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IOE 511 - Continuous Optimization Methods
Amijo Wolfe Line Search

As I mentioned in class, for the project, you can implement the **strong Wolfe line search** described in the textbook (*Numerical Optimization, Chapter 3, Algorithm 3.5-3.6).* Alternatively, you can implement the **weak Wolfe line search** whose pseudocode is given below.

Strong Wolfe line search conditions:

$$f(x_k + \alpha_k d_k) \leq f(x_k) + c_1 \alpha_k \nabla f(x_k)^T d_k \tag{1}$$

$$|\nabla f(x_k + \alpha_k d_k)^T d_k| \leq c_2 |\nabla f(x_k)^T d_k| \tag{2}$$

Algorithm 3.5 (and 3.6), Chapter 3, Numerical Optimization.

Weak Wolfe line search conditions:

$$f(x_k + \alpha_k d_k) \leq f(x_k) + c_1 \alpha_k \nabla f(x_k)^T d_k \tag{3}$$

$$\nabla f(x_k + \alpha_k d_k)^T d_k \geq c_2 \nabla f(x_k)^T d_k \tag{4}$$

Pseudo-code:

[0] **Inputs:** x_k (current iterate), d_k (search direction)

[0] **Inputs:** (c_1, c_2) (line search parameters)

[0] **Inputs:** $(\alpha, \alpha_{high}, \alpha_{low}, c)$ (subroutine parameters)

[1] **While** 1

[2] Evaluate f at point $x_k + \alpha d_k$

[3] **If** (3) holds for $\alpha_k = \alpha$

[4] Evaluate ∇f at point $x_k + \alpha d_k$

[5] **If** (4) holds for $\alpha_k = \alpha$

[6] **Break**

[7] **End If** (line 5)

[8] **End If** (line 3)

[9] **If** (3) holds for $\alpha_k = \alpha$

[10] $\alpha_{low} = \alpha$

[11] **Else**

[12] $\alpha_{high} = \alpha$

[13] **End If** (line 9)

[14] **Set** $\alpha = c\alpha_{low} + (1 - c)\alpha_{high}$

Constants:

$0 < c_1 < c_2 < 1$; $\alpha > 0$ (initial α; default: 1); $\alpha_{high} > \alpha_{low} \geq 0$ (defaults: $\alpha_{low} = 0$, $\alpha_{high} = 1000$); $c \in (0, 1)$ (default: 0.5)