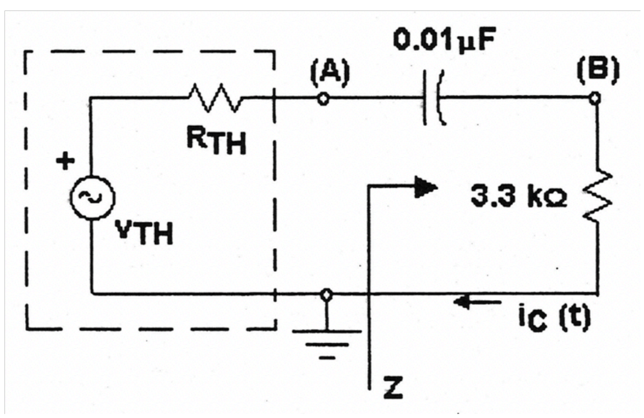
Lab#6 Sinusoidal-Steady-State Response of RC & RL Circuits

The focus of this lab is to study the frequency response of the simple RC and RL circuits. When given a sinusoidal AC source with a frequency, $\omega = 2\pi f$, the impedance of a capacitor and an inductor becomes:

Capacitor Impedance	Inductor Impedance
$Z_C = \frac{1}{j\omega C} = -\frac{j}{\omega C}$	$Z_L = j\omega L$
$X_C = -\frac{1}{\omega C}$	$X_L = \omega L$

NOTE: Triangle wave and square wave are multi-component signals. They have more than one frequency. In this study, we will use a sinusoidal wave since it has only one frequency.



Impedance

Since the impedance Z of the series circuit contains a resistor and a capacitor, thus

$$Z = R + jX_C = 3.3k - \frac{j}{\omega C} = |Z|\angle\theta_Z^o$$

Given RTH = 600Ω, the total impedance of the circuit can be written in phaser form as:

$$Z_{Total} = RTH + R + jX_C = 3.9k - \frac{j}{\omega C} = |Z_{Total}|\angle\theta^o$$

where $|Z_{Total}|$ is the magnitude of the total impedance and $\theta^o = \tan^{-1}(-\frac{1}{\omega R_{Total} C})$ is its phase angle.

Voltage Source

The **phaser form** is expressed as a function of cosine. If the voltage source is given as a sine function, first convert it into cosine. For $v_{TH} = 5\sin 2\pi1000t = 5\cos(2\pi1000t - 90) \rightarrow$ $|V|\angle\phi^o = 5\angle{-90^o}$ V with $\omega = 2\pi1000$ rad/s and magnitude.

Current

Since this is a series circuit, the theoretical estimation of the current passing through the capacitor is with $\omega = 2\pi1000$ rad/s is

$$I_C = \frac{|V|\angle\phi^o}{|Z_{Total}|\angle\theta^o} = \frac{5\angle{-90}}{16.4k\angle{-76^o}} = 0.3\angle{-24^o} \text{ mA}$$

That means the current has a **magnitude |I$_C$| of 0.3 mA** and a phase angle of -24°. In other words, the sinusoidal-steady-state

$$i_C(t) = 0.3\cos(2\pi1000t - 24) \text{ mA} \quad \text{or} \quad i_C(t) = 0.3\sin(2\pi1000t + 76) \text{ mA.}$$

Inductor

If you replace the capacitor with a practical inductor where its resistive element is non-zero

$$Z_L = R_L + jX_L = 700 + j\omega L$$

Thus,

$$Z = 3.3k + Z_L = 4k + j\omega L$$

Given RTH = 600Ω, the total impedance of the circuit can be written in phaser form as:

$$Z_{Total} = RTH + R + Z_L = 4.6k + j\omega L = |Z_{Total}|\angle\theta^o$$

where $|Z_{Total}|$ is the magnitude of the impedance and $\theta^o = \tan^{-1}(\frac{\omega L}{R_{Total}})$ is its phase angle.

Similarly, you can use the phaser from to estimate the inductor current.

Summary

From the impedance definition for the capacitor (-ve imaginary value) and inductor (+ve imaginary value), it can be concluded that the current, $I = \frac{V\angle\phi^o}{Z\angle\theta^o} = \left|\frac{V}{Z}\right|\angle(\phi^o - \theta^o)$ of a RC or RL circuit is leading or lagging the voltage.

	RC circuit	RL circuit
Impedance	-ve imaginary	+ve imaginary
θ	-ve phase	+ve phase
Current Phase	Leads (+phase shift)	Lags (-phase shift)

Multisim

Please follow the steps below to setup the simulation environment and build your circuits

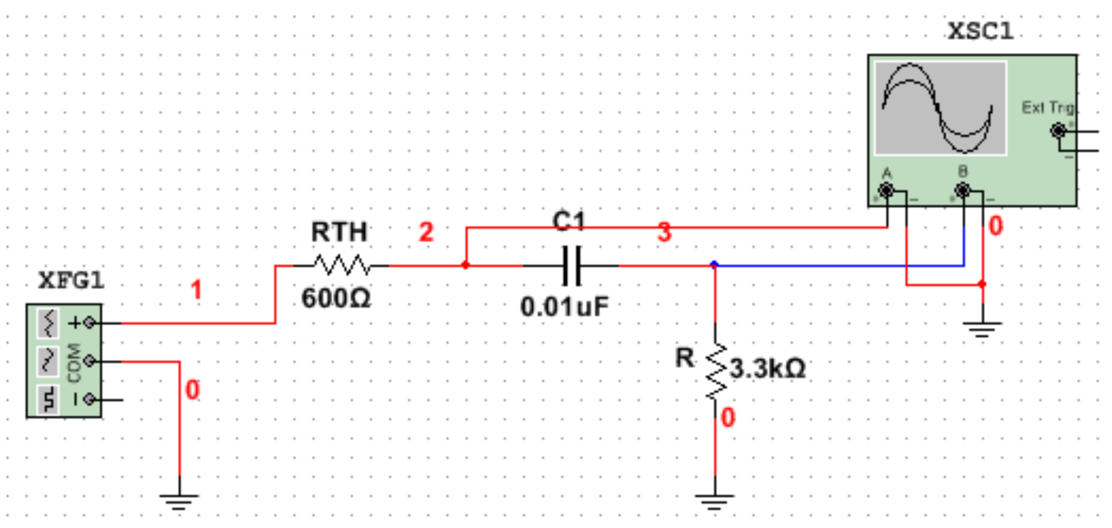
1. Setting up **Interactive simulation** environment
   - Set **End time (TSTOP): 0.1 s**
   - Save the setting
2. Use the following components to build your circuits:
   - Resistor: RESISTOR_RATED
   - Capacitor: CAPACITOR_RATED
   - Inductor: INDUCTOR_RATED
   - Ground: GROUND
3. You only need to use:
   - Function generator as voltage source
   - Oscilloscope to measure voltage signals

Lab Procedure

Part I: RC-circuit

NOTE: In a physical lab, you will check your AC power using the oscilloscope first before connecting to the circuit. This step is not needed in Multisim, since what you set is what you get.

Step 1: Construct the circuit with Channel A connected to node A to measure voltage across Z and Channel B to node B to measure current. The current amplitude with, $|I_C| = \frac{|V_B|}{3.3k}$.



Set the function generator to provide a **sinusoidal** wave at **500 H**z frequency with **5 V$_P$**.

Use default oscilloscope settings with the following changes:

Timebase	Channel A	Channel B	Trigger
Scale: 200 µs/Div	Scale: 2 V/Div	Scale: 200 mV/Div	Use Single to display 1 duty cycle.

NOTE: Every time you change the frequency, you will need to adjust the Timebase and Channel so you can see the voltage signal better.

Step 2: Measure voltage peak value using the T1 (Channel A Peak) and T2 (Channel B Peak):

- $|V_A| \approx 4.95$ V
- $|V_B| \approx 515$ mV and $I_C \approx \frac{|V_B|}{3.3k} = \frac{515m}{3.3k} = 0.16$ mA.

Step 3: Measure impedance Z, $|Z| = \frac{|V_A|}{|I_C|}$, and $\theta_Z^o = \angle\phi_A - \phi_B$. Channel B is the current phase and impedance is V/I, thus its phase is the voltage phase – current phase.

Use the cursors to measure DeltaT = |T2-T1|

$$\theta_Z^o = DeltaT * Freq * 360$$

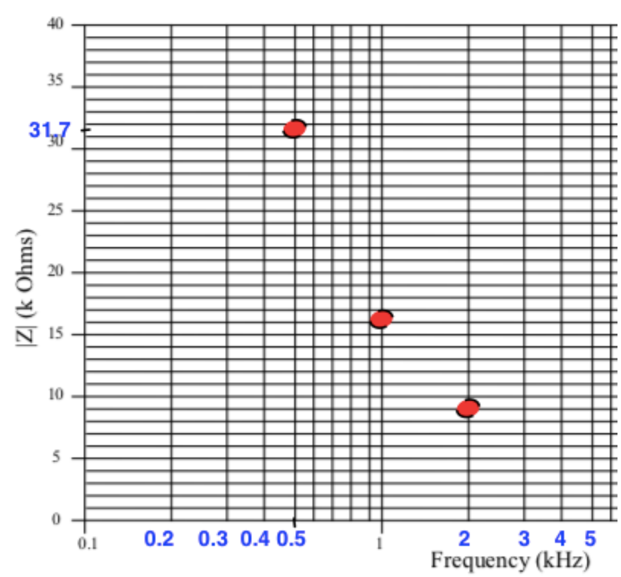
NOTE: Phase angle is always negative for RC circuits

Step 4: Repeat the steps to complete Table (6.3). You might want to use an Excel or Google spreadsheet with the following formula to speed up the process:

	$|V_A|$ V	$|V_B|$ V	$|I_C|$ mA	$|Z|$ kΩ	DeltaT	θ_Z^o
Equation	Measured	Measured	**=$|V_B|$/3.3k**	**=$|V_A|$/$|I_C|$**	Measured	**=DeltaT*Freq*360**

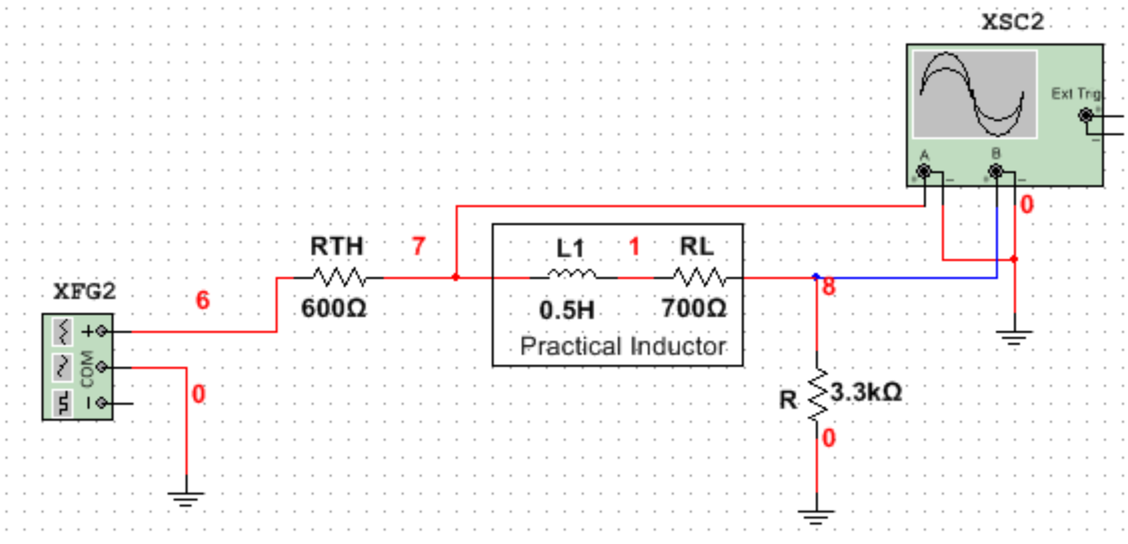
Step 5: Plotting semi-log (only one axis is in log scale) graphs in Graph (6.1). Example:

Freq (kHz)	$|Z|$ kΩ
0.5	31.72
1	16.29
2	8.42



Part II: RL-Circuit

Step 6: In order to compare with the pre-lab, we will also add a 700 Ω resistor to represent the practical inductor and complete Table (6.4).



Start with 0.1 kHz sinusoidal wave with the following oscilloscope setting:

Timebase	Channel A	Channel B	Trigger
Scale: 1 ms/Div	Scale: 2 V/Div	Scale: 2 V/Div	Use Single to display 1 duty cycle.

Step 7: Plot the semi-log graphs in Graph (6.2)

NOTE: Please change the vertical axis for the Phase (deg) graph to the range of [0,90°]