MATH 472/572 Computational Statistics - Spring 2020

Homework 3 - Due February 6, Thursday

Instructor: Leming Qu

Rules for HW:

- You are allowed to discuss HW with fellow students in the course, but the work you hand in must be your own.
- You have to write your own Python code by yourself. You are prohibited from sharing, copying or editing any Python code from other students.

How to turn in your coding portion of the HW?

- Submit your code in Jupyter Notebook format (.ipynb file) through the blackboard HW link. The deadline for code submission is the class starting time 1:30PM of the due date.
- Required output (prints and plots) must be included in the Jupyter Notebook - do not expect the Grader to run the code to see the required output. If the required output is not included in the Jupyter Notebook, the grader will take points off accordingly.

How to turn in your non-coding portion of the HW?

- If you submit your answer in hard copy, turn it in to the instructor during class on the due date.
- If you submit your answer in electronic copy (.pdf file), submit it through the blackboard HW link.

Coding Assignments:

(1) Implement iteratively reweighted least squares (IRLS) algorithm to reproduce the result in Example 2.5 of *Computation Statistics* book using. Note that the data set is available from the book's webpage `http://www.stat.colostate.edu/computationalstatistics/`.

   (a) Print your result in the format of table 2.1, using starting value $\beta^{(0)} = (\beta_0^{(0)}, \beta_1^{(0)})^T = (0.95913, 0)^T$, which means $\pi_i = 775/1072$ for all i at iteration 0.

   (b) Print your result in the format of table 2.1, using starting value $\beta^{(0)} = (\beta_0^{(0)}, \beta_1^{(0)})^T = (0, 0)^T$, which means $\pi_i = 0.5$ for all i at iteration 0.

   (c) Making a single plot similar to Figure 2.7 showing the solution paths in part (a) and part (b).

(2) Problem 2.1 on page 54 of the *Computation Statistics* book.

(3) Problem 2.4 on page 56 of the *Computation Statistics* book. Required output: A single Plot including (a) the Gamma(2,1) probability density, (2) the lower and the upper end of the interval clearly marked on the x-axis, (3) the region under the density curve and above the between the 95% highest posterior density interval.

Non-Coding Assignments:

(4) Observations $Y_1, \ldots, Y_n$ are described by the relationship $Y_i = \theta x_i^2 + \epsilon_i$, where $x_1, \ldots, x_n$ are fixed constants and $\epsilon_1, \ldots, \epsilon_n$ are iid $N(0, \sigma^2)$.

(a) Find the least squares estimator of θ.

(b) Find the MLE of θ.