$A_1: \; x_1(t) = \cos\left(\frac{3\pi}{10}t\right) + \frac{1}{2}\cos\left(\frac{\pi}{10}t\right)$

Is it periodic?

$\frac{3\pi}{10} \times \frac{10}{\pi} = 3$ Yes $\qquad w_0 = \frac{\pi}{10}$

$x_1(t) = \sum_{-\infty}^{\infty} D_n e^{jw_0 n t}$

$\cos\theta = \frac{1}{2}e^{jw} + \frac{1}{2}e^{-jw}$

$x_1(t) = \cos\left(\frac{3\pi}{10}t\right) + \frac{1}{2}\cos\left(\frac{\pi}{10}t\right)$

$x_1(t) = \frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{4}e^{j\frac{\pi}{10}t} + \frac{1}{4}e^{-j\frac{\pi}{10}t}$

$\frac{3}{10}\pi = \frac{\pi}{10}n$	$\frac{-3}{10}\pi = \frac{\pi}{10}n$	$\frac{\pi}{10} = n\frac{\pi}{10}$	$-\frac{\pi}{10} = n\frac{\pi}{10}$
$n=3$	$n=-3$	$n=1$	$n=-1$
$D_3 = \frac{1}{2}$	$D_{-3} = \frac{1}{2}$	$D_1 = \frac{1}{4}$	$D_{-1} = \frac{1}{4}$

$$\boxed{D_3 = D_{-3} = \frac{1}{2}, \quad D_1 = D_{-1} = \frac{1}{4}}$$

$$D_n = \frac{1}{20}\int_{-10}^{10}\left[\frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{4}e^{j\frac{\pi}{10}t} + \frac{1}{4}e^{-j\frac{\pi}{10}t}\right]e^{-jn\frac{\pi}{10}t}\,dt$$

$$D_n = \frac{1}{20}\int_{-10}^{10} \frac{1}{2}e^{j(3-n)\frac{\pi}{10}t} + \frac{1}{2}e^{-j(3+n)\frac{\pi}{10}t} + \frac{1}{4}e^{j(1-n)\frac{\pi}{10}t} + \frac{1}{4}e^{-j(1-n)\frac{\pi}{10}t}\,dt$$

$$D_n = \frac{1}{20}\left[\frac{10}{2j(3-n)\pi}\left(e^{j(3-n)\pi} - e^{-j(3-n)\pi}\right) + \frac{10}{2j(3+n)}\left(e^{j(3+n)\pi} - e^{-j(3+n)\pi}\right)\right.$$
$$+ \frac{10}{4j(1-n)\pi}\left(e^{j(1-n)\pi} - e^{-j(1-n)\pi}\right)$$
$$\left. + \frac{10}{4j(1+n)\pi}\left(e^{j(1+n)\pi} - e^{-j(1+n)\pi}\right)\right]$$

$$\boxed{\begin{aligned} D_n = \frac{1}{2}\Big[& \operatorname{sinc}((3-n)\pi) + \operatorname{sinc}((3+n)\pi) \\ & + \frac{1}{2}\sin((1-n)\pi) + \frac{1}{2}\operatorname{sinc}((1+n)\pi) \Big] \end{aligned}}$$

A_2:

$x_2(t) =$

$T_0 = 20 \quad \omega_0 = \frac{2\pi}{20} = \frac{\pi}{10}$

$$D_n = \frac{1}{20}\int_{-5}^{5} (1)\, e^{-jn\pi/10\, t}\, dt = \frac{1}{20}\left[\frac{-10}{jn\pi} e^{-jn\frac{\pi}{10}t}\right]_{-5}^{5}$$

$$D_n = \frac{1}{20}\left[\frac{-10}{jn\pi} e^{-jn\pi/2} + \frac{10}{jn\pi} e^{jn\pi/2}\right] = \frac{1}{n\pi}\sin\left(\frac{\pi n}{2}\right)$$

$$\boxed{D_n = \frac{\sin\left(\frac{\pi n}{2}\right)}{n\pi}}$$

$x_3(t)$:

$T_0 = 40 \quad \omega_0 = \frac{2\pi}{40} = \frac{\pi}{20}$

$$D_n = \frac{1}{40}\int_{-5}^{5} e^{-jn\frac{\pi}{20}t}\, dt$$

$$D_n = \frac{1}{40}\left[\frac{-20}{jn\pi} e^{-jn\pi/20\, t}\right]_{-5}^{5} = \frac{1}{40}\left[\frac{-20}{jn\pi} e^{-jn\pi/4} + \frac{20}{jn\pi} e^{jn\pi/4}\right]$$

$$\boxed{D_n = \frac{\sin\left(\frac{n\pi}{4}\right)}{n\pi}}$$

B_1 : x_1 : $\frac{\text{HCF of top}}{\text{LCM of bottom}} = \frac{\pi}{10}$

x_2 : $T_0 = 20$ $\quad w_0 = \frac{2\pi}{20} = \frac{\pi}{10}$

x_3 : $T_0 = 40$ $\quad w_0 = \frac{2\pi}{40} = \frac{\pi}{20}$