5 \\ 0 & 1 \end{bmatrix}$$

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Exercise # 1.6

More on Linear systems and invertible matrices

A system of linear equations has zero one, or infinitely many solutions. There are no other possibilities.

Solution of a linear system using A^{-1}

$$x_1 + 2x_2 + 3x_3 = 5$$
$$2x_1 + 5x_2 + 3x_3 = 3$$
$$x_1 \qquad\quad + 8x_3 = 17$$

$$Ax = b$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 3 \\ 17 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

$$x = A^{-1}b = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 17 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$x_1 = 1$$
$$x_2 = -1$$
$$x_3 = 2$$

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Solving Two linear system at once

$x_1 + 2x_2 + 3x_3 = 4$ $\quad\quad$ $x_1 + 2x_2 + 3x_3 = 1$

$2x_1 + 5x_2 + 3x_3 = 5$ $\quad\quad$ $2x_1 + 5x_2 + 3x_3 = 6$

$x_1 + 8x_3 = 9$ $\quad\quad$ $x_1 + 8x_3 = -6$

$$\left[\begin{array}{ccc|c|c} 1 & 2 & 3 & 4 & 1 \\ 2 & 5 & 3 & 5 & 6 \\ 1 & 0 & 8 & 9 & -6 \end{array}\right]$$

Reducing this matrix to reduced row echlon form yields

$$\left[\begin{array}{ccc|c|c} 1 & 2 & 3 & 4 & 1 \\ 2 & 5 & 3 & 5 & 6 \\ 1 & 0 & 8 & 9 & -6 \end{array}\right]$$

$2R_1 - R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c|c} 1 & 2 & 3 & 4 & 1 \\ 0 & -1 & 3 & 3 & -4 \\ 1 & 0 & 8 & 9 & -6 \end{array}\right]$$

$1R_1 - R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c|c} 1 & 2 & 3 & 4 & 1 \\ 0 & -1 & 3 & 3 & -4 \\ 0 & 2 & -5 & -5 & 7 \end{array}\right]$$

$2R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c|c} 1 & 2 & 3 & 4 & 1 \\ 0 & -1 & 1 & 3 & -4 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right]$$

$-R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c|c} 1 & 2 & 3 & 4 & 1 \\ 0 & 1 & -3 & -3 & 4 \\ 0 & 0 & 1 & 1 & -1 \end{array}\right]$$

$3R_3 + R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c|c} 1 & 2 & 3 & 4 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 \end{array}\right]$$

$-3R_3 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 \end{array}\right]$$

$2R_2 - R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c|c} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 \end{array}\right]$$

a) $x_1 = 1$
$x_2 = 0$
$x_3 = 1$

b) $x_1 = 2$
$x_2 = 1$
$x_3 = -1$

Properties of invertible matrices

An $n \times n$ matrix A is invertible it has been necessary to find $n \times n$ matrix

$$AB = I \text{ and } BA = I$$

Theorem

let A be a square matrix

(a) If B is a square matrix satisfying $BA = I$ then $B = A^{-1}$

b) if B is a square matrix satisfying $AB = I$ then $B = A^{-1}$.

Equivalent statements

If A is an $n \times n$ matrix then following are equivalent.

(a) A is invertible
(b) $Ax = 0$ has only trivial solution
(c) The reduced row echlon form of A in I_n.
(d) A is ~~invertible~~ expressible as a product of E matrix
(e) $Ax = b$ is consistent for every $n \times 1$ matrix b
(f) $Ax = b$ has exactly one solution for every $n \times 1$ matrix b.

Example 3

Determinig consistency by elimination

$$x_1 + x_2 + 2x_3 = b_1$$
$$x_1 \quad\quad + x_3 = b_2$$
$$2x_1 + x_2 + 3x_3 = b_3$$

The augumented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & b_1 \\ 1 & 0 & 1 & b_2 \\ 2 & 1 & 3 & b_3 \end{array}\right]$$

$-R_1 + R_2 \rightarrow R_2$
$-2R_1 + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & b_1 \\ 0 & -1 & -1 & b_1 - b_2 \\ 0 & -1 & -1 & b_3 - 2b_1 \end{array}\right]$$

$R_2 \times -1 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & b_1 \\ 0 & 1 & 1 & b_1 - b_2 \\ 0 & -1 & -1 & b_3 - 2b_1 \end{array}\right]$$

$R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{cccc} 1 & 1 & 2 & b_1 \\ 0 & 1 & 1 & b_1 - b_2 \\ 0 & 0 & 0 & b_3 - b_2 - b_1 \end{array}\right]$$

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$b_3 - b_2 - b_1 = 0$

$b_3 = b_2 + b_1$

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_1 + b_2 \end{bmatrix}$$

where b_1 and b_2 are arbitrary.

Ex # 4

$$x_1 + 2x_2 + 3x_3 = b_1$$
$$2x_1 + 5x_2 + 3x_3 = b_2$$
$$x_1 \qquad + 8x_3 = b_3$$

Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & b_1 \\ 2 & 5 & 3 & b_2 \\ 1 & 0 & 8 & b_3 \end{array}\right]$$

Exercise 1.6

Example

Question

Solve the system by inverting the coefficient matrix and .

(1)

$x_1 + x_2 = 2$
$5x_1 + 6x_2 = 9$

$Ax = b$

$A = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix}$

Identity matrix was adjoined

$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 5 & 6 & 0 & 1 \end{array}\right]$

$\Rightarrow -5R_1 + R_2 \longleftrightarrow R_2$

$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -5 & 1 \end{array}\right]$

$\Rightarrow -1R_2 + R_1 \longrightarrow R_1$

$\left[\begin{array}{cc|cc} 1 & 0 & 6 & -1 \\ 0 & 1 & -5 & 1 \end{array}\right]$

since $A^{-1} = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix}$

$$x = A^{-1} b$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$x_1 = 3$

$x_2 = -1$

Q2/

$$4x_1 - 3x_2 = -3$$
$$2x_1 - 5x_2 = 9$$

$$Ax = b$$

$$A = \begin{bmatrix} 4 & -3 \\ 2 & -5 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad b = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$$

Adjoined the identity matrix

$$\left[\begin{array}{cc|cc} 4 & -3 & 1 & 0 \\ 2 & -5 & 0 & 1 \end{array}\right]$$

$R_1 \longleftrightarrow R_2$

$$\left[\begin{array}{cc|cc} 2 & -5 & 0 & 1 \\ 4 & -3 & 1 & 0 \end{array}\right]$$

$-2R_1 + R_2 \longrightarrow R_2$

$$\left[\begin{array}{cc|cc} 2 & -5 & 0 & 1 \\ 0 & 7 & 1 & -2 \end{array}\right]$$

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$\frac{1}{2}R_1 \longrightarrow R_1$

$\frac{1}{7}R_2 \longrightarrow R_2$

$$\left[\begin{array}{cc|cc} 1 & -5/2 & 0 & 1/2 \\ 0 & 1 & 1/7 & -2/7 \end{array}\right]$$

$\frac{5}{2}R_2 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{cc|cc} 1 & 0 & 5/14 & -3/14 \\ 0 & 1 & 1/7 & -2/7 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 5/14 & -3/14 \\ 1/7 & -2/7 \end{bmatrix}$$

$$x = A^{-1}b$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5/14 & -3/14 \\ 1/7 & -2/7 \end{bmatrix} \begin{bmatrix} -3 \\ 9 \end{bmatrix}$$

$x_1 = -3$

$x_2 = -3$

$x_1, x_2 = -3$

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Question #3

$Ax = b$

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right]$$

$-2R_1 + R_2 \longrightarrow R_2$

$-2R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & -3 & -1 & -2 & 0 & 1 \end{array}\right]$$

$-1R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right]$$

$R_2 \longleftrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & -4 & -1 & -2 & 1 & 0 \end{array}\right]$$

$4R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -2 & -3 & 4 \end{array}\right]$$

$-1R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array}\right]$$

$-1R_3 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & -1 & -3 & 4 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array}\right]$$

$-3R_2 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix}$$

$Ax = b$

$x = A^{-}b$

$$= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -7 \end{bmatrix}$$

$x_1 = -1$

$x_2 = 4$

$x_3 = -7$

$Ax = b$

$$A = \begin{bmatrix} 5 & 3 & 2 \\ 3 & 3 & 2 \\ 0 & 1 & 1 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

Q4

$$\left[\begin{array}{ccc|ccc} 5 & 3 & 2 & 1 & 0 & 0 \\ 3 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\Rightarrow -1R_2 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & -1 & 0 \\ 3 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\Rightarrow \frac{1}{2}R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 0 \\ 3 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\Rightarrow R_2 \longleftrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 3 & 2 & -3/2 & 5/2 & 0 \end{array}\right]$$

$\Rightarrow -3R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -3/2 & 5/2 & -3 \end{array}\right]$$

$\Rightarrow -1R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3/2 & -5/2 & 3 \end{array}\right]$$

$\Rightarrow -R_3 + R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 0 \\ 0 & 1 & 0 & -3/2 & 5/2 & -2 \\ 0 & 0 & 1 & 3/2 & -5/2 & 3 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 0 \\ -3/2 & 5/2 & -2 \\ 3/2 & -5/2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 0 \\ -3/2 & 5/2 & -2 \\ 3/2 & -5/2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -11 \\ 16 \end{bmatrix}$$

$x_1 = 1$

$x_2 = -11$

$x_3 = 16$

Q5/ $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -4 \\ -4 & 1 & 1 \end{bmatrix}$ $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $b = \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix}$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & -4 & 0 & 1 & 0 \\ -4 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$-1R_1 + R_2 \to R_2$
$4R_1 + R_3 \to R_3$
$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -5 & -1 & 1 & 0 \\ 0 & 5 & 5 & 4 & 0 & 1 \end{array}\right]$$

$R_2 \longleftrightarrow R_3$
$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 5 & 5 & 4 & 0 & 1 \\ 0 & 0 & -5 & -1 & 1 & 0 \end{array}\right]$$

$\frac{1}{5}R_2 \to R_2$
$-\frac{1}{5}R_3 \to R_3$
$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 4/5 & 0 & 1/5 \\ 0 & 0 & 1 & 1/5 & -1/5 & 0 \end{array}\right]$$

$-R_3 + R_2 + R_1 \to R_2, R_1$
$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 4/5 & 1/5 & 0 \\ 0 & 1 & 0 & 3/5 & 1/5 & 1/5 \\ 0 & 0 & 1 & 1/5 & -1/5 & 0 \end{array}\right]$$

$-1R_2 + R_1 \to R_1$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/5 & 0 & -1/5 \\ 0 & 1 & 0 & 3/5 & 1/5 & 1/5 \\ 0 & 0 & 1 & 1/5 & -1/5 & 0 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1/5 & 0 & -1/5 \\ 3/5 & 1/5 & 1/5 \\ 1/5 & -1/5 & 0 \end{bmatrix}$$

$x = A^{-1}b$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1/5 & 0 & -1/5 \\ 3/5 & 1/5 & 1/5 \\ 1/5 & -1/5 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$$

$x_1 = 1 \quad y = 5 \quad z = -1$

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Q#6

$$A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 1 & 4 & 4 \\ 1 & 3 & 7 & 9 \\ -1 & -2 & -4 & -6 \end{bmatrix}$$

$$x = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 7 \\ 4 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|cccc} 0 & -1 & -2 & -3 & 1 & 0 & 0 & 0 \\ 1 & 1 & 4 & 4 & 0 & 1 & 0 & 0 \\ 1 & 3 & 7 & 9 & 0 & 0 & 1 & 0 \\ -1 & -2 & -4 & -6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \longleftrightarrow R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 4 & 4 & 0 & 1 & 0 & 0 \\ 0 & -1 & -2 & -3 & 1 & 0 & 0 & 0 \\ 1 & 3 & 7 & 9 & 0 & 0 & 1 & 0 \\ -1 & -2 & -4 & -6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$-2R_1 + R_3 \longrightarrow R_3$

$R_1 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 4 & 4 & 0 & 1 & 0 & 0 \\ 0 & -1 & -2 & -3 & 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 5 & 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & -2 & 0 & 1 & 0 & 1 \end{array}\right]$$

$-1R_2 \longrightarrow R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 4 & 4 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 & -1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 5 & 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & -2 & 0 & 1 & 0 & 1 \end{array}\right]$$

$-2R_2 + R_3 \longrightarrow R_3$

$R_1 + R_4 \longrightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 4 & 4 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 & -1 & 1 & 0 & 1 \end{array}\right]$$

$-1R_3 \longrightarrow R_3$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 4 & 4 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 & -1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 & 0 & \\ 0 & 0 & 2 & 1 & -1 & 1 & & \end{array}\right]$$

$-2R_3 + R_4 \rightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 4 & 4 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 3 & -1 & 2 & 1 \end{array}\right]$$

$-1R_4 \rightarrow R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 4 & 4 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -3 & 1 & -2 & -1 \end{array}\right]$$

$-1R_4 + R_3 \rightarrow R_3$

$-3R_4 + R_2 \rightarrow R_2$

$-4R_4 + R_1 \rightarrow R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 4 & 0 & 12 & -3 & 8 & 4 \\ 0 & 1 & 2 & 0 & 8 & -3 & 6 & 3 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -3 & 1 & -2 & -1 \end{array}\right]$$

$-2R_3 + R_2 \rightarrow R_2$

$-4R_3 + R_1 \rightarrow R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 8 & -3 & 4 & 0 \\ 0 & 1 & 0 & 0 & 6 & -3 & 4 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -3 & 1 & -2 & -1 \end{array}\right]$$

$-1R_2 + R_1 \rightarrow R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 6 & -3 & 4 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -3. & 1 & -2 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 6 & -3 & 4 & 1 \\ 1 & 0 & 1 & 1 \\ -3 & 1 & -2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 6 & -3 & 4 & 1 \\ 1 & 0 & 1 & 1 \\ -3 & 1 & -2 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 7 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} -6 \\ 1 \\ 10 \\ 7 \end{bmatrix}$$

Q#7

$Ax = b$

$$A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 3 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right]$$

$R_1 \longleftrightarrow R_2$

$$\left[\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 3 & 5 & 1 & 0 \end{array}\right]$$

$-3R_1 + R_2 \longrightarrow R_2$

$$\left[\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 0 & -1 & 1 & -3 \end{array}\right]$$

$-1R_2 \longrightarrow R_2$

$$\left[\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 0 & 1 & -1 & 3 \end{array}\right]$$

$-2R_2 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{cc|cc} 1 & 0 & 2 & -5 \\ 0 & 1 & -1 & 3 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$= \begin{bmatrix} 2b_1 - 5b_2 \\ -1b_1 + 3b_2 \end{bmatrix}$$

$x_1 = 2b_1 - 5b_2$

$x_2 = -b_1 + 3b_2$

Q#8/ $Ax = b$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 5 & 8 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 5 & 0 & 1 & 0 \\ 3 & 5 & 8 & 0 & 0 & 1 \end{array}\right]$$

$-2R_1 + R_2 \longrightarrow R_2$

$-3R_1 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & -1 & -1 & -3 & 0 & 1 \end{array}\right]$$

$R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & -2 & -5 & 1 & 1 \end{array}\right]$$

$-\frac{1}{2}R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5/2 & -1/2 & -1/2 \end{array}\right]$$

$R_3 + R_2 \longrightarrow R_2$

$-3R_3 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -13/2 & 3/2 & 3/2 \\ 0 & 1 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 & 5/2 & -1/2 & -1/2 \end{array}\right]$$

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$-2R_2 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -15/2 & 1/2 & 5/2 \\ 0 & 1 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 & 5/2 & -1/2 & -1/2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -15/2 & 1/2 & 5/2 \\ 1/2 & 1/2 & -1/2 \\ 5/2 & -1/2 & -1/2 \end{bmatrix}$$

$$x = A^{-1} b$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -15/2 & 1/2 & 5/2 \\ 1/2 & 1/2 & -1/2 \\ 5/2 & -1/2 & -1/2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$x_1 = -\frac{15 b_1}{2} + 1/2\, b_2 + 5/2\, b_3$$

$$x_2 = 1/2\, b_1 + 1/2\, b_2 - 1/2\, b_3$$

$$x_3 = 5/2\, b_1 - 1/b_2 - 1/2\, b_3$$

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Q#9 $\left[\begin{array}{cc|c|c} 1 & -5 & 1 & -2 \\ 3 & 2 & 4 & 5 \end{array}\right]$

$-3R_1 + R_2 \longrightarrow R_2$

$\left[\begin{array}{cc|c|c} 1 & -5 & 1 & -2 \\ 0 & 17 & 1 & 11 \end{array}\right]$

$\frac{1}{17} R_2 \longrightarrow R_2$

$\left[\begin{array}{cc|c|c} 1 & -5 & 1 & -2 \\ 0 & 1 & 1/17 & 11/17 \end{array}\right]$

$5R_2 + R_1 \longrightarrow R_1$

$\left[\begin{array}{cc|c|c} 1 & 0 & 22/17 & 21/17 \\ 0 & 1 & 1/17 & 11/17 \end{array}\right]$

(i) $x_1 = \frac{22}{17}$ $x_2 = 1/17$

(ii) $x_1 = 21/17$ $x_2 = 11/17$

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$$\left[\begin{array}{ccc|c|c} -1 & 4 & +1 & 0 & -3 \\ 1 & 9 & -2 & 1 & 4 \\ 6 & 4 & -8 & 0 & -5 \end{array}\right]$$

$-1R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c|c} 1 & -4 & -1 & 0 & 3 \\ 1 & 9 & -2 & 1 & 4 \\ 6 & 4 & -8 & 0 & -5 \end{array}\right]$$

$-1R_1 + R_2 \longrightarrow R_2$

$-6R_1 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c|c} 1 & -4 & -1 & 0 & 3 \\ 0 & 13 & -1 & 1 & 1 \\ 0 & 28 & -2 & 0 & -23 \end{array}\right]$$

$R_2\ 1/13 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c|c} 1 & -4 & -1 & 0 & 3 \\ 0 & 1 & -1/13 & 1/13 & 1/13 \\ 0 & 28 & -2 & 0 & -23 \end{array}\right]$$

$-28R_2 + R_3 \longrightarrow R_3$

$13/2\ R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c|c} 1 & -4 & -1 & 0 & 3 \\ 0 & 1 & -1/13 & 1/13 & 1/13 \\ 0 & 0 & 1 & -14 & -\frac{327}{2} \end{array}\right]$$

$1/13\ R_3 \rightarrow R_2 \longrightarrow R_2$

$R_3 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c|c} 1 & -4 & 0 & -14 & -321/2 \\ 0 & 1 & 0 & -1 & -25/2 \\ 0 & 0 & 1 & -14 & -327/2 \end{array}\right]$$

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$4R_2$

(i) x_1 =

x_2

x

(ii)

11

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$4R_2 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|c|c} 1 & 0 & 0 & -18 & -421/2 \\ 0 & 1 & 0 & -1 & -25/2 \\ 0 & 0 & 1 & -14 & -327/2 \end{array}\right]$$

(i) $x_1 = -18$
$x_2 = -1$
$x_3 = -14$

(ii) $x_1 = -421/2$
$x_2 = -25/2$
$x_3 = -327/2$

11

$$\left[\begin{array}{cc|c|c|c|c} 4 & -7 & 0 & -4 & -1 & -5 \\ 1 & 2 & 1 & 6 & 3 & 1 \end{array}\right]$$

$R_1 \Longleftrightarrow R_2$

$$\left[\begin{array}{cc|c|c|c|c} 1 & 2 & 1 & 6 & 3 & 1 \\ 4 & -7 & 0 & -4 & -1 & -5 \end{array}\right]$$

$-4R_1 + R_2 \rightarrow R_2$

$$\left[\begin{array}{cc|c|c|c|c} 1 & 2 & 1 & 6 & 3 & 3 \\ 0 & -15 & -4 & -28 & -13 & -9 \end{array}\right]$$

$R_2 \div -1/15$

$$\left[\begin{array}{cc|c|c|c|c} 1 & 2 & 1 & 6 & 3 & 3 \\ 0 & 1 & 4/15 & 28/15 & 13/15 & 9/15 \end{array}\right]$$

$-2R_2 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{cc|c|c|c|c} 1 & 0 & 7/15 & 34/15 & 19/15 & -1/5 \\ 0 & 1 & 4/15 & 28/15 & 13/15 & 3/5 \end{array}\right]$$

1 $x_1 = 7/15$, $x_2 = 4/15$

2 $x_1 = 34/15$ $x_2 = 28/15$

3 $x_1 = 29/15$, $x_2 = 13/15$

4 $x_1 = -1/15$ $x_2 = 3/8$

Q 12/

$$\left[\begin{array}{ccc|c|c|c} 1 & 3 & 5 & 1 & 0 & -1 \\ -1 & -2 & 0 & 0 & 1 & -1 \\ 2 & 5 & 4 & -1 & 1 & 0 \end{array}\right]$$

$R_1 + R_2 \rightarrow R_2$

$-2R_1 + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c|c|c} 1 & 3 & 5 & 1 & 0 & -1 \\ 0 & 1 & 5 & 1 & 1 & -2 \\ 0 & -1 & -6 & -3 & 1 & 2 \end{array}\right]$$

$R_2 + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c|c|c} 1 & 3 & 5 & 1 & 0 & -1 \\ 0 & 1 & 5 & 1 & 1 & -2 \\ 0 & 0 & -1 & -2 & 2 & 0 \end{array}\right]$$

$-1 R_3 \rightarrow R_3$

$-5R_3 + R_1 + R_2 \rightarrow R_1 + R_2$

$$\left[\begin{array}{ccc|c|c|c} 1 & 3 & 0 & -9 & 10 & -1 \\ 0 & 1 & 0 & -9 & 11 & -2 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{array}\right]$$

$-3R_2 + R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c|c|c} 1 & 0 & 0 & 18 & -23 & 5 \\ 0 & 1 & 0 & 9 & 11 & -2 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{array}\right]$$

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Q13/

$$\left[\begin{array}{cc|c} 1 & 3 & b_1 \\ -2 & 1 & b_2 \end{array}\right]$$

$2R_1 + R_2 \rightarrow R_2$

$$\left[\begin{array}{cc|c} 1 & 3 & b_1 \\ 0 & 7 & 2b_1 + b_2 \end{array}\right]$$

$R_2 \ 1/7 \rightarrow R_2$

$$\left[\begin{array}{cc|c} 1 & 3 & b_1 \\ 0 & 1 & \frac{2}{7} b_1 + 1/7\, b_2 \end{array}\right]$$

The system is consistent foo all values of b.

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$$\left[\begin{array}{cc|c} 6 & -4 & b_1 \\ 3 & -2 & b_2 \end{array}\right]$$

$1/6 R_1 \longrightarrow R_1$

$$\left[\begin{array}{cc|c} 1 & -2/3 & 1/6\, b_1 \\ 3 & -2 & b_2 \end{array}\right]$$

$-3 R_1 + R_2 \longrightarrow R_2$

$$\left[\begin{array}{cc|c} 1 & -2/3 & 1/6\, b_1 \\ 0 & 0 & -1/2\, b_1 + b_2 \end{array}\right]$$

The system is consistent

if $-1/2\, b_1 + b_2 = 0$

$b_1 = 2b_2$

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15/
$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & b_1 \\ 4 & -5 & 8 & b_2 \\ -3 & 3 & -3 & b_3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & b_1 \\ 0 & 3 & -12 & -4b_1+b_2 \\ 0 & -3 & 12 & 3b_1+b_3 \end{array}\right] \quad \begin{array}{l} -4R_1+R_2 \to R_2 \\ 3R_1+R_3 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & b_1 \\ 0 & 3 & -12 & -4b_1+b_2 \\ 0 & 0 & 0 & -b_1+b_2+b_3 \end{array}\right] \quad R_2+R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & b_1 \\ 0 & 1 & -4 & -4/3\, b_1+1/3\, b_2 \\ 0 & 0 & 0 & -b_1+b_2+b_3 \end{array}\right] \quad R_2\, 1/3 \to R_2$$

Thes system is consistent iff and only if

$$-b_1+b_2+b_3=0$$
$$b_1=b_2+b_3$$

16/
$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & b_1 \\ -4 & 5 & 2 & b_2 \\ -4 & 7 & 4 & b_3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & b_1 \\ 0 & -3 & -2 & 4b_1+b_2 \\ 0 & -1 & 0 & 4b_1+b_3 \end{array}\right] \quad \begin{array}{l} 4R_1+R_2+R_3 \\ \to R_2, R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & b_1 \\ 0 & -1 & 0 & 4b_1+b_3 \\ 0 & -3 & -2 & 4b_1+b_2 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

17/

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & b_1 \\ 0 & 1 & 0 & -4b_2 - b_3 \\ 0 & -3 & -2 & 4b_1 + b_2 \end{array}\right] \quad -R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & b_1 \\ 0 & 1 & 0 & -4b_1 - b_3 \\ 0 & 0 & -2 & -8b_1 + b_2 - 3b_3 \end{array}\right] \quad 3R_2 + R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & b_1 \\ 0 & 1 & 0 & -4b_1 - b_3 \\ 0 & 0 & 1 & 4b_1 - \tfrac{1}{2}b_2 + \tfrac{3}{2}b_3 \end{array}\right]$$

$$-1/3\, R_3 \to R_3$$

17/

$$\left[\begin{array}{cccc|c} 1 & -1 & 3 & 2 & b_1 \\ -2 & 1 & 5 & 1 & b_2 \\ -3 & 2 & 2 & -1 & b_3 \\ 4 & -3 & 1 & 3 & b_4 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 3 & 2 & b_1 \\ 0 & -1 & 11 & 5 & 2b_1 + b_2 \\ 0 & -1 & 11 & 5 & 3b_1 + b_3 \\ 0 & 1 & -11 & -5 & -4b_1 + b_4 \end{array}\right] \quad \begin{array}{l} 2R_1 + R_2 \to R_2 \\ 3R_1 + R_3 \to R_3 \\ -4R_1 + R_4 \to R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 3 & 2 & b_1 \\ 0 & 1 & -11 & -5 & -2b_1 - b_2 \\ 0 & -1 & 11 & 5 & 3b_1 + b_3 \\ 0 & 1 & -11 & -5 & -4b_1 + b_4 \end{array}\right] \quad -1R_2 \to R_2$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 3 & 2 & b_1 \\ 0 & 1 & -11 & -5 & 2b_1 - b_2 \\ 0 & 0 & 0 & 0 & b_1 - b_2 + b_3 \\ 0 & 0 & 0 & 0 & -2b_1 + b_2 + b_4 \end{array}\right] \quad \begin{array}{l} R_2 + R_3 \to R_3 \\ -1R_2 + R_4 \to R_4 \end{array}$$

The system is consistent for all values of b_1, b_2, b_3, b_4 that satisfy equation

$b_1 - b_2 + b_3 = 0$

$-2b_1 + b_2 + b_4 = 0$

Q18/ $X = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$

The identity matrix was adjust in matrix

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$-2R_1 + R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 5 & -2 & -2 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$-2R_3 + R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$-2R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & -1 & 4 & -2 & 5 \end{array}\right]$$

$-1R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & -4 & 2 & -5 \end{array}\right]$$

$-1R_3 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 5 & -2 & 5 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & -4 & 2 & -5 \end{array}\right]$$

$R_2 + R_1 \longrightarrow R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & 3 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & -4 & 2 & -5 \end{array}\right]$$

using $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} +3 & -1 & 3 \\ -2 & 1 & -2 \\ -4 & 2 & -5 \end{bmatrix}$

$$X = \begin{bmatrix} 3 & -1 & 3 \\ -2 & 1 & -2 \\ -4 & 2 & -5 \end{bmatrix} \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 12 & -3 & 27 & 26 \\ -6 & -8 & 1 & -18 & -17 \\ -15 & -21 & 9 & -38 & -35 \end{bmatrix}$$

20/ $X = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & -4 \end{bmatrix}^{-1} \begin{bmatrix} 4 & 3 & 2 & 1 \\ 6 & 7 & 8 & 9 \\ 1 & 3 & 7 & 9 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} -2 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 1 & 1 & -4 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \longleftrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -4 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ -2 & 0 & 1 & 1 & 0 & 0 \end{array}\right]$$

$2R_1 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -4 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 0 & 2 & -7 & 1 & 0 & 2 \end{array}\right]$$

$-1R_2 \longrightarrow R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -4 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & -7 & 1 & 0 & 2 \end{array}\right]$$

$-2R_2 + R_3 \longrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -4 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & -9 & 1 & 2 & 2 \end{array}\right]$$