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Paper (2016) Discrete

1. Verify whether $(P \wedge Q) \rightarrow (P \vee Q)$ is tautology or not?

Solution:-

P	Q	$P \wedge Q$	$P \vee Q$	$(P \wedge Q) \rightarrow (P \vee Q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Hence $(P \wedge Q) \rightarrow (P \vee Q)$ is tautology.

2. What is the decimal expansion of the number with hexadecimal-expansion $(2AE0B)_{16}$?

Sol:-

$$A = 10, \quad E = 14, \quad B = 11$$

$$2 \times 16^4 + 10 \times 16^3 + 14 \times 16^2 + 0 \times 16^1 + 11 \times 16^0$$

$$2 \times 65536 + 10 \times 4096 + 14 \times 256 + 0 + 11 \times 1$$

$$131072 + 40960 + 3584 + 11$$

$$(175627)_{10} \quad \text{Ans}$$

Q.3 How can you produce the terms of a sequence if first 10 terms are 5, 11, 17, 23, 29, 35, 41, 47, 53, 59?

Sol:-

$a_1 = 5$, $d = 6$, $a_{10} = 59$

By applying formula we can find the next terms

$a_{11} = a_1 + 10d = 5 + 10(6) = 65$

$a_{12} = a_1 + 11d = 5 + 11(6) = 71$

$a_{13} = a_1 + 12d = 5 + 12(6) = 77$

$a_n = a_1 + nd = 5 + 6n$

So

$65, 71, 77, \ldots, 5 + 6n$

Q.4 Determine that 10, 17 & 21 are relatively prime?

$10 = 2 \times 5 \times 1$	$10 = 2 \times 5 \times 1$	$17 = 17 \times 1$
$17 = 17 \times 1$	$21 = 3 \times 7 \times 1$	$21 = 3 \times 7 \times 1$
g.c.d = 1	g.c.d = 1	g.c.d =

As the g.c.d of (10,17), (10,21) & (17,21) is 1. So these are relatively prime.

Q.5 write the names of properties of an algorithm?

Ans A step by step procedure to solve any problem is call algorithm.

1 Finiteness:-
Algorithm must always terminate after finite number of steps.

2 Definiteness:-
Each step must be precisely defined.

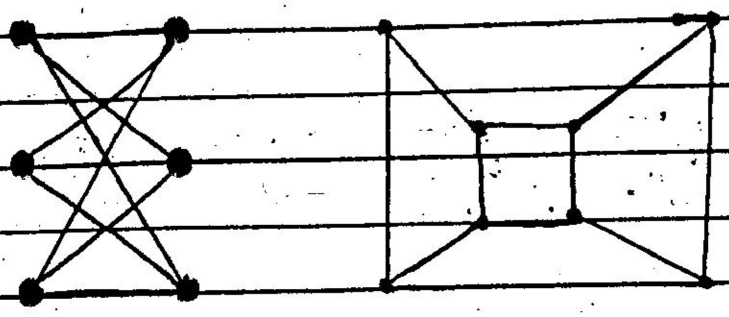
Q.6 what are connected components of a graph?

Ans A connected components of a graph is a sub-graph in which any two vertices are connected to each other by paths. A vertex with no incident edge is itself a connected componer

Q.7 what is bipartite graph?

Ans A bipartite graph is also called a bigraph. It is a set of graph vertices decomposed into two disjoint sets such that no two graph vertices with

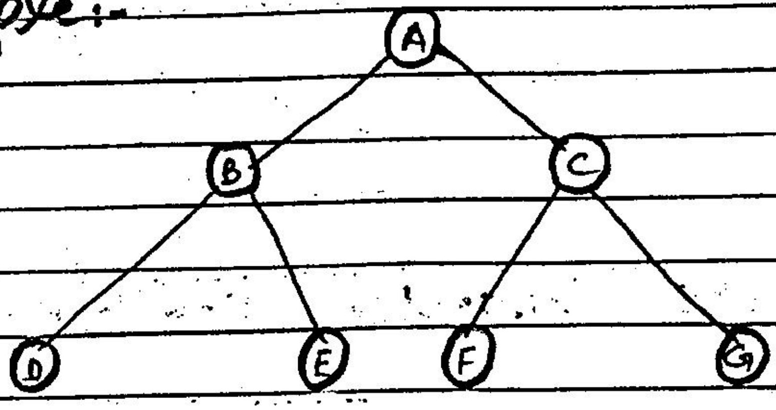
Example:-



Q.8 What is the height of a rooted tree?

Ans The number of edges from a particular node to a root node is called height of rooted tree.

Example:-



F

Height A to D = 2

Q.9 Define Partial ordering?

Ans It means transitive anti-symmetric relation between the elements of a set. Which does not necessarily apply to each pair of element.

Q.10 Define spanning tree of a graph?

Ans A spanning tree of a graph G (undirected), is a sub-graph that is a tree which includes all of the vertice of G with minimum possible number of edges.

Q.11 How many permutations of letters ABCDEFG contain the string CFGA?

Ans Number of strings(n) = 7
The string CFGA ~~r =~~ 4

$$^7P_4 = \frac{7!}{(7-4)!}$$

x

Q.11 solution
If the string contain CFGA then

$\boxed{CFGA, B, D, E}$

So the permutation will be = 4!

= 4·3·2·1

= 24 Ans

Q.12 How many subsets with more than two elements does a set with 100 elements have?

Ans If set has 100 elements then the subsets $= 2^{100}$

$2^{100} - 1$ (non empty subsets)

${}^{100}C_1$ & ${}^{100}C_2$ have 2 or less elements

Now subtract from $2^{100} - 1$

$2^{100} - 1 - 100 - 4950 = 2^{100} - 5051$

$\boxed{\text{The subsets are} = 2^{100} - 5051}$

Let $Q(x)$ denote the statement "$x = x+1$". What are the truth values of quantification $\exists x Q(x)$, where the domain consists all real number?

If x belongs to real numbers then putting $x = 0, 1, 2, 3, \ldots$

$Q(0)$ -	$Q(1)$
$x = x+1$	$x = x+1$
$0 = 0+1$	$1 = 1+1$
$0 \neq 1$	$1 \neq 2$

So the truth values of $Q(x)$

"$x = x+1$" are "false" for all real numbers.

Q.14 Find recurrence relation of the sequence $S(n) = 5^n$

Ans putting $n = 1$

$S(1) = 5$ ← (i)

$S(n) = 5\, S(n-1)$ for $n \geq 5$

~~$S(n) = 5(5\,S(n-1))_k$~~

Now using the recurrence relation everytime

$S(n) = 5\,S(n-1)$

$\Rightarrow S(n) = 5[5\,S(n-2)] = 5^2\, S(n-2)$

$\Rightarrow S(n) = 5^2[5\,S(n-3)] = 5^3\, S(n-3)$

After 'k' expansion the question has the form

$S(n) = 5^k\, S(n-k)$

It would stop when $n-k = 1$

$\therefore n-1 = k$

$S(n) = 5^{n-1}\, S(1)$

using eq I

$S(1) = 5$

$= 5^{n-1}(5)$

Q.15 Encypt the message WATCH YOUR STEP By ....

Solution:-

A → 0	G → 6	M → 12	S → 18	Y → 24
B → 1	H → 7	N → 13	T → 19	Z → 25
C → 2	I → 8	O → 14	U → 20	
D → 3	J → 9	P → 15	V → 21	
E → 4	K → 10	Q → 16	W → 22	
F → 5	L → 11	R → 17	X → 23	

Now

The Statement is

WATCH YOUR STEP

22-0-19-2-7 24-14-20-17 18-19-4-15

~~About~~

$f(p) = (14p + 21) \bmod 26$

WATCH = 17-0-1-23-15

YOUR = 19-9-15-25

STEP = 13-1-25-23

Now letters will be

"RABXP TJPZ NBZX"

16 Suppose the domain of propositinal function $p(x)$ consiststs integers 1, 2, 3, 4, & 5 .... "Some students have no ID cards"?

(i) $\sim \exists x\, p(x) \sim (p(1) \wedge p(2) \wedge p(3) \wedge p(4) \wedge p(5))$

Ans "There is no x for which $p(x)$ is not true".

(ii) Some students have no ID card.

Ans:- $\{\exists x \sim p(x)\}$

where $p(x)$ denotes ID card.

Q.1 m+n and n+p is even, So m+p is even.

By definition the sum of two even integers is even.

if m+n is even so

$m+n = 2c$ —— (i) where $c \in R$

$n+p = 2d$ —— (ii) where $d \in R$

if m+n & n+p both are even then

$$m+p = (m+n)+(n+p)$$
$$= 2c + 2d$$
$$m+p = 2(c+d)$$
$$\frac{m+p}{2} = c+d$$

As 2 divides m+p, so m+p is also even.

Q.2 Find each of these values?

(a) (992 mod 32)3 mod 15

= (0)3 mod 15

= 0 mod 15

= 0 Ans

(b) (34 mod 17)2 mod 11

= (0)2 mod 11

= 0 mod 11

= 0 Ans

(c) (193 mod 23)2 mod 31
= (9)2 mod 31
= 18 mod 31
= 18 Ans

(d) (893 mod 79)4 mod 24
= (24) 4 mod 24
= 96 mod 24
= 0 Ans

Q.7 A Coin is flipped 10 times where each film comes up either heads or tail. How many possible outcomes?

(a) Are there in total?

Total out comes $= 2^{10} = 1024$

(b) Contain exactly two heads?

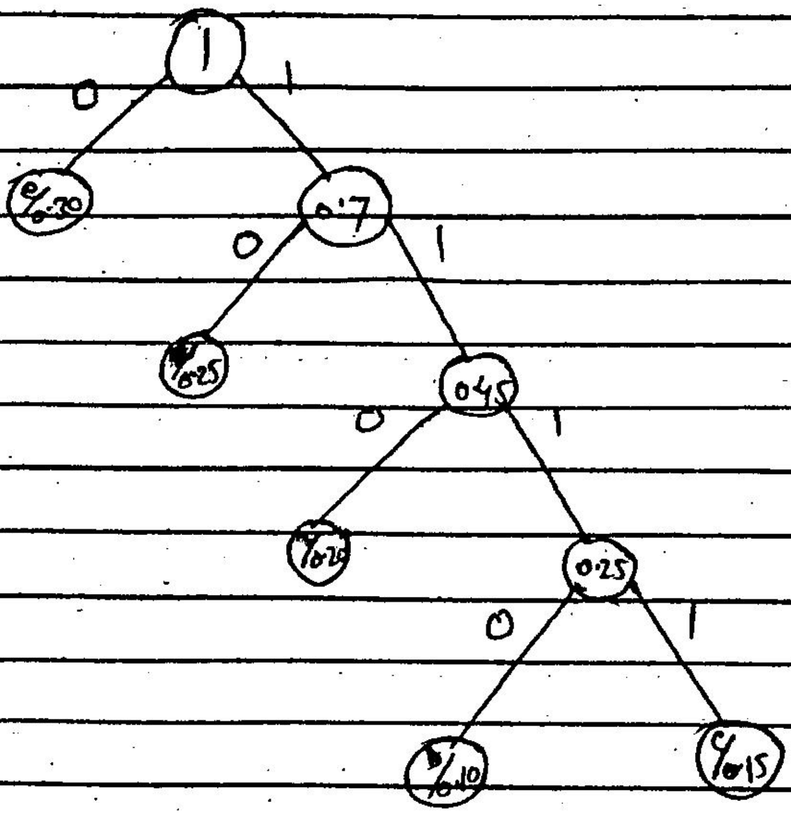
$$^{10}C_2 = \frac{10!}{(10-2)!\,2!}$$

$$= \frac{10 \times 9 \times 8!}{8! \times 2} = 45$$

(C) Contain at most three tails?

$$^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3$$

Tree.



$$Avg = \frac{0.20 + 0.10 + 0.15 + 0.25 + 0.30}{5}$$

$$= \frac{1}{5}$$

$= 0.2$ Ans