LECTURE NOTE

PHY 101/1311: General Physics I (Mechanics)

DEPARTMENT OF PHYSICS

FACULTY OF NATURAL AND APPLIED SCIENCES

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Mechanics (2 Units C: LH 30)

Learning Outcome

At the end of the course, students should be able to:

1. identify and deduce the physical quantities and their units;
2. differentiate between vectors and scalars;
3. describe and evaluate motion of systems on the basis of the fundamental laws of mechanics;
4. apply Newton's laws to describe and solve simple problems of motion
5. evaluate work, energy, velocity, momentum, acceleration, and torque of moving or rotating objects;
6. explain and apply the principles of conservation of energy, linear and angular momentum;
7. describe the laws governing motion under gravity; and
8. explain motion under gravity and quantitatively determine behaviour of objects moving under gravity.

Course Contents

Space and time. Units and dimension. Vectors and scalars. Differentiation of vectors (displacement, velocity and acceleration). Kinematics. Newton's laws of motion (Inertial frames, impulse, force and action at a distance, momentum conservation). Relative motion. Application of Newtonian mechanics. Equations of motion. Conservation principles in physics (conservative forces, conservation of linear momentum, kinetic energy and work, potential energy). System of particles. Centre of mass. Rotational motion (torque, vector product, moment, rotation of coordinate axes and angular momentum). Coordinate systems. Polar coordinates. Conservation of angular momentum. Circular motion. Moments of inertia (gyroscopes, and precession). Gravitation (Newton's Law of Gravitation, Kepler's laws of planetary motion, gravitational potential energy, escape velocity, satellites motion and orbits)

THE CONCEPT OF UNIT

Assuming, a teacher measured the length of a wire and found it to be *20cm*. Instead of writing 20cm, he mistakenly wrote **20** only on the board. He then asked three of his students to tell him what was the length of the wire measured. As such, based on what was written on the board, student **A** said it was 20mm, student **B** said it was 20m, student **C** said it was 20 inches. What brought about the different answers was the failure of the teacher to attach a *unit* to the quantity measured. This has really illustrated an important point. Therefore, quoting the result of a calculation or measurement without attaching a **UNIT** to it is **useless**. Frankly, this is a common reason for many students losing marks in examinations. So, the concept of UNIT is of paramount importance not only in physics.

To make accurate and reliable measurements, we need units of measurement that do not change and that can be duplicated by observers in various locations around the globe. The system of units used by scientists and engineers around the world is commonly called "The metric system", but since 1960 it has been known officially as the **International system of units**, or **SI** (the abbreviation for its French name is *Systeme International).* The main advantage of using a set of agreed units is that scientists from all over the world can exchange ideas and designs for experiments without having to translate specifications into different units. It is rather like having a *common language*.

There are two types of units namely:

- Base or Fundamental Units.
- Derived Units.

Fundamental Units

Scientists have worked out that they can measure any quantity in nature in terms of a small number of base units. The challenge is to reduce the number of base/fundamental units to a minimum and still be able to measure anything that can come up. Thus, it has been possible to keep to combinations of the *seven base units* as listed below:

Quantity	Name of unit	Symbol
	SI base units	
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

NOTE: Any number that is used to describe a physical phenomenon quantitatively is called a **physical quantity.**

Derived Units

From the word '**derive**', the derived units are obtained from combination of two or more fundamental units. They enable us to measure more than the basic quantities of length, time, mass, etc. For instance, there is no unit for speed among the base units. However, a suitable unit can be derived from the equation for speed as:

$$average\ speed = \frac{distance\ (m)}{time\ (s)}$$

This suggests that the unit of speed is metres divide by seconds; unfortunately, such a division is impossible. Division can only happen when the quantities are of the same type. We now say instead that the unit of speed is metres per second and denoted as ms^{-1}.

Acceleration is another important quantity. It is the rate at which speed is changing:

$$average\ accelaration = \frac{final\ speed\ (ms^{-1}) - initial\ speed\ (ms^{-1})}{time\ (s)}$$

The top line is the difference between two quantities in ms^{-1}. The bottom line is in seconds. Thus, the units of acceleration are 'metre per second per second', or ms^{-2}.

The following table shows some other examples of the derived units:

Quantity	**Derived Unit**	**Name**
force	$kg\ m\ s^{-2}$	Newton (N)
pressure	$kg\ m^{-1}\ s^{-2}$	Pascal (Pa)
energy	$kg\ m^{2}\ s^{-2}$	Joule (J)
charge	A s	Coulomb(C)
volume	m^3	-

Unit Prefixes

Once we have defined the fundamental units, it is easy to introduce larger and smaller units for the same physical quantities. In the metric system, these other units are related to the fundamental units by multiples of 10 or $\frac{1}{10}$. Thus, one kilometer (1 km) is 1000 meters, and one centimetre (1 cm)is $\frac{1}{100}$ meter. Weusually express multiples ofl0 or $\frac{1}{10}$in exponential notation; 1000 = $l0^3$, $\frac{1}{1000}$ = $10^3$and so on. With this notation, 1 km = $l0^3$mand 1cm = 10^{-2} m. The names of additional units are derived by adding a prefix to the name of the fundamental unit. For example, the prefix "kilo-," abbreviated k, always means a unit larger by a factor of1000; thus

1kilometer = 1km = 10^3meters = 10^3m.
1 kilogram = 1kg = 10^3gram = 10^3g.
1 kilowatt =1 kW = 10^3 watts = 10^3 W.
The table below shows the standard prefixes:

Prefix	Symbol	Multiplies unit by Multiples
kilo-	k	10^3
mega-	M	10^6
giga-	G	10^9
tera-	T	10^{12}
Peta-	P	10^{15}
exa-	E	10^{18}
		Multiplies unit by Submultiples
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}
pico-	p	10^{-12}
femto-	f	10^{-15}
atto-	a	10^{-18}
Other common prefixes		
deci-	d	10^{-1}
centi-	c	10^{-2}

THE CONCEPT OF DIMENSION

Let consider a simple arithmetic equation such as:

$$3 + 5 = 8$$

The equation balances, because the value of the numbers on the left-hand side is equal to the value of numbers on the right-hand side. In physics, equations usually equate quantities that have magnitude (values), dimensions and units. All three must balance for the equation to be meaningful.

However, the word *dimension* has a special meaning in physics. It denotes the physical nature of a quantity. Whether a distance is measured in units of feet or metres or inches, it is still a distance. We say its dimension is *length.*

In other words, *dimension* is the type of quantity we are dealing with independent of its units or value. For instance, 100cm, 1m, 2 miles and 3 light-years all have the dimension of length, but are expressed in different units. Simple numbers, as in the above arithmetic equation, are dimensionless, whereas dimensions are valueless.

The examples below show situations in which values balance but dimensions and/or units do not.

3 kg + 5 kg = 8 m(1)

3 m + 5 m = 8 cm (2)

- ✓ Equation (1) is meaningless because the left-hand side has the dimension of mass while the right-hand side has the dimensions of length.
- ✓ Equation (2) is also incorrect. Both sides of the equation have the same dimension of length but the units are different.

Equations whose dimensions and units balance are said to be **homogeneous.** Although being homogeneous does not guarantee that an equation is correct. An example of homogeneous but incorrect equation is F = 2ma with all quantities measured SI units.

However, all the quantities used to describe mass and motion are combinations of the three fundamental dimensions of mass (M), length (L) and time (T). To find dimensions of any particular quantity, you must find an equation that relates it to quantities you do know and then balance the dimensions.

It helps to introduce square brackets to mean 'the dimensions of ….' For example: [v] means 'the dimensions of velocity'. Then $[v] = LT^{-1}$ means 'the dimensions of velocity are length divided by time'.

The units of a quantity follow from its dimensions. In principle, there is a free choice of all possible units for each dimension. That is, velocity could be measured in metres per second, or centimetres per century, or feet per hour. In practice, SI is usually adopted, so that all lengths are in metres, masses in kilograms and times in seconds.

Example1. Find the dimensions and units for energy.

Solution

Any equation for energy would do, for example

$$KE = \frac{1}{2}mv^2$$

Hence,

$[energy] = \left[\frac{1}{2}\right][m][V]^2 = ML^2T^{-2}$

Note that the number $\frac{1}{2}$ is simply replaced by unity (1) because it is dimensionless. The suitable SI units for energy are kg m^2 s^{-2}, which is renamed the Joule (J).

Using Dimensions

Although dimensions are more fundamental than units, either can be used in a similar way to check equations and to predict the form of new relationship. This can be illustrated in the followings two examples:

Example 1: A student derives an expression for frictional drag on a car body and comes up with $F = \frac{1}{2}C\rho Av$

Where ρ is the density of the air through which the vehicle moves at a velocity v, A is the cross-sectional area of the vehicle, C is a dimensionless constant of proportionality.

Solution

LHS: [F] = M L T^{-2} (taken from the dimensions of $F = ma$)

RHS: $\left[\frac{1}{2}\right][C][\rho][A][v] = 1 \times 1 \times ML^{-3} \times L^2 \times LT^{-1} = MT^{-1}$

The two sides have different dimensions by LT^{-1}. The difference corresponds to the dimensions of velocity. Therefore, the correct relation is $F = \frac{1}{2}C\rho Av^2$.

Example 2: Suppose a small mass is suspended from a long thread so as to form a simple pendulum. We may reasonably suppose that the period, T, of the oscillations depends only on the mass m, the length l of the thread, and the acceleration, g, of free-fall at the place concerned. Suppose then that

$T = km^x l^y g^z$(1)

Where $x.y, z, and\, k$ are unknown numbers. The dimensions of g are LT^{-2}. Now the dimensions of both sides of equation (1) must be the same.

$$\therefore T \equiv M^x L^y (LT^{-2})^z \quad (2)$$

Equating the indices of M, L, T on both sides, we have

$x = 0$

$y + z = 0$ and

$-2z = 1$

$\therefore z = -\frac{1}{2}, y = \frac{1}{2}, x = 0$

Thus, from equation (1), the period T is given by:

$T = kl^{\frac{1}{2}}g^{-\frac{1}{2}}$ or $T = k\sqrt{\frac{l}{g}}$

We cannot find the magnitude of k by the method of dimensions, since it is a number. A complete mathematical investigation shows that $k = 2\pi$ in this case, and hence, $T = 2\pi\sqrt{\frac{l}{g}}$.

Note that T is independent of the mass m.

VECTORS AND SCALARS QUANTITIES

Physical quantities which have only magnitude (numerical value) are called **SCALARS**. On other hands, the physical quantities having both magnitude and direction in space are called **VECTORS**. However, scalars and vectors are both very important in physics. The followings are examples of scalars and vectors:

- ✓ **Scalars**: mass, energy, distance, speed, temperature, potential, time, density.
- ✓ **Vectors**: displacement, velocity, acceleration, force, momentum, field strength.

Working with scalars is easy, because they obey the rules of simple arithmetic. On the other hands, combining vectors require a different set of operations. The simplest way to tackle vector problems is to start with a <u>vector diagram</u>. The beauty of vectors is that they all behave in the same way, thus, anything we say about combining displacements for example, will also hold for forces, field strengths, momenta. The combined effect (sum) of two or more vectors is called the **resultant vector**.

Moreover, in some textbooks, a boldface letter, such as **A**, is used to represent a vector quantity. Another notation is useful when boldface notation is difficult, such as when writing on paper or on a chalkboard—an arrow is written over the symbol for the vector such as $\vec{\boldsymbol{A}}$. The letter represents the magnitude while the arrow head shows direction. The magnitude of the vector $\vec{\boldsymbol{A}}$ is written either as A or $|A|$. The magnitude of a vector has physical units, such as metres for displacement or metres per second for velocity. It is always a positive number.

Vector Addition

Suppose a particle undergoes a displacement $\vec{A}$ followed by a second displacement $\vec{B}$ (Fig. 1.11a). The final result is the same as if the particle had started at the same initial point and undergone a single displacement $\vec{C}$, as shown. We call displacement $\vec{C}$ the **vector sum,** or **resultant,** of displacements $\vec{A}$ and $\vec{B}$. We express this relationship symbolically as

$$\vec{C} = \vec{A} + \vec{B} \tag{1.2}$$

The boldface plus sign emphasizes that adding two vector quantities requires a geometrical process and is not the same operation as adding two scalar quantities such as $2 + 3 = 5$. In vector addition we usually place the *tail* of the *second* vector at the *head,* or tip, of the *first* vector (Fig. 1.11a).

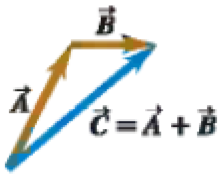
If we make the displacements $\vec{A}$ and $\vec{B}$ in reverse order, with $\vec{B}$ first and $\vec{A}$ second, the result is the same (Fig. 1.11b). Thus

$$\vec{C} = \vec{B} + \vec{A} \quad \text{and} \quad \vec{A} + \vec{B} = \vec{B} + \vec{A} \tag{1.3}$$

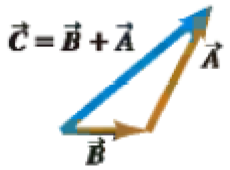
This shows that the order of terms in a vector sum doesn't matter. In other words, vector addition obeys the commutative law.

Figure 1.11c shows another way to represent the vector sum: If vectors A and $\vec{B}$ are both drawn with their tails at the same point, vector $\vec{C}$ is the diagonal of a parallelogram constructed with $\vec{A}$ and $\vec{B}$ as two adjacent sides.

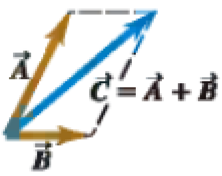
(a) We can add two vectors by placing them head to tail.



(b) Adding them in reverse order gives the same result.



(c) We can also add them by constructing a parallelogram.



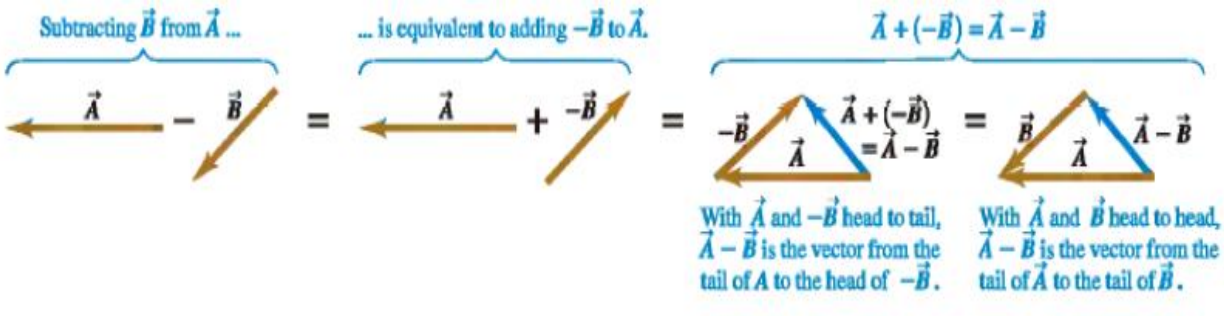
1.11 Three ways to add two vectors. As shown in (b), the order in vector addition doesn't matter; vector addition is commutative.

Subtraction of Vectors

We can *subtract* vectors as well as add them. To see how, recall that the vector $-\vec{A}$ has the same magnitude as $\vec{A}$ but the opposite direction. We define the difference $\vec{A}-\vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ to be the vector sum of $\vec{A}$ and $-\vec{B}$:

$$\vec{A}-\vec{B}=\vec{A}+(-\vec{B}) \tag{1.4}$$

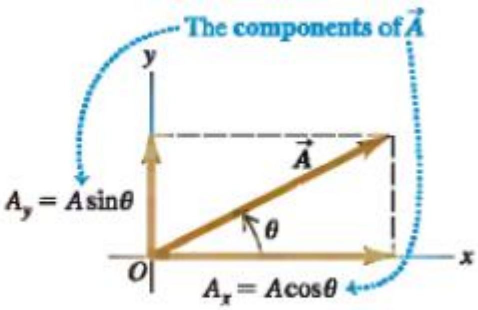
Figure 1.14 shows an example of vector subtraction.



1.14 To construct the vector difference $\vec{A}-\vec{B}$, you can either place the tail of $-\vec{B}$ at the head of $\vec{A}$ or place the two vectors $\vec{A}$ and $\vec{B}$ head to head.

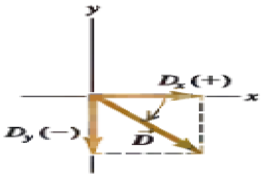
Components of Vectors

Any vector quantity can be resolved (split) into components along any axes. This helps because components along any given axis add like ordinary number. To find a resultant of several vectors, it is simplest to resolve them along a common set of axes, find the resultant components, and then reconstruct the resultant vector.



Resolving a Vector

Example 1: what are the x- and y- components of vector $\vec{D}$ in the figure below? The magnitude of the vector is D = 3.00m and the angle $\theta = 45^0$.



Example 2: What is the resultant displacement when a ship sails 200km due north and then 150km due east.

Solution

Using Pythagoras's theorem, we have

$R^2 = 200^2 + 150^2 = 62500$

Therefore, R = 250km.

Since the resultant displacement is a vector, its magnitude and direction must be stated. This can be done by giving a bearing (angle to the east of north) in this case.

From trigonometry,

$\tan\theta = \dfrac{150km}{2000km} = 0.75$

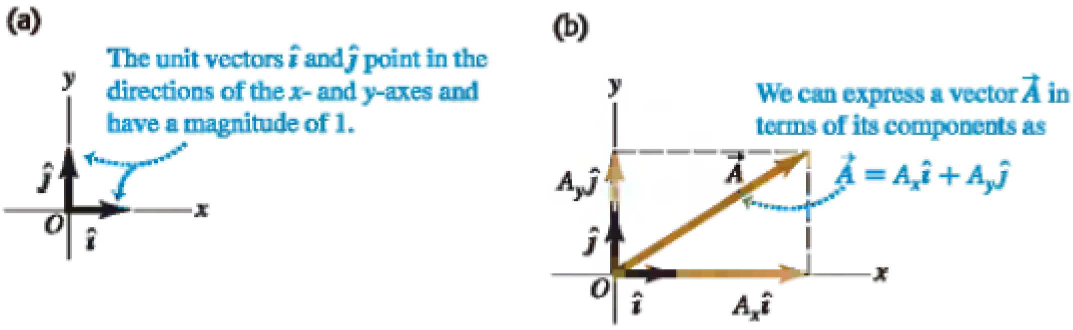
$$\therefore\ \theta = 37^0$$

Unit Vector

This is a vector having unit magnitude (a magnitude of 1), with no units. Its only purpose is to points-that is to describe a direction in space. A caret or "hat" (ʌ) is included in the symbol for a unit vector to distinguish it from ordinary vectors whose magnitude may or may not be equal to 1.

In an x-y coordinate system, we can define a unit vector $\hat{\imath}$ that points in the direction of the positive x-axis and a unit vector $\hat{\jmath}$that pointsin the direction of positive y-axis. We can express a vector $\vec{\boldsymbol{A}}$in terms of its components as:

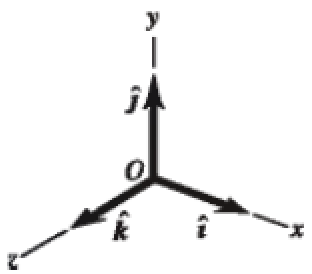
$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$



When two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are represented in terms of their components, we can express the vector sum $\vec{\mathbf{R}}$ using unit vectors as follows:

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$
$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$$
$$\begin{aligned}\vec{R} &= \vec{A} + \vec{B}\\ &= (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath})\\ &= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}\\ &= R_x\hat{\imath} + R_y\hat{\jmath}\end{aligned}$$

If the vectors do not lie in the xy-plane, then we need a third component. We introduce a third unit vector $\hat{k}$ that pointsin the direction of positive z-axis as shown below:



The unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

Then, we can write

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\vec{R} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$
$$= R_x\hat{\imath} + R_y\hat{\jmath} + R_z\hat{k}$$

<u>Example 1:</u>

Given the two displacements

$$\vec{D} = (6\hat{\imath} + 3\hat{\jmath} - \hat{k})\ \text{m} \qquad \text{and} \qquad \vec{E} = (4\hat{\imath} - 5\hat{\jmath} + 8\hat{k})\ \text{m}$$

find the magnitude of the displacement $2\vec{D} - \vec{E}$.

<u>Solution</u>

Letting $\vec{F} = 2\vec{D} - \vec{E}$, we have

$$\vec{F} = 2(6\hat{\imath} + 3\hat{\jmath} - \hat{k})\ \text{m} - (4\hat{\imath} - 5\hat{\jmath} + 8\hat{k})\ \text{m}$$
$$= [(12 - 4)\hat{\imath} + (6 + 5)\hat{\jmath} + (-2 - 8)\hat{k}]\ \text{m}$$
$$= (8\hat{\imath} + 11\hat{\jmath} - 10\hat{k})\ \text{m}$$

The units of the vectors $\vec{D}$, $\vec{E}$, and $\vec{F}$ are meters, so the components of these vectors are also in meters. From Eq. (1.12),

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$
$$= \sqrt{(8\ \text{m})^2 + (11\ \text{m})^2 + (-10\ \text{m})^2} = 17\ \text{m}$$

Product of Vectors

Vectors are not ordinary numbers, so ordinary multiplication is not directly applicable to vectors. Lets define two different kinds of products of vectors, The SCALAR PRODUCT which yields a result that is a scalar quantity and the VECTOR PRODUCT which yields another vector.

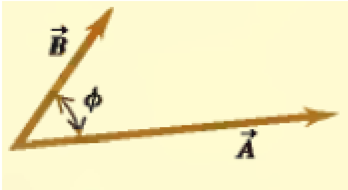
Scalar Product:

The **scalar product** of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A} \cdot \vec{B}$. Because of this notation, the scalar product is also called the **dot product.** Although $\vec{A}$ and $\vec{B}$ are vectors, the quantity $\vec{A} \cdot \vec{B}$ is a scalar.

It is defined as the product of the magnitude of the two vectors ($\vec{\mathbf{A}}$and $\vec{\mathbf{B}}$) and the cosine of the angle ϕbetween them.

$$\vec{A} \cdot \vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi$$

$$0 \leq \phi \leq \pi$$



The scalar product is a scalar quantity, not a vector, and may be positive, negative, or zero. When ϕ = 90^0, then $\vec{A} \cdot \vec{B} = 0$. The scalar product of two perpendicular vectors is always zero.

Vector Product

The vector product of two vectors $\vec{A}$ and $\vec{B}$, also called the **cross product**, is denoted by $\vec{A} \times \vec{B}$. As the name suggests, the vector product is itself a vector.

The vector product of vectors$\vec{\boldsymbol{A}}$and$\vec{\boldsymbol{B}}$is a vector $\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$ (reads $\vec{\boldsymbol{A}}$cross$\vec{\boldsymbol{B}}$). The magnitude of $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$ is defined as the product of the magnitude of $\vec{\boldsymbol{A}}$and$\vec{\boldsymbol{B}}$and the sine of the angle ϕbetween them. Symbolically,

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \mathrm{AB} \sin \phi \, \mathbf{u}, \qquad 0 \le \phi \le \pi$$

Where **u**is a unit vector indicating the direction of$\vec{\mathbf{A}} \times \vec{\mathbf{B}}$**.**

Note: If $\vec{A} = \vec{B}$, or $\vec{A}$ is parallel to $\vec{B}$, then $\sin \phi = 0$ and we define $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = 0$.

CONCEPT OF FORCE: Newton's Laws of motion, mass, weight, Conservation of momentum, elastic and inelastic collisions of particles along a straight line, Impulse of a force.

1.0 – Forces

In everyday language, a force is a push or a pull, or is a push or pull that one object exerts on another which produces or tends to produce motion, stops or tends to stop motion, or change the direction of motion. A force is something that is capable of changing an object's state of motion, that is, changing its velocity or producing an acceleration. *Examples of common types of force are: contact force, normal force, friction force, weight, tension force, electric force, magnetic force, gravitational force and resistance (also known as viscous force), etc.*

1.1 – Newton's Laws of motion

ISAAC NEWTON *English Physicist and Mathematician* (1642–1727). Newton was one of the most brilliant and greatest scientists in history. Before he was 30, he formulated the basic concepts and laws of mechanics, discovered the law of universal gravitation, and invented the mathematical methods of the calculus. As a consequence of his theories, Newton was able to explain the motions of the planets, the ebb and flow of the tides, and many special features of the motions of the Moon and Earth. He also interpreted many fundamental observations concerning the nature of light. His contributions to physical theories dominated scientific thought for two centuries and remain important today.

1.1.1 – Newton's First Law of Motion: *If no net force acts on a body, the body's velocity cannot change; that is, the body cannot accelerate. In other word, In the absence of an unbalanced applied force, (F_{net} = 0) a body at rest remains at rest, and a body in motion remains in motion with a constant velocity (constant speed and direction).*

The First Law tells us two things:

- If there is no resultant force acting on an object at rest, the object will remain at rest.
- If there is no resultant force acting on a moving object, the moving object will continue to move at a constant speed in a straight line i.e. velocity is constant.

The First Newton's law is sometimes called the law of inertia. *Inertia is the natural tendency of an object to maintain a state of rest or to remain in uniform motion in a straight line (constant velocity) unless acted upon by an external force*. The inertia of an object is determined by its mass. The greater the mass of an object, the greater is its opposition to a change in its state of rest or uniform motion.

1.1.2 – Newton's Second Law of Motion

What happens when there is a resultant force or net force acting on an object? Newton's Second Law explains how the motion of an object changes when the net force is not zero.

"If a net external force acts on a body, the body accelerates. The direction of acceleration is the same as the direction of the net force. In other words, the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. The direction of the acceleration is in the direction of the applied net force".

In symbols, $$\sum \vec{F} = m\vec{a} \quad or \quad \vec{a} = \frac{1}{m}\sum \vec{F}$$ *(Newton's second law of motion)*

Where F is the net force, m is the mass in kg and a is the acceleration of an object in m/s^2. Force is a vector quantity and its SI unit is newton (N). Newton's second law tells us that when there is a resultant force acting on an object, the object will either slow down (decelerate) or speed up (accelerate). Newton's second law is a fundamental law of nature, the basic relationship between force and motion.

Mass: is a quantitative measure of inertia of a body. That is, a massive object has more inertia, or more resistance to a change in motion, than does a less massive object. For example, a car has more inertia than a bicycle. Mass is constant all over the world; it would be the same at the equator or at the poles, or even if it were landed on the moon as it is on the earth. Mass doesn't depend on the value of g, but weight does. Mass is measured in kilogrammes, kg, by using a chemical balance, beam balance, lever balance and sliding mass balance.

Weight: is defined as the gravitational force of attraction that a celestial body exerts on an object. In other ward, the magnitude of the gravitational force acting on an object of mass *m* near Earth's surface is called the *weight*, *w,* of the object, given by

$$W = mg$$

where g is the acceleration of gravity. **SI unit: newton (N),** even though g varies but a constant value is used as 9.80 m/s^2 near the earth surface and on the surface of the moon is about 1.625 m/s^2.

The value of g varies from place to place on the earth, therefore, the weight of an object differs in different part of the world, it will be greater at the north and south poles of the earth, for example, than at the equator. So the weight is greater at the poles.

Example 01: A car of mass 1200 kg accelerates from rest to 25 m/s in a time of 7 sec. Compute the forward thrust of the car. (Assume there is no friction.)

SOLUTION

m = 1200 kg u = 0 v = 25 m/s t = 7 sec

we have to find acceleration first, using first equation of motion. a = $\frac{v-u}{t}$ = $\frac{(25-0)\ m/s}{7\ sec}$ = 3.6 m/s^2

Applying 2_{nd} Newton's law, the forward thrust $F = ma$ = 1200 kg x 3.6 m/s^2 = 4286 kg. m/s^2 = 4286 N

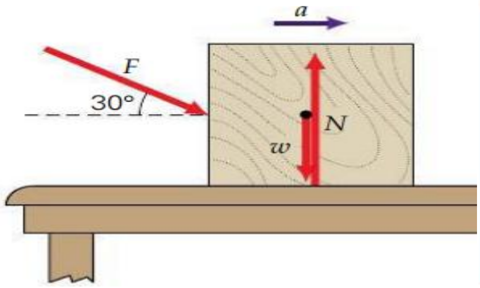
Example 02: Compute the least acceleration with which a 37-kg boy can slide down a rope if the rope can withstand a tension of only 200 N.

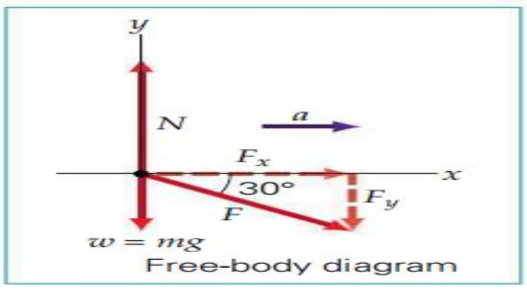
SOLUTION

The weight of boy is $W = mg$ = (37 kg) (9.8 m/s^2) = 363 N. the rope can support only 200 N, the unbalance downward F on the boy must be at least 363 N – 200 N = 163 N.

Her minimum downward acceleration is then a = $\frac{F}{m}$ =163/37 =5.2 m/s^2

Example 03: A force of 23.0 N is applied at an angle of to the horizontal on a 15-kg block initially at rest on a frictionless surface, as illustrated in Fig. below. (a) What is the magnitude of the block's acceleration? (b) What is the magnitude of the normal force?





ANS = (a) = 1.33 m/s^2 (b) = 158.5 N.

Example 04: Abox rests on a frozen pond, which serves as a frictionless horizontal surface. If a fisherman applies a horizontal force with magnitude 48.0 N to the box and produces an acceleration of magnitude 3.00 m/s^2, what is the mass of the box? ANS = 16 kg

Example 05: A force of 20 N is directed at an angle of 60 ° above the *x*-axis. A second force of 20 N is directed at an angle of 60 ° below the *x*-axis. What is the vector sum of these two forces?

ANS = 20 N in the +ve x-direction

Example 06: A 70.0-kg man stands on a bathroom scale in an elevator. Compute its weight, if the elevator is slowing down at a rate of 3.00 m/s2 while descending? (Take g as 9.80 m/s2)

ANS = 896 N

Example 07: At the surface of Jupiter's moon, the acceleration due to gravity is g =1.81 m/s^2. A watermelon weighs 44.0 N at the surface of the earth. (a) What is the watermelon's mass on the earth's surface? (b) What are its mass and weight on the surface of Jupiter's moon?

ANS = (a) watermelon's mass on the earth's surface = 4.5 kg (b) m = 4.5 kg, w = 8.1 N

Example 08: A Consider a 2.0-kg ball and a 6.0-kg ball in free fall. (a) What is the net force acting on each? (b) What is the acceleration of each?

ANS = (a) 2.0 kg = 20 N, 6.0 kg = 59 N (b) each have the same acceleration of 9.80 m/s^2

Example 09*:* Aisha, who has a mass of 50.0 kg, is riding at 35.0 m/s in her red sports car when she must suddenly slam on the brakes to avoid hitting a deer crossing the road. She strikes the air bag that brings her body to a stop in 0.500 s. What average force does the seat belt exert on her?

ANS = 3500 N.

Example 10*:* A 0.50-kg cart (#1) is pulled with a 1.0-N force for 1 second; another 0.50 kg cart (#2) is pulled with a 2.0 N-force for 0.50 seconds. Which cart (#1 or #2) has the greatest acceleration? *Explain*.

ANS =**Cart #2 has the greatest acceleration**. Recall that acceleration depends on force and mass. They each have the same mass, yet cart #2 has the greater force.

1.1.3 − Newton's Third Law

Force acting on a body is always the result of its interaction with another body, so forces always come in pairs. You can't pull on a doorknob without the doorknob pulling back on you. When you kick a football, the forward force that your foot exerts on the ball launches it into its trajectory, but you also feel the force the ball exerts back on your foot. If you kick a boulder, the pain you feel is due to the force that the boulder exerts on your foot.

In each of these cases, the force that you exert on the other body is in the opposite direction to the force that body exerts on you. Experiments show that whenever two bodies interact, the two forces that they exert on each other are always *equal in magnitude* and *opposite in direction.* This fact is called *Newton's third law of motion:*

$$F_{12} = -F_{21}$$

Newton's third law of motion states that: If body A exerts a force on body B (an "action"), then body B exerts a force on body A (a "reaction"). These two forces have the same magnitude but are opposite in direction. These two forces act on *different* bodies.

1.2 − Momentum, Impulse, and Collisions

The term *momentum* may bring to mind a football player running down the field, knocking down players who are trying to stop him. Or you might have heard someone say that a team lost its momentum (and so lost the game). Such everyday usages give some insight into the meaning of momentum. They suggest the idea of mass in motion and therefore inertia. We tend to think of heavy or massive objects in motion as having a great deal of momentum, even if they move very slowly. However, according to the technical definition of momentum, a light object can have just as much momentum as a heavier one, and sometimes more.

1.2.1 − Momentum:

Newton referred to what modern physicist's term linear momentum (p) as "the quantity of motion arising from velocity and the quantity of matter conjointly." In other words, the momentum of a body is proportional to both its mass and velocity. By definition,

"The linear momentum p of an object of mass m moving with velocity v is the product of its mass and velocity".

$$\vec{p} = m\vec{v}$$

The SI unit of momentum is kilogram meter per second (kg . m/s)

It is common to refer to linear momentum as simply *momentum.* Momentum is a vector quantity that has the same direction as the velocity, and *x*- and *y*-components with magnitudes of $p_x = mv_x$ and, $p_y = mv_y$ respectively.

Equation above, expresses the momentum of a single object or particle. For a system of more than one particle, the **total linear momentum ($\vec{\mathrm{P}}$)** of the system is the vector sum of the *momenta* of the individual particles:

$$\vec{\mathrm{P}} = \vec{\mathrm{P}}_1 + \vec{\mathrm{P}}_2 + \vec{\mathrm{P}}_3 + \ . \ . \ . = \sum \vec{\mathrm{P}}_\mathrm{i}$$

$$\Delta\vec{P} = F_{net}\ x\ \Delta t \qquad or \qquad F_{net} = \frac{\Delta\vec{P}}{\Delta t} \quad Newton's\ 2nd\ law\ of\ motion\ in\ terms\ of\ momentum$$

1.2.2 − Impulse:

When two objects—such as a hammer and a nail, a golf club and a golf ball, or even two cars—collide, they can exert a large force on one another for a short period of time, or an *impulse*. The force is not constant in this situation. However, Newton's second law in momentum form is still useful for analyzing such situations by using average values.

Impulse is the product of the average force acting on the object and the time of action of the force. The term $F_{avg}\ \Delta t$is known as the ***Impulse***($\vec{\mathrm{I}}$)of the force:

$$\vec{I} = F_{avg}\,\Delta t = \Delta\vec{P} = m\vec{v}_f - m\vec{v}_i$$

Where F_{avg} is the average force and t or Δt is the time taken or interval. Thus, *the impulse exerted on an object is equal to the change in the object's momentum.* This statement is referred to as the **impulse–momentum theorem**. The SI unit of impulse is newton-second (N.s) or kilogram metre per second (kg ms^{-1}), which are also units of momentum.

Example 11: A 5 kg mass is sitting on a frictionless surface. An unknown constant force pushes the mass for 3 seconds until the mass reaches a velocity of 7 m/s. (a) What are the initial and the final momentum of the mass? (b) What are the force and the impulse acting on the mass?

ANS = (a) 0, 35 kg m/s (b) 11.6 N, 34.8 Ns.

Example 12:(A) - cue stick hits a cue ball with an average force of 24 N for a duration of 0.028 s. If the mass of the ball is 0.16 kg, how fast is it moving after being struck? ANS = 4.2 m/s

(B) - A ball of mass 5.0 kg moving with a speed of 2.0 m/s in the +*x*-direction hits a wall and bounces back with the same speed in the −*x*-direction. What is the change of momentum of the ball?

ANS = 20 kg m/s in the –ve x direction

Example 13: A system consists of three particles with these masses and velocities: mass 3.0 kg, moving north at 3.0 m/s; mass 4.0 kg, moving south at 5.0 m/s; and mass 7.0 kg, moving north at 2.0 m/s. What is the total momentum of the system? ANS = 3 kg m/s north

Example 14:(A) - When tossed upward and hit horizontally by a batter, a 0.20-kg softball receives an impulse of 3.0 Ns with what horizontal speed does the ball move away from the bat? ANS = 15 m/s

(B) - An automobile with a linear momentum of 3.0 x 10^4 kg.m/s is brought to a stop in 5.0 s. What is the magnitude of the average braking force?

ANS = 6000 N.

(C) - A pool player imparts an impulse of 3.2 Ns to a stationary 0.25-kg cue ball with a cue stick. What is the speed of the ball just after impact? ANS = 12.8 m/s

Example 15:(A) - An object of mass 3.0 kg is allowed to fall from rest under the force of gravity for 3.4 s. What is the change in its momentum? Ignore air resistance. ANS = 100 kg m/s, downward in the direction of motion

(B) - What average force is necessary to bring a 0.05-g sled from rest to a speed of 3.0 m/s in a period of 0.02 min? Assume frictionless ice. ANS = 0.000125 N.

1.2.3 Conservation of Linear Momentum

The principle conservation of linear momentum is an important concept when dealing with problems that involve particles in collisions or explosive event such as the firing of a gun.

"The principle conservation of linear momentum states that, if no external force act on a system of colliding objects, the total momentum of the objects in a given direction before collision is equal to the total momentum in same direction after collision".

If the net force acting on a particle is zero, that is,

$$\vec{\mathrm{F}}_{\mathrm{net}} = \frac{\Delta\vec{P}}{\Delta t} = 0$$

Then

$$\Delta\vec{P} = 0 = \vec{P}_f - \vec{P}_i$$

where $\vec{P}_i$is the initial momentum and $\vec{P}_f$ is the momentum at some later time. Since these two values are equal, the momentum is conserved, and

$$\vec{P}_f = \vec{P}_i \quad \text{or} \quad m\vec{v}_f = m\vec{v}_i$$

final momentum = initial momentum

Note that this conservation is consistent with Newton's first law: An object remains at rest ($\vec{P} = 0$), or in motion with a *uniform* velocity ($\vec{P} \neq 0$), unless acted on by a net external force.

Mathematically, the principle conservation of linear momentum is:-

$$m_1u_{1i} + m_2u_{2i} = m_1v_{1f} + m_2v_{2f}$$

or

$$m_Au_A + m_Bu_B = m_Av_A + m_Bv_B$$

When no net external force acts on a system, the total linear momentum of the system remains constant with time. The principle conservation of linear momentum is a direct consequence of Newton's Second and Third Laws.

In general, a *collision* may be defined as a meeting or interaction of particles or objects that causes an exchange of energy momentum. In collisions, momentum is conserved. Kinetic energy may or may not be remain the same after a collision. Kinetic energy is the energy possessed by a body as a result of its motion.

Kinetic energy, K= $\frac{1}{2}mv^2$

Collision can be classified into two types: *Elastic and Inelastic collisions*.

1.2.4 − Elastic Collisions

An *elastic* collision is defined as one in which both momentum and kinetic energy are conserved. For example, two steel balls (collisions between the steel balls in 'Newton's cradle') or two billiard balls may have a nearly elastic collision, with each ball having the same shape afterward as before; that is, there is no permanent deformation. The collisions of air molecules with the walls of a container at ordinary temperatures are highly elastic.

$$total\ K\ after\ = total\ K\ before$$

$$K_f\ =\ K_i \qquad condition\ for\ an\ elastic\ collision$$

The principle conservation of Kinetic energy is

$$\frac{1}{2}m_1u_{1i}^2 + \frac{1}{2}m_2u_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

1.2.5 − Inelastic Collisions

An *Inelastic collision* is defined as a collision in which momentum is conserved, but total kinetic energy is *not* conserved. Because some of the kinetic energy is converted to sound energy, internal energy, heat energy or frictional heat may be generated and some kinetic energy is lost, and the work needed to permanently deform the objects involved, such as cars in a car crash. The collision of a rubber ball with a hard surface is inelastic, because some of the kinetic energy is lost when the ball is deformed during contact with the surface.

For another example, is a hollow aluminium ball that collides with a solid steel ball may be dented. Permanent deformation of the ball takes work, and that work is done at the expense of the original kinetic energy of the system.

$$total\ K\ after\ < total\ K\ before$$

$$K_f < K_i \qquad condition\ for\ an\ inelastic\ collision$$

Everyday collisions are inelastic. **For isolated systems, momentum is conserved in both elastic and inelastic collisions**. *For an inelastic collision, only an amount of kinetic energy consistent with the conservation of momentum may be lost.*

A special case of inelastic collision is the *perfectly* inelastic collision. In a *perfectly* inelastic collision, momentum is conserved, kinetic energy is not, and the two bodies stick together after the collision, so their final velocities are the same.

$$m_1 u_{1i} + m_2 u_{2i} = (\ m_1 + \ m_2) v_f$$

To solve for the final velocity using conservation of momentum alone: $v_f = \frac{m_1 u_{1i} + m_2 u_{2i}}{(\ m_1 + \ m_2)}$

Example 16*:* A 3.0-kg ball with a speed of 2500 mm/s strikes a 7.0-kg stationary ball. If the collision is completely inelastic, (a) what are the speeds of the balls after the collision? (b) What percentage of the initial kinetic energy do the balls have after the collision? (c) What is the total momentum after the collision?

SOLUTION

(a) The final speed of the balls after collision is?

The momentum is conserved and

$$p_i = p_f$$

$m_1 u_i = (\ m_1 + \ m_2) v_f$ The balls stick together and have the same speed after collision.

Substitute the values and solve for the final velocity, vf:

This speed is then, $v_f = \left(\frac{m_1}{(\ m_1 + \ m_2)}\right) u_i = 0.75\ m/s$

(b) The percentage of the initial kinetic energy do the balls have after the collision is

Calculate the initial kinetic energy of the system:

$$\frac{K_f}{K_i}(x\ 100\%)$$

$$\frac{K_f}{K_i} = \frac{½\,(m_1 \ + \ m_2)v_f^2}{½\,m_1 u_{1i}^2} = \frac{½\,(3.0\,kg + \ 7\,kg)(0.75\,m/s)^2}{½\,(3.0\,kg)(2.5\,m/s)^2} = 0.3$$

$$Hence, \quad \frac{K_f}{K_i}(x\ 100\%) = \ 30\%$$

(c) The total momentum is conserved in all collisions (in the absence of external forces), so the total momentum after collision is the same as before collision. That value is the momentum of the incident ball, with a magnitude of

$$\vec{P}_f = \vec{P}_i \text{ but } \quad \vec{P}_i = m_1 u_{1i} = (3.0\,kg)(2.5\,kg) = \ 7.5\,kg\,m/s$$

and the same direction as that of the incoming ball. Also,

$$\vec{P}_f = \ (m_1 \ + \ m_2)v_f = (10\,kg)(0.75\,m/s) = \ 7.5\,kg\,m/s$$

***Example 17**:* A 1200-kg car moving to the right with a speed of 25 m/s collides with a 1500-kg truck and locks bumpers with the truck. Calculate the velocity of the combination after the collision if the truck is initially (a) at rest, (b) moving to the right with a speed of 20 m/s.

SOLUTION

(a) The truck is initially at rest,

$$v_f = \frac{m_1 u_{1i}}{(m_1 + m_2)} = \frac{30000}{2700} = 11\,m/s \text{ to the right.}$$

(b) The velocity of the combination after the collision if the truck is moving to the right with a speed of 20 m/s is

$$v_f = \frac{m_1 u_{1i} + \ m_2 u_{2i}}{(m_1 + \ m_2)} = \frac{60000}{2700} = 22\,m/s\text{to the right.}$$

***Example 18**:* A 2.0-kg block is moving to the right at 1.0 m/s just before it strikes and sticks to a 1.0-kg block initially at rest. What is the total momentum of the two blocks after the collision?

SOLUTION

(a) The total momentum of the two blocks after the collision will be?

$$But\ recall, momentum\ is\ conserved \qquad \vec{P}_f = \ \vec{P}_i$$

$$The\ total\ momentum\ \ is = 2\ kg\ m/s, \qquad \textit{moving to the right.}$$

Example 19: A 75-kg man is at rest on ice skates. A 0.20-kg ball is thrown to him. The ball is moving horizontally at 25 m/s just before the man catches it. How fast is the man moving just after he catches the ball?

SOLUTION

From the relation, $m_1u_{1i} + m_2u_{2i} = (\ m_1 + \ m_2)v_f$

$$v_f = \frac{m_2u_{2i}}{(\ m_1 + \ \ m_2)} = \frac{0.5}{75.2} = 0.0066\ m/s$$

Example 20: A 5.0-kg ball is at rest when it is struck head-on by a 2000-g ball moving along a track at 10.0 m/s. If the 2000-g ball is at rest after the collision, what is the speed of the 5.0-kg ball after the collision?

SOLUTION

Applying relation and recall the momentum is elastic

$$m_1u_{1i} + m_2u_{2i} = m_1v_{1f} + \ m_2v_{2f}$$

After cancellation of some terms the equation becomes

$$m_2u_{2i} = m_1v_{1f}$$

$$v_{1f} = \frac{m_2u_{2i}}{m_1} = \frac{20}{5} = 4.0\ m/s$$

(-2-) - WORK, ENERGY AND POWER: Units, Conservation of energy, Conservative and dissipative forces, Hook's law. Elastic spring. Equilibrium and Stability.

Introduction

There are many cases where motion involves changing forces and accelerations. In this chapter, we introduce the important physical concepts of **work, energy** and **power**. These powerful concepts enable us to "shortcut" the detailed application of Newton's law to analyze these more complex situations. We begin with the concept of work.

2.1 – WORK

The word *work* is commonly used in a variety of ways: We go to work; work on projects; work at our desks or on computers; carrying a piece of furniture upstairs involves work. The heavier the furniture or the higher the stairs, the greater the work. Pushing a stalled car involves work. The harder you push or the farther you push the more work you do. A man pushing against a solid wall, the wall remains stationary, in this case this man is not doing work, because work is said to be done on an object when it moves the influence of force. In physics, however, *work* has a very specific meaning.

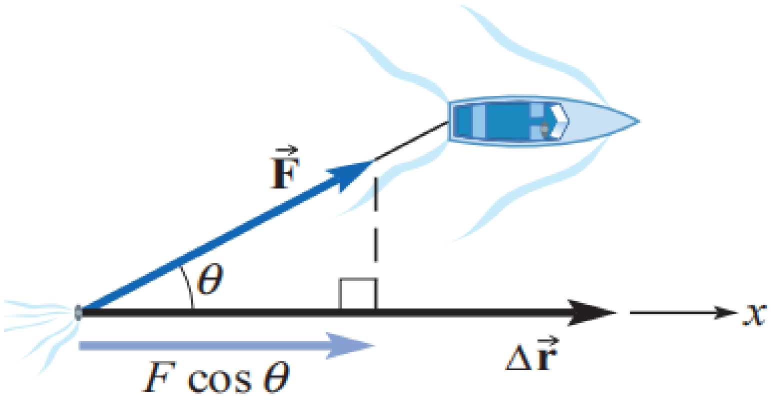
Mechanically, work involves force and displacement, and the word *work* is used to describe quantitatively what is accomplished when a force acts on an object as it moves through a distance. In the simplest case of a *constant* force acting on an object, work that the force does is defined as follows:

*"The **work** done by a constant force acting on an object is equal to the product of the magnitudes of the displacement and the force, or component of the force, parallel to that displacement."*

Work then involves a force acting on an object and moving it through a distance. A force may be applied, as in Fig. below. *Work is an energy transfer that occurs when a force acts on an object that moves. If there is no displacement, (on motion) no work is done and no energy is transferred.*

Work done by a constant force $F^{\rightarrow}$ acting on an object whose displacement is $\Delta r^{\rightarrow}$:

$$W = F \cdot \Delta r \qquad (constant\ force\ in\ direction\ of\ straight - line\ displacement)$$



Mathematically, Work is also given by

$$W = F\ cos\theta\ x\ \Delta r$$

Notice that θ is the angle *between* the force and the displacement vectors. When the angle between $\boldsymbol{F}^{\rightarrow}$ and $\boldsymbol{\Delta r}^{\rightarrow}$ is less than 90 °, $cos\ \theta$, is positive, so the work done by the force is positive ($W > 0$). If the angle between $\boldsymbol{F}^{\rightarrow}$ and $\boldsymbol{\Delta r}^{\rightarrow}$ is greater than 90 °, $cos\ \theta$is negative and the work done by the force is negative ($W < 0$). If the force is perpendicular to the displacement, $\theta = 90°$ and $cos\ 90° = 0$, so the work done is zero.

Total Work

When several forces act on an object, the total work is the sum of the work done by each force individually:

$$W_{total} = W_1 + W_2 + W_3 \dots + W_N$$

Total work is sometimes called *net* work because the work done by each force can be positive, negative, or zero, so the total work is often smaller than the work done by any one of the forces. Because we assume a rigid object with no rotational or internal motion, another way to calculate the total work is to find the work done by the *net* force as if there were a single force acting:

$$W_{total} = F_{net}\, \Delta r \cos \theta$$

To show that these two methods give the same result, let's choose the *x*-axis in the direction of the displacement. Then the work done by each individual force is the x -component of the force times Δx. From Eq. above

$$W_{total} = F_{1x}\Delta x + F_{2x}\Delta x + F_{3x}\Delta x \dots + F_{Nx}\Delta x$$

Factoring out the Δx from each term,

$$W_{total} = (F_{1x} + F_{2x} + F_{3x} \dots + F_{Nx})\Delta x = \left(\sum F_x\right)\Delta x$$

$\sum F_x$is the x −component of the net force. In Eq., $F_{net} cos\ \theta$is the component of the net force in the direction of the displacement, which is the *x*-component of the net force. The two methods give the same total work.

Work is a scalar quantity, its SI unit is **joule** (abbreviated J, pronounced "jool," and named in honor of the 19th-century English physicist James Prescott Joule). From Eq. above, we see that in any system of units, the unit of work is the unit of force multiplied by the unit of distance. In SI units the unit of force is the newton and the unit of distance is the meter, so 1 joule is equivalent to 1 *newton-meter* (N · m).

2.2 – ENERGY

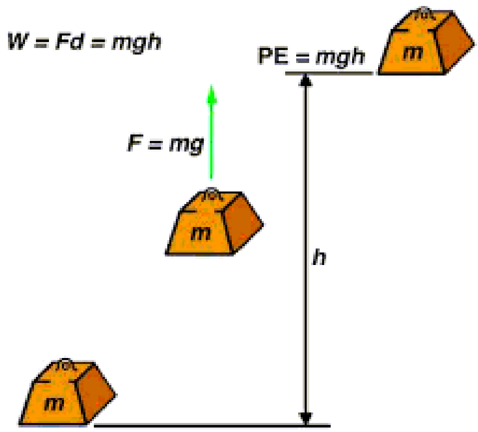
Energy is defined as the capacity of a body to do work. The SI unit of energy is therefore the same as that of work i.e. the joule (J). Without a supply of energy, neither people nor machines can do work. There are many different forms (or types) of energy. Chemical energy, nuclear energy, radiant energy, electrical energy, internal energy, heat energy, sound energy, light energy, magnetic energy, mechanical energy etc.

Mechanical energy is the type of energy which comprises *kinetic energy* and *potential energy*.

2.2.1 – Potential Energy

An object in motion has kinetic energy. However, whether an object is in motion or not, it may have another form of energy—potential energy. As the name implies, an object having potential energy has the *potential* to do work. Potential energy *(P.E. or U)* is the energy possessed by a body due to its position or condition. For example, an object raised above the ground has gravitational *P.E.* due to its raised position. While a stretched rubber band has *elastic P.E.* due to its stretched condition.

An object of mass *m* raised to a height *h* above the ground.



The work done W by the gravitational force F is given by $W = F\ x\ h$, Force F is the weight of the object given by mg. Hence, $W = mgh$

The potential energy or gravitational potential energy P.E. of a body near the surface of the earth is defined as the product of its weight mg and its height h or d above a reference level, in this case, the ground. SI unit of gravitational potential energy: joule (J)

$$\text{Gravitational P.E.} \qquad U = mgh$$

The work done in lifting is then equal to the change in potential energy

$$work\ done\ by\ external\ force\ =\ change\ in\ gravitational\ potential\ energy$$

$$\text{or} \qquad W = F\,\Delta d = mg\left(d_f - d_i\right) = \ mgd_f - mgd_i = \ \Delta U = \ U_f - U_i$$

Elastic Potential Energy

The work done by an ideal spring depends on the initial and final positions of the moveable end, but *not* on the path that was taken. Therefore, the force exerted by an ideal spring is *conservative* and we can associate a potential energy with it. The kind of potential energy stored in a spring is called *elastic potential energy*.

Recall that the work done in such a case is $W = \frac{1}{2}kx^2$ (with$x_o = x_i = 0$). Note that the amount of work done depends on the amount of stretching$(x = x_f)$. Because work is done, there is a *change* in the spring's potential energy, which is equal to the work done *by the applied force* in stretching (or compressing) the spring:

$$W = \ \Delta U = U_f - U_i = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

Thus, with $x_i = 0$and $U_i = 0$, as they are commonly taken for convenience, the *potential energy of a spring* is

$$U_{elastic} = U_f = \frac{1}{2}kx_f^2 \qquad (potential\ energy\ for\ a\ spring)$$

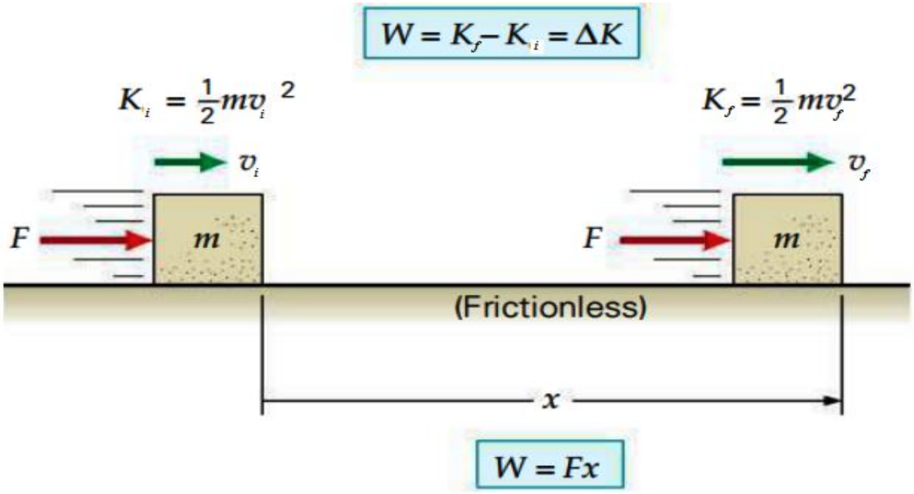
2.2.2 − Kinetic Energy

Kinetic energy *(K.E. or K)* is the energy possessed by a body due to its motion. In other words, any moving object has *K.E.* In symbol $K = \frac{1}{2}mv^2$

Where the Kinetic energy(in J), m is the mass of the body (in kg) and $v$is the speed of the body (in $/s$), SI unit of energy: joule (J)

Relation between total work and kinetic energy

Therefore, the total work done is



$$W_{total} = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The total work done is equal to the change in the quantity $K = \frac{1}{2}mv^2$, which is called the object's **translational kinetic energy** (symbol *K*). Translational kinetic energy is the energy associated with motion of the object as a whole; it does not include the energy of rotational or internal motion.

$$\textbf{Work} - \textbf{kinetic energy theorem}: \quad W_{total} = \Delta K$$

Kinetic energy is a scalar quantity and is always positive if the object is moving or zero if it is at rest. Kinetic energy is never negative, although a *change* in kinetic energy can be negative.

Example 21: How much work must Aisha do to drag her basket of laundry of mass 5.0 kg a distance of 5.0 m along a floor, if the force she exerts is a constant 30.0 N at an angle of 60.0 ° with the horizontal?

Hint: Applying, $= F\,cos\theta\ x\ \Delta r$, ANS = 75 J

Example 22: (a) -Umar pushes a 10.0-kg sack of bread flour on a frictionless horizontal surface with a constant horizontal force of 2.0 N starting from rest. (i) What is the kinetic energy of the sack after Umar has pushed it a distance of 35 cm? (ii) What is the speed of the sack after Umar has pushed it a distance of 35 cm?

(b) - A 7.0 kg object is lifted 3.5 m. How much work is done against the Earth's gravity?

SOLUTION

Hint: For (a) part, (i) applying $W = F \, x \, s$ and (ii) $K = \frac{1}{2}mv^2$ (b) $P = mgh$

ANS = (a) (i) 0.70 J, (ii) 0.37 m/s (b) 240 J.

Example 23: A ball of mass 0.10 kg moving with speed of 2.0 m/s hits a wall and bounces back with the same speed in the opposite direction. What is the change in the ball's kinetic energy? ANS = 0.

Example 24: Justin moves a desk 5.0 m across a level floor by pushing on it with a constant horizontal force of 340 N. (It slides for a negligibly small distance before coming to a stop when the force is removed.) Then, changing his mind, he moves it back to its starting point, again by pushing with a constant force of 340 N. (a) What is the change in the desk's gravitational potential energy during the round-trip? (b) How much work has Justin done on the desk? (c) If the work done by Justin is not equal to the change in gravitational potential energy of the desk, then where has the energy gone?

ANS = (a) 0 (b) 3400 J (b) Dissipated as heat

Example 25: Emil is tossing an orange of mass 0.30 kg into the air. (a) Emil throws the orange straight up and then catches it, throwing and catching it at the same point in space. What is the change in the potential energy of the orange during its trajectory? Ignore air resistance. (b) Emil throws the orange straight up, starting 1.0 m above the ground. He fails to catch it. What is the change in the potential energy of the orange during this flight? ANS = (a) 0 (b) applying, P = mgh, – 2.94 J

Example 26: A 69.0-kg short-track ice skater is racing at a speed of 11.0 m/s when he falls down and slides across the ice into a padded wall that brings him to rest. Assuming that he doesn't lose any speed during the fall or while sliding across the ice, how much work is done by the wall while stopping the ice skater? ANS = applying, $K = \frac{1}{2}mv^2 = –4175$ J, the workdone by the wall is in opposite direction.

2.2.3 − Conservation of Energy

The importance of the energy idea stems from the *principle of conservation of energy*: Energy is a quantity that can be converted from one form to another but cannot be created or destroyed. In an automobile engine, chemical energy stored in the fuel is converted partially to the energy of the automobile's motion and partially to thermal energy. In a microwave oven, electromagnetic energy obtained from your power company is converted to thermal energy of the food being cooked. In these and all other processes, the *total* energy—the sum of all energy present in all different forms—remains the same. No exception has ever been found.

The principal Law of conservation of energy states:

$$\text{“}The\ total\ energy\ in\ the\ universe\ is\ unchanged\ by\ any\ physical\ process\text{”}$$

$$total\ energy\ before\ =\ total\ energy\ after$$

Conservation of Total Mechanical Energy: The idea of a conservative force allows us to extend the conservation of energy to the special case of mechanical energy. *The sum of the kinetic and potential energies is called the* **total mechanical energy**:

$$E\ \text{(mechanical energy)} = K(kinetic\ energy) + U(potential\ energy)$$

For a **conservative system** (that is, a system in which only conservative forces do work), the total mechanical energy is constant, or conserved:

$$E_f = E_i$$

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}mv_f^2 + U_f = \frac{1}{2}mv_i^2 + +U_i$$

Equation above, is a mathematical statement of the **law of the conservation of mechanical energy**:

“In a conservative system, the sum of all types of kinetic energy and potential energy is constant and equals the total mechanical energy of the system at any time.”

$$(K_f - K_i) + (U_f - U_i) = 0$$

$$\Delta K + \Delta U = 0 \qquad (for\ conservative\ system).$$

The following are example of the conversion and conservation of energy that can be explained by the principal of conservation of energy.

(a) A cyclist going up to the top of a hill: Stored *chemical energy* in the body of the cyclist allows him to do work against gravity. At the top of the hill, he will possess *gravitational P.E.,* which will allow him to go down the hill with increasing *K.E.,* even without pedaling.

(b) The burning of fuels: Stored *chemical energy* in fuels such as oil, coal or wood is converted into *heat energy* and *light energy* by burning them.

(c) Connecting a battery to a filament lamp: Stored *chemical energy* in the battery is converted into *electrical energy*, which in turn is converted in the filament into *heat energy* and *light energy*. Similarly, in an electrical torch, the stored *chemical energy* is converted into *electrical energy* and then *light energy*.

2.3 – POWER

The time rate at which work is done or energy is transferred is called power. Like work and energy, power is a scalar quantity. The average power is the work done divided by the time it takes to do the work, or work per unit of time:

$$Power = \frac{Work\ done\ or\ energy\ expended}{Time\ taken} \Rightarrow P = \frac{W}{t} = \frac{E}{t} = \frac{F\ x\ d}{t} = F\left(\frac{d}{t}\right) = F \cdot v$$

If the force and displacement are not in the same direction, then we can write

$$P = \frac{F(\cos\theta)\, d}{t} = F \cdot \vec{v} \cos\theta$$

where θ is the angle between the force and the displacement.

Where P is the power, W is the work done (*in J*), E is the energy converted in (in J) and t is the time taken (*in second*). The SI unit of power is joule per second (Js^{-1}) is called the Watt (W). In symbols, 1 Watt $= 1\, Js^{-1}$. The unit of power includes:

1 kilowatt (1 kW) $= 10^3 W$

1 megawatt (1 MW) $= 10^6 W$

1 horsepower (1 hp) $= 746\ W$

Power tells how fast work is being done *or* how fast energy is transferred. For example, motors have power ratings commonly given in horsepower. A $2 - hp$ motor can do a given amount of work in half the time that a $1 - hp$ motor would take, or twice the work in the same amount of time. That is, a $2 - hp$ motor is twice as “powerful” as a $1 - hp$ motor.

Example 27: A pump lifts 200 kg of water per hour a height of 5.0 m. What is the minimum necessary power output rating of the water pump in watts and horsepower?

Hint: $P = \frac{mgh}{t}$, ANS = 2.72 watt, 0.003646 hp.

Example 28: A race car is driven at a constant velocity of 200 km/h on a straight, level track. The power delivered to the wheels is 150 kW. What is the total resistive force on the car? ANS = 2.7×10^3 N

Example 29: An electric heater is rated at 150 W. Compute the quantity of heat generated in 12 minutes.
Hint: E = Pt, ANS = 108 kJ

Example 30: A construction hoist exerts an upward force of 500 N on an object with a mass of 50 kg. If the hoist started from rest, determine the power it expended to lift the object vertically for 10 s to a height of 30 cm.
ANS = 14.7 watt

2.7 − Equilibrium and Stability

Human constructions are supposed to be stable in spite of the forces that act on them. A building, for example, should be stable in spite of the gravitational force and wind forces on it, and a bridge should be stable in spite of the gravitational force pulling it downward and the repeated jolting it receives from cars and trucks. One focus of physics is on what allows an object to be stable in spite of any forces acting on it.

2.7.1 − Equilibrium

Consider these objects: (1) a book resting on a table, (2) a hockey puck sliding with constant velocity across a frictionless surface, (3) the rotating blades of a ceiling fan, and (4) the wheel of a bicycle that is travelling along a straight path at constant speed. For each of these four objects, the four objects mentioned only one—the book resting on the table—is in static equilibrium *(a body at rest with no net force acting on it is said to be in static equilibrium)*. While the other object are in dynamic equilibrium *(if the body maintains constant linear velocity or rotates with constant angular velocity, is said to be in dynamic equilibrium).*

2.7.1.1− Conditions for Equilibrium: The particle is in *equilibrium*− if the particle does not accelerate− in an inertial frame of reference if the vector sum of all the forces acting on the particle is zero, $\Sigma F = 0$. For an *extended* body, the equivalent statement is that the centre of mass of the body has zero acceleration if the vector sum of all external forces acting on the body is zero. This is often called the **first condition for equilibrium.** In vector and component forms,

$$\Sigma F = 0, \Sigma F_x = 0,\ \Sigma F_y = 0 \qquad \Sigma F_z = 0 \textit{(first condition for equilibrium)}$$

A second condition for an extended body to be in equilibrium is that the body must have no tendency to *rotate.* This condition is based on the dynamics of rotational motion in exactly the same way that the first condition is based on Newton's first law. A rigid body that, in an inertial frame, is not rotating about a certain point has zero angular momentum about that point. If it is not to start rotating about that point, the rate of change of angular momentum must *also* be zero. this means that the sum of torques due to all the external forces acting on the body must be zero. A rigid body in equilibrium can't have any tendency to start rotating about *any* point, so the sum of external torques must be zero about any point. This is the **second condition for equilibrium:**

$$\Sigma \tau = 0 \text{about any point} \quad \textit{(second condition for equilibrium)}$$

The sum of the torques due to all external forces acting on the body, with respect to any specified point, must be zero.

By apply the first and second conditions for equilibrium to situations in which a rigid body is at rest (no translation or rotation). Such a body is said to be in **static equilibrium**. But the same conditions apply to a rigid body in uniform *translational* motion (without rotation), such as an airplane in flight with constant speed, direction, and altitude. Such a body is in equilibrium but is not static. To be in static equilibrium, a body at rest must satisfy *both* conditions for equilibrium: It can have no tendency to accelerate as a whole or to start rotating.

2.7.2 − The Center of Gravity

The gravitational force on an extended body is the vector sum of the gravitational forces acting on the individual elements (the atoms) of the body. Instead of considering all those individual elements, we can say that:

The gravitational force F_g on a body effectively acts at a single point, called the ***center of gravity*** *(cog) of the body.*

Here the word "effectively" means that if the gravitational forces on the individual elements were somehow turned off and the gravitational force F_g at the center of gravity were turned on, the net force and the net torque (about any point) acting on the body would not change. Assumed that the gravitational force F_g acts at the center of mass (com) of the body. This is equivalent to assuming that the center of gravity is at the center of mass. Recall that, for a body of mass M, the force F_g is equal to Mg, where g is the acceleration that the force would produce if the body were to fall freely.

If is the same for all elements of a body, then the body's center of gravity (cog) is coincident with the body's center of mass (com). ***The center of gravity coincides with the center of mass when the gravitational field is uniform.***

2.7.3 − Stability

If a body is disturbed from equilibrium, it generally experiences nonzero torques or forces that cause it to accelerate. Figure below shows two very different possibilities for the subsequent motion of two cones initially in equilibrium. Tip the cone on the left slightly, and a torque develops that brings it quickly back to equilibrium. Tip the cone on the right, and over it goes. The torque arising from even a slight displacement swings the cone permanently away from its original equilibrium. The former situation is an example of **stable equilibrium**, the latter of **unstable equilibrium**. Nearly all the equilibria we encounter in nature are stable, since a body in unstable equilibrium won't remain so. The slightest disturbance will set it in motion, bringing it to a very different equilibrium state.

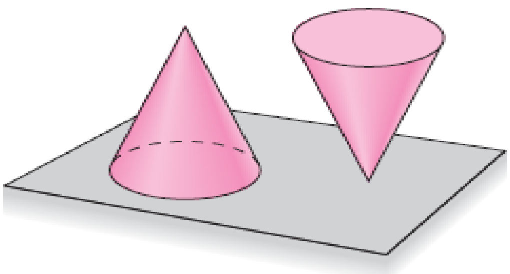


Fig 3: Stable (left) and unstable (right) equilibria.

Stability is closely associated with potential energy. Because gravitational potential energy is directly proportional to height, the shapes of the hills and valleys in the potential-energy curves. Figure below shows example in four different equilibrium situations. Stable, unstable and the situation neither stable nor unstable; it's called **neutrally stable**. But in metastable state, for small disturbances, the ball or object will return to its original state, so the equilibrium is stable. But for larger disturbances—large enough to push the ball or object over the highest points on the hill—it's unstable. Such an equilibrium is **conditionally stable** or **metastable**.

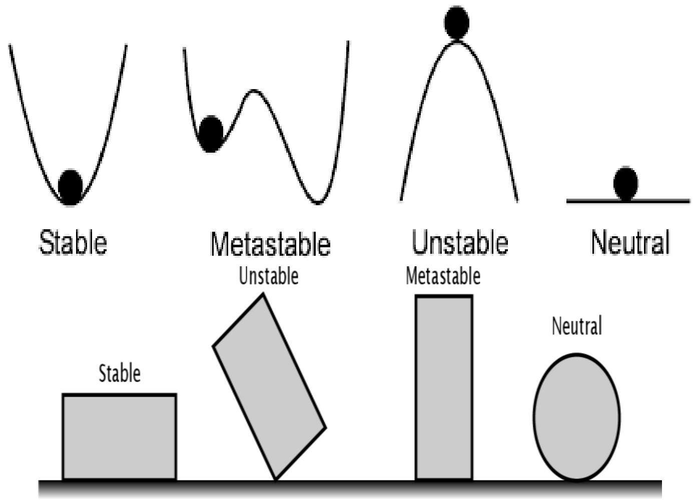


Figure: Stable, unstable, neutrally stable, and metastable equlilibria.

(-3-) - CIRCULAR MOTION: Angular velocity and Acceleration, Relation between Linear Velocity and Linear Acceleration, Angular Momentum, Conservation of Angular Momentum, Torque, Centripetal and Centrifugal forces with their applications (centrifugal), Motion of a vehicle in a curve and banking of roads.

Introduction

If you were to tire a small stone to the end of a string and whirl it above your head in a horizontal circle, you will feel the pull of the string on your hand. Since a taut string can only exerts a pulling forces on the objects at its ends. We can deduce that there is a pulling force (known as the *tension force*) exerted on the stone by the string. If the speed v of the stone moving around the circle (or orbit) is constant, we say that the stone is undergoing *uniform circular motion.*

3.1 − Angular velocity

There are numerous cases of objects moving in a curve about some fixed point. The earth and the moon revolve continuously round the sun, for example, and the rim of the balance-wheel of a watch moves to-and-fro ina circular path about the fixed axis of the wheel.

If the object moves from A to B so that the radius OA moves through an angle θ, its angular velocity, ω,about O is defined as the change of the angle per second. Thus if t is the time taken by the object to move from A to B,

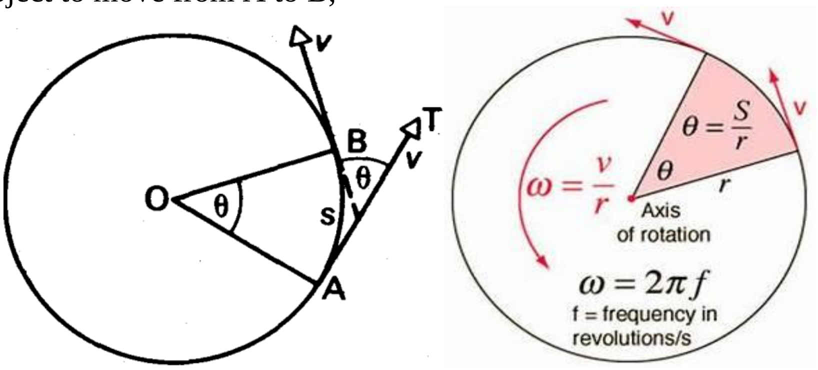


Fig: Circular Motion

$$\omega = \frac{\theta}{t}$$

Angular velocity is usually expressed in 'radian per second' (rad s^{-1}) from above,

$$\theta = \omega t$$

Which is analogous to formula 'distance = uniform velocity x time' for motion in a straight line. It will be noted that the time T to be describe the circle once, known as the period of the motion, is given by

$$T = \frac{2\pi}{\omega}$$

since $2\pi\, radians = 360^{\circ}$ by definition.

If s is the length of the arc AB, then $s/r = \theta$, by definition of an angle in radians.

$$\therefore s = r\theta.$$

Dividing by t, the time taken to move from A to B,

$$\therefore \frac{s}{t} = r\frac{\theta}{t}.$$

But s/t = the velocity, v, of the rotating object, and θ/t is the angular velocity.

$$\therefore v = r\omega$$

Angular velocity is the rate of velocity at which an object or a particle is rotating around a center or a specific point in a given time period. It is also known as rotational velocity. Angular velocity is measured in angle per unit time or radians per second (rad/s).

3.2− Angular Acceleration

When the angular velocity of a rotating objects changes, object undergoes angular acceleration (it has an *angular acceleration).* When you pedal your bicycle harder to make the wheels turn faster or apply the brakes to bring the wheels to a stop, you're giving the wheels an angular acceleration. You also impart an angular acceleration whenever you change the rotation speed of a piece of spinning machinery such as an automobile engine's crankshaft.

The defined analogously to linear acceleration:

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

The SI units of angular acceleration are rad/s^2 although we sometimes use other units such as rpm/s or rev/s^2. Angular acceleration has the same direction as angular velocity—clockwise (CW) or counter clockwise (CCW).

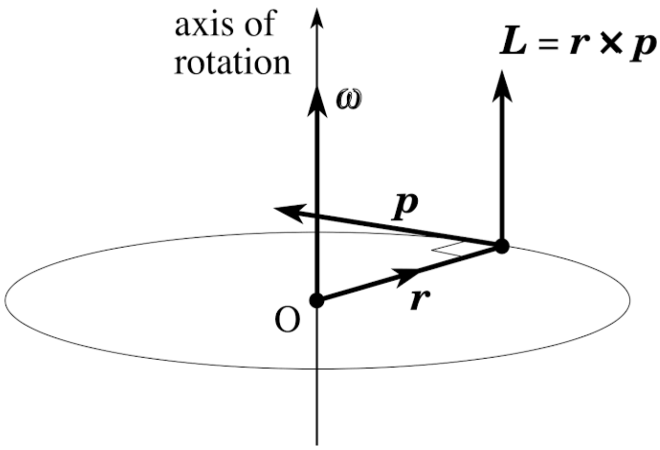
3.3− Equations for Uniformly Accelerated Angular Motion: are exactly analogous to those for uniformly accelerated linear motion. In the usual notation we have:

Linear	Angular
$v_{av} = \frac{1}{2}(v_i + v_f)$ $s = v_{av} t$ $v_f = v_i + at$ $v_f^2 = v_i^2 + 2as$ $s = v_i t + \frac{1}{2}at^2$	$\omega_{av} = \frac{1}{2}(\omega_i + \omega_f)$ $\theta = \omega_{av} t$ $\omega_f = \omega_i + \alpha t$ $\omega_f^2 = \omega_i^2 + 2\alpha\theta$ $\theta = \omega_i t + \frac{1}{2}\alpha t^2$

Take alone, the second of these equations is just the definition of average speed, so it is valid whether the acceleration is constant or not.

3.4− Angular Momentum

is a vector quantity that is a measure of the rotational momentum of a rotating body or system, that is equal in classical physics to the product of the angular velocity of the body or system and its moment of inertia with respect to the rotation axis, and that is directed along the rotation axis.



$$Angular\ momentum, \qquad L = r\ x\ p = r\ m\ v = m\ \omega\ r^2, \qquad recall\ v = \omega r$$

3.5− Conservation of Angular Momentum

The principle of conservation of angular momentum is useful for dealing with large rotating bodies such as the earth, as well as tiny, spinning particles such as electrons, protons, neutrons.

The earth is an object which rotates about an axis passing through its geographic north and south poles with a period of 1 day. If it is struck by meteorites, then since action and reaction are equal, no external couple acts on the earth and meteorites. Their total angular momentum is thus conserved.

Neglecting the angular momentum of the meteorites about the earth's axis before collision compared with that of the earth. Then, Angular momentum of earth plus meteorites after collision = angular momentum of earth before collision.

Hence the principle of angular momentum states that:

"The angular momentum of a system of particles around a point in a fixed inertial reference frame is conserved if there is no net external torque around that point".

Or

"The total angular momentum of a system remains constant provided no external torque acts on the system rigid or otherwise".

Mathematically we have that,

$$\frac{d\vec{L}}{dt} = 0$$

Or

$$\vec{L} = l_1 + l_2 + l_3 + l_4 + \cdots + l_N = constant$$

Note that the total angular momentum $\vec{L}$ is conserved. Any of the individual angular momenta can change as long as their sum remains constant. *This law is analogous to linear momentum being conserved when the external force on a system is zero*.

The angular momentum is constant, so that

$$L' = L \quad or \vec{L}_i = \vec{L}_f$$

or

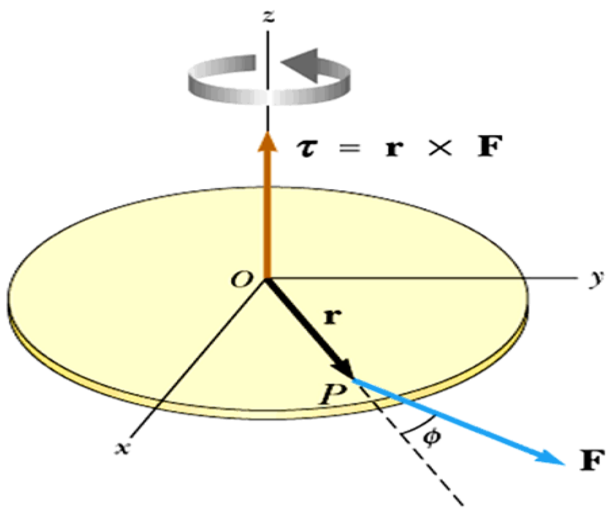
$$I'\omega' = I\omega,$$

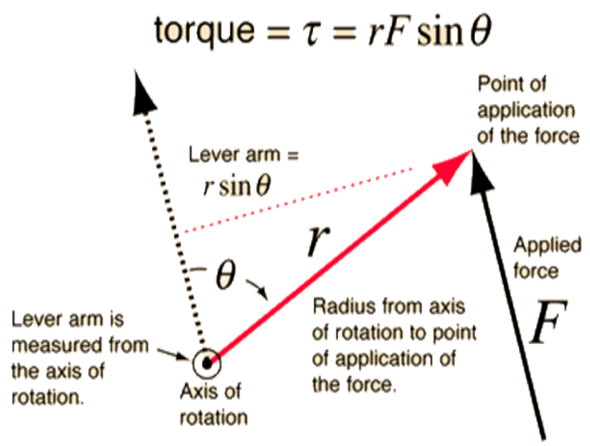
The solar system is an example of how conservation of angular momentum works in our universe. Our solar system was born from a huge cloud of gas and dust that initially had rotational energy. Gravitational forces caused the cloud to contract, and the rotation rate increased as a result of conservation of angular momentum. Another example of conservation of angular momentum is seen in an ice skater executing a spin. The net torque on her is very close to zero, because 1) there is relatively little friction between her skates and the ice, and 2) the friction is exerted very close to the pivot point.

3.6 − Torque

Torque is the tendency of a force to cause or change the rotational motion of a body. It is a twist or turning force on an object. Torque is calculated by multiplying force and distance. It is a vector quantity, meaning it has both a direction and a magnitude. Either the angular velocity for the moment of inertia of an object is changing, or both. *Also known as: moment or moment of force.*

The SI units of torque are newton-meters or N*m. Even though this is the same as Joules, torque isn't work or energy so should just be newton-meters. *Torque* is represented by the Greek letter tau: τ in calculations. When it is called moment of force, it is represented by M.





3.7– The Centripetal and Centrifugal forces

3.7.1– Centripetal Force (F_c)

Is the force that keeps an object moving with a uniform speed along a circular path. is the force that must act on a mass m moving in a circular path of radius r to give it the required centripetal accelerationv^2/r. From $F = ma$, we have

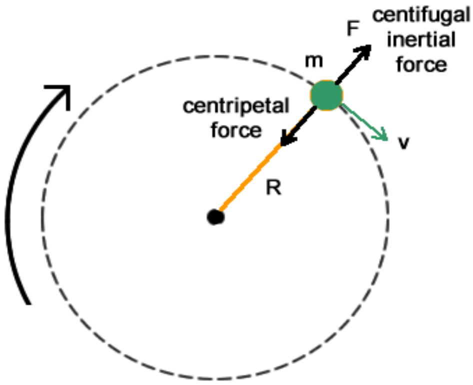
$$F_C = \frac{mv^2}{r} = mr\omega^2$$

Where F_C is directed toward the centre of the circular path. Centripetal force is not a new kind of force; it's just the name given to whatever force (be it gravity, the tension in a string, magnetism, friction, etc.) that causes an object to move (off it's straight-line inertial path) along an arc. ***Examples of centripetal force*** (1) The orbiting motion of the Moon around the Earth: Centripetal force is the gravitational force exerted by the earth on the Moon. (2) A bicycle turning around a corner on a horizontal rough surface: Centripetal force is given by the frictional force exerted by the ground on the wheels of the bicycle towards the centre of the circular path (3) The orbiting motion of the electrons around an atomic nucleus: Centripetal force is the electrostatic force exerted by the nucleus on the electrons.

3.7.2 – Centrifugal force

(Latin for "centre fleeing") describes the tendency of an object following a curved path to fly outwards, away from the centre of the curve. It's not really a force; it results from inertia — the tendency of an object to resist any change in its state of rest or motion.

The centrifugal force is directed outwards; in the same direction as the velocity of the object. For circular motion, the velocity at any given point in time is at a tangent to the arc of movement.



Examples of centrifugal forceis when a stone tied to a string is whirled in a circle, the force acting on the passengers outwards in a car when the car is taking a turn, children pushed out on a roundabout. The equation of centrifugal force is the same with that of centripetal force but opposite in sign.

$$F_C = -\frac{mv^2}{r} = -mr\omega^2$$

Example 33: A 33-kg rock swings in a circle of radius 5 m. If its constant speed is 8 m/s, what is the centripetal acceleration and centripetal force?

SOLUTION

$$a_c = \frac{v^2}{R} \qquad m = 3kg \qquad R = 5m; \quad v = 8m/s$$

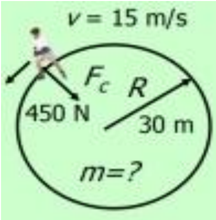
$$a_c = \frac{(8\,m/s)^2}{5\,m} = 12.8\,m/s^2$$

$$F_c = ma_c = \frac{mv^2}{R}; \;\; F_c = (3kg)(12.8\,m/s^2) = 38.4N$$

Example 34: A skater moves with 15 m/s in a in a circle of radius 30 m. The ice exerts a central force of 450 N. What is the mass of the skater?

SOLUTION

$$F_c = \frac{mv^2}{R}; \ m = \frac{F_c R}{v^2} = \frac{(450\ N)(30\ m)}{(15\ m/s)^2}$$



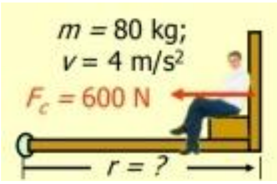
$$m = 60.0\ kg$$

Example 35: The wall exerts a 600 N force on an 80-kg person moving at 4 m/s on a circular platform. What is the radius of the circular path?

SOLUTION

Applying Newton's *2nd* law for circular motion

$$F = \frac{mv^2}{r}; \quad r = \frac{mv^2}{F} = \frac{(80\ kg)(4\ m/s)^2}{600\ N}$$



$$r = 2.13\ m$$

3.7.3– Applications

Knowledge of centrifugal and centripetal forces can be applied to many everyday problems. For example, it is used when designing roads to prevent skidding and improve traction on curves and access ramps. It also allowed for the invention of the centrifuge, which separates particles suspended in fluid by spinning test tubes at high speeds. Banking of the roads, washing machine dryer, the cream separator are some examples of centripetal and centrifugal force.

1.0 ROTATIONAL MOTION

Rotational motion is a kind of motion in which the body follows a curved path. Examples are:

1. The rotation of the earth about its axis
2. The rotation of worm drive over worm gear
3. The motion of the blades of hand blender
4. The motion of a wheel of a car or train.

1.1MOMENT OF INERTIA, I

The moment of inertia (I) of a body is a measure of the rotational inertia of the body. If an object that is free to rotate about an axis is difficult to set into rotation, its moment of inertia about that axis is large. An object with small I have little rotational inertia.

If a body is considered to be made up of tiny masses m_1, m_2, m_3, ……, at respective distances r_1, r_2, r_3, ……, from its axis. Its moment of inertia from the axis is …

$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots\ldots\ldots$

$I = \sum m_i\, r_i^2$

The units of moments of inertia, I are kgm^2.

1.2RADIUS OF GYRATION

This is defined as the distance a point mass M must be from the axis if the point mass is to have the same moment of inertia I as the object. The radius of gyration for an object about an axis is given by the relation ….

$I = Mk^2$

$k = \sqrt{\frac{I}{M}}$

Where I is the moment of inertia of the object, M is the total mass of the object, and k is called the radius of gyration.

Therefore, the moment of inertia $\sum mr^2$ is sometimes written as Mk^2.

Let us discuss the moment of inertia of these three objects

1. **MOMENT OF INERTIA OF UNIFORM ROD:**

   (i) About axis through middle: The moment of inertia of a small element δx about an axis PQ through its centre O perpendicular to the length = $(\frac{\delta x}{l} M)\, x^2$, where l is the length of the rod, M is its mass, and x is the distance of the small element from O, figure 1.1.∴

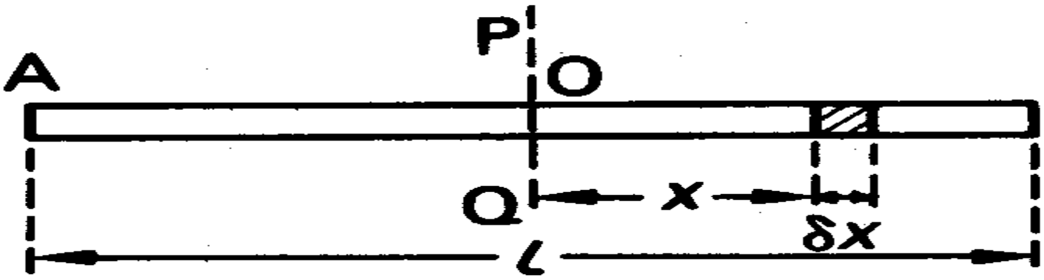


Figure 1.1: Moment of inertia of a uniform rod

∴ The moment of inertia, I, = $2\int_0^{1/2}(\frac{dx}{l}M)x^2$,

$= \frac{2M}{l}\int_0^{1/2} x^2 \mathrm{d}x = \frac{Ml^2}{12}$……………………..(1)

(ii) About the axis through one end, A: In this case, measuring distance x from A instead of O, the moment of inertia I is

Moment of inertia, I, = $\int_0^{l}(\frac{dx}{l}M)x^2$,

$= \frac{M}{l}\int_0^{l} x^2 \mathrm{d}x = \frac{Ml^2}{3}$…………………….(2)

Example (1)

Suppose a rod has a mass of 60g and a length of 20cm, calculate the moment of inertia of the rod about an axis through middle and about axis through one end.

Solution: M = 60g = 60/100 = 6×10^{-2}kg, l = 20cm = 20/100 = 0.2m

The moment of inertia about axis through middle, I = $\frac{Ml^2}{12}$

I = $\frac{6\times10^{-2}\times(0.2)^2}{12}$ = 2×10^{-4}kgm^2.

The moment of inertia about axis through one end = $\frac{Ml^2}{3} = \frac{6\times10^{-2}\times(0.2)^2}{3}$

I = 8×10^{-4}kgm^2.

2. MOMENT OF INERTIA OF A RING:

Every element of the ring is the same distance from the centre. Hence, the moment of inertia about an axis through the centre perpendicular to the plane of the ring = Ma^2, where M is the mass of the ring and a is its radius.

3. MOMENT OF INERTIA OF A CIRCULAR DISC:

Consider the moment of inertia of a circular disc about an axis through its centre perpendicular to its plane, figure 1.2.

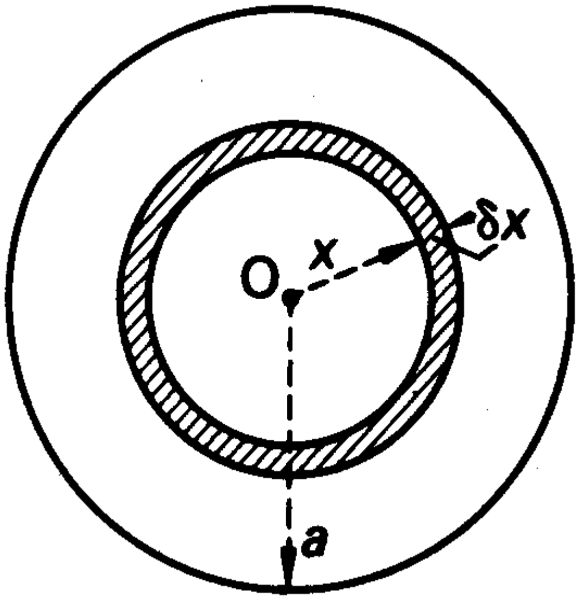


Figure 1.2: Moment of inertia of a circular disc

If we take a small ring of the disc enclosed between radii x and $x+\delta x$, its mass = $\frac{2\pi x\delta x}{\pi a^2}$ M, where a is the radius of the disc and M is its mass. Each element of the ring is distant x from the centre, and hence the moment of inertia of the ring about the axis through O = $(\frac{2\pi x\delta x}{\pi a^2}M)x^2$.

∴ The moment of inertia of the whole disc $=\int_0^a Mx^2\,\frac{2\pi x dx}{\pi a^2}$

$= \frac{2\pi M}{\pi a^2}\int_0^a x^3\ dx$

$= \frac{Ma^2}{2}$ …………….(3)

Example (2)

If a ring and a disc weighs 60g and has a radius of 10cm, calculates its moment of inertia of both the ring and the disc.

Solution: m = 60g = $6\text{x}10^{-2}$kg

a = 10cm = 0.1m

Moment of inertia of the ring =Ma^2

$= 6x10^{-2}$ x $(0.1)^2 = 0.06x0.01 = 6x10^{-4}kgm^2$.

Moment of inertia (I) of the disc = $\frac{Ma^2}{2}$

$= \frac{6x10^{-4}}{2} = 3x10^{-4}kgm^2$.

Example (3)

Consider a disc of mass 100g and radius 10cm is rotating freely about axis O through its centre at 40rpm. Calculate the moment of inertia and angular momentum of the disc.

Solution: m = 100g = 0.1kg, a = 10cm = 0.1m

The moment of inertia, I, of a disc = $\frac{Ma^2}{2} = \frac{0.1x(0.1)^2}{2}$

$$I = 5x10^{-4}kgm^2.$$

And angular momentum = Iw,

But w is given in rev. per min., so we need to convert it to rad/s.

$w = (\frac{40rpm}{1min})(\frac{1}{60s})(\frac{2\pi rad}{1rev}) = 4.2rad/s$

Hence, angular momentum (A. M.) = $5x10^{-4}x4.2$

$$A.M. = 21x10^{-4}kgm^2/s$$

1.6 TORQUE

A torque τ acting on a body of moment of inertia I produces in it an angular acceleration α given by

$\tau = I\,\alpha$

Here τ is in Nm, I is in kgm^2 and α must be in rad/s^2.

1.7. KINETIC ENERGY OF ROTATION

The kinetic energy of rotation (KE_r) of a mass whose moment of inertia about an axis is I and which is rotating about the axis with angular velocity w, is given by

$K.E_r = \frac{1}{2}Iw^2$

Where $K.E_r$ is the kinetic energy in Joules.

Example (4): A wheel of mass 6000g and radius of gyration 40cm is rotating at 300rpm. Find its moment of inertia and its rotational kinetic energy.

Soln: I = 0.96kgm^2

K. E_r = 473J = 0.473kJ

Example (5): An airplane propeller has a mass of 70000g and a radius of gyration of 75cm. Calculate its moment of inertia. How large a torque is needed to give it an angular acceleration of 4.0rev/s^2.

Soln: I = 39kgm^2

τ= 980Nm = 0.98kNm.

GRAVITATION

Sir Isaac Newton deduced the law of universal gravitation in 1686 from speculations concerning the fall of an apple toward the earth. His proposal was, that the gravitational attraction of the sun for the planets is the source of the centripetal force which maintains the orbital motion of the planets round the sun. Newton also affirms that this was similar to the attraction of the earth for the apple. Thus, gravity (the attraction the earth has for an object) which you are already familiar with, was a particular case of gravitation. According to Newton also, there is a gravitational force between all objects in the universe. It is this universal gravitational force that is responsible for the orbital motion of the heavenly bodies.

Newton's law of universal gravitation, states that *Every particle of matter in the universe attracts other particles with a force which is directly proportional to the product of their masses and inversely proportional to the square of their distances apart.* This means there is gravitational attraction between you and any object in the room where you are. The gravitational force attraction between two bodies is given by

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = \frac{G m_1 m_2}{r^2}$$

Where F is the magnitude of the gravitational force on either particle, $m_1 and\ m_2$ are their masses, r is the distance between them, and G is a fundamental physical constant called the universal gravitational constant.It is assumed to have the same value everywhere for all matter. (6.67 X 10^{-11}Nm^{2}kg^{-2})

For a system of particles, we can find the gravitational force on any one of them from the others by using the principle of superposition. This is a general principle that says a net effect is the sum of the individual effects. This means that we first compute the individual gravitational forces that act on selected particle due to each of the other particles and then find the net force by adding these forces vectorially, as it is done when adding forces. For *n* interacting particles, we can write the principle of superposition for the gravitational forces on particle 1 as

$$\vec{F}_{1net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{13} + \ldots\ldots\ldots\ldots + \vec{F}_{1n}$$

Where $\vec{F}_{1net}$ is the net force on particle 1 due to the other particles.This equation can be expressed as a vector sum:

$$\vec{F}_{1net} = \sum_{i=2}^{n} \vec{F}_{1i}$$

Example

1. The mass of one of the small spheres of a Cavendish balance is 10 g, the mass of the nearest large sphere is 500g, and the center-to-center distance between them is 0.0500 m. Find the gravitational force on each sphere due to the other. (Ans is 1.33×10^{-10}N)
2. Calculate the gravitational force of attraction between the Earth and a 70 kg man if he is standing at a sea level,6.38×10^{6} m from the earth's centre. (Ans is 685N)

Mass and weight

The weight of a body is the total gravitational force exerted on the body by all other bodies in the universe. When the body is near the surface of the earth, we can neglect all other gravitational forces and consider the weight as just the earth's gravitational attraction. Similarly, if an object is on the surface of the moon or any other planet, its weight will be from the gravitational attraction of the moon or the planet. If the earth is assumed to be a spherically symmetric body with radius r and mass M, the weight w of a small body of mass m at the earth's surface (a distance r from its center) is given by

$$w = F = \frac{GMm}{r^2}$$

Then, when the body is allowed to fall freely towards the centre of the earth, the force accelerating it is its weight, w and the acceleration produced by this force is that due to gravity known as acceleration due to gravity denoted by g. From Newton's second law, force that causes the acceleration g of free fall is given by

$$w = mg$$

The magnitude of the gravitational force from Earth on a particle of mass m, located outside Earth a distance r from Earth's center, is then given by

$$w = \frac{GMm}{r^2}$$

Equating the two equations we get

$$g = \frac{GM}{r^2}$$

where r = radius of the earth, G = the universal gravitational constant and g is the acceleration due to gravity.

This shows that the acceleration due to gravity is the same for all bodies or objects. From the above equation, mas of the earth is then

$$M = \frac{r^2 g}{G}$$

Example

A robotic lander with an earth weight of 3430 N is sent to Mars, which has radius$r_m = 3.40 \times 10^6 m$ and mass $M = 6.42 \times 10^{23} m$. Find the weight of the lander on the Martian surface and the acceleration there due to gravity.

Gravitational potential`

When a body of mass (m) is moved from infinity to a point inside the gravitational influence of a source mass without accelerating it, the amount of work done in displacing it into the source field is stored in the form of potential energy, this is known as gravitational potential energy.

To find the expression for the gravitational potential energy, we consider a body of mass m outside the earth, and first compute the work done W by the gravitational force when the body moves directly away from or toward the center of the earth from r to ∞. This work is given by

$$W = \int_r^\infty \vec{F}(r) \cdot \mathrm{d}\vec{r}$$

The integral contains the scalar (or dot) product of the force $\vec{F}$(r) and the differential displacement vector d$\vec{r}$ along the ball's path. We can expand that product as

$$\vec{F}(r) \cdot \mathrm{d}\vec{r} = F(r) dr cos\varphi$$

Where φ is the angle between the directions of $\vec{F}(r)$ and d$\vec{r}$ which is 180°. The force is given by:$\vec{F}(r) = \frac{GMm}{r^2}$. Therefore,

$$\vec{F}(r) \cdot \mathrm{d}\vec{r} = -\frac{GMm}{r^2} dr$$

Where M is Earth's mass and m is the mass of the ball. Substituting the above equation in the integral we get

$$W = -GMm\int_r^{\infty} \frac{1}{r^2}dr$$

$$W = -\frac{GMm}{r}$$

where W is the work required to move the ball from point P (at distance r) to infinity. But the work done in terms of potential energies is

$$\Delta U = -W$$

$$U_{\infty} - U = -W$$

The potential energy U at infinity is zero, therefore $U = W$

$$U = -\frac{GMm}{r}$$

Potential Energy of a System of particles

If a system contains more than two particles, its total gravitational potential energy U is the sum of terms representing the potential energies of all the pairs. (we consider each pair of particles in turn, calculate the gravitational potential energy of that pair with as if the other particles were not there, and then algebraically sum the results). As an example, for three particles of masses m_1, m_2 and m_3, the potential energy can be written as

$$U = -(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_{13}}{r_{13}} + \frac{Gm_2m_3}{r_{23}})$$

Artificial satellite

Artificial satellites orbiting the earth are a familiar part of modern technology. But how do they stay in orbit, and what determines the properties of their orbits? We can use Newton's laws and the law of gravitation to provide the answers.

Satellites: Circular Orbits

A circular orbit is the simplest case. It is also an important case since many artificial satellites have nearly circular orbits and the orbits of the planets around the sun are also circular. The only force acting on a satellite in circular orbit around the earth is the earth's gravitational attraction, which is directed toward the center of the earth and hence toward the center of the orbit. This means that the satellite is in Uniform circular motion and its speed is constant. The satellite isn't falling toward the earth; rather, it's constantly falling around the earth.

Orbital velocity

To find the velocity *v* of a satellite in a circular orbit, let r be the radius of the orbit measured from the center of the earth; a be the acceleration of the satellite with magnitude $\frac{v^2}{r}$ and is always directed toward the center of the circle. By the law of gravitation, the net force (gravitational force) on the satellite of mass m has magnitude$F = \frac{GMm}{r^2}$.

From Newton's second law,$F = ma$, Substituting $a = \frac{v^2}{r}$ in, we have$F = m\frac{v^2}{r}$.

Equating the two equations of force we get

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Solving for *v* we have

$$v = \sqrt{\frac{GM}{r}}$$

The above equation is called the orbital velocity and it is the velocity with which a satellite revolves round the earth. This shows that the motion of a satellite round the earth does not depend on its mass.

Energy of a circular orbit

The total energy of a satellite of mass *m* moving in a circular orbit with radius ris the sum of both kinetic energy K and potential energy *U* given by

$$E = K + U$$

Where $U = -\frac{GMm}{r}$ and $K = \frac{1}{2}mv^2$. v is the orbital velocity and is equal to $\sqrt{\frac{GM}{r}}$

$$E = \frac{1}{2}m\left(\frac{GM}{r}\right) - \frac{GMm}{r}$$

$$E = -\frac{GMm}{2r}$$

This means that the total energy is just one-half the potential energy fora circular orbit.

Escape velocity

Escape velocity is the minimum velocity with which a body must be projected vertically upwards from the earth's surface so that it just crosses the earth's gravitational field. Escape velocity does not depend upon the mass or shape or size of the body as well as the direction of projection of the body.

Consider a projectile of mass m, leaving the surface of a planet (or some other astronomical body or system) with escape speed v. The projectile has a kinetic energy K given by $\frac{1}{2}mv^2$ and a potential energy U given by $-\frac{GMm}{r}$, where M is mass of the planet and r is its radius. When the projectile reaches infinity, it stops and thus has no kinetic energy. It also has no potential energy because an infinite separation between two bodies is our zero-potential-energy configuration. Therefore, Its total energy at infinity is zero.

$$K + U = 0$$

$$\frac{1}{2}mv^2 + (-\frac{GMm}{r}) = 0$$

$$v = \sqrt{\frac{2GM}{r}}$$

But $g = \frac{GM}{r^2}$Substituting, we get

$$v = \sqrt{2gr}$$

The equation above gives the expression for the velocity of escape.

Example

1. Assuming you wish to put a 1000-kg satellite into a circular orbit 300 km above the earth's surface. (a) What speed, period, and radial acceleration will it have? (b) How much work must be done to the satellite to put it in orbit? (c) How much additional work would have to be done to make the satellite escape the earth?
2. A particle of mass m moves in a circular orbit of radius r under the influence of the gravitational force due to a point object of mass M>>m. Calculate the total energy of the particle as a function of r.

Weightlessness

It is a situation in which the effective weight of the body becomes zero, Weightlessness is achieved

(i) during freely falling under gravity

(ii) inside a space craft or satellite

(iii) at the centre of the earth

(iv) when a body is lying in a freely falling lift.

Variation of acceleration due to gravity with height

Let us assume that g' is the acceleration due to gravity at a distance a from the centre of the earth where a >r. r is the radius of the earth. Then g'can be written as

$$g' = \frac{GM}{a^2}$$

But we know that

$$g = \frac{GM}{r^2}$$

Dividing we have

$$\frac{g'}{g} = \frac{r^2}{a^2}$$

$$g' = \frac{r^2}{a^2} g$$

From this equation we conclude that, above the earth's surface, the acceleration due to gravity g' varies inversely as the square of the distance, a between the object and the center of the earth. Also r and g are constant and g' decreases with height.

At height h above the earth's surface, a = r + h,

$$g' = \frac{r^2}{(r+h)^2} g = \frac{1}{\left(1 + \frac{h}{r}\right)^2} g$$

$$g' = \left(1 + \frac{h}{r}\right)^{-2} g$$

We see that if h is very small compared to r (where r is 6400km) we neglect the powers of higher than the first

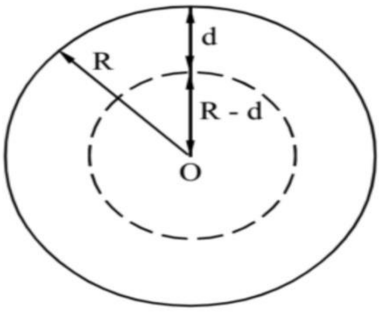
Hence,

$$g' = \left(1 - \frac{2h}{r}\right) g$$

$$g - g' = \frac{2h}{r} g$$

$g - g'$= reduction in acceleration due to gravity

Variation of g with depth



The acceleration due to gravity on the earth surface is given by

$$g = \frac{GM}{r^2}$$

Let ρ be the density of material of the earth, then

$$mass = volume \times density$$

$$M = \frac{3}{4}\pi r^3 \rho$$

Substituting M we have

$$g = \frac{G}{r^2}\frac{3\pi r^3 \rho}{4}$$

$$g = \frac{3}{4}\ \pi G r \rho \qquad (a)$$

Let the body be taken to a depth d below the earth surface, the distance from the centre of the earth will be (r-d). Assuming the earth has a uniform density, the acceleration due to gravity at depth d below the surface of the earth will be given by

$$g_d = \frac{3}{4}\ \pi G(r-d)\rho \qquad (b)$$

Dividing (b) by (a) we have

$$\frac{g_d}{g} = \frac{r-d}{r}$$

$$g_d = \left[1 - \frac{d}{r}\right] g$$

This is the expression for the acceleration due to gravity at d below the surface of the earth and it shows that g increases for all depths below the surface of the earth.