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HW 10.

$$f(x) = Unif(0,1) \quad , \quad g(x) = y = \log\left(\frac{x}{1-x}\right)$$

$$\Rightarrow \frac{x}{1-x} = e^{y} \Rightarrow x = e^{y} - xe^{y} \Rightarrow x + xe^{y} = e^{y} \Rightarrow x(1+e^{y}) = e^{y}$$

$$\Rightarrow x = \frac{e^{y}}{1+e^{y}} = g^{-1}(y) \Rightarrow 0 \le g^{-1}(y) \le 1 \Rightarrow f(g^{-1}(y)) = 1 \quad (1)$$

$$\frac{dg^{-1}(y)}{dy} = \frac{e^{y}(1+e^{y}) - e^{2y}}{(1+e^{y})^2} = \frac{e^{y} + e^{2y} - e^{2y}}{(1+e^{y})^2} = \frac{e^{y}}{(1+e^{y})^2} \quad (2)$$

and $\Rightarrow f_y(y) = f_y(g^{-1}(y)) \left| \frac{dg^{-1}(y)}{dy} \right| \overset{(1),(2)}{\Rightarrow} f_y(y) = \frac{e^{y}}{(1+e^{y})^2}$