The Multi-Industry Trade Model with Scale Effects

International Trade (PhD), Fall 2025

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Introduction and Roadmap

- This lecture reviews quantitative trade models with industry-level scale effects.
- We consider on a *generalized* multi-industry Krugman model that
  - nests the multi-industry Armington/Krugman models as a special case
  - is isomorphic to the multi-industry Melitz-Pareto model and Eaton-Kortum model with Marshallian externalities. (Kucheryavyy, Lyn, Rodriguez-Clare, 2022, AEJ-Macro)

References:

- multi-industry model *with* scale effects: Lashkaripour, Lugovskyy (2023, AER)
- multi-industry model *without* scale effects: Donaldson, Costinot, Komunjer (2012, ReStud)

Main Implications of Multi-Industry Models

- Multi-industry models predict larger gains from trade than single-industry variants, narrowing the gap between the gains implied by structural models and reduced-form estimation.

- While the single-industry Armington/EK/Krugman/Melitz models are efficient, the multi-industry model with scale effects describes an *inefficient* economy:
  - too little output in high-return-to-scale industries $\longrightarrow$ allocative inefficiency

- Trade can improve or worsen allocative inefficiency:
  - trade induces specialization in low-return-to-scale industries $\longrightarrow$ allocative inefficiency $\downarrow$
  - trade induces specialization in high-return-to-scale industries $\longrightarrow$ allocative inefficiency $\uparrow$

Environment

- Many countries indexed by $i,\ n = 1, ..., N$

- Many industries indexed by $k,\ g = 1, ..., K$

- Each country hosts many symmetric firms
  - firms are indexed by ω
  - firms supply differentiated varieties and are monopolistically competitive

- Labor is the only factor of production

- Country i is endowed with L_i (inelastically-supplied) units of labor

- Trade is balanced: $D_i = 0 \longrightarrow E_i = Y_i \quad (\forall i)$

Demand: Three-Tier Utility Function

- The representative consumer in country i has a three-tier utility function:

$$\text{[cross-indutry]} \qquad U_i = \prod_{k=1}^{K} \left(Q_{i,k}/\beta_{i,k}\right)^{\beta_{i,k}}$$

$$\text{[cross-national]} \qquad Q_{i,k} = \left(\sum_{n=1}^{N} Q_{ni,k}{}^{\frac{\sigma_k - 1}{\sigma_k}}\right)^{\frac{\sigma_k}{\sigma_k - 1}}$$

$$\text{[sub-national]} \qquad Q_{ni,k} = \left[\int_{\omega \in \Omega_{n,k}} q_{ni,k}\left(\omega\right)^{\frac{\gamma_k - 1}{\gamma_k}} d\omega\right]^{\frac{\gamma_k}{\gamma_k - 1}}$$

- $\beta_{i,k}$ is country i's *constant* expenditure share on industry k.
- $\sigma_k \geq 1$ is the cross-national elasticity of substitution
- $\gamma_k \geq \sigma_k$ is the sub-national elasticity of substitution b/w firm-level varieties
- $q_{ni,k}\left(\omega\right)$ is the quantity of firm variety ω from origin n–industry k.

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Demand: Nested-CES Preferences

- The representative consumer maximizes utility subject to their budget constraint:

$$\max_{\mathbf{q}_i} U_i(\mathbf{q}_{1i}, ..., \mathbf{q}_{Ni}) \qquad s.t. \qquad \sum_{k=1}^{K} \sum_{n=1}^{N} \left[\int_{\omega \in \Omega_{n,k}} p_{ni,k}(\omega)\, q_{ni,k}(\omega) \right] \leq E_i$$

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$$p_{ni,k}(\omega)\, q_{ni,k}(\omega) = \underbrace{\left(\frac{p_{ni,k}(\omega)}{P_{ni,k}} \right)^{1-\gamma_k}}_{\text{sub-national share}} \times \underbrace{\left(\frac{P_{ni,k}}{P_{i,k}} \right)^{1-\sigma_k}}_{\text{cross-national share}} \times \beta_{i,k} E_i$$

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where $P_{ni,k}$ and $P_{i,k}$ are CES price indexes:

$$P_{ni,k} = \left[\int_{\Omega_{n,k}} p_{ni,k}(\omega)^{1-\gamma_k} d\omega \right]^{\frac{1}{1-\gamma_k}} \qquad\qquad P_{i,k} = \left[\sum_n P_{ni,k}^{1-\sigma_k} \right]^{\frac{1}{1-\sigma_k}}$$

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The non-nested CES demand in Krugman (1980) is a special case where $\gamma_k = \sigma_k$:

$$\gamma_k = \sigma_k \qquad \longrightarrow \qquad p_{ni,k}(\omega)\, q_{ni,k}(\omega) = \left(\frac{p_{ni,k}(\omega)}{P_{i,k}} \right)^{1-\sigma_k} \beta_{i,k} E_i$$

Supply: Technology and Production

- There is a pool of *ex-ante* identical firms in country i, each of which can pay an entry cost $(w_i f_{i,k})$ to independently draw a productivity φ from distribution $G_{i,k}(\varphi)$.
- Upon entry, firm ω with productivity $\varphi_{i,k}(\omega)$ can sell to country n with a constant marginal cost:

$$\mathrm{MC}_{in,k}(\omega) = \underbrace{\frac{1}{\varphi_{i,k}(\omega)}}_{\text{productivity}} \times \underbrace{\tau_{in,k}}_{\text{trade cost}} \times \underbrace{w_i}_{\text{wage}}$$

- For now, we assume no fixed overhead cost for serving individual markets $\rightarrow$ non firm-selection into export markets
- The total cost faced by firm ω from country i–industry k:

$$\mathrm{TC}_{i,k}(\omega) = w_i f_{i,k} + \sum_{n=1}^{N} \frac{1}{\varphi_{i,k}(\omega)} \tau_{in,k} w_i q_{in,k}(\omega)$$

Supply: Optimal Pricing

- Productivity, φ, uniquely determines the firm-level outcomes $\longrightarrow$ we can specify firm-level variables in terms of φ.
- Firms are monopolistically competitive and set prices to maximize variable profits:

$$p_{in,k}(\varphi) = \arg\max_{p} \left[p - \frac{1}{\varphi} \tau_{in,k} w_i \right] D_{in,k}(p),$$

  where $D_{in,k}(p) = p^{-\gamma_k} \Phi_{in,k}$ denotes the CES demand function facing firm varieties, with $\Phi_{in,k} \equiv P_{in,k}^{\gamma_k} Q_{n,k}$ encompassing market-level shifters that firms take as given.
- The optimal price exhibits a constant markup over marginal cost

$$p_{in,k}(\varphi) = \underbrace{\frac{\gamma_k}{\gamma_k - 1}}_{\text{markup}} \times \frac{1}{\varphi} \tau_{in,k} w_i$$

Supply: Firm Entry

- The mass $M_{i,k} \equiv | \Omega_{i,k} |$ of firms that pay the entry cost to operate from country i is determined by free entry (*i.e.*, firms enter until profits are dissipated)

$$\text{expected profits} \sim \sum_{n=1}^{N} \underbrace{\mathbb{E}_{\varphi}\left[\left(p_{in,k}\left(\varphi\right) - \frac{1}{\varphi}\tau_{in,k}w_i\right) q_{in,k}\left(\varphi\right)\right]}_{\text{variable profits from sales to } n} - w_i f_{i,k} = 0$$

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- Since revenues from sales equal the input cost (or wage bill) per industry, we can derive a simple expression for $M_{i,k}$:

$$\begin{cases} \overbrace{M_{i,k} \sum_{n} \mathbb{E}_{\varphi}\left[p_{in,k}(\varphi)\, q_{in,k}(\varphi)\right]}^{\text{revenues from sales}(i,k)} = \overbrace{w_i L_{i,k}}^{\text{input cost}} \\ \underbrace{\sum_{n=1}^{N} \mathbb{E}_{\varphi}\left[p_{in,k}(\varphi)\, q_{in,k}(\varphi)\right] = \gamma_k w_i f_{i,k}}_{\text{free entry condition}} \end{cases} \longrightarrow \quad M_{i,k} = \frac{1}{\gamma_k f_{i,k}} L_{i,k}$$

Aggregate Price Indexes

- The price index of the composite good sold by origin n to destination i in industry k is

$$P_{ni,k} = \left(\int_{\Omega_{n,k}} p_{ni,k}(\omega)^{1-\gamma_k} d\omega \right)^{\frac{1}{1-\gamma_k}} = \left(M_{n,k} \int_{\varphi} \left(\frac{\gamma_k}{\gamma_k - 1} \frac{\tau_{ni,k} w_n}{\varphi} \right)^{1-\gamma_k} dG_{n,k}(\varphi) \right)^{\frac{1}{1-\gamma_k}}$$

$$= M_{n,k}^{\frac{1}{1-\gamma_k}} \left(\frac{\gamma_k}{\gamma_k - 1} \right) \frac{\tau_{ni,k} w_n}{\varphi_{n,k}}$$

where $\varphi_{n,k} \equiv \left[\int_{\varphi} \varphi^{\gamma_k - 1} dG_{n,k}(\varphi) \right]^{\frac{1}{\gamma_k - 1}}$ denotes average firm productivity.

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- Plugging $M_{n,k} = \frac{1}{\gamma_k f_{n,k}} L_{n,k}$ into the above equation yields:

$$P_{ni,k} = \tau_{ni,k}\left(\frac{\gamma_k}{\gamma_k-1}\right)\frac{\left(\gamma_k f_{n,k}\right)^{\frac{1}{\gamma_k-1}}}{\varphi_{n,k}} w_n\, L_{n,k}^{\frac{1}{1-\gamma_k}}$$

The Scale Elasticity

- Let $Q_{i,k} = \sum_n \tau_{in,k} Q_{in,k}$ denote the output of country i in industry k.
- Given that $P_{ii,k} Q_{i,k} = w_i L_{i,k}$, the TFP can be obtained as

$$\mathrm{TFP}_{i,k} \sim \frac{Q_{i,k}}{L_{i,k}} = \frac{w_i}{P_{ii,k}} = \left(1 - \frac{1}{\gamma_k}\right) \varphi_{i,k} \left(\gamma_k f_{i,k}\right)^{\frac{1}{1-\gamma_k}} \times L_{i,k}^{\frac{1}{\gamma_k - 1}}$$

where the last line uses $P_{ii,k} = \left(\frac{\gamma_k}{\gamma_k - 1}\right) \frac{\left(\gamma_k f_{i,k}\right)^{\frac{1}{\gamma_k - 1}}}{\varphi_{i,k}} w_i L_{i,k}^{\frac{1}{1-\gamma_k}}$, as previously derived.

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- We refer to the elasticity of TFP *w.r.t.* employment size as the *scale elasticity*

$$\frac{\partial \ln \text{TFP}_{i,k}}{\partial \ln L_{i,k}} = \frac{1}{\gamma_k - 1} \sim \text{ scale elasticity}$$

Aggregate Expenditure Shares

- National-level expenditure shares (within industry k) can be calculated using CES demand:

$$\lambda_{in,k} = \left(\frac{P_{in,k}}{P_{i,k}}\right)^{1-\sigma_k} = \frac{P_{in,k}^{1-\sigma_k}}{\sum_{j=1}^{N} P_{jn,k}^{1-\sigma_k}}$$

  where the price indexes are given by

$$P_{in,k} = \tau_{in,k}\left(\frac{\gamma_k}{\gamma_k - 1}\right)\frac{(\gamma_k f_{i,k})^{\frac{1}{\gamma_k - 1}}}{\varphi_{i,k}} w_i\, L_{i,k}^{\frac{1}{1-\gamma_k}}$$

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- Consolidating the above equations yields the industry-level gravity equation:

$$\lambda_{in,k} = \frac{\left(\frac{L_{i,k}}{f_{i,k}}\right)^{\frac{\sigma_k - 1}{\gamma_k - 1}} \varphi_{i,k}^{\sigma_k - 1} \left(\tau_{in,k} w_i\right)^{1-\sigma_k}}{\sum_{j=1}^{N} \left(\frac{L_{j,k}}{f_{j,k}}\right)^{\frac{\sigma_k - 1}{\gamma_k - 1}} \varphi_{j,k}^{\sigma_k - 1} \left(\tau_{jn,k} w_j\right)^{1-\sigma_k}}$$

Aggregate Expenditure Shares

- To economize on notation, we use μ_k and ϵ_k to denote the scale and trade elasticities:

$$\mu_k \equiv \frac{\partial \ln \text{TFP}_{i,k}}{\partial \ln L_{i,k}} = \frac{1}{\gamma_k - 1} \quad \sim \ \text{scale elasticity}$$

$$\epsilon_k \equiv -\frac{\partial \ln \left(\lambda_{ni,k} / \lambda_{ii,k}\right)}{\partial \ln \tau_{ni,k}} = \sigma_k - 1 \quad \sim \ \text{trade elasticity}$$

- With this choice of notation, the industry-level gravity equation can be specified as

$$\lambda_{in,k} = \frac{\left(L_{i,k} / f_{i,k}\right)^{\mu_k \epsilon_k} \ \varphi_{i,k}^{\epsilon_k} \ \left(\tau_{in,k} w_i\right)^{-\epsilon_k}}{\sum_{j=1}^{N} \left(L_{j,k} / f_{j,k}\right)^{\mu_k \epsilon_k} \ \varphi_{j,k}^{\epsilon_k} \ \left(\tau_{jn,k} w_j\right)^{-\epsilon_k}}$$

General Equilibrium

For a given set of parameters, $\{\tau_{in,k}, \varphi_{i,k}, f_{i,k}, \beta_{i,k}, \bar{L}_i, \mu_k, \epsilon_k\}_{i,n,k}$, equilibrium is a vector of wages and labor allocations, $\{w_i, L_{i,k}\}_{i,k}$, such that labor markets clear in all countries. Namely,

$$\underbrace{\sum_{n=1}^{N} \lambda_{in,k}\left(\mathbf{w}, \mathbf{L}_k\right) \times \beta_{i,k} E_n\left(w_n\right)}_{\text{demand for country } i\text{'s labor services in industry } k} = w_i L_{i,k} \qquad \sum_{k=1}^{K} L_{i,k} = \bar{L}_i, \qquad (\forall i)$$

with the expenditure shares ($\lambda_{in,k}$) and national expenditure levels (E_n) given by

$$\begin{cases} \lambda_{in,k}\left(\mathbf{w}, \mathbf{L}_k\right) = \dfrac{\chi_{i,k}\, L_{i,k}^{\mu_k \epsilon_k}\, \left(\tau_{in,k} w_i\right)^{-\epsilon_k}}{\sum_{j=1}^{N} \chi_{j,k}\, L_{j,k}^{\mu_k \epsilon_k}\, \left(\tau_{jn,k} w_j\right)^{-\epsilon_k}} & (\forall i, n, k) \\ E_n\left(w_n\right) = w_n \bar{L}_n & (\forall i, \text{ balance budegt}) \end{cases}$$

where $\chi_{i,k} \equiv f_{i,k}^{-\psi_k \epsilon_k} \varphi_{i,k}^{\epsilon_k}$ is a constant specific to origin i–industry k.

National-Level Welfare

- The indirect utility or welfare of the representative consumer in country i is

$$W_i = \frac{E_i}{P_i} = \frac{Y_i}{P_i} = \frac{w_i L_i}{P_i}$$

where P_i is the Cob-Douglas-CES consumer price index:

$$P_i = \prod_{k=1}^{K} \left[\sum_{n=1}^{N} P_{ni,k}^{1-\sigma_k} \right]^{\frac{\beta_{i,k}}{1-\sigma_k}} \sim P_i = \prod_{k=1}^{K} \left[\sum_{n=1}^{N} P_{ni,k}^{-\epsilon_k} \right]^{-\frac{\beta_{i,k}}{\epsilon_k}}$$

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$$P_i = C \times \prod_{k=1}^{K} \left[\sum_{n=1}^{N} \chi_{n,k} \, L_{n,k}^{\mu_k \epsilon_k} \, (\tau_{ni,k} w_n)^{-\epsilon_k} \right]^{-\frac{\beta_{i,k}}{\epsilon_k}}$$

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encapsulates non-country-specific constants

Accounting for Melitz-Style Firm-Selection

The generalized multi-industry Krugman model is isomorphic to the generalized multi-industry Melitz Pareto model (Kucheryavyy et. al, 2023)

- Suppose firms incur a fixed cost to serve each market $\longrightarrow$ endogenous aggregate productivity
- With a Pareto productivity distribution, $G_{i,k}(\varphi) = 1 - (A_{i,k}/\varphi)^{\theta_k}$, equilibrium has the same macro-level representation, but χ and τ have different interpretations.
- Also, the trade and scale elasticities depend on the productivity distribution parameter θ_k:

$$\mu_k \equiv \frac{\partial \ln \mathrm{TFP}_{i,k}}{\partial \ln L_{i,k}} = \frac{1}{\theta_k} \sim \text{ scale elasticity}$$

$$\epsilon_k \equiv -\frac{\partial \ln (\lambda_{ni,k}/\lambda_{ii,k})}{\partial \ln \tau_{ni,k}} = \frac{\theta_k}{1+\theta_k\left(\frac{1}{\sigma_k - 1} - \frac{1}{\gamma_k - 1}\right)} \sim \text{ trade elasticity}$$

Performing Counterfactuals using Exact Hat-Algebra

- To perform hat-algebra it is useful to write the equilibrium conditions in terms of nominal output or GDP (Y_i) and industry-level output shares ($y_{i,k}$):

$$Y_i = w_i L_i, \qquad\qquad y_{i,k} \equiv \frac{Y_{i,k}}{Y_i} = \frac{w_i L_{i,k}}{w_i L_i}$$

- Under autarky, $y_{i,k}^{(autarky)} = \beta_{i,k}$, but under trade, $y_{i,k} \neq \beta_{i,k}$.

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- We can re-write the equilibrium conditions in terms of $\{Y_i\}$ and $\{y_{i,k}\}$:

$$Y_{i,k} \sim y_{i,k} Y_i = \sum_{n=1}^{N} \left[\lambda_{in,k} \beta_{n,k} Y_n\right] \quad (\forall i, k)\,; \qquad \sum_{k=1}^{K} y_{i,k} = 1$$

Performing Counterfactuals using Exact Hat-Algebra

- To perform hat-algebra it is useful to write the equilibrium conditions in terms of nominal output or GDP (Y_i) and industry-level output shares ($y_{i,k}$):

$$Y_i = w_i L_i, \qquad y_{i,k} = \frac{Y_{i,k}}{Y_i} = \frac{w_i L_{i,k}}{w_i L_i} \longrightarrow L_{i,k} = y_{i,k} L_i$$

- Under autarky, $y_{i,k}^{(autarky)} = \beta_{i,k}$, but under trade, $y_{i,k} \neq \beta_{i,k}$.

- We can re-write the equilibrium conditions in terms of $\{Y_i\}$ and $\{y_{i,k}\}$:

$$y_{i,k} Y_i = \sum_{n=1}^{N} \left[\frac{\tilde{\chi}_{i,k} y_{i,k}^{\mu_k \epsilon_k} \left(\tau_{in,k} Y_i\right)^{-\epsilon_k}}{\sum_{j=1}^{N} \tilde{\chi}_{j,k} y_{j,k}^{\mu_k \epsilon_k} \left(\tau_{jn,k} Y_j\right)^{-\epsilon_k}} \beta_{n,k} Y_n \right] \quad (\forall i, k)\,; \qquad \sum_{k=1}^{K} y_{i,k} = 1 \quad (\forall i)$$

where $\tilde{\chi}_{i,k} \equiv \chi_{i,k} L_i^{(1+\mu_k)\epsilon_k}$ encompasses constants specific to origin i–industry k.

Performing Counterfactuals using Exact Hat-Algebra

- The welfare impacts of an arbitrary trade cost shock $\{\hat{\tau}_{in}\}_{i,n}$, can be calculated as

$$\hat{W}_i = \frac{\hat{Y}_i}{\hat{P}_i}, \qquad \widehat{P}_i = \prod_{k=1}^{K} \left[\sum_{n=1}^{N} \lambda_{ni,k} \hat{y}_{n,k}^{\mu_k \epsilon_k} \left(\hat{\tau}_{ni,k} \hat{Y}_n \right)^{-\epsilon_k} \right]^{-\frac{\beta_{i,k}}{\epsilon_k}}$$

where $\hat{Y}_n$ and $\hat{y}_{n,k}$ can be calculated with data on baseline expenditure shares,$\lambda_{in,k}$, GDP levels, Y_i, and industry output shares, $y_{i,k}$, via the following system:

$$\hat{y}_{i,k} \hat{Y}_i \, y_{i,k} Y_i = \sum_{n=1}^{N} \left[\frac{\lambda_{in,k} \, \hat{y}_{i,k}^{\mu_k \epsilon_k} \left(\hat{\tau}_{in,k} \hat{Y}_i \right)^{-\epsilon_k}}{\sum_{j=1}^{N} \lambda_{jn,k} \, \hat{y}_{j,k}^{\mu_k \epsilon_k} \left(\hat{\tau}_{jn,k} \hat{Y}_j \right)^{-\epsilon_k}} \beta_{n,k} \hat{Y}_n Y_n \right] \qquad (\forall i, k)$$

$$\sum_{k=1}^{K} \left[\hat{y}_{i,k} y_{i,k} \right] = 1 \qquad (\forall i, \quad \text{adding up constraint})$$

Performing Counterfactuals using Exact Hat-Algebra

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$$\hat{W}_i = \frac{\hat{Y}_i}{\hat{P}_i}, \qquad \hat{P}_i = \prod_{k=1}^{K}\left[\sum_{n=1}^{N}\lambda_{ni,k}\hat{y}_{n,k}^{\mu_k\epsilon_k}\left(\hat{\tau}_{ni,k}\hat{Y}_n\right)^{-\epsilon_k}\right]^{-\frac{\beta_{i,k}}{\epsilon_k}}$$

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$$\overbrace{\hat{y}_{i,k}\hat{Y}_i\, y_{i,k}Y_i = \sum_{n=1}^{N}\left[\frac{\lambda_{in,k}\,\hat{y}_{i,k}^{\mu_k\epsilon_k}\left(\hat{\tau}_{in,k}\hat{Y}_i\right)^{-\epsilon_k}}{\sum_{j=1}^{N}\lambda_{jn,k}\,\hat{y}_{j,k}^{\mu_k\epsilon_k}\left(\hat{\tau}_{jn,k}\hat{Y}_j\right)^{-\epsilon_k}}\beta_{n,k}\hat{Y}_nY_n\right]}^{N\times K \text{ equations}} \qquad (\forall i,k)$$

$$\underbrace{\sum_{k=1}^{K}\left[\hat{y}_{i,k}y_{i,k}\right] = 1}_{N \text{ equations}} \qquad (\forall i, \quad \text{adding up constraint})$$

Growth Accounting with
Multiple Industries & Scale Effects

Growth Accounting: Multi-Industry Krugman Model

- The welfare impacts of a shock to trade costs $\{\mathrm{d}\ln\tau_{in,k}\}_{i,n,k}$ and aggregate productivity, $\{\mathrm{d}\ln\varphi_{i,k}\}_{i,k}$ can be specified as

$$\mathrm{d}\ln W_i = \mathrm{d}\ln Y_i - \mathrm{d}\ln P_i$$

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$$\mathrm{dln}W_i = \mathrm{d}\ln Y_i - \sum_{k=1}^{K}\sum_{n=1}^{N} \lambda_{ni,k}\beta_{i,k}\mathrm{d}\ln P_{ni,k}$$

- We can simplify the above expression by appealing to the CES demand structure:

$$\mathrm{d}\ln \lambda_{ni,k} - \mathrm{dln}\lambda_{ii,k} = -\epsilon_k\left(\mathrm{d}\ln P_{ni,k} - \mathrm{d}\ln P_{ii,k}\right)$$

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- We can simplify the above expression by appealing to the CES demand structure:

$$\mathrm{d}\ln P_{ni,k} = \mathrm{d}\ln P_{ii,k} - \frac{1}{\epsilon_k}\left(\mathrm{d}\ln\lambda_{ni,k} - \mathrm{dln}\lambda_{ii,k}\right)$$

where $P_{ii,k} = C \times \frac{1}{\varphi_{i,k}} Y_i\, y_{i,k}^{-\mu_k}$, implying that

$$\mathrm{d}\ln P_{ii,k} = -\mathrm{d}\ln\varphi_{i,k} + \mathrm{d}\ln Y_i - \mu_k \mathrm{d}\ln y_{i,k}$$

Growth Accounting: Multi-Industry Krugman Model

- Plugging the expression for $\mathrm{d}\ln P_{ii,k}$ into the welfare equation yields

$$\begin{aligned}
\mathrm{dln} W_i &= \mathrm{d}\ln Y_i - \sum_{k=1}^{K}\sum_{n=1}^{N}\beta_{i,k}\lambda_{ni,k}\mathrm{d}\ln P_{ni,k} \\
&= \mathrm{d}\ln Y_i - \sum_{k}\beta_{i,k}\mathrm{d}\ln P_{ii,k} + \sum_{k}\sum_{n}\left[\frac{1}{\epsilon_k}\beta_{i,k}\lambda_{ni,k}\left(\mathrm{dln}\,\lambda_{ni,k} - \mathrm{d}\ln\lambda_{ii,k}\right)\right]
\end{aligned}$$

Growth Accounting: Multi-Industry Krugman Model

- Plugging the expression for $\mathrm{d}\ln P_{ii,k}$ into the welfare equation yields

$$
\begin{aligned}
\mathrm{dln} W_i &= \mathrm{d}\ln Y_i - \sum_{k=1}^{K} \sum_{n=1}^{N} \beta_{i,k} \lambda_{ni,k} \mathrm{d}\ln P_{ni,k} \\
&= \mathrm{d}\ln Y_i - \sum_{k} \beta_{i,k} \mathrm{d}\ln P_{ii,k} + \sum_{k} \sum_{n} \left[\frac{1}{\epsilon_k} \beta_{i,k} \lambda_{ni,k} \left(\mathrm{dln}\, \lambda_{ni,k} - \mathrm{d}\ln \lambda_{ii,k} \right) \right]
\end{aligned}
$$

- Noting that $\sum_n \lambda_{ni,k} \mathrm{d}\ln \lambda_{ni,k} = 0$ and $\sum_n \lambda_{ni,k} = 1$, the last line yields

$$
\mathrm{dln} W_i = \sum_{k} \left[\beta_{i,k} \left(\mathrm{d}\ln \varphi_{i,k} + \mu_k \mathrm{d}\ln y_{i,k} - \frac{1}{\epsilon_k} \mathrm{d}\ln \lambda_{ii,k} \right) \right]
$$

Growth Accounting: Multi-Industry Model with Scale Effects

We can decompose the resulting welfare impacts as

$$\mathrm{dln}W_i = \overbrace{\sum_k y_{i,k} \mathrm{d}\ln\varphi_{i,k}}^{\text{Hulten}} + \overbrace{\sum_k y_{i,k}\mu_k \mathrm{d}\ln y_{i,k}}^{\Delta\text{allocative efficiency}}$$
$$+ \underbrace{\sum_k \left[\left(\beta_{i,k} - y_{i,k}\right)\left(\mu_k \mathrm{d}\ln y_{i,k} + \mathrm{d}\ln\varphi_{i,k}\right) - \frac{\beta_{i,k}}{\epsilon_k}\mathrm{d}\ln\lambda_{ii,k} \right]}_{\text{Terms of trade effects}}$$

Growth Accounting: Multi-Industry Model with Scale Effects

We can decompose the resulting welfare impacts as

$$\mathrm{d}\ln W_i = \overbrace{\sum_k y_{i,k} \mathrm{d}\ln \varphi_{i,k}}^{\text{Hulten}} + \overbrace{Cov\left(\mu_k, \mathrm{d}\ln y_{i,k}\right)}^{\Delta\text{allocative efficiency}}$$

$$+ \underbrace{\sum_k \left[\left(\beta_{i,k} - y_{i,k}\right)\left(\mu_k \mathrm{d}\ln y_{i,k} + \mathrm{d}\ln \varphi_{i,k}\right) - \frac{\beta_{i,k}}{\epsilon_k} \mathrm{d}\ln \lambda_{ii,k}\right]}_{\text{Terms of trade effects}}$$

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Allocative efficiency

- high-returns-to-scale (high-μ) industries produce below efficient levels $\longrightarrow$ trade/productivity shocks improve efficiency if they direct resources to these sectors, $Cov\left(\mu_k, \mathrm{d}\ln y_{i,k}\right) > 0$

Terms of trade effects ($\Delta\frac{\text{export prices}}{\text{import prices}}$)

- ToT effects depend on the divergence between expenditure and output $(\beta_{i,k} - y_{i,k})$ and the change in trade openness $(\mathrm{d}\ln \lambda_{ii,k})$, echoing the ACR formula.

Growth Accounting: Multi-Industry Model with Scale Effects

- Calculating welfare effects requires industry-level estimates for scale elasticities (μ_k):
  - Lashkaripour & Lugovskyy (2023) and Bartelme et al. (2025) provide such estimates
- The previous welfare formula holds non-parametrically if we treat μ_k and ϵ_k as local (and possibly variable) scale and trade elasticities.
- In the CES model, where μ_k and ϵ_k are constant structural elasticities, the same formula describes the impact of large changes or shocks to productivity and trade costs:

$$\Delta \ln W_i = \overbrace{\sum_k y_{i,k} \Delta \ln \varphi_{i,k}}^{\text{Hulten}} + \overbrace{Cov\left(\mu_k, \Delta \ln y_{i,k}\right)}^{\Delta \text{allocative efficiency}}$$
$$+ \underbrace{\sum_k \left[\left(\beta_{i,k} - y_{i,k}\right) \left(\mu_k \Delta \ln y_{i,k} + \Delta \ln \varphi_{i,k}\right) - \frac{\beta_{i,k}}{\epsilon_k} \Delta \ln \lambda_{ii,k} \right]}_{\text{Terms of trade effects}}$$

Immizerising Growth Effects

Corrective Industry Policy in a Closed Economy

- The multi-industry model with scale effects is *inefficient* because high-returns-to-scale industries have too little entry/output $\longrightarrow$ there's an efficiency rationale for industrial policy
- However, corrective industrial policy (IP) can lead to negative terms-of-trade effects that offset efficiency gains, resulting in *immiserizing growth* effects (Lashkaripour & Lugovskyy, 2023).

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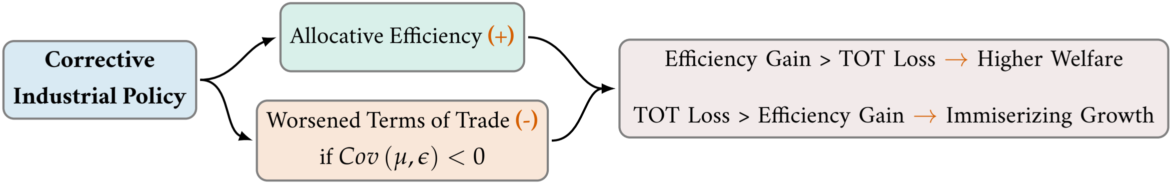

Corrective Industry Policy in a Closed Economy Setting

To understand this effect consider the ***closed economy*** case, where $y_i = \beta_i$:

- Efficient IP provides a subsidy $(1+\mu_k)$ to each industry k
  - it relocates resources from *low-returns* to *high-returns to scale* industries
- The resulting welfare gains are given by $(\tilde{\mu} \equiv 1+\mu)$

$$\Delta \ln W_i^{(closed)} = \sum_k \left[y_{i,k}\tilde{\mu}_k \ln\left(\tilde{\mu}_k\right)\right] - \sum_k \left[y_{i,k}\tilde{\mu}_k\right] \sum_k \left[y_{i,k} \ln\left(\tilde{\mu}_k\right)\right]$$

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  - it relocates resources from *low-returns* to *high-returns to scale* industries

- The resulting welfare gains are given by $(\tilde{\mu} \equiv 1 + \mu)$

$$\Delta \ln W_i^{(closed)} = \mathbb{E}_{y_i}\left[\tilde{\mu} \ln\left(\tilde{\mu}\right)\right] - \mathbb{E}_{y_i}\left[\tilde{\mu}\right] \ln \mathbb{E}_{y_i}\left[\tilde{\mu}\right] > 0$$

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- The welfare gains represent the **Bregman distance** (with $\phi\left(\tilde{\mu}\right) = \tilde{\mu}\ln\left(\tilde{\mu}\right)$) between scale elasticities and their mean, measuring the sectoral dispersion in scale elasticities

$$\Delta \ln W_i^{(closed)} \approx Var_{y_i}\left[\mu\right]$$

Corrective Industry Policy in an Open Economy

- The gains from corrective IP in open economies include ToT effects

$$\Delta \ln W_i = \overbrace{\mathbb{E}_{y_i}\left[\tilde{\mu}\ln\left(\tilde{\mu}\right)\right] - \mathbb{E}_{y_i}\left[\tilde{\mu}\right]\ln \mathbb{E}_{y_i^*}\left[\tilde{\mu}\right] + Cov_{y_i}\left(\tilde{\mu}, \Delta \ln y_i\right)}^{\text{efficiency gains}}$$
$$\underbrace{- \sum_k \frac{\beta_{i,k}}{\epsilon_k}\Delta \ln \lambda_{ii,k}}_{\Delta\text{ gains from trade}} \underbrace{- \sum_k \left(y_{i,k} - e_{i,k}\right)\Delta \ln \mathrm{TFP}_{i,k}}_{\text{transfer to foreign}}$$

Corrective Industry Policy in an Open Economy

- The gains from corrective IP in open economies include ToT effects

$$\Delta \ln W_i = \overbrace{\mathbb{E}_{y_i}\left[\tilde{\mu}\ln\left(\tilde{\mu}\right)\right] - \mathbb{E}_{y_i}\left[\tilde{\mu}\right]\ln \mathbb{E}_{y_i^*}\left[\tilde{\mu}\right] + Cov_{y_i}\left(\tilde{\mu}, \Delta \ln y_i\right)}^{\text{efficiency gains}}$$

$$- \underbrace{\sum_k \frac{\beta_{i,k}}{\epsilon_k} \Delta \ln \lambda_{ii,k}}_{\Delta \text{ gains from trade}} - \underbrace{\sum_k \left(y_{i,k} - e_{i,k}\right) \Delta \ln \mathrm{TFP}_{i,k}}_{\text{transfer to foreign}}$$

- $Cov_{y_i}\left(\mu, \Delta \ln y_i\right) > 0$, since corrective IP raises output in high-μ industries
  - in a closed economy sectoral output shares ($y_i = \beta_i$) are invariant to policy
- $\sum_k \frac{\beta_{i,k}}{\epsilon_k} \Delta \ln \lambda_{ii,k}$ accounts for the change in gains from trade
  - λ_{ii} and ϵ are the sufficient statistics à la ACR
- $\sum_k \left(y_{i,k} - \beta_{i,k}\right) \Delta \ln \mathrm{TFP}_{i,k}$ represents the TFP gains passed onto foreign consumers

Corrective Industry Policy in an Open Economy

- The gains from corrective IP in an open economy include terms of trade effects

$$\Delta \ln W_i = \overbrace{\mathbb{E}_{y_i}\left[\tilde{\mu} \ln\left(\tilde{\mu}\right)\right] - \mathbb{E}_{y_i}\left[\tilde{\mu}\right] \ln \mathbb{E}_{y_i}\left[\tilde{\mu}\right] + Cov_{y_i}\left(\mu, \Delta \ln y_i\right)}^{\text{efficiency gains}}$$

$$- \sum_k \left(y_{i,k} - \beta_{i,k}\right) \Delta \ln \mathrm{TFP}_{i,k} - \underbrace{\sum_k \frac{\beta_{i,k}}{\epsilon_k} \Delta \ln \lambda_{ii,k}}_{\Delta \text{ gains from trade}}$$

Corrective Industry Policy in an Open Economy

- The gains from corrective IP in an open economy

$$\Delta \ln W_i = \overbrace{\mathbb{E}_{y_i}\left[\tilde{\mu}\ln\left(\tilde{\mu}\right)\right] - \mathbb{E}_{y_i}\left[\tilde{\mu}\right]\ln \mathbb{E}_{y_i}\left[\tilde{\mu}\right] + Cov_{y_i}\left(\mu, \Delta \ln y_i\right)}^{\text{efficiency gains}}$$

$$- \sum_k \left(y_{i,k} - \beta_{i,k}\right)\Delta \ln \mathrm{TFP}_{i,k} - \underbrace{\sum_k \frac{\beta_{i,k}}{\epsilon_k}\Delta \ln \lambda_{ii,k}}_{\Delta \text{ gains from trade}}$$

Proposition: if $Cov\left(\epsilon_k, \mu_k\right)$ is sufficiently negative, then corrective IP worsens the terms of trade, leading to possible *immiserizing* welfare effects.

Corrective Industry Policy in an Open Economy

- The gains from corrective IP in an open economy

$$\Delta \ln W_i = \overbrace{\mathbb{E}_{y_i}\left[\tilde{\mu}\ln\left(\tilde{\mu}\right)\right] - \mathbb{E}_{y_i}\left[\tilde{\mu}\right]\ln \mathbb{E}_{y_i}\left[\tilde{\mu}\right] + Cov_{y_i}\left(\mu, \Delta \ln y_i\right)}^{\text{efficiency gains}}$$
$$- \sum_k \left(y_{i,k} - \beta_{i,k}\right)\Delta \ln \mathrm{TFP}_{i,k} - \underbrace{\sum_k \frac{\beta_{i,k}}{\epsilon_k}\Delta \ln \lambda_{ii,k}}_{\Delta \text{ gains from trade}}$$

Sketch of proof:

1. Corrective IP distorts prices in favor of domestic varieties in high-μ industries
   - $Cov\left(\epsilon_k, \mu_k\right) < 0 \longrightarrow \lambda_{ii} \uparrow$ in low-ϵ industries $\longrightarrow Cov\left(\frac{1}{\epsilon}, \Delta \ln \lambda_{ii}\right) > 0$
2. Aggregate domestic expenditure goes up $\longrightarrow \mathbb{E}_{\beta_i}\left[\Delta \ln \lambda_{ii,k}\right] > 0$

Corrective Industry Policy in an Open Economy

- The gains from corrective IP in an open economy

$$\Delta \ln W_i = \overbrace{\mathbb{E}_{y_i}\left[\tilde{\mu} \ln\left(\tilde{\mu}\right)\right] - \mathbb{E}_{y_i}\left[\tilde{\mu}\right] \ln \mathbb{E}_{y_i}\left[\tilde{\mu}\right] + Cov_{y_i}\left(\mu, \Delta \ln y_i\right)}^{\text{efficiency gains}}$$

$$- \sum_k \left(y_{i,k} - \beta_{i,k}\right) \Delta \ln \mathrm{TFP}_{i,k} - \underbrace{\sum_k \frac{\beta_{i,k}}{\epsilon_k} \Delta \ln \lambda_{ii,k}}_{\Delta \text{ gains from trade}}$$

Sketch of proof:

1. Corrective IP distorts prices in favor of domestic varieties in high-μ industries
   - $Cov\left(\epsilon_k, \mu_k\right) < 0 \longrightarrow \lambda_{ii} \uparrow$ in low-ϵ industries $\longrightarrow Cov\left(\frac{1}{\epsilon}, \Delta \ln \lambda_{ii}\right) > 0$
2. Aggregate domestic expenditure goes up $\longrightarrow \mathbb{E}_{\beta_i}\left[\Delta \ln \lambda_{ii,k}\right] > 0$

(1) & (2) $\longrightarrow$ $\displaystyle\sum_k \frac{\beta_{i,k}}{\epsilon_k} \Delta \ln \lambda_{ii,k} = Cov\left(\frac{1}{\epsilon}, \Delta \ln \lambda_{ii}\right) + E_{\beta_i}\left[\frac{1}{\epsilon}\right] E_{\beta_i}\left[\Delta \ln \lambda_{ii}\right] > 0$

Projected Immiserizing Growth Effects from IP

- Lashkaripour & Lugovskyy (2023) estimate scale and trade elasticities across various industries and find that they exhibit a negative correlation:

$$Cov\left(\mu_k, \epsilon_k\right) \approx -0.65$$

$\longrightarrow$ non-coordinated scale-correcting IP may lead to immiserizing growth effects.

- Counterfactual simulations reveal that for the average country:
  - non-coordinated corrective IP leads to immiserizing welfare effects
  - coordinated IP delivers sizable welfare gains

TABLE 5—IMMISERIZING EFFECTS OF NONCOORDINATED INDUSTRIAL POLICIES

	Restricted entry		Free entry	
	Unilateral	Coordinated	Unilateral	Coordinated
Gains from *corrective* industrial policies	−0.32%	1.67%	−2.78%	3.42%

Notes: The data source is the 2014 World Input-Output Database (Timmer et al. 2015; WIOD 2021). The columns titled "Unilateral" report welfare gains when a country unilaterally adopts industrial subsidies that restore marginal cost pricing in the domestic economy. The columns titled "Coordinated" report welfare gains when all countries simultaneously adopt industrial subsidies that restore marginal cost pricing globally. The average gains are calculated as the simple average across all 43 countries in the WIOD sample. Country-level results are reported in online Appendix X.

The Gains from Trade with
Multiple Industries & Scale Effects

Deriving the Gains From Trade Formula

- Define the gains from trade as the ex-post gains from trade openness relative to autarky ($\tau = \infty$)

$$\mathrm{GT}_i \equiv \frac{W_i - W_i^A}{W_i} = 1 - \exp\left(\int_\tau^\infty \mathrm{d}\ln W_i\right)$$

Deriving the Gains From Trade Formula

- Define the gains from trade as the ex-post gains from trade openness relative to autarky ($\tau = \infty$)

$$\mathrm{GT}_i = 1 - \exp\left(\int_{\tau}^{\infty} \sum_k \beta_{i,k} \left(\mu_k \mathrm{d}\ln y_{i,k} - \frac{1}{\epsilon_k} \mathrm{d}\ln \lambda_{ii,k}\right)\right)$$

Deriving the Gains From Trade Formula

- Define the gains from trade as the ex-post gains from trade openness relative to autarky ($\tau = \infty$)

$$\mathrm{GT}_i = 1 - \exp\left(\int_{\tau}^{\infty} \sum_k \beta_{i,k}\left(\mu_k \mathrm{d}\ln y_{i,k} - \frac{1}{\epsilon_k}\mathrm{d}\ln\lambda_{ii,k}\right)\right)$$

- We can calculate the gains from trade using our previous accounting formula by noting that autarky corresponds to $\lambda_{ii} = 1$:

$$\begin{aligned}\mathrm{GT}_i &= 1 - \exp\left(\sum_k \beta_{i,k}\left[\mu_k \int_{y_{i,k}}^{e_{i,k}} \mathrm{d}\ln y_{i,k} - \frac{1}{\epsilon_k}\int_{\lambda_{ii,k}}^{1} \mathrm{d}\ln\lambda_{ii,k}\right]\right) \\ &= \ 1 - \exp\left(\sum_k \beta_{i,k}\left[-\mu_k \ln\left(\frac{y_{i,k}}{e_{i,k}}\right) + \frac{1}{\epsilon_k}\ln\lambda_{ii,k}\right]\right)\end{aligned}$$

Deriving the Gains From Trade Formula

- Define the gains from trade as the ex-post gains from trade openness relative to autarky $(\tau = \infty)$

$$\mathrm{GT}_i = 1 - \exp\left(\int_{\tau}^{\infty} \sum_k \beta_{i,k} \left(\mu_k \mathrm{d} \ln y_{i,k} - \frac{1}{\epsilon_k} \mathrm{d} \ln \lambda_{ii,k}\right)\right)$$

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$$\mathrm{GT}_i = 1 - \prod_{k=1}^{K} \left(\frac{\beta_{i,k}}{y_{i,k}}\right)^{\mu_k \beta_{i,k}} \prod_{k=1}^{K} \lambda_{ii,k}^{\frac{\beta_{i,k}}{\epsilon_k}}$$

Deriving the Gains From Trade Formula

- Define the gains from trade as the ex-post gains from trade openness relative to autarky ($\tau = \infty$)

$$\mathrm{GT}_i = 1 - \exp\left(\int_\tau^\infty \sum_k \beta_{i,k}\left(\mu_k \mathrm{d}\ln y_{i,k} - \frac{1}{\epsilon_k}\mathrm{d}\ln\lambda_{ii,k}\right)\right)$$

- We can calculate the gains from trade using our previous accounting formula by noting that autarky corresponds to $\lambda_{ii} = 1$:

$$\mathrm{GT}_i = 1 - \underbrace{\prod_{k=1}^{K}\left(\frac{\beta_{i,k}}{y_{i,k}}\right)^{\mu_k\beta_{i,k}}}_{\text{scale effects}} \times \prod_{k=1}^{K}\lambda_{ii,k}^{\frac{\beta_{i,k}}{\epsilon_k}}$$

Deriving the Gains From Trade Formula

- Define the gains from trade as the ex-post gains from trade openness relative to autarky ($\tau = \infty$)

$$\mathrm{GT}_i = 1 - \exp\left(\int_{\tau}^{\infty} \sum_k \beta_{i,k} \left(\mu_k \mathrm{d}\ln y_{i,k} - \frac{1}{\epsilon_k}\mathrm{d}\ln \lambda_{ii,k}\right)\right)$$

- We can calculate the gains from trade using our previous accounting formula by noting that autarky corresponds to $\lambda_{ii} = 1$:

$$\mathrm{GT}_i = 1 - \underbrace{\prod_{k=1}^{K} \left(\frac{\beta_{i,k}}{y_{i,k}}\right)^{\mu_k \beta_{i,k}}}_{\text{scale effects}} \times \prod_{k=1}^{K} \lambda_{ii,k}^{\frac{\beta_{i,k}}{\epsilon_k}}$$

- We need the following *sufficient statics* to compute the (ex-post) gains from trade

$$\mathbb{D} = \{\lambda_{ii,k}, e_{i,k}, y_{i,k}, \mu_k, \epsilon_k\}_{i,k}$$

How do Scale Economies Modify the Gains From Trade?

- The gains from trade formula feature the following shifter that accounts for scale effects

$$\ln \prod_{k=1}^{K} \left(\frac{\beta_{i,k}}{y_{i,k}} \right)^{\mu_k \beta_{i,k}} = Cov_{\beta} \left(\mu_k, \ln \left(\frac{\beta_{i,k}}{y_{i,k}} \right) \right) + \mathbb{E}_{\beta} \left[\mu_k \right] \cdot D_{KL} \left(\mathbf{y}_i \;||\; \boldsymbol{\beta}_i \right)$$

where $D_{KL} \left(\mathbf{y}_i \;||\; \boldsymbol{\beta}_i \right)$ denotes the Kullback-Leibler divergence of $\mathbf{y}_i$ from $\boldsymbol{\beta}_i$.

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- **With scale distortions (heterogeneous μ_k):** the gains from trade are larger if trade integration elevates output in high-returns-to-scale industries—*i.e.*, $Cov_\beta \left(\mu_k, \ln \left(\frac{\beta_{i,k}}{y_{i,k}} \right) \right) < 0$.

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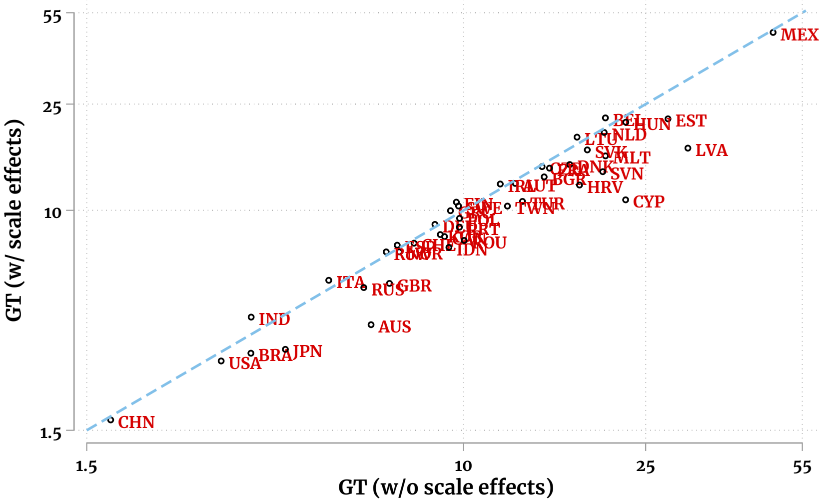
- **Without scale distortions ($\mu_k = \overline{\mu}$):** scale economies dampen the gains from trade:

$$\ln \prod_{k=1}^{K} \left(\frac{e_{i,k}}{y_{i,k}} \right)^{\overline{\mu} e_{i,k}} = \overline{\mu} D_{KL} \left(\mathbf{y}_i \mid\mid \boldsymbol{\beta}_i \right) \geq 0 \quad \longrightarrow \quad GT_i \leq \underbrace{1 - \prod_{k=1}^{K} \lambda_{ii,k}^{\frac{e_{i,k}}{\epsilon_k}}}_{\text{GT w/o scale economies}}$$

Gains from Trade: *with* and *without* Scale Economies



trade and Scale elasticities: Lashkaripour-Lugovskyy (2022); Data: WIOD 2014

Multi-Industry Models Predict Larger Gains from Trade

- The gains from trade in multi-industry models *w/o* scale economies ($\mu_k = 0, \forall k$):

$$GT_i^{(\text{multi})} = 1 - \prod_{k=1}^{K} \lambda_{ii,k}^{\frac{\beta_{i,k}}{\epsilon_k}} = 1 - \lambda_{ii}^{\frac{1}{\tilde{\epsilon}_i}}$$

  where $\frac{1}{\tilde{\epsilon}_i} \equiv \sum_k \frac{\beta_{i,k}}{\epsilon_k} \frac{\ln \lambda_{ii,k}}{\ln \lambda_{ii}} \approx$ Harmonic mean.

- Gains from trade in single industry models:

$$GT_i^{(\text{single})} = 1 - \lambda_{ii}^{\frac{1}{\epsilon}}$$

  where $\epsilon \equiv \mathbb{E}(\epsilon_k)$ is a (weighted) arithmetic mean, implicitly estimated when using aggregate data to recover the trade elasticity.

- Jensen's Inequality $\longrightarrow \tilde{\epsilon}_i < \epsilon \longrightarrow GT_i^{(\text{multi})} > GT_i^{(\text{single})}$

Gains predicted by Multi-Industry *vs.* Single-Industry Model

without scale economies

	% GT	
	single-industry	multi-industry
Ireland	8%	23.5%
Belgium	7.8%	32.7%
Germany	4.5%	12.7%
China	2.6%	4%
U.S.	1.8%	4.4%

Source: *Costinot & Rodriguez-Clare (2014)* based on data from WIOD 2008.