2020

INDE 431: Production Planning and Inventory Control

PROJECT 2

JAD ATWEH & AHMAD SHMAYSSANI

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In order to solve this scheduling problem, we decided to model it as an Integer Linear Program (ILP) in order to get an optimal solution for both the first quarter and the year of 2021. In order to get the optimal solution for our ILP we decided to use python (google or tools library) in order to find a feasible schedule.

Note that: An optimal solution for this ILP does not mean that is the optimal solution for this problem, since the problem can be formulated in different ways and each way will lead to a feasible schedule.

Furthermore, we decided to use google OR tools built in Coin-or branch and cut (CBC) solver for Mixed-Integer Programming since this solver allows some of our variables to be integers which is the case for our Boolean variable that will take values of either 0 or 1, whereas the glop linear solver would allow this Boolean to get any value between 0 or 1 without forcing it to be an integer.

Our goal is to balance the load across the workers within each category for the entire schedule, by balancing the number of shifts worked on the weekdays and weekends separately.

Formulation of the Integer Linear Program for the first quarter of 2021:

1. Decision Variables:

$shifts_{i,w,d,s}$ *is a binary decision variable that takes the value of* $\begin{cases} 1 \text{ if the worker } i \text{ takes shift } s \text{ on day } d \text{ in week } w \\ 0 \text{ if the worker } i \text{ doesn't take shift } s \text{ on day } d \text{ in week } w \end{cases}$

a_j: *Balancing variable for weekdays shifts*

b_j : *Balancing variable for weekends shifts*

2. Parameters:
i: worker index (1,2,3,4,5,6,7,8,9,10,11,12)

Note:

Category	Index (i)	Worker	Lp Representation
Senior Associate (SA)	1	CR	$shifts_{1,w,d,s}$ *refers to the shift worked by CR*
	2	FM	$shifts_{2,w,d,s}$ *refers to the shift worked by FM*
	3	MY	$shifts_{3,w,d,s}$ *refers to the shift worked by MY*
	4	BB	$shifts_{4,w,d,s}$ *refers to the shift worked by BB*
Junior Worker (JW)	5	JB	$shifts_{5,w,d,s}$ *refers to the shift worked by JB*
	6	RK	$shifts_{6,w,d,s}$ *refers to the shift worked by RK*
	7	CK	$shifts_{7,w,d,s}$ *refers to the shift worked by CK*
	8	CY	$shifts_{8,w,d,s}$ *refers to the shift worked by CY*
Senior Worker (SW)	9	HT	$shifts_{9,w,d,s}$ *refers to the shift worked by HT*
	10	AY	$shifts_{10,w,d,s}$ *refers to the shift worked by AY*
	11	CA	$shifts_{11,w,d,s}$ *refers to the shift worked by CA*
	12	GH	$shifts_{12,w,d,s}$ *refers to the shift worked by GK*

w: number of the week (1,2,3,4,5,6,7,8,9,10,11,13) for 1 quarter and 1..52 for the entire year.

d: day of the week (1,2,3,4,5,6,7) where 1 is Monday and 7 is Sunday

s: shift number (1,2,3,4,5,6,7)

j: index of balancing variable (1,2,3)

3. Objective Function

$$Min\ Z = (\sum_{j=1}^{3} a_j + \sum_{j=1}^{3} b_j)$$

4. Constraints:

a. Shifts can only be done once per day

i. *For weekdays:*

$$\sum_{w=1}^{13}\sum_{d=1}^{5}\sum_{s=1}^{7}(shifts_{i,w,d,s}) = 1 \quad \forall i = 1 \dots 12$$

ii. *For weekends:*

$$\sum_{w=1}^{13}\sum_{d=6}^{7}(shifts_{i,w,d,1}) = 1$$

$$\sum_{w=1}^{13}\sum_{d=6}^{7}(shifts_{i,w,d,2}) = 1$$

$$\sum_{w=1}^{13}\sum_{d=6}^{7}(shifts_{i,w,d,7}) = 1$$

b. Each worker covers at most one shift per day.

i. *For weekdays:*

$$\sum_{i=1}^{12}\sum_{w=1}^{13}\sum_{d=1}^{7}\sum_{s=1}^{7}\left(shifts_{i,w,d,s}\right) \le 1$$

c. If shift C7 is covered the worker has the next day off.

i. *For each day of the week except the last:*

$$\sum_{i=1}^{12}\sum_{w=1}^{13}\sum_{d=1}^{6}\left(shifts_{i,w,d,7} + shifts_{i,w,d,s}\right) \le 1 \quad \forall s = 1 \dots 7$$

ii. *For Sundays (Last day of the week):*

$$\sum_{i=1}^{12}\sum_{w=1}^{13}\left(shifts_{i,w,7,7} + shifts_{i,(w+1),d,s}\right) \leq 1 \quad \forall s = 1 \ldots 7$$

d. Senior Associate (SA):
Weekdays:

i. Each worker can work at most 1 shift per day:

$$\sum_{i=1}^{4}\sum_{w=1}^{13}\sum_{d=1}^{6}\left(shifts_{i,w,d,1} + shifts_{i,w,d,4}\right) \leq 1$$

$$\sum_{i=1}^{4}\sum_{w=1}^{13}\sum_{d=1}^{6}\left(shifts_{i,w,d,2} + shifts_{i,w,d,3} + shifts_{i,w,d,5} + shifts_{i,w,d,6} + shifts_{i,w,d,7}\right) = 0$$

Weekends:

$$\sum_{i=1}^{4}\sum_{w=1}^{13}\sum_{d=1}^{6}\left(shifts_{i,w,d,1} + shifts_{i,w,d,2}\right) \leq 1$$

$$\sum_{i=1}^{4}\sum_{w=1}^{13}\sum_{d=1}^{7}\sum_{s=3}^{7}\left(shifts_{i,w,d,s}\right) = 0$$

e. Junior Workers (JW):
i. Each worker can work at most 1 shift per day:
Weekday:

$$\sum_{i=5}^{8}\sum_{w=1}^{13}\sum_{d=1}^{6}\left(shifts_{i,w,d,2} + shifts_{i,w,d,3} + shifts_{i,w,d,5} + shifts_{i,w,d,6} + shifts_{i,w,d,7}\right) \leq 1$$

$$\sum_{i=5}^{8}\sum_{w=1}^{13}\sum_{d=1}^{6}\left(shifts_{i,w,d,1} + shifts_{i,w,d,4}\right) = 0$$

Weekend:

$$\sum_{i=5}^{8}\sum_{w=1}^{13}\sum_{d=1}^{6}\left(shifts_{i,w,d,1} + shifts_{i,w,d,2} + shifts_{i,w,d,7}\right) \leq 1$$

$$\sum_{i=5}^{8}\sum_{w=1}^{13}\sum_{d=1}^{7}\sum_{s=3}^{6}\left(shifts_{i,w,d,s}\right)=0$$

f. Senior Workers:

i. <u>One worker can work one shift per day:</u>

<u>*Weekday:*</u>

$$\sum_{i=9}^{12}\sum_{w=1}^{13}\sum_{d=1}^{7}\sum_{s=1}^{6}\left(shifts_{i,w,d,s}\right)\leq 1$$

$$\sum_{i=9}^{4}\sum_{w=1}^{13}\sum_{d=1}^{6}\left(shifts_{i,w,d,7}\right)=0$$

<u>Weekend:</u>

$$\sum_{i=9}^{12}\sum_{w=1}^{13}\sum_{d=1}^{6}\left(shifts_{i,w,d,1}+shifts_{i,w,d,2}\right)\leq 1$$

$$\sum_{i=9}^{12}\sum_{w=1}^{13}\sum_{d=1}^{7}\sum_{s=3}^{7}\left(shifts_{i,w,d,s}\right)=0$$

g. Balancing Constraints

For these constraints, the variable a_j is assigned for the weekdays and b_j for the weekends. The index j is assigned for each category, thus j=1,2,3 (SA=1, JW=2,SW=3). For example, for the SA workers during weekdays the sum of the shifts worked by each worker on all weeks and days is $\leq a_1$. Since the a_j is shared across workers of the same category and our objective function aims on minimizing the variable a_j. The program will give the variable the smallest possible value by making the total shifts of all the workers balanced (for the weekdays of this category). The same logic is applied for the other categories and for the weekends shifts.

For Senior Associates

Weekdays:

$$\sum_{w=1}^{13}\sum_{d=1}^{5}\sum_{s=1}^{7}\left(shifts_{1,w,d,s}\right) \leq a_1$$

$$\sum_{w=1}^{13}\sum_{d=1}^{5}\sum_{s=1}^{7}\left(shifts_{2,w,d,s}\right) \leq a_1$$

$$\sum_{w=1}^{13}\sum_{d=1}^{5}\sum_{s=1}^{7}\left(shifts_{3,w,d,s}\right) \leq a_1$$

$$\sum_{w=1}^{13}\sum_{d=1}^{5}\sum_{s=1}^{7}\left(shifts_{4,w,d,s}\right) \leq a_1$$

Weekdays:

$$\sum_{w=1}^{13}\sum_{d=6}^{7}\sum_{s=1}^{7}\left(shifts_{1,w,d,s}\right) \leq b_1$$

$$\sum_{w=1}^{13}\sum_{d=6}^{7}\sum_{s=1}^{7}\left(shifts_{2,w,d,s}\right) \leq b_1$$

$$\sum_{w=1}^{13}\sum_{d=6}^{7}\sum_{s=1}^{7}\left(shifts_{3,w,d,s}\right) \leq b_1$$

$$\sum_{w=1}^{13}\sum_{d=6}^{7}\sum_{s=1}^{7}\left(shifts_{4,w,d,s}\right) \leq b_1$$

For Junior Workers

Weekdays:

$$\sum_{w=1}^{13}\sum_{d=1}^{5}\sum_{s=1}^{7}\left(shifts_{5,w,d,s}\right) \leq a_2$$

$$\sum_{w=1}^{13}\sum_{d=1}^{5}\sum_{s=1}^{7}(shifts_{6,w,d,s}) \leq a_2$$

$$\sum_{w=1}^{13}\sum_{d=1}^{5}\sum_{s=1}^{7}(shifts_{7,w,d,s}) \leq a_2$$

$$\sum_{w=1}^{13}\sum_{d=1}^{5}\sum_{s=1}^{7}(shifts_{8,w,d,s}) \leq a_2$$

Weekdays:

$$\sum_{w=1}^{13}\sum_{d=6}^{7}\sum_{s=1}^{7}(shifts_{5,w,d,s}) \leq b_2$$

$$\sum_{w=1}^{13}\sum_{d=6}^{7}\sum_{s=1}^{7}(shifts_{6,w,d,s}) \leq b_2$$

$$\sum_{w=1}^{13}\sum_{d=6}^{7}\sum_{s=1}^{7}(shifts_{7,w,d,s}) \leq b_2$$

$$\sum_{w=1}^{13}\sum_{d=6}^{7}\sum_{s=1}^{7}(shifts_{8,w,d,s}) \leq b_2$$

For Senior Workers

Weekdays:

$$\sum_{w=1}^{13}\sum_{d=1}^{5}\sum_{s=1}^{7}(shifts_{9,w,d,s}) \leq a_3$$

$$\sum_{w=1}^{13}\sum_{d=1}^{5}\sum_{s=1}^{7}(shifts_{10,w,d,s}) \leq a_3$$

$$\sum_{w=1}^{13}\sum_{d=1}^{5}\sum_{s=1}^{7}\left(shifts_{11,w,d,s}\right) \leq a_3$$

$$\sum_{w=1}^{13}\sum_{d=1}^{5}\sum_{s=1}^{7}\left(shifts_{12,w,d,s}\right) \leq a_3$$

Weekdays:

$$\sum_{w=1}^{13}\sum_{d=6}^{7}\sum_{s=1}^{7}\left(shifts_{9,w,d,s}\right) \leq b_3$$

$$\sum_{w=1}^{13}\sum_{d=6}^{7}\sum_{s=1}^{7}\left(shifts_{10,w,d,s}\right) \leq b_3$$

$$\sum_{w=1}^{13}\sum_{d=6}^{7}\sum_{s=1}^{7}\left(shifts_{11,w,d,s}\right) \leq b_3$$

$$\sum_{w=1}^{13}\sum_{d=6}^{7}\sum_{s=1}^{7}\left(shifts_{12,w,d,s}\right) \leq b_3$$

where $\boldsymbol{ai, b_i \geq 0}$

Note that: To get the feasible schedule for the entire year we would only need to change the index of the weeks from 1-13 to 1-52.

5. Feasible Schedule:

The optimal solution for this ILP in python for the first quarter was found to be:

Total Shifts per Category

Senior Associates Shifts= 132.0

Junior Workers Shifts= 130.0

Senior Workers Shifts= 271.0

Total Shifts per Worker on Weekdays

Senior Associates

Total Shifts worked by CR=31.0

Total Shifts worked by FM=31.0

Total Shifts worked by MY=31.0

Total Shifts worked by BB=31.0

Junior Workers

Total Shifts worked by JB=25.0

Total Shifts worked by RK=25.0

Total Shifts worked by CK=25.0

Total Shifts worked by CY=25.0

Senior Workers

Total Shifts worked by HT 57.0

Total Shifts worked by AY 58.0

Total Shifts worked by CA 58.0

Total Shifts worked by GK 58.0

Total Shifts per Worker on Weekends

Senior Associates

Total Shifts worked by CR=2.0

Total Shifts worked by FM=2.0

Total Shifts worked by MY=2.0

Total Shifts worked by BB=2.0

Junior Workers

Total Shifts worked by JB=8.0

Total Shifts worked by RK=7.0

Total Shifts worked by CK=8.0

Total Shifts worked by CY=7.0

Senior Workers

Total Shifts worked by HT=10.0

Total Shifts worked by AY=10.0

Total Shifts worked by CA=10.0

Total Shifts worked by GK=10.0

The entire schedule can be found on the csv file ***QuarterSolution_BalancedWithin.xlsx***

The optimal solution for this ILP in python for the year was found to be:

Total Shifts per Category

Senior Associates Shifts= 480.0

Junior Workers Shifts= 712.0

Senior Workers Shifts= 940.0

Total Shifts per Worker on Weekdays

Senior Associates

Total Shifts worked by CR=105.0

Total Shifts worked by FM=105.0

Total Shifts worked by MY=105.0

Total Shifts worked by BB=105.0

Junior Workers

Total Shifts worked by JB=146.0

Total Shifts worked by RK=146.0

Total Shifts worked by CK=146.0

Total Shifts worked by CY=146.0

Senior Workers

Total Shifts worked by HT=204.0

Total Shifts worked by AY=204.0

Total Shifts worked by CA=204.0

Total Shifts worked by GK=204.0

Total Shifts per Worker on Weekends

Senior Associates

Total Shifts worked by CR=15.0

Total Shifts worked by FM=15.0

Total Shifts worked by MY=15.0

Total Shifts worked by BB=15.0

<u>Junior Workers</u>

Total Shifts worked by JB=32.0

Total Shifts worked by RK=32.0

Total Shifts worked by CK=32.0

Total Shifts worked by CY=32.0

<u>Senior Workers</u>

Total Shifts worked by HT=31.0

Total Shifts worked by AY=31.0

Total Shifts worked by CA=31.0

Total Shifts worked by GK=31.0

The entire schedule can be found on the csv file ***YearSolution_BalancedWithin.xlsx***

6. References

1. Mixed-Integer Programming | OR-Tools | Google Developers. (n.d.). Retrieved May 02, 2020, from https://developers.google.com/optimization/mip/integer_opt