CHE260: Heat Transfer

October 2021

Contents

1 Introduction

- How is heat transfer different from thermodynamics? In thermodynamics, we assume quasi-equilibrium processes i.e. the time was not an important parameter. In heat transfer, time is an important parameter and we are interested in the rate of heat transfer.
- What is the relationship between $\dot{Q}$ and ΔT? What are the mechanisms of heat transfer?
- Conduction: Transfer of heat through a medium that is stationary.
- Convection: Transfer of heat from a solid surface and an adjacent fluid that is moving. Example: a fan blowing air over a hot plate. There is heat transfer from the hot plate into the fluid.
- Radiation: Energy emitted by matter in the form of electromagnetic waves.
- Radiation does not need a medium. In a vacuum, we can have radiation but not convection or condcution.
- Different mechanisms of heat transfer can take place simultaneously.
- Applications
  - Power Generation
    * Power plant: steam generation, condenser
    * Automobiles: engine cooling, space heating/cooling
  - Buildings
    * Heating / Cooling
    * Hot water
  - Refrigeration
  - Manufacturing
    * Casting / Heat treatment
    * Injection Moulding

2 Electronic Cooling

- $>$ 99 % of the electrical energy supplied to a circuit is dissipated as heat
- Heat has to be dissipated to the environment while keeping the temperature of the chip in a certain range

- Heat is lost from the surface of the chip
- Important parameter is heat flux $= \frac{\text{Heat Transfer Rate}}{\text{Unit Area}}$ (in $\frac{W}{\text{cm}^2}$)
- To reduce heat flux, we can reduce heat generation and increase the surface area
- As size increases, it becomes more difficult to lose heat
- Water cooling is more efficient for large systems compared to air cooling

3 Radiation

- Radiation is energy emitted by all matter in the form of e.m. radiation
- Thermal radiation is emitted by all bodies at a finite temperature
- Opaque objects emit only from the surface
- Amount of radiation depends on the surface temperature. Summarized by the Stefan Boltzmann Law: $\dot{Q}_{emit} = \sigma A T_s^4$ where σ is the Boltzmann constant ($5.67 \times 10^{-8}\ \frac{W}{m^2 k^4}$), T_s is the surface temperature in Kelvin and A is the surface area.
- A surface that emits as much radiation as this is called a "Blackbody". A real surface emits less than this.: $\dot{Q}_{emit} = \epsilon \sigma A T_s^4$ where ϵ is the emissivity and $0 \leq \epsilon \leq 1$
- Black paint has $\epsilon = 0.99$ which is very close to 1. Aluminum foil has a low emissivity of around 0.07.
- If radiation is incident on a surface some will be absorbed. The fraction absorbed is a surface property known as the absorptivity α such that $\dot{Q}_{absorbed} = \alpha \cdot \dot{Q}_{incident}$ and $\dot{Q}_{reflected} = (1 - \alpha) \cdot \dot{Q}_{incident}$
- Kirchoff's law says that $\alpha = \epsilon$
- Note: α and ϵ vary over different wavelengths
- Consider a special case of radiation
  - Small surface which is completely surrounded by a much larger surface
  - T_s, A_s are temperature and area of the small surface (which is also the boundary), T_{surr} is the temperature of the surrounding surface. Both surfaces are emitting and we are interested in the net emission
  - $\dot{Q}_{rad} = \epsilon \sigma A_s (T_s^4 - T_{surr}^4)$
- Example
  - Chip with an area of $15 \times 15mm$, $\epsilon = 0.6$, $T_{surr} = 25$.
  - Two methods of heat transfer

- Natural convection
  - $h = c(T_s - T_\infty)^{\frac{1}{4}}$
  - $c = 4.2 \frac{W}{m^2 K^{\frac{5}{4}}}$
  - $q_{conv} = hA(T_S - T_\infty)$
  - $q_{rad} = \epsilon A(T_s^4 - T_{surr}^4)$
- Forced convetion: h is constant at 250 $\frac{W}{m^2 K}$
  - $q_{conv} = hA(T_S - T_\infty)$

4 Heat Conduction

- Heat Conduction Equation
  - x, y, z components of $\dot{Q}$
  - T is a function of (x, y, z, t)
  - $\vec{\dot{Q}} = \dot{Q}_x \hat{i} + \dot{Q}_y \hat{j} + \dot{Q}_z \hat{k}$
  - $\dot{Q}_x = -kA_x \frac{dT}{dx}$ (similar expressions for $\dot{Q}_y$ and $\dot{Q}_z$
- One dimensional heat conductivity can model more complicated situations. For example if $\Delta x << \Delta y, z$, $\frac{dT}{dx} >> \frac{dT}{dy}, \frac{dT}{dz}$ so that $\dot{Q}_y$ and $\dot{Q}_z$ can be neglected
- One dimensional heat conduction
  - Cross sectional area is $A(x)$ where x is the coordinate along which heat transfer occurs
  - $\dot{Q}_x$ at the entry and $\dot{Q}_{x+\Delta x}$ at the exit
  - Want to find $T(x)$ inside the object
  - Rate of increase of enthalpy $= mc_p \frac{\partial T}{\partial t} = \rho V c_p \frac{\partial T}{\partial t} = \rho c_p A \Delta x \frac{\partial T}{\partial t}$
  - Energy balance:
    - $\rho c_p A \Delta x \frac{\partial T}{\partial t} = \dot{Q}_x - \dot{Q}_{x+\Delta x}$
    - After simplifying and taking the limit as Δx approaches 0, we get $\rho c_p \frac{\partial T}{\partial t} = \frac{-1}{A} \frac{\partial(\dot{q}A)}{\partial x}$
    - A depends on the coordinate system and we use Fourier's law for $\dot{q}$: $\dot{q} = -k \frac{dT}{dx}$
- Cartesian Coordinates
  - A is a constant
  - $pc_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}[k \frac{\partial T}{\partial x}]$
  - Assume k is constant. Then $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$ where $\alpha = \frac{k}{pc_p}$.
  - If steady state i.e. $\frac{\partial T}{\partial t} = 0$ then $\frac{d^2 T}{dx^2} = 0$.
  - Units of $\alpha = \frac{k}{pc_p}$, the thermal diffusivity is $\frac{m^2}{s}$.

  - High k means the material conducts well. High pc_p means that the material stores energy

- Cylindrical Coordinates
  - Heat being conducted radially so $\dot{q} = -k\frac{\partial T}{\partial r}$ and $A = 2\pi rL$
  - $pc_p\frac{\partial T}{\partial t} = \frac{-1}{2\pi rL}[\frac{\partial}{\partial r} \cdot \frac{\partial T}{\partial r}]$
  - $\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(r \cdot \frac{\partial T}{\partial r})$
  - At steady state, $\frac{d}{dr}(r\frac{dT}{dr}) = 0$.

- Spherical Coordinates
  - $A = 4\pi r^2$ and $\dot{q} = -k\frac{\partial T}{\partial r}$ where r is the radial spherical coordinate
  - $\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r})$

- In general $\frac{1}{r^n}\frac{\partial}{\partial r}(r^n\frac{\partial T}{\partial r}) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$ where cartesian has $n = 0$, cylindrical has $n = 1$ and spherical has $n = 2$.

5 Thermal Resistance

- At steady state, $\frac{d^2T}{dx^2} = 0$
- Heat flux: $\dot{q} = -k\frac{dT}{dx}$.
- Heat flux is a constant
- Heat transer rate: $\dot{Q} = \dot{q}A = \frac{-kA(T_2-T_1)}{L}$
- $\dot{Q} = \frac{T_1-T_2}{R_{wall}}$ where T_1 and T_2 are the temperatures of the walls
- $R_{cond} = R_{wall} = \frac{L}{kA}$
- Similar to current with voltage and Resistance
- $\dot{Q}_{conv} = hA(T_s - T_\infty)$
- $R_{conv} = \frac{T_s-T_\infty}{Q_{conv}} = \frac{1}{hA}$
- Radiation is more complicated. $\dot{Q}_{rad} = \epsilon\sigma A(T_s^4 - T_{sur}^4)$
- We need to define a heat transfer coefficient for radiation. $h_{rad} = \frac{\epsilon\sigma A(T_s^4-T_{sur}^4)}{A(T_s-T_{sur})}$
- $h_{rad} = \epsilon\sigma(T_s^2 + T_{sur}^2)(T_s + T_{sur})$
- Can treat it as a resistance. $R_{rad} = \frac{T_s-T_{sur}}{\dot{Q}_{rad}} = \frac{1}{h_{rad}A}$
- Multilayer Plane Wall

- Each layer has the same surface Area
- Layers have thicknesses L_i
- Temperature varies as T_1 on the outside, T_2, ..., T_{n+1} where n is the number of surfaces
- Treat each layer seperately as resistances in series
- For first wall $\dot{Q} = \frac{T_1 - T_2}{R_1}$ so $T_1 - T_2 = \dot{Q}R_1$. In general, $T_i - T_{i+1} = \dot{Q}R_i$
- Summing all of them gives $T_1 - T_{n+1} = \dot{Q}(R_1 + ... + R_n)$ so that $\dot{Q} = \frac{T_1 - T_4}{R_{total}}$
- Therefore you can sum up resistances similar to electric circuits
- We can find each R_i as $\frac{L_i}{k_i A}$

- $\dot{Q} = UA(T_{\infty,1} - T_{\infty,4})$ where U is the overall heat transfer
- Example: Refrigerator Wall
  - 1 mm thick insulation on the outside and width of refrigerator is L.
  - $T_{room} = 25C$ and $T_{refrig} = 3C$
  - $h_0 = 9\frac{W}{m^2C}$ and $h_i = 4\frac{W}{m^2C}$
  - $k_{steel} = 15.1\frac{W}{m^2C}$ and $k = 0.035\frac{W}{m^2C}$ inside the refrigerator
  - Constraint is that $T_{s,out} > 20$. We assume heat transfer from the outside room to the inside of the refrigerator
  - What is L to ensure $T_{s,out} > 20$ to prevent condensation on the outside of the refrigerator
  - Thermal circuit consists of a convective resistance outside the refrigerator followed by 3 conductive resistance on the surfaces and then one convective resistance at the end
  - INSERT PICTURE FROM LEC NOTES
  - $\dot{Q} = \frac{T_{room} - T_{s,out}}{R_{conv,0}} = \frac{T_{room} - T_{s,out}}{\frac{1}{h_0 A}}$
  - Consider unit area. Then $\dot{Q} = h_0(T_{room} - T_{s,out}) = 9(25 - 20) = 45W$
  - $R_{total} = \frac{1}{h} + (\frac{L}{k})_{metal} + (\frac{L}{k})_{insulation} + (\frac{L}{k})_{metal} + \frac{1}{h_i} = \frac{1}{9} + \frac{10^{-3}}{15.1} + \frac{L_2}{0.035} + \frac{10^{-3}}{15.1} + \frac{1}{4} = 0.361 + \frac{L_2}{0.035}$
  - $\dot{Q} = \frac{T_{room} - T_{refrig}}{R_{total}} \rightarrow 45(0.361 + \frac{L_2}{0.035}) = 25 - 3$ so that $L = 45mm$

6 Thermal Resistance Networks

- Multiple layers e.g. in an electric chip each with different thermal properties
  - We are interested in $\dot{Q} = \dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} + \frac{T_1 - T_2}{R_3}$
  - $= (T_1 - T_2)(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}) = \frac{T_1 - T_2}{R_{total}}$

  - This is the electrical analog to parallel resistances

- Thermal contact resistance
  - So far we have been assuming perfect contact between different boundaries
  - In reality, there is a rough surface at the boundary
  - We can always define a thermal contact resistance $R_c = \frac{T_2 - T_1}{\dot{q}}$ (units are $\frac{m^2 C}{W}$ (Note: Resistanc per unit area)
  - The reciprocal of R_C is known as the thermal contact conductance h_c.
  - $h = \frac{1}{R_c} = \frac{\dot{q}}{\Delta T}$ so that $\dot{q} = h_c \Delta T$
  - h_c is thus similar to the heat transfer coefficient

- Heat conduction in cylinders & spheres
  - INSERT DRAWING FROM THE SLIDEs
  - For a long pipe, the main temperature gradient is in the radial direction i.e. $\frac{dT}{dx} << \frac{dT}{dR}$
  - Therefore we can assume 1-D radial conduction
  - INSERT r_1, r_2 diagram
  - Solve heat conduction equation in cylindrical coordinates to get $T(r)$
  - Steady state: $\frac{d}{dr}(r\frac{dT}{dr}) = 0$, at $r = r_1, T = T_1$ and at $r = r_2, T = T_2$
  - Integrate this to get $T(r) = c_1 \ln r + c_2$
  - Using the boundary conditions gives $c_1 = \frac{T_1 - T_2}{\ln(\frac{r_1}{r_2})}$ and $c_2 = T_2 - \frac{T_1 - T_2}{\ln(\frac{r_1}{r_2})} \ln r_2$

- Define thermal resistance of a cylinder
  - $R_{cyl} = \frac{T_1 - T_2}{\dot{Q}_{cond}} = \frac{\ln(\frac{r_2}{r_1})}{2\pi LK}$

7 Conduction in cylinders and spheres, Insulation

- Inner temperature and then two surface layers
- $R_C = \frac{1}{hA} = \frac{1}{2\pi r L h}$
- $R_{total} = R_{c,1} + R_{cond} + R_{c,2} = \frac{1}{2\pi r_1 L h_1} + \frac{\ln(\frac{r_2}{r_!})}{2\pi LR} + \frac{1}{2\pi r_2 L h_2}$
- Insulation
  - $R_{total} = R_{c,1} + R_{cyl,1} + R_{cyl,2} + R_{c,2}$ where $R_{cyl,2}$ is for the insulation
  - For a sphere
    - Insulation around a spherical metal tank
    - $R_{total} = R_{c,1} + R_{sph,1} + R_{sph,2} + R_{c,2}$

* $= \frac{1}{4\pi r_1^2 h_1} + \frac{r_2-r_1}{4\pi r_1 r_2 k_1} + \frac{r_3-r_2}{4\pi r_3 r_2 k} + \frac{1}{4\pi r_3^2 h_2}$

- R-value
  - Thermal resistance. Thickness L, Surface area A and thermal conductivity k
  - $R = \frac{L}{k}$ is the R-value
  - $\dot{Q} = \frac{\Delta T}{R} \times A$
  - Units here are in imperial units i.e. L is in feet, k is in $\frac{Btu}{hftF}$
- Critical Radius of insulation
  - Consider the area for heat loss
  - Insulation: increase thickness, increasing conduction resistance and decreasing convective resistance
  - Can we increase heat transfer?
    * Plot $\dot{Q}$ against r_2 to find the critical value of Resistance
    * Equivalently set $\frac{d\dot{Q}}{dr_2} = 0$ to find the critical radius

8 Heat Transfer from Finned Surfaces

- Read Chapter 17.6
- How to find $R_{heatsink}$ for finned surfaces e.g. heat sinks in computers
- Add diagram from notes: We have a cylinder with cross sectional area $A_c(x)$ and heat transfer coeff h.
  - Consider a thin slice of this with thickness Δx some distance x away from the end
  - Energy balance $\dot{Q}_{cond,x}$ in and $\dot{Q}_{cond,x+\Delta x}$ out
  - Energy in = Energy out: $\dot{Q}_{cond,x} = \dot{Q}_{cond,x+\Delta x} + \dot{Q}_{conv}$
  - Let the perimeter of fin be P. Then surface area of element is $P\Delta x$ so that $\dot{Q}_{conv} = hP\Delta x(T - T_\infty)$
  - Simplifying the energy balance by taking the limit as $\Delta x \to 0$: $\frac{d\dot{Q}_{cond}}{dx} + hP(T - T_\infty) = 0$
  - Using Fourier's law $\dot{Q}_{cond} = -kA_C\frac{dT}{dx}$: $\frac{d}{dx}(kA_c\frac{dT}{dx}) - hP(T - T_\infty) = 0$
  - Assuming A_C, k, P constant: $\frac{d^2T}{dx^2} - \frac{hP}{kA_C}(T - T_\infty) = 0$
  - Define $\Theta = T - T_\infty$ and $a^2 = \frac{hP}{kA_c}$ (constant) so that $\frac{d^2\Theta}{dx^2} - a^2\Theta = 0$ where the solution is $\Theta(x) = c_1e^{ax} + c_2e^{-ax}$
  - Boundary conditions: $T = T_b$ at the left end while on the right end as $L \to \infty$, $T = T_\infty$

- Therefore we simplify by having $T = T_\infty$ at $x = L$
- In terms of Θ: At $x = 0$, $\Theta = T_b - T_\infty = \Theta_b$ and $\Theta(\infty) = 0$
- This gives $c_1 = 0$ and $c_2 = \Theta_b$ so that the solution is $\Theta(x) = \Theta_b e^{-ax}$

- What is the heat loss from the fin? $\dot{Q}_b = -kA_c \frac{dT}{dx}|_{x=0} \dot{Q}_{fin}$

- $\dot{Q}_{fin,long} = \sqrt{hPkA_C}(T_b - T_\infty)$

- Finite fin length: What is the boundary condition at the open end
  - We can heat transfer is negligible so that adiabatic and $\frac{dT}{dx} = 0$ at the boundary
  - At $x = L$, $\frac{dT}{dx} = 0$ so that $\frac{d\Theta}{dx} = 0$: $c_1 e^{aL} - c_2 e^{-aL} = 0$
  - At $x = 0$, $\Theta = \Theta_b$
  - Solve for c_1 & c_2 and the following solution will be obtained: $\frac{T(x)-T_\infty}{T_b-T_\infty} = \frac{\cosh a(L-x)}{\cosh aL}$

- Can do the same for $\dot{Q}_{fin,insulated} = -kA_C \frac{dT}{dx}|_{x=0}$ which will give $\dot{Q}_{fin,insulated} = \sqrt{hPkA_c}(T_b - T_\infty)\tanh(aL)$ where $a = \sqrt{\frac{hP}{kA_c}}$

- To account for heat transfer from the tip, we can add a length ΔL at the end and the area there will be $A_c = \Delta LP$ (P is the perimeter of the fin) so that the corrected length is $L_c = L + \frac{A_c}{P}$

9 Heat transfer from finned surfaces (contd)

- Fin efficiency
  - ΔT given by the difference between the fin temperature and the surrounding
  - Most efficient fin would have a uniform temperature T_b everywhere
  - This would imply an infinite thermal conductivity
  - In this case, $\dot{Q} = hA_{fin}(T_b - T_\infty) = hPL(T_b - T_\infty)$
  - We define $\eta_{fin} = \frac{dotQ_{fin}}{\dot{Q}_{fin,max}}$
  - For an infinitely long fin, $\eta_{fin,long} = \frac{\sqrt{hPkA_c}(T_b-T_\infty)}{hPL(T_b-T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hP}} = \frac{1}{aL}$ which is in terms of physical properties
  - $\dot{Q}_{fin} = \eta_{fin}\dot{Q}_{fin,max} = \eta_{fin} hA_{fin}(T_b - T_\infty) = h\eta_{fin}A_{fin}(T_b - T_\infty)$
  - $\eta_{fin}A_{fin}$ can be treated as the corrected area
  - For an insulated tip, perform the same steps with the original definition to get $\eta_{insulatedtip} = \frac{\tanh aL}{aL}$

- Fin effectiveness
  - How much has the fin increased heat transfer by?

- $\epsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{nofin}}$
- $\dot{Q}_{nofin} = hA_c(T_b - T_\infty)$
- $\dot{Q}_{longfin} = \sqrt{hPkA_c}(T_b - T_\infty)$ so that $\epsilon_{longfin} = \sqrt{\frac{kP}{hA_c}}$
- To increase the effectiveness, make k as large as possible and maximize $\frac{P}{A_c}$
- Fins are most effective with low h so they are used for gases, hot liquids
- Generally we use fins if $\epsilon \geq 2$

- When can we assume fins are infinitely long?
  - $\frac{\dot{Q}_{fin,insulated}}{\dot{Q}_{fin,long}} = \tanh(aL)$. tanh asymptotically approaches 1 as aL approaches ∞
  - In practice, if $aL \geq 5$ we can assume an infinitely long fin. But even $aL = 1$ has tanh $= 0.76$ so it gives 76 % of heat transfer of an infinitely long fin. Therefore $L = \frac{1}{a}$ is a reasonable length for a fin

- Designing a heat sink
  - $\dot{Q}_{total} = \dot{Q}_{unfinned} + \dot{Q}_{fin}$
  - From the definition of efficience $\dot{Q}_{fin} = \eta_{fin} \cdot hA_{fin}(T_b - T_\infty)$
  - $\Rightarrow \dot{Q}_{total} = hA_{unfinned}(T_b - T_\infty) + h\eta_{fin}A_{fin}(T_b - T_\infty) = h\left[A_{unfinned} + \eta_{fin}A_{fin}\right](T_b - T_\infty)$
  - We can define a thermal resistance $R_{fin} = \frac{T_b - T_\infty}{\dot{Q}_{total}} = \frac{1}{h\left[A_{unfinned} + \eta_{fin}A_{fin}\right]}$

10 Transient Heat Conduction

- Consider a solid at temperture T_i and a liquid at $T_\infty < T_i$. The solid is dropped into the liquid. How does T vary over time?
  - We would expect the temperature T to asymptotically approach T_∞
  - Heat Conduction equation: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$
  - We set the first three terms equal to 0 by using the lumped capacitance approximation i.e. no temperature gradient in the body
  - We would expect this to be valid when the object is small and has a high thermal conductivity
  - Using an energy balance: $\dot{E}_{store} = -\dot{Q}_{conv}$
  - $\dot{E}_{store} = mc_p\frac{dT}{dt} = \rho V c_p \frac{dT}{d}$ and $\dot{Q}_{conv} = hA(T - T_\infty)$
  - Equating the two, we get $\frac{d(T-T_\infty)}{T-T_\infty} = -\frac{hA}{\rho V c_p}dt$ so that $\ln(T - T_\infty) = \frac{-hA}{\rho V c_p}t + C_1$
  - Using $T = T_i$ at $t = 0$, we get $\ln\left[\frac{T-T_\infty}{T_i - T_\infty}\right] = \frac{-hA}{\rho V c_p}t$
  - We define a "time constant" $\tau = \frac{\rho V c_p}{hA}$ so that $\frac{T-T_\infty}{T_i-T_\infty} = \exp\left[\frac{-t}{\tau}\right]$

- The LHS starts at 1 and decays to 0 as $t \to \infty$. Moreoever at $t = \tau$, the value is $\frac{1}{e} \approx 0.368$

- The response time of a thermometer is usually 3τ. However it is important to note that τ is a function of h so it varies in different environments

- Moreover τ depends on $\frac{V}{A} = \frac{r}{3}$ for a sphere so to get a fast response time you would make it very thin

- When is a lumped capacitance valid?

  - At steady state, the conduction in a solid must be equal to the convection in a fluid i.e. $kA\frac{(T_1-T_2)}{L} = hA(T_2 - T_\infty)$ i.e. $\frac{T_1-T_2}{T_2-T_\infty} = \frac{hL}{k}$
  - $\frac{hL}{k}$ is a dimensionless number and is known as a Biot number.
  - Note: k is the thermal conductivity of the solid, L is the length scale in the direction of conduction
  - Suppose the Biot number is large i.e. $>> 1$ so that $T_1 - T_2 >> T_2 - T_\infty$
  - If Biot number is very small, then $T_1 - T_2 << T_2 - T_\infty$ so we can neglect T change inside the body (we assume uniform temp in the body) and use the lumped capacitance model
  - By $<<$ we typically mean a Biot value < 0.1
  - In an irregular body, the length scale used is $L = \frac{V}{A}$

11 Transient Heat Conduction in 2 and 3 Dimensions

- A ball of volume V,mass m and SA A and heat transfer coefficient h is droppped into a fluid

- Assume that the temperature T is uniform in the body

- We had previously assumed that if $Bi < 0.1$ we have a lumped capacitance

- Example: Steel shaft, $k = 51.2$, $\rho = 7832$, $c = 541$ and $T_i = 300$ is placed into a furnace with $T_\infty = 1200$.

  - How long before the shaft temperature reaches 800?
  - We first calculate the Biot number as $Bi = \frac{hL}{k}$ where $L = \frac{V}{A} = \frac{\pi r^2 L}{2\pi r L} = \frac{r}{2}$
  - Then $Bi = \frac{h\frac{r}{2}}{k} = \frac{100\times\frac{0.05}{2}}{51.2} = 0.05$ so we can apply the lumped value
  - $\frac{T-T_\infty}{T_i-T_\infty} = \exp\left[-\frac{hA}{\rho V c}t\right]$ where $\frac{A}{V} = \frac{2}{r}$
  - This gives $\ln\left[\frac{800-1200}{300-1200}\right] = \ldots$ and solving gives $t = 859s$

- Transient heat conduction in 3 dimensions e.g. plane walls, cylinders, spheres

- What happens if $Bi > 0.1$?
- In this case we cannot neglect the temperature gradients inside the body
- Have to solve the complete heat conduction equation
- Consider a solid wall with temperature T_i on one side which then instantly becomes lowered to a temperature T_∞ as it is placed into a fluid.
- As time increases, the temperature inside the wall e.g. at the center decreases
- So in this case, T is a function of x and t
- For a plane wall $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$ where $\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity
- Second order wrt x so two boundary conditions are needed there. First order wrt t so one initial condition is neeeded there
- This can be solved analytically but we will not do that
- We instead consider the lumped capacitance solution $\ln\left[\frac{T-T_\infty}{T_i-T_\infty}\right] = \frac{-hA}{\rho V c_p}t$
- We take the characteristic length $L = \frac{V}{A}$.
- $\frac{hA}{\rho V c_p}t = \frac{h}{\rho L c_p}t = (\frac{h}{\rho L c_p}t)(\frac{L}{L}\cdot\frac{k}{k}) = (\frac{hL}{k})(\frac{k}{\rho c_p})(\frac{t}{L^2}) = (\frac{hL}{k})(\frac{\alpha t}{L^2})$ where $Bi = \frac{hL}{k}$ which is unitless
- We define the Fourier number $Fo = \frac{\alpha t}{L^2}$ which is also dimensionless
- The dimensionless temperature $\Theta = \frac{T-T_\infty}{T_i-T_\infty}$ so that the lumped capacitance solution can be written as $\Theta = \exp(-Bi \cdot Fo)$
- A physical interpreation of the Fourier number can be found by considering a cube with side length L. Then $\dot{Q}_{cond} = kA\frac{\partial T}{\partial x} = kL^2\frac{\Delta T}{L} = kL\Delta T$
- $\dot{Q}_{store} = mc_p\frac{\partial T}{\partial t} = \rho L^3 c_p\frac{\Delta T}{t}$ so that $\frac{\dot{Q}_{cond}}{\dot{Q}_{store}} = \frac{k}{\rho c_p}\cdot\frac{t}{L^2} = \frac{\alpha t}{L^2} = Fo$
- Even when we cannot assume lumped capacitance and get an exact solution of the heat conduction equation, the solution is of the form $\Theta = \Theta(Bi, Fo)$
- We can define $\Theta_0 = \frac{T_0-T_\infty}{T_i-T_\infty}$ so that the solution is of the form $\Theta_0 = A_1 e^{-\lambda_1^2 Fo}$ where A_1, λ_1 are function of Bi

• Example: Carbon steel plate with $T_i = 440$ is placed in a furnace at $T_\infty = 600$. We need to heat to a minimum temperature of 520. What is the time t required.

- $h = 200$, $k = 40$ and $\alpha = 8 \times 10^{-6}$
- $Bi = \frac{hL}{k} = \frac{200\times 0.04}{40} = 0.2$
- Since $Bi > 0.1$, we cannot use the lumped capacitance and instead use the analytical solution which we obtain from tables
- From Table 18.2, $Bi = 0.2 \implies \lambda_1 = 0.4328, A_1 = 1.0311$
- $\Theta_0 = \frac{T_0-T_\infty}{T_i-T_\infty} = A_1 e^{-\lambda_1^2 Fo}$
- $\Theta_0 = \frac{520-600}{440-600} = 0.5$ so that $0.5 = 1.0311\exp(-(0.4328)^2 Fo)$ and so $Fo = 3.864$
- $Fo = \frac{\alpha t}{L^2} \implies t = \frac{Fo L^2}{\alpha} = \frac{3.864\times(0.04)^2}{8\times 10^{-6}} = 773s$

12 Transient Heat Conduction in Semi-Infinite Solids

- The general problem consists of a body at temperature T_i with the surface temperature suddenly being changed to T_s.
  - Temperature variation would go from T_s down to T_i. δ corresponds to a skin depth and after that we are in the core
  - How does the skin depth δ vary with time?
  - Heat conduction equation with one dimension: $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$
  - An estimate of $\frac{\partial^2 T}{\partial x^2}$ can be found as $\frac{(\frac{\partial T}{\partial x})_{x\sim\delta} - (\frac{\partial T}{\partial x})_{x\sim 0}}{\delta}$
  - $(\frac{\partial T}{\partial x})_{x=0} \sim \frac{T_i - T_s}{\delta}$ so that $\frac{\partial^2 T}{\partial x^2} \sim 0 - \frac{T_i - T_s}{\delta^2}$, $\frac{\partial T}{\partial t} \sim \frac{T_s - T_i}{t}$
  - From the heat conduction equation $-\frac{T_i - T_s}{\delta^2} \sim \frac{1}{\alpha}\frac{T_s - T_i}{t}$
  - Time for effect to be felt throughout the body is $t_c \sim \frac{r_0^2}{\alpha}$
  - $\delta \sim \sqrt{\alpha t}$
  - For a short time $t << t_c$, we can treat the body as being semi infinite
  - The exact solution is given by $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$
  - δ grows as a function of time
  - Boundary conditions: At $x = 0$, $T(0,t) = T_s$ and as $x \to \ \infty T(\infty, t) = T_i$
  - Initial conditions: $T(x,0) = T_i$
  - Define a similarity variable $\eta = \frac{x}{\delta}$ such that $0 < \eta < 1$
  - $\eta = \frac{x}{2\sqrt{\alpha t}}$ so that $\frac{\partial T}{\partial t} = \frac{dT}{d\eta} \cdot \frac{\partial \eta}{\partial t} = \frac{d}{dt}\left[\frac{-x}{4t\sqrt{\alpha t}}\right]$
  - $\frac{\partial T}{\partial x} = \frac{dT}{d\eta} \cdot \frac{\partial \eta}{\partial x} = \frac{dT}{d\eta}\left[\frac{1}{2\sqrt{\alpha t}}\right]$
  - $\frac{\partial^2 T}{\partial x^2} = \frac{d}{d\eta}(\frac{\partial T}{\partial \eta}) \cdot \frac{\partial \eta}{\partial x} = \frac{d^2 T}{d\eta} = \frac{1}{4\alpha t}$
  - Transforming the heat conduction equation, we get $\frac{d^2 T}{d\eta^2} = \frac{1}{\alpha}\frac{dT}{d\eta}(\frac{-x}{4t\sqrt{\alpha t}}) = -2\eta\frac{dT}{d\eta}$
  - The PDE is now an ODE
  - At $x = 0$, $\eta = 0$, $T(0) = T_s$ and as $x \to \infty$, $\eta \to \infty$, $T(\infty) = T_i$
  - Let $w = \frac{dT}{d\eta}$ so that $\frac{dw}{d\eta} = -2\eta w$ which gives $\ln w = -\eta^2 + c_0$
  - $\frac{dT}{d\eta} = w = c_0 e^{-\eta^2}$ and integrating, $T = c_0 \int_0^\eta e^{-u^2}\, du + c_1$
  - Boundary conditions give $T_i = c_0 \int_0^\infty e^{-u^2}\, du + T_s$ which gives the solution $\frac{T - T_s}{T_i - T_s} = \frac{2}{\sqrt{\pi}}\int_0^\eta e^{-u^2}\, du = erf(\eta)$ where the RHS is the error function
  - Can also be written as $1 - \frac{T - T_s}{T_i - T_s} = 1 - erf(\eta)$ or $\frac{T - T_i}{T_s - T_i} = erfc(\eta)$
- Heat Flux at Surface
  - $\dot{q}_s = -k\frac{\partial T}{\partial x}|_{x=0} = 0k\frac{\partial T}{\partial \eta}\frac{\partial \eta}{\partial x}|_{\eta=0}$

- $\frac{d\eta}{dx} = \frac{1}{2\sqrt{\alpha t}}$
- $\dot{q} = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}}$

- Contact of two semi-infinite bodies
  - One body with temperature $T_{A,i}$ and conductivity k_A and the other with $T_{B,i}$ and k_B
  - We require that $T_{S,A} = T_{S,B}$
  - The heat flux must be the same i.e. $\dot{q}_{S,A} = \dot{q}_{S,B}$
  - $-\frac{k_A(T_s - T_{A,i})}{\sqrt{\pi(\alpha_a t)}} = \frac{k_B(T_s - T_{B,i})}{\sqrt{\pi(\alpha_B t)}}$
  - $\implies \frac{T_{A,i} - T_s}{T_s - T_{B,i}} = \frac{\sqrt{(k\rho c)_B}}{\sqrt{(k\rho c)_A}}$
  - Define effusivity as $\gamma = \sqrt{k\rho c}$

13 Forced convection, Velocity and Thermal boundary layers, Reynolds, Prandtl and Nusselt numbers

- Forced Convection
  - Convection is the heat transfer from a surface to a moving fluid
  - Forced part means that motion is imposed by external means
  - A surface at a temperature T_s and a fluid with velocity V_∞ and temperature T_∞
  - $\dot{Q}_{conv} = hA(T_s - T_\infty)$
  - How do we determine h?
    * Quite complicated
    * Depends on physical properties of fluid: viscosity μ, density ρ, thermal conductivity k and specific heat c_p
    * Depends on the fluid velocity V_∞
    * Depends on the shape and size of body: Characteristic length i.e. for a plate it is the length L whereas for a cylinder of sphere it is the diameter D. Differences for objects of irregular shapes
    * Type of flow - laminar, turbulent

- Velocity Boundary layer
  - Incoming flow at velocity V_∞ and temperature T_∞ and a solid plate
  - There is a no slip condition at the interface between the fluid and the solid. i.e. the velocity at the interface must be 0 but if you move away, it will go back to V_∞
  - We can define a boundary layer thickness by choosing where the velocity is $0.99V_\infty$

- At $y = 0$ (plate surface), $V = 0$. The heat transfer there is thus by conduction only so that Fouriers law applies
- Here $\dot{q}_{cond} = -k_{fluid}\frac{\partial T}{\partial y}|_{y=0}$. We have also defined $\dot{q}_{conv} = h(T_s - T_\infty)$
- At $y = 0$, $\dot{q}_{conv} = \dot{q}_{cond}$ so that $h = \frac{-k_{fluid}\frac{\partial T}{\partial y}|_{y=0}}{T_s - T_\infty}$
- Therefore h depends on k_{fluid} and the temperature gradient $\frac{\partial T}{\partial y}|_{y=0}$. This leads to thermal boundary layers

- Thermal Boundary layer
  - Temperature of the surface is $T_s > T_\infty$
  - Fluid coming at temperature T_∞
  - When the fluid comes in contact with the plate, there cannot be a discontinuity so it must be equal to T_s at the contact point
  - INSERT PIC FROM NOTES. We define the thermal boundary layer in a similar way to be the point where $T - T_s = 0.99(T_\infty - T_s)$
  - $\frac{\partial T}{\partial y}|_{y=0}$ is the temperature gradient.
  - h changes with position as the local $\frac{\partial T}{\partial y}|_{y=0}$ changes
  - We define a local heat transfer coefficient $h(x)$
  - We can average: $\bar{h} = \frac{1}{L}\int_0^L h(x)\,dx$
  - Instead of worrying about local variations, we use the average value of h

- Velocity Boundary layer flow
  - The difference between the boundary and the surfaces increases linearly as a straight path in laminar flow
  - There is a transition region then where the path will start becoming unstable which eventually becomes a turbulent region with turbulent flow
  - Fluid exerts a drag on the plate. Measured in terms of the shear stress (force per unit area)
  - $\tau = \mu\frac{\partial V}{\partial y}|_{y=0}$ where μ is the fluid viscosity ($\frac{kg}{ms}$)
  - Velocity gradient can be solved but it is very complicated and practically we define a friction coefficient
  - We imagine a fluid coming to rest at a stagnation point. Using Bernoulli's equation, $\frac{P}{\rho} = \frac{V_\infty^2}{2} + \frac{P_\infty}{\rho} \implies P - P_\infty = \frac{\rho V_\infty^2}{2}$. This is the pressure rise and the force felt by plate
  - We define the friction coefficient c_f so that $\tau = c_f\frac{\rho v_\infty^2}{2}$
  - Generally we expect $c_f \sim 1$ but this depends on the shape

- Laminar & Turbulent Flow

- Heat transfer is greater in turbulent flow i.e. with an increased velocity
- However when the heat transfer goes up, shear goes up and so bigger fans (more energy) are needed
- The transition to turbulence depends on the ratio of fluid inertia to viscosity
- A high inertia drives random motion and therefore turbulence
- A high viscosity damps turbulence
- Consider a mass with diameter D and a fluid coming in around it at velocity V_∞
  * There is an inertial force F_i and a viscous force F_v
  * Take the characteristic distance to be D
  * The inertial force $F_i = ma$ where the mass $m = \rho D^3$ and the acceleration $a = \frac{V_\infty^2}{D}$
  * The viscous force $F_v = \tau A = \mu \frac{\partial V}{\partial y} \cdot A$
  * $F_v \sim \mu \frac{V_\infty}{D} \cdot D^2 \implies \frac{F_i}{F_v} = \frac{\rho V_\infty D}{\mu}$
  * This number is known as the Reynolds number: $Re = \frac{\rho V_\infty D}{\mu}$

14 Forced Convection Currents

- Fluid with velocity V_∞ and density ρ_{mu}. What forces are exerted onto the body (with characteristic length being the diameter D)
  - Inertial force $F_i = ma$
    * $m \sim \rho D^3$
    * The fluid starts with velocity V_∞ and is brought to rest over a distance D
    * $t = \frac{D}{V_\infty}$
    * $\Delta V = V_\infty$ so $a \sim \frac{\Delta v}{t} = \frac{V_\infty}{\frac{D}{V_\infty}} = \frac{V_\infty^2}{D}$
    * Thus $F_i \sim \rho D^3 (\frac{V_\infty^2}{D}) = \rho D^2 V_\infty^2$
  - Viscous Force
    * $F_v = \tau A = \mu \frac{dV}{dy} \cdot A$
    * $F_v \sim \mu \frac{V_\infty}{D} \cdot D^2$
  - We are interested in the ratio
    * $\frac{F_i}{F_v} \sim \frac{\rho V_\infty D}{\mu}$
    * This is a very important number called the Reynolds Number $Re = \frac{\rho V_\infty D}{\mu}$
    * The kinematic viscosity is defined as $\nu = \frac{\mu}{\rho}$ so that he Reynolds number can also be written as $Re = \frac{V_\infty D}{\nu}$
    * For small Re, viscous forces are dominant. Fluctuations in the flow are damped. This leads to laminar flow.

- * For large Re, inertial forces are dominant. Fluctuations in the flow become amplified and this leads to turbulent flow
  * For every geometry, there is a critical value of Re at which a transition to turbulence occurs e.g. $Re_{critical,flatplate} = 5 \times 10^5$

- Two boundary layers are developing - velocity, thermal
  - The velocity goes from V_∞ down to 0 and the temperature goes down from T_∞ to T_s
  - Let δ_v be the velocity boundary layer and δ_t be the thermal boundary layer
  - δ_t may be smaller or larger than δ_v. How do we tell?
    * This depends on the physical properties of the fluid
  - Fluids with high viscosity ν (oils) have thick velocity BL i.e. δ_v is a large fraction of V_∞
  - Fluids with high thermal diffusivity ()$\alpha = \frac{k}{\rho c_p}$) have thick thermal BL
  - The ratio $\frac{\delta_v}{\delta_t}$ is given by the ratio $\frac{\nu}{\alpha}$
  - The Prandtl Number is defined to be this ratio: $Pr = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$
  - Pr is a fluid property
  - For $Pr \ll 1$ (e.g. a liquid metal) $\delta_v \ll \delta_t$
  - For $Pr \gg 1$ (e.g. oils) $\delta_v \gg \delta_t$
  - For $Pr \sim 1$ (e.g. gases) $\delta_t \sim \delta_v$

- We have two dimensionless parameters $Re = \frac{\rho V_\infty D}{\mu}$ and $Pr = \frac{\nu}{\alpha}$

- Need to non dimensionalize h
  - $\dot{Q}_{conv} = hA(T_s - T_\infty) \sim hD^2(T_s - T_\infty)$
  - Suppose the fluid was not moving
  - Then heat transfer is by conduction only
  - $\dot{Q}_{cond} = k_{fluid} A \frac{dT}{dr} \sim k_{fluid} D^2 \frac{T_s - T_\infty}{D}$
  - How much is heat transfer enhanced due to convection
  - This is given by the ratio $\frac{\dot{Q}_{conv}}{\dot{Q}_{cond}} = \frac{hD}{k_{fluid}}$ which is a dimensionless number
  - This ratio is known as the Nusselt number $Nu = \frac{hD}{k_{fluid}}$
  - Do not confuse with $Bi = \frac{hD}{k_{solid}}$

- We started with $h = f(D, V_\infty, \rho, \mu, c_p, k)$ and dependent on the geometry

- This can now be written as $\frac{hD}{k} = f\left(\frac{\rho D V_\infty}{\mu}, \frac{\mu c_p}{k}\right)$ and dependent on the geometry

- $\implies Nu = f(Re, Pr)$ and geometry

- Can do experiments
  - Consider a plate with a uniform heat flux due to an electrical current through it
  - Can put it into a wind tunnel with V_∞ and T_∞
  - We know that $\dot{Q} = hA(T_s - T_\infty) = P = EI$ (electrical power)
  - We know the other variables and can thus solve for h and plot it as a function of V_∞
  - Repeat for different plate sizes, different T_s and different T_∞
  - Can plot Nu vs Re and all the data should fall on the same curve
  - Repeat for different fluids to get different curves for different fluids
  - Can then plot $\ln(\frac{Nu}{Pr^n})$ against $\ln Re$ and the data for different fluids will all lie on the same line
  - $\ln(\frac{Nu}{Pr^n}) = m \ln Re + C$ or equivalently $Nu = cRe^m Pr^n$ where n, m, C are determined experimentally for each geometry

15 Forced convection correlations

- Read Ch 19.3, 19.4. 19.5 - 19.8 will not be covered
- Dimensionless Analysis
  - Consider a tube with velocity from the bottom
    * Z (height) is in terms of P, V
    * What are relevant parameters?
      · P, V, z
      · Fluid properties: ρ
      · Gravity g
    * Develop an equation relating P, V, z, ρ, g
    * If we develop an equation, the dimension of both sides must be equal
    * $P[\frac{N}{m^2}] = \frac{kg\frac{m}{s^2}}{m^2} = [\frac{kg}{ms^2}]$
    * $\frac{P}{\rho}$ is in $[\frac{m^2}{s^2}] \implies \frac{P}{\rho V^2}[\frac{m^2}{s^2} \times \frac{s^2}{m^2}]$
    * $\frac{g}{V^2}$ is also dimensionless
    * Therefore $\frac{P}{\rho V^2} = f\left(\frac{gz}{v}\right)$
    * We plot $\frac{P}{\rho V^2}$ against $\frac{gz}{V^2}$ and we discover that all data falls on one line. If it doesnt fall on one line, the analysis was done incorrectly
    * Thus $\frac{P}{\rho V^2} = -\frac{gz}{v^2} + C_1$ which when simplified gives $\frac{P}{\rho} + gz + \frac{V^2}{2}$ which is a constant. We can derive Bernoulli's equation experimentally in this way
- Forced Convection Correlation
  - Flow over a flate plate

- V_∞ flowing from the left, laminar region, then transition and then turbulenc
- $\tau = \mu \frac{\partial V}{\partial y} \sim \mu \frac{V_\infty}{\delta_V}$
- As δ_v goes up, τ goes down
- Local frictional coefficient $c_{f,x}$ goes down initially while in laminar, then goes up while in transition and finally goes down again in turbulence region
- For laminar flow $c_{f,x} = \frac{0.664}{Re_x^{\frac{1}{2}}}$
- Local Reynolds number $= Re_x = \frac{V_\infty x}{\nu}$
- Average friction coefficient over length L of plate: $c_f = \frac{1}{L}\int_0^L c_{f,x}\,dx = \frac{1}{L}\int_0^L \frac{0.664}{Re_x^{\frac{1}{2}}}\,dx = \frac{1}{L}0.664(\frac{\nu}{V_\infty})^{\frac{1}{2}}\int_0^L \frac{dx}{x^{\frac{1}{2}}} = \frac{2}{L}\times 0.664(\frac{\nu}{V_\infty})^{\frac{1}{2}}\cdot L^{\frac{1}{2}}$
- This gives $c_f = \frac{1.328}{Re_L^{\frac{1}{2}}}$
- The heat transfer coefficient $h = -\frac{k}{T_s - T_\infty}\frac{\partial T}{\partial y}|_{y=0}$ so that $\frac{\partial T}{\partial y} \sim \frac{T_s - T_\infty}{\delta_t}$
- As δ_t increases, $\frac{\partial T}{\partial y}$ goes down implying that h decreases
- So h decreases while in laminar flow, increases in the transition and then decreases again in turbulent region
- We define a Local Nusselt Number: $Nu_x = \frac{h_x x}{k}$
- For laminar flow, $Nu_x = 0.332 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}$ where $Pr \geq 0.6$
- The average Nusselt number is $Nu = \frac{hL}{k} = \frac{1}{L}\int_0^L Nu_x\,dx$
- This gives $Nu = 0.664 Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}}$
- For turbulent flow, $Re_x > 5\times 10^5$ and $c_{f,x} = \frac{0.0592}{Re_x^{\frac{1}{2}}}$

- Example
  - Air with $T_\infty = 300C$, $V_\infty = 10$ on a tube with $T_s = 50$ and $L = 0.5$. Find $\dot{Q}_{cooling}$
  - The film temperature is used where $T_f = \frac{T_\infty + T_s}{2}$
  - In this case, $T_f = 175$ and using $\nu = 3.18\times 10^{-5}$, $Pr = 0.7$ and $k = 0.0363$ (using tables)
  - Therefore $Re_L = \frac{V_\infty L}{\nu} = 1.57\times 10^5$
  - Flow is laminar since $Re_L < 5\times 10^5$
  - $Nu = 0.664 Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}} = 233.6$
  - $Nu = \frac{hL}{k} \implies h = \frac{Nuk}{L} = 16.9\frac{W}{m^2C}$
  - Cooling per unit width of the plate $\dot{Q} = hA(T_\infty - T_s) = 16.9\times(0.5\times 1)(300-50) = 2112.5\frac{W}{m}$
- Flows over cylinders and spheres

- Flow seperation causes wake
- Not to be confused with turbulence
- Turbulence occurs when $Re = \frac{V_\infty D}{\nu} > 2 \times 10^5$
- For flow accross cylinders, $Nu = cRe^m Pr^n$
- The values of c, m, n depend on the range of the Reynolds number - given in Table 19.2
- Churchill & Bernstein correlation is valid for $RePr > 0.2$
- Fluid properties evaluated at $T_f = \frac{T_s + T_\infty}{2}$
- Flow over a sphere has $Nu = 2 + [0.4Re^{\frac{1}{2}} + 0.06Re^{\frac{2}{3}}] \cdot Pr^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{0.25}$
- All properties evaluated at T_∞ and mu_s is evaluated at T_s
- Valid for $3.5 \leq Re \leq 80000$ and $0.7 \leq Pr \leq 380$

16 Thermal Radiation, Black body radiation, Radiative properties

- Read Ch 21.1 - 21.4
- Radiation
  - Consider a solid at temperature T_s with an enclosure outside at T_{surr} and a vacuum in between
  - Initially $T_s > T_{surr}$
  - No conduction, convection since vacuum so heat transfer is by radiation
  - Solid will lose heat to surrounding surface and cool until $T_s = T_{surr}$
  - Thermal radiation: Energy is emitted by matter as a result of its finite temperature
  - Energy is in the form of electromagnetic waves
  - Radiation has wave properties: frequency ν and wavelength λ related by $\lambda = \frac{c}{\nu}$
  - Thermal radiation has a wavelength from 0.1 to 100 μm. This includes ultraviolet $(0.1 - 0.4\mu m)$, visible light $(0.4 - 0.7\mu m)$ and infrared $(0.7 - 100\mu m)$
  - For opaque objects, radiation occurs from the surface
- Black body radiation
  - A block body is a perfect emitter of radiation
  - At a given temperature, no surface can emit more energy than a Blackbody
  - A blackbody absorbs all radiation incident on
  - It emits equally in all directions. It is a "diffuse" service

- The Stefan-Boltzmann law: The radiation energy emitted by a blackbody per unit time and per unit surface area is: $E_b = \sigma T^4$ (in $\frac{W}{m^2}$) where $\sigma = 5.67 \times 10^{-8}$, E_b is the blackbody emissive power and T is the temperature in K
- A surface painted black is close to a blackbody for visible radiation
- A white surface absorbs infrared light - can be considered a blackbody in IR
- Depending on the wavelength, different objects can be considered blackbodies
- To get a perfect blackbody, you take a box with a small opening and the opening is the blackbody
- This is because the aperture absorbs all light (anything that goes inside the opening reflects inside there and is thus absorbed)
- Radiation from a real surface encompasses a range of wavelengths. This is described by a "spectral distribution"
- Can plot Emission energy $E_{b,\lambda}$ against λ to obtain a curve described by Planck's Law: $E_{b,\lambda} = \frac{c_1}{\lambda^5\left[\exp(\frac{c_2}{\lambda T})-1\right]}$
- There may also be a directional distribution
- Surface emits more in a given direction

- Radiation properties
  - For a real surface, the emissive power is less than that of a blackbody
  - We define a surface property - emissivity
  - $\varepsilon(T) = \frac{E(t)}{E_b(t)}$ where $E(t)$ is integrated over all λ and θ and $E_b(t)$ is integrated over all λ
  - We will assume that ε is independent of λ. This is known as a gray surface
  - ϵ is independent of θ: A diffuse surface
  - The emissive power of a real surface is $E(t) = \varepsilon\sigma T^4$

- Surface Absorption, Reflection and Transmission
  - G is the incident radiation in $\frac{W}{m^2}$
  - Part of it will be reflected (G_{ref}), part will be absorbed G_{abs} and the rest will be transmitted ($G_{transmitted}$)
  - G-radiation is the radiant energy incident on a surface per unit surface area per unit time
  - Define 3 properties: Absorptivity $\alpha = \frac{G_{abs}}{G}$, the reflectivity $\rho = \frac{G_{ref}}{G}$ and the transmittivity $\tau = \frac{G_{tr}}{G}$
  - In general $\alpha + \rho + \tau = 1$
  - For opaque objects, $\tau = 0$, for a blackbody $\alpha = 1$
  - The "Gray Body assumption" is that α, ρ, τ are independent of λ
  - The "Diffuse surface assumption" is that α, ρ, τ do not depend on direction

17 Radiation heat transfer - black surfaces

- Consider a small body inside an enclosre
  - Let the small body have surface area A, emissivity ϵ and absorptivity α
  - The temperature inside the enclosure is T and at equilibirum both the surface and the inside are at the same temperature
  - G is the incident radiation per unit area
  - The large cavity acts as a black body
  - We neglect absorption by the small body
  - The cavity surfaces absorb all incident radiation
  - We can assume that $G = \sigma T^4$
  - For a small body $E_{abs} = \alpha G A = \alpha \sigma T^4 A$ is the energy absorbed
  - $E_{emit} = \epsilon \sigma T^4 A$
  - At equilibrium, $E_{abs} = E_{emit}$ so that $\sigma = \epsilon$
  - This is Kirchoff's law which says that for any surface the emissivity is equal to the absorptivity
- Radiation between surfaces
  - Two surfaces which are exchanging energy due to radiation
  - Amount of radiation incident on a surface depends on the orientation of the surfaces
- The View Factor
  - Consider two surfaces i and j
  - We define the view facor F_{ij} as the fraction of radiation leaving surface i that reaches surface j directly
  - e.g. $F_{1,3}$ is the fraction of energy leaving surface 1 which reaches surface 3 directly
  - Can also define F_{ii} for a concave surface which would be the fraction leaving it that reaches it on the other end
  - If the surface is convex, $F_{ii} = 0$
  - If one surface i is completely enclosed by another surface j, $F_{ii} = 0$ and $F_{ij} = 1$
- Calculation of view factors
  - Done by integrating geometric shapes
  - Can calculate the energy exchanges dA_1 and dA_2
  - For common geometries, the view factor calculations can be done based on charts and tables

- Table 21.3, 21.4, Fig 21.33 can be used
- In general, for an enclosure with N sides, $\sum_{i=1} NF_{ij} = 1$
- All radiation leaving surface i must be intercepted by other surfaces
- For an enclosure of N surfaces, a total of N^2 view factors
- These can be put into a matrix
- Not all view factors are independent. The summation rule says that $\sum_{j=1}^{N} F_{i,j} = 1$
- The reciprocity rule
  * If A_1, A_2 are blackbodies then all incident radiation is absorbed so that $\alpha = \epsilon = 1$
  * Then the energy leaving A_1 and reaching A_2 is $E_{b1}A_1F_{12}$
  * Doing the same in the reverse direction, the energy leaving A_2 and reaching A_1 is $E_{b2}A_2F_{21}$
  * The net energy exchange Q_{12} is $Q_{12} = E_{b1}A_1F_{12} - E_{b2}A_2F_{21}$
  * If both bodies are at the same temperature, there is thermal equilibrium so $Q_{12} = 0$
  * Moreover $E_{b1} = E_{b2} = \sigma T^4$
  * Thus $A_1F_{12} = A_2F_{21}$
  * A_1, A_2, F_{12}, F_{21} are all geometric properties independent of T, ϵ
  * Thus in general for any two surface i, j, $A_iF_{ij} = A_jF_{ji}$
- Superposition
  * Suppose you break up surface j in two parts a and b
  * Then $F_{ij} = F_{i,ja} + F_{1,jb}$
- Symmetry
  * Consider a geometry such that surface j and k are identical
  * Then $F_{ij} = F_{ik}$ and $F_{ji} = F_{ki}$

• Example: Long duct of length L

- A 3/4 circle with radius R with area A_2 and the slice with area A_1
- $F_{12} = 1$. Note $F_{21} \neq 1$ since 2 is concave and not all of the radiation goes from there to A_1
- $A_2F_{21} = A_1F_{12} \implies F_{21} = \frac{A_1}{A_2}F_{12}$
- $\implies F_{21} = \frac{2RL}{\frac{3}{4}(2\pi R)L} = \frac{4}{3\pi} \approx 0.424$
- $F_{22} + F_{21} = 1 \implies F_{22} = 1 - F_{21} = 0.576$

• Example: Small sphere under a concentric hemisphere

- A_1 is the area of the small sphere and A_2 is that of the hemisphere so that $A_2 = 2A_1$

- We can extend the hemisphere to a sphere and let A_3 be the area of the added region (ADD DIAGRAM)
- We know that $F_{11} + F_{12} + F_{13} = 1$ where $F_{11} = 0$ since it is a convex surface
- By symmetric, $F_{12} = F_{13} \implies F_{12} = 0.5$
- By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{A_1}{2A_1} 0.5$ so $F_{21} = 0.25$

- Radiation from black surfaces
  - Black surface means no reflection
  - All incident radiation is absorbed
  - Two surface A_1 and A_2 with temperatures T_1 and T_2 respectively
  - Energy from $A_1 \to A_2 = A_1 E_{b1} F_{12}$
  - Energy from $A_2 \to A_1 = A_2 E_{b2} F_{21}$
  - Net radiation exchange $\dot{Q}_{12} = A_1 E_{b1} F_{12} - A_2 E_{b2} F_{21}$
  - Since $A_1 F_{12} = A_2 F_{21}$, $\dot{Q}_{12} = A_1 F_{12}(E_{b1} - Eb2)$
  - $\implies \dot{Q}_{12} = A_1 F_{12} \sigma (T_1^4 - T_2^4)$
  - Since $F_{12} = 1$, $\dot{Q}_{12} = A_1 \sigma (T_1^4 - T_2^4)$

18 Radiation Heat Transfer - Diffuse, Grey Surfaces

- Read Chapter 21.8
- Not all surfaces are black
- For a more realistic analysis, assume that surfaces are 1. Isothermal, 2. Opaque, 3. Diffuse (ϵ is independent of Θ) and 4. Grey (ϵ is independent of λ)
- New Terms
  - Define G as the irradiation: total radiation that is incident on a surface per unit time and per unit area
  - Define J as the radiosity: total radiation energy which leaves a surface per unit area and per unit time
  - $E = \epsilon E_b$. $J = \epsilon E_b + \rho G$ where $E_b = \sigma T^4$
  - $\rho = 1 - \alpha = 1 - \epsilon$
  - We can simplify J as $\epsilon E_b + (1 - \epsilon) G$
  - Net energy leaving the surface per unit area: $\frac{\dot{Q}}{A} = J - G$
  - The total energy is then $\dot{Q} = A(J - G)$ and since $J = \epsilon E_b + (1 - \epsilon) G$, we can rearrange to get $G = \frac{J - \epsilon E_b}{1 - \epsilon}$
  - Substituting this gives $\dot{Q} = A\left(\frac{\epsilon E_b - J}{1 - \epsilon}\right)$ or $\dot{Q} = \frac{E_b - J}{\frac{1-\epsilon}{\epsilon A}}$

- Electrical analogy
  - Can think of $R = \frac{1-\epsilon}{\epsilon A}$ as a resistance
  - We call it the surface resistance to radiation
  - For a blackbody $\epsilon = 1 \implies R = 0$ and so there is no resistance
- Radiation exchange between grey surfaces
  - Two grey surfaces, surface i and surface j
  - Net energy leaving surface i is J_i and the net energy leaving surface j is J_j
  - We are interested in the net exchange between these two surfaces
  - Radiation from i that reaches j is $J_i A_i F_{ij}$
  - Radiation from j that reaches i is $J_j A_j F_{ji}$
  - Net heat transfer $\dot{Q}_{ij} = J_i A_i F_{ij} - J_j A_j F_{ji}$
  - Using the reciprocity equation, $A_i F_{ij} = A_j F_{ji}$ which then means that $\dot{Q}_{ij} = A_i F_{ij}(J_i - J_j) = \frac{J_i - J_j}{(A_i F_i j)^{-1}}$
  - We refer to $\frac{1}{A_i F_{ij}}$ as the space resistance and this depends on the shape of the surfaces and how they are oriented
- Two surface enclosure
  - Insert diagram and resistance
  - $\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$
  - Can write $\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{R_{total}} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{A_2\epsilon_2}}$
- Example
  - Two surfaces. Top surface has $T_1 = 1000K$, $\epsilon_1 = 1$ and bottom surface has $T_2 = 500K$ and $\epsilon_2 = 0.8$
  - What is the net radiation exchange between plates per unit area?
  - $A_1 = A_2 = A$, $F_{12} = 1 - (large plates)$
  - Then $\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{A_2\epsilon_2}}$ so that $\frac{\dot{Q}_{12}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1} + 1 + \frac{1-\epsilon_2}{\epsilon_2}}$
  - Simplifying, $\frac{\dot{Q}_{12}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_2}} = \epsilon_2\sigma(T_1^4 - T_2^4)$
- Example 2
  - See Lecture notes!