MATH 472/572 Computational Statistics - Spring 2020

Homework 7 - Due March 19, Thursday

Instructor: Leming Qu

Rules for HW:

- You are allowed to discuss HW with fellow students in the course, but the work you hand in must be your own.
- You have to write your own Python code by yourself. You are prohibited from sharing, copying or editing any Python code from other students.

How to turn in your coding portion of the HW?

- Submit your code in Jupyter Notebook format (.ipynb file) through the blackboard HW link. The deadline for code submission is the class starting time 1:30PM of the due date.
- Required output (prints and plots) must be included in the Jupyter Notebook - do not expect the Grader to run the code to see the required output. If the required output is not included in the Jupyter Notebook, the grader will take points off accordingly.

Coding Assignments:

1. (20 Points)Implementing Example 6.1 on page 157 of *Computational Statistics* for $r = 2$, produce a plot similar to FIGURE 6.2., report the acceptance rate of your algorithm. Show the generated random samples by superimposing the following three items in another plot, using different line type for different curves, and display of legend.

   (a) a relative frequency histogram with appropriate number of bars for your generated random numbers with sample size n = 5000;

   (b) an estimated probability density curve based on the generated random numbers;

   (c) the true Gamma(2,1) probability density curve.

2. (20 Points) Generate a random sample with sample size $n = 5000$ according to the following probability density function (PDF)

$$f(x) \propto \frac{1}{12}(1+x)\exp\left\{-\frac{(x-1)^2}{2x}\right\}, \quad x > 0.$$

   Produce a plot similar to FIGURE 6.2. Report the acceptance rate of your algorithm. your algorithm's acceptance rate must be higher than 50% - otherwise, you will receive only half of the full points. Show the generated random samples by superimposing the following three items in another plot, using different line type for different curves, and display of legend.

   (a) a relative frequency histogram with appropriate number of bars for your generated random numbers with sample size n = 5000;

   (b) an estimated probability density curve based on the generated random numbers;

   (c) the true PDF.

Non-Coding Assignments:

3. (10 Points) Let $U_1, U_2, \ldots, U_n$ be a random sample from UNIF[0,1] distribution. Let

$$U_{(n)} = \max\{U_1, U_2, \ldots, U_n\}$$

Prove that $U_{(n)}$ converges to 1 in probability.