Chapter 1: Logic and Proofs

Page 110 All definitions of chapter 1

1. **Construct truth table for the both** $p \vee \backsim q$ $(p \vee q) \wedge \backsim p$

p	q	$\backsim p$	$p \vee q$	$(p \vee q) \wedge \backsim p$
T	T	F	T	F
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

2. **Construct a truth table for** $p \oplus (p \vee q)$ **Q32(c) Page 15**

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

$(q \rightarrow \neg p) \leftrightarrow (q \leftrightarrow p)$

p	q	$\neg p$	$q \rightarrow \neg p$	$q \leftrightarrow p$	$(q \rightarrow \neg p) \leftrightarrow (q \leftrightarrow p)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	T	T	F	F
F	F	T	T	T	F

$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$\neg q$	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	T	F	T
T	F	F	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Assignment Q 30, 31, 32, 33, 34, 35, 36 Page 15

3. **Construct the truth table for** $(p \leftrightarrow Q) \oplus (p \leftrightarrow \backsim p)$? **Assignment: By Own**

4. **Show the equivalence** $(A \vee B) \rightarrow A \equiv B \rightarrow A$ **Page 6**

A	B	$A \vee B$	$(A \vee B) \rightarrow A$	$B \rightarrow A$
T	T	T	T	T
T	F	T	T	T
F	T	T	F	F
F	F	F	T	T

5. **Logically equivalence? Page 25 Definition 2**
6. **Show that ¬(p∨q) and ¬p∧¬q are logically equivalent.**

   Example 2 Page 26(Draw a Truth Table to Show Equivalent) Assignment Example 3 Page 26

7. **Show that** $\neg(p \oplus q) and\ (p \leftrightarrow q)$ **are logically equivalent**

p	q	$p \oplus q$	$\neg(p \oplus q)$
T	T	F	T
T	F	T	F

F	T	T	F
F	F	F	T

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Assignment Q16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27 Page 35

8. **Define predicate   page 37**
9. **Differentiate preposition and Predicate with examples** **Page 2 Preposition**
   OR differentiate between predicate and prepositional logic?

1	It is the basic and most widely used logic. Also known as Boolean logic.	It is an extension of propositional logic covering predicates and quantification.
2	A proposition has a specific truth value, either true or false.	A predicate's truth value depends on the variables' value.
3	Scope analysis is not done in propositional logic.	Predicate logic helps analyze the scope of the subject over the predicate. There are three quantifiers: Universal Quantifier (∀) depicts for all, Existential Quantifier (∃) depicting there exists some and Uniqueness Quantifier (∃!) depicting exactly one.
4	Propositions are combined with Logical Operators or Logical Connectives like Negation (¬), Disjunction(∧), Conjunction(∨), Exclusive OR(⊕), Implication(⇒), Bi-Conditional or Double Implication(⇔).	Predicate Logic adds by introducing quantifiers to the existing proposition.

10. **Differentiate Predicate and Quantifiers? Page 110**
    Quantifiers are words that refer to quantities such as "some" or "all" and tell for how many elements a given predicate is true.   Universal Quantifier and Existential Quantifier???
    Answer: Chapter 1 The Foundations Logic and Proofs.pdf slide 79 to 89
11. Define Existential Quantifier.   Page 110 definition
12. Define Universal Quantifier?   Page 40 definition 1
13. Define the term counter example.   **Page 40 Defination1**
14. **Give an Example of Contingency?**   Page 25 Definition 1 and Example 1

A proposition that is neither a tautology nor contradiction is called a contingency.

Answer: Chapter 1 The Foundations Logic and Proofs.pdf slide 59

15. **Prove by contradiction with example OR Define Contradiction page 86 Example 9**

Answer: Chapter 1 The Foundations Logic and Proofs.pdf slide 58

16. **State the idempotent law Page 27 (Also Learn Table 6 Logical Equivalence)**

Idempotence is the property of certain operations in mathematics and computer science that they can be applied multiple times without changing the result beyond the initial application. ... Both 0 and 1 are idempotent under multiplication, because 0 x 0 = 0 and 1 x 1 = 1.

$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws

17. **What are absorption laws for sets? Page 27 Table OR Example 10 Page 816**

The **absorption law or absorption** identity is an identity linking a pair of binary operations. Two binary operations, + and X, are said to be connected by the absorption law if: a + (a X b) =a and a X (a + b) = a.

$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws

18. **Define idempotent and absorption laws? Above Questions are Answer**

Table 1 Page 130

19. "**Every student in the class has studied calculus" Translate them in to mathematical form**

"For every student x in this class, x has studied calculus."

Continuing, we introduce $C(x)$, which is the statement "x has studied calculus." Consequently, if the domain for x consists of the students in the class, we can translate our statement as $\forall x C(x)$.

If $S(x)$ represents the statement that person x is in this class, we see that our statement can be expressed as $\forall x(S(x) \rightarrow C(x))$. [*Caution!* Our statement *cannot* be expressed as $\forall x(S(x) \wedge C(x))$ because this statement says that all people are students in this class and have studied calculus!]

OR

This statement is a universal quantification, namely, $\forall x P(x)$

Where $P(x)$ is the statement

"x has taken a course in calculus."

20. **Translate into English the statement $\forall x \forall y((x > 0) \wedge (y < o) \rightarrow (x.y < 0))$, where the domain for both variables consists of all real numbers. Example 2 Page 58**

Solution: This statement says that for every real number x and for every real number y, if $x > 0$ and $y < 0$, then $xy < 0$. That is, this statement says that for real numbers x and y, if x is positive and y is negative, then xy is negative. This can be stated more succinctly as "The product of a positive real number and a negative real number is always a negative real number." ◀

21. Translate these statements into English, where C(x) is "x is comedian" and F(x) is "x is funny" and the domain consists of all people.

a) $\forall x(C(x) \rightarrow F(x))$

b) $\forall x(C(x) \wedge F(x))$

c) $\exists x(C(x) \rightarrow F(x))$

d) $\exists x(C(x) \wedge F(x))$

Answer: Q7 Page 53

a) All comedians are funny

b) Every person is a comedian and funny

c) There exists a person such that, if a person is a comedian, then the person is funny

d) There exists a person that is a comedian and funny

22. Let P(x) be the statement "Student x knows calculus" and let Q(y) be the statement "Class y contains a student who knows calculus." Express each of these as quantifications of p(x) and Q(y). **Q20 Page 112 Assignment**

a) Some students knows calculus.
b) Not every student knows calculus.
c) Every class has a student in it who knows calculus
d) Every student in every class knows calculus.
e) There at least one class with not students who know calculus

(a) $\exists x P(x)$
(b) $\neg\forall x P(x)$
(c) $\forall y Q(y)$
(d) $\forall x P(x) \wedge \forall y Q(y)$
(e) $\exists y \neg Q(y)$

23. Let P(x) be the statement "x spend more than 5 hours every work day in the class" where the domain consist of all the students. Express of these Quantification in English

a) $\exists x P(x)$ **b)** $\forall x P(x)$
c) $\exists x \neg P(x)$ **d)** $\forall x \neg P(x)$

a) There exists a student that spends more than five hours every week day in the class
b) All students spend more than five hours every week day in the class
c) There exists s student that does not spend more than five hours every week day in the class
d) All students do not spend more than five hours every week day in the class

24. Let N(x) be the statement "x has visited North Dakota" where domain consists of the students in your school. Express each of these quantifications in English.

a) $\exists x N(x)$ **b)** $\forall x N(x)$ **c)** $\neg\exists x N(x)$
d) $\exists x \neg N(x)$ **e)** $\neg\forall x N(x)$ **f)** $\forall x \neg N(x)$

a) There exists a student in your school who has visited North Dakota
b) All students in your school have visited North Dakota
c) There does not exist a student in your school who has visited North Dakota.
d) There exist a student your school who has not visited North Dakota
e) Not all students in your school have visited North Dakota
f) All students in your school have not visited North Dakota

25. Write the introduction and elimination rules for universal quantifier in structured proof?

Introduction Rule

To use a universally quantified formula: if we know $\forall x.P(x)$, then we can deduce $P(v)$ for any v (of the appropriate domain)

...

$m.$ $\forall\ x.P(x)$ from ...

...

$n.$ $P(v)$ from m by $\forall$-elimination

Elimination Rule:

To prove a universally quantified formula $\forall\ x.P(x)$, consider an arbitrary fresh variable x (ranging over the appropriate domain) and prove $P(x)$, then discharge the assumption.

...

$m.$ Consider an arbitrary x (from domain ...)
...
$n.$ $P(x)$ by ...

$n+1.$ $\forall\ x.P(x)$ from m–n by $\forall$-introduction

Introduction Rules:

To prove an existentially quantified formula $\exists\ x.P(x)$, prove $P(v)$ for some witness v (from the appropriate domain).

...

$m.$ $P(v)$

...

$n.$ $\exists\ x.P(x)$ from m by $\exists$-introduction with witness $x = v$

Elimination Rules:

To use an existentially quantified formula $\exists\, x.P(x)$, introduce a fresh variable (ranging over the appropriate domain) x_1, about which we know only $P(x_1)$. The elimination rule for existential quantifiers is reminiscent of the elimination rule for disjunction.

$l.\ \exists\, x.P(x)$

...

$m.$ For some actual x_1, $P(x_1)$
...
$n.$ Q (where x_1 not free in Q)

...

$o.$ Q from l, m–n, by $\exists$-elimination

Also Read Introduction and Elimination rules.pdf from Google Drive

26. Determine the truth value of the following statements if the domain for all variables consists of all integers. Q15 Page 53 Topic: Page 4

$$\forall n(n^2 \geq n)$$

$a)$ The squaring of any integer results in a positive number. Thus, all integers are greater than or equal to zero.

$b)$ There are two numbers on on the Reals that would satisfy this equation: $\sqrt{2}$ and $-\sqrt{2}$. However, these do not lie on the Integers. Thus, this is false.

$c)$ This would break down only if n^2 were negative, which is impossible. Thus, this holds true for all integers.

$d)$ The squaring of n results in all numbers being greater than or equal to zero. Thus, this is not possible.

$a)$ True
$b)$ False
$c)$ True
$d)$ False

Assignment Q1, 2, 3, 4 Page 12, Q13 Page 53

27. Determine the truth value of the following statement if the domain consists of all real numbers $\exists x(x^4 < x^2)$

(a) $\exists x(x^3 = -1)$ is true, because the statement $x^3 = -1$ is true for $x = -1$ $((-1)^3 = -1)$

(b) $\exists x(x^4 < x^2)$ is true, because the statement $x^4 < x^4$ is true for $x = 0.5$ $(0.0625 = 0.5^4 < 0.5^2 = 0.25)$.

(c) $\forall x((-x)^2 = x^2)$ is true, because the statement $(-x)^2 = x^2$ is true for all real values of x.

(d) $\forall x(2x > x^2)$ is false, because the statement $2x > x^2$ is not true for $x = 0$ $(0 = 2(0) > 0^2 = 0$ is not true).

(a) True
(b) True
(c) True
(d) False

28. Determine truth value $\exists n(2n = 3n)$ if the domain is set of integers.

Page 53 Exercise Q 13

$\exists n(2n = 3n)$

True, e.g., for n = 0.

Since 2 · 0 = 3 · 0, this is true.

29. Let Q(x) denote the statement "x= x+1" what is the truth value of the quantification $\exists x Q(x)$ where the domain consist of all real numbers.

Page 43 example 15

Solution: Because $Q(x)$ is false for every real number x, the existential quantification of $Q(x)$, which is $\exists x\, Q(x)$, is false.

If x belongs to real numbers then putting x = 0, 1, 2, 3.

Q (0)	Q (1)
x= x + 1	x = x + 1
0 = 0 + 1	1 = 1 + 1
0≠1	1≠2

So the truth values of Q(x) "x= x + 1" are "false" for all real numbers

30. Give an indirect proof of the theorem? "If 3n + 2 is odd then n is odd"

Page 83 example 3

31. Give a proof by contradiction of theorem? "If 3n + 2 is odd then n is odd"

Page 87 example 11

32. Prove that $p \to (q \to r)$ *and* $(p \wedge \sim r) \to \sim q$

p	q	r	$q \to r$	$p \to (q \to r)$	~r	$\sim q$	$(p \wedge \sim r)$	$(p \wedge \sim r) \to \sim q$
T	T	T	T	T	F	F	F	T
T	T	F	F	F	T	F	T	F
T	F	T	T	T	F	T	F	T
T	F	F	T	T	T	T	T	T
F	T	T	T	T	F	F	F	T
F	T	F	F	T	T	F	T	F
F	F	T	T	T	F	T	F	T
F	F	F	T	T	T	T	T	T

33. Verify whether $(P \wedge Q) \to (P \vee Q)$ tautology is or not.

P	Q	$(P \vee Q)$	$(P \wedge Q)$	$(P \wedge Q) \to (P \vee Q)$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	F	F	T

Hence Proved Above statement is Tautology

34. **Show the conditional statement is tautology? $\backsim (p \to q) \to p$? Q 10 Assignment Page 35**

a) $(p \wedge q) \to p$ **b)** $p \to (p \vee q)$
c) $\neg p \to (p \to q)$ **d)** $(p \wedge q) \to (p \to q)$
e) $\neg(p \to q) \to p$ **f)** $\neg(p \to q) \to \neg q$

A proposition is a tautology, if the proposition is always true (thus true for any combination of truth values for the variables $p, q, r, ...$).

TRUTH TABLE

A conjunction $p \wedge q$ is true, if both (sub)propositions (p and q) are true.
A disjunction $p \vee q$ is true, if either of the (sub)propositions (p or q) are true.
A negation $\neg p$ is true, if the (sub)proposition p is false.
A conditional statement $p \to q$ is true, if p is false, or if both (sub)propositions are true.
A biconditional statement $p \leftrightarrow q$ is true, if both (sub)propositions are true or if both (sub)propositions are false.

(a) The conditional statement is a tautology, because the last column of the following truth table contains only true T (and thus does not contain false F).

p	q	p ^ q	(p ^ q) -> p
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

(b) The conditional statement is a tautology, because the last column of the following truth table contains only true T (and thus does not contain false F).

p	q	p v q	p -> (p v q)
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

(c) The conditional statement is a tautology, because the last column of the following truth table contains only true T (and thus does not contain false F).

p	q	not p	p -> q	not p -> (p -> q)
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

(d) The conditional statement is a tautology, because the last column of the following truth table contains only true T (and thus does not contain false F).

p	q	p ^ q	p -> q	(p ^ q) -> (p -> q)
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

(e) The conditional statement is a tautology, because the last column of the following truth table contains only true T (and thus does not contain false F).

p	q	p -> q	not (p -> q)	not (p -> q) -> p
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

(f) The conditional statement is a tautology, because the last column of the following truth table contains only true T (and thus does not contain false F).

p	q	p -> q	not (p -> q)	not q	not (p -> q) -> not q
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

All Conditional Statements are Tautology

35. Use truth table to determine whether ((¬p∧ (p→q))→¬q)) is a tautology?

p	Q	¬p	¬q	p→q	(¬p∧ (p→q)	(¬p∧ (p→q))→¬q
F	F	T	T	T	T	T
F	T	T	F	T	T	T
T	F	F	T	F	F	T
T	T	F	F	T	F	T

Hence proved above statement is tautology

36. Difference between tautology and contradictions?

A compound proposition that is always true for all possible truth values of the propositions is **called a tautology**. A compound proposition that is always false is **called a contradiction**. A proposition that is neither a tautology nor contradiction is **called a contingency**. Example: p ∨ ¬p is a tautology.

37. Suppose that domain of propositional function p(x) consist of the integers 1, 2, 3, 4 and 5. Express these statements without using Quantifies (instead using only disjunction, conjunction and Negation) $\backsim \exists x\, P(x) \neg(p(1) \wedge p(2) \wedge p(3) \wedge p(4) p(5))$ write in symbolic form "Some students have no ID Card". Answer: Q19 Page 54

Q17 Exercise page 53 Answer page 27

$\backsim \exists x\, P(x) \neg(p(1) \wedge p(2) \wedge p(3) \wedge p(4) p(5))$ {"There is no x for which P(x) is not true"}

"Some students have no ID Card"= {$\exists x \backsim P(x)$} where P(x) denotes ID Card.

Existential quantifiers are like disjunctions, and universal quantifiers are like conjunctions.

See Examples 11 page 42 and example 17 page 44 Q20 Page 54 Assignment

a) $\exists x P(x)$ **b)** $\forall x P(x)$
c) $\neg\exists x P(x)$ **d)** $\neg\forall x P(x)$
e) $\forall x((x \neq 3) \rightarrow P(x)) \vee \exists x \neg P(x)$

a) There exists a value between 1 and 5 such that $P(x)$ is true, this is thus equivalent with $P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5)$

b) For every value between 1 and 5 we need $P(x)$ is true, this is thus equivalent with $P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5)$

c) This statement is the negation of the statement in a, and is thus equivalent with $\neg(P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5))$

d) This statement is the negation of the statement in b, and is thus equivalent with $\neg(P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5))$

e) The statement is equivalent with $(P(1) \wedge P(2) \wedge P(4) \wedge P(5)) \vee (\neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4) \vee \neg P(5))$, because the first statement means that P(x) is true when x is not equal to 3 and the second statement means that there has to be a value for x where $P(x)$ is false and thus $\neg P(x)$ is true.

38. Suppose that domain consists of Q(x, y, z) consists of triples x, y, z where x= 0, 1 or 2 and y= 1,2 or 3. Write this expression $\exists x p(x, 3)$ using conjunction and disjunction.
Page 54 Exercise Question 30

$\exists$ x P(x , 3)
P(1,3)V P(2,3) VP(3,3)

a) $\exists x\, P(x, 3)$ **b)** $\forall y\, P(1, y)$
c) $\exists y \neg P(2, y)$ **d)** $\forall x\, \neg P(x, 2)$

a) $P(1,3) \vee P(2,3) \vee P(3,3)$	x can be 1,2 or 3 and there has to exists 1 value out of the three for the propositional function.
b) $P(1,1) \wedge P(1,2) \wedge P(1,3)$	y can be 1,2 or 3 and all values have to be possible.
c) $\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)$	y can be 1,2 or 3 and there has to exists 1 value out of the three for the propositional function.
d) $\neg P(1,2) \wedge \neg P(2,2) \wedge \neg P(3,2)$	x can be 1,2 or 3 and all values have to be possible.

a) $P(1,3) \vee P(2,3) \vee P(3,3)$
b) $P(1,1) \wedge P(1,2) \wedge P(1,3)$
c) $\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)$
d) $\neg P(1,2) \wedge \neg P(2,2) \wedge \neg P(3,2)$

39. **Differentiate b/w free and bound variables with the help of examples. Page 44 for Example 18 page 45**
    All variables in a predicate must be bound to turn a predicate into a proposition. We bind a variable by assigning it a value or quantifying it. Variables which are not bound are free.
40. **Find the negation of this expression $\exists w \forall a \exists f\,(P(w,f) \vee Q(f,a))$**

    Example 13 page 63 and Example 15 is the negation of the statement

    Negation of above Expression is $\forall w \exists a \forall f(\neg P(w, f) \vee \neg Q(f, a)).$

    OR

    $\neg \exists w \forall a \exists f\,(P(w, f) \wedge Q(f, a))$
41. **Find the negation of this statement** $\forall x \forall y((x > 0) \wedge (y < 0) \rightarrow (xy < 0))$
    Negation is $\exists x \exists y \neg((x > 0) \wedge (y < 0) \rightarrow (xy < 0))$
    Example 2 page 58
    Exercise Q31, 32 Page 67 Assignment
42. **Write simplification rule of inference? Page 69**
    Chapter 1 The Foundations Logic and Proofs.pdf Slide 102

Mathematical logic is often used for logical proofs. **Proofs** are valid arguments that determine the truth values of mathematical statements.
An **argument is a sequence** of statements. The **last statement** is the **conclusion** and all its **preceding statements** are called **premises (or hypothesis**). The **symbol "∴∴", (read therefore**) is placed **before the conclusion**. A valid argument is one where the conclusion follows from the truth values of the premises.
Rules of Inference provide the **templates or guidelines** for constructing valid arguments from the statements that we already have

43. **Show that the premises " Student in this class has not read the book," and Everyone in this class passed the first exam| Imply the conclusion " Someone who has passed the first exam has not read the book," Example 13 Page 77**

Solution: Let $C(x)$ be "x is in this class," $B(x)$ be "x has read the book," and $P(x)$ be "x passed the first exam." The premises are $\exists x(C(x) \wedge \neg B(x))$ and $\forall x(C(x) \rightarrow P(x))$. The conclusion is $\exists x(P(x) \wedge \neg B(x))$. These steps can be used to establish the conclusion from the premises.

Step	Reason
1. $\exists x(C(x) \wedge \neg B(x))$	Premise
2. $C(a) \wedge \neg B(a)$	Existential instantiation from (1)
3. $C(a)$	Simplification from (2)
4. $\forall x(C(x) \rightarrow P(x))$	Premise
5. $C(a) \rightarrow P(a)$	Universal instantiation from (4)
6. $P(a)$	Modus ponens from (3) and (5)
7. $\neg B(a)$	Simplification from (2)
8. $P(a) \wedge \neg B(a)$	Conjunction from (6) and (7)
9. $\exists x(P(x) \wedge \neg B(x))$	Existential generalization from (8)

44. What is bi-implication? State with an example   page 9

Chapter 1 The Foundations Logic and Proofs.pdf slide 40, 41, 42

45. What is compound preposition? Give an example

Chapter 1 The Foundations Logic and Proofs.pdf slide 9

Propositions may be modified by means of one or more *logical operators* to form what are called ***compound propositions***.

There are three logical operators:

- conjunction: meaning AND
- disjunction: ∨ meaning OR
- negation: ¬ meaning NOT

46. Negate the following compound Prepositions

John is six feet tall and he weight at least 200 pounds

Negation is: John is **not** six feet tall **or** he weight **less** 200 pounds

The bus was late or Tom's watch was slow

Negation is: The bus was **not** late **and** Tom's watch was **not** slow

47. Find the conjunction of the preposition p and q where p is preposition "Today is Friday" and q is preposition " it is raining today"

The conjunction is the proposition "Today is Friday and it is raining today". The proposition is true on rainy Fridays

48. Find the conjunctions of the propositions p and q where p is proposition "Today is Holiday" and q is the preposition "It is not Raining Today".   Practice Example 5 Page 4

The conjunction is the proposition "Today is Holiday and it is not Raining Today". The proposition is true on holidays that's are not Rainy

49. What is the negation of the statements "All goats are mammals".

It is not the case that all goats are mammals

50. How many rows appear in a truth table for each of these compound prepositions?

$$(\boldsymbol{p} \rightarrow \boldsymbol{r}) \vee (\neg \boldsymbol{s} \rightarrow \boldsymbol{t}) \vee (\neg \boldsymbol{u} \rightarrow \boldsymbol{v})$$

a) $(q \rightarrow \neg p) \vee (\neg p \rightarrow \neg q)$

b) $(p \vee \neg t) \wedge (p \vee \neg s)$

c) $(p \rightarrow r) \vee (\neg s \rightarrow \neg t) \vee (\neg u \rightarrow v)$

d) $(p \wedge r \wedge s) \vee (q \wedge t) \vee (r \wedge \neg t)$

A: 4 Rows B: 8 Rows C: 16 Rows D: 32 Rows	The number of rows can be obtained by N= 2^n where n = the number of propositional variables. Note however that a negated proportional variable is not evaluated as a new variable if its standalone has already been brought up.

Q30 Page 15 (c Part in Paper), Q29 is assignment Page 15

51. Define The following Terms **Answers: Page 8 practice Example 9**

    Converse, Inverse and Contrapositive

    Converse inverse.pdf in Google Drive

52. **What are the contra-positive, the converse and inverse of the conditional statement? "The Home teams win whenever it is raining? " Answer: page 8 Example 9**

Some terminology, for an implication p →q:

- Its converse is: q → p.
- Its inverse is: ¬p→ ¬q.
- Its contrapositive: ¬q → ¬ p.

Example of Conditional Statement − "If you do your homework, you will not be punished." Here, "you do your homework" is the hypothesis, p, and "you will not be punished" is the conclusion, q.

Example − The inverse of "If you do your homework, you will not be punished" is "If you do not do your homework, you will be punished."

Example − The converse of "If you do your homework, you will not be punished" is "If you will not be punished, you do your homework".

Example − The Contra-positive of " If you do your homework, you will not be punished" is "If you are punished, you do your homework".

Write converse, inverse and contrapositive of each statement:

(a) If it rains, then you will not play.

***Converse* — If you will not play then it will rain.**

***Inverse* — If it doesn't rain then you will play.**

***Contrapositive* — If you will play then it will not rain.**

(b) If you work hard, then you will pass.

***Converse* — If you will pass then you worked hard.**

***Inverse* — If you don't work hard, then you will not pass.**

***Contrapositive* — If you will not pass then you didn't work hard.**

(c) If I run fast, then I will win the race.

***Converse* — If I will win the race then I ran fast.**

***Inverse* — If I don't run fast, then I will not win the race.**

***Contrapositive* — If I will not win the race then I didn't run fast.**

53. **Write the converse of this statement. " if we prepare the exam then we will get the good grade".**

    Answer: the converse of statement is "if we prepare the exam, then we will get good grade." Will be: "If we get good grade then we did prepare the exam"

    converse of *p*→*q*: the conditional statement $q \to p$

    contrapositive of *p*→*q*: the conditional statement $\neg q \to \neg p$

    inverse of *p*→*q*: the conditional statement $\neg p \to \neg q$

    Some Examples from exercise Q27, Q28 Page 15
54. **What is the addition rule of inference? Page 72**

    Chapter 1 The Foundations Logic and Proofs.pdf slide 104

    The tautology basis on $p \to (p \vee q)$ of the rules of inference is called addition rule of inference.
55. **Define Modus Rule of Inference? Page 71 Example and Rule Page 72**

    Assignment: Example 3, 4, 5 Page 72 to 74
56. Define fallacies? Page 69 page 75
57. Give an Example of Fallacy? page 75
58. State resolution rule. Page 74
59. **Define vacuous? OR What is Vacuous Proof**

    Topic Page 84 also explain with answer Example 5

We can quickly prove that a conditional statement $p \rightarrow q$ is true when we know that p is false, because $p \rightarrow q$ must be true when p is false. Consequently, if we can show that p is false , then we have a proof called a vacuous proof the conditional statement $p \rightarrow q$.

Chapter 1:- Logic and Proofs: Long Question

1. **Show that is premises "It is not sunny this afternoon and it is colder than yesterday. We will go swimming if it is sunny; if we don't go swimming then we will take a canoe trip. If we take a canoe trip, then we will be home by sun set, lead to conclusion: we will be home by sun set."**

   Example 6 page 73 Table 1 Page 72 You can solve this problem

$\dfrac{p \wedge q}{\therefore\ p}$	$(p \wedge q) \rightarrow p$	Simplification
p $\dfrac{p \rightarrow q}{\therefore\ q}$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens
$\neg q$ $\dfrac{p \rightarrow q}{\therefore\ \neg p}$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus tollens

2. **Prove that if m + n and n-p are even integers, where m, n and p are integers, then m-p is even. What kind of proof did you use? Q5 Page 91**

Direct Proof

m, n, and p are integers
m + n and n - p are even integers
to proof m- p are even integer

OR

If Question is
m, n, and p are integers
m + n and n + p are even integers
to proof m+ p are even integer Then Below Solution is correct
Otherwise just change n + p to n – p and m + p to m – p in below solution
Result is m – p = 2 (y + z - n)
If we take even values like y = 6, z= 4, and n=8 then m – p = 4 which is even integer

DIRECT PROOF

Properties odd and even integer:
If x is an odd integer, then there exists an integer y such that $x = 2y + 1$.
If x is an even integer, then there exists an integer y such that $x = 2y$.

Let $m+n$ and $n+p$ be even integers, then there exists integers y and z such that:

$$m + n = 2y$$

$$n + p = 2z$$

Let us add the two even integers:

$$m + n + n + p = 2y + 2z$$

Combine like terms in the previous equation:

$$m + 2n + p = 2y + 2z$$

We are interested in $m + p$, thus subtract $2n$ from each side of the previous equation:

$$m + p = 2y + 2z - 2n$$

Factor out the 2 in the right side of the equation:

$$m + p = 2(y + z - n)$$

Since y, z and n are integers, $y + z - n$ is also an integer and thus $m + p$ is even (using the above property for even integers).

3. Explain prove by contradiction with example to prove that $\sqrt{2}$ is irrational. **Page 86**

Suppose $\sqrt{2}$ is **rational.** That means it can be written as the ratio of two integers p and q

$$\sqrt{2} = \frac{p}{q} \qquad (1)$$

where we may assume that p **and** q **have no common factors.** (If there are any common factors we cancel them in the numerator and denominator.) Squaring in (1) on both sides gives

$$2 = \frac{p^2}{q^2} \qquad (2)$$

which implies

$$p^2 = 2q^2 \qquad (3)$$

Thus p^2 is even. The only way this can be true is that p itself is even. But then p^2 is actually divisible by 4. Hence q^2 and therefore q must be even. So p and q are both even which is a contradiction to our assumption that they have no common factors. The square root of 2 cannot be rational!

√2 is irrational is false then we suppose √2 is rational

A rational number can be written as a ratio or a fraction

Any fraction can be simplified to its reducible form

In its simplification form at least one term of the fraction is odd

An irrational number cannot be expressed as a fraction or ratio

If $P^2 = 2q^2$ if we take p =2 and q=2 then we get 4 = 8 If we take p = 2 and q = 1 then 2 = 1 its shows that p is even and q is odd, hence we say that p is even and q is even then fraction is not in simplest form, we can say that it is irreducible

Contradiction means the statement cannot be proven false. It must be true

Truth and falsity are opposites if one exists, then the other cannot this is the rule of contradiction

A statement cannot be true or false at the same time it must be true or it must be false

p = 1, 2, 3, 4 to 10 then

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

$9^2 = 81$

$10^2 = 100$

So prove p must be itself even then p^2 is even and we can see that 4, 16, 36, 64, 100 are all divisible by 4.if p is then odd then its square must be odd. If we take q = 1, 2, 3, to 10 q must be even, so not possible both p and q are even possibility q is even and p is not even so **https://tutors.com/math-tutors/geometry-help/proof-by-contradiction-definition-examples**

4. **Proof that sum of two rational number is rational Example 7 Page 85**

Rational numbers that are expressed in p/q form

Like 2/3 + 4/6 if we add them then $\frac{2(6)+4(3)}{3(6)} = \frac{12+12}{18} = \frac{24}{18} = \frac{4}{3}$

5. **Use rule of inference to show that the hypothesis "Randy works hard"," If randy works hard then he is dull boy", and "If randy is dull boy then he will not get the job", imply the conclusion "Randy will not get good job". Q5 Page 78**

$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$	$(p \wedge q) \to p$	Simplification
$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$	$(p \wedge (p \to q)) \to q$	Modus ponens

"If it is raining, then I will make tea." $p \to q$
"It is raining." p
"Therefore, I will make tea." $\therefore q$

It is raining is input (p) and therefore I will make tea is output (q)

Let us assume:

p = "Randy Works Hard"

q = "Randy is a dull boy"

r = "Randy will get the job"

We can then rewrite the given statements (1), (2) and 3 using the above interpretations

	Step	Reason
1.	p	*Premise*
2.	$p \to q$	*Premise*
3.	$q \to \neg r$	*Premise*
4.	q	Modens ponens from (1) and (2)
5.	$\neg r$	Modens ponens from (3) and (4)

$\neg r$ Means "Randy will not get the job" and thus we obtained the implied conclusion

Use modus ponens twice

Q6 Page 78 Assignment

https://calcworkshop.com/logic/rules-inference/

Important link for Understanding Rule of inference

Watch video Arguments building using rules of interference

6. **What is wrong with this argument? Let H(x) be "x Is happy" Given the premises $\exists x H(x)$, we conclude that H(Waqar). Therefore, Lola is happy**

Let us assume:
H(x) = "x is happy"
The premise $\exists x H(x)$ is given and thus we assume that $\exists x H(x)$ is true
We know that *some x* exists that makes *H (x)* true, but we cannot conclude that Maria is one such *x.* Maybe only Waqar is happy and everyone else is not happy. Then *3x H(x)* is true, but H(Maria) is false mean Maria is not happy.
We do not know if Maria is the person for which H(x) is true

Q18 Page 79 Assignment

7. **Translate the statements in to quantifiers and predicate logic. " For all x, for all y if x is greater than zero and y is less than zero, then multiply them together will produce a negative number"**

Answer: $\forall \boldsymbol{x} \forall \boldsymbol{y}((\boldsymbol{x} > 0) \wedge (\boldsymbol{y} < 0) \rightarrow (\boldsymbol{xy} < 0))$

8. **Give Structured Proofs or Counter Examples for the following:**

Both symbols are equal or both Having the same Meaning

$\Rightarrow = \rightarrow$

$(A \Rightarrow B) \wedge (C \Rightarrow \neg B) \Rightarrow (A \Rightarrow \neg C)$
$(\exists x. A(x) \wedge \forall y. A(y) \Rightarrow y = x) \Rightarrow (\exists x. \forall y. (A(y) \Leftrightarrow y = x))$
Counter example: element for which p(x) is false is called counter example

9. **Which of the following formulas are tautologies? Explain what is meant by "tautology" and write down the truth table to justify your answers**

$p \Rightarrow q$ $\qquad$ $(p \Rightarrow q) \Rightarrow p$ $\qquad$ $((p \Rightarrow q) \Rightarrow p) \Rightarrow p$

Implication (denoted ⇒ or →)
Biconditional (denoted ⇔ or ↔)

Assignment by own

10. **Prove that following are logical equivalent by developing a series of logically equivalence?**

$$\neg(p \vee (\neg p \wedge q)) \text{and } \neg p \wedge \neg q$$

$$\neg p \leftrightarrow q \Leftrightarrow p \leftrightarrow \neg q$$

Example 7 Page 30 & Q19 Page 35

We will use logical equivalences to show that the two expressions are logically equivalent:

$$
\begin{aligned}
\neg p \leftrightarrow q &\equiv (\neg p \to q) \wedge (q \to \neg p) && \text{Logical equivalence (5)}\\
&\equiv (\neg(\neg p) \vee q) \wedge (\neg q \vee \neg p) && \text{Logical equivalence (2)}\\
&\equiv (p \vee q) \wedge (\neg q \vee \neg p) && \text{Double negation law}\\
&\equiv (\neg q \vee \neg p) \wedge (p \vee q) && \text{Commutative law}\\
&\equiv (\neg p \vee \neg q) \wedge (q \vee p) && \text{Commutative law}\\
&\equiv (\neg p \vee \neg q) \wedge (\neg(\neg q) \vee p) && \text{Double negation law}\\
&\equiv (p \to \neg q) \wedge (\neg q \to p) && \text{Logical equivalence (2)}\\
&\equiv p \leftrightarrow \neg q && \text{Logical equivalence (5)}
\end{aligned}
$$

We can then conclude that $\neg p \leftrightarrow q$ is logically equivalent with $p \to \neg q$.

11. Prove that following are logically equivalent by developing a series of logically equivalence
    $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ Is a tautology
    $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$

The conditional statement is a tautology, if the conditional statement is equivalent with true T.

Use logical equivalence (2) (twice):

$$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r) \equiv \neg[(p \vee q) \wedge (\neg p \vee r)] \vee (q \vee r)$$

Use De Morgan's law (twice):

$$\equiv [\neg(p \vee q) \vee \neg(\neg p \vee r)] \vee (q \vee r)$$

$$\equiv [(\neg p \wedge \neg q) \vee (\neg(\neg p) \wedge \neg r)] \vee (q \vee r)$$

Use double negation law:

$$\equiv [(\neg p \wedge \neg q) \vee (p \wedge \neg r)] \vee (q \vee r)$$

Use distributive law (twice):

$$\equiv [((\neg p \wedge \neg q) \vee p) \wedge ((\neg p \wedge \neg q) \vee \neg r)] \vee (q \vee r)$$

$$\equiv [((\neg p \vee p) \wedge (\neg q \vee p)) \wedge ((\neg p \vee \neg r) \wedge (\neg q \vee \neg r))] \vee (q \vee r)$$

Use negation law:

$$\equiv [(T \wedge (\neg q \vee p)) \wedge ((\neg p \vee \neg r) \wedge (\neg q \vee \neg r))] \vee (q \vee r)$$

Use identity law:

$$\equiv [(\neg q \vee p) \wedge ((\neg p \vee \neg r) \wedge (\neg q \vee \neg r))] \vee (q \vee r)$$

Use associative law:

$$\equiv ([(\neg q \vee p) \wedge ((\neg p \vee \neg r) \wedge (\neg q \vee \neg r))] \vee q) \vee r$$

Use distributive law:

$$\equiv [((\neg q \vee p) \vee q) \wedge (((\neg p \vee \neg r) \vee q) \wedge ((\neg q \vee \neg r) \vee q))] \vee r$$

Use associative and commutative law:

$$\equiv [((\neg q \vee p) \vee q) \wedge (((\neg p \vee \neg r) \vee q) \wedge ((\neg q \vee \neg r) \vee q))] \vee r$$

Use associative and commutative law:

$$\equiv [((\neg q \vee q) \vee p) \wedge (((\neg p \vee \neg r) \vee q) \wedge ((\neg q \vee q) \vee \neg r))] \vee r$$

Use negation law:

$$\equiv [(T \vee p) \wedge (((\neg p \vee \neg r) \vee q) \wedge (T \vee \neg r))] \vee r$$

Use domination law:

$$\equiv [T \wedge (((\neg p \vee \neg r) \vee q) \wedge T)] \vee r$$

Use identity law (twice):

$$\equiv [T \wedge ((\neg p \vee \neg r) \vee q)] \vee r$$

$$\equiv ((\neg p \vee \neg r) \vee q) \vee r$$

Use associative law:

$$\equiv (\neg p \vee \neg r) \vee (q \vee r)$$

Use commutative law:

$$\equiv (\neg p \vee \neg r) \vee (r \vee q)$$

Use associative law (twice):

$$\equiv ((\neg p \vee \neg r) \vee r) \vee q$$

$$\equiv (\neg p \vee (\neg r \vee r)) \vee q$$

Use negation law:

$$\equiv (\neg p \vee T) \vee q$$

Use domination law (twice):

$$\equiv T \vee q$$

$$\equiv T$$

We have shown that the conditional statement is equivalent with true T and thus the conditional statement is a tautology.

$$[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r) \equiv T$$

$(p \wedge q) \rightarrow r$ is not logically equivalent with $(p \rightarrow r) \wedge (q \rightarrow r)$, because the last two columns (yellow columns) of the following truth table do not contain the same truth value in each row.

p	q	r	p ^ q	p -> r	q -> r	(p ^ q) -> r	(p -> r) ^ (q -> r)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	F	T	T	F
F	T	T	F	T	T	T	T
F	T	F	F	T	F	T	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

$(p \wedge q) \rightarrow r$ is not logically equivalent with $(p \rightarrow r) \wedge (q \rightarrow r)$

12. Prove that following are logically equivalent by developing a series logically equivalences

    $\neg p \rightarrow (q \rightarrow r) amd\ q \rightarrow (p \vee r)$

    $p \leftrightarrow q\ and \neg p \leftrightarrow \neg q$

Use logical equivalence (1) (twice):

$$\neg p \to (q \to r) \equiv \neg(\neg p) \vee (q \to r)$$

$$\equiv \neg(\neg p) \vee (\neg q \vee r)$$

Use double negation law:

$$\equiv p \vee (\neg q \vee r)$$

Use associative law:

$$\equiv (p \vee \neg q) \vee r$$

Use commutative law:

$$\equiv (\neg q \vee p) \vee r$$

Use associative law:

$$\equiv \neg q \vee (p \vee r)$$

Use logical equivalence (1):

$$\equiv q \to (p \vee r)$$

We have thus derived that $\neg p \to (q \to r)$ is logically equivalent with $q \to (p \vee r)$.

$$\neg p \to (q \to r) \equiv q \to (p \vee r)$$

Use logical equivalence (1):

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$

Use distributive law (twice):

$$\equiv [p \vee (\neg p \wedge \neg q)] \wedge [q \vee (\neg p \wedge \neg q)]$$

$$\equiv [(p \vee \neg p) \wedge (p \vee \neg q)] \wedge [(q \vee \neg p) \wedge (q \vee \neg q)]$$

Use negation law:

$$\equiv [T \wedge (p \vee \neg q)] \wedge [(q \vee \neg p) \wedge T]$$

Use identity law:

$$\equiv (p \vee \neg q) \wedge (q \vee \neg p)$$

Use double negation law:

$$\equiv (\neg(\neg p) \vee \neg q) \wedge (\neg(\neg q) \vee \neg p)$$

Use logical equivalence (2):

$$\equiv (\neg p \to \neg q) \wedge (\neg q \to \neg p)$$

Use logical equivalence (3):

$$\equiv \neg p \leftrightarrow \neg q$$

We have thus derived that $p \leftrightarrow q$ is logically equivalent with $\neg p \leftrightarrow \neg q$.

$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$

13. Show that using law's of logic $\backsim \{\{\backsim \boldsymbol{p} \wedge \boldsymbol{q}\} \vee \{\backsim \boldsymbol{p} \wedge \backsim \boldsymbol{q}\}\} \vee \{\boldsymbol{p} \wedge \boldsymbol{q}\} \cong \boldsymbol{p}$ **page 27**

Or $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (\boldsymbol{p} \wedge \boldsymbol{q}) \cong \boldsymbol{p}$

$$\neg((\neg p) \vee (\neg p)) \vee (\boldsymbol{p} \wedge \boldsymbol{q}) \cong$$

Equivalence	Name
$p \wedge T \equiv p, \quad p \vee F \equiv p$	Identity laws
$p \vee T \equiv T, \quad p \wedge F \equiv F$	Domination laws
$p \vee p \equiv p, \quad p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \vee q) \equiv \neg p \wedge \neg q$ $\neg(p \wedge q) \equiv \neg p \vee \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv T, \quad p \wedge \neg p \equiv F$	Negation laws

14. **Check the validity of the following argument? Section 1.3 pdf in Google drive discrete**

$$p \wedge q \backsim \boldsymbol{q} \longrightarrow \boldsymbol{r}$$
$$p \wedge q$$
$$\boldsymbol{q} \rightarrow \boldsymbol{r}$$
$$\therefore \boldsymbol{r}$$

15. **Give a logic gate implementation of** $(\backsim \boldsymbol{x}).(\backsim \boldsymbol{y}) + \boldsymbol{y}.(\backsim \boldsymbol{x})$
    Using Symbols we have to create Gate that are created in DLD
16. **You can upgrade your operating system only if you have a 32 bit processor running at 1 GHz or Faster, at least 1 GB RAM, and 16 GB free Hard Disk Space, or a 64 bit Processor running at 2 GHz or faster, at least 2 GB RAM, and At Least 32 GB Free Hard Disk space. Expression your answer in terms of u: "You can upgrade your operating system," b_{32}: "You have a 32 bit Processor," b_{64}: "You have a 64 – bit processor," g_1: Your Processors funs at 1 GHz or faster," g2: "Your Processor has at least 1 GB RAM," r_2: "Your Processor has at least 2GB RAM," h_{16}: "You Have at least 16 GB of Free Hard disk space," and h_{32}: "You have at least 32 GB free hard disk space."** **Answer Page 22 Q6**

Given:
u="You can upgrade your operating system"
b_{32}="You have a 32-bit processor"
b_{64}=You have a 64-bit processor"
g_1="Your processor runs at 1 GHz or faster"
g_2="Your processor runs at 2 GHz or faster"
r_1="Your processor has at least 1 GB RAM"
r_2="Your processor has at least 2 GB RAM"
h_{16}="You have at least 16 GB free hard disk space"
h_{32}="You have at least 16 GB free hard disk space"

Let us replace the substatements with their symbols in the given statement:

$$u \text{ only if } b_{32} \text{ and } g_1,\ r_1, \text{ and } h_{16}, \text{ or } b_{64},\ g_2,\ r_2, \text{ and } h_{32})$$

Note that the "or" statement refers to the 32-bit processor and the 64-processors along with their corresponding characteristics, while the comma's without an "and" or "or" imply "and". Let us then rewrite the statement:

$$u \text{ only if } (b_{32} \text{ and } g_1 \text{ and } r_1 \text{ and } h_{16}) \text{ or } (b_{64} \text{ and } g_2 \text{ and } r_2 \text{ and } h_{32})$$

"p only if q" is logically equivalent with "if p, then q":

$$\text{if } u, \text{ then } (b_{32} \text{ and } g_1 \text{ and } r_1 \text{ and } h_{16}) \text{ or } (b_{64} \text{ and } g_2 \text{ and } r_2 \text{ and } h_{32})$$

Disjunction $p \vee q$: p or q
Conjunction $p \wedge q$: p and q
Conditional statement $p \to q$: if p, then q

$$u \to ((b_{32} \wedge g_1 \wedge r_1 \wedge h_{16}) \vee (b_{64} \wedge g_2 \wedge r_2 \wedge h_{32}))$$

17. Find a counter example, if possible to these universally quantified statements, where the domain for all variables consists of all integers. Ans: Q39 Page 68

$\forall x \forall y (x^2 = y^2 \to x = y)$
$\forall x \exists y (y^2 = x)$
$\forall x \forall y (xy \geq x)$
Assignment Q35 Page 55

a) The given sentence means: If the squares of any two integers are equal, then the two integers have to be equal as well.

The statement is false. For example, if we take the integers $x = -1$ and $y = 1$, then there squares are equal while the integers are unequal.

$$x^2 = (-1)^2 = 1 = 1^2 = y^2$$
$$x = -1 \neq 1 = y$$

---

b) Note: $y^2 = x$ is equivalent with $y = \pm\sqrt{x}$

The given statement means: For all integers, the (positive/negative) square root of the integer is also an integer.

The statement is false, for example if $x = 3$, then its square root $y = \sqrt{3}$ is not integer.

$$y^2 = (\sqrt{3})^2 = 3 = x$$

---

c) The given statement means: For all two integers, the product of the two integers is larger than or equal to one of the integers.

The statement is false. For example, if $x = 1$ and $y = -2$, we have that the product is smaller than one of the integers.

$$xy = 1(-2) = -2 < 1 = x$$

---

a) For example: $x = -1$ and $y = 1$
b) For example: $x = 3$ and $y = \sqrt{3}$ ($\sqrt{3}$ is not an integer)
c) For example: $x = 1$ and $y = -2$

Chapter 2: set, functions, sequence sum and matrices

Definitions 185

1. **State the principal of inclusion and exclusion page128 Detail in chapter 6 and 8 Question page 553**
   When counting the possibilities, we cannot include a give out come more than once?
   $|A_1 \cup A_2| = |A_1|+|A_2|-|A_1 \cap A_2|$
   Let A_1 have 5 elements, A_2 have 3 elements and 1 element in both A_1 and A_2
   Total in the union is 5 + 3 -1 = 7 not 8
2. **Intersection of sets and Union of sets page 133**

3. **What is cardinality of each of these sets? {a, {a}, {a, {a}}} page 126 Q19 hw3-sol.pdf of Google drive**

   What is the cardinality of each of these sets?

   a) ∅ 0

   b) {∅} 1

   c) {∅, {∅}} 2

   d) {∅, {∅}, {∅, {∅}}} 3

   |*S*| (the cardinality of *S*): the number of elements in *S*

4. **How many different elements does A x B x C have if A has m elements, B has n elements and C has P elements Answer: Q36 Page 126**

   There are m different values the first member of the tuple can take, n different values for the second member to the tuple and p different values for the third member, yielding that

   $|A\ x\ B\ x\ C|$= m.n.p

   Assignment Q35, 27 Page 126

5. **Cardinality of set page 170 chapter 2 sets.pdf Slide 36**

   Let S be a set. If there are exactly n distinct elements in S, where n is a nonnegative integer, we say S is a finite set and that n is the **cardinality of S**. The cardinality of S is denoted by **| S |.**

   V={1 2 3 4 5}

   | V | = 5

6. Let $\{t_n\}$ be a sequence, where $\{t_n\}$ = 7+3n what is common ratio/difference and what are the terms of a sequence?

   A set of numbers arranged in a definite order according to some definite rule is called a sequence .A sequence in which the difference between two consecutive terms remains constant. Like 2, 5, 8, 11, 14, ...is a arithmetic sequence.

   A series in which the difference between any term and its previous term is constant as 2 + 5 + 8 + 11 + 14 + ... is a arithmetic series

   Page 157 example 3

7. **List the members of the following sets.**

   {*x* | *x* is a real number such that x^2 = 1}

   Ans: Now, there are two real numbers whose squares equal 1, i.e. 1 and -1.

   So, {1,-1} and {2, -2}

   {x| x is a real numbers such that x^2=4}

   Ans: The members of set {x | x is a real number such that x^2 =4} are {2}

   {x | x is an integer such that x^2=2}

   Ans: The Members of the set {x | x is an integer such that x2 =2 } are **{∅}**

   It's Null set as there's no integer whose square is equal to 2.

. List the members of these sets.

a) $\{x \mid x \text{ is a real number such that } x^2 = 1\}$

b) $\{x \mid x \text{ is a positive integer less than } 12\}$

c) $\{x \mid x \text{ is the square of an integer and } x < 100\}$

d) $\{x \mid x \text{ is an integer such that } x^2 = 2\}$

{-1, 1}

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

{0, 1, 4, 9, 16, 25, 36, 47, 64, 81}

{Ø}

8. **How can you produce the terms of the sequence if the first 10 terms are 5,11, 17, 23, 29, 35, 41, 47, 53, 69?**

Page 161 Example 14

Solution: Note that each of the first 10 terms of this sequence after the first is obtained by adding 6 to the previous term. (We could see this by noticing that the difference between consecutive terms is 6.) Consequently, the nth term could be produced by starting with 5 and adding 6 a total of $n - 1$ times; that is, a reasonable guess is that the nth term is $5 + 6(n - 1) = 6n - 1$. (This is an arithmetic progression with $a = 5$ and $d = 6$.) ◀

By Putting values in formula 6n – 1 calculate the 11^{th} term and after that terms

9. **Find the formula for this sequence? 1, 1/2, 1/3, 1/4, 1/5, …………………………..**

Formula is: $A_n=1/n$

Example 1 page 156 Topic: Sequence Page 156

10. **Differentiate b/w function and relation?**

Relation: Let A and B be sets. A binary relation from A to B is a subset of A x B. Let A and B be sets. The binary relation R from A to B is a subset of A x B where $(a, b) \in R$, we say 'a' is related to 'b' by R, written aRb.

Function: A function F from a set X to a set Y is a relation from X to Y that satisfies the following properties.

a. For every element x in X, there is an element y in Y such that $(x, y) \in F$. In other words every element of X is the first element of some ordered pair of F.
b. For all element x in X and y in Y, if $(x, y) \in F$ and $(x, z) \in F$,then y =z in other words no two distinct ordered pairs in F have the same first element.

A relationship between the objects or some set of information is called the relation. But, a well-defined relation is called a function.

In any relation, an input can have more than one output but in function, this is not possible. In function, there is one to one correspondence between the inputs and outputs. For example, { (1,2), (3,4), (3,5), (4,1) } is not a function since there is two ordered pairs which have the same first element 3.

In a relation, if there is none of the ordered pair present in the relation which have same first element, then that relation is known as function.

With the help of function, we have come to know about the dependence between the independent variable and the dependent variable but not in relation.

11. **Is the function f(x) =x^2 from the set of integers to the set of integers one to one? Page 142 example 9**

<u>Graph in Page 142 Figure 3</u>

<u>Proper Understanding must see Figure 5 Page 144 Example 16 2nd Paragraph is the understanding of Graph.</u>

<u>If it is both one to one and On to we say that such function is bijective</u>

Solution: The function $f(x) = x^2$ is not one-to-one because, for instance, $f(1) = f(-1) = 1$, but $1 \neq -1$.

12. **Determine whether the function f from {a, b. c, d} to {1, 2, 3, 4, 5} with f(a) = 4, f(b) = 5, f(c) = 1 and f(d)= 3 is one to one   Answer: Example 8 Page 142**

The function f is one to one because f takes on different values at the four elements of its domain. Illustrated below

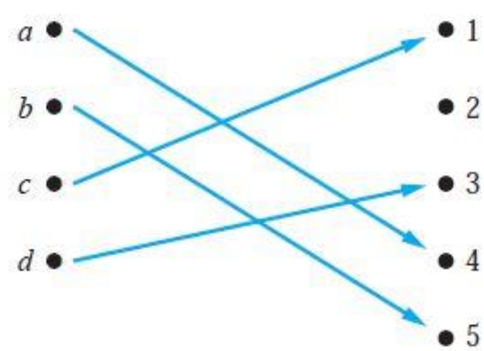


FIGURE 3 **A One-to-One Function.**

13. **Let f be a function from {a, b, c, d} to {1,2,3} defined by f(a)= 3, f(b)=2, f(c)=1 and f(d) =3. If f an onto function   Example 12 page 143**

Solution: Because all three elements of the codomain are images of elements in the domain, we see that f is onto. This is illustrated in Figure 4. Note that if the codomain were {1, 2, 3, 4}, then f would not be onto.

14. **Is the function f(x)=x2 from the set of integers to the set of integer on to?**

Example 13 page 143

The function f is not onto because there is no integer x with $x^2 = -1$, for instance.

15. **Define the function f(x) = (x^2+1)/(x^2+2) one to one or onto. Domain consist of all integers**

Q23 page 153

a) $f(x) = 2x + 1$
b) $f(x) = x^2 + 1$
c) $f(x) = x^3$
d) $f(x) = (x^2 + 1)/(x^2 + 2)$

a) Yes, all real numbers can be doubled and increased by one to equal unique real number.
b) No, the combination completely leaves out all negative numbers and zero, not to mention, it is not one to one
c) Yes, all real numbers can be cubed and equal to unique real number.
d) No, though the domain includes all real numbers, the numerator and denominators both output positive numbers individual, so all outputs must be positive.(Graphing function also reveals that it is not one to one).

16. **Define the function f(x) = (x+1)/(x^2-4) one to one or onto. Domain consist of all integers**

    Topic Page 141 One to one OR Injective, Onto OR Surjective,

    Definition 5, 7, 8 page 141 – 144, Practice Example 8, 9, 12, 13, 14, 15, 16

    Above Question is Assignment

17. **Differentiate b/w subset the proper subset?** **Subset page 119**

    Subset is a set containing some or all members of another set.
    For example, if the set S is defined as { a, b, c }, then { a }, { a, c } and { a, b, c } are all subsets of S.
    A **proper subset** is a subset which contains fewer elements of its parent set. For example, if S is defined as { a, b, c }, { a, b, c }and { a, b, c } are subsets of S, but { a, b, c } is not, since it is equal to S.
    In other words, the term proper subset can be read as "subset of but not equal to".

18. **Define disjoint set?** **Page 128**

    Definition 3 Example 5 Page 128

19. **If A = {1, 3, 5,}, B = {1, 2, 3} are A & B disjoint?**

    No, A & B are not disjoint sets because more than one elements (1,3) are common

20. **What is the Cartesian product of A={a, b, c} and B={1, 2}**

    Example 17, 18 page 123

21. **What is the Cartesian product of A ={a, b} and B={1, 2, 3, 4}.**

    Same as above but elements in sets are more than above

22. **What is the Cartesian product A x B x C, where a = { 0, 1}, B = { 1, 2}, and C ={ 0, 1, 2}?**

    Example 19 Page 124

$$A \times B \times C = \{(0,1,0), (0,1,1), (0,1,2), (0,2,0), (0,2,1), (0,2,2),\\ (1,1,0), (1,1,1), (1,1,2), (1,2,0), (1,2,1), (1,2,2)\}.$$

23. **What is the difference b/w geometric progression and arithmetic progression?** **Page 157**

    An <u>arithmetic progression</u> is a sequence of numbers in which each term is derived from the preceding term by adding or subtracting a fixed number called the common difference "d"
    For example, the sequence 9, 6, 3, 0,-3, ... is an arithmetic progression with -3 as the

common difference. The progression -3, 0, 3, 6, 9 is an Arithmetic Progression (AP) with 3 as the common difference.

A <u>geometric progression</u> is a sequence in which each term is derived by multiplying or dividing the preceding term by a fixed number called the common ratio. For example, the sequence 4, -2, 1, - 1/2,.... is a Geometric Progression (GP) for which - 1/2 is the common ratio.

Definition 1 and definition 3 definitions 186 Chapter 2

24. Let $\{t_n\}$ be a sequence where $t_n = 7-3n$. what type of progression is this?

This a an A.P because if we put n = 1, 2, 3, …………

$T_1 = 7 – 3 = 4$, $T_2 = 7 – 6 = 1$, $T3 = 7 – 9 = -2$, so common difference is 3 so it is A.P

25. What is recurrence relation?

page 157 and Definition 4 Page 158

26. Find the recurrence relation of the sequence $S(n)= 5^n$ Q14 Page 168

a) $a_n = 3$ **b)** $a_n = 2n$
c) $a_n = 2n + 3$ **d)** $a_n = 5^n$
e) $a_n = n^2$ **f)** $a_n = n^2 + n$
g) $a_n = n + (-1)^n$ **h)** $a_n = n!$

(a) Given:

$$a_n = 3$$

Let us first determine the first term by replacing n in the given expression for a_n by 0:

$$a_0 = 3$$

We note that each term is the same as the previous term (as all terms are equal to 3):

$$a_n = a_{n-1}$$

Thus a recurrence relation for a_n is then:

$$a_0 = 3$$

$$a_n = a_{n-1}$$

Note: There are infinitely many different recurrence relations that satisfy any sequence.

(b) Given:

$$a_n = 2n$$

Let us first determine the first term by replacing n in the given expression for a_n by 0:

$$a_0 = 2(0) = 0$$

Let us similarly determine the next few terms as well:

$$a_1 = 2(1) = 2 = a_0 + 2$$

$$a_2 = 2(2) = 4 = a_1 + 2$$

$$a_3 = 2(3) = 6 = a_2 + 2$$

We note that each term is the previous term increased by 2:

$$a_n = a_{n-1} + 2$$

Thus a recurrence relation for a_n is then:

$$a_0 = 0$$

$$a_n = a_{n-1} + 2$$

Note: There are infinitely many different recurrence relations that satisfy any sequence.

(c) Given:

$$a_n = 2n + 3$$

Let us first determine the first term by replacing n in the given expression for a_n by 0:

$$a_0 = 2(0) + 3 = 3$$

Let us similarly determine the next few terms as well:

$$a_1 = 2(1) + 3 = 5 = a_0 + 2$$

$$a_2 = 2(2) + 3 = 7 = a_1 + 2$$

$$a_3 = 2(3) + 3 = 9 = a_2 + 2$$

We note that each term is the previous term increased by 2:

$$a_n = a_{n-1} + 2$$

Thus a recurrence relation for a_n is then:

$$a_0 = 3$$

$$a_n = a_{n-1} + 2$$

Note: There are infinitely many different recurrence relations that satisfy any sequence.

(d) Given:

$$a_n = 5^n$$

Let us first determine the first term by replacing n in the given expression for a_n by 0:

$$a_0 = 5^0 = 1$$

Let us similarly determine the next few terms as well:

$$a_1 = 5^1 = 5 = 5a_0$$

$$a_2 = 5^2 = 25 = 5a_1$$

$$a_3 = 5^3 = 125 = 5a_2$$

We note that each term is the previous term multiplied by 5:

$$a_n = 5a_{n-1}$$

Thus a recurrence relation for a_n is then:

$$a_0 = 1$$

$$a_n = 5a_{n-1}$$

Note: There are infinitely many different recurrence relations that satisfy any sequence.

(e) Given:

$$a_n = n^2$$

Let us first determine the first term by replacing n in the given expression for a_n by 0:

$$a_0 = 0^2 = 0$$

Let us similarly determine the next few terms as well:

$$a_1 = 1^2 = 1 = a_0 + 2(1) - 1$$

$$a_2 = 2^2 = 4 = a_1 + 2(2) - 1$$

$$a_3 = 3^2 = 9 = a_2 + 2(3) - 1$$

$$a_4 = 4^2 = 16 = a_3 + 2(4) - 1$$

$$a_5 = 5^2 = 25 = a_4 + 2(5) - 1$$

$$a_6 = 6^2 = 36 = a_5 + 2(6) - 1$$

We note that each term is the previous term increased by $2n - 1$:

$$a_n = a_{n-1} + 2n - 1$$

Thus a recurrence relation for a_n is then:

$$a_0 = 0$$

$$a_n = a_{n-1} + 2n - 1$$

Note: There are infinitely many different recurrence relations that satisfy any sequence.

(f) Given:

$$a_n = n^2 + n$$

Let us first determine the first term by replacing n in the given expression for a_n by 0:

$$a_0 = 0^2 + 0 = 0$$

Let us similarly determine the next few terms as well:

$$a_1 = 1^2 + 1 = 2 = a_0 + 2(1)$$

$$a_2 = 2^2 + 2 = 6 = a_1 + 2(2)$$

$$a_3 = 3^2 + 3 = 12 = a_2 + 2(3)$$

$$a_4 = 4^2 + 4 = 20 = a_3 + 2(4)$$

$$a_5 = 5^2 + 5 = 30 = a_4 + 2(5)$$

$$a_6 = 6^2 + 6 = 42 = a_5 + 2(6)$$

We note that each term is the previous term increased by $2n$:

$$a_n = a_{n-1} + 2n$$

Thus a recurrence relation for a_n is then:

$$a_0 = 0$$

$$a_n = a_{n-1} + 2n$$

Note: There are infinitely many different recurrence relations that satisfy any sequence.

(g) Given:

$$a_n = n + (-1)^n$$

Let us first determine the first term by replacing n in the given expression for a_n by 0:

$$a_0 = 0 + (-1)^0 = 0 + 1 = 1$$

Let us similarly determine the next few terms as well:

$$a_1 = 1 + (-1)^1 = 1 - 1 = 0 = a_0 - 1$$

$$a_2 = 2 + (-1)^2 = 2 + 1 = 3 = a_1 + 3$$

$$a_3 = 3 + (-1)^3 = 3 - 1 = 2 = a_2 - 1$$

$$a_4 = 4 + (-1)^4 = 4 + 1 = 5 = a_3 + 3$$

$$a_5 = 5 + (-1)^5 = 5 - 1 = 4 = a_4 - 1$$

$$a_6 = 6 + (-1)^6 = 6 + 1 = 7 = a_5 + 3$$

We note that each term is the previous term increased by 3 if n is even:

$$\text{If } n \text{ even: } a_n = a_{n-1} + 3$$

We note that each term is the previous term decreased by 1 if n is odd:

$$\text{If } n \text{ odd: } a_n = a_{n-1} - 1$$

Thus a recurrence relation for a_n is then:

$$a_0 = 1$$

$$a_n = \begin{cases} a_{n-1} + 3 & \text{if } n \text{ even} \\ a_{n-1} - 1 & \text{if } n \text{ odd} \end{cases}$$

Note: There are infinitely many different recurrence relations that satisfy any sequence.

(h) Given:

$$a_n = n!$$

Let us first determine the first term by replacing n in the given expression for a_n by 0:

$$a_0 = 0! = 1$$

Let us similarly determine the next few terms as well:

$$a_1 = 1! = 1 = 1a_0$$

$$a_2 = 2! = 2 = 2a_1$$

$$a_3 = 3! = 6 = 3a_2$$

$$a_4 = 4! = 24 = 4a_3$$

$$a_5 = 5! = 120 = 5a_4$$

$$a_6 = 6! = 720 = 6a_5$$

We note that each term is the previous term multiplied by n:

$$a_n = na_{n-1}$$

Thus a recurrence relation for a_n is then:

$$a_0 = 1$$

$$a_n = na_{n-1}$$

Note: There are infinitely many different recurrence relations that satisfy any sequence.

27. Define commutative law with the help of example? Page 130

The Law that says you can swap numbers around and still get the same answer when you add. We say $A \cup B = B \cup A$ *and* $A \cap B = B \cap A$ Commutative laws

Examples: You can swap when you add: 6 + 3 = 3 + 6.
Examples: You can swap when you multiply: 6 * 3 = 3 * 6.

28. Find the value for $\left\lfloor \frac{1}{2} \cdot \left\lfloor \frac{5}{2} \right\rfloor \right\rfloor$

Definition Page 186 For Understanding Examples 26, 27, 28 Page 149

Q8, 9 Page 153 Assignment

Find these values.

a) $\lceil \frac{3}{4} \rceil$
b) $\lfloor \frac{7}{8} \rfloor$
c) $\lceil -\frac{3}{4} \rceil$
d) $\lfloor -\frac{7}{8} \rfloor$
e) $\lceil 3 \rceil$
f) $\lfloor -1 \rfloor$
g) $\lfloor \frac{1}{2} + \lceil \frac{3}{2} \rceil \rfloor$
h) $\lfloor \frac{1}{2} \cdot \lfloor \frac{5}{2} \rfloor \rfloor$

First divide or multiply then apply floor and Ceiling Function

Ceiling(x) = smallest integer greater than x ;
Floor(x) = greatest integer less than x

Result

(a) 1 ; (b) 0 ; (c) 0 ; (d) -1 ; (e) 3 ; (f) -1 ; (g) Floor (0.5 + 2) = 2 ; (h) Floor (0.5 x 2) = 1

a) 1 **b)** 0 **c)** 0 **d)** -1 **e)** 3 **f)** -1 **g)** $\lfloor \frac{1}{2} + \lceil \frac{3}{2} \rceil \rfloor = \lfloor \frac{1}{2} + 2 \rfloor = \lfloor 2\frac{1}{2} \rfloor = 2$
h) $\lfloor \frac{1}{2} \lfloor \frac{5}{2} \rfloor \rfloor = \lfloor \frac{1}{2} \cdot 2 \rfloor = \lfloor 1 \rfloor = 1$

29. Give an example of Indefinitely Count able set? **Page 171, 173**

A set is countably infinite if its elements can be put in one-to-one correspondence with the set of natural numbers. ... For example, the set of integers {0,1,−1,2,−2,3,−3,...} is clearly infinite. However, as suggested by the above arrangement, we can count off all the integers. Counting off every integer will take forever.

Give an example of two UN countable sets A and B such as A∩B is Infinite, countably infinite, and uncountable

(a) For example, the set $A = \mathbf{R}$ (real numbers) is uncountable and the set $B = \mathbf{R} - \{0\}$ (nonzero real numbers) is also uncountable.

Using the definition of the difference and the fact that 0 is the only element of A that is not in B:

$$A - B = \mathbf{R} - (\mathbf{R} - \{0\}) = \{0\}$$

Since the set $\{0\}$ contains only 1 element, the set $\{0\}$ is finite and thus $A - B$ is finite as well.

(b) For example, the set $A = (0,1) \cup \mathbf{N}$ (all real numbers between 0 and 1 and all nonnegative integers) is uncountable and the set $B = (0,1)$ (all real numbers between 0 and 1) is also uncountable.

Using the definition of the difference and the fact thatthe nonnegative integers are the only element of A that is not in B:

$$A - B = ((0,1) \cup \mathbf{N}) - (0,1) = \mathbf{N}$$

The set $\mathbf{N}$ of nonnegative integers is countably infinite.

(c) For example, the set $A = \mathbf{R}$ (real numbers) is uncountable and the set $B = \mathbf{R}^- \cup \{0\}$ (negative real numbers and zero) is also uncountable.

Using the definition of the difference and the fact that the positive real numbers are the only element of A that is not in B:

$$A - B = \mathbf{R} - (\mathbf{R}^- \cup \{0\}) = \mathbf{R}^+$$

The set $\mathbf{R}^+$ of positive real numbers is uncountable.

Q11 Page 176 Assignment

30. Find $A \odot B$ Answer Q26 Page 185

Practice: Example 1 to 9 Page 178 to 183

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Find

a) $\mathbf{A} \vee \mathbf{B}$. b) $\mathbf{A} \wedge \mathbf{B}$. c) $\mathbf{A} \odot \mathbf{B}$.

Given:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(a) $\mathbf{A} \vee \mathbf{B}$ takes the disjunction of each element of $\mathbf{A}$ with the corresponding element (same row and same column) of $\mathbf{B}$.

$$\mathbf{A} \vee \mathbf{B} = \begin{bmatrix} 1 \vee 0 & 1 \vee 1 \\ 0 \vee 1 & 1 \vee 0 \end{bmatrix}$$

$b_1 \vee b_2$ is equal to 1 if $b_1 = 1$ or $b_2 = 1$ (else $b_1 \vee b_2$ is equal to 0).

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

(b) $\mathbf{A} \wedge \mathbf{B}$ takes the conjunction of each element of $\mathbf{A}$ with the corresponding element (same row and same column) of $\mathbf{B}$.

$$\mathbf{A} \wedge \mathbf{B} = \begin{bmatrix} 1 \wedge 0 & 1 \wedge 1 \\ 0 \wedge 1 & 1 \wedge 0 \end{bmatrix}$$

$b_1 \wedge b_2$ is equal to 1 if $b_1 = b_2 = 1$(else $b_1 \wedge b_2$ is equal to 0).

$$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

(c) The ijth element of the Boolean product **A** $\odot$ **B** takes the disjunction of all conjunctions of corresponding elements **A** and **B** in the ith row of **A** and in the jth column of **B**.

$$\mathbf{A} \odot \mathbf{B} = \begin{bmatrix} (1 \wedge 0) \vee (1 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 0) \\ (0 \wedge 0) \vee (1 \wedge 1) & (0 \wedge 1) \vee (1 \wedge 0) \end{bmatrix}$$

$b_1 \wedge b_2$ is equal to 1 if $b_1 = b_2 = 1$(else $b_1 \wedge b_2$ is equal to 0).

$$= \begin{bmatrix} 0 \vee 1 & 1 \vee 0 \\ 0 \vee 1 & 0 \vee 0 \end{bmatrix}$$

$b_1 \vee b_2$ is equal to 1 if $b_1 = 1$ or $b_2 = 1$ (else $b_1 \vee b_2$ is equal to 0).

$$= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Chapter 2: set, functions, sequence sum and matrices Long Questions

Definition Page 185

1. **Let A={ a,b, c, d} and B={ 1, 2 , a, 3, c} and C={ 2, b, a, 1}**
   Write $|AUC|$ and write $(\boldsymbol{A} \cap \boldsymbol{B})\ \boldsymbol{x}\ (\boldsymbol{A} \cap \boldsymbol{C})$

   Definition 1, 2, … 5 Page 127, 128. Example 1 to 9 Page 127, 128, Page 133 Example 15 definition 6, 7, Cartesian Product Example 16, 17 Page 123, definition 7 Page 122, definition 8 Page 123

2. **Let A= {a, b, c, d} B= {1, 2, a, 3, c} and C= {2, a, 1} write C-B, find P(C) $|AUB|$**

   <u>P(C) = Power set of C is the set of all subset of set C</u>

   Page 121 Definition 4, 5, 6 Example 10 to 15

3. **For all set A and B prove that $((A^cUB^c)-A)^c = A$**

   Prove by using Table 1 Page 130

4. **Suppose that B is a Boolean Algebra then prove that for all $\boldsymbol{x \epsilon B}$ $\boldsymbol{x.x = x\ and\ x + x = x}$**

   $x.x = x$

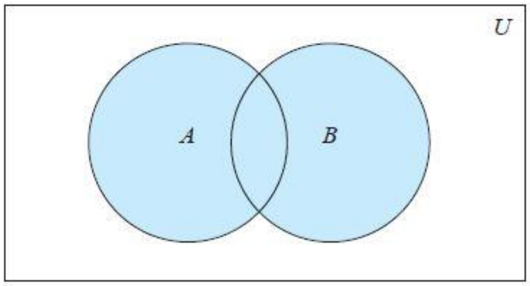
x.x + 0	by identity element if add 0 no change in equation
x.x + x.$\bar{x}$	x.$\bar{x}$=0 Put x.$\bar{x}$ in place of 0(complement Property)
x(x+$\bar{x}$)	after taking common x then its equal to (x+$\bar{x}$)
x(1)	(x+$\bar{x}$) by complement property equal to 1
x	hence Proved

   B Part

   x + x = x

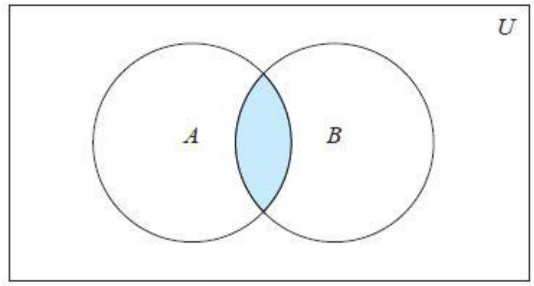
x.1 + x.1	Multiply both 1 is identity element and no change in equation
x(1 + 1)	take x common
x(1)	logical addition 1 + 1 = 1
x	hence proved

5. **Draw a Venn diagrams that represents that union and intersection of A and B sets Figure 1 and 2 Page 127**



$A \cup B$ is shaded.

FIGURE 1 Venn Diagram of the Union of A and B.



$A \cap B$ is shaded.

FIGURE 2 Venn Diagram of the Intersection of A and B.

6. **In a school, 100 students have access to three software Packages, A, B, C. 28 did not use any software 8 used only packages A, 26 used package B, 7 used Package C, 10 used all three packages, 13 used both A and B.**
   - **Draw a venn diagram with all sets enumerated as for as possible. Label the two subsets which cannot be enumerated as x and y in any order**
   - **If twice as many students used package B as Package A, write down a pair of simultaneous equations in x and y.**
   - **Solve these equations to find x and y.**
   - **How many students used package C**

   Solution:

A only = 8

B only = 26

C only = 7

All three = 10

both A and B = 13

both A and B but not C = 13 - 10 = 3

both A and C but not B = x

both B and C but not A = y

None = 28

Total = 100

8 + 26 + 7 + 10 + 3 + x + y + 28 = 100

82 + x + y = 100

=> x + y = 18

twice as many students used package B as package A,

B used = 26 + 13 + y = 39 + y

A used = 8 + 13 + x = 21 + x

=> 39 + y = 2(21 + x)

=> 39 + y = 42 + 2x

=> y = 2x + 3

x + y = 18

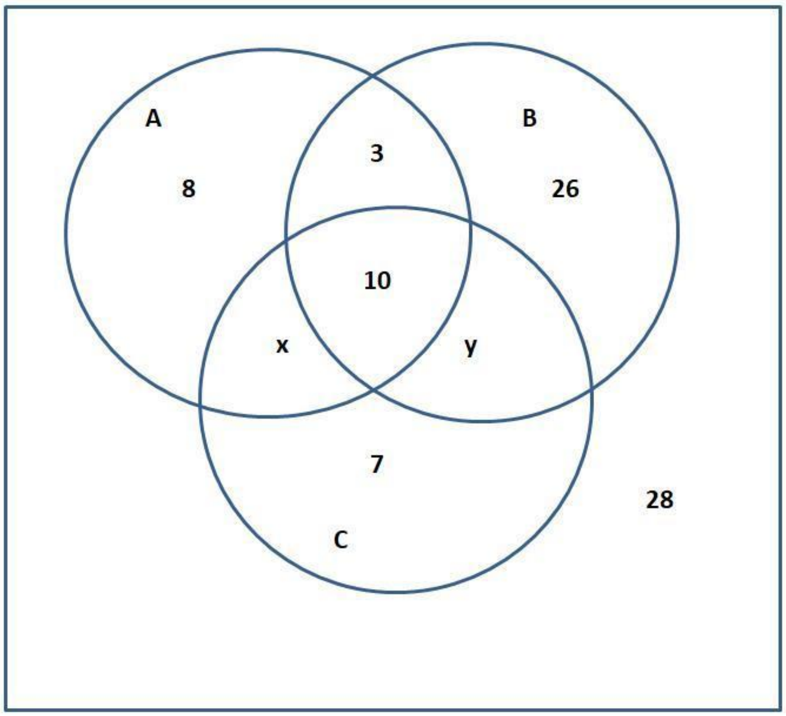
=> x + 2x + 3 = 18

=> 3x = 15

=> **x = 5**

Hence **y = 13**

C used = 7 + 10 + x + y = 17 + 18 = 35



7. **Define the following with example.**

   One to one function on to function, bijective function

   Definition 5 Page 141 Example 8, 9 Page 142, definition 7 Page 143 Example 12, 13, Definition 8 Page 144 Example 16 Page 144

8. **Let f: $R \rightarrow R$ be defined by the rule f(x) = 4x -1 for all $x \in R$ is f onto? Prove or give a counter example.**

   Assignment: Solve by using above examples and definitions

9. **Determine each of the function is bijection from R to R** **page 153Q23**

   F(x)= x^2+ 1 **f(x)= x^3**

   No, the combination completely leaves out all negative numbers and zero, not to mention, it is not one to one

   Yes, all real numbers can be cubed and equal to unique real number.

10. **Let f: $R \rightarrow R$ be defined by the rule f(x) = x^3. Show that f is bijective**

    Assignment Example 16 Page 144 to solve above Question, Above 6 Question solved as same

11. **Determine whether each of these functions is bijection from R to R** **Q22 Page 153**

    f(x)= -5x/2+4 f(x)= -3x^2+7 f(x)= (x+1)/(x^3+2)

    f(x)=x^5+1

a) $f(x) = -3x + 4$
b) $f(x) = -3x^2 + 7$
c) $f(x) = (x + 1)/(x + 2)$
d) $f(x) = x^5 + 1$

a) f (x) = -3x + 4 is a bijection
   it is one to one as f (x) = f(y) ⇒ -3x + 4 = -3y + 4 ⇒ x = y
   it is onto as $f\left(\frac{4-x}{x}\right) = x$
b) f (-x) = f(x). Hence function is not a bijection
c) there is no real number x such that f(x) = $\frac{x+1}{x+2} = 1$
d) f(x) = x5 + 1 is a bijection
   it is a strictly increasing

12. Let f be a functions from {1, 2, 3, 4} to {a, b, c, d} and g be function from {a, b, c, d} to {1,2,3,4}, respectively. Such that f(1)= d , f(2)=c, f(3)= a, f(4)= b, and g(a)= 2, g(b)=1, g(c) =3, g(d)= 2. **Answer Page 187 Q13**

Is f one to one , if g one to one **if f onto , if g on to**

does either f or g have a inverse if so find this inverse

Ans: The function is onto if and only if every element b ϵB there exist an element a ϵ A such that f (a) = b. The function f is one to one if and only if f (a) = f (b) implies that a = b for all a and b in the domain. f is one to one correspondence if and only if is one to one and onto. If fi is one to one correspondence, then f is invertible

f: {1, 2, 3, 4} →{a, b, c, d}

f(1) = d

f (2) = c

f (3) = a

f (4) = b

g: {a, b, c, d}→ {1, 2, 3, 4}

g(a) = 2

g(b) = 1

g(c) =3

g(d) = 2

a) f is one to one, because f (x) = f(y) is possibly only when x= y as no two elements in {1, 2, 3, 4} have the same image
   g is not one to one because a and d have the same image g(a) = g(d) with a≠ d.
b) f is onto, because every element in {a, b, c, d} is the image of some element in {1, 2, 3, 4}.
   G is not onto, because 4 is not the image of any element in {a, b, c, d}

c) A function is invertible if and only if the function is a one to one correspondence if and only if the function is onto and one to one
   Since f is onto and one to one f has an inverse
   Since g is not onto and not one to one, g does not have an inverse.
   The inverse function is f-1 of f is defined as $f^{-1}(x) = y$ if and only if $f(y) = x$
   $f^{-1} : \{a, b, c, d\} \to \{1, 2, 3, 4\}$
   $f^{-1}(a) = 3$
   $f^{-1}(b) = 4$
   $f^{-1}(c) = 2$
   $f^{-1}(d) = 1$

13. **Find the inverse function $H(x) = \frac{x+1}{x-1}$ inverse function from Google drive**

Let $y = \frac{x+1}{x-1}$
y (x-1) = x + 1
$yx - y = x + 1$
$yx - x = y + 1$
$x(y-1) = y + 1$
$x = \frac{y+1}{y-1}$
There for the inverse function is given by the rule $f^{-1}(y) = \frac{y+1}{y-1}$. Equivalently by the rule $f^{-1}(y) = \frac{x+1}{x-1}$ since the variable in the definition is just a dummy variable.
Definition 9 page 145 Examples 18, 19, 20 Page 146, Q69 Page 155

14. **What are the symmetric matrix?**
 Page181 Definition 7 Example 6

Chapter 3: Algorithm
Definitions Page 232

1. **Write the names of algorithm properties.**
 Page 191 and properties 193 Algorithm?
2. **What are the Properties of Algorithm**
 Page 191 and properties 193
3. **Define the correctness of all algorithms?** **Page 193 from properties**
 Algorithm should be produced correct output values for each set of input values
4. **Pseudo code?** **Page 192**
 Pseudocode is an artificial and informal language that helps programmers develop algorithms. Pseudocode is a "text-based" detail (algorithmic) design tool. The rules of Pseudocode are reasonably straightforward.

Step 1

We call the algorithm "selectionsort" and a list of natural numbers $a_1, a_2, ..., a_n$

procedure selectionsort($a_1, a_2, ...a_n$: natural numbers with $n \geq 2$)

The algorithm will require $n-1$ steps, since we don't need to determine the minimum of the remaining list if the remaining list only contains 1 element (that element will then already be ordered)..

for i:=1 **to** $n-1$

We initially define the minimum as the first element in the list (if it is not the minimum, then this value will be adjusted later in the algorithm).

min:=a_i
k:=i

For the 2nd to nth element in the list, we then compare it with the current minimum in that step. If the value is smaller, then we reassign the value to the minimum. We record the position of the minimum by k.

for $j := i+1$ to n
  if a_j <min **then**
    min:=a_j
    k:=j

The found minimum will then be place at the beginning of the remaining list.

for $m := i$ to $k-1$
  $a_{i+k-1-m} := a_{i+k-m}$
a_i:=min

Finally we return the ordered list.

return $a_1, a_2, ..., a_n$

Step 2

Combining all these steps, we then obtain the algorithm:

procedure selectionsort($a_1, a_2, ...a_n$: natural numbers with $n \geq 2$)
for i:=1 **to** $n-1$
  min:=a_i
  k:=i
  for $j := i+1$ to n
    if a_j <min **then**
      min:=a_j
      k:=j
  for $m := i$ to $k-1$
    $a_{i+k-1-m} := a_{i+k-m}$
  a_i:=min
return $a_1, a_2, ..., a_n$

5. **Define Greedy Algorithm with Example? OR Define Greedy algorithm?**
Page 198, example 6 Page 199
Greedy algorithm that is used in optimization problems. The algorithm makes the optimal choice at each step as it attempts to find the overall optimal way to solve the entire problem. However, in many problems, a greedy strategy does not produce an optimal solution.
6. **Define the term halting a problem** **Page 201**
Unsolvable algorithmic problem is the halting problem, which states that no program can be written that can predict whether or not any other program halts after a finite number of steps. .Condition known as the "halting problem.
It asks whether there is a procedure that does this. It takes as input a computer and input to the program and determine whether the program will eventually stops to run with this input.

It would be convenient to have such procedure, if existed. Certainly being able to test whether a program entered into a finite loop would be helpful when written and debugging programs. However 1936 Alan Turing showed that no such procedure exists.

7. **What is average case complexity of linear search algorithm?**
   Example 4 Page 221
8. **What is the worst complexity of bubble sort?**
   Page 221 example 5, Last two lines Page 222
9. What is the space complexity of linear search algorithm?
   The amount of space in computer memory required for an algorithm to solve a problem
10. **State division algorithm.**
    Definition 2 and Theorem 2 Page 239
11. **What is the use of best case and worst case complexity of algorithm?**
    The worst-case complexity of an algorithm is the greatest number of operations needed to solve the problem over all inputs of size n. The best-case complexity of an algorithm is the least number of operations needed to solve the problem over all inputs of size n

Chapter 3: Algorithm Long Question
Definition Page 232

1. **Define the following terms**

   Growth of function

   Topic Page 204 (Chapter 3 Algorithm Growth of function.pdf file in Google Drive)

   Complexities of Algorithm **Topic Page 218**

   Given an algorithm, how efficient is this algorithm for solving a problem given input of a particular size?

   - How much time does this algorithm use to solve a problem?
   - How much computer memory does this algorithm use to solve a problem?

   We measure time complexity in terms of the number of operations an algorithm uses and use big-O and big-Theta notation to estimate the time complexity.

   Compare the efficiency of different algorithms for the same problem. We focus on the worst-case time complexity of an algorithm (Derive an upper bound on the number of operations an algorithm uses to solve a problem with input of a particular size)

   Here: Ignore implementation details and hardware properties.

   Asymptotic notations:

   Asymptotic notations are mostly used in computer science to describe the asymptotic running time of an algorithm. As an example, an algorithm that takes an array of size n as input and runs for time proportional to n2 is said to take O(n2) time.

   Asymptotic running time: The limiting behavior of the execution time of an algorithm when the size of the problem goes to infinity. This is usually denoted in big-O notation.

2. **Define and Explain Big –O, Big – O Omega and Big Theta Notations.**

Big O Represent Upper Bound Mean Bigger value

Page 205 advantages of big O

Asymptotic Notations - Theta, Big O and Omega.pdf BSCS folder of discrete Mathematics

Asymptotic Notations file print and given to Students

3. **Show that $2x^3 + 3x^2 + 1$ is $O(x^3)$ and Use O notation to prove that $10x^3 + x^2 - 5x + 6$ is $O(x^3)$**

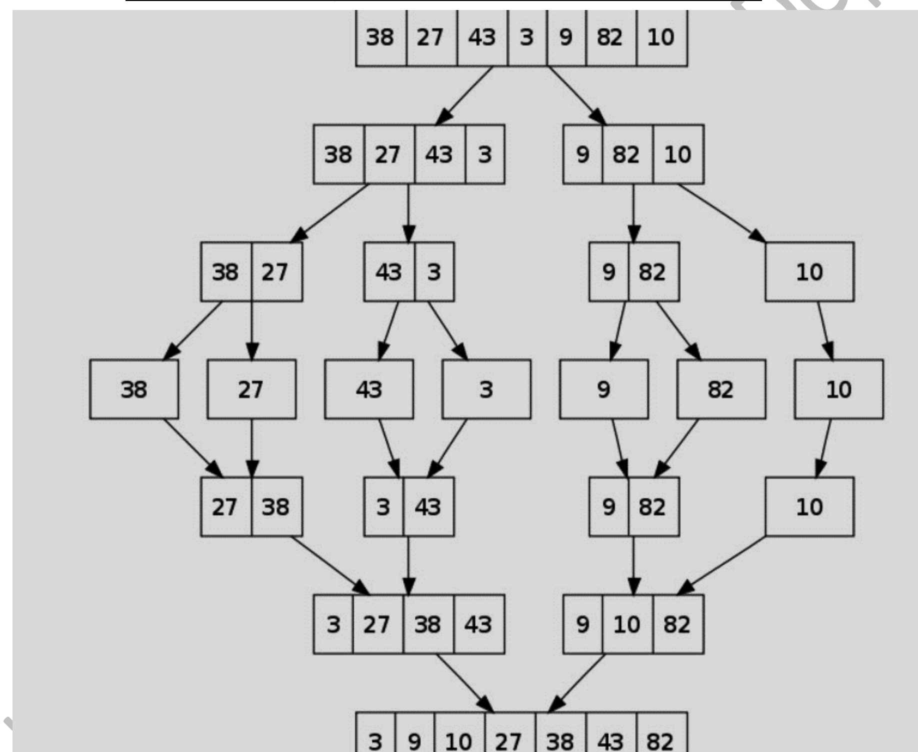
   For understanding Example 1 page 206, Example 2, 3, 4 Page 208

   https://www.youtube.com/watch?v=BPTCK9J3qPw important link for Big O

   https://www.youtube.com/watch?v=7dz8Iaf_weM

   Important link for Graph in Asymptotic Notations

4. **Use divide and conquer algorithm to put 6, 1, 2, 5, -7, 23, 11, 12, 4, and 3 into decreasing order?** Use below Graph to Sort Above List of Values



   https://www.geeksforgeeks.org/divide-and-conquer/

   For proper understanding divide.ppt from Google drive discrete folder

5. **Use divide and conquer algorithm to put { 1, 6, 15, 9, -1, 23, 11, 2, 41, 13} into decreasing order.**

   https://www.geeksforgeeks.org/divide-and-conquer/

6. **Use the insertion sort to sort 3, 1, 5, 5, 7, 4 in decreasing order showing the lists obtained at each step** Solve this Question as Assignment by Understanding Procedure below.

   Procedure Given below Sort above using insertion sort

Use the insertion sort to put 3, 2, 4, 1, 5 into increasing order

- Insert the 2nd element 2 in the right position:
  - $3 > 2 \Rightarrow$ put 2 in front of 3. $\Rightarrow$ 2, 3, 4, 1, 5
- Insert the 3rd element 4 in the right position:
  - $4 > 2 \Rightarrow$ do nothing. Move to the next comparison.
  - $4 > 3 \Rightarrow$ do nothing. Done. $\Rightarrow$ 2, 3, 4, 1, 5
- Insert the 4th element 1 in the right position:
  - $2 > 1 \Rightarrow$ put 1 in front of 2. $\Rightarrow$ 1, 2, 3, 4, 5
- Insert the 5th element 5 in the right position:
  - $5 > 1 \Rightarrow$ do nothing. Move to the next comparison.
  - $5 > 2 \Rightarrow$ do nothing. Move to the next comparison.
  - $5 > 3 \Rightarrow$ do nothing. Move to the next comparison.
  - $5 > 4 \Rightarrow$ do nothing. Done. $\Rightarrow$ 1, 2, 3, 4, 5

7. Devise an algorithm that finds the closest pair of integers in a sequence of n integers and determine the worst case complexity of your algorithm Q10 Page 233

Step 1

Let us call the algorithm "closest pair" and the input of the algorithm is a list of integers $a_1, a_2, ..., a_n$.

procedure closest pair($a_1, a_2, ...a_n$: integers with $n \geq 2$)

We first sort of the list of integers in increasing order (for example, you could use insertion sort of bubble sort).

Sort the list of integers in increasing order.

The closest pair is the pair of values with the smallest difference. Since the list is sorted in increasing order, the smallest difference will occur between two consecutive terms in the list. Note that any element also has to be larger than or equal to the previous term.
The variable "diff" will keep track of the smallest difference. Initially we define the smallest difference as the difference between the two first terms.
The variable "location" will keep track of the locations of the closest pairs.

diff:$=a_2 - a_1$
location:$=\{1, 2\}$

Next we compare the difference between all other two consecutive elements with the current smallest difference "diff". If the difference is smaller than "diff", then "diff" is changed to the value of the smallest difference and "location" is changed to the location of the current pair.

for i:=3 **to** n
  if $a_i - a_{i-1}$ <diff **then**
    diff:=$a_i - a_{i-1}$
    location:=$\{i-1, i\}$

Finally, we return the location of the closest pair.

return location

Step 2

Combining these steps, we then obtain the algorithm:

procedure closest pair($a_1, a_2, ...a_n$: integers with $n \geq 2$)
Sort the list of integers in increasing order.
diff:=$a_2 - a_1$
location:=$\{1, 2\}$
for i:=3 **to** n
  if $a_i - a_{i-1}$ <diff **then**
    diff:=$a_i - a_{i-1}$
    location:=$\{i-1, i\}$
return location

Step 3

Worst-case time complexity

Given: Sorting the elements has a worst-case time complexity of $O(n \log n)$.

The algorithm only makes a comparison when "$a_i - a_{i-1}$ <diff" is executed, which each occurs once in every iteration of the **for**-loop and thus there is 1 comparison in every iteration of the **for**-loop.

Since i can take on values from 3 to n, there are $n-2$ iterations of the **for**-loop and thus $(n-2)$ comparisons are made, while $n-2$ is $O(n)$.

Combining the worst-case time complexity of sorting and the remaining algorithm, we then obtain $O(n \log n + n) = O(n \log n)$ as the sorting and the remaining algorithm aren't nested.

8. Describe an algorithm based on the linear search for determining the correct position in which to insert a new element in an already sorted list

Answer Q 43 page 203   Assignment Q44, 45, 46 Page 203

We carry out the linear search algorithm given as Algorithm 2(Page 194) in this section, except that we replace $x \neq a_i$ by x < ai, and we replace the else clause with else location: = n + 1

9. **Find the least integer n (Only d Part is in Paper 2019) Answer Q7 Page 216**
   Q8 Page 216 Is Assignment

Find the least integer n such that $f(x)$ is $O(x^n)$ for each of these functions.

a) $f(x) = 2x^3 + x^2 \log x$
b) $f(x) = 3x^3 + (\log x)^4$
c) $f(x) = (x^4 + x^2 + 1)/(x^3 + 1)$
d) $f(x) = (x^4 + 5 \log x)/(x^4 + 1)$

The choice C and K is not unique

a) **n = 3, C= 3 and k = 1**
b) **n= 3, C= 4 and k = 1**
c) **n =1, C= 2, and k = 1**
d) **n = 0, C =2 , k =1**

a) Since $\log x$ grows more slowly than x, $x^2 \log x$ grows more slowly than x^3, so the first term dominates. Therefore this function is $O(x^3)$ but not $O(x^n)$ for any $n < 3$. More precisely, $2x^3 + x^2 \log x \leq 2x^3 + x^3 = 3x^3$ for all x, so we have witnesses $C = 3$ and $k = 0$.

b) We know that $\log x$ grows so much more slowly than x that *every* power of $\log x$ grows more slowly than x (see Exercise 58). Thus the first term dominates, and the best estimate is $O(x^3)$. More precisely, $(\log x)^4 < x^3$ for all $x > 1$, so $3x^3 + (\log x)^4 \leq 3x^3 + x^3 = 4x^3$ for all x, so we have witnesses $C = 4$ and $k = 1$.

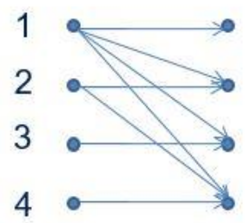
c) By long division, we see that $f(x) = x +$ lower order terms. Therefore this function is $O(x)$, so $n = 1$. In fact, $f(x) = x + \frac{1}{x+1} \leq 2x$ for all $x > 1$, so the witnesses can be taken to be $C = 2$ and $k = 1$.

d) Again by long division, this quotient has the form $f(x) = 1 +$ lower order terms. Therefore this function is $O(1)$. In other words, $n = 0$. Since $5 \log x < x^4$ for $x > 1$, we have $f(x) \leq 2x^4/x^4 = 2$, so we can take as witnesses $C = 2$ and $k = 1$.

Chapter 4: Number theory and cryptography

1. **Let A = {1, 2, 3, 4, 5}, and R is a relation defined by "a divides b". Write R as a set of ordered pair, draw directed graph**
   Practice: Definition 1, example 1, 2 Page 238
   Order Pair: An ordered pair is a composition of the x coordinate (abscissa) and the y coordinate (ordinate), having two values written in a fixed order within parentheses. It helps to locate a point on the Cartesian plane for better visual comprehension. The numeric values in an ordered pair can be integers or fractions.

R={(1,1),(1,2),(1,3),(1,4),(2,2),(2,4),(3,3),(4,4)}



R	1	2	3	4
1	X	X	X	X
2		X		X
3			X	
4				X

2. **What is the decimal expansion of the number with hexadecimal expansion $(2AE0B)_{16}$**

   page 247 Example 3

Solution: Using the definition of a base b expansion with $b = 16$ tells us that

$$(2AE0B)_{16} = 2 \cdot 16^4 + 10 \cdot 16^3 + 14 \cdot 16^2 + 0 \cdot 16 + 11 = 175627.$$

Each hexadecimal digit can be represented using four bits. For instance, we see that $(1110\ 0101)_2 = (E5)_{16}$ because $(1110)_2 = (E)_{16}$ and $(0101)_2 = (5)_{16}$. **Bytes**, which are bit strings of length eight, can be represented by two hexadecimal digits.

<u>**Same as in Old course of BA/ B.Sc**</u>

3. **Convert the following Hexadecimal Expansion $(80E)_{16}$ integers to binary expansion**

Convert the hexadecimal expansion of each of these integers to a binary expansion.

a) $(80E)_{16}$ **b)** $(135AB)_{16}$
c) $(ABBA)_{16}$ **d)** $(DEFACED)_{16}$

(a) $(1000\ 0000\ 1110)_2$
(b) $(1\ 0011\ 0101\ 1010\ 1011)_2$
(c) $(1010\ 1011\ 1011\ 1010)_2$
(d) $(1101\ 1110\ 1111\ 1010\ 1100\ 1110\ 1101)_2$

4. **Convert the octal expansion of $(1604)_8$ integers to binary expansion Q5 Part c Page 255**

   Remaining Parts is Assignment

Convert the binary expansion of each of these integers to an octal expansion.

a) $(1111\ 0111)_2$
b) $(1010\ 1010\ 1010)_2$
c) $(111\ 0111\ 0111\ 0111)_2$
d) $(101\ 0101\ 0101\ 0101)_2$

(a) $(367)_8$
(b) $(5252)_8$
(c) $(73567)_8$
(d) $(52525)_8$

Convert the octal expansion of each of these integers to a binary expansion.

a) $(572)_8$ **b)** $(1604)_8$
c) $(423)_8$ **d)** $(2417)_8$

(a) $(1\ 0111\ 1010)_2$
(b) $(11\ 1000\ 0100)_2$
(c) $(1\ 0001\ 0011)_2$
(d) $(101\ 0000\ 1111)_2$

<u>**Remaining Conversions You will learn in DLD**</u>

5. **Determine whether integers 10, 17 and 21 are pair wise relatively prime?**

   <u>265 Example 13</u>

   Integers a and b are relatively prime if their greatest common divisor is 1

   Answer: Yes, pair wise relatively prime

   Example 12 Page 265 Practice

6. **Encrypt the message WATCH YOUR STEP by translating the letters into numbers applying the given encryption function then translate them back in to letter, f(p)= (14p+ 21) mod 26 In each case, we translate the letters to numbers from 0 to 25, then apply the function, then translate back.** **<u>Q3 Page 304</u>**

   a) $f(p) = (p + 14)$ **mod** 26
   b) $f(p) = (14p + 21)$ **mod** 26
   c) $f(p) = (-7p + 1)$ **mod** 26

   In each case, the numerical message is 22-0-19-2-7 24-14-20-17 18-19-4-15.

   a) Adding 14 to each number modulo 26 yields 10-14-7-16-21 12-2-8-5 6-7-18-3. Translating back into letters yields KOHQV MCIF GHSD.

   b) Multiplying each number by 14, adding 21, and reducing modulo 26 yields 17-21-1-23-15 19-9-15-25 13-1-25-23. Translating back into letters yields RVBXP TJPZ NBZX.

   c) Multiplying each number by - 7, adding 1, and reducing modulo 26 yields 3-1-24-13-4 15-7-17-12 5-24-25-0. Translating back into letters yields DBYNE PHRM FYZA.

   Q2 Page 304 Assignment

<u>Chapter 4: Number theory and cryptography Long Questions</u>

1. **What are the quotient and remainder when -11 is dividend by 3** **<u>page 240 example 4</u>**

   <u>Page 239 Example 3 Practice, Definition 2 Page 239</u>

2. **Find Quotient & Remainder of 101 is divided by 11?**

   Example 3 Page 239

3. **Find each of these values. $(99^2$ mod32$)^3$ mod 15 $(34$ mod $17)^2$ mod 11 $(193$ mod $23)^2$ mod 31 $(893$ mod $79)^4$ mod 26** **<u>page 245 Q33</u>**

   a) $(99^2$ **mod** $32)^3$ **mod** 15 = $(3^2$ **mod** $32)^3$ **mod** 15 = 9^3 **mod** 15 = 729 **mod** 15 = 9

   b) $(3^4$ **mod** $17)^2$ **mod** 11 = $(81$ **mod** $17)^2$ **mod** 11 = 13^2 **mod** 11 = 2^2 **mod** 11 = 4

   c) $(19^3$ **mod** $23)^2$ **mod** 31 = $((-4)^3$ **mod** $23)^2$ **mod** 31 = $(-64$ **mod** $23)^2$ **mod** 31 = 5^2 **mod** 31 = 25

   d) $(89^3$ **mod** $79)^4$ **mod** 26 = $(10^3$ **mod** $79)^4$ **mod** 26 = $(1000$ **mod** $79)^4$ **mod** 26 = 52^4 **mod** 26= 0^4 **mod** 26 = 0

Find each of these values.
a) $(19^2$ **mod** $41)$ **mod** 9
b) $(32^3$ **mod** $13)^2$ **mod** 11
c) $(7^3$ **mod** $23)^2$ **mod** 31
d) $(21^2$ **mod** $15)^3$ **mod** 22

99 mod 32 give us 3 because 32 x 3 = 96 remainder is 3

Q32 page 245

(a)

$$(19^2 \textbf{ mod } 41) \textbf{ mod } 9$$

First evaluate the square:

$$= (361 \textbf{ mod } 41) \textbf{ mod } 9$$

Let us next determine 361 **mod** 41

$$a = 361 = 328 + 33 = 8 \cdot 41 + 33 = 8d + 33$$

The remainder is the constant in the final expression: 361 **mod** 41 = 33

$$= 33 \textbf{ mod } 9$$
$$= 6$$

(since $a = 33 = 27 + 6 = 3 \cdot 9 + 6 = 3d + 6$).

(b)

$$(32^3 \textbf{ mod } 13)^2 \textbf{ mod } 11$$

Use theorem 5:

$$= ((32 \textbf{ mod } 13)^3 \textbf{ mod } 13)^2 \textbf{ mod } 11$$

Let us next determine 32 **mod** 13

$$a = 32 = 26 + 6 = 2 \cdot 13 + 6 = 2d + 6$$

The remainder is the constant in the final expression: 32 **mod** 13 = 6

$$= (6^3 \textbf{ mod } 13)^2 \textbf{ mod } 11$$
$$= (216 \textbf{ mod } 13)^2 \textbf{ mod } 11$$

Let us next determine 216 **mod** 13

$$a = 216 = 208 + 8 = 16 \cdot 13 + 8 = 16d + 8$$

The remainder is the constant in the final expression: 213 **mod** 13 = 8

$$= 8^2 \textbf{ mod } 11$$
$$= 64 \textbf{ mod } 11$$
$$= 9$$

(since $64 = 55 + 9 = 11 \cdot 5 + 6 = 11d + 6$).

(c)

$$(7^3 \textbf{ mod } 23)^2 \textbf{ mod } 31$$

First evaluate the cube:

$$= (343 \textbf{ mod } 23)^2 \textbf{ mod } 31$$

Let us next determine 343 **mod** 23

$$a = 343 = 322 + 21 = 14 \cdot 23 + 21 = 14d + 21$$

The remainder is the constant in the final expression: $343 \textbf{ mod } 23 = 21$

$$= 21^2 \textbf{ mod } 31$$

$$= 441 \textbf{ mod } 31$$

$$= 7$$

(since $a = 441 = 434 + 7 = 14 \cdot 31 + 7 = 14d + 7$).

(d)

$$(21^2 \textbf{ mod } 15)^3 \textbf{ mod } 22$$

Use theorem 5:

$$= ((21 \textbf{ mod } 15)^2 \textbf{ mod } 15)^3 \textbf{ mod } 22$$

Let us next determine 21 **mod** 15

$$a = 21 = 15 + 6 = 1 \cdot 15 + 6 = 1d + 6$$

The remainder is the constant in the final expression: 21 **mod** 15 = 6

$$= (6^2 \textbf{ mod } 15)^3 \textbf{ mod } 22$$
$$= (36 \textbf{ mod } 15)^3 \textbf{ mod } 22$$

Let us next determine 36 **mod** 15

$$a = 36 = 30 + 6 = 2 \cdot 15 + 6 = 2d + 6$$

The remainder is the constant in the final expression: 36 **mod** 15 = 6

$$= 6^3 \textbf{ mod } 22$$
$$= 216 \textbf{ mod } 22$$
$$= 18$$

(since $216 = 198 + 18 = 9 \cdot 22 + 18 = 9d + 18$).

4. **What is the GCD of 45 and 60**
   Page 265 definition 2 Example 10, 11 Page 265
5. **Encrypt the message UPLOAD using RSA System with n=53 . 61 and e=17.** **PAGE305 Q25**
   Topic 299 and Example 8 Page 300 Before solving above Question

   Encrypt the message ATTACK using RSA system with n=43. 59 and e = 13. Translating each letter into integers and grouping together pairs of integers as done in Example 8

Given text: ATTACK

$$n = 43 \cdot 59 = 2537$$

$$e = 13$$

Let A=00, B=01, C=02, D=03, E=04, F=05, G=06, H=07, I=08, J=09, K=10, L=11, M=12, N=13, O=14, P=15, Q=16, R=17, S=18, T=19, U=20, V=21, W=22, X=23, Y=24, Z=25

00 19 19 00 02 10

We then group the numbers in blocks of four digits (since 2525 < 2537< 252525).

0019 1900 0210

Encrypt each block using the mapping $C = M^{13} \textbf{ mod } 2537$

$$C_1 = 0019^{13} \textbf{ mod } 2537 = 2299$$

$$C_2 = 1900^{13} \textbf{ mod } 2537 = 1317$$

$$C_3 = 0210^{13} \textbf{ mod } 2537 = 2117$$

The encryption is then:

2299 1317 2117

6. **Find the prime factorization of 10! Q5 Page 272 Practice Example 2, 3 Page 258. Example 14 Page 266, Assignment Page 272 Q1, 2, 3, 4, 6**

By the definition of factorial:

$$\begin{aligned}10! &= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\ &= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2\end{aligned}$$

2, 3, 5 and 7 are primes. Let us determine the prime factorization of all other factors.

To determine the prime factorization, we divide the given integer by successive primes. If the remainder is zero, then the prime is part of the factorization and we continue the prime factorization with the quotient.

$$\begin{aligned}10 &= 2 \cdot 5 \\ 9 &= 3 \cdot 3 \\ 8 &= 2 \cdot 2 \cdot 2 \\ 6 &= 2 \cdot 3 \\ 4 &= 2 \cdot 2\end{aligned}$$

We use these prime factorization in the above found product:

$$\begin{aligned}10! &= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \\ &= (2 \cdot 5) \cdot (3 \cdot 3) \cdot (2 \cdot 2 \cdot 2) \cdot 7 \cdot (2 \cdot 3) \cdot 5 \cdot (2 \cdot 2) \cdot 3 \cdot 2\end{aligned}$$

Finally combine like terms:

$$10! = 2^8 \cdot 3^4 \cdot 5^2 \cdot 7$$

Chapter 5: Induction and recursion

1. **Monotonic function**

   Chapter 2 Function in Mathematics.pdf Slide 21 to 26

   A monotonic function is a function that is only either decreasing or increasing from negative Infinity to infinity. A function is said to be monotone function, if it is either increasing or Decreasing.

   A function y = g(x) is called monotonic on the interval I if it is decreasing, increasing or constant

2. **Recursive algorithm? Page 360**

   An algorithm is recursive if it solves a problem by reducing it to an instant of the same problem with smaller input.

   Example 1 page 360

3. Write down the base case and recursive case for sum of array elements. Also solve for array of length 5 via tree convention.

Chapter 5: Induction and recursion Long Questions

1. **Define principle of mathematical induction using quantifier Topic Page 313 Answer Page 377 definitions**

The Principle of Mathematical induction: The statement $\forall n p(n)$ is true if P(1) is true $\forall k[P(k) \to P(k+1)]$ true

Basis step: the proof of P(1) in a proof by mathematical induction of $\forall n p(n)$

Inductive step: the proof of $P(k) \to P(k+1)$ for all positive integers k in a proof by mathematical induction of $\forall n p(n)$

2. **Differentiate b/w mathematical induction and strong induction? OR What is strong induction Topic Page 312,334 Weak induction Assignment**

Strong induction is a proof technique that is a slight variation on mathematical (regular) induction. Just like **regular induction**, have to prove base case and inductive step, but inductive step is slightly different

Regular induction: assume P(k) holds and prove P(k + 1)

Strong induction: assume P(1),P(2), ..,P(k); prove P(k + 1)

<u>Regular induction and strong induction are equivalent, but strong induction can sometimes make proofs easier</u>

Definition of Strong induction: the statement $\forall n p(n)$ is true if P(1) is true and $\forall k[P(1) \wedge \dots \dots \wedge\to P(k+1)]$ is true

<u>Example 1 Page 335</u>

3. **Prove by induction method 2^{2n}-1 is divisible by 3 for all $n \geq 1$**

Basis step: For n=1

$2^{2\cdot(1)}-1=2^2-1=4-1=3$. 3 is a multiple of 3. So it's true for n=1 , 3 is divisible by 3

We must prove that if you know it's true for integer n=k, then it will be true for n=k+1. Assume it's true for n=k.

So inductive step:

$P(k+1) = 2^{2(k+1)}-1$

$=2^{2k+2} - 1$

$= 2^{2k}. 2^2 -1$ using formula $a^{m+n} = a^m . a^n$

$= 2^{2k}. 4 -1$

$=2^{2k}. (3+1) -1$

$= 3.2^{2k} + 2^{2k} - 1$

$= 3.2^{2k}$ is divisible by 3 and $2^{2k} - 1$ is divisible by 3 for above basis step

$P(k+1) = 2^{2(k+1)}-1$ whole term is divisible by 3

Therefore since the theorem is true for n=k=1, it's true for n=k+1=2.

And since the theorem is true for n=k=2, it's true for n=k+1=3

Thus since the theorem is true for n=k=3, it's true for n=k+1=4

<u>Important Link</u>

https://www.doubtnut.com/question-answer/if-22n-1-is-divisible-by-3-then-show-that-22n-1-1-is-also-divisible-by-3-121711720

4. **What is proving by mathematical induction? Use mathematical induction to prove that k^3 – k is divisible by 3 whenever k is positive integer?**

Page 311 Definition Page 313 and Example 8 Page 321

Is the integer number 0 divisible by 3? 0 is divisible by 3. 3 | 0. Zero is divisible by any integer number.

5. **Use mathematical induction to prove that 1 + 3 + 5 + …………. + (2n -1) = n^2 for all integers $n \geq 1$ Page 317 Top of Page**

Let P (n): 1 + 3 + 5 + ... + (2n - 1) = n^2 be the given statement

Step 1: Basis Step: Put n = 1

Then, L.H.S = (2n - 1) = (2(1) - 1) = (2 - 1) = 1

R.H.S = $(1)^2$ = 1

∴. L.H.S = R.H.S .

⇒ P(n) is true for n = 1

Inductive Step: **Step 2:** Assume that P(n) is true for n = k.

∴ 1 + 3 + 5 + ... + (2k - 1) = k^2

Adding 2k + 1 on both sides, we get

1 + 3 + 5 . . . . . + (2k - 1) + (2k + 1) = k^2 + (2k + 1) = $(k + 1)^2$

OR

1 + 3 + 5 . . . . . + (2k - 1) + (2k + 1) = k^2 + 2k + 1 = $(k + 1)^2$

Using formula $(a + b)^2 = a^2 + b^2 + 2ab$

∴ 1 + 3 + 5 + ... + (2k -1) + (2(k + 1) - 1) = $(k + 1)^2$ =(2(k + 1) - 1) = 2k + 2 -1 = 2k + 1

⇒ P (n) is true for n = k + 1.

∴ by the principle of mathematical induction P (n) is true for all natural numbers 'n'

Hence, 1 + 3 + 5 + ... + (2n - 1) =n^2, for all n ϵ n

Example 3 Assignment page 318

6. **Use mathematical induction prove that for any natural numbers n, n>0, n>=0**
1^2+ 2^2+……………….+ n^2= n (n+1) (2n+1)/6

Q3 Exercise page 329

a) **What is the statement P (1)?**
If n = 1 we have P (1) is the statement 1^2 = 1. 2. 3/6
b) **Show that P(1) is true, completing the basis step of the proof**
Both sides of P (1) how in part (a) equal to 1
c) **What is the inductive hypothesis?**
Inductive hypothesis is the statement that
$1^2 + 2^2 + \ldots\ldots\ldots + k^2 + (k + 1)^2 = \frac{k(k+1)(2k+1)(2k+3)}{6}$
d) **What do you need to prove in the inductive step?**
For the inductive step, we want to show for each k≥ 1 that p(k) implies p(k+1). In other words, we want to show that assuming the inductive hypothesis (see part c) we can show
$1^2 + 2^2 + \ldots\ldots\ldots + k^2 = \frac{k(k+1)(2k+1)}{6}$
e) **Complete the inductive step, identifying where you use the inductive hypnosis?**
The left hand side of the equation in part (d) is equals, by the inductive hypothesis, k (k+1) (2k + 1)/ 6 + $(k+1)^2$. We need only do a bit of algebraic manipulation to get this expression inti the desired form: factor out (k+1)/6 and tem factor the rest in detail.
$(1^2 + 2^2 + \ldots\ldots\ldots + k^2) + (k + 1)^2 = \frac{k(k+1)(2k+1)}{6} + (k + 1)^2$ by the inductive hypothesis

$=\frac{(k+1)}{6}\ (k\ (2k+1)+6(k+1)$

$=\frac{(k+1)}{6}\ (2k^2+7k+6)$

$=\frac{(k+1)}{6}\ (k+2)(2k+3)$ after factorization of above equation

$=\frac{(k+1)(k+2)(2k+3)}{6}$

Therefore by the principle of mathematical induction the give statement is true for any positive integer n

Inductive.pdf Page 1 in Google Drive

f) Explain why these steps show that this formula is true whenever n is positive integer

We have completed both the basis step and the inductive step, so by the principle of mathematical induction the statement is true for every positive integer n.

7. Give a Recursive definition of the sequence a_n= 4n -2 , n=1, 2, 3…………

Topic Page 345, Example 1 Q8 page 358

a) a_n = 4n-2
b) a_n = 1 + $(-1)^n$
c) a_n = n (n + 1)
d) a_n = n^2

an = 4n – 2 a2 = 4(2) – 2 a2 = 8 - 2 a2 = 6 **I can see they go up by 4 for part a, but do I leave it like this?**	**If n is 2 and if n is 4** an = 4n – 2 a4 = 4(4) – 2 a4 = 16 - 2 a4 = 14 So, we add 4 in equation

https://mathhelpforum.com/threads/give-a-recursive-definition.80196/

Link for understanding above calculations

(a) Given:

$$a_n = 4n - 2$$

Let us first determine the first value at $n = 1$:

$$a_1 = 4(1) - 2 = 4 - 2 = 2$$

Next we determine the recursive definition by writing a_n in terms of a_{n-1}.

$$a_n = 4n - 2 = 4n - 4 + 4 - 2 = [4(n-1) - 2] + 4 = a_{n-1} + 4$$

Thus the recursive definition is then:

$$a_1 = 2$$

$$a_n = a_{n-1} + 4 \text{ when } n \geq 2$$

(b) Given:

$$a_n = 1 + (-1)^n$$

Let us first determine the first two values at $n = 1$ and $n = 2$:

$$a_1 = 1 + (-1)^1 = 1 - 1 = 0$$

$$a_2 = 1 + (-1)^2 = 1 + 1 = 2$$

Next we determine the recursive definition by writing a_n in terms of a_{n-2}.

$$a_n = 1 + (-1)^n = 1 + (-1)^2 \cdot (-1)^{n-2} = 1 + 1 \cdot (-1)^{n-2} = 1 + (-1)^{n-2} = a_{n-2}$$

Thus the recursive definition is then:

$$a_1 = 0$$

$$a_2 = 2$$

$$a_n = a_{n-2} \text{ when } n \geq 3$$

(c) Given:

$$a_n = n(n+1)$$

Let us first determine the first value at $n = 1$:

$$a_1 = 1(1+1) = 1(2) = 2$$

Next we determine the recursive definition by writing a_n in terms of a_{n-1}.

$$a_n = n(n+1) = n^2 + n = n^2 - n + n + n = n(n-1) + n + n = (n-1)n + 2n = a_{n-1} + 2n$$

Thus the recursive definition is then:

$$a_1 = 2$$

$$a_n = a_{n-1} + 2n \text{ when } n \geq 2$$

(d) Given:

$$a_n = n^2$$

Let us first determine the first value at $n = 1$:

$$a_1 = 1^2 = 1$$

Next we determine the recursive definition by writing a_n in terms of a_{n-1}.

$$a_n = n^2 = n^2 - 2n + 2n - 1 + 1 = (n^2 - 2n + 1) + 2n - 1 = (n-1)^2 + 2n - 1 = a_{n-1} + 2n - 1$$

Thus the recursive definition is then:

$$a_1 = 1$$

$$a_n = a_{n-1} + 2n - 1 \text{ when } n \geq 2$$

Q7 Assignment Page 358

8. Give A Recursive definition of the Fibonacci number

Page 347 before Example 4 definition 5 Page 158

The Fibonacci sequence or number, f_0, f_1, f_2……………, is defined by the initial conditions or Equations $f_0 = 0$, $f_1 = 1$ and recursive relation

$F_n = f_n -1 + f_n -2$

For n=2, 3, 4, …………..

$$f_n = f_{n-1} + f_{n-2}$$

for $n = 2, 3, 4, \ldots$. [We can think of the Fibonacci number f_n either as the nth term of the sequence of Fibonacci numbers $f_0, f_1, \ldots$ or as the value at the integer n of a function $f(n)$.]

We can use the recursive definition of the Fibonacci numbers to prove many properties of these numbers. We give one such property in Example 4.

9. **Show that whenever $n \geq 3, f_n > a^{n-2}$, Where f_n is nth Fibonacci number and $a = (1 + \sqrt{5})/2$.Answer Q7 Page 378**
   Answer: Example 3 Page 346, Example 4 Page 347

Chapter 6: Counting

Definitions 439 Page

1. **Differentiate b/w sum and product rule with the help of an example?**
   Topic: Page 386 and Example 1 to 5 Page 386, 387 and Topic Page 389 Example 12, 13
2. **In certain countries car number plates is formed by four digits from the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 followed by 3 letters from the alphabet. How many number plates can be formed if neither the digits nor the letters are repeated? (Discuss the Imran Sab)**
   Answer: There are 9^4*26^3 = 115316136 different plates possible.
   OR
   ${}_9P_4 \times {}_{26}P_3 = 47,174,400$
   Practice Example 5 Page 387
3. **How many bits strings of the length eight either start with 0 bit or end with two bits 11?**
   Page 393 Example 18, Page 393 Example 19 Practice At Home
   How many bits strings of length 7 either begin with 2 zeros or end with three 1s.
   If the string start with two 0's then 7 – 2 = 5. The possible combinations will be $2^5 = 32$,
   If the string start with three 1's then 7 – 3 = 4. The possible combinations will be $2^4 = 16$,
   Total combination of bit string = 32 + 16 = 48
4. **How many positive integers less than 100**

Division rule If a finite set A is the union of pairwise disjoint subsets with d elements each, then $n = \frac{|A|}{d}$

Product rule If one event can occur in m ways AND a second event can occur in n ways, the number of ways the two events can occur in sequence is then $m \cdot n$.

Subtraction rule If an event can occur either in m ways OR in n ways (overlapping), the number of ways the event can occur is then $m + n$ decreased by the number of ways that the event can occur commonly to the two different ways.

Sum rule If an event can occur either in m ways OR in n ways (non-overlapping), the number of ways the event can occur is then $m + n$.

   a) **Are divisible by 7**

(a) Let A be the positive integers less than 1000.

We then note that A contains 999 integers, while we are interested in integers divisible by 7.

$$|A| = 999$$

$$d = 7$$

Using the quotient rule (round down!):

$$n_7 = \frac{|A|}{d} = \frac{999}{7} \approx 142.7143 \approx 142$$

Thus 142 integers are divisible by 7.

b) Are divisible by 7 but not by 11

Number of integers divisible by 7 and by 11

Let A be the positive integers less than 1000.

We then note that A contains 999 integers, while we are interested in integers divisible by 7 and divisible by 11, thus divisible by $7 \cdot 11 = 77$.

$$|A| = 999$$

$$d = 77$$

Using the quotient rule (round down!):

$$n_{77} = \frac{|A|}{d} = \frac{999}{77} \approx 12.9740 \approx 12$$

Thus 12 integers are divisible by 77.

Number of integers divisible by 7 but not by 11

Integers divisible by 7 but not by 11 are integers divisible by 7 that are not integers that are divisible by 7 and 11.

$$n_{7,\text{ not }11} = n_7 - n_{11} = 142 - 12 = 130$$

c) Are divisible of 7 and 11?

(c) Let A be the positive integers less than 1000.

We then note that A contains 999 integers, while we are interested in integers divisible by 7 and divisible by 11, thus divisible by $7 \cdot 11 = 77$.

$$|A| = 999$$

$$d = 77$$

Using the quotient rule (round down!):

$$n_{77} = \frac{|A|}{d} = \frac{999}{77} \approx 12.9740 \approx 12$$

Thus 12 integers are divisible by 77.

d) Are divisible by 7 or 11

Number of integers divisible by 11

Let A be the positive integers less than 1000.

We then note that A contains 999 integers, while we are interested in integers divisible by 11.

$$|A| = 999$$

$$d = 11$$

Using the quotient rule (round down!):

$$n_{11} = \frac{|A|}{d} = \frac{999}{11} \approx 90.8182 \approx 90$$

Thus 90 integers are divisible by 11.

Number of integers divisible by either 7 or 11

Use the subtraction rule:

$$n_{7 \text{ or } 11} = n_7 + n_{11} - n_{77} = 142 + 90 - 12 = 220$$

e) Divisible by exactly one of 7 and 11

Number of integers divisible by 11, but not by 7

Integers divisible by 11 but not by 7 are integers divisible by 11 that are not integers that are divisible by 7 and 11.

$$n_{11,\text{ not }7} = n_{11} - n_{77} = 90 - 12 = 78$$

Number of integers divisible by exactly one of 7 or 11

Use the sum rule:

$$n = n_{7,\text{ not }11} + n_{11,\text{ not }7} = 130 + 78 = 208$$

f) Are divisible by neither 7 or 11

(f) Integers divisible by neither 7 nor 11 is the integers in the set that are not divisible by either 7 or 11

$$n_{not(7\text{ or }11)} = |A| - n_{7\text{ or }11} = 999 - 220 = 779$$

g) Have distinct digits

(g) Integers below 1000 have 3 possible digits or less.

1 digit There are 9 possible positive integers less than 1000 with 1 digit: 1, 2, 3, 4, 5, 6, 7, 8, 9

2 digits

First digit: 9 ways (since the first digit cannot be 0)
Second digit: 9 ways (since there are 10 digits, while the digit cannot be the same as the first digit).

Use the product rule: $9 \cdot 9 = 81$ ways

3 digits

First digit: 9 ways (since the first digit cannot be 0)
Second digit: 9 ways (since there are 10 digits, while the digit cannot be the same as the first digit).
Third digit: 8 ways (since there are 10 digits, while the digit cannot be the same as the first nor the second digit).

Use the product rule: $9 \cdot 9 \cdot 8 = 648$ ways

In total Use the sum rule:

$$9 + 81 + 648 = 738$$

h) Have distinct digits are even

(h) Integers below 1000 have 3 possible digits or less.

1 digit There are 4 possible even integers: 2, 4, 6, 8

2 digits

First digit: 9 ways (since the first digit cannot be 0) of which 5 are odd and 4 are even.
Second digit:

- 5 ways if the first digit is odd (since the number needs to be even, thus the last digit has to be 0, 2, 4, 6 or 8)
- 4 ways if the first digit is even (since the number needs to different from the first digit)

Use the product rule and sum rule: $5 \cdot 5 + 4 \cdot 4 = 25 + 16 = 41$ ways

3 digits

First digit: 9 ways (since the first digit cannot be 0) of which 5 are odd and 4 are even
Second digit: 9 ways (since there are 10 possible digits and the second digit cannot be the same as the first)

- If first digit is odd: 4 odd and 5 even
- If first digit is even: 5 odd and 4 even

Third digit:

- If the first two digits are odd: 5 ways (since the number needs to be even, thus the last digit has to be 0, 2, 4, 6 or 8).
- If the first two digits are even: 3 ways
- If the first digit is even and the second odd: 4 ways
- If the first digit is odd and the second even: 4 ways

Use the product rule:

First two digits odd: $5 \cdot 4 \cdot 5 = 100$ ways
First two digits even: $4 \cdot 4 \cdot 3 = 48$ ways
First digit is even and the second odd: $4 \cdot 5 \cdot 4 = 80$ ways
First digit is odd and the second even: $5 \cdot 5 \cdot 4 = 100$ ways

Use the sum rule: $100 + 48 + 80 + 100 = 328$ ways

Use the sum rule: $100 + 48 + 80 + 100 = 328$ ways

In total Use the sum rule:

$$4 + 41 + 328 = 373$$

Q23 Page 396 Assignment

5. **What is pigeonhole principle? OR State pigeon hole principle?**

Page 399, Example 1, 2, 3 Page 400

6. **What is the minimum number of students required in the class be sure that 6 will receive the same grade if there is five possible grades A, B, C, D and F?**

Example 6 page 402

7. **Permutation?** **Page 407**

A permutation is an arrangement of some elements in which order matters. In other words a Permutation is an ordered Combination of elements.

P (n, r) = P (5, 5)

$$= \frac{n!}{(n-r)!} = \frac{5!}{(5-5)!} = \frac{5!}{(0)!} = \frac{5!}{(1)}$$

8. **How many permutations of the letters ABCDEFG contain the string CFGA?**

Topic Page 407 Answer: Q21 page 414, Example 4, 5, 6, 7 Page 409, Q22 Page 414, Assignment To solve Q3 Follow Example 7 Page 409

P (n, r) = P (5, 5)

$$= \frac{n!}{(n-r)!} = \frac{5!}{(5-5)!} = \frac{5!}{(0)!} = \frac{5!}{(1)}$$

a) The String BCD

If *BCD* is to be a substring, then we can think of that block of letters as one super letter, and the problem is to count permutations of five items ~the letters *A, E, F,* and *G,* and the super letter *BCD.* Therefore the answer is *P (5,* 5) = 5! = 120.

b) The string CFGA?

Reasoning as in part (a), we see that the answer is P (4, 4) = 4! = 24.

c) The String BA and GF?

As in part (a), we glue BA into one item and glue GF into one item. Therefore we need to permute five items, and there are P (5, 5) = 5! = 120 ways to do it.

d) The String ABC and DE

This is similar to part (c). Glue ABC into one item and glue DE into one item, producing four items, so the answer is P (4, 4) = 4! = 24.

e) The String ABC and CDE?

If both ABC and CDE are substrings, then ABCDE has to be a substring. So we are really just Permuting three items: ABCDE, F, and G. Therefore the answer is P (3, 3) = 3! = 6.

f) The String CBA and BED?

There are no permutations with both of these substrings, since *B* cannot be followed by both *A* and *E* at the same time.

9. **How many permutations of letter ASSESSINATION contain the string SES?**

AS**SES**SINATION

P (11, 11) = 11! = 39916800

10. **Pascal's Triangle?** **Q9 Page 440 Assignment**

Pascal's triangle: a representation of the binomial coefficients where the *i*th row of the triangle contains $\binom{i}{j}$ for $j = 0, 1, 2, \ldots, I$ **Topic: Definition <u>Page 439 and Triangle 419</u>**

11. **How many subsets with more than two elements does a set with 100 elements have?**

<u>Chapter Q 17 Page 413 pdf from Google drive discrete structure</u>

A combination is a mathematical technique that determines the number of possible arrangements in a collection of items where the order of the selection does not matter. In combinations, you can select the items in any order. Combinations can be confused with permutations.

Q13 and 15 Page 413 Assignment

We know that there are 2^{100} subsets of a set with 100 elements. All of them have more than two elements except the empty set, the 100 subsets consisting of one element each, and the set with two elements C(l00, 2) = 4950 subsets with two elements. Therefore the answer is 2^{100} - 5051 ≈ 1.3 x 10^{30} .

We subtract from the total number of subsets those that have 0, 1, or 2 elements:

2^{100} – C(100, 0) – C(100, 1) – C(100, 2) = 2^{100} – 1 – 100 – 4950 = ≈ 1.3 x 10^{30}

Exercise Q17 page 413

12. How many bit strings of length 12 contain exactly three 1s?

a) at most three 1s?

the order of the bits is not important (since we are interested in the number of ones, not the order of ones) thus we need to use the definition of combination

n= 12, r = 3,

evaluate the definition of combination C (12, 3) = $\frac{12!}{3!(12-3)!}$ = $\frac{12!}{3!(9!)}$ = 220

b) At least three 1s?

n = 12, $n \geq 3$

Evaluate the definition of combination C (12, 3) = $\frac{12!}{3!(12-3)!}$ = $\frac{12!}{3!(9!)}$ = 220

C (12, 2) = $\frac{12!}{2!(12-2)!}$ = $\frac{12!}{3!(10!)}$ = 66

C (12, 0) = $\frac{12!}{0!(12-0)!}$ = $\frac{12!}{0!(12!)}$ = 1

Add the number of bits strings for each value of r: 220 + 66 + 12 + 1 = 299

c) an equal number of 0x and 1s?

n = 12, $n \leq 3$

Evaluate the definition of combination C (12, 3) = $\frac{12!}{3!(12-3)!}$ = $\frac{12!}{3!(9!)}$ = 220

C (12, 4) = $\frac{12!}{4!(12-4)!}$ = $\frac{12!}{4!(8!)}$ = 495

C (12, 5) = $\frac{12!}{5!(12-5)!}$ = $\frac{12!}{3!(7!)}$ = 792

C (12, 6) = 924

C (12, 7) = 792

C (12, 8) = 492

C (12, 9) = 220

C (12, 10) = 66

C (12, 11) = 12

C (12, 12) = 1

Add the number of bit string for each value of r: 220 + 595 + 924 -----------12 + 1 = 4017

d) An equal number of 0s and 1s

An equal number of 0s and 1s in a string of length 12, means that there are six 0s and six 1s. n= 12, r = 6.

Evaluate the definition of a combination: C (12, 6) = $\frac{12!}{6!(12-6)!}$ = $\frac{12!}{6!(6!)}$ = 924

13. How many lines you can draw using 3 non collinear (not in a single line) point A, B, and C on a plane.

The lines are: AB, BC and AC ; 3 lines only. So in fact we can draw 3 lines and not 6 and that's because in this problem the order of the points A, B and C is not important.

You need two points to draw a line. The order is not important. Line AB is the same as line BA. The problem is to select 2 points out of 3 to draw different lines. If we proceed as we did with permutations, we get the following pairs of points to draw lines.

AB , AC

BA , BC

CA , CB

There is a problem: line AB is the same as line BA, same for lines AC and CA and BC and CB.

The lines are: AB, BC and AC ; 3 lines only.

14. How many Triangles can you make using 6 non collinear points on a plane?

$$C(n,r) = C(6,3)$$

$$= \frac{6!}{(3!(6-3)!)}$$

$$= \frac{6!}{3! \times 3!}$$

$$= 20$$

$$C(n,r) = \binom{n}{r} = \frac{n!}{(r!(n-r)!)} = ?$$

Where n! = n. (n-1)……….2.1

As we have 6 non-collinear points, we have to choose 3 out of 6.

This can be done in 6C3

https://www.doubtnut.com/questions/how-many-triangles-can-you-make-using-6-non-collinear-points-on-a-plane-140452

15. What is the difference between an r – Combination and an r – permutation of a set with n elements Answer Q8 Page 440

Page 410 definition & Example 9, Page 423 Theorem 1, Example 1 Page 427

(a) The order of the r elements in a r-combination is not important (thus changing the order of the elements results in the same combination), while the order of the r elements in a r-permutation is important (thus changing the order of the elements results in a different permutation).

b) Derive an equation that relates the number of r- combinations and the number of r-permutations of a set with n elements

(b) Use the definition of combination and permutation:

$$C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{1}{r!} \cdot \frac{n!}{(n-r)!} = \frac{1}{r!} \cdot P(n,r)$$

Thus $C(n,r) = \frac{1}{r!} \cdot P(n,r)$ or equivalently $P(n,r) = r! \cdot C(n,r)$.

c) How many ways are there to select six students from a class of 25 to serve on a committee?

(c) We want to select 6 students from a class of 25:

$$n = 25$$

$$r = 6$$

The order of the selected students is not important (as they all receive the same position), thus we need to use the definition of a **combination**:

$$C(25,6) = \frac{25!}{6!(25-6)!} = \frac{25!}{6!19!} = 177,100$$

d) How many ways are there to select six students from a class of 25 to hold six different executive positions on a committee?

(d) We want to select 6 students from a class of 25:

$$n = 25$$

$$r = 6$$

The order of the selected students is important (as they all receive a different position), thus we need to use the definition of a **permutation**:

$$P(25,6) = \frac{25!}{(25-6)!} = \frac{25!}{19!} = 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 = 127,512,000$$

Chapter 6: Counting Long Questions

1. **Each use on the computer system has password each password is six to 8 characters long, each character is upper case letter or a digit and each password must contain at least 1 digit. How many possible passwords are there?**

   Example 16 page 391

   Example 15 Assignment Page 391

2. **In certain countries, car number plate is formed by four digits from the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9 followed by 3 letters from the alphabet. How many number plates can be formed if neither the digit nor letter is repeated?**

   Answer: There are 9^4*26^3 = 115316136 different plates possible.

   OR

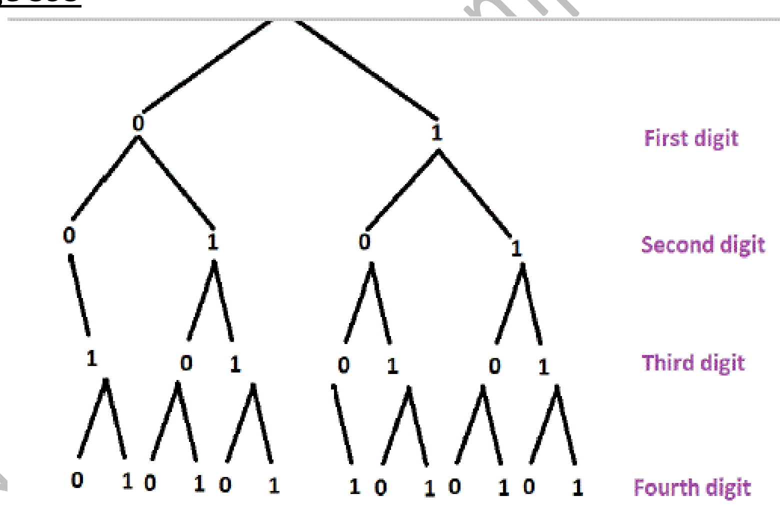
   ${}_9P_4 \times {}_{26}P_3 = 47{,}174{,}400$

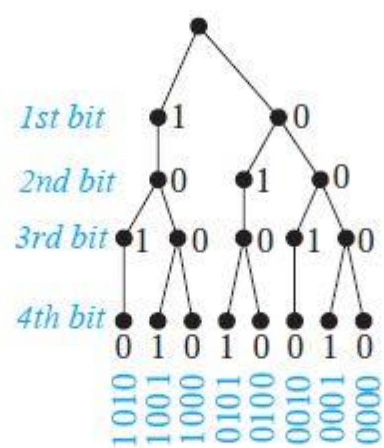
   OR

   First has all nine choices
   Second has eight remaining choices
   Third has seven remaining choices
   Fourth has six remaining choices
   Fifth has twenty six choices
   Sixth has twenty five remaining choices
   Seventh has twenty four remaining choices
   9 x 8 x 7 x 6 x 26 x 25 x 24 = 47,174,400 different license plates
   <u>Must sure that it is combination or permutation problem from Imran sab</u>

3. **Use diagram to represents how many bit strings of length four don't have 3 consecutive 0's?**

   <u>Proper understanding of this Q understand example tree diagram page 394 and example 21 page 395</u>





Example 23 Page 395 Assignment

4. **How many integers from 1 through 1000 are multiples of 3 or multiples of 5?**

   Topic: Page 393 Principle of Inclusion Exclusion

   Let A = the numbers from 1 through 1000 that are multiples of 3
   Let B = the numbers from 1 through 1000 that are multiples of 5
   |A| = 333 (they go from 3 = 1*3 to 333 = 333*3) |B| = 200 (they go from 5 = 1*5 to 200 = 200*5)

A ∩ B is the number of numbers which are multiples of 15. There are 66 of these (from 15 = 1 * 15 to 990 = 66 *15), because we need multiples of 3 and 5 both.

So |A U B| = |A| + |B| - | A ∩ B | = 333 + 200 – 66 = 467

Example 18,19 Assignment Page 393

5. **A committee is formed consisting of one representative from each of the 25 states in the united state, where the representative from the state is either the governor or one of the two senators from the United States. How many ways are there to form this committee?**

   Exercise Q27 page 397 this is for 50 states by using product rule 3^{25} is the required answer.

   There are 50 choices to make, each of which can be done in 3 ways, namely by choosing the governor, choosing the senior senator, or choosing the junior senator. By the product rule the answer is therefore $3^{50} = 7.2\text{x}10^{23}$

   OR

   There are 25 choices to make, each of which can be done in 3 ways, namely by choosing the governor, choosing the senior senator, or choosing the junior senator. By the product rule the answer is therefore $3^{25} = 8.5 \times 10^{11}$

   Q25, 29 Assignment Page 397

6. **How many license plates can be made using either three letters followed by 3 digits or 4 letters followed by two digits? OR How many license plates can be made using either three Uppercase English letters followed by three digits or four uppercase English letters followed by two digits.** **Page 397 Q 30**

   Product Rule: if one event occur in m ways AND a second event occur in n ways, the number of ways the two events can occur in sequence is them m.n

   Sum Rule: if an event occur either in m ways OR in n ways (non overlapping), the number of ways the event can occur is then m+n.

   Solution: Three letters followed by 3 digits

   There are 26 possible letters and 10 possible digits

   First letter = 26 ways, 2nd letter = 26 ways, 3rd letter = 26 ways. First digit = 10 ways, 2nd digit = 10 ways, 3rd digit = 10 ways

   Using product Rule: 26.26.26. 10.10.10 = 26^3. 10^3 = 17,576,000

   Four letters followed by two digits

   There are 26 possible letters and 10 possible digits

   First letter = 26 ways, 2nd letter = 26 ways, 3rd letter = 26 ways, 4th letter = 26 ways First digit = 10 ways, 2nd digit = 10 ways

   Using Product Rule: 26.26.26.26. 10.10 = 26^4. 10^2 = 45,697,600

   Three letters followed by 3 digits or four letters followed by two digits

   Using the sum rule: 17, 576,000 + 45697, 600= 63,273,000

7. **Suppose that a password of computer system must have at least 8, but no more than 12 characters. Where each character in the password is a lower case English letter, an upper case English letter, a digit, or one of the six special characters *, >, <, !, + and =.**

   Exercise Question 55 page 398 C Part is not a part of Paper

a) **How many different password for the computer system.**

We are told that there are 26 + 26 + 10 + 6 = 68 available characters. A password of length *k* using these characters can be formed in 68^k ways. Therefore the number of passwords with the specified length restriction is $68^8 + 68^9 + 68^{10} + 68^{11} + 68^{12} =$ 9,920,671,339,261,325,541,376, which is about 9.9 x 1021 or about ten sextillion.

b) **How many of these passwords contain at least one occurrence of at least one of six special characters.**

For a password *not* to contain one of the special characters, it must be constructed from the other 62 characters. There are $62^8 + 62^9 + 62^{10} + 62^{11} + 62^{12}$ = 3,279,156,377,874,257,103,616 of these. Thus there are 6,641,514,961,387,068,437,760 ≈ 6.6 x 1021 (about seven sextillion) passwords that do contain at least one Occurrence of one of the special symbols.

c) **Using your answer to part (a), determine how long it takes a hacker to try every possible password, assuming that it takes one Nano second for a hacker to check each possible password**

Assuming no restrictions, it will take one nanosecond (one billionth of a second, or 10^{-9} sec) for each password. We just multiply this by our answer in part (a) to find the number of seconds the hacker will require. We can convert this to years by dividing by 60 · 60 · 24 · 365.2425 (the average number of seconds in a year). It will take about 314,374 years.

8. **State pigeon hole principal. Explain using suitable example**

Page 399, Example 1, 2, 3, 4 Page 399 to 401

9. **Suppose that a sales man has to visit eight different places. She must begin her trip in specified city, but she can visit the other seven cities in any order. How many possible orders a sales women use when visiting these places?**

Page 409 example 6

Practice Example 4, 5 Page 409

10. **A CSE senior need 5 CSE course to graduate. There a 12 computer science courses that she can take next semester that would count toward his degree how many different sets of 5 courses should she take?**

Discuss with Imran sab

11. **Suppose that there are eight runners in a race. Gold Silver and bronze medals are given. How many different ways are there to award these medals if all possible outcomes of the race can occur?**

Example 5 page 409

12. **A coin flipped 10 times where each flip comes up either head or tail. How many possible outcomes** **<u>413 Q19</u>**

a) Are there in total

Each flip can be either heads or tails, so there are 2^{10} = 1024 possible outcomes.

b) Contain exactly two heads

To specify an outcome that has exactly two heads, we simply need to choose the two flips that came up heads. There are C (10, 2) = 45 such outcomes.

c) Contain at most three tails

To contain at most three tails means to contain three tails, two tails, one tail, or no tails. Reasoning as in part **(b),** we see that there are C (10, 3) + C (10, 2) + C (10, 1) + C (10, 0) = 120 + 45 + 10 + 1 = 176 such outcomes.

d) Contain the same number of head and tails.

To have an equal number of heads and tails in this case means to have five heads. Therefore the answer is C (10, 5) = 252.

Q18 and 20 Page 413 Assignment

Q29 and 31 Page 397 Assignment

13. **One hundred tickets, numbered 1, 2, 3, … … , 100, are sold to 100 different people for a drawing. Four different prizes are awarded, included a grand prize (a trip to Tahiti). How Many ways are there to ward the prizes if (a, c, h, j in Paper) Answer Q25 Page 414**

a) **There is no restriction?**

Since the prize are different, we want an order arrangement of four numbers from the set of the first 100 positive integers. Thus there are P (100, 4) = 94, 109,400 ways to award the prizes

b) **The person holding ticket 47 wins the grand prize?**

If the grand prize winner is specified, then we need to choose an order set of three tickets to win the other prizes. This can be done is P (99, 3) = 941, 094 ways

c) **The person holding ticket 47 wins one of the prizes?**

We can first determine which prize the person holding ticket 47 will win (this can be done in 4 ways) and then we can determine the winners of the other three prizes, exactly as in part (b). Therefore the answer is 4P (99, 3) = 3, 764, 376

d) **The person holding ticket 47 does not win a prize?**

This is the same calculation as in part (a), except there are only 99 available tickets. There for answer is P (99, 4) = 90, 345, 024. Note that this answer plus the answer to part (c) equals to the part (a), since person holding ticket 47 either wins a prize or does not win a prize.

e) **The people holding tickets 19 and 47 both win prizes?**

This is similar to part (c). There are 4.3 =12 ways to determine which prizes these two lucky people will win, after which there are P (98, 2) = 9506 ways to award the other two prizes. Therefore the answer is 12. 9506 = 114, 072

f) **The person holding tickets 19, 47 and 73 will win prizes?**

This is like part (e). There are P (4, 3) =24 ways to choose the prizes for the three people mentioned, and then 97 ways to choose the other winner. This gives 24. 97 = 2328 ways in all

g) **The people holding ticket 19, 47, 73 and 97 will win prizes?**

Here it is just a matter of ordering the prizes for these four people, so the answer is P(4, 4)= 24.

h) None of the people holding tickets 19, 47, 73, and 97 will win prizes?

This is similar to part (d), except that this time the pool of viable numbers has only 96 numbers in it. Therefore the answer is P(96, 4) = 79, 727,040

i) The grand prize winner is a person holding ticket 19, 47, 73 or 97

There are four ways to determine the grand prize winner under these conditions. Then there are P(99, 3) ways to award the remaining prizes. This gives an answer of 4P(99, 3)= 3, 764,376

j) The people holding tickets 19 and 47 win prizes, but the people holding tickets 73 and 97 do not win prizes?

First we need to choose the prizes for the holder of 19 and 47. Since there are four prizes, there are P(4, 2) = 12 ways to do this. Then there are 96 people who might win the remaining prizes, and there are P (96, 2) = 9120 ways to award these prizes. Therefore the answer is 12. 9120 = 109, 440.

14. **How many lines can you draw using 3 non collinear (Not in single line) points A, B and C on a plane?**

You need two points to draw a line. The order is not important. Line AB is the same as line BA. The problem is to select 2 points out of 3 to draw different lines. If we proceed as we did with permutations, we get the following pairs of points to draw lines.
AB , AC
BA , BC
CA , CB
There is a problem: line AB is the same as line BA, same for lines AC and CA and BC and CB.
The lines are: AB, BC and AC ; 3 lines only.
So in fact we can draw 3 lines and not 6 and that's because in this problem the order of the points A, B and C is not important.
This is a combination problem: combining 2 items out of 3 and is written as follows:

$$_nC_r = n! / [\,(n - r)!\, r!\,]$$

The number of combinations is equal to the number of permutations divided by r! to eliminates those counted more than once because the order is not important.

15. **How many triangles can you make using 6 non collinear points on the plane?**

$$C(n,r) = C(6,3) = \frac{6!}{(3!(6-3)!)} = \frac{6!}{3! \times 3!} = 20$$

$$C(n,r) = \binom{n}{r} = \frac{n!}{(r!(n-r)!)} = ?$$

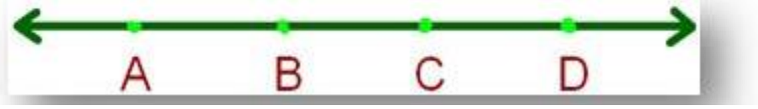
Where n! = n. (n-1)……….2.1

As we have 6 non-collinear points, we have to choose 3 out of 6.

This can be done in 6C3

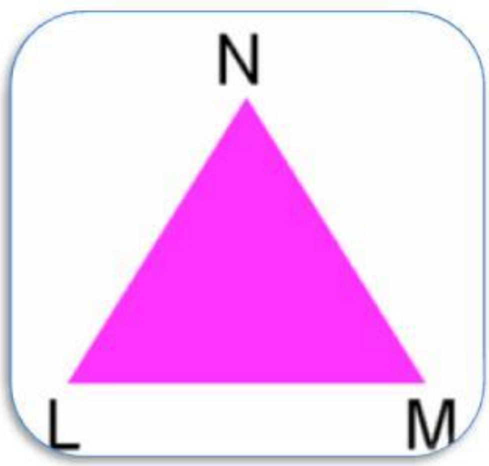
Triangle is one of the basic shape in geometry.
We know that we can mark many points on any given line.

Three or more points which lie on the same line are called **collinear points**.



Above, points A, B, C and D which lie on the same line *collinear points*.
But in the figure below, only two points A and D lied on the line. Points B, E, C and F do not lie on that line.

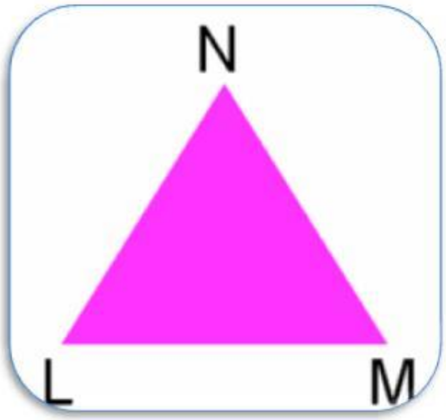
Hence, these points A, B, C, D, E, F are called **non - collinear points**.
If we join three non - collinear points L, M and N lie on the plane of paper, then we will get a closed figure bounded by three line segments LM, MN and NL. This closed figure is called a **Triangle**.
The three line segments of a triangle are also known as sides of the triangle.



This triangle is named as Δ LMN with its side as LM, MN and NL and three vertices as L, M and N.

The three angles named as ∠ LMN, ∠MNL and ∠ NLM are the angles of the triangle.
Three angles are denoted by ∠M, ∠N and ∠L respectively.
`The three angles and the three sides of a triangle are together called the six parts or **elements** of the triangle.

Thus, a closed figure bounded by three line segments is called a triangle. **Δ** is the symbol to denote a triangle.
Note: A triangle has 6 elements: Three sides and three angles.



Permutations (partial permutations) of n objects taken k at a time is: **n!/(n - k)!**
So, through 6 random points, since 2 points are needed to define a line,
6! / (6–2)! = 30 pairs of points, resulting in 30/2 = **15 lines**, otherwise each line would be taken twice.
Now, **15 random lines form** {15 objects taken 3 at a time} triangles:
15!/(15–3)! = **2730 triangles**.

Chapter 7: Discrete Probability Definitions Page 494

1. **Define Sample space? Page 446**

   Sample space of experiment is the set of possible outcomes.

   Experiment? Event? Page 446

2. **What is conditional probability? Page 456 and Example 4 Page 457**

   The **conditional probability** of an event B is the probability that the event will occur given an event A has already occurred. This is written as P(B|A). If event A and B are mutually exclusive, then the conditional probability of event B after the event A will be the probability of event B that is P(B).

3. **Random variables?** **Page 460 and Example 10 Page 460**

   A random variable is a function from the sample space of an experiment to the set of real Numbers. That is, a random variable assigns a real number to each possible outcome.
   A **random variable** is a variable whose value is a numerical outcome of a random phenomenon.
   ▪ a random variable is denoted with a capital letter
   ▪ the probability distribution of a random variable X tells what the possible values of X are and how probabilities are assigned to those values
   ▪ a random variable can be discrete or continuous

4. **What is the probability of getting a number greater than 4 when a dice is tossed?**

Answer: 1/3.If you roll a single die there are 6 possible outcomes (1, 2, 3, 4, 5, 6), 2 of which are greater than 4. So in a single roll the probability of getting a number greater than 4 is 2/6 = 1/3.

5. **A bag contains 6 white, 5 black and 4 red balls. Find the probability of getting a white or a black ball in a single draw**

   6 white balls, 4 red and 5 black. Therefore probability of black ball when 1 is drawn is 5/5+4+6 (that color/ total) => 5/15 or 1/3.

6. **Mention whether the following problems are permutation and combination problems in how many ways can six tosses of coin yield 2 heads and 4 tails**

   <u>Assignment from Home is it Correct if Not Correct it, if Correct then how It is correct.</u>

   Here, the number of ways is ${}^6C_2 = 15$. There are 15 cases.

   In each case, we have six flips, and each one is either H or T. Both have a probability of 1/2, so any one case would have a probability of

   $$\frac{1}{2^6}=\frac{1}{64}$$

   Since there are 15 cases with identical probabilities, the total probability is

   $$\frac{15}{2^6}=\frac{15}{64}$$

<u>Chapter 7: Discrete Probability Long Questions</u>

1. **A bit string of length four is generated at random so that each of the 16 bit strings of length four is equally likely. What is the probability that it contains at least two consecutive 0s, given that its first bit is a 0? (We assume that 0 bits and 1 bits are equally likely Page 456**

   Using Truth table combinations of 4 bits is generated so we can easily solve

   E∩F contain those combination that contains two consecutive zeros

<u>Chapter 8: Advance counting techniques</u>

1. **How many different comparisons are needed for a binary search tree in a set of 64 elements? Q1 Page 535**

   Let f(n) be the number of comparisons needed in a binary search of a list of n elements. From **Example 1 Page 528** we know that f satisfies the divide and conquer recurrence relation f(n) = f(n/2) + 2. Also 2 comparisons are needed for a list with one element i.e. f(1) = 2 (**see Example 3 Page 220** in section 3.3 for further discussion). Thus f(64) = f(32) + 2 = f(16) + 4 = f(8) + 6 = f(4) + 8 = f(2)+ 10 = f(1) + 12 = 2 + 12 = 14

2. **Derangements Page 562 Example 4 Page 562**

   A **derangement** is a permutation of objects that leaves no object in its original position. We will need to determine the number of derangements of a set of *n* objects.

   The permutation 21453 is a derangement of 12345 because no number is left in its original Position. However, 21543 is not a derangement of 12345, because this permutation leaves 4 Fixed.

   *D*3 =2, because the derangements of 123 are 231 and 312.We will evaluate *Dn*, for all positive integers *n*, using the principle of inclusion–exclusion.

Chapter 9: Relation Short Questions

Page 633 definitions

1. **Define reflexive relation** <u>page 576</u>

   Example 7, 8, 9 Page 576

2. **Give an Example of Antisymmetric Relation on the set {1, 2, 3}**

   Definition 4 Page 577 and Example 7 Page 576 and Answer Example 10 Page 577

   Practice Example 11 Page 577

3. **How many relations are there on the set? S={a, b, c}** <u>page 583 Q45</u>

   There are 3 x 3 =9 relations in a set S.

   <u>Q43 Page 583 Assignment</u>

4. **How many relation are there on the set {a, b, c, d**}

   <u>Similar to example 6 Page 576 and Example 16 Page 578</u>.

   A relation on a set S with n elements is subset of S x S has n^2 elements, we are asking for the number of subsets of a set with n^2 elements, which is 2^{n^2}. In our case n=4, so the answer is 2^{16} = 65,536.

5. **How many relations are there on the set {a, b, c, d} that contain the pair (a, a)?**

   <u>Similar to example 16 Page 578.</u>

   In solving Q4(above), we had 16 binary choices to make – whether to include a pair (x, y) in the relation or not as x and y ranged over the set {a, b, c, d}. One of those choices has been made for us: we must include (a,a). We are free to make other 15 choices. So the answer is 2^{15} = 32, 768

6. **Differentiate b/w join and projection?**

   <u>Page 586 Definition 2, Definition 3 586, 587</u>

   Projection: a function that produces relations of smaller degree from an *n*-ary relation by deleting fields

   Join: a function that combines *n*-ary relations that agree on certain fields

7. **What is relational Data Model? Answer Page 633 Topic Page 584 and 585**

   Answer: Model for representing database using n-ary relations

   <u>Table? Primary key? Extension? Intension? Record? Fields?</u>

   <u>Example 5, 6 Page 585 and 586</u>

8. **Define reflexive or symmetric closure? OR What is Reflexive or Symmetric Closure**

   Topic: Definitions Page 598 & Example 1, 2 Page 598

9. **Define transitive closure** <u>page 600 definition 2</u>

   The **transitive closure** of a binary relation R on a set X is the smallest relation on X that contains R

   For example, if X is a set of airports and xRy means "there is a direct flight from airport x to airport y" (for x and y in X), then the transitive closure of R on X is the relation R+ such that x R+ y means "it is possible to fly from x to y in one or more flights".

   Example 4, 5, 6, theorem 2, LEMMA 1 from 600 to 602

10. **Define a Poset OR Partial Ordering? Or Define Partial Ordering with Example?**

    Page 618 and Example 1, 2, 4 Page 619, 620

    It means transitive anti-symmetric relation between the elements of a set, which does not necessarily apply to each pair of element

Chapter 9: Relation Long Question

1. **What is relation on a set? How many relations are there on a set with n elements?**

   Page 575 page Definition 2, Example 4, 5, 6 Page 575, 576

   Properties of Relation????

2. **Which of these is symmetric, anti- symmetric and reflexive?**

Example 7 page 576 and Example 10 page 577

R1= {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)}

R2= {(1,1), (1,2), (2,1)}

R3= {(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)}

R4= { (2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}

R5= {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}

R6= {((3,4)}

Assignment Example 8, 9 Page 576, Example 11, 12 Page 577

3. **Find either the relation is reflexive, symmetric, anti-symmetric or transitive?**

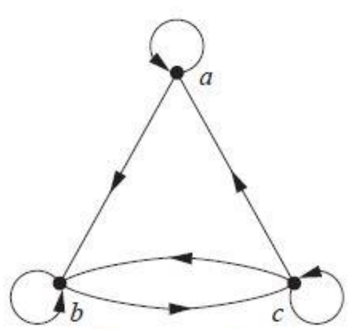
   R= {(1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,3),(4,1), (4,3)}

   Assignment by Own with the help of above Questions, Q3 Page 581

4. **According to digraph representing R** **Answer Example 10 Page 595**

   Page 594 Topic:

   Practice Example 7, 8, 9 Page 594, 595



(a) Directed graph of *R*

Is R reflexive ?

Is R symmetric

Is R anti symmetric

Is R transitive?

- **Reflexivity**: In a digraph, the represented relation is reflexive *iff* every vertex has a self loop
- **Symmetric**: In a digraph, the represented relation is symmetric *iff* for every directed edge from a vertex x to a vertex y there is also an edge from y to x
- **Anti-symmetric** : A represented relation is anti-symmetric *iff* there is never a back edge for any directed edges between two distinct vertices
- **Transitivity**: A digraph is transitive if for every pair of directed edges (x,y) and (y,z) there is also a directed edge (x,z)

$\rightarrow$ This may be harder to visually verify in more complex graphs

5. **Find the smallest relation containing the relation (1,2), (1, 4), (3,3), (4,1) that is**

607 page Q29

a) Reflexive and transitive

{(1,1), (1,2), (1,4), (2,2), (3,3), (4,1), (4,2), (4,4) }

b) Symmetric and transitive

{(1,1), (1,2), (1,4), (2,1), (2,2), (2,4), (3,3), (4,1), (4,2), (4,4) }

c) **Reflexive symmetric and transitive**

{(1,1), (1,2), (1,4), (2,1), (2,2), (2,4), (3,3), (4,1), (4,2), (4,4) }

Help odd solutions

6. **List the orders pairs in the relation S and R from A= {1,2,3,4} to itself, where (a, b) ∈ R if and only if**

   a) **R={{(a, b) l a + b = 4} and S={(a, b)a l b}}**

   A = {1, 2, 3, 4}

   B = {1, 2, 3, 4}

   R= {{(a, b) l a + b=4} = {(1, 3), (3, 1), (2, 2)}

   S= {(a, b) a l b}} ={(1, 1), (1, 2), (2, 2), (1, 3), (3, 3), (1, 4), (2, 4), (4, 4) }

   b) **Find S - R and R U S**

   Note: Created two supposed relations from set A= {1, 2, 3, 4}

   Let S = {(1, 3), (3, 1), (2, 2)}

   Let R ={(1, 1), (1, 2), (2, 2), (1, 3), (3, 3), (1, 4), (2, 4), (4, 4) }

   R – S = {(3, 1)}

   S – R = {(1, 1), (1, 2), (3, 3), (1, 4), (2, 4), (4, 4) }

   c) **Find relation S is reflexive, symmetric and anti-symmetric or transitive.**

   S = {(a, b)a l b}} ={(1, 1), (1, 2), (2, 2), (1, 3), (3, 3), (1, 4), (2, 4), (4, 4) }

   Yes Reflexive because it contains all the pairs of form (a, a)

   Not Symmetric because it does not contain relation (b, a) belongs to (a, b)

   Yes Antisymmetric **Chapter 9 Relation.pdf**

   Combining Relations Example 17, 18 Page 579

   For Understand Check Example below Q1 Page 581

Given:

$$A = \{0, 1, 2, 3, 4\}$$

$$B = \{0, 1, 2, 3\}$$

(a)

$$R = \{(a, b) | a = b\}$$

We note that the elements common by A and B are $0, 1, 2$ and 3.

R then contains the points with $a = b = 0$, $a = b = 1$, $a = b = 2$ and $a = b = 3$

$$R = \{(0, 0), (1, 1), (2, 2), (3, 3)\}$$

(b)

$$R = \{(a, b) | a + b = 4\}$$

We note that the sum of the element of A and the element of B needs to be equal to 4:

$$a = 1, b = 3$$

$$a = 2, b = 2$$

$$a = 3, b = 1$$

$$a = 4, b = 0$$

R then contains the points:

$$R = \{(1, 3), (2, 2), (3, 1), (4, 0)\}$$

(c)

$$R = \{(a, b) | a > b\}$$

We note that the element of A needs to be larger than the element of B:

$$a = 1, b = 0$$

$$a = 2, b = 0$$

$$a = 3, b = 0$$

$$a = 4, b = 0$$

$$a = 2, b = 1$$

$$a = 3, b = 1$$

$$a = 4, b = 1$$

$$a = 3, b = 2$$

$$a = 4, b = 2$$

$$a = 4, b = 3$$

R then contains the points:

$$R = \{(1, 0), (2, 0), (3, 0), (4, 0), (2, 1), (3, 1), (4, 1), (3, 2), (4, 2), (4, 3)\}$$

(d)

$$R = \{(a, b) | a | b\}$$

We note that the element of A needs to be a divisor of the element of B.

All nonnegative integers are divisors of 0 (as 0 divided by any nonnegative integer is 0

$$a = 1, b = 0$$

$$a = 2, b = 0$$

$$a = 3, b = 0$$

$$a = 4, b = 0$$

The only divisor of 1 is 1 itself.

$$a = 1, b = 1$$

The divisors of 2 are 1 and 2.

$$a = 1, b = 2$$

$$a = 2, b = 2$$

The divisors of 3 are 1 and 3

$$a = 1, b = 3$$

$$a = 3, b = 3$$

R then contains the points:

$$R = \{(1, 0), (2, 0), (3, 0), (4, 0), (1, 1), (1, 2), (2, 2), (1, 3), (3, 3)\}$$

(e)

$$R = \{(a,b)|gcd(a,b) = 1\}$$

We note that the greatest common divisor of the element of A and the element of B needs to be equal to 1. In other words, 1 can be the only common divisor of the two elements.

Since 0 is divisible by any nonnegative integer and since 1 is only divisible by 1, the greatest common divisor of 0 and 1 is 0.

$$a = 1, b = 0$$

$$a = 0, b = 1$$

Since 1 is only divisible by 1, the greatest common divisor between 1 and any positive integer is always 1.

$$a = 1, b = 1$$

$$a = 1, b = 2$$

$$a = 1, b = 3$$

$$a = 2, b = 1$$

$$a = 3, b = 1$$

$$a = 4, b = 1$$

The greatest common divisor of 2 and 3 is 1.

$$a = 2, b = 3$$

$$a = 3, b = 2$$

The greatest common divisor of 2 and 4 is 2.

The greatest common divisor of 3 and 4 is 1.

$$a = 4, b = 3$$

R then contains the points:

$$R = \{(1,0), (0,1), (1,1), (1,2), (1,3), (2,1), (3,1), (4,1), (2,3), (3,2), (4,3)\}$$

(f)

$$R = \{(a, b) | lcm(a, b) = 2\}$$

We note that the least common multiple of the element of A and the element of B needs to be equal to 2.

Since the least common multiple is 2, neither element can be larger than 2.

Since any multiple of 0 is always 0 itself, the least common multiple of 0 and any other integer does not exist.

If both elements are 1, then the least common multiple is 1.

The only remaining possibility is then that at least one of the two elements is a 2 and the other element is a 1 or 2. In each of these cases, the least common multiple is 2.

$$a = 1, b = 2$$

$$a = 2, b = 1$$

$$a = 2, b = 2$$

R then contains the points:

$$R = \{(1, 2), (2, 1), (2, 2)\}$$

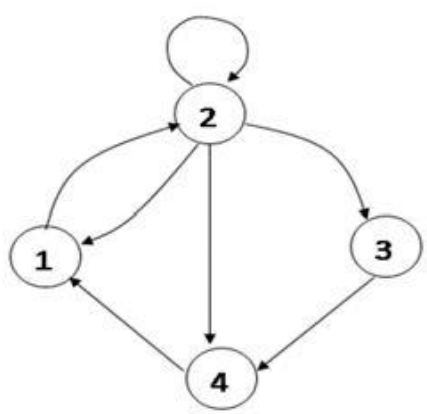
7. **Represent this relation on a set A= {1, 2, 3, 4} with the help of diagraph?**

 Page 594 diagraph page 595 Example 8 and 9

The resulting pictorial representation of R is called a directed Graph or digraph of R. Let A = {1, 2, 3, 4}

R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (2, 4), (3, 4), (4, 2)}

Then the diagraph is as shown below:



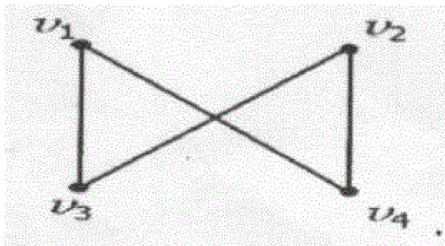
Diagraph and very Good Explanation for In degree Out degree Paths and Types of Relations from Google Drive Discrete Mathematics Folder

Chapter 10: Graphs

Page 735 definitions

1. Define graph **Page 641**

2. **Multi-graph? Or Define Multi Graph?** **<u>Page 642 or page 643 directed Multi graph</u>**
3. Define the isolated vertex in graph? **<u>Page 652</u>**
4. List down the names of 3 graph models? **<u>Page 644</u>**
5. **How many edges are there in a graph of with 10 vertices each of degree six?**
   Slide 35 chapter 10 slides of adil aslam and <u>Page 653 Example 3</u>
6. **Compute the degree of each vertex in the graph** **<u>page 652 definition 3</u>**



   Degree of vertex is the number of edges connecting it. The degree of vertex v1=2, v2=2, v3=2 v4-2
7. Complete graph **<u>Page 655 Table 1 Page 644</u>**
8. What is bi partite graph? **<u>Page 656</u>**
9. **What are two different ways of Representation of Graph?**
   A graph can be represented either as an adjacency matrix or adjacency list and Incident Matrix
10. **How many storage is needed to represent a simple graph with n vertices and m edges using an adjacency matrix?**
    Q70 Page 678

2

SOLUTION

The entries in row i of the incidence matrix represent the edges that are incident to v_i.

A 1 represents that an edge is incident with v_i, while a 0 represents that no edge is incident with v_i.
The sum of the entries in row i then is the number of edges that are incident with v_i.

Using the definition of the degree of a vertex, we then note that the sum of the entries in row i is the degree of vertex v_i decreased by the number of loops. Note: We decrease by the number of loops, because each loop contributes 2 to the degree.

If we then multiply the incidence matrix by the its transpose, the resulting matrix will contain the number of edges between a pair vertices when the vertices are different and the matrix will contain the number edges incident with the vertex on the main diagonal.

RESULT

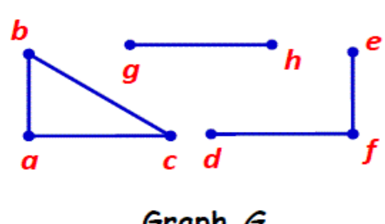
Main diagonal: Number of edges incident with the vertex
Not on the main diagonal: Number of edges between the pair of vertices

11. **What are the connected components of the graph?** **<u>Page 682 and Example 5 and page 683</u>**

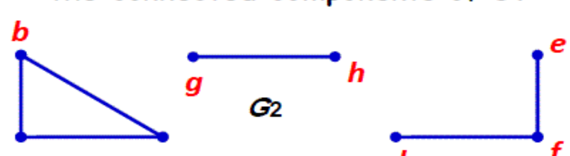
A connected components of a graph is a sub graph in which any two vertices are connected to each other by path. A vertex with no incident edge is itself a connected component.

- A connected component of a graph G is a connected subgraph of G that is not a proper subgraph of another connected subgraph of G.
- A connected component of a graph G is a maximal connected subgraph of G.
- A graph G that is not connected has two or more connected components that are disjoint and have G as their union.

<u>Example</u>

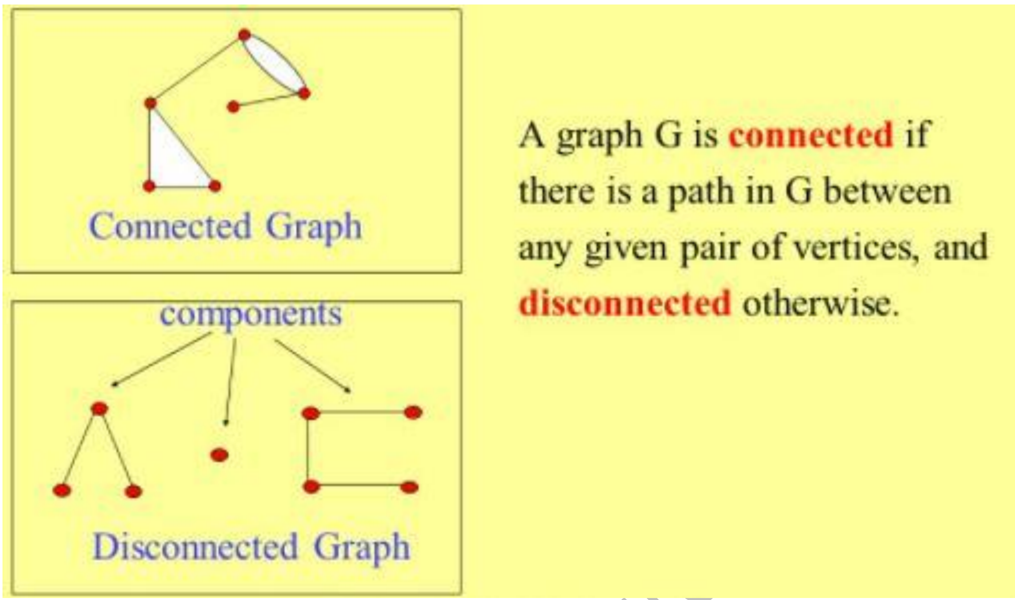


The graph G is the union of three disjoint connected subgraphs $G1$, $G2$, and $G3$. These subgraphs are the connected components of G.



12. **What is connected Graph?** **Page 683**

A **connected graph** is an undirected graph in which every unordered pair of vertices in the graph is connected. Otherwise, it is called a disconnected graph. In a directed graph, an ordered pair of vertices (x, y) is called strongly connected if a directed path leads from x to y.

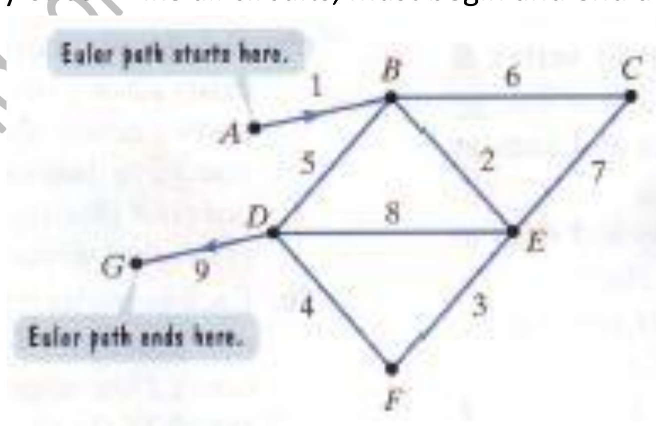


13. **What is meant by Euler path and Euler circuit?** **Page 693**

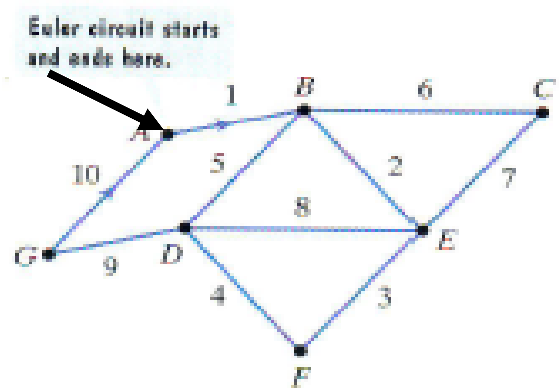
Path vs Circuit???

An **Euler path** in a graph G is a simple path containing every edge of G (Travels through every edge once and only once – Each edge must be traveled and no edge can be retraced.)

•An **Euler circuit** in a graph G: is a simple circuit containing every edge of G (Travels through every edge once and only once – Like all circuits, must begin and end at the same vertex.)



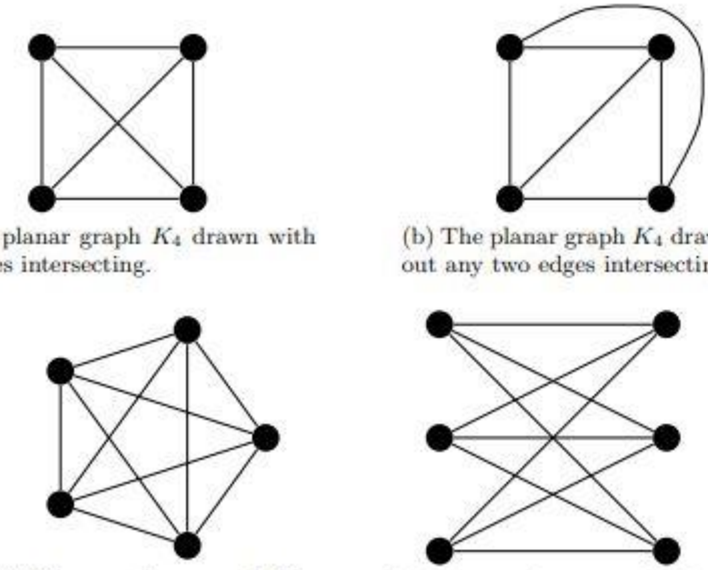
The path A, B, E, F, D, B, C, E, D, G is an Euler path because each edge is traveled once. Trace this path with your pencil. Now try using the numbers along the edges. The voice balloon indicates a starting vertex, A, and the arrow shows which way to trace first. When you arrive at the next vertex, B, take the next numbered edge, 2. When you arrive at the next vertex, E, take the next numbered edge, 3. Continue in this manner until numbered edge 9 ends the path at vertex G.



The path A, B, E, F, D, B, C, E, D, G, A, shown with numbered edges 1 through 10, is an Euler circuit. Do you see why? Each edge is traced only once. Furthermore, the path begins and ends at the same vertex, A. Notice that **every Euler circuit is an Euler path.** However, **not every Euler path is an Euler circuit.**

Some graphs have no Euler paths. Other graphs have several Euler paths. Furthermore, some graphs with Euler paths have no Euler circuits. **Euler's Theorem** is used to determine if a graph contains Euler paths or Euler circuits

14. Define Euler path? **Page 694**
15. Define Euler circuit? **Page 694**
16. **Give an Example of Graph that has Euler Circuit?**
    Example 1 Page 694 OR Long Question Below have Euler Circuit
17. **Draw the graph k_4? Page 719**



(a) The planar graph K_4 drawn with two edges intersecting.

(b) The planar graph K_4 drawn without any two edges intersecting.

(c) The nonplanar graph K_5.

(d) The nonplanar graph $K_{3,3}$

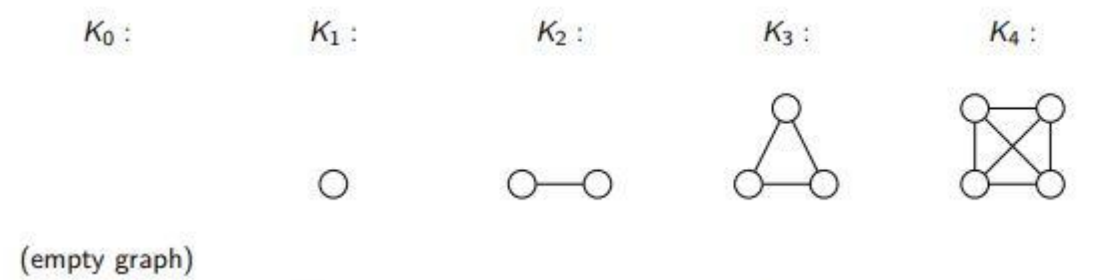
Figure 19.1: Some examples of planar and nonplanar graphs.

18. **Define the graph k_4? This graph, denoted K_4 is defined as the complete graph on a set of size four. page 719**

Definition

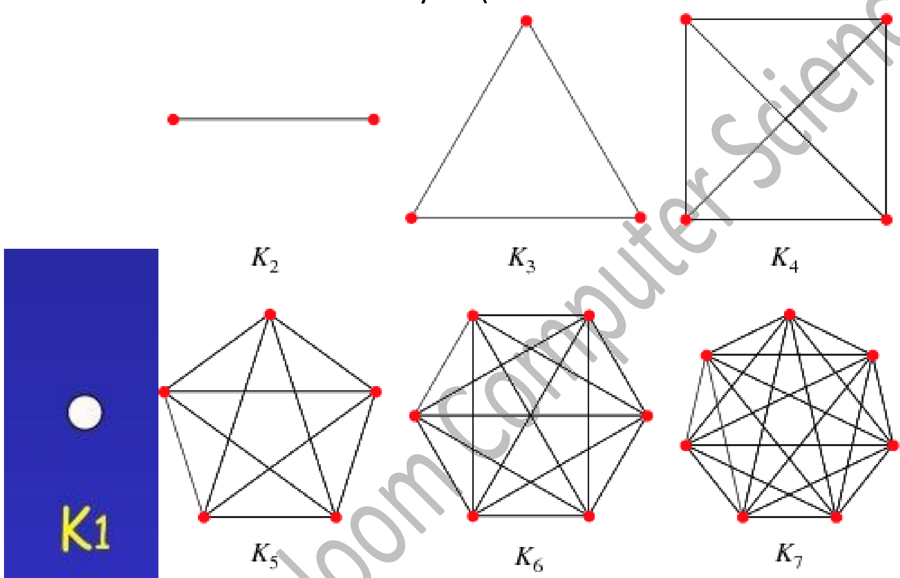
For $n \in \mathbb{N}$, the *complete graph* on n vertices, denoted K_n, is the undirected graph which contains n vertices and exactly one edge between any distinct pair of vertices.

Example



19. Draw graph K4

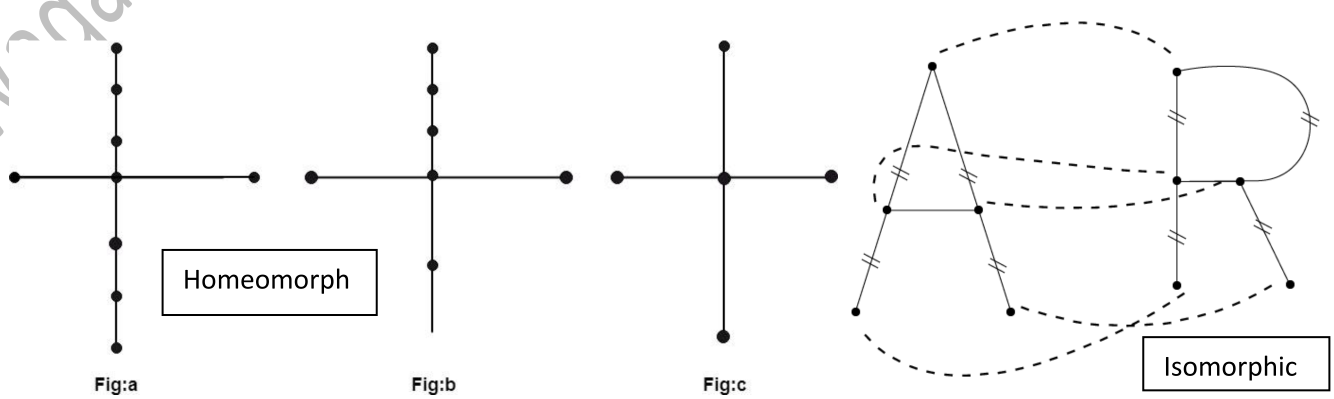
A complete graph on n vertices is a simple graph in which each vertex is connected to every other vertex and is denoted by K_n (k_n means that there are n vertices)



20. Define chromatic number of graph. **<u>Page 728</u>**

21. When Two Graphs are Homeomorphic? Definition Page 736

Two graphs G and G* are said to homeomorphic if they can be obtained from the same graph or isomorphic graphs by this method. The graphs (a) and (b) are not isomorphic, but they are homeomorphic since they can be obtained from the graph (c) by adding appropriate vertices.



Consider a graph G(V, E) and G* (V*,E*) are said to be isomorphic if there exists one to one correspondence i.e. f:V→V* such that {u, v} is an edge of G if and only if {f(u), f(v)} is an edge of G*.

Number of vertices of graph (a) must be equal to graph (b), i.e., one to one correspondence some goes for edges.

22. In Directed Graph, what is the relationship between the vertices and number of edges?

Chapter 10: Graphs & Tree Long Questions

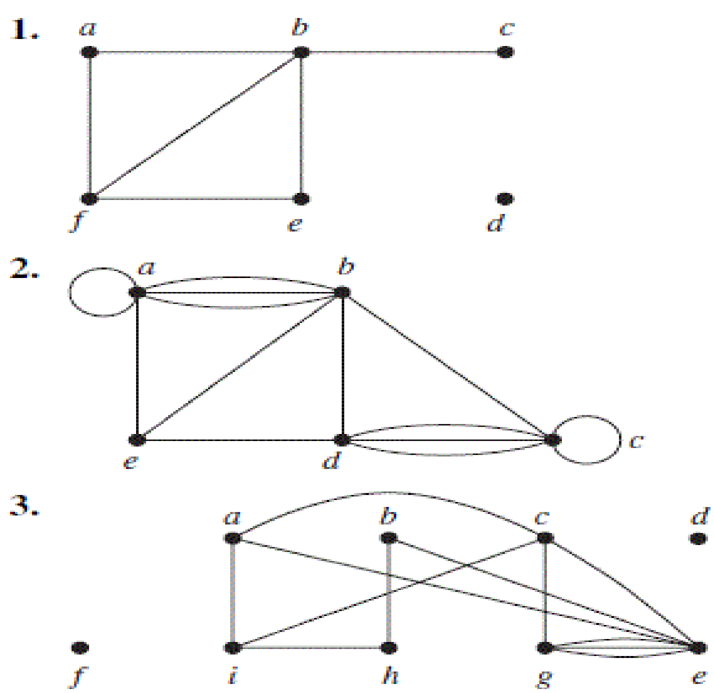
DEFINATIONS 735

Applications of Special Type of Graph??

1. Find the degree of each vertex in the graph below?

Definition 3 Page 652, Example 1 and 2 Page 652 and 653

Answer: Q1, 2, 3 Page 665



The **number of vertices** is the number of points (or number of letters) in the Un-directed graph. Number **of vertices = 6**, the vertices are a, b, c, d, e, f.

Number **of edges is the number** of lines between any pair of vertices, **Number of edges = 6**.

The degree of a vertex is the number of edges that connect to the vertex. A loop at a vertex counts as two edges.

deg (a) = 2, deg (b) = 4, deg (c) = 1, deg (d) = 0, deg (e) = 2, deg (f) = 3

A **vertex is isolated** if the vertex has degree 0. Isolated vertex = d.

A **vertex is pendent** if the vertex had degree 1. Pendent vertex = c.

The **number of vertices** is the number of points (or number of letters) in the Un-directed graph. Number **of vertices = 5**, the vertices are a, b, c, d, e.

Number **of edges is the number** of lines between any pair of vertices, **Number of edges = 11 Lines + 2 Loops = 13**.

The degree of a vertex is the number of edges that connect to the vertex. A loop at a vertex counts as two edges.

deg (a) = 4(lines) + 2 (loop) =6, deg (b) = 6, deg (c) = 4 (lines) + 2(Loop) = 6, deg (d) = 5, deg (e) = 3.

A **vertex is isolated** if the vertex has degree 0. Isolated vertex = none

A **vertex is pendent** if the vertex had degree 1. Pendent vertex = none

The **number of vertices** is the number of points (or number of letters) in the Un-directed graph. Number **of vertices = 9**, the vertices are a, b, c, d, e, f, g, h, I.

Number **of edges is the number** of lines between any pair of vertices, **Number of edges = 12.**

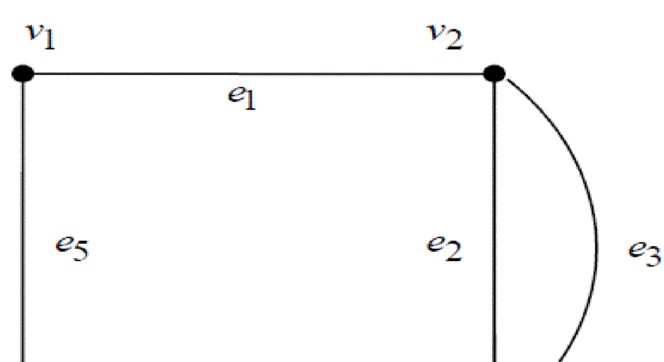
The degree of a vertex is the number of edges that connect to the vertex. A loop at a vertex counts as two edges.

deg (a) = 3, deg (b) = 2, deg (c)= 4, deg (d) = 0, deg (e) = 6, deg (f) = 0, deg (g) = 4, , deg (h) = 2, deg (i) = 3.

A **vertex is isolated** if the vertex has degree 0. Isolated vertex = d, f

A **vertex is pendent** if the vertex had degree 1. Pendent vertex = None

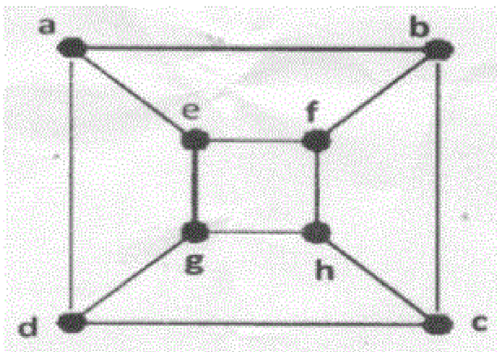
2. Draw a graph if possible with four vertices of degree 1, 2, 3, and 4

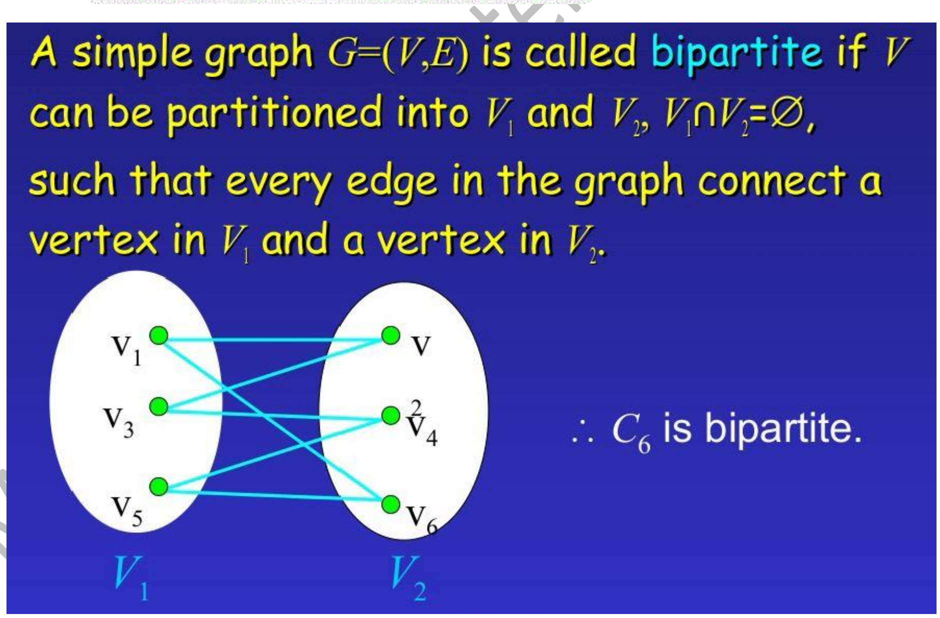


3. **Determine whether the graph is bipartite?**

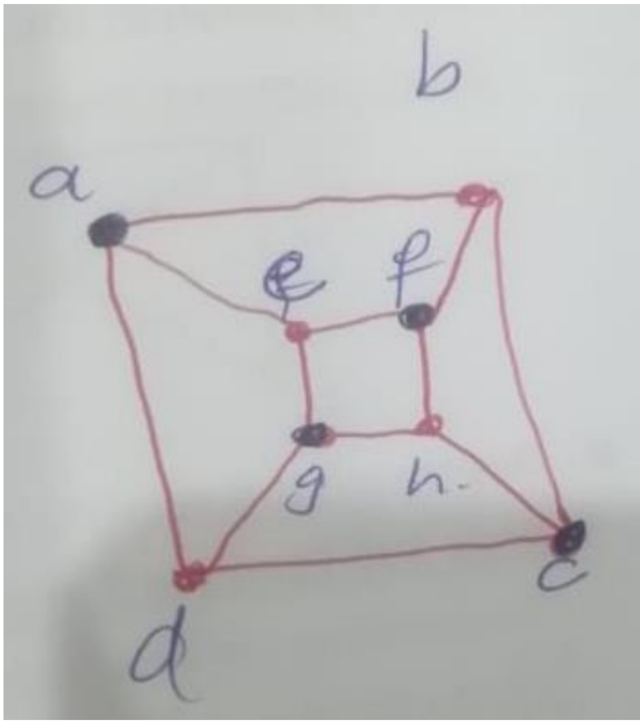
   <u>Complete Bipartite Graph?</u>

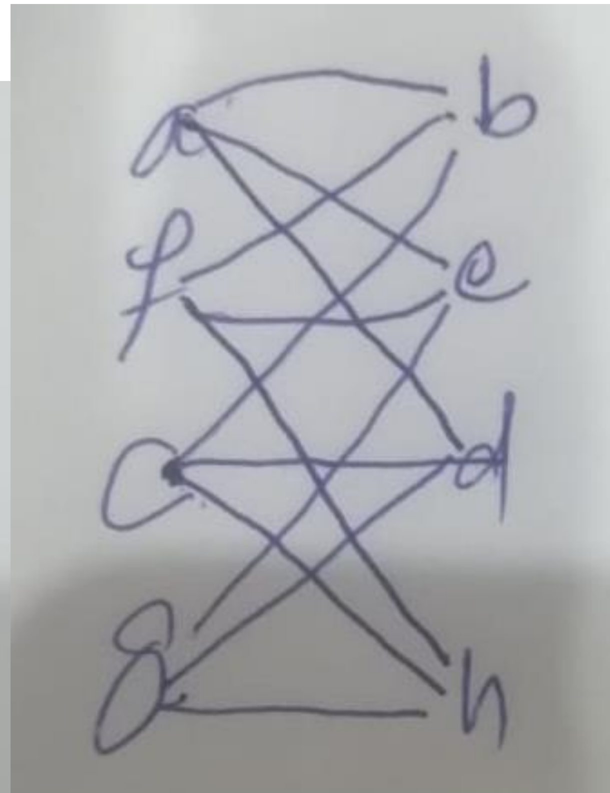
   <u>Topic: Page 656, Definition =6, Example 9, 10, 11, 12, 13 Page 656 to 658</u>



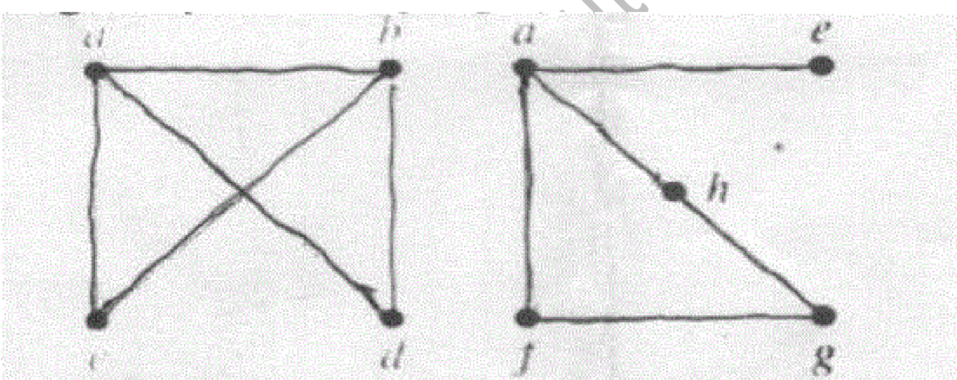


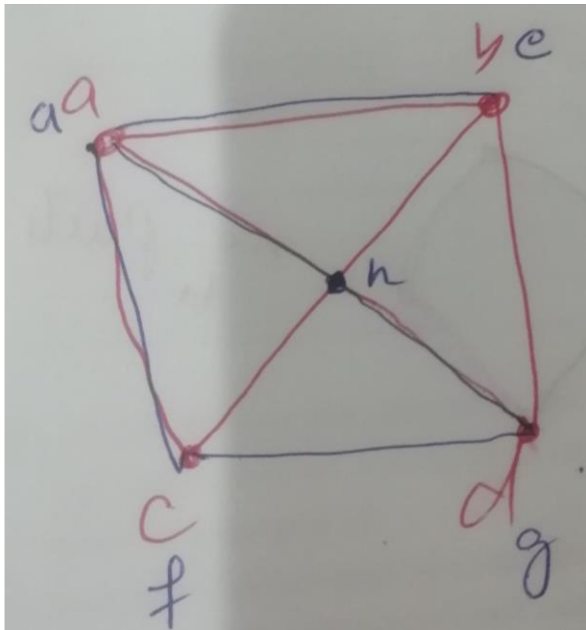
So Above Graph is Bipartite because given graph is divided into two half's as below



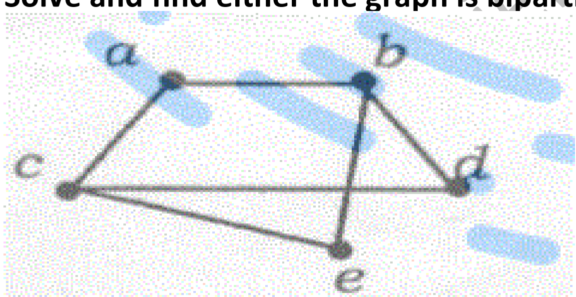


4. **Find the union of the given pair of simple graph?** **<u>Page 664 Definition 9 Example 10</u>**
   <u>For Understanding read carefully slide (54) of chapter 10</u>



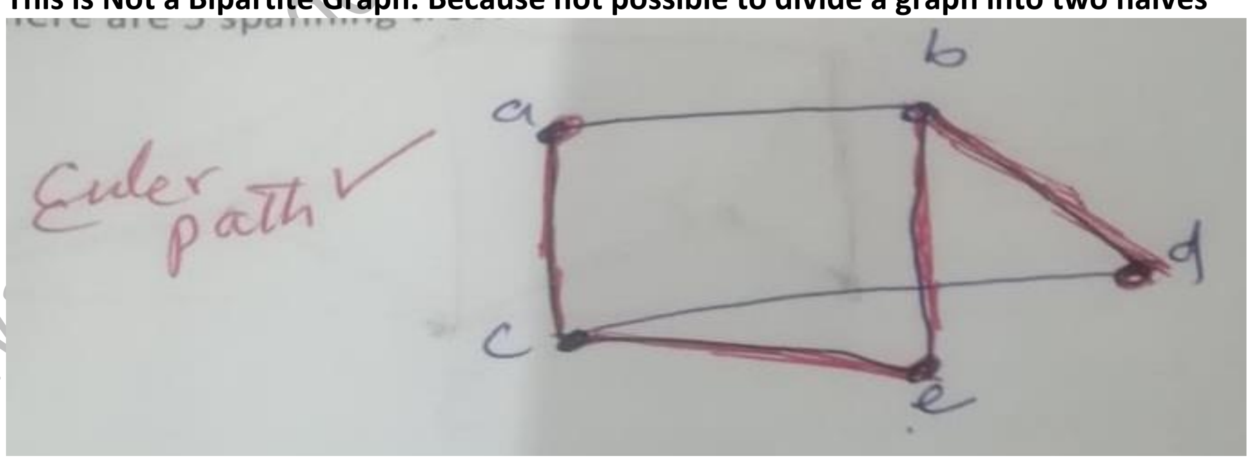


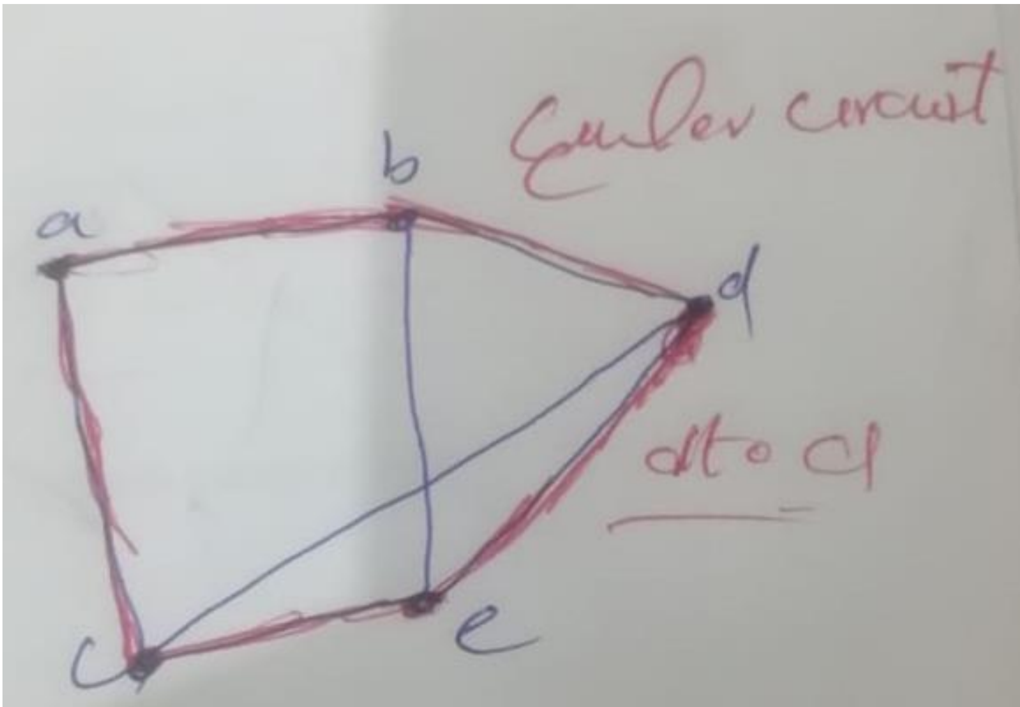
5. **Determine the graph have Hamilton circuit and Hamilton path. Construct Hamilton circuit and path if one exist. Solve and find either the graph is bipartite**



<u>Topic: 698</u>
<u>Definition 2</u>
<u>Example 5, 6, 7</u>
<u>Page 699 to 702</u>
<u>Applications 702</u>
<u>Topic: Page 656</u>
<u>Definition =6</u>
<u>Example 9, 10, 11, 12, 13 Page 656 to 658</u>

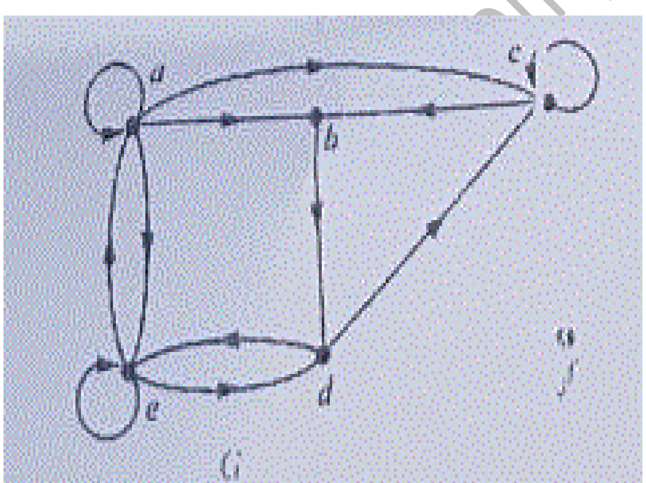
To traverse a tree means to visit all the nodes in some specified order.
This is Not a Bipartite Graph. Because not possible to divide a graph into two halves





6. **Find out the in degree and out degree of the given graph? Also find whether the given graph has Hamiltonian or Euler Tour. Show visually also represent the graph into adjacency matrix.**

<u>**Topic Page :** 654,Example 5, 6, 7, 8 Page 654 to 656,**Topic Page=** 693, Definition 1= Page 694, Examples 1, 2, 3, 4, **Applications, Theorem 1**, 2 Page 694 to 698, **Adjacency Matrix: 669** Page Example 3, 4, 5 Page 669 to 672, **Topic: Page 668**, Example 1, 2 Page 668, 669</u>



Adjacency Matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In – Degrees in Graph G are deg$^-$(a)=2, deg$^-$(b)=2, c=3, d=2, e= 3, f=0

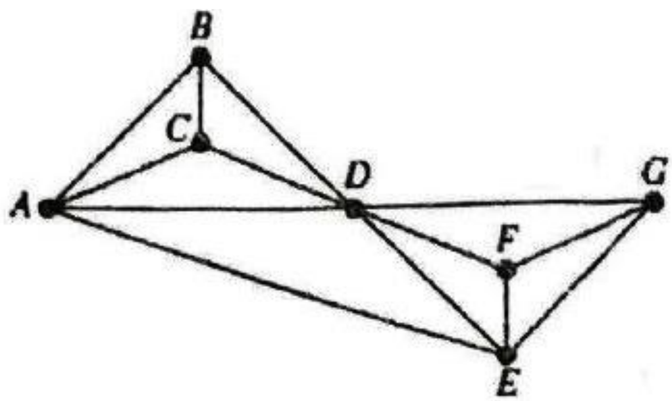
Out – Degrees in Graph G are deg$^+$(a)=4, deg$^+$(b)=1, c=2, d=2, e= 3, f=0

A Hamiltonian Path: in a graph G is a simple circuit that passes through every vertex in G exactly once. <u>a, c, b, d, e but there is no path toward f so there is no Hamiltonian Path</u>

•**A Hamiltonian Circuit** in a graph G is a simple path that passes through every vertex in G exactly once and which start and ends on the same vertexes. a , c, b, d, e but there is no Path to visit f and move back to start point a so there is no Hamiltonian Circuit

Page 654 Example 4 PAGE 669 for Adjacency Matrix 688 Hamiltonian page 698

7. **Consider the following graph?**
   Definition 4 Page 685, Definition 5 Page 686, Example 10, 11, 12 Page 686, 687
   Answer: Relevant to This Graph is: Figure 3 Page 682, Definition Page 736
   Strongly Connected Component of an undirected graph G: a maximal strongly connected subgraph of g.



Final all strongly connected components of the graph

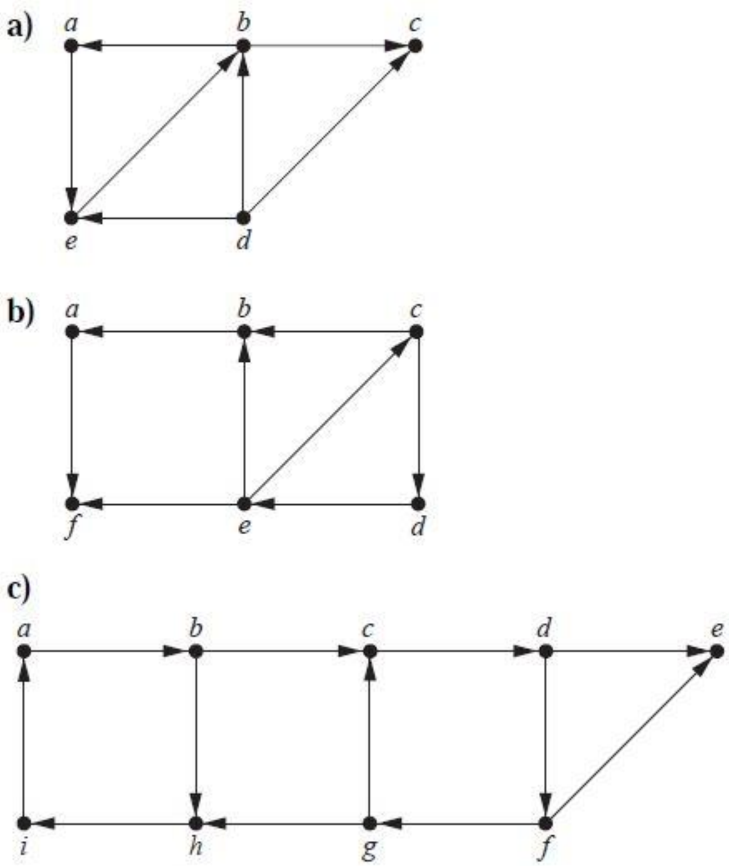
Q11, 13, 15 Page 689, 690 Assignment

A path in a directed graph G is a sequence of edges in G

A directed graph is **strongly connected**, if for every pair of vertices a and b in the graph, there exists a path from a to b and there exists a path from b to a.

A **strongly connected component** is a part of the graph where ever pair of element in this part is connected by a path (in both directions)

8. **Find the strongly connected component of the Graph** **Q14 Answer: 689 Page**



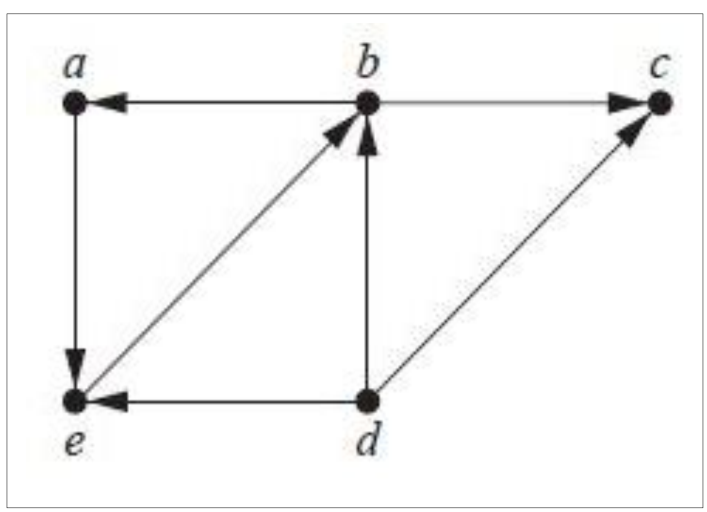
Let us first determine the vertices and edges from the given graph

V = {a, b, c, d, e}

E = {(a, e), (b, a), (b, c), (d, b), (d, c), (d, e), (e, b)}

Let us determine a path between every pair of vertices (if it exists)

Vertices	**Path**
	a, e, b
From a to b	a, e, b, c
From a to c	Does not Exist
From a to d	a, e
From a to e	b, a
From b to e	b, c
From b to c	Does not Exist
From b to d	a, a, e
From b to e	Does not Exist
From c to a	Does not Exist
From c to b	Does not Exist
From c to d	Does not Exist
From c to e	d, b, a
From d to a	d, b
From d to b	d, c
From d to c	d, e
From d to e	e, b, a



From e to a	e, b
From e to b	e, b, c
From e to c	Does not Exist
From e to d	

We then note that there are no paths from c to another vertex, thus c forms a **strongly connected component on itself** {c}

We then note that there are no paths from a vertex to d, thus d forms a strongly connected component on itself. {d}

The remaining vertices are a, e and b. there are paths between every pair of these vertices (in both directions) and this these vertices are together in a strongly connected component. {a, b, e}

Thus the strongly connected components are {a, b, e}, {c}, {d}

B Part: Let us first determine the vertices and edges from the given graph

V = {a, b, c, d, e, f}

E = {(a, f), (b, a), (c, b), (c, d), (d, e), (e, b), (e, c), (e, f)}

Let us determine a path between every pair of vertices (if it exists)

Vertices	**Path**
From a to b	Does not Exist
From a to c	Does not Exist
From a to d	Does not Exist
From a to e	Does Not Exist
From a to f	a, f
From b to a	b, a
From b to c	Does not Exist
From b to d	Does not Exist
From b to e	Does not Exist
From b to f	b, a, f
From c to a	c, b, a
From c to b	c, b
From c to d	c, d
From c to e	c, d, e
From c to f	c, d, e, f
From d to a	d, e, b, a
From d to b	d, e, b
From d to c	d, e, c
From d to e	d, e
From d to f	d, e, f
From e to a	e, b, a
From e to b	e, b
From e to c	e, c

From e to d	e, c, d
From e to f	e, f
From f to a	Does not Exist
From f to b	Does not Exist
From f to c	Does not Exist
From f to d	Does not Exist
From f to e	Does not Exist

We then note that there are no paths from f to another vertex, thus f forms a strongly connected component on itself {f}

The only path from a to another vertex is from a to f. since f forms a strongly connected component on itself, a also has to form a strongly connected component on itself. {a}

The only path from b to another vertex are from b to a and from b to f. since f and a form strongly connected component on themselves, b also has to form a strongly connected component on itself {b}

The remaining vertices are c, d and e. there are paths between every pair of these vertices (in both directions) and this these vertices are together in a strongly connected component. {c, d, e}

Thus the strongly connected components are {a}, {b}, {c, d, e}, {f}

Let us first determine the vertices and edges from the given graph

V = {a, b, c, d, e, f, g, h, i}

E = {(a, b), (b, c), (b, h), (c, b), (d, e), (d, f), (f, g), (g, c), (g, h), (h, i), (i, a)   }

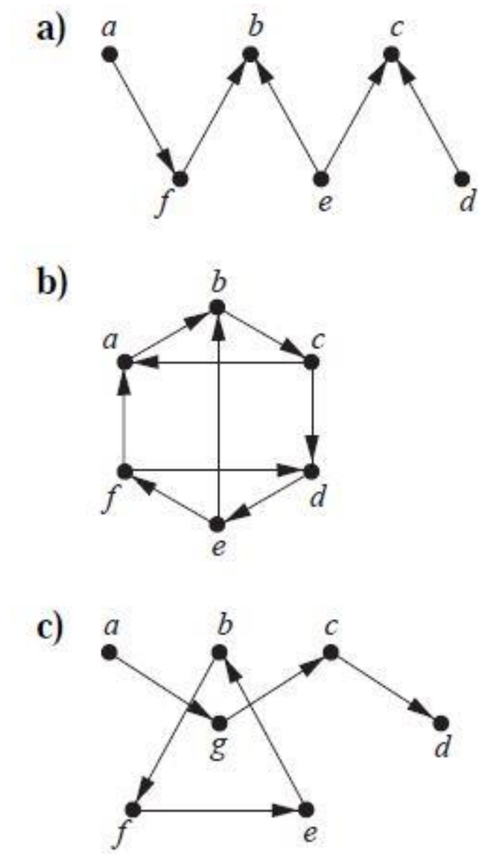
Let us determine a path between every pair of vertices (if it exists)

We then note that there are no paths from e to another vertex, thus e forms a strongly connected component on itself {e}

The remaining vertices are a, b, c, d, f, g, h and i. there are paths between every pair of these vertices (in both directions), a, b, c, d, f, g, h, I, a forms a circuit. Thus these vertices are together in a strongly connected component. {a, b, c, d, f, g, h, i}

Thus the strongly connected components are {a, b, c, d, f, g, h, i}, {e}

9. **Determine whether each of these graphs is strongly connected or not, whether it is weekly connected   Page 690 Q12**



A Path in a directed Graph G is a sequence of edges in G?

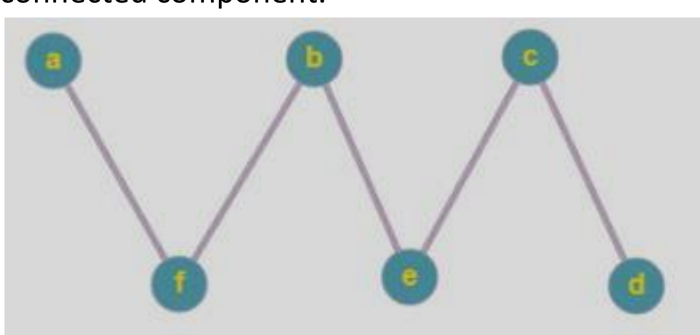
A **directed graph is strongly connected**, if for every pair of vertices a and b in the graph, there exists a path from a to b and there exists a path from b to a.

A **directed graph is weakly** connected if for every pair of vertices a and b in the graph, there exist a path from a to b in the underlying undirected graph.

A **connected component** is a part of the graph where every pair of elements in this part is connected by the path.

a) **Let us first determine the vertices and edges from the given graph. V = {a, b, c, d, e, f}, E = {(a, f), (d, c), (e, b), (e, c), (f, b)}.** The graph is not strongly connected, because there is no path from c to d as there are no edge with c as their initial vertex.

The directed graph is weekly connected, because the underlying undirected graph (given below) has only 1 connected component.



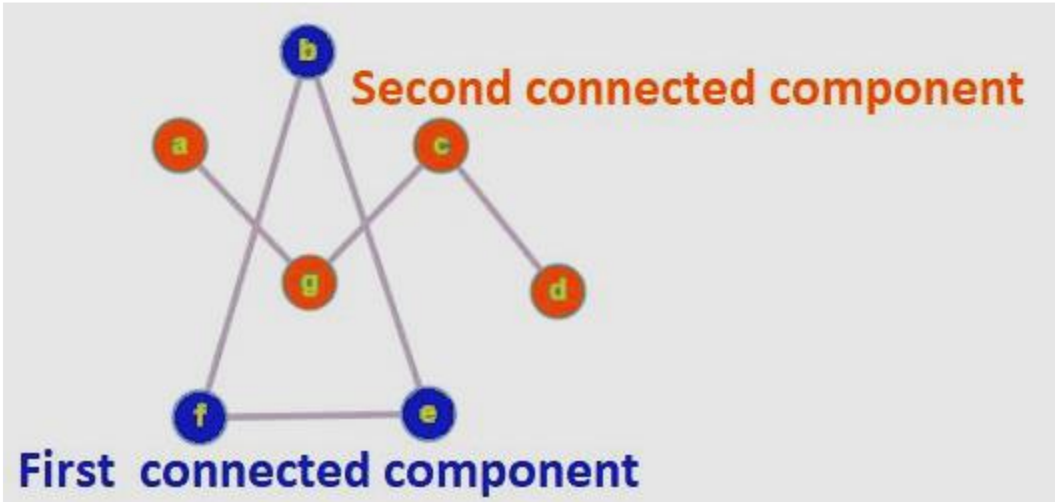
b) **Let us first determine the vertices and edges from the given graph. V = {a, b, c, d, e, f}, E = {(a, b), (b, c), (c, d), (d, e), (e, b), (e, f), (f, a), (f, d)}.** Let us determine a path between every pair of vertices (if it exist).

Vertices	**Path**
From a to b	a, b
From a to c	a, b, c
From a to d	a, b, c, d
From a to e	a, b, c, d, e
From a to f	a, b, c, d, e, f
From b to a	b, c, d, e, f, a
From b to c	b, c
From b to d	b, c, d
From b to e	b, c, d, e
From b to f	b, c, d, e, f
From c to a	c, d, e, f, a
From c to b	c, d, e, f, a, b
From c to d	c, d
From c to e	e, d, e
From c to f	c, d, e, f
From d to a	d, e, f, a
From d to b	d, e, b
From d to c	d, e, b, c
From d to e	d, e
From d to f	d, e, f
From e to a	c, f, a
From e to b	e, b
From e to c	e, b, c
From e to d	e, b, c, d
From e to f	e, f
From f to a	f, a
From f to b	f, a, b
From f to c	f, a, b, c
From f to d	f, d
From f to e	f, d, e

The directed graph is strongly connected, because there is path from every vertex to every other vertex.

c) **Let us first determine the vertices and edges from the given graph. V = {a, b, c, d, e, f, g}, E = {(a, g), (b, f), (c, d), (e, b), (f, e), (g, c)}.**

The **directed graph is not strongly** connected because there is no path from d to c as there are no edges with d as their initial vertex. **The directed graph** is not weekly connected, because the underlying undirected graph (give below) has more than 1 connected component.



Chapter 11: Trees

1. Define tree **page 746 Definition and Example 1**
2. Leaf? Page 748
3. What is the height of the rooted tree? OR How the Height of Tree is calculated? Page 753
4. **Height of tree is seven? Find the sum of the height.** Example 10 Page 753
   The height of a tree: length of the longest path from root to a leaf.
   Tree from image of height seven
   Exercise Q45 page 757
5. Tree transversal? OR Define Tree Transversal Page 772
6. **Define Pre Order Transversal in a tree?**
   Page 772 and Understand Tree.pdf slide 17 file in Google drive
   Assignment Q16, 17, 22, 23, 24 Page 784
7. **Define Binary Search Tree**
   Page 757, Example 1 Page 758
8. What are cyclic graph? Give an example of Cyclic Graph?
9. **What is the prefix notation of the given expression (a / b (b * c + d) ^f – e)**
   Topic Page 779, 780 Example 6, 7, 8, 9 Page 780 to & 782
10. **Define the spanning tree of the graph? Page 785**
    A spanning tree of a Graph is a sub graph of G that is a tree containing every vertex of G.
    <u>A spanning tree of a Graph G (undirected) is a sub graph that is a tree which includes all of the vertices of G with minimum possible number of edges.</u>
11. **Difference b/w tree and graph?**

BASIS FOR COMPARISON	TREE	GRAPH
Path	Only one between two vertices.	More than one path is allowed.
Root node	It has exactly one root node.	Graph doesn't have a root node.
Loops	No loops are permitted.	Graph can have loops.
Complexity	Less complex	More complex comparatively
Traversal techniques	Pre-order, In-order and Post-order.	Breadth-first search and depth-first search.
Number of edges	n-1 (where n is the number of nodes)	Not defined
Model type	Hierarchical	Network

Chapter 11: Trees Long Questions

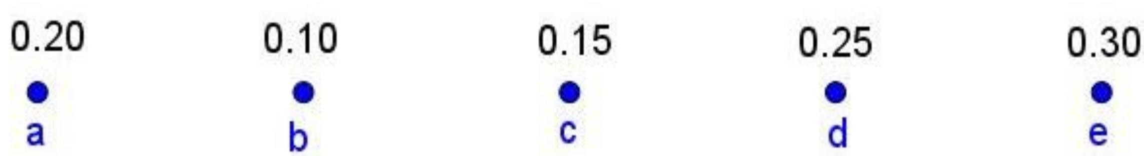
1. **Use Huffman coding to encode the symbols with given frequencies. a:0.2, b:0.10, c:0.15, d:0.25, e:0.30. what is the average number of characters required to encode a character.**

   <u>Example 5 page 764 for proper understanding Huffman Code data compression video</u>

   Exercise Q23 page 770 Assignment Question Q22 and 24 Page 770

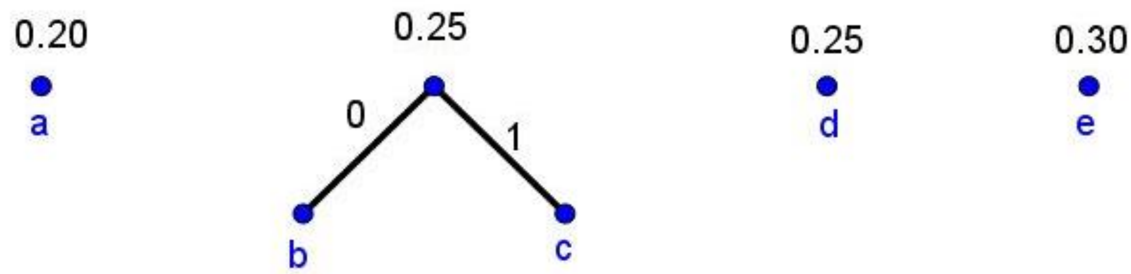
   Huffman Coding: a: 0.20, b: 0.10, c: 0.15, d: 0.25, e: 0.30

   We have been given 5 symbols. We will then draw a forest of 5 trees containing 1 vertex each. While the vertex has the above names and are labelled with the given values.



   We note that b and c has the smallest labels. We will then replace their trees with a new tree that has a root with a left and right child. The edge to the left child is labeled 0 and the edge to the right child is labeled 1. The left child is named c (highest label of the two) and the right child is named b (lowest label of the two).

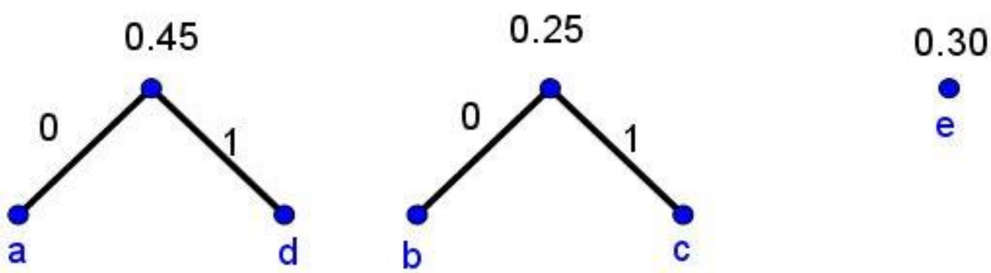
   The label of the new tree is the sum of the labels of the trees that were replaced.



We then note that the trees with the smallest labels are a, b + c and d. Note: b + c represents the tree with leaves b and c.

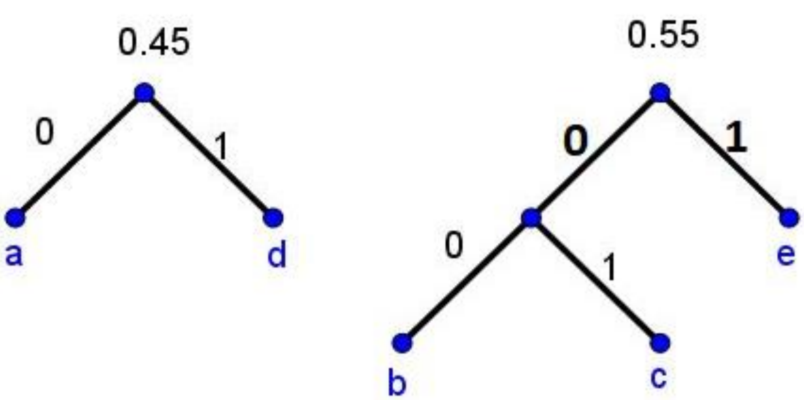
I will choose a and b next (Note: you could also choose a and b + c, which will result in different coding).

We will then replace their trees with a new tree that has a root with a left and right child. The edge to the left child is labeled 0 and the edge to the right child is labeled 1. The left child is named d (highest label of the two) and the right child is name a(lowest label of the two). The label of the new tree is the sum of the labels of the tree that were replaced.



We then note that the trees with the smallest labels are b + c and e.

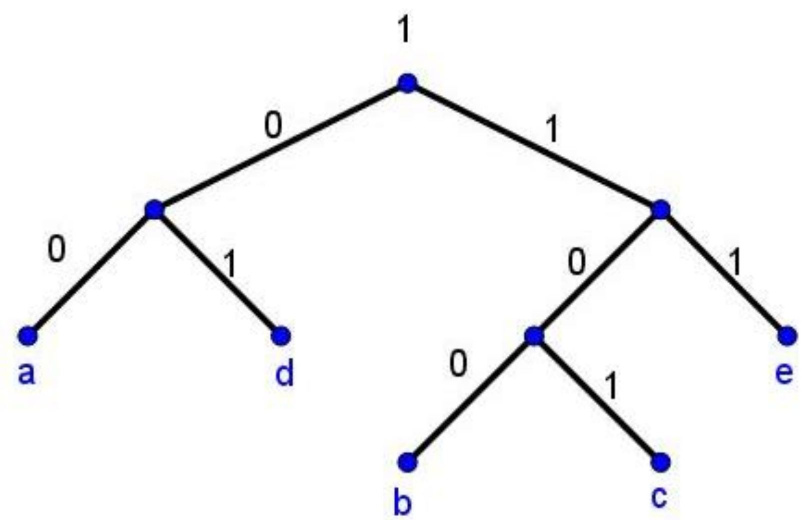
We will then replace their trees with a new tree that has a root with a left and right child. The edge to the left child is labeled 0 and the edge to the right child is labeled 1. The left child is named as e (highest label of the two) and the right child is the tree b + c (lowest label of the two). The label of the new tree is the sum of the labels of the trees that were replaced



We then note that the trees with smallest labels are b + c+ e and a + d.

We will then replace their trees with a new tree that has a root with a left and right child. The edge to the left child is labeled 0 and the edge to the right child is labeled as 1. The left child is named b + c+ e (highest label of the two) and the right child is the tree a + d(lowest label of the two).

The label of the new tree is the sum of the labels of the trees that were replaced.



The encoding of the letter is then sequence of the labels of the edges in the path from the root of the letter. a: 00, b: 100, c: 101, d: 01,e: 11.

The weight of the letter is the number of bits in the code of the letter

W_a: 2, W_b: 3, W_c: 3, W_d: 2, W_e: 2.

The average number of bits required is then the sum of the products of the weight and the frequencies.

Average number of bits required=

$$\begin{aligned}\text{Average number of bits required} &= \sum_{i\in\{a,b,c,d,e\}} w_i \cdot f_i \\ &= 2\cdot 0.20 + 3\cdot 0.10 + 3\cdot 0.15 + 2\cdot 0.25 + 2\cdot 0.30 \\ &= 2.25\end{aligned}$$

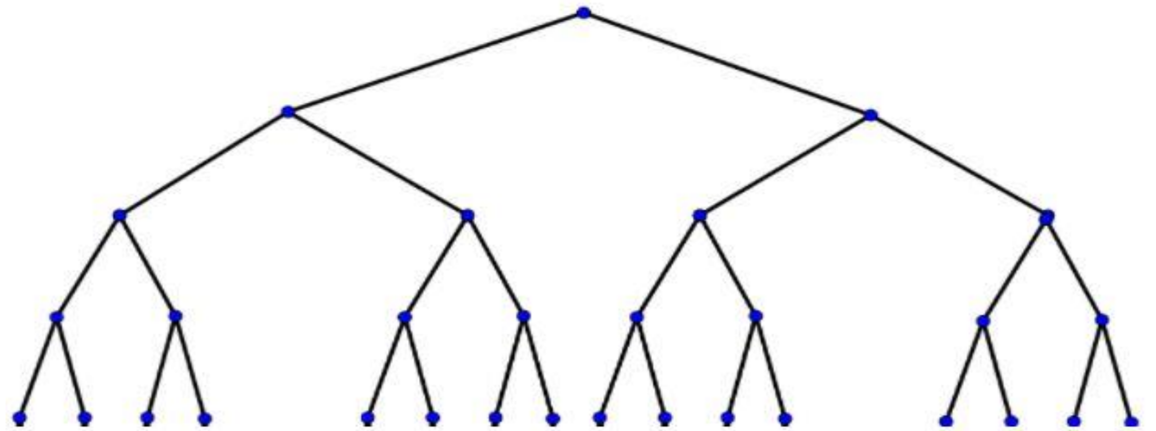
2. **Construct a complete binary tree of height 4 and complete 3 ary tree of height 3.**

Page 756 Q27 Answer S-69

Complete binary Tree of height 4:

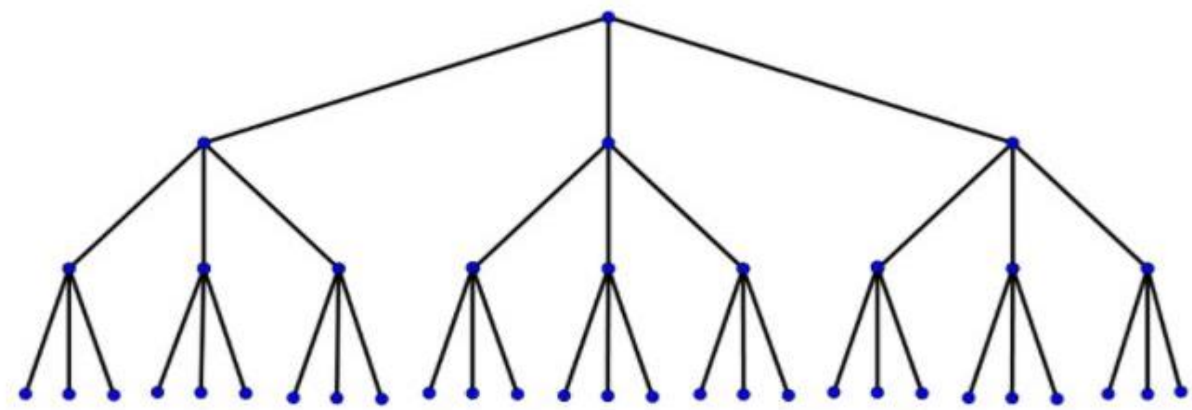
A binary tree is a 2 – ary tree. m = 2

The complete binary tree of height 4 has only leaves in the last 4th level and all vertices in the previous level need to have exactly 2 children.



Complete 3 – ary tree of hight 3. m=3

The complete 3 – ary tree of hight 3 has only leaves in the last 3rd level and all vertices in the previous levels need to have exactly 3 children.



<u>Assignment Q28 Page 756</u>

<u>Construct a complete binary tree of height 5 and complete 3 ary tree of height 2.</u>

3. **Using alphabetical order construct a binary search tree for the words in the sentence. " The Quick brown fox jump over the lazy dog" 769 Q5**

Alphabetical order

Alphabetical or *lexicographic order* is the order of the dictionary:

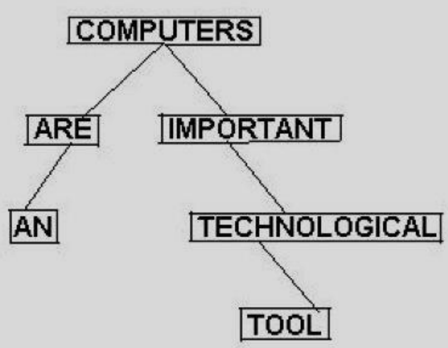
a) start with an ordered set of symbols X = {a,b,c, …}. X can be infinite or finite.

b) Let $\alpha = x_1x_2\ldots x_m$ and $\beta = y_1y_2\ldots y_n$ be strings over X. Then define $\alpha < \beta$ if

- $x_1 < y_1$
- or if $x_j = y_j$ for all j, $1 \leq j \leq k$, for some k such that $1 \leq k \leq \min\{m,n\}$ and $x_{j+1} < y_{j+1}$
- or if $m \leq n$ and $x_j = y_j$ for all j, $1 \leq j \leq m$

Binary search trees

- Data are associated to each vertex
- Order data alphabetically, so that for each vertex v, data to the left of v are less than data in v
- and data to the right of v are greater than data in v
- Example: "Computers are an important technological tool"



Given words: *the*, *quick*, *brown*, *fox*, *jumps*, *over*, *the*, *lazy*, *dog*

We start with the word *the*. This is the first word and then we will assign this word as the root of the tree.

The next word in the given list is *quick*. *quick* occurs before *the* in alphabetical order, thus *quick* then needs to be the left child of *the*.

The next word in the given list is *brown*. *brown* occurs before *the* in alphabetical order, thus *brown* needs to be to the left of *the*. *brown* occurs before *quick* in the alphabetic order, thus *brown* needs to be the left child of *quick*.

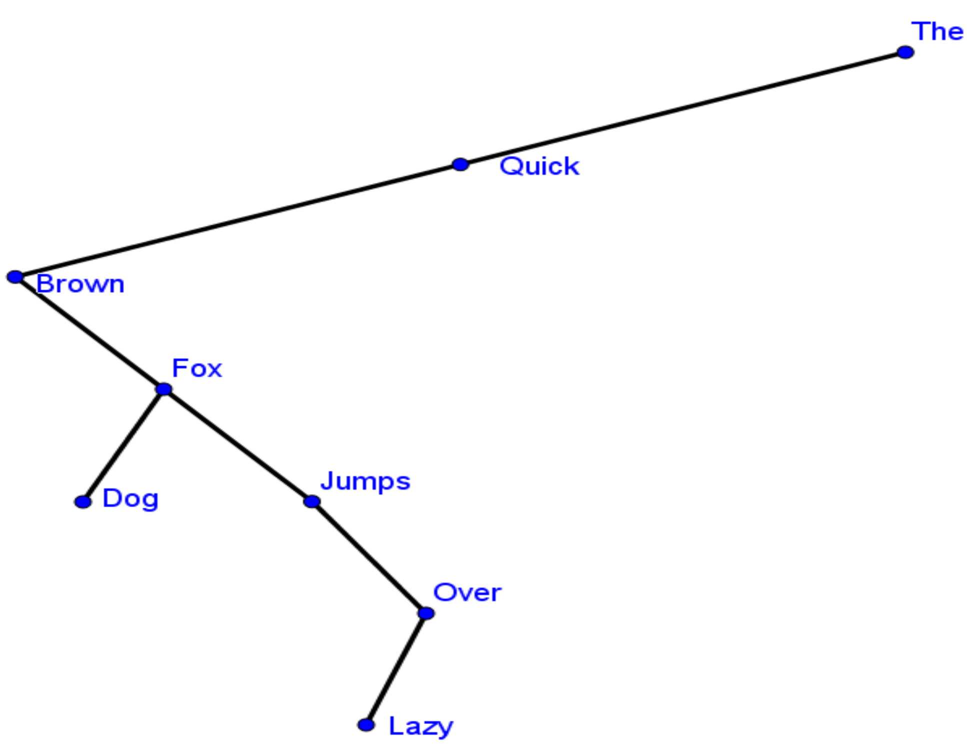
The next word in the given list is *fox*. *fox* occurs before *the* in alphabetical order, thus *fox* needs to be to the left of *the*. *fox* occurs before *quick* in alphabetical order, thus *fox* needs to be to the left of *quick*. *fox* occurs after *brown* in the alphabet order, thus *fox* needs to be the right child of *brown*.

The next word in the given list is *jumps*. *jumps* occurs before *the* in alphabetical order, thus *jumps* needs to be to the left of *the*. *jumps* occurs before *quick* in alphabetical order, thus *jumps* needs to be to the left of *quick*. *jumps* occurs after *brown* in alphabetical order, thus *jumps* needs to be to the right of *brown*. *jumps* occurs after *fox* in alphabetical order, thus *jumps* needs to be the right child of *fox*.

The next word in the given list is *over*. *over* occurs before *the* in alphabetical order, thus *over* needs to be to the left of *the*. *over* occurs before *quick* in alphabetical order, thus *over* needs to be to the left of *quick*. *over* occurs after *brown* in alphabetical order, thus *over* needs to be to the right of *brown*. *over* occurs after *fox* in alphabetical order, thus *over* needs to be to the right of *fox*. *over* occurs after *jumps* in alphabetical order, thus *over* needs to be the right child of *jumps*.

The next word in the given list is *lazy*. *lazy* occurs before *the* in alphabetical order, thus *lazy* needs to be to the left of *the*. *lazy* occurs before *quick* in alphabetical order, thus *lazy* needs to be to the left of *quick*. *lazy* occurs after *brown* in alphabetical order, thus *lazy* needs to be to the right of *brown*. *lazy* occurs after *fox* in alphabetical order, thus *lazy* needs to be to the right of *fox*. *lazy* occurs after *jumps* in alphabetical order, thus *lazy* needs to be to the right of *jumps*. *lazy* occurs before *over* in alphabetical order, thus *lazy* needs to be the left child of *over*.

The final word in the given list is *dog*. *dog* occurs before *the* in alphabetical order, thus *dog* needs to be to the left of *the*. *dog* occurs before *quick* in alphabetical order, thus *dog* needs to be to the left of *quick*. *dog* occurs after *brown* in alphabetical order, thus *dog* needs to be to the right of *brown*. *dog* occurs before *fox* in alphabetical order, thus *dog* needs to be the left child of *fox*.

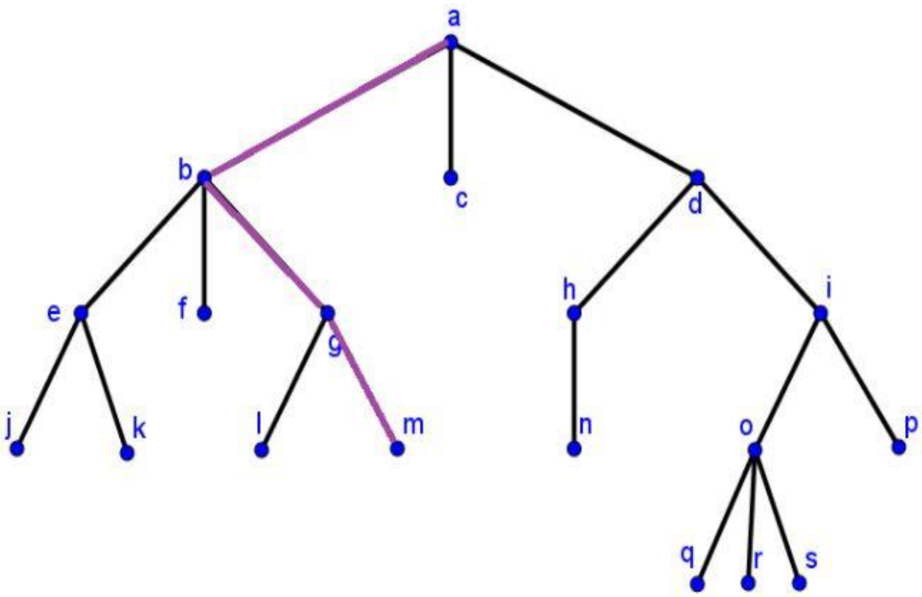


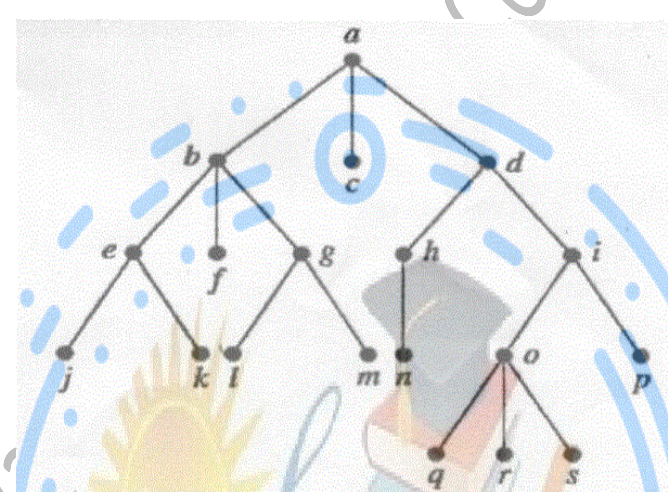
4. **Write these about the rooted as illustrated? Q4 PAGE 755**

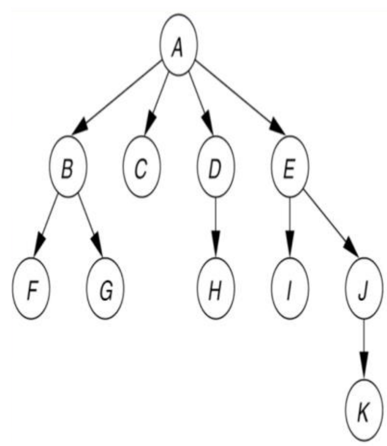
Depth of a node: Number of ancestors (Depth of “a” is zero, depth of “d” is 1, depth of “i” is 2 and “o” is 3 and q is 4)

Height of a tree: Maximum depth of any node

- What is the height of the given tree? (Height of below tree is 4)
- what are the internal vertices of the tree
- which vertices are ancestors of q **(a, d, i, o)**
- WHICH vertices are descendants of q: **Descendants of q are all vertices whose ancestor is q (There is No Descendants of q)**







Node	Height	Depth
A	3	0
B	1	1
C	0	1
D	1	1
E	2	1
F	0	2
G	0	2
H	0	2
I	0	2
J	1	2
K	0	3

a) **The** root of the tree is the vertex at the top of the tree. Root =a.

b) **The** internal vertices are the vertices that have children.
Internal vertices = a, b, d, e, g, h, I, o

c) **The l**eaves are all vertices with no children. Leaves = c, f, j, k, l, m, n, p, q, r, s

d) The children of j are all vertices below j that are connected to j by an edge. Since j is not connected to vertices lower than j in the tree, j does not have any children.
Children of j = None

e) The parent of h is the vertex above h that is connect to h by an edge. Parent of h = d.

f) **The sibling** of o are the vertices who have the same parent as o. siblings of o = p

g) The ancestors of m are all vertices in the path from the root to m (except m itself).
Ancestors of m = a, b, g.

h) **The descendants o**f b are all vertices whose ancestor is b.
Descendants of b = e, f, g, j, k, l, m

Q3: Page 755

a) Which vertex is the root?

b) Which vertices are internal?

c) Which vertices are leaves?

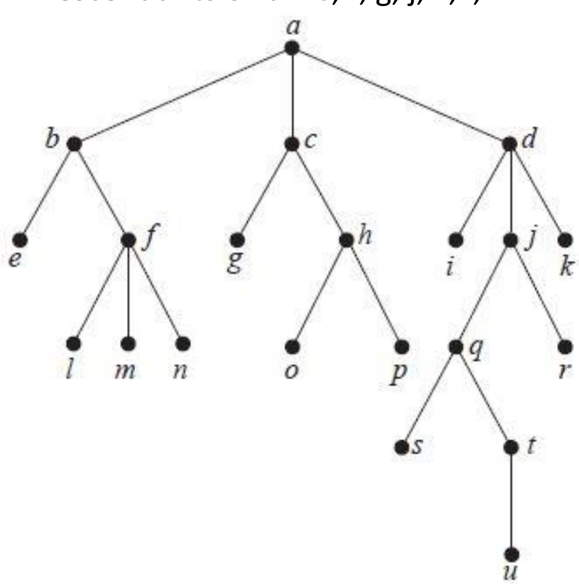
d) Which vertices are children of *j*?

e) Which vertex is the parent of *h*?

f) Which vertices are siblings of *o*?

g) Which vertices are ancestors of *m*?

h) Which vertices are descendants of *b*?



a) Vertex *a* is the root, since it is drawn at the top.
b) The internal vertices are the vertices with children, namely *a*, *b*, *c*, *d*, *f*, *h* , *j* , *q* and *t*.
c) The leaves are the vertices without children, namely e, *g*, i, *k*, *l*, m, n, o, p, *r*, *s*, and *u*.
d) The children of *j* are the vertices adjacent to *j* and below *j*, namely *q* and *r*.
e) The parent of *h* is the vertex adjacent to *h* and above *h*, namely c.
f) Vertex *o* has only one sibling, namely *p*, which is the other child of o's parent, *h*.
g) The ancestors of *m* are all the vertices on the unique simple path from *m* back to the root, namely *f*, *b*, and *a*.
h) The descendants of *b* are all the vertices that have *b* as an ancestor, namely e, *f*, *l*, m, and *n*

10. **How many different spanning trees does each of these simple graphs have** page 785 Q11 OR For spanning Tree Understand Tree slides and understand counting.pdf in Google drive Page 795 Q7,11 and video minimum spanning tree No of spanning trees video

(Q8, 9, 10, 12 Assignment)

K_3 k_4 $k_{2,2}$ C_5

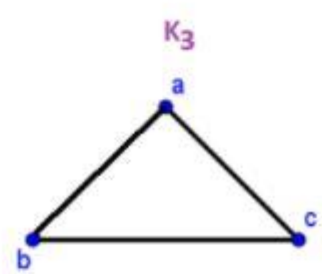
(a) 3
(b) 16
(c) 4
(d) 5

(a) K_3 is a graph with 3 vertices and an edge between every pair of vertices, which results in 3 edges (see graph below).

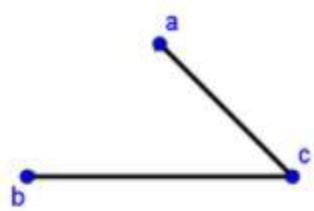
The spanning tree will then contain the same 3 vertices and $3 - 1 = 2$ edges. Thus one of the three edges should be removed.

A tree cannot contain any simple circuits, while we note that K_3 contains a triangle and thus we will have to remove one of the three sides of the triangle.

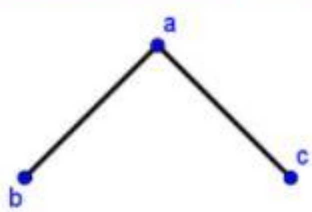
Since we have the choice of deleting one of 3 sides of the triangle, there are then 3 possible spanning trees.



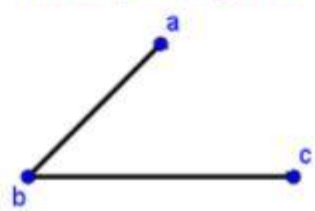
First spanning tree



Second spanning tree



Third spanning tree



(b) K_4 is a graph with 4 vertices and an edge between every pair of vertices, which results in 6 edges (see graph below).

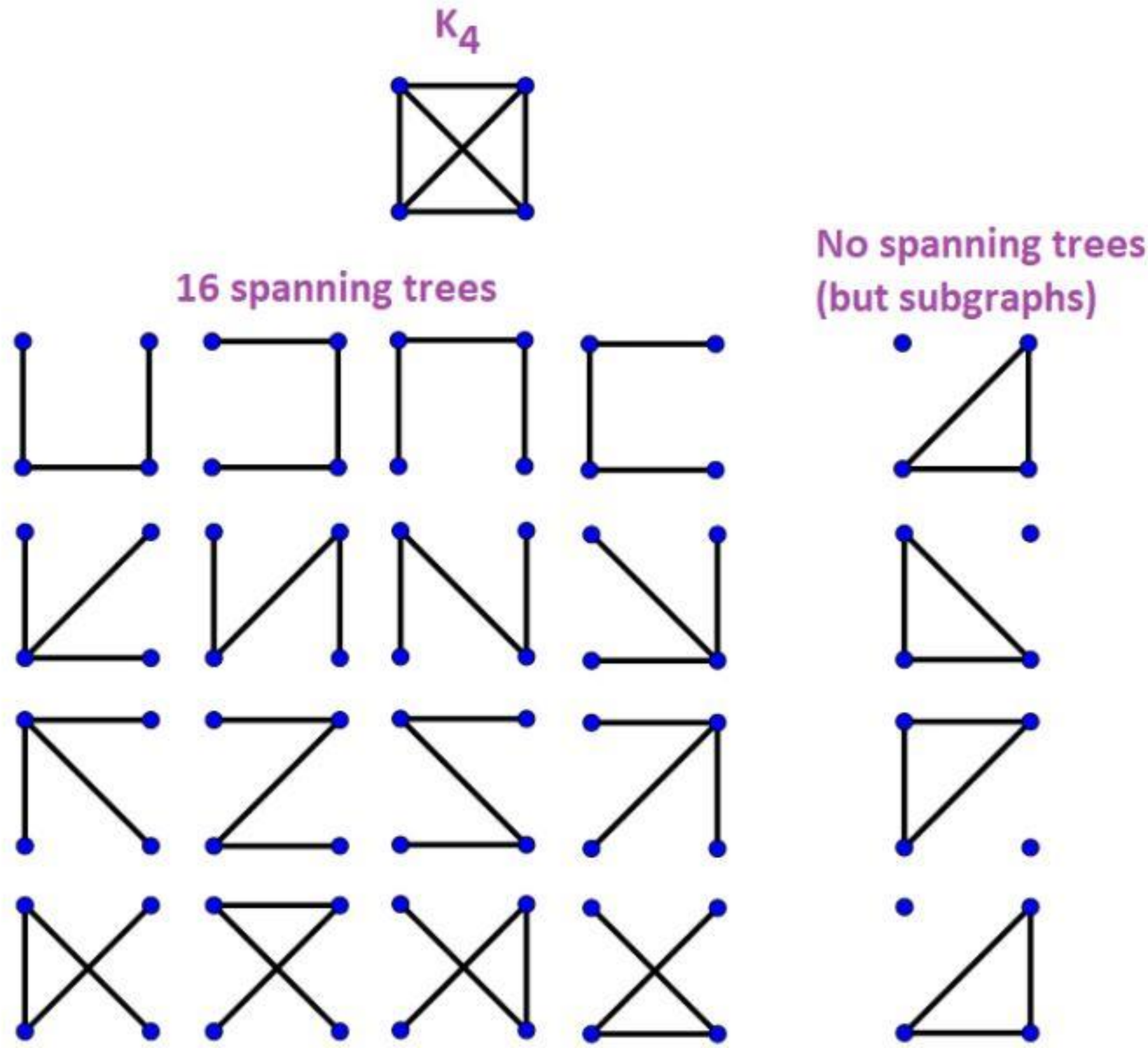
The spanning tree will then contain the same 4 vertices and $4-1=3$ edges. Thus $6-3=3$ edges should be removed.

We thus need to select 3 of the 6 edges to be removed. Since the order of the edges does not matter (as a different order of the edges results in the same edges to be removed), we need to use the definition of a **combination**.

$$C(6,3) = \frac{6!}{3!(6-3)!} = \frac{6!}{3!3!} = 20$$

Thus there are 20 possible subgraphs of K_4 with 3 edges.

However, it is possible to form a triangle by removing all three edges at one vertex (which is not a tree as a triangle is a simple circuit). Since K_4 has 4 vertices with 3 edges connecting to the vertex each, there are then 4 possible triangles and thus 4 of the 20 subgraphs are NOT spanning trees. However, this then means that $20-4=16$ subgraphs are spanning trees.

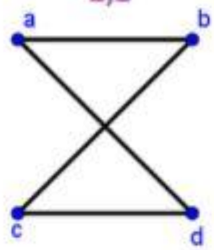

K_4
16 spanning trees
No spanning trees
(but subgraphs)

(c) $K_{2,2}$ is a graph with a set of 2 vertices $M = \{a, c\}$ and another set of 2 vertices $N = \{b, d\}$. All vertices in M are connected to a vertex in N and thus each vertex has two edges connecting to it, which results in 4 edges in total (see image below).

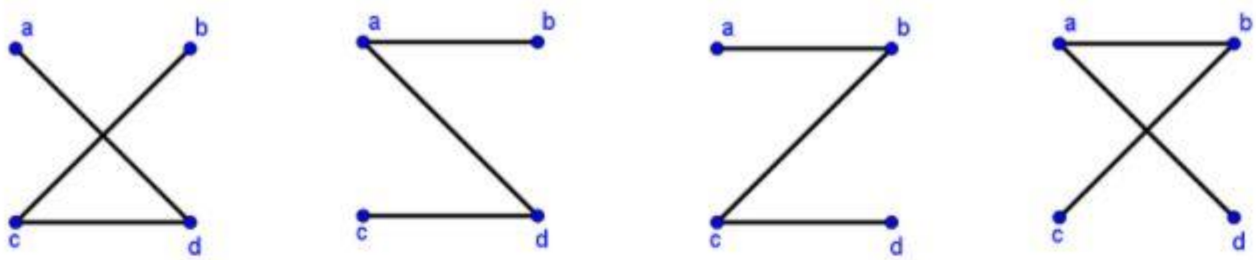
The spanning tree will then contain the same 4 vertices and $4 - 1 = 3$ edges. Thus we then need to remove $4 - 3 = 1$ edges.

It does not matter which of the four edges is removed, because it will always result in a spanning tree. Since we have 4 possible edges to be removed and we need to select one of the four, we then have 4 possible spanning trees.

$K_{2,2}$



4 spanning trees

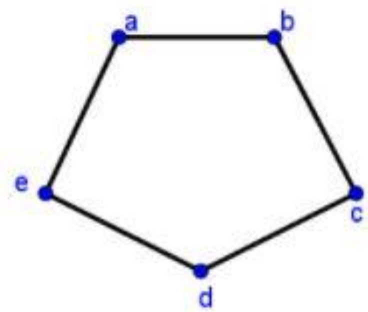


(d) C_5 is a graph with 5 vertices a, b, c, d, e and 5 edges $\{a, b\}, \{b, c\}, \{c, d\}, \{d, e\}, \{e, a\}$

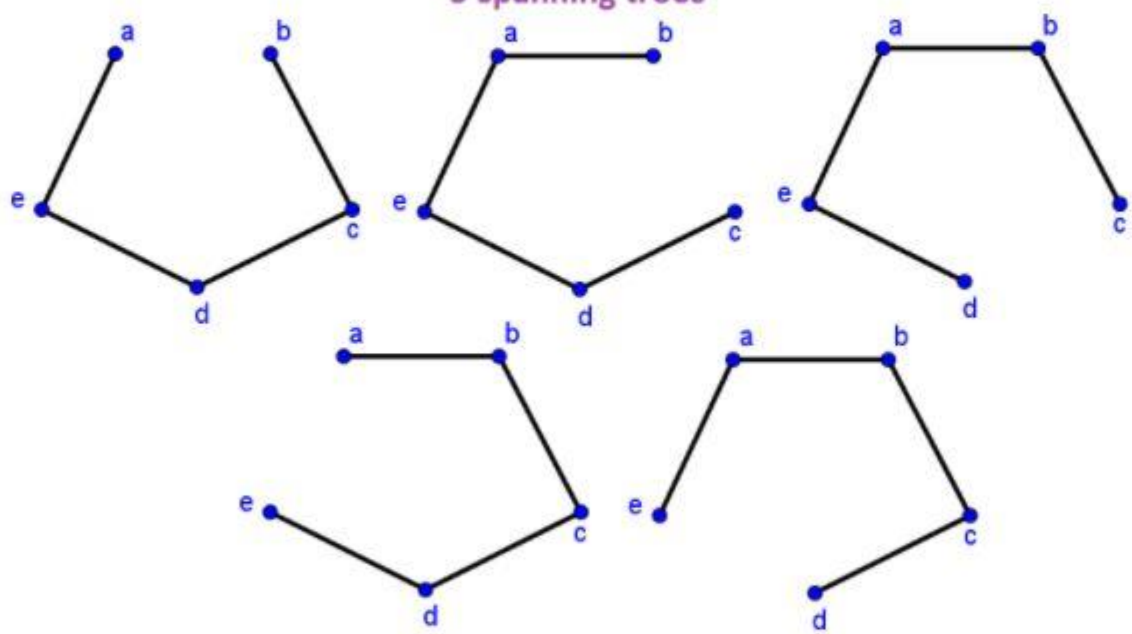
The spanning tree will then contain the same 5 vertices and $5 - 1 = 4$ edges. Thus we need to remove one of the edges.

It doesn't matter which edge you remove, you will always obtain a spanning tree if you remove one of the five edges. Thus there are then 5 possible spanning trees.

C_5



5 spanning trees



We approach this problem in a rather ad hoc way.

a) Every pair of edges in K3 forms a spanning tree, so there are C (3, 2) = 3 such trees.

b) There are 16 spanning trees; careful counting is required to see this. First, let us note that the Trees can take only two shapes: the star *K* 1,3 and the simple path of length 3. There are 4 Different spanning trees of the former shape, since any of the four vertices can be chosen as the Vertex of degree 3. There are *P(* 4, 4) = 24 orders in which the vertices can be listed in a simple Path of length 3, but since the path can be traversed in either of two directions to yield the same Tree, there are only 12 trees of this shape. Therefore there are 4 + 12 = 16 spanning trees of *K 4* Altogether.

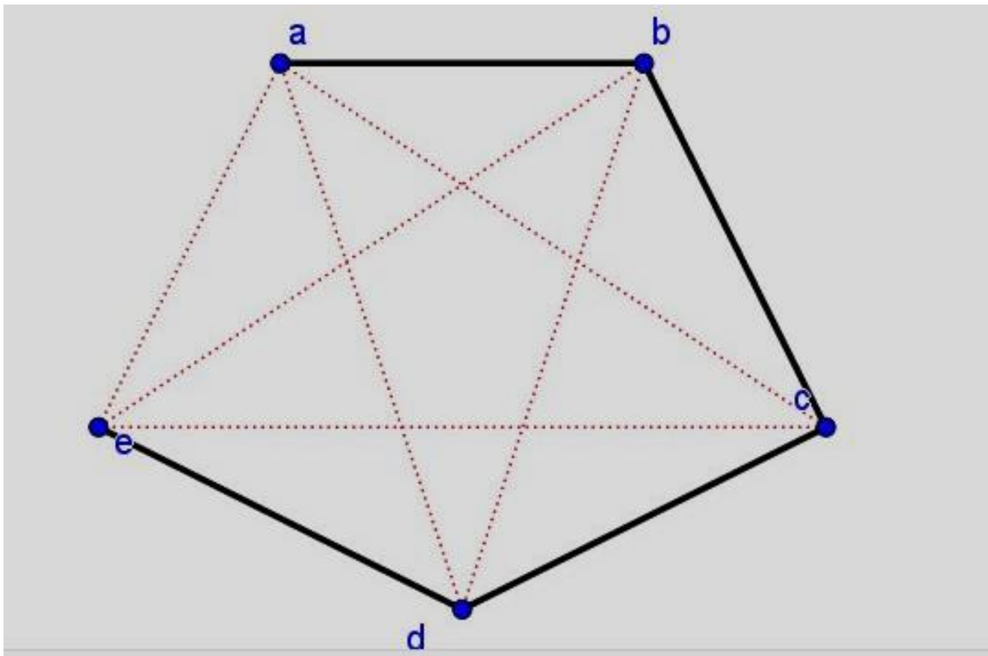
c) Note that K2,2 = C4 . A tree is determined simply by deciding which of the four edges to Remove. Therefore there are 4 spanning trees.

d) By the same reasoning as in part (c), there are 5 spanning trees.

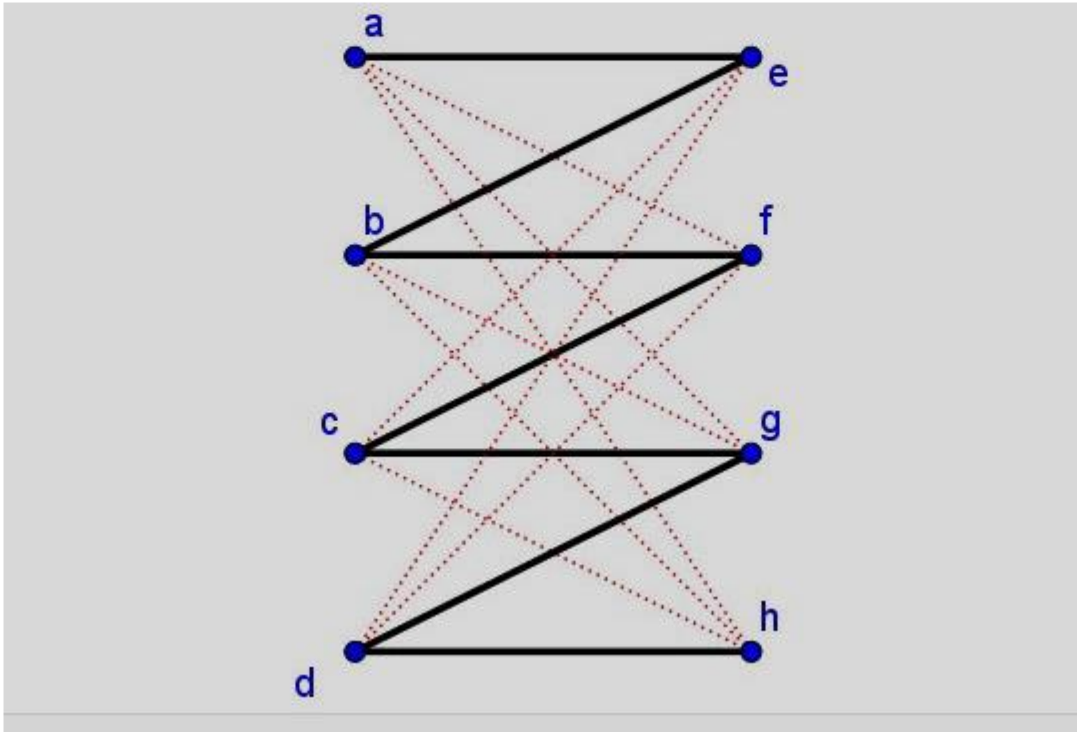
11. How many spanning tree for each if these graphs:

K_5 $K_{4,4}$ $K_{1,6}$ Q_3 C_5 W_5

K_5 is a graph with 5 vertices and an edges between every pair of vertices. The spanning tree will then contain the same 5 vertices and 5 – 1 = 4 edges. The easiest way to create the spanning tree is by creating a path that pass through all vertices exactly one. For example, a possible spanning tree is the outer circuit of K_5 with one edge remove.

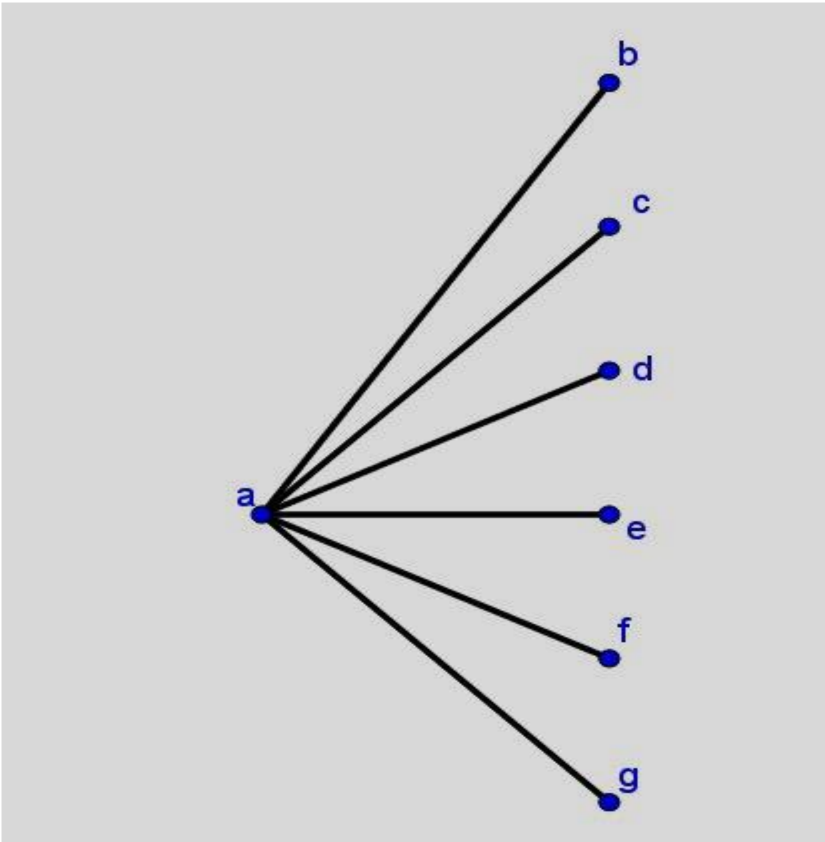


K4, 4 is a graph with a set of 4 vertices M = {a, b, c, d} and another set of 4 vertices N = {e, f, g, h}. all vertices in M are connected to a vertex in N. the spanning tree with contain the same 8 vertices and 8 – 1 = 7 edges. The easiest way to create the spanning tree is by creating a path that passes through all vertices exactly once. for example, a possible spanning tree of k4,4 is then given by the path a, e, b, f, c, g, d, h:

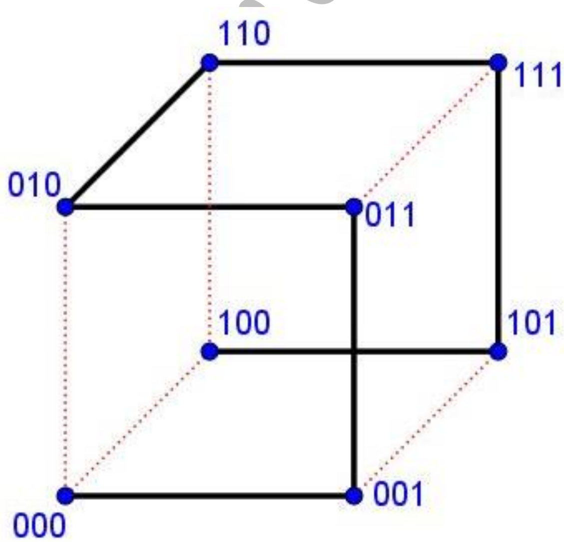


(c) $K_{1,6}$ is a graph with a set of 1 vertex $M = \{a\}$ and another set of 6 vertices $N = \{b, c, d, e, f, g\}$. All vertices in M are connected to a vertex in N.

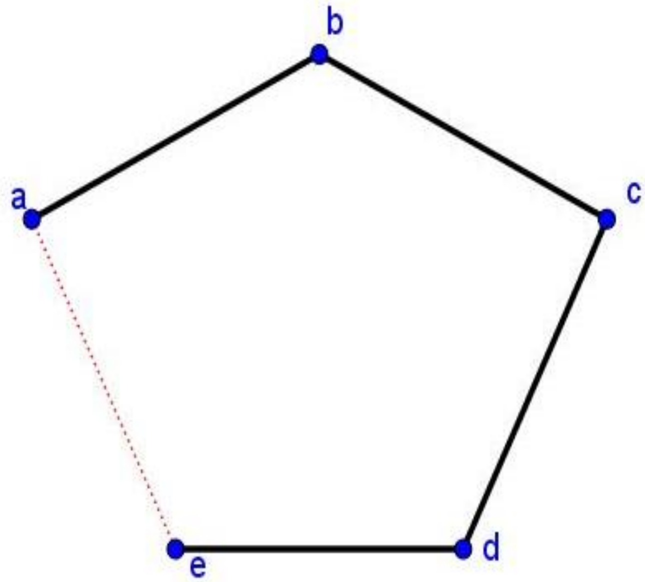
The spanning tree will then contain the same 7 vertices and $7 - 1 = 6$ edges. However, $K_{1,6}$ contains 6 edges and $K_{1,6}$ is already a tree, thus the only spanning tree of $K_{1,6}$ is $K_{1,6}$ itself.



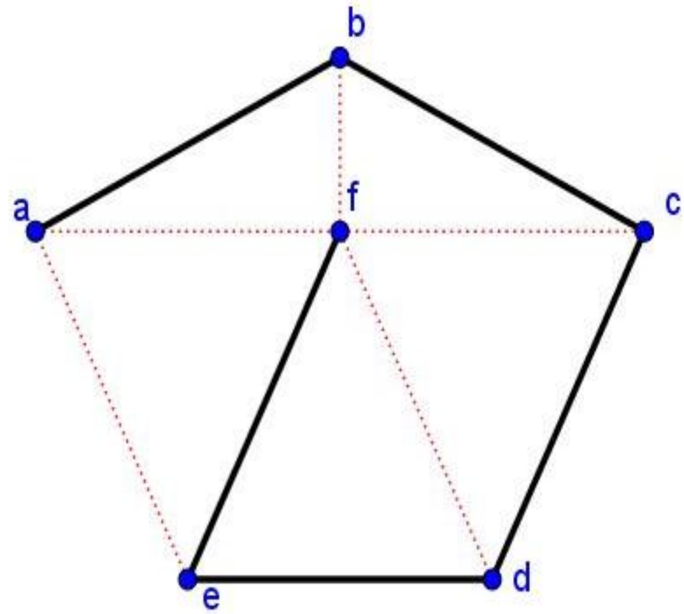
Q_3 is a given graph with $2^3 = 8$ vertices (which are the bit strings of length 3) and an edges between two vertices if the corresponding bits strings differ by 1 bit. The spanning tree will then contain the same 8 vertices and 8-1 = 7 edges. The easiest way to create the spanning tree is by creating a path that pass through all vertices exactly one. For example, a possible spanning tree Q_3 is the given by the path 000, 001, 011, 010, 110, 111, 101, 100:



C_5 is graph with 5 vertices a, b, c, d, e and edges {a, b}, {b, c}, {c, d}, {d, e}, {e, a}. the spanning tree will then contain the same 5 vertices and 5 – 1 = 4 edges. Thus we need to remove one of the edges. It does not matter which edge you remove, you will always obtain a spanning tree if you remove one of the five edges. For example, let us remove the edge {e, a}.



W_5 is graph with 6 vertices a, b, c, d, e, f and edges {a, b}, {b, c}, {c, d}, {d, e}, {e, a} and all edges connecting f to another vertex. The spanning tree will then contain the same vertices and 6 – 1 = 5 edges. Thus we need to remove one of the edge. For example, let us use the spanning tree found in part (e) and then add one of the edges connecting a vertex to f.



12. Draw a binary search Tree by inserting the following members from left to right?

<u>Topic: Page 757, Example 1 Page 758</u>

Compare the ITEM with the root node.

If ITEM>ROOT NODE, proceed to the right child, and it becomes a root node for the right subtree.

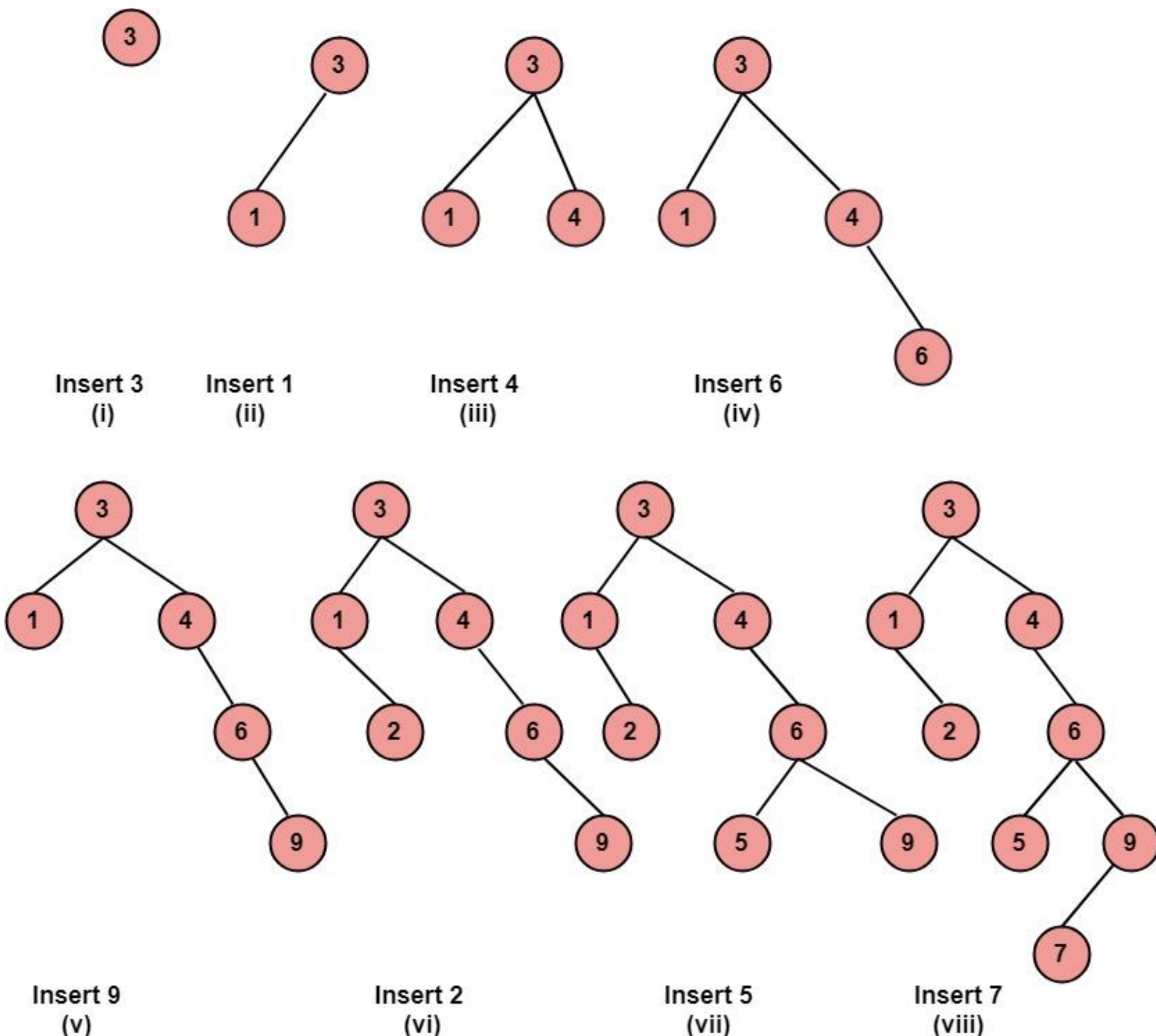
If ITEM<ROOT NODE, proceed to the left child.

Repeat the above steps until we meet a node which has no left and right subtree.

Now if the ITEM is greater than the node, then the ITEM is inserted as the right child, and if the ITEM is less than the node, then the ITEM is inserted as the left child.

Example: Show the binary search tree after inserting 3, 1,4,6,9,2,5,7 into an initially empty binary search tree.

Solution: The insertion of the above nodes in the empty binary search tree is shown in fig:



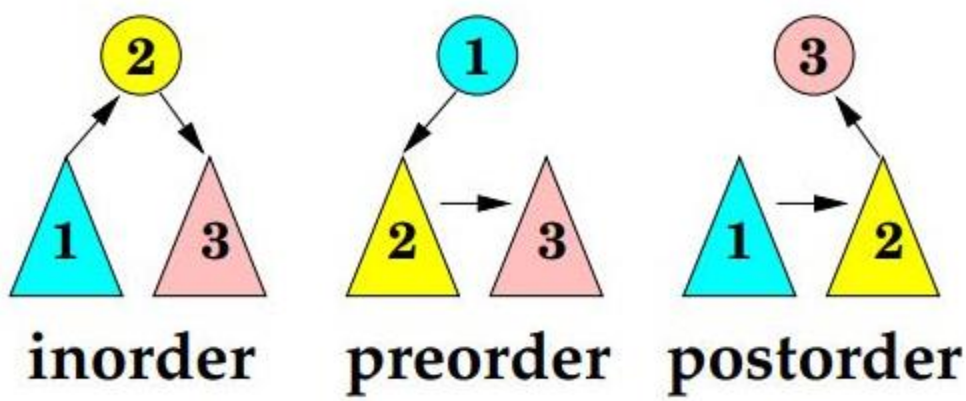
<u>19, 10, 8, 17, 4, 10, 6, 13, 43, 47, 33 OR 11, 6, 8, 19, 4, 10, 5, 17, 43, 49, 31</u>

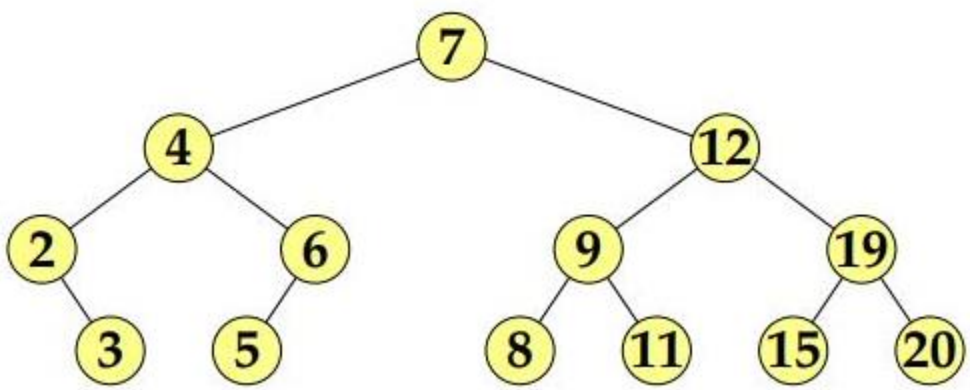
<u>Determine the order, in which the vertices of the following binary trees will be visited under</u>

<u>Preorder In order Post Oder</u>

<u>Assignment: By using up and down Question solve these Questions</u>

13. **Draw a tree for the following and run in order tree traversal 1, 9, 8, 12, 20, 15, 2**





In order traversal gives: 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 15, 19, and 20.

Preorder traversal gives: 7, 4, 2, 3, 6, 5, 12, 9, 8, 11, 19, 15, and 20.

Post order traversal gives: 3, 2, 5, 6, 4, 8, 11, 9, 15, 20, 19, 12, 7.

Q8, 9, 10 Page 795 Assignment of making spanning tree

Below are the Answers of Q8, 9, 10 Page 795

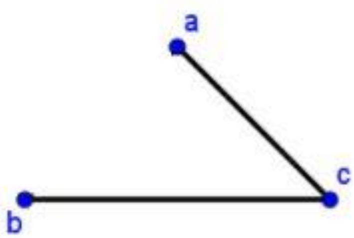
We note that the given graph contains 3 vertices and 3 edges.

The spanning tree will then contains the same 3 vertices and $3-1=2$ edges. Thus we will need to remove 1 edge from the given graph.

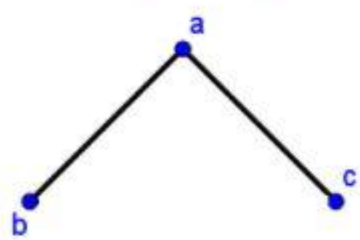
Since a tree cannot contain any simple circuits and since there is a triangle (abc) present in the graph, it then suffices to remove one edge from the triangle.

However it doesn't matter which edge is removed, thus we could remove any of the three edges, which will result in 3 possible spanning trees.

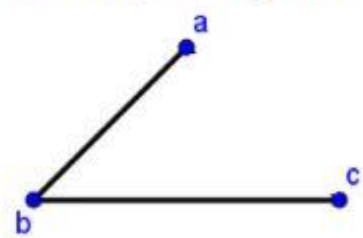
First spanning tree



Second spanning tree



Third spanning tree



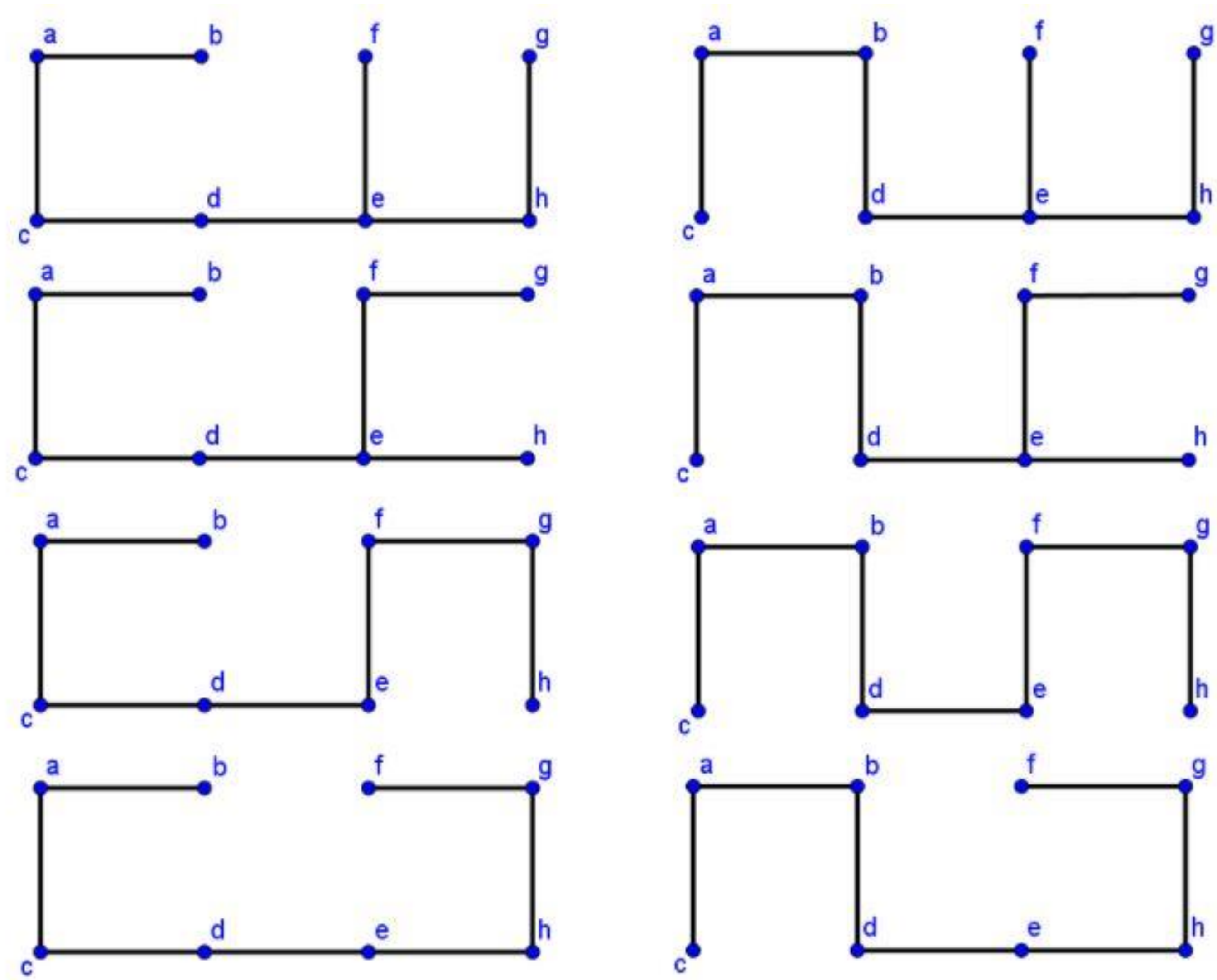
RESULT

Since a tree cannot contain any simple circuits and since there is a triangle (abc) present in the graph, it suffices to remove one edge from the triangle.

The spanning tree will then contains the same 8 vertices and $8-1=7$ edges. Thus we will need to remove 2 edges from the given graph.

Since a tree cannot contain any simple circuits and since there are two squares ($abcd$ and $efgh$) present in the graph, it then suffices to remove one edge from each square.

However it doesn't matter which edge is removed as long as one edge is removed from each square, thus we could remove any of the four edges in both squares, which will result in $4 \cdot 4 = 16$ possible spanning trees.


a b f g
d e h
c
a b f g
d e h
c
a b f g
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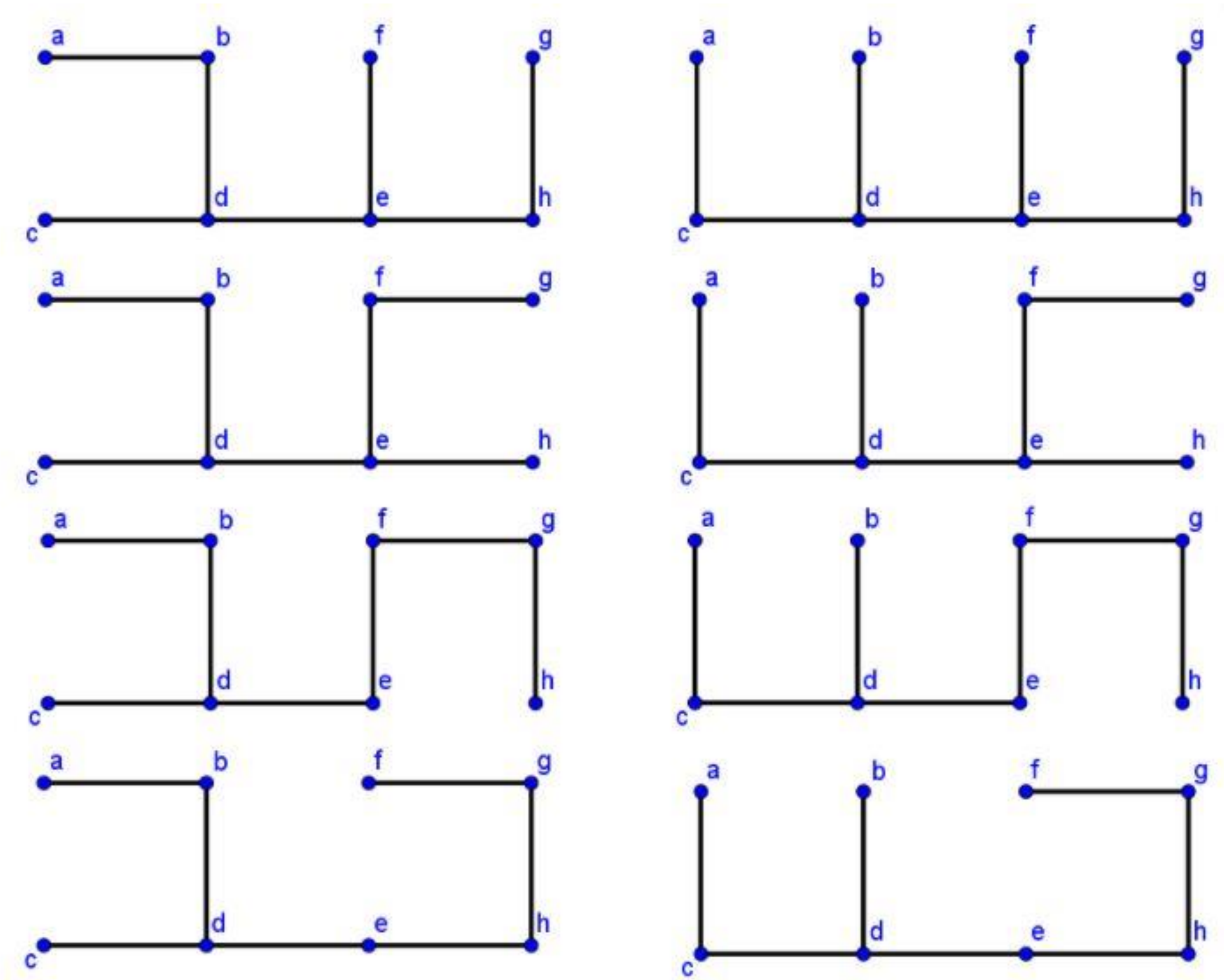


RESULT

It doesn't matter which edge is removed as long as one edge is removed from each square, thus we could remove any of the four edges in both squares, which will result in $4 \cdot 4 = 16$ possible spanning trees.

DEFINITIONS

A **spanning tree** of a simple graph G is a subgraph of G that is a tree and that contains all vertices of G.

A **tree** is an undirected graph that is connected and that does not contain any simple circuits.

A tree with n vertices has $n-1$ edges.

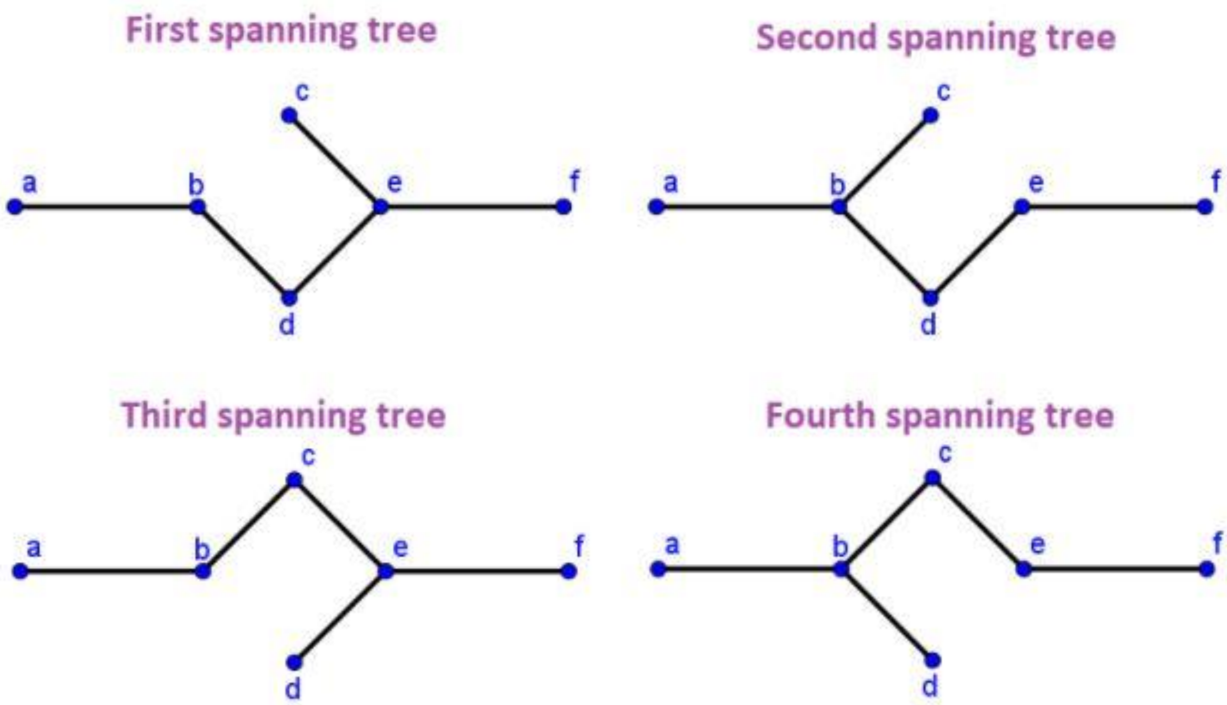
SOLUTION

We note that the given graph contains 6 vertices and 6 edges.

The spanning tree will then contains the same 6 vertices and $6-1=5$ edges. Thus we will need to remove 1 edge from the given graph.

Since a tree cannot contain any simple circuits and since there is a square ($bced$) present in the graph, it then suffices to remove one edge from this square.

However it doesn't matter which edge is removed as long as one edge is removed from the square, thus we could remove any of the four edges in the squares, which will result in 4 possible spanning trees.

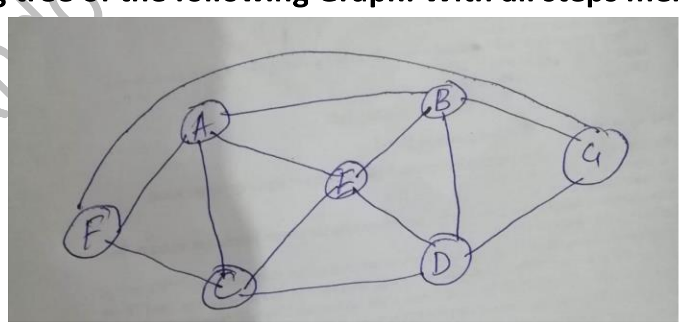


RESULT

Since a tree cannot contain any simple circuits and since there is a square ($bced$) present in the graph, it then suffices to remove one edge from this square.

However it doesn't matter which edge is removed as long as one edge is removed from the square, thus we could remove any of the four edges in the squares, which will result in 4 possible spanning trees.

14. Make the spanning tree of the following Graph. With all steps mentioned.



<u>Chapter 12:</u> <u>Definitions 843</u>

1. Full adder? **<u>Page 843 and 827 Figure adders</u>**

   Half adder: a circuit that adds two bits, producing a sum bit and a carry bit

   Full adder: a circuit that adds two bits and a carry, producing a sum bit and a carry bit

2. K-Map **page 830**

   To reduce the number of terms in a Boolean expression representing a circuit, it is necessary to find terms to combine. There is a graphical method, called a **Karnaugh map** or **K-map**, for finding terms to combine for Boolean functions involving a relatively small number of variables.

3. Postulates? Page 83

A proof is a valid argument that establishes the truth of a theorem. The statements Used in a proof can include **axioms** (or **postulates**), which are statements we assume to be true. A statement, also known as an axiom, which is taken to be true without proof. **Postulates** are the basic structure from which lemmas and theorems are derived.