Asset Allocation with a Carbon Objective

Samuel Brito Rodrigues, Eliot Letac, Ahmad Roukain

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1 Introduction

The global climate crisis is one of the biggest challenges of our generation, threatening the environment, societies around the world and their economies. It's never been more obvious that measures to combat climate change have to be put in place. One concept that has gained popularity in the last years to fight climate change is Sustainable Finance. Sustainable finance refers to the integration of environmental, social, and governance (ESG) criteria into financial decisions. One of the roles of Sustainable Finance is to drive us into a low-carbon economy. One way to do so is to adapt our portfolios by directing capital towards projects and companies that prioritize sustainability. In this paper, we will construct portfolios with carbon constraints in order to make them more sustainable. We will start with two classic asset allocation methods: Markowitz's portfolio and the value-weighted portfolio. The goal will be then to add a 50% carbon footprint constraint to these portfolios in order to make them more sustainable. Finally, we will construct a portfolio with a net zero objective in which the carbon footprint of the portfolio is reduced by 10% per year every year.

2 Data

The original dataset obtained from S&P Trucost contained a total of 2051 firms with the four following regions: North America, Europe, Pacific, Emerging markets. These firms belonged to the following sectors: Energy, Materials, Financials, Industrials, Utilities, Real Estate, Consumer Discretionary, Health Care, Information Technology, Consumer Staples and Communication Services. The data covers three distinct scopes of carbon emissions. Scope 1 includes direct emissions from sources that are owned or controlled by the company, such as those resulting from burning fossil fuels and production processes. Scope 2 pertains to indirect emissions from the consumption of purchased electricity, heat, or steam by the company. Scope 3 covers all other indirect emissions that occur in the value chain of the reporting company, both upstream and downstream. This includes emissions from the extraction and production of purchased materials and fuels, transportation activities in vehicles not owned or controlled by the company, electricity-related activities not included in Scope 2, outsourced activities, waste disposal, and other relevant sources. The emissions for the three scopes cover the years 2005 to 2021 and are expressed in tonnes of CO_2 as well as in tonnes CO_2/million USD for carbon intensity. The rest of the data contained annual revenues (in million USD), yearly and monthly market capitalization (in million USD) and yearly and monthly price data (in USD).

Our group was assigned the ACWI (All Country World Index) for the region. We had to look at highly polluting industries and take into account Scope 1+2+3. We decided to pick the firms from 4 different sectors: Energy, Materials, Utilities and Consumer Staples. These sectors tended to have the highest emissions when looking at the data so we only picked the firms from these industries. From the original 2051 firms we were left with 595 firms to construct our portfolios. We also decided to remove 3 firms from our data due to the presence of infinite values, these firms were: PT Indofood Sukses Makmur Tbk, PT Indocement Tunggal Prakarsa Tbk, PT Astra Agro Lestari Tbk. This left us with a total of 592 firms to conduct our analysis.

3 Standard Asset Allocation

3.1 Methodology

For the first part of our analysis, we want to compare the minimum variance portfolio with the value weighted portfolio. For the minimum variance portfolio, we use the monthly returns from the last 6 years to compute the expected returns and the covariance matrix for a specific year. The allocation for 2006, which is the first year we start investing, is computed at the end of December 2005 using monthly returns from January 2000 to December 2005. The allocation is determined by using the Markowitz's approach:

$$\begin{aligned}
&\underset{\{\alpha_Y\}}{\text{minimize}} && \sigma^2_{p,Y+1} = \alpha'_Y \Sigma_{Y+1} \alpha_Y \\
&\text{subject to} && \alpha'_Y e = 1 \\
&\text{subject to} && \alpha_{i,Y} \geq 0 \quad \text{for all } i
\end{aligned}$$

By rolling the window by one year and iterating until the end of the sample, we get optimal weights for 2006 until 2022. To evaluate the Ex-post performance, we use monthly stock returns and the optimal weights we got from Markowitz's optimization. Each month, a new weight is calculated which accounts for the relative change in the value of the securities in the portfolio. The portfolio return is computed at the end of every month using these monthly weights and the monthly stock returns. This gives us a time series of ex post portfolio returns which we use to evaluate the characteristics of the portfolio.

For the value weighted portfolio, the optimal weights are simply computed by dividing the market capitalization of each firm by the sum of the market capitalization of all the firms. The performance is then given by :

$$R_{t+1}^{(vw)} = \sum_{i=1}^{N} w_{i,t} R_{i,t+1}$$

3.2 Results

3.2.1 Minimum variance portfolio

Metric	$P^{(mv)}$
Annualized Average Return (%)	11.91
Annualized Volatility (%)	11.49
Annualized Sharpe Ratio	0.9290
Minimum Return (%)	-11.20
Maximum Return (%)	11.09
Maximum Drawdown(%)	-23.03

Table 1: $P^{(mv)}$ Financial Metrics

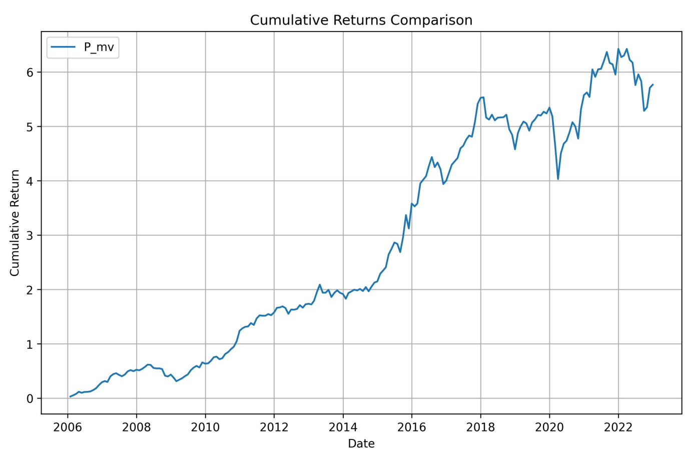

Figure 1: Cumulative returns

3.2.2 Value weighted portfolio

As shown in Table 1 and Figure 1, the cumulative returns plot highlights a significant underperformance of the P^{vw} portfolio compared to the P^{mv} portfolio, with the P^{vw} portfolio showing a 12% return and the P^{mv} portfolio a 7.8% return. The plot clearly illustrates how the portfolios performed during crisis periods. As a world index, the P^{vw} portfolio was impacted by region-specific events.

During the 2008 financial crisis, the cumulative return of the P^{vw} portfolio dropped back to its starting point, unlike the P^{mv} portfolio, which allowed the P^{vw} portfolio to miss out on the compounding effect. A similar pattern was observed in 2011 during the European debt crisis and the US Black Monday. From 2016 onwards, and especially during the 2020 market correction, the P^{mv} portfolio exhibited much higher volatility compared to the P^{vw} portfolio. This may be due to the compounding effect, since the P^{mv} portfolio has a higher value, it experiences larger absolute gains and losses, though not larger in relative terms, due to compounding effect. Indeed, the 2021 correction was more severe in relative terms for the P^{vw} portfolio than for the P^{mv} portfolio.

The Sharpe ratio for the P^{vw} portfolio is about half that of the P^{mv} portfolio, indicating much lower risk-adjusted returns. The P^{vw} portfolio has an overall volatility of 15.38%, compared to 11.5% for the P^{mv} portfolio. Additionally, the absolute values of minimum and maximum returns, as well as the maximum drawdown, are smaller for the P^{vw} portfolio, suggesting it is more stable.

Metric	$P^{(vw)}$	$P^{(mv)}$
Annualized Average Return (%)	7.81	11.91
Annualized Volatility (%)	15.38	11.49
Annualized Sharpe Ratio	0.43	0.93
Minimum Return (%)	-18.44	-11.20
Maximum Return (%)	12.90	11.09
Maximum Drawdown (%)	-46.85	-23.03

Table 2: $P^{(vw)}$ and $P^{(mv)}$ Financial Metrics Comparison

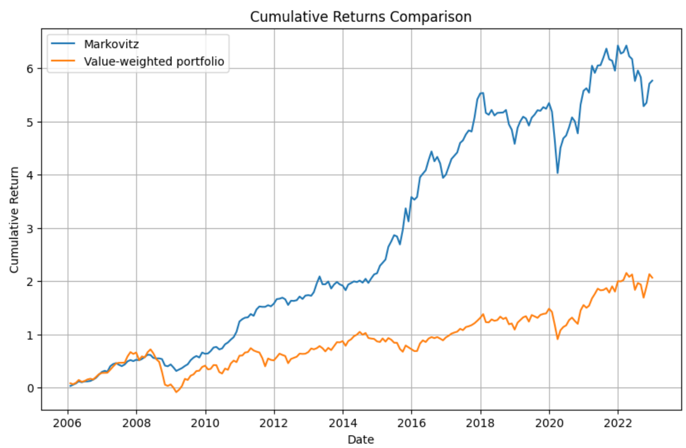

Figure 2: Cumulative Returns Comparison

4 Allocation with a 50% Reduction in Carbon Emissions

4.1 Methodology

Now, we want to add a environmental component to our Markowitz portfolio. We consider carbon intensity for all firms and we compute the weighted average carbon intensity and the carbon footprint which are given by the following formulas:

$$WACI_Y^{(p)} = \sum_{i=1}^{N} \alpha_{i,Y} CI_{i,Y}$$

$$CF_Y^{(p)} = \frac{1}{V_Y} \sum_{i=1}^{N} o_{i,Y} E_{i,Y}$$

where $o_{i,Y} = \frac{V_{i,Y}}{\mathrm{Cap}_{i,Y}}$ measures the fraction of the equity of the firm owned by the portfolio, with $V_{i,Y} = \alpha_{i,Y} V_Y$ the dollar value invested in firm i and $V_Y = \sum_{i=1}^{N} V_{i,Y}$ the dollar value of the portfolio.

We know use this carbon footprint metric to construct a new portfolio with a 50% carbon reduction. We take our initial mean variance allocation to which we add this new constraint:

$$
\begin{aligned}
\min_{\{\alpha_Y\}} \quad & \sigma^2_{p,Y} = \alpha'_Y \Sigma_{Y+1} \alpha_Y \\
\text{s.t.} \quad & CF^{(p)}_Y \leq 0.5 \times CF^{(P,mv)}_Y \\
\text{s.t.} \quad & \alpha_{i,Y} \geq 0 \quad \text{for all } i
\end{aligned}
$$

4.2 Results

4.2.1

Results for WACI and CF are in Table 3 and Table 4 in the next page.

Our WACI is quite high compared to what we generally observe. This is likely due to the composition of our stock universe, which includes the ACWI index that features highly polluting sectors with significant Scope 1, 2, and 3 emissions. It is logical that our WACI is higher than the average because we are exposed to the most carbon-intensive industries, particularly utilities and materials in emerging markets.

Regarding the carbon footprint, it is significantly higher for the P^{mv} compared to the value-weighted portfolio. This is probably due to the lesser diversification in the P^{mv}, where we sometimes allocate 10% or more to a single stock. If that stock is from a company with a small market value, we account for a large portion of its carbon emissions. This can explain the particularly high carbon footprint observed, especially in 2019.

$Date$	$P^{(mv)}$	$P^{(mv)}(0.5)$	$P^{(vw)}$	$P^{(vw)}(0.5)$	$P^{(vw)}(NZ)$
2005	2473.5889	1857.6851	1436.4715	812.7607	1273.8988
2006	2261.1383	1496.3451	1390.4448	755.1883	1309.0027
2007	1393.7773	931.0548	1339.3000	829.5789	1359.7109
2008	881.0818	545.1207	1143.6731	661.2109	724.8205
2009	1982.9993	1527.5025	1270.1271	881.8082	1065.4662
2010	2199.3356	1829.1895	1140.1999	749.1228	905.6925
2011	2091.7017	1938.2796	1019.1338	665.7646	674.9065
2012	1925.9870	1805.7273	1007.3699	658.8893	653.9934
2013	2018.9582	1627.8757	966.9887	622.0857	607.4522
2014	4069.6308	3623.2769	1073.3035	742.6918	659.0414
2015	1871.7999	1053.9999	1008.1519	683.0956	560.4595
2016	1713.1963	1126.4143	1150.4603	780.2171	644.9188
2017	1049.0276	793.3725	1067.1489	803.7779	662.5416
2018	1489.9667	1214.7185	1006.8019	784.3739	532.2121
2019	1982.7907	1420.6669	957.4985	753.0278	537.4859
2020	2175.8346	1909.4131	940.8957	713.6889	532.3825
2021	1203.3451	834.9643	900.4307	683.1496	485.6219

Table 3: All portfolios : Evolution of WACI

$Date$	$P^{(mv)}$	$P^{(mv)}(0.5)$	$P^{(vw)}$	$P^{(vw)}(0.5)$	$P^{(vw)}(NZ)$
2005	1204.4408	602.2204	1368.6175	684.3087	1230.8068
2006	950.5969	475.2985	1140.0631	570.0315	1092.0221
2007	899.9214	449.9607	971.3118	485.6559	968.1579
2008	931.1102	465.5551	1585.3366	792.6683	897.9499
2009	1655.9087	827.9544	1181.7840	590.8920	808.1549
2010	1502.2013	751.1006	1090.6814	545.3407	727.3394
2011	1583.9731	791.9865	1285.3162	642.6581	654.6055
2012	1918.8680	959.4340	1187.8781	593.9391	589.1450
2013	2158.1626	1079.0813	1095.0876	547.5438	530.2305
2014	6052.5756	3026.2878	1100.6940	550.3470	477.2074
2015	1671.2256	835.6128	1125.9579	562.9789	429.4867
2016	1793.1695	896.5847	1035.9455	517.9728	386.5380
2017	1215.4636	607.7318	945.4124	472.7062	347.8842
2018	7556.5437	3778.2719	1110.4014	555.2007	313.0958
2019	16394.7409	8197.3705	924.3492	462.1746	281.7862
2020	8404.5350	4202.2675	817.6989	408.8494	253.6076
2021	3028.8222	1514.4111	825.6759	412.8379	228.2468

Table 4: All Portfolios: Evolution of Carbon Footprint

4.2.2

The performance of the reduced carbon footprint $P^{mv}(0.5)$ is almost identical to that of the P^{mv}. However, its carbon footprint is effectively reduced by 50%. This shows that it is indeed possible to effectively decarbonize a portfolio without impacting its financial performances. Although this comes with others inconveniences as we will discuss later in the section 4.2.4.

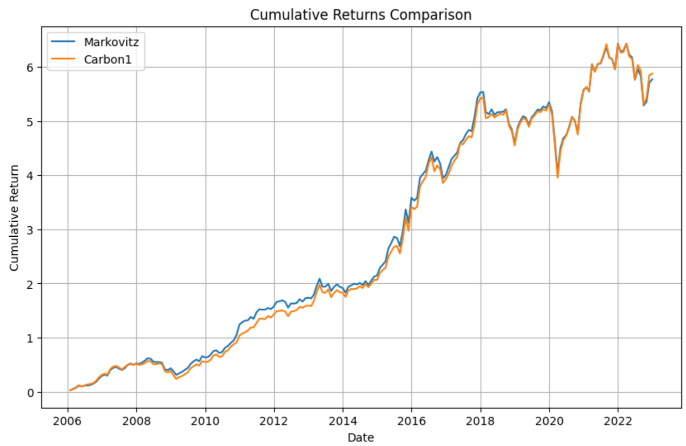

Figure 3: Cumulative returns comparaison

Metric	$P_{oos}^{(mv)}$	$P^{(mv)}$
Annualized Average Return (%)	12.08	11.91
Annualized Volatility (%)	11.69	11.49
Annualized Sharpe Ratio	0.92	0.93
Minimum Return (%)	-11.38	-11.20
Maximum Return (%)	11.09	11.09
Maximum Drawdown (%)	-22.99	-23.03

Table 5: $P_{oos}^{(mv)}$ and $P^{(mv)}$ Financial Metrics Comparison

4.2.3 Methodology and Results

Another strategy that we are going to explore is to maximize the minimum variance criterion for the tracking error every year while reducing the carbon footprint of the value weighted portfolio by 50%. This is done in the following way:

$$\begin{aligned} \min_{\{\alpha_Y\}} \quad & (TE_{p,Y})^2 = (\alpha_Y - \alpha_Y^{(vw)})' \Sigma_{Y+1} (\alpha_Y - \alpha_Y^{(vw)}) \\ \text{s.t.} \quad & CF_Y^{(p)} \leq 0.5 \times CF_Y^{(P,vw)} \\ \text{s.t.} \quad & \alpha_{i,Y} \geq 0 \quad \text{for all } i \end{aligned}$$

where $CF_Y^{(P,vw)} = \frac{1}{\text{Cap}_Y} \sum_{i=1}^{N} o_{i,Y} E_{i,Y}$ denotes the carbon footprint of the value-weighted portfolio, with $\text{Cap}_Y = \sum_{i=1}^{N} \text{Cap}_{i,Y}$ the total market value of the investment set.

The tracking error portfolio $P_{oos}^{(vw)}(0.5)$ has a higher average return compared to the benchmark. This could be due to the portfolio's allocation towards assets with lower carbon footprints, which might have performed better over the period. The volatility of $P_{oos}^{(vw)}(0.5)$ is slightly higher than the benchmark. This indicates that while the portfolio aims to closely follow the benchmark, the adjustments made to reduce the carbon footprint introduced a small amount of additional risk. The (almost) similar Sharpe Ratio for the tracking error portfolio suggests that the $P_{oos}^{(vw)}(0.5)$ manages to achieve its goal of staying close to the benchmark while meeting the carbon footprint constraint, demonstrating that it is possible to construct a low-carbon portfolio without significantly sacrificing performance.

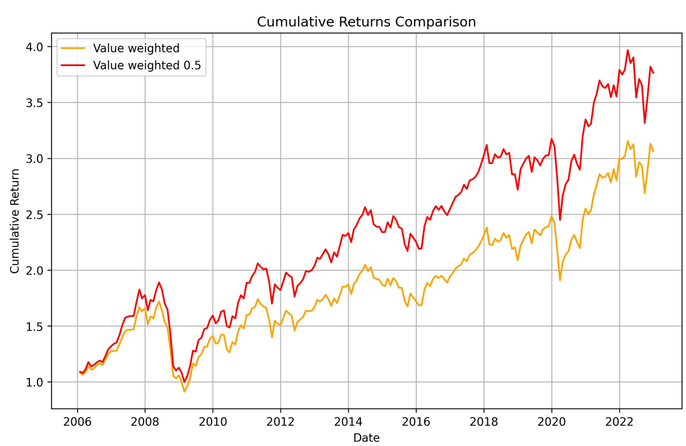

Figure 4: Cumulatives returns comparaison

Metric	$P_{oos}^{(vw)}(0.5)$	$P^{(vw)}$
Annualized Average Return (%)	9.05	7.81
Annualized Volatility (%)	15.51	15.38
Annualized Sharpe Ratio	0.50	0.42
Minimum Return (%)	-20.37	-18.44
Maximum Return (%)	12.16	12.90
Maximum Drawdown (%)	-47.14	-46.85

Table 6: $P_{oos}^{(vw)}$ (0.5) and $P^{(vw)}$ Financial Metrics Comparison

4.2.4

As mentioned earlier, financial performance can negatively be impacted when decarbonizing a portfolio, especially in periods where highly polluting industries perform well. But due to the highly skewed distribution of firms' carbon intensity, it is possible to significantly reduce the carbon footprint of a portfolio by excluding firms with the highest carbon intensity. Our $P^{(mv)}$ still performed as well as the $P^{(mv)}(0.5)$ in 2021, where the energy sector performed the best. A possible explanation is that given our universe of stocks, we still had some exposure to it, as there more polluting industries, especially utilities and materials.

Regarding the $P^{(vw)(0.5)}$, it's performance was a bit better. As mentioned in 4.2.3, this is due to the initial allocation. We believe that it is purely luck though, and we could have borne the negative sides of this allocation. Nevertheless, it has also worked really well and has given us very close volatility to its benchmark, while effectively reducing the ex-ante carbon footprint of the portfolio by half.

It is also interesting to mention that such reallocation can change our sector exposure, but also our regional exposure. Hence in a world portfolio like ours, this could cause poorer performance due to being less exposed in the emerging markets, where the carbon emissions are very high but so is the growth.

5 Allocation with a Net Zero Objective

5.1 Methodology

The last strategy we are going to explore is a progressive decarbonization strategy where the carbon footprint of the portfolio is reduced by $\theta = 10\%$ per year every year from Dec. 2005 to Dec. 2021. We also maximize the minimum variance criterion for the tracking error like we did earlier but this time we replace the old constraint with the new following constraint:

$$CF_Y^{(p)} \leq (1-\theta)^{Y-Y_0+1} \times CF_{Y_0}^{(P,vw)} \quad \text{for } Y = 2005, \ldots, 2021$$

with $Y_0 = 2005$.

5.2 Results

This portfolio shows similar results than the $P^{(vw)}$ for the same reasons explained in the previous point. The optimization has effectively reduced the carbon footprint.

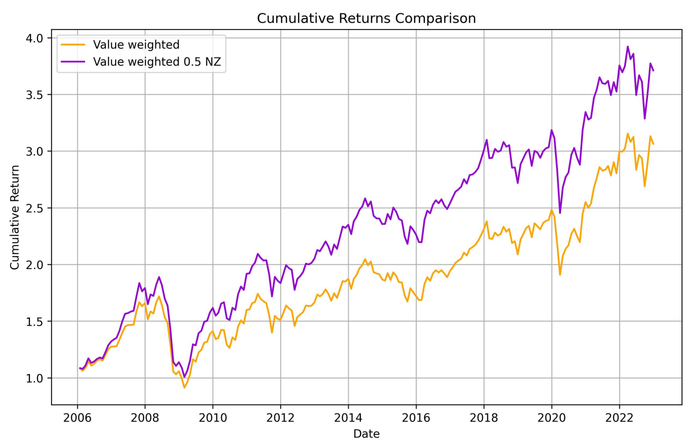

Figure 5: Cumulatives returns comparison

Metric	$P_{oos}^{(vw)}(NZ)$	$P^{(vw)}$
Annualized Average Return (%)	8.97	7.81
Annualized Volatility (%)	15.50	15.38
Annualized Sharpe Ratio	0.50	0.42
Minimum Return (%)	-20.12	-18.44
Maximum Return (%)	12.37	12.90
Maximum Drawdown (%)	-46.63	-46.85

Table 7: $P_{oos}^{(vw)}(NZ)$ and $P^{(vw)}$ Financial Metrics Comparison

6 Conclusion

In our results, the $P^{(mv)}$ outperformed the $P^{(vw)}$ portfolio in terms of returns, while having a higher Sharpe ratio and lower volatility. The introduction of a carbon constraint to the $P^{(mv)}$, reducing its carbon footprint by 50%, did not significantly impact its financial performance. The carbon-reduced $P^{(mv)}(0.5)$

portfolio maintained nearly identical returns and volatility compared to the unconstrained $P^{(mv)}$ portfolio, proving that it is possible to achieve substantial carbon reductions without sacrificing financial performance.

Additionally, we explored the tracking error minimization strategy, which reduced the carbon footprint by 50% while maintaining performance closely aligned with the benchmark. This approach yielded higher average returns and a better Sharpe ratio than the $P^{(vw)}$, demonstrating that sustainable investments can also enhance returns in some cases.

Overall, our results highlight the possibility and benefits of incorporating carbon constraints into investment strategies. The significant reduction in carbon footprint achieved by reallocating our weights into firms with smaller emissions, coupled with the preservation of financial performance, show the potential of sustainable finance to drive the transition to a low-carbon economy. These results reinforce the idea that investors can contribute to mitigate climate change by adopting investment strategies that balance financial returns with environmental responsibility.

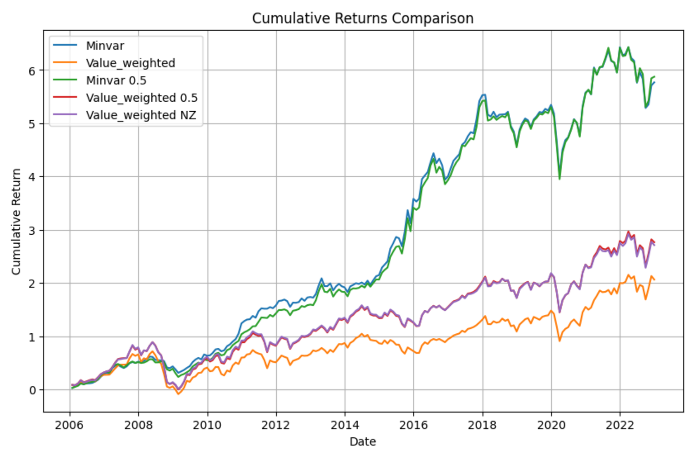

Figure 6: Cumulatives returns comparaison