بسم الله الرَّحمن الَّرحیم

تمرین سری اول درس شبکه عصبی و یادگیری عمیق

Hagan:

E2.3 Given a two-input neuron with the following weight matrix and input vector: $\mathbf{W} = \begin{bmatrix} 3 & 2 \end{bmatrix}$ and $\mathbf{p} = \begin{bmatrix} -5 & 7 \end{bmatrix}^T$, we would like to have an output of 0.5. Do you suppose that there is a combination of bias and transfer function that might allow this?

i. Is there a transfer function from Table 2.1 that will do the job if the bias is zero?

ii. Is there a bias that will do the job if the linear transfer function is used? If yes, what is it?

iii. Is there a bias that will do the job if a log-sigmoid transfer function is used? Again, if yes, what is it?

iv. Is there a bias that will do the job if a symmetrical hard limit transfer function is used? Again, if yes, what is it?

Name	Input/Output Relation	Icon	MATLAB Function
Hard Limit	$a = 0 \quad n < 0$ $a = 1 \quad n \geq 0$		hardlim
Symmetrical Hard Limit	$a = -1 \quad n < 0$ $a = +1 \quad n \geq 0$		hardlims
Linear	$a = n$		purelin
Saturating Linear	$a = 0 \quad n < 0$ $a = n \quad 0 \leq n \leq 1$ $a = 1 \quad n > 1$		satlin
Symmetric Saturating Linear	$a = -1 \quad n < -1$ $a = n \quad -1 \leq n \leq 1$ $a = 1 \quad n > 1$		satlins
Log-Sigmoid	$a = \frac{1}{1 + e^{-n}}$		logsig
Hyperbolic Tangent Sigmoid	$a = \frac{e^{n} - e^{-n}}{e^{n} + e^{-n}}$		tansig
Positive Linear	$a = 0 \quad n < 0$ $a = n \quad 0 \leq n$		poslin

Table 2.1 Transfer Functions

E2.5 Consider the following neuron.

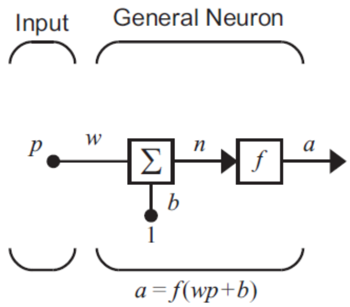


Figure P15.1 General Neuron

Sketch the neuron response (plot a versus p for $-2<p<2$) for the following cases.

i. $w = 1$, $b = 1$, $f = hardlims$.

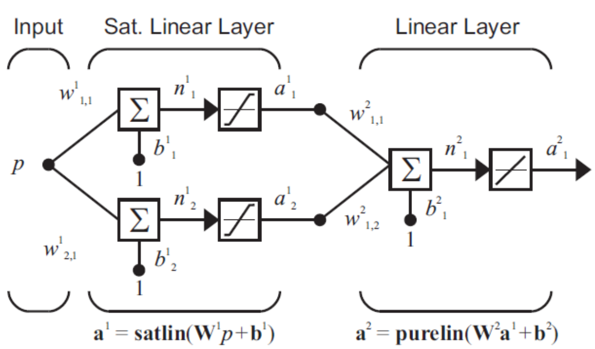
ii. $w = -1$, $b = 1$, $f = hardlims$.

iii. $w = 2$, $b = 3$, $f = purelin$.

iv. $w = 2$, $b = 3$, $f = satlins$.

v. $w = -2$, $b = -1$, $f = poslin$.

E2.6 Consider the following neural network.



$w^1_{1,1} = 2$, $w^1_{2,1} = 1$, $b^1_1 = 2$, $b^1_2 = -1$, $w^2_{1,1} = 1$, $w^2_{1,2} = -1$, $b^2_1 = 0$

Sketch the following responses (plot the indicated variable versus p for $-3 < p < 3$).

i. n^1_1.

ii. a^1_1.

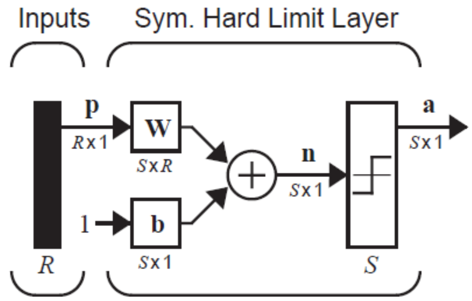
iii. n^1_2

iv. a^1_2.

v. n^2_1.

vi. a^2_1.

E3.4 Consider the following perceptron network.



$$\mathbf{W} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$$\mathbf{a} = \mathbf{hardlims}(\mathbf{Wp} + \mathbf{b})$$

i. How many different classes can this network classify?

ii. Draw a diagram illustrating the regions corresponding to each class. Label each region with the corresponding network output.

iii. Calculate the network output for the following input.

$$\mathbf{p} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

iv. Plot the input from part iii in your diagram from part ii, and verify that it falls in the correctly labeled region.

E4.2 Consider the classification problem defined below.

$$\left\{\mathbf{p}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, t_1 = 1\right\} \left\{\mathbf{p}_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, t_2 = 1\right\} \left\{\mathbf{p}_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, t_3 = 0\right\} \left\{\mathbf{p}_4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, t_4 = 0\right\}.$$

i. Design a single-neuron perceptron to solve this problem. Design the network graphically, by choosing weight vectors that are orthogonal to the decision boundaries.

ii. Test your solution with all four input vectors.

iii. Classify the following input vectors with your solution.

$$\mathbf{p}_5 = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \qquad \mathbf{p}_6 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \mathbf{p}_7 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \mathbf{p}_8 = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

iv. Which of the vectors in part (iii) will always be classified the same way, regardless of the solution values for $\mathbf{W}$ and b? Which may vary depending on the solution? Why?

E4.6 We have four categories of vectors.

$$\text{Category I: } \left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right\}, \text{ Category II: } \left\{ \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$$

$$\text{Category III: } \left\{ \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}, \text{ Category IV: } \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right\}$$

i. Design a two-neuron perceptron network (single layer) to recognize these four categories of vectors. Sketch the decision boundaries.

ii. Draw the network diagram.

iii. Suppose the following vector is to be added to Category I.

$$\begin{bmatrix} -1 \\ -3 \end{bmatrix}$$

Perform one iteration of the perceptron learning rule with this vector. (Start with the weights you determined in part i.) Draw the new decision boundaries.