

CHE260: Heat Transfer

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1 Introduction

- How is heat transfer different from thermodynamics? In thermodynamics, we assume quasi-equilibrium processes i.e. the time was not an important parameter. In heat transfer, time is an important parameter and we are interested in the rate of heat transfer.
- What is the relationship between \dot{Q} and ΔT ? What are the mechanisms of heat transfer?
- Conduction: Transfer of heat through a medium that is stationary.
- Convection: Transfer of heat from a solid surface and an adjacent fluid that is moving. Example: a fan blowing air over a hot plate. There is heat transfer from the hot plate into the fluid.
- Radiation: Energy emitted by matter in the form of electromagnetic waves.
- Radiation does not need a medium. In a vacuum, we can have radiation but not convection or conduction.
- Different mechanisms of heat transfer can take place simultaneously.
- Applications
 - Power Generation
 - * Power plant: steam generation, condenser
 - * Automobiles: engine cooling, space heating/cooling
 - Buildings
 - * Heating / Cooling
 - * Hot water
 - Refrigeration
 - Manufacturing
 - * Casting / Heat treatment
 - * Injection Moulding

2 Electronic Cooling

- > 99 % of the electrical energy supplied to a circuit is dissipated as heat
- Heat has to be dissipated to the environment while keeping the temperature of the chip in a certain range
- Heat is lost from the surface of the chip
- Important parameter is heat flux = $\frac{\text{Heat Transfer Rate}}{\text{Unit Area}}$ (in $\frac{W}{\text{cm}^2}$)

- To reduce heat flux, we can reduce heat generation and increase the surface area
- As size increases, it becomes more difficult to lose heat
- Water cooling is more efficient for large systems compared to air cooling

3 Radiation

- Radiation is energy emitted by all matter in the form of e.m. radiation
- Thermal radiation is emitted by all bodies at a finite temperature
- Opaque objects emit only from the surface
- Amount of radiation depends on the surface temperature. Summarized by the Stefan Boltzmann Law: $\dot{Q}_{emit} = \sigma AT_s^4$ where σ is the Boltzmann constant ($5.67 \times 10^{-8} \frac{W}{m^2 K^4}$), T_s is the surface temperature in Kelvin and A is the surface area.
- A surface that emits as much radiation as this is called a "Blackbody". A real surface emits less than this: $\dot{Q}_{emit} = \epsilon \sigma AT_s^4$ where ϵ is the emissivity and $0 \leq \epsilon \leq 1$
- Black paint has $\epsilon = 0.99$ which is very close to 1. Aluminum foil has a low emissivity of around 0.07.
- If radiation is incident on a surface some will be absorbed. The fraction absorbed is a surface property known as the absorptivity α such that $\dot{Q}_{absorbed} = \alpha \cdot \dot{Q}_{incident}$ and $\dot{Q}_{reflected} = (1 - \alpha) \cdot \dot{Q}_{incident}$
- Kirchoff's law says that $\alpha = \epsilon$
- Note: α and ϵ vary over different wavelengths
- Consider a special case of radiation
 - Small surface which is completely surrounded by a much larger surface
 - T_s , A_s are temperature and area of the small surface (which is also the boundary), T_{surr} is the temperature of the surrounding surface. Both surfaces are emitting and we are interested in the net emission
 - $\dot{Q}_{rad} = \epsilon \sigma A_s (T_s^4 - T_{surr}^4)$
- Example
 - Chip with an area of $15 \times 15 mm$, $\epsilon = 0.6$, $T_{surr} = 25$.
 - Two methods of heat transfer
 - * Natural convection
 - $h = c(T_s - T_\infty)^{\frac{1}{4}}$
 - $c = 4.2 \frac{W}{m^2 K^{\frac{5}{4}}}$

- $q_{conv} = hA(T_S - T_\infty)$
- $q_{rad} = \epsilon A(T_s^4 - T_{surr}^4)$
- * Forced convection: h is constant at $250 \frac{W}{m^2K}$
- $q_{conv} = hA(T_S - T_\infty)$

4 Heat Conduction

- Heat Conduction Equation
 - x, y, z components of \vec{Q}
 - T is a function of (x, y, z, t)
 - $\vec{Q} = \dot{Q}_x \hat{i} + \dot{Q}_y \hat{j} + \dot{Q}_z \hat{k}$
 - $\dot{Q}_x = -kA_x \frac{dT}{dx}$ (similar expressions for \dot{Q}_y and \dot{Q}_z)
- One dimensional heat conductivity can model more complicated situations. For example if $\Delta x \ll \Delta y, z, \frac{dT}{dx} \gg \frac{dT}{dy}, \frac{dT}{dz}$ so that \dot{Q}_y and \dot{Q}_z can be neglected
- One dimensional heat conduction
 - Cross sectional area is $A(x)$ where x is the coordinate along which heat transfer occurs
 - \dot{Q}_x at the entry and $\dot{Q}_{x+\Delta x}$ at the exit
 - Want to find $T(x)$ inside the object
 - Rate of increase of enthalpy $= mc_p \frac{\partial T}{\partial t} = \rho V c_p \frac{\partial T}{\partial t} = \rho c_p A \Delta x \frac{\partial T}{\partial t}$
 - Energy balance:
 - * $\rho c_p A \Delta x \frac{\partial T}{\partial t} = \dot{Q}_x - \dot{Q}_{x+\Delta x}$
 - * After simplifying and taking the limit as Δx approaches 0, we get $\rho c_p \frac{\partial T}{\partial t} = \frac{-1}{A} \frac{\partial(\dot{Q}A)}{\partial x}$
 - * A depends on the coordinate system and we use Fourier's law for \dot{Q} : $\dot{Q} = -k \frac{dT}{dx}$
- Cartesian Coordinates
 - A is a constant
 - $\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} [k \frac{\partial T}{\partial x}]$
 - Assume k is constant. Then $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$ where $\alpha = \frac{k}{\rho c_p}$.
 - If steady state i.e. $\frac{\partial T}{\partial t} = 0$ then $\frac{d^2 T}{dx^2} = 0$.
 - Units of $\alpha = \frac{k}{\rho c_p}$, the thermal diffusivity is $\frac{m^2}{s}$.
 - High k means the material conducts well. High ρc_p means that the material stores energy
- Cylindrical Coordinates

- Heat being conducted radially so $\dot{q} = -k \frac{\partial T}{\partial r}$ and $A = 2\pi r L$
- $p c_p \frac{\partial T}{\partial t} = \frac{-1}{2\pi r L} [\frac{\partial}{\partial r} \cdot \frac{\partial T}{\partial r}]$
- $\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} (r \cdot \frac{\partial T}{\partial r})$
- At steady state, $\frac{d}{dr} (r \frac{dT}{dr}) = 0$.
- Spherical Coordinates
 - $A = 4\pi r^2$ and $\dot{q} = -k \frac{\partial T}{\partial r}$ where r is the radial spherical coordinate
 - $\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial T}{\partial r})$
- In general $\frac{1}{r^n} \frac{\partial}{\partial r} (r^n \frac{\partial T}{\partial r}) = \frac{1}{\alpha} \frac{\partial T}{\partial t}$ where cartesian has $n = 0$, cylindrical has $n = 1$ and spherical has $n = 2$.

5 Thermal Resistance

- At steady state, $\frac{d^2 T}{dx^2} = 0$
- Heat flux: $\dot{q} = -k \frac{dT}{dx}$.
- Heat flux is a constant
- Heat transfer rate: $\dot{Q} = \dot{q} A = \frac{-k A (T_2 - T_1)}{L}$
- $\dot{Q} = \frac{T_1 - T_2}{R_{wall}}$ where T_1 and T_2 are the temperatures of the walls
- $R_{cond} = R_{wall} = \frac{L}{k A}$
- Similar to current with voltage and Resistance
- $\dot{Q}_{conv} = h A (T_s - T_\infty)$
- $R_{conv} = \frac{T_s - T_\infty}{\dot{Q}_{conv}} = \frac{1}{h A}$
- Radiation is more complicated. $\dot{Q}_{rad} = \epsilon \sigma A (T_s^4 - T_{sur}^4)$
- We need to define a heat transfer coefficient for radiation. $h_{rad} = \frac{\epsilon \sigma A (T_s^4 - T_{sur}^4)}{A (T_s - T_{sur})}$
- $h_{rad} = \epsilon \sigma (T_s^2 + T_{sur}^2) (T_s + T_{sur})$
- Can treat it as a resistance. $R_{rad} = \frac{T_s - T_{sur}}{\dot{Q}_{rad}} = \frac{1}{h_{rad} A}$
- Multilayer Plane Wall
 - Each layer has the same surface Area
 - Layers have thicknesses L_i
 - Temperature varies as T_1 on the outside, T_2, \dots, T_{n+1} where n is the number of surfaces

- Treat each layer separately as resistances in series
- For first wall $\dot{Q} = \frac{T_1 - T_2}{R_1}$ so $T_1 - T_2 = \dot{Q}R_1$. In general, $T_i - T_{i+1} = \dot{Q}R_i$
- Summing all of them gives $T_1 - T_{n+1} = \dot{Q}(R_1 + \dots + R_n)$ so that $\dot{Q} = \frac{T_1 - T_4}{R_{total}}$
- Therefore you can sum up resistances similar to electric circuits
- We can find each R_i as $\frac{L_i}{k_i A}$
- $\dot{Q} = UA(T_{\infty,1} - T_{\infty,4})$ where U is the overall heat transfer
- Example: Refrigerator Wall
 - 1 mm thick insulation on the outside and width of refrigerator is L .
 - $T_{room} = 25^\circ C$ and $T_{refrig} = 3^\circ C$
 - $h_0 = 9 \frac{W}{m^2 C}$ and $h_i = 4 \frac{W}{m^2 C}$
 - $k_{steel} = 15.1 \frac{W}{m^2 C}$ and $k = 0.035 \frac{W}{m^2 C}$ inside the refrigerator
 - Constraint is that $T_{s,out} > 20$. We assume heat transfer from the outside room to the inside of the refrigerator
 - What is L to ensure $T_{s,out} > 20$ to prevent condensation on the outside of the refrigerator
 - Thermal circuit consists of a convective resistance outside the refrigerator followed by 3 conductive resistance on the surfaces and then one convective resistance at the end
 - INSERT PICTURE FROM LEC NOTES
 - $\dot{Q} = \frac{T_{room} - T_{s,out}}{R_{conv,0}} = \frac{T_{room} - T_{s,out}}{\frac{1}{h_0 A}}$
 - Consider unit area. Then $\dot{Q} = h_0(T_{room} - T_{s,out}) = 9(25 - 20) = 45W$
 - $R_{total} = \frac{1}{h} + (\frac{L}{k})_{metal} + (\frac{L}{k})_{insulation} + (\frac{L}{k})_{metal} + \frac{1}{h_i} = \frac{1}{9} + \frac{10^{-3}}{15.1} + \frac{L_2}{0.035} + \frac{10^{-3}}{15.1} + \frac{1}{4} = 0.361 + \frac{L_2}{0.035}$
 - $\dot{Q} = \frac{T_{room} - T_{refrig}}{R_{total}} \rightarrow 45(0.361 + \frac{L_2}{0.035}) = 25 - 3$ so that $L = 45mm$

6 Thermal Resistance Networks

- Multiple layers e.g. in an electric chip each with different thermal properties
 - We are interested in $\dot{Q} = \dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} + \frac{T_1 - T_2}{R_3}$
 - $= (T_1 - T_2)(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}) = \frac{T_1 - T_2}{R_{total}}$
 - This is the electrical analog to parallel resistances
- Thermal contact resistance
 - So far we have been assuming perfect contact between different boundaries

- In reality, there is a rough surface at the boundary
- We can always define a thermal contact resistance $R_c = \frac{T_2 - T_1}{\dot{q}}$ (units are $\frac{m^2 C}{W}$)
(Note: Resistance per unit area)
- The reciprocal of R_c is known as the thermal contact conductance h_c .
- $h = \frac{1}{R_c} = \frac{\dot{q}}{\Delta T}$ so that $\dot{q} = h_c \Delta T$
- h_c is thus similar to the heat transfer coefficient
- Heat conduction in cylinders & spheres
 - INSERT DRAWING FROM THE SLIDES
 - For a long pipe, the main temperature gradient is in the radial direction i.e. $\frac{dT}{dx} \ll \frac{dT}{dr}$
 - Therefore we can assume 1-D radial conduction
 - INSERT r_1, r_2 diagram
 - Solve heat conduction equation in cylindrical coordinates to get $T(r)$
 - Steady state: $\frac{d}{dr}(r \frac{dT}{dr}) = 0$, at $r = r_1, T = T_1$ and at $r = r_2, T = T_2$
 - Integrate this to get $T(r) = c_1 \ln r + c_2$
 - Using the boundary conditions gives $c_1 = \frac{T_1 - T_2}{\ln(\frac{r_1}{r_2})}$ and $c_2 = T_2 - \frac{T_1 - T_2}{\ln(\frac{r_1}{r_2})} \ln r_2$
- Define thermal resistance of a cylinder
 - $R_{cyl} = \frac{T_1 - T_2}{\dot{Q}_{cond}} = \frac{\ln(\frac{r_2}{r_1})}{2\pi L K}$

7 Conduction in cylinders and spheres, Insulation

- Inner temperature and then two surface layers
- $R_C = \frac{1}{hA} = \frac{1}{2\pi r L h}$
- $R_{total} = R_{c,1} + R_{cond} + R_{c,2} = \frac{1}{2\pi r_1 L h_1} + \frac{\ln(\frac{r_2}{r_1})}{2\pi L K} + \frac{1}{2\pi r_2 L h_2}$
- Insulation
 - $R_{total} = R_{c,1} + R_{cyl,1} + R_{cyl,2} + R_{c,2}$ where $R_{cyl,2}$ is for the insulation
 - For a sphere
 - * Insulation around a spherical metal tank
 - * $R_{total} = R_{c,1} + R_{sph,1} + R_{sph,2} + R_{c,2}$
 - * $= \frac{1}{4\pi r_1^2 h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k_1} + \frac{r_3 - r_2}{4\pi r_2 r_3 k_2} + \frac{1}{4\pi r_3^2 h_2}$
- R-value
 - Thermal resistance. Thickness L , Surface area A and thermal conductivity k

- $R = \frac{L}{k}$ is the R-value
- $\dot{Q} = \frac{\Delta T}{R} \times A$
- Units here are in imperial units i.e. L is in feet, k is in $\frac{Btu}{hftF}$
- Critical Radius of insulation
 - Consider the area for heat loss
 - Insulation: increase thickness, increasing conduction resistance and decreasing convective resistance
 - Can we increase heat transfer?
 - * Plot \dot{Q} against r_2 to find the critical value of Resistance
 - * Equivalently set $\frac{d\dot{Q}}{dr_2} = 0$ to find the critical radius

8 Heat Transfer from Finned Surfaces

- Read Chapter 17.6
- How to find $R_{heatsink}$ for finned surfaces e.g. heat sinks in computers
- Add diagram from notes: We have a cylinder with cross sectional area $A_c(x)$ and heat transfer coeff h .
 - Consider a thin slice of this with thickness Δx some distance x away from the end
 - Energy balance $\dot{Q}_{cond,x}$ in and $\dot{Q}_{cond,x+\Delta x}$ out
 - Energy in = Energy out: $\dot{Q}_{cond,x} = \dot{Q}_{cond,x+\Delta x} + \dot{Q}_{conv}$
 - Let the perimeter of fin be P . Then surface area of element is $P\Delta x$ so that $\dot{Q}_{conv} = hP\Delta x(T - T_\infty)$
 - Simplifying the energy balance by taking the limit as $\Delta x \rightarrow 0$: $\frac{d\dot{Q}_{cond}}{dx} + hP(T - T_\infty) = 0$
 - Using Fourier's law $\dot{Q}_{cond} = -kA_C \frac{dT}{dx}$: $\frac{d}{dx}(kA_c \frac{dT}{dx}) - hP(T - T_\infty) = 0$
 - Assuming A_C, k, P constant: $\frac{d^2T}{dx^2} - \frac{hP}{kA_C}(T - T_\infty) = 0$
 - Define $\Theta = T - T_\infty$ and $a^2 = \frac{hP}{kA_C}$ (constant) so that $\frac{d^2\Theta}{dx^2} - a^2\Theta = 0$ where the solution is $\Theta(x) = c_1e^{ax} + c_2e^{-ax}$
 - Boundary conditions: $T = T_b$ at the left end while on the right end as $L \rightarrow \infty$, $T = T_\infty$
 - Therefore we simplify by having $T = T_\infty$ at $x = L$
 - In terms of Θ : At $x = 0$, $\Theta = T_b - T_\infty = \Theta_b$ and $\Theta(\infty) = 0$
 - This gives $c_1 = 0$ and $c_2 = \Theta_b$ so that the solution is $\Theta(x) = \Theta_b e^{-ax}$

- What is the heat loss from the fin? $\dot{Q}_b = -kA_c \frac{dT}{dx}|_{x=0} \dot{Q}_{fin}$
- $\dot{Q}_{fin,long} = \sqrt{hPkA_c}(T_b - T_\infty)$
- Finite fin length: What is the boundary condition at the open end
 - We can heat transfer is negligible so that adiabatic and $\frac{dT}{dx} = 0$ at the boundary
 - At $x = L$, $\frac{dT}{dx} = 0$ so that $\frac{d\Theta}{dx} = 0$: $c_1 e^{aL} - c_2 e^{-aL} = 0$
 - At $x = 0$, $\Theta = \Theta_b$
 - Solve for c_1 & c_2 and the following solution will be obtained: $\frac{T(x)-T_\infty}{T_b-T_\infty} = \frac{\cosh a(L-x)}{\cosh aL}$
- Can do the same for $\dot{Q}_{fin,insulated} = -kA_c \frac{dT}{dx}|_{x=0}$ which will give $\dot{Q}_{fin,insulated} = \sqrt{hPkA_c}(T_b - T_\infty) \tanh(aL)$ where $a = \sqrt{\frac{hP}{kA_c}}$
- To account for heat transfer from the tip, we can add a length ΔL at the end and the area there will be $A_c = \Delta LP$ (P is the perimeter of the fin) so that the corrected length is $L_c = L + \frac{A_c}{P}$

9 Heat transfer from finned surfaces (contd)

- Fin efficiency
 - ΔT given by the difference between the fin temperature and the surrounding
 - Most efficient fin would have a uniform temperature T_b everywhere
 - This would imply an infinite thermal conductivity
 - In this case, $\dot{Q} = hA_{fin}(T_b - T_\infty) = hPL(T_b - T_\infty)$
 - We define $\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,max}}$
 - For an infinitely long fin, $\eta_{fin,long} = \frac{\sqrt{hPkA_c}(T_b-T_\infty)}{hPL(T_b-T_\infty)} = \frac{1}{L} \sqrt{\frac{kA_c}{hP}} = \frac{1}{aL}$ which is in terms of physical properties
 - $\dot{Q}_{fin} = \eta_{fin} \dot{Q}_{fin,max} = \eta_{fin} hA_{fin}(T_b - T_\infty) = h\eta_{fin} A_{fin}(T_b - T_\infty)$
 - $\eta_{fin} A_{fin}$ can be treated as the corrected area
 - For an insulated tip, perform the same steps with the original definition to get $\eta_{insulatedtip} = \frac{\tanh aL}{aL}$
- Fin effectiveness
 - How much has the fin increased heat transfer by?
 - $\epsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{nofin}}$
 - $\dot{Q}_{nofin} = hA_c(T_b - T_\infty)$
 - $\dot{Q}_{longfin} = \sqrt{hPkA_c}(T_b - T_\infty)$ so that $\epsilon_{longfin} = \sqrt{\frac{kP}{hA_c}}$

- To increase the effectiveness, make k as large as possible and maximize $\frac{P}{A_c}$
- Fins are most effective with low h so they are used for gases, hot liquids
- Generally we use fins if $\epsilon \geq 2$
- When can we assume fins are infinitely long?
 - $\frac{\dot{Q}_{fin,insulated}}{\dot{Q}_{fin,long}} = \tanh(aL)$. \tanh asymptotically approaches 1 as aL approaches ∞
 - In practice, if $aL \geq 5$ we can assume an infinitely long fin. But even $aL = 1$ has $\tanh = 0.76$ so it gives 76 % of heat transfer of an infinitely long fin. Therefore $L = \frac{1}{a}$ is a reasonable length for a fin
- Designing a heat sink
 - $\dot{Q}_{total} = \dot{Q}_{unfanned} + \dot{Q}_{fin}$
 - From the definition of efficiency $\dot{Q}_{fin} = \eta_{fin} \cdot hA_{fin}(T_b - T_\infty)$
 - $\Rightarrow \dot{Q}_{total} = hA_{unfanned}(T_b - T_\infty) + h\eta_{fin}A_{fin}(T_b - T_\infty) = h[A_{unfanned} + \eta_{fin}A_{fin}](T_b - T_\infty)$
 - We can define a thermal resistance $R_{fin} = \frac{T_b - T_\infty}{\dot{Q}_{total}} = \frac{1}{h[A_{unfanned} + \eta_{fin}A_{fin}]}$

10 Transient Heat Conduction

- Consider a solid at temperature T_i and a liquid at $T_\infty < T_i$. The solid is dropped into the liquid. How does T vary over time?
 - We would expect the temperature T to asymptotically approach T_∞
 - Heat Conduction equation: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$
 - We set the first three terms equal to 0 by using the lumped capacitance approximation i.e. no temperature gradient in the body
 - We would expect this to be valid when the object is small and has a high thermal conductivity
 - Using an energy balance: $\dot{E}_{store} = -\dot{Q}_{conv}$
 - $\dot{E}_{store} = mc_p \frac{dT}{dt} = \rho V c_p \frac{dT}{dt}$ and $\dot{Q}_{conv} = hA(T - T_\infty)$
 - Equating the two, we get $\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA}{\rho V c_p} dt$ so that $\ln(T - T_\infty) = \frac{-hA}{\rho V c_p} t + C_1$
 - Using $T = T_i$ at $t = 0$, we get $\ln \left[\frac{T - T_\infty}{T_i - T_\infty} \right] = \frac{-hA}{\rho V c_p} t$
 - We define a "time constant" $\tau = \frac{\rho V c_p}{hA}$ so that $\frac{T - T_\infty}{T_i - T_\infty} = \exp \left[\frac{-t}{\tau} \right]$
 - The LHS starts at 1 and decays to 0 as $t \rightarrow \infty$. Moreover at $t = \tau$, the value is $\frac{1}{e} \approx 0.368$
- The response time of a thermometer is usually 3τ . However it is important to note that τ is a function of h so it varies in different environments

- Moreover τ depends on $\frac{V}{A} = \frac{r}{3}$ for a sphere so to get a fast response time you would make it very thin
- When is a lumped capacitance valid?
 - At steady state, the conduction in a solid must be equal to the convection in a fluid i.e. $kA\frac{(T_1-T_2)}{L} = hA(T_2 - T_\infty)$ i.e. $\frac{T_1-T_2}{T_2-T_\infty} = \frac{hL}{k}$
 - $\frac{hL}{k}$ is a dimensionless number and is known as a Biot number.
 - Note: k is the thermal conductivity of the solid, L is the length scale in the direction of conduction
 - Suppose the Biot number is large i.e. $\gg 1$ so that $T_1 - T_2 \gg T_2 - T_\infty$
 - If Biot number is very small, then $T_1 - T_2 \ll T_2 - T_\infty$ so we can neglect T change inside the body (we assume uniform temp in the body) and use the lumped capacitance model
 - By \ll we typically mean a Biot value < 0.1
 - In an irregular body, the length scale used is $L = \frac{V}{A}$

11 Transient Heat Conduction in 2 and 3 Dimensions

- A ball of volume V , mass m and SA A and heat transfer coefficient h is dropped into a fluid
- Assume that the temperature T is uniform in the body
- We had previously assumed that if $Bi < 0.1$ we have a lumped capacitance
- Example: Steel shaft, $k = 51.2$, $\rho = 7832$, $c = 541$ and $T_i = 300$ is placed into a furnace with $T_\infty = 1200$.
 - How long before the shaft temperature reaches 800?
 - We first calculate the Biot number as $Bi = \frac{hL}{k}$ where $L = \frac{V}{A} = \frac{\pi r^2 L}{2\pi r L} = \frac{r}{2}$
 - Then $Bi = \frac{h\frac{r}{2}}{k} = \frac{100 \times \frac{0.05}{2}}{51.2} = 0.05$ so we can apply the lumped value
 - $\frac{T-T_\infty}{T_i-T_\infty} = \exp\left[-\frac{hA}{\rho V c} t\right]$ where $\frac{A}{V} = \frac{2}{r}$
 - This gives $\ln\left[\frac{800-1200}{300-1200}\right] = \dots$ and solving gives $t = 859s$
- Transient heat conduction in 3 dimensions e.g. plane walls, cylinders, spheres
 - What happens if $Bi > 0.1$?
 - In this case we cannot neglect the temperature gradients inside the body
 - Have to solve the complete heat conduction equation
 - Consider a solid wall with temperature T_i on one side which then instantly becomes lowered to a temperature T_∞ as it is placed into a fluid.

- As time increases, the temperature inside the wall e.g. at the center decreases
- So in this case, T is a function of x and t
- For a plane wall $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$ where $\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity
- Second order wrt x so two boundary conditions are needed there. First order wrt t so one initial condition is needed there
- This can be solved analytically but we will not do that
- We instead consider the lumped capacitance solution $\ln \left[\frac{T - T_\infty}{T_i - T_\infty} \right] = \frac{-hA}{\rho V c_p} t$
- We take the characteristic length $L = \frac{V}{A}$.
- $\frac{hA}{\rho V c_p} t = \frac{h}{\rho L c_p} t = \left(\frac{h}{\rho L c_p} t \right) \left(\frac{L}{L} \cdot \frac{k}{k} \right) = \left(\frac{hL}{k} \right) \left(\frac{k}{\rho c_p} \right) \left(\frac{t}{L^2} \right) = \left(\frac{hL}{k} \right) \left(\frac{\alpha t}{L^2} \right)$ where $Bi = \frac{hL}{k}$ which is unitless
- We define the Fourier number $Fo = \frac{\alpha t}{L^2}$ which is also dimensionless
- The dimensionless temperature $\Theta = \frac{T - T_\infty}{T_i - T_\infty}$ so that the lumped capacitance solution can be written as $\Theta = \exp(-Bi \cdot Fo)$
- A physical interpretation of the Fourier number can be found by considering a cube with side length L . Then $\dot{Q}_{cond} = kA \frac{\partial T}{\partial x} = kL^2 \frac{\Delta T}{L} = kL \Delta T$
- $\dot{Q}_{store} = mc_p \frac{\partial T}{\partial t} = \rho L^3 c_p \frac{\Delta T}{t}$ so that $\frac{\dot{Q}_{cond}}{\dot{Q}_{store}} = \frac{k}{\rho c_p} \cdot \frac{t}{L^2} = \frac{\alpha t}{L^2} = Fo$
- Even when we cannot assume lumped capacitance and get an exact solution of the heat conduction equation, the solution is of the form $\Theta = \Theta(Bi, Fo)$
- We can define $\Theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty}$ so that the solution is of the form $\Theta_0 = A_1 e^{-\lambda_1^2 Fo}$ where A_1, λ_1 are function of Bi
- Example: Carbon steel plate with $T_i = 440$ is placed in a furnace at $T_\infty = 600$. We need to heat to a minimum temperature of 520. What is the time t required.
 - $h = 200$, $k = 40$ and $\alpha = 8 \times 10^{-6}$
 - $Bi = \frac{hL}{k} = \frac{200 \times 0.04}{40} = 0.2$
 - Since $Bi > 0.1$, we cannot use the lumped capacitance and instead use the analytical solution which we obtain from tables
 - From Table 18.2, $Bi = 0.2 \implies \lambda_1 = 0.4328, A_1 = 1.0311$
 - $\Theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 Fo}$
 - $\Theta_0 = \frac{520 - 600}{440 - 600} = 0.5$ so that $0.5 = 1.0311 \exp(-(0.4328)^2 Fo)$ and so $Fo = 3.864$
 - $Fo = \frac{\alpha t}{L^2} \implies t = \frac{Fo L^2}{\alpha} = \frac{3.864 \times (0.04)^2}{8 \times 10^{-6}} = 773s$

12 Transient Heat Conduction in Semi-Infinite Solids

- The general problem consists of a body at temperature T_i with the surface temperature suddenly being changed to T_s .

- Temperature variation would go from T_s down to T_i . δ corresponds to a skin depth and after that we are in the core
- How does the skin depth δ vary with time?
- Heat conduction equation with one dimension: $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$
- An estimate of $\frac{\partial^2 T}{\partial x^2}$ can be found as $\frac{(\frac{\partial T}{\partial x})_{x \sim \delta} - (\frac{\partial T}{\partial x})_{x \sim 0}}{\delta}$
- $(\frac{\partial T}{\partial x})_{x=0} \sim \frac{T_i - T_s}{\delta}$ so that $\frac{\partial^2 T}{\partial x^2} \sim 0 - \frac{T_i - T_s}{\delta^2}$, $\frac{\partial T}{\partial t} \sim \frac{T_s - T_i}{t}$
- From the heat conduction equation $-\frac{T_i - T_s}{\delta^2} \sim \frac{1}{\alpha} \frac{T_s - T_i}{t}$
- Time for effect to be felt throughout the body is $t_c \sim \frac{r_0^2}{\alpha}$
- $\delta \sim \sqrt{\alpha t}$
- For a short time $t \ll t_c$, we can treat the body as being semi infinite
- The exact solution is given by $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$
- δ grows as a function of time
- Boundary conditions: At $x = 0$, $T(0, t) = T_s$ and as $x \rightarrow \infty$, $T(\infty, t) = T_i$
- Initial conditions: $T(x, 0) = T_i$
- Define a similarity variable $\eta = \frac{x}{\delta}$ such that $0 < \eta < 1$
- $\eta = \frac{x}{2\sqrt{\alpha t}}$ so that $\frac{\partial T}{\partial t} = \frac{dT}{d\eta} \cdot \frac{\partial \eta}{\partial t} = \frac{d}{dt} \left[\frac{-x}{4t\sqrt{\alpha t}} \right]$
- $\frac{\partial T}{\partial x} = \frac{dT}{d\eta} \cdot \frac{\partial \eta}{\partial x} = \frac{dT}{d\eta} \left[\frac{1}{2\sqrt{\alpha t}} \right]$
- $\frac{\partial^2 T}{\partial x^2} = \frac{d}{d\eta} \left(\frac{\partial T}{\partial \eta} \right) \cdot \frac{\partial \eta}{\partial x} = \frac{d^2 T}{d\eta^2} = \frac{1}{4\alpha t}$
- Transforming the heat conduction equation, we get $\frac{d^2 T}{d\eta^2} = \frac{1}{\alpha} \frac{dT}{d\eta} \left(\frac{-x}{4t\sqrt{\alpha t}} \right) = -2\eta \frac{dT}{d\eta}$
- The PDE is now an ODE
- At $x = 0$, $\eta = 0$, $T(0) = T_s$ and as $x \rightarrow \infty$, $\eta \rightarrow \infty$, $T(\infty) = T_i$
- Let $w = \frac{dT}{d\eta}$ so that $\frac{dw}{d\eta} = -2\eta w$ which gives $\ln w = -\eta^2 + c_0$
- $\frac{dT}{d\eta} = w = c_0 e^{-\eta^2}$ and integrating, $T = c_0 \int_0^\eta e^{-u^2} du + c_1$
- Boundary conditions give $T_i = c_0 \int_0^\infty e^{-u^2} du + T_s$ which gives the solution $\frac{T - T_s}{T_i - T_s} = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-u^2} du = \text{erf}(\eta)$ where the RHS is the error function
- Can also be written as $1 - \frac{T - T_s}{T_i - T_s} = 1 - \text{erf}(\eta)$ or $\frac{T - T_i}{T_s - T_i} = \text{erfc}(\eta)$

- Heat Flux at Surface

- $\dot{q}_s = -k \frac{\partial T}{\partial x} \Big|_{x=0} = 0k \frac{\partial T}{\partial \eta} \frac{\partial \eta}{\partial x} \Big|_{\eta=0}$
- $\frac{d\eta}{dx} = \frac{1}{2\sqrt{\alpha t}}$
- $\dot{q} = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}}$

- Contact of two semi-infinite bodies

- One body with temperature $T_{A,i}$ and conductivity k_A and the other with $T_{B,i}$ and k_B
- We require that $T_{S,A} = T_{S,B}$
- The heat flux must be the same i.e. $\dot{q}_{S,A} = \dot{q}_{S,B}$
- $$-\frac{k_A(T_s - T_{A,i})}{\sqrt{\pi(\alpha_A t)}} = \frac{k_B(T_s - T_{B,i})}{\sqrt{\pi(\alpha_B t)}}$$
- $$\implies \frac{T_{A,i} - T_s}{T_s - T_{B,i}} = \frac{\sqrt{(k\rho c)_B}}{\sqrt{(k\rho c)_A}}$$
- Define effusivity as $\gamma = \sqrt{k\rho c}$

13 Forced convection, Velocity and Thermal boundary layers, Reynolds, Prandtl and Nusselt numbers

• Forced Convection

- Convection is the heat transfer from a surface to a moving fluid
- Forced part means that motion is imposed by external means
- A surface at a temperature T_s and a fluid with velocity V_∞ and temperature T_∞
- $\dot{Q}_{conv} = hA(T_s - T_\infty)$
- How do we determine h ?
 - * Quite complicated
 - * Depends on physical properties of fluid: viscosity μ , density ρ , thermal conductivity k and specific heat c_p
 - * Depends on the fluid velocity V_∞
 - * Depends on the shape and size of body: Characteristic length i.e. for a plate it is the length L whereas for a cylinder or sphere it is the diameter D . Differences for objects of irregular shapes
 - * Type of flow - laminar, turbulent

• Velocity Boundary layer

- Incoming flow at velocity V_∞ and temperature T_∞ and a solid plate
- There is a no slip condition at the interface between the fluid and the solid. i.e. the velocity at the interface must be 0 but if you move away, it will go back to V_∞
- We can define a boundary layer thickness by choosing where the velocity is $0.99V_\infty$
- At $y = 0$ (plate surface), $V = 0$. The heat transfer there is thus by conduction only so that Fourier's law applies
- Here $\dot{q}_{cond} = -k_{fluid} \frac{\partial T}{\partial y}|_{y=0}$. We have also defined $\dot{q}_{conv} = h(T_s - T_\infty)$

- At $y = 0$, $\dot{q}_{conv} = \dot{q}_{cond}$ so that $h = \frac{-k_{fluid} \frac{\partial T}{\partial y}|_{y=0}}{T_s - T_\infty}$
- Therefore h depends on k_{fluid} and the temperature gradient $\frac{\partial T}{\partial y}|_{y=0}$. This leads to thermal boundary layers
- Thermal Boundary layer
 - Temperature of the surface is $T_s > T_\infty$
 - Fluid coming at temperature T_∞
 - When the fluid comes in contact with the plate, there cannot be a discontinuity so it must be equal to T_s at the contact point
 - INSERT PIC FROM NOTES. We define the thermal boundary layer in a similar way to be the point where $T - T_s = 0.99(T_\infty - T_s)$
 - $\frac{\partial T}{\partial y}|_{y=0}$ is the temperature gradient.
 - h changes with position as the local $\frac{\partial T}{\partial y}|_{y=0}$ changes
 - We define a local heat transfer coefficient $h(x)$
 - We can average: $\bar{h} = \frac{1}{L} \int_0^L h(x) dx$
 - Instead of worrying about local variations, we use the average value of h
- Velocity Boundary layer flow
 - The difference between the boundary and the surfaces increases linearly as a straight path in laminar flow
 - There is a transition region then where the path will start becoming unstable which eventually becomes a turbulent region with turbulent flow
 - Fluid exerts a drag on the plate. Measured in terms of the shear stress (force per unit area)
 - $\tau = \mu \frac{\partial V}{\partial y}|_{y=0}$ where μ is the fluid viscosity ($\frac{kg}{ms}$)
 - Velocity gradient can be solved but it is very complicated and practically we define a friction coefficient
 - We imagine a fluid coming to rest at a stagnation point. Using Bernoulli's equation, $\frac{P}{\rho} = \frac{V_\infty^2}{2} + \frac{P_\infty}{\rho} \implies P - P_\infty = \frac{\rho V_\infty^2}{2}$. This is the pressure rise and the force felt by plate
 - We define the friction coefficient c_f so that $\tau = c_f \frac{\rho v_\infty^2}{2}$
 - Generally we expect $c_f \sim 1$ but this depends on the shape
- Laminar & Turbulent Flow
 - Heat transfer is greater in turbulent flow i.e. with an increased velocity
 - However when the heat transfer goes up, shear goes up and so bigger fans (more energy) are needed

- The transition to turbulence depends on the ratio of fluid inertia to viscosity
- A high inertia drives random motion and therefore turbulence
- A high viscosity damps turbulence
- Consider a mass with diameter D and a fluid coming in around it at velocity V_∞
 - * There is an inertial force F_i and a viscous force F_v
 - * Take the characteristic distance to be D
 - * The inertial force $F_i = ma$ where the mass $m = \rho D^3$ and the acceleration $a = \frac{V_\infty^2}{D}$
 - * The viscous force $F_v = \tau A = \mu \frac{\partial V}{\partial y} \cdot A$
 - * $F_v \sim \mu \frac{V_\infty}{D} \cdot D^2 \implies \frac{F_i}{F_v} = \frac{\rho V_\infty D}{\mu}$
 - * This number is known as the Reynolds number: $Re = \frac{\rho V_\infty D}{\mu}$

14 Forced Convection Currents

- Fluid with velocity V_∞ and density ρ_{mu} . What forces are exerted onto the body (with characteristic length being the diameter D)
 - Inertial force $F_i = ma$
 - * $m \sim \rho D^3$
 - * The fluid starts with velocity V_∞ and is brought to rest over a distance D
 - * $t = \frac{D}{V_\infty}$
 - * $\Delta V = V_\infty$ so $a \sim \frac{\Delta v}{t} = \frac{V_\infty}{\frac{D}{V_\infty}} = \frac{V_\infty^2}{D}$
 - * Thus $F_i \sim \rho D^3 (\frac{V_\infty^2}{D}) = \rho D^2 V_\infty^2$
 - Viscous Force
 - * $F_v = \tau A = \mu \frac{dV}{dy} \cdot A$
 - * $F_v \sim \mu \frac{V_\infty}{D} \cdot D^2$
 - We are interested in the ratio
 - * $\frac{F_i}{F_v} \sim \frac{\rho V_\infty D}{\mu}$
 - * This is a very important number called the Reynolds Number $Re = \frac{\rho V_\infty D}{\mu}$
 - * The kinematic viscosity is defined as $\nu = \frac{\mu}{\rho}$ so that the Reynolds number can also be written as $Re = \frac{V_\infty D}{\nu}$
 - * For small Re , viscous forces are dominant. Fluctuations in the flow are damped. This leads to laminar flow.
 - * For large Re , inertial forces are dominant. Fluctuations in the flow become amplified and this leads to turbulent flow
 - * For every geometry, there is a critical value of Re at which a transition to turbulence occurs e.g. $Re_{critical, flatplate} = 5 \times 10^5$

- Two boundary layers are developing - velocity, thermal
 - The velocity goes from V_∞ down to 0 and the temperature goes down from T_∞ to T_s
 - Let δ_v be the velocity boundary layer and δ_t be the thermal boundary layer
 - δ_t may be smaller or larger than δ_v . How do we tell?
 - * This depends on the physical properties of the fluid
 - Fluids with high viscosity ν (oils) have thick velocity BL i.e. δ_v is a large fraction of V_∞
 - Fluids with high thermal diffusivity ($\alpha = \frac{k}{\rho c_p}$) have thick thermal BL
 - The ratio $\frac{\delta_v}{\delta_t}$ is given by the ratio $\frac{\nu}{\alpha}$
 - The Prandtl Number is defined to be this ratio: $Pr = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$
 - Pr is a fluid property
 - For $Pr \ll 1$ (e.g. a liquid metal) $\delta_v \ll \delta_t$
 - For $Pr \gg 1$ (e.g. oils) $\delta_v \gg \delta_t$
 - For $Pr \sim 1$ (e.g. gases) $\delta_t \sim \delta_v$
- We have two dimensionless parameters $Re = \frac{\rho V_\infty D}{\mu}$ and $Pr = \frac{\nu}{\alpha}$
- Need to non dimensionalize h
 - $\dot{Q}_{conv} = hA(T_s - T_\infty) \sim hD^2(T_s - T_\infty)$
 - Suppose the fluid was not moving
 - Then heat transfer is by conduction only
 - $\dot{Q}_{cond} = k_{fluid}A\frac{dT}{dr} \sim k_{fluid}D^2\frac{T_s - T_\infty}{D}$
 - How much is heat transfer enhanced due to convection
 - This is given by the ratio $\frac{\dot{Q}_{conv}}{\dot{Q}_{cond}} = \frac{hD}{k_{fluid}}$ which is a dimensionless number
 - This ratio is known as the Nusselt number $Nu = \frac{hD}{k_{fluid}}$
 - Do not confuse with $Bi = \frac{hD}{k_{solid}}$
- We started with $h = f(D, V_\infty, \rho, \mu, c_p, k)$ and dependent on the geometry
- This can now be written as $\frac{hD}{k} = f\left(\frac{\rho DV_\infty}{\mu}, \frac{\mu c_p}{k}\right)$ and dependent on the geometry
- $\implies Nu = f(Re, Pr)$ and geometry
- Can do experiments
 - Consider a plate with a uniform heat flux due to an electrical current through it
 - Can put it into a wind tunnel with V_∞ and T_∞

- We know that $\dot{Q} = hA(T_s - T_\infty) = P = EI$ (electrical power)
- We know the other variables and can thus solve for h and plot it as a function of V_∞
- Repeat for different plate sizes, different T_s and different T_∞
- Can plot Nu vs Re and all the data should fall on the same curve
- Repeat for different fluids to get different curves for different fluids
- Can then plot $\ln(\frac{Nu}{Pr^n})$ against $\ln Re$ and the data for different fluids will all lie on the same line
- $\ln(\frac{Nu}{Pr^n}) = m \ln Re + C$ or equivalently $Nu = cRe^m Pr^n$ where n, m, C are determined experimentally for each geometry

15 Forced convection correlations

- Read Ch 19.3, 19.4. 19.5 - 19.8 will not be covered
- Dimensionless Analysis
 - Consider a tube with velocity from the bottom
 - * Z (height) is in terms of P, V
 - * What are relevant parameters?
 - P, V, z
 - Fluid properties: ρ
 - Gravity g
 - * Develop an equation relating P, V, z, ρ, g
 - * If we develop an equation, the dimension of both sides must be equal
 - * $P[\frac{N}{m^2}] = \frac{kg \frac{m}{s^2}}{m^2} = [\frac{kg}{ms^2}]$
 - * $\frac{P}{\rho}$ is in $[\frac{m^2}{s^2}] \implies \frac{P}{\rho V^2} [\frac{m^2}{s^2} \times \frac{s^2}{m^2}]$
 - * $\frac{g}{V^2}$ is also dimensionless
 - * Therefore $\frac{P}{\rho V^2} = f(\frac{gz}{v})$
 - * We plot $\frac{P}{\rho V^2}$ against $\frac{gz}{v^2}$ and we discover that all data falls on one line. If it doesn't fall on one line, the analysis was done incorrectly
 - * Thus $\frac{P}{\rho V^2} = -\frac{gz}{v^2} + C_1$ which when simplified gives $\frac{P}{\rho} + gz + \frac{V^2}{2}$ which is a constant. We can derive Bernoulli's equation experimentally in this way
 - Forced Convection Correlation
 - Flow over a flat plate
 - V_∞ flowing from the left, laminar region, then transition and then turbulence
 - $\tau = \mu \frac{\partial V}{\partial y} \sim \mu \frac{V_\infty}{\delta_v}$
 - As δ_v goes up, τ goes down

- Local frictional coefficient $c_{f,x}$ goes down initially while in laminar, then goes up while in transition and finally goes down again in turbulence region
- For laminar flow $c_{f,x} = \frac{0.664}{Re_x^{\frac{1}{2}}}$
- Local Reynolds number $= Re_x = \frac{V_\infty x}{\nu}$
- Average friction coefficient over length L of plate: $c_f = \frac{1}{L} \int_0^L c_{f,x} dx = \frac{1}{L} \int_0^L \frac{0.664}{Re_x^{\frac{1}{2}}} dx = \frac{1}{L} 0.664 \left(\frac{\nu}{V_\infty}\right)^{\frac{1}{2}} \int_0^L \frac{dx}{x^{\frac{1}{2}}} = \frac{2}{L} \times 0.664 \left(\frac{\nu}{V_\infty}\right)^{\frac{1}{2}} \cdot L^{\frac{1}{2}}$
- This gives $c_f = \frac{1.328}{Re_L^{\frac{1}{2}}}$
- The heat transfer coefficient $h = -\frac{k}{T_s - T_\infty} \frac{\partial T}{\partial y}|_{y=0}$ so that $\frac{\partial T}{\partial y} \sim \frac{T_s - T_\infty}{\delta_t}$
- As δ_t increases, $\frac{\partial T}{\partial y}$ goes down implying that h decreases
- So h decreases while in laminar flow, increases in the transition and then decreases again in turbulent region
- We define a Local Nusselt Number: $Nu_x = \frac{h_x x}{k}$
- For laminar flow, $Nu_x = 0.332 Re_x^{\frac{1}{2}} Pr^{\frac{1}{3}}$ where $Pr \geq 0.6$
- The average Nusselt number is $Nu = \frac{hL}{k} = \frac{1}{L} \int_0^L Nu_x dx$
- This gives $Nu = 0.664 Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}}$
- For turbulent flow, $Re_x > 5 \times 10^5$ and $c_{f,x} = \frac{0.0592}{Re_x^{\frac{1}{4}}}$

• Example

- Air with $T_\infty = 300C$, $V_\infty = 10$ on a tube with $T_s = 50$ and $L = 0.5$. Find $\dot{Q}_{cooling}$
- The film temperature is used where $T_f = \frac{T_\infty + T_s}{2}$
- In this case, $T_f = 175$ and using $\nu = 3.18 \times 10^{-5}$, $Pr = 0.7$ and $k = 0.0363$ (using tables)
- Therefore $Re_L = \frac{V_\infty L}{\nu} = 1.57 \times 10^5$
- Flow is laminar since $Re_L < 5 \times 10^5$
- $Nu = 0.664 Re_L^{\frac{1}{2}} Pr^{\frac{1}{3}} = 233.6$
- $Nu = \frac{hL}{k} \implies h = \frac{Nuk}{L} = 16.9 \frac{W}{m^2 C}$
- Cooling per unit width of the plate $\dot{Q} = hA(T_\infty - T_s) = 16.9 \times (0.5 \times 1)(300 - 50) = 2112.5 \frac{W}{m}$

• Flows over cylinders and spheres

- Flow separation causes wake
- Not to be confused with turbulence
- Turbulence occurs when $Re = \frac{V_\infty D}{\nu} > 2 \times 10^5$

- For flow across cylinders, $Nu = cRe^m Pr^n$
- The values of c, m, n depend on the range of the Reynolds number - given in Table 19.2
- Churchill & Bernstein correlation is valid for $RePr > 0.2$
- Fluid properties evaluated at $T_f = \frac{T_s + T_\infty}{2}$
- Flow over a sphere has $Nu = 2 + [0.4Re^{\frac{1}{2}} + 0.06Re^{\frac{2}{3}}] \cdot Pr^{0.4} \left(\frac{\mu_\infty}{\mu_s} \right)^{0.25}$
- All properties evaluated at T_∞ and μ_s is evaluated at T_s
- Valid for $3.5 \leq Re \leq 80000$ and $0.7 \leq Pr \leq 380$

16 Thermal Radiation, Black body radiation, Radiative properties

- Read Ch 21.1 - 21.4
- Radiation
 - Consider a solid at temperature T_s with an enclosure outside at T_{surr} and a vacuum in between
 - Initially $T_s > T_{surr}$
 - No conduction, convection since vacuum so heat transfer is by radiation
 - Solid will lose heat to surrounding surface and cool until $T_s = T_{surr}$
 - Thermal radiation: Energy is emitted by matter as a result of its finite temperature
 - Energy is in the form of electromagnetic waves
 - Radiation has wave properties: frequency ν and wavelength λ related by $\lambda = \frac{c}{\nu}$
 - Thermal radiation has a wavelength from 0.1 to 100 μm . This includes ultraviolet (0.1 – 0.4 μm), visible light (0.4 – 0.7 μm) and infrared (0.7 – 100 μm)
 - For opaque objects, radiation occurs from the surface
- Black body radiation
 - A black body is a perfect emitter of radiation
 - At a given temperature, no surface can emit more energy than a Blackbody
 - A blackbody absorbs all radiation incident on
 - It emits equally in all directions. It is a "diffuse" surface
 - The Stefan-Boltzmann law: The radiation energy emitted by a blackbody per unit time and per unit surface area is: $E_b = \sigma T^4$ (in $\frac{W}{m^2}$) where $\sigma = 5.67 \times 10^{-8}$, E_b is the blackbody emissive power and T is the temperature in K

- A surface painted black is close to a blackbody for visible radiation
- A white surface absorbs infrared light - can be considered a blackbody in IR
- Depending on the wavelength, different objects can be considered blackbodies
- To get a perfect blackbody, you take a box with a small opening and the opening is the blackbody
- This is because the aperture absorbs all light (anything that goes inside the opening reflects inside there and is thus absorbed)
- Radiation from a real surface encompasses a range of wavelengths. This is described by a "spectral distribution"
- Can plot Emission energy $E_{b,\lambda}$ against λ to obtain a curve described by Planck's Law: $E_{b,\lambda} = \frac{c_1}{\lambda^5 [\exp(\frac{c_2}{\lambda T}) - 1]}$
- There may also be a directional distribution
- Surface emits more in a given direction
- Radiation properties
 - For a real surface, the emissive power is less than that of a blackbody
 - We define a surface property - emissivity
 - $\varepsilon(T) = \frac{E(t)}{E_b(t)}$ where $E(t)$ is integrated over all λ and θ and $E_b(t)$ is integrated over all λ
 - We will assume that ε is independent of λ . This is known as a gray surface
 - ε is independent of θ : A diffuse surface
 - The emissive power of a real surface is $E(t) = \varepsilon \sigma T^4$
- Surface Absorption, Reflection and Transmission
 - G is the incident radiation in $\frac{W}{m^2}$
 - Part of it will be reflected (G_{ref}), part will be absorbed G_{abs} and the rest will be transmitted ($G_{transmitted}$)
 - G -radiation is the radiant energy incident on a surface per unit surface area per unit time
 - Define 3 properties: Absorptivity $\alpha = \frac{G_{abs}}{G}$, the reflectivity $\rho = \frac{G_{ref}}{G}$ and the transmittivity $\tau = \frac{G_{tr}}{G}$
 - In general $\alpha + \rho + \tau = 1$
 - For opaque objects, $\tau = 0$, for a blackbody $\alpha = 1$
 - The "Gray Body assumption" is that α, ρ, τ are independent of λ
 - The "Diffuse surface assumption" is that α, ρ, τ do not depend on direction