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HW 7

3. $U_1, U_2, \dots, U_n$ are random sample from Unif $[0,1]$

$U_n = \max\{U_1, U_2, \dots, U_n\}$

we want to prove, As $n \to \infty$: $p(|U_n - 1| < \epsilon) = 1$, for $\epsilon > 0$

If we prove that $p(|U_n - 1| \geq \epsilon) = 0$, is the same as $p(|U_n - 1| < \epsilon) = 1$

then, $p(|U_n - 1| \geq \epsilon) = p(U_n \leq 1 - \epsilon)$

$= p(U_i \leq 1 - \epsilon, i = 1, \dots, n) = \underbrace{(1-\epsilon)(1-\epsilon)\dots(1-\epsilon)}_{n \text{ times}}$

$= (1-\epsilon)^n$ and as $1 - \epsilon < 1$

$\Rightarrow \lim_{n \to \infty} (1-\epsilon)^n = 0$

Then $p(|U_n - 1| \geq \epsilon) = 0$ and $\boxed{p(|U_n - 1| < \epsilon) = 1}$