Assignment #5

1\. a) $A = \begin{bmatrix} -2 & 5 \\ 3 & 1 \\ -1 & 0 \end{bmatrix}$ $A^T = \begin{bmatrix} -2 & 3 & -1 \\ 5 & 1 & 0 \end{bmatrix}$

$$AA^T = \begin{bmatrix} -2 & 5 \\ 3 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -2 & 3 & -1 \\ 5 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 4+25 & -6+5 & 2+0 \\ -6+5 & 9+1 & -3+0 \\ 2+0 & -3+0 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -1 & 2 \\ -1 & 10 & -3 \\ 2 & -3 & 1 \end{bmatrix}$$

size of AA^T is a 3×3 matrix

b) $(AA^T)^2 - 7(I_3)$

$$= \begin{bmatrix} 29 & -1 & 2 \\ -1 & 10 & -3 \\ 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} 29 & -1 & 2 \\ -1 & 10 & -3 \\ 2 & -3 & 1 \end{bmatrix} - 7\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 841+1+4 & -29-10-6 & 58+3+2 \\ -29-10-6 & -10+100+9 & -2-30-3 \\ 58+3+2 & -6-30-3 & 4+9+1 \end{bmatrix} - \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 846 & -45 & 63 \\ -45 & 99 & -35 \\ 63 & -39 & 14 \end{bmatrix} - \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 839 & -45 & 63 \\ -45 & 92 & -35 \\ 63 & -39 & 7 \end{bmatrix}$$

Assignment #3

Question #2

2. ① $x^2 = (AX^T)^T + BX$

② $x^2 = A^T X + BX$ $(Ax^T)^T \neq A^T x \quad (Ax^T)^T = XA^T$

③ $x^2 - A^T X - BX = 0$

- Line 4 to 5 you can't factor out that x like that because $x(A^T+B) \neq (A^T+B)x$

④ $x^2 - ((A^T)+B)x = 0$

⑤ $x(x-(A^T+B)) = 0$

- Line 6 you can't conclude that $x=0$. $XA=0$ doesn't mean either $A=0$ or $X=0$

⑥ $x = 0$ or $X = A^T + B$

Assignment #5

3. a) $\begin{bmatrix} 6 & -2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $R_1+3R_2 \to R_1$ $\begin{bmatrix} 0 & 3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ $R_1 \leftrightarrow R_2$

$\begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix}$ $-\frac{1}{2}R_1 \to R_1$ $\begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & -\frac{1}{2} \\ 1 & 3 \end{bmatrix}$ $\frac{1}{3}R_2 \to R_2$

$\begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{1}{3} & 1 \end{bmatrix}$ $R_1+\frac{1}{2}R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix}$

-2: inverse DNE

b) $\begin{bmatrix} 1 & 4 & -5 \\ 2 & 0 & 1 \\ 3 & 4 & -4 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $R_2-2R_1 \to R_2$, $R_3-3R_1 \to R_3$ [redacted] $\begin{bmatrix} 1 & 4 & -5 \\ 0 & -8 & 11 \\ 0 & -8 & 11 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$ $R_3-R_2 \to R_3$

$\begin{bmatrix} 1 & 4 & -5 \\ 0 & -8 & 11 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$

A can't be transformed into I so A is not invertible. This is because of the zero row.

c) $\begin{bmatrix} 1 & -1 & 2 \\ -1 & 4 & 3 \\ 2 & 1 & 10 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $R_2+R_1 \to R_2$, $R_3-2R_1 \to R_3$ $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 5 \\ 0 & 3 & 6 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ $R_3-R_2 \to R_3$

$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -3 & -1 & 1 \end{bmatrix}$ $R_1-2R_3 \to R_1$, $R_2-5R_3 \to R_2$ $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 7 & 2 & -2 \\ 16 & 6 & -5 \\ -3 & -1 & 1 \end{bmatrix}$ $R_1+\frac{1}{3}R_2 \to R_1$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{37}{3} & 4 & -\frac{11}{3} \\ 16 & 6 & -5 \\ -3 & -1 & 1 \end{bmatrix}$ $\frac{1}{3}R_2 \to R_2$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{37}{3} & 4 & -\frac{11}{3} \\ \frac{16}{3} & 2 & -\frac{5}{3} \\ -3 & -1 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{37}{3} & 4 & -\frac{11}{3} \\ \frac{16}{3} & 2 & -\frac{5}{3} \\ -3 & -1 & 1 \end{bmatrix}$

-7: need to use inverse & wrong result

4. $Ax=b$, A invertible, Solution: $x=A^{-1}b$

$A=\begin{bmatrix}2 & -4\\ -3 & 1\end{bmatrix}\begin{pmatrix}-22\\ 13\end{pmatrix}=b$ $\quad\begin{bmatrix}2 & -4\\ -3 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} R_1+R_2\to R_1$

$\begin{bmatrix}-1 & -3\\ -3 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} -R_1\to R_1 \begin{bmatrix}1 & 3\\ -3 & 1\end{bmatrix}\begin{bmatrix}-1 & -1\\ 0 & 1\end{bmatrix} R_2+3R_1$

$\begin{bmatrix}1 & 3\\ 0 & 10\end{bmatrix}\begin{bmatrix}-1 & -1\\ -3 & -2\end{bmatrix} \frac{1}{10}R_2\to R_2 \begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}\begin{bmatrix}-1 & -1\\ \frac{-3}{10} & \frac{-2}{10}\end{bmatrix} R_1-3R_2\to R_1$

$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\frac{-1}{10} & -\frac{2}{5}\\ \frac{-3}{10} & \frac{-1}{5}\end{bmatrix} \quad A^{-1}=\begin{bmatrix}\frac{-1}{10} & -\frac{2}{5}\\ \frac{-3}{10} & \frac{1}{5}\end{bmatrix}$

$x=\begin{bmatrix}\frac{-1}{10} & \frac{-2}{5}\\ \frac{-3}{10} & \frac{-1}{5}\end{bmatrix}\begin{bmatrix}-22\\ 13\end{bmatrix}=\begin{bmatrix}\frac{22}{10} & \frac{-26}{5}\\ \frac{66}{10} & \frac{-13}{5}\end{bmatrix}=\begin{bmatrix}\frac{11}{5} & \frac{-26}{5}\\ \frac{33}{5} & \frac{-13}{5}\end{bmatrix}$

Solution $=\begin{bmatrix}\frac{11}{5} & \frac{-26}{5}\\ \frac{33}{5} & \frac{-13}{5}\end{bmatrix}$

5. $A_3=I$ show A is invertible.

if $AB=I$ and $BA=I$ $B=A^{-1}$, unique

then A is invertible

$A\cdot A\cdot A=I$ $\quad A(A^{-1})=I$

$A(A^2)=I$

$A(A^2)=A(A^{-1})$ divide out A

$A^2=A^{-1}$

6. Find a 2×2 matrix X

$$\left(X^T + \begin{bmatrix}3 & 1\\ 11 & 4\end{bmatrix}\right)^{-1} = \frac{1}{2}\begin{bmatrix}3 & 1\\ 11 & 4\end{bmatrix}$$

$$A = \begin{bmatrix}a & b\\ c & d\end{bmatrix} \qquad A = \begin{bmatrix}3 & 1\\ 11 & 4\end{bmatrix}$$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$$

$$A^{-1} = \frac{1}{(3)(4)-(1)(11)}\begin{bmatrix}4 & -1\\ -11 & 3\end{bmatrix}$$

$$A^{-1} = \frac{1}{1}\begin{bmatrix}4 & -1\\ -11 & 3\end{bmatrix}$$

$$X^T + \begin{bmatrix}3 & 1\\ 11 & 4\end{bmatrix} = \left(\frac{1}{2}\begin{bmatrix}3 & 1\\ 11 & 4\end{bmatrix}\right)^{-1}$$

$$X^T + \begin{bmatrix}3 & 1\\ 11 & 4\end{bmatrix} = 2\,(1)\begin{bmatrix}4 & -1\\ -11 & 3\end{bmatrix}$$

$$X^T + \begin{bmatrix}3 & 1\\ 11 & 4\end{bmatrix} = \begin{bmatrix}8 & -2\\ -22 & 6\end{bmatrix}$$

$$X^T = \begin{bmatrix}8 & -2\\ -22 & 6\end{bmatrix} - \begin{bmatrix}3 & 1\\ 11 & 4\end{bmatrix}$$

$$X^T = \begin{bmatrix}5 & -3\\ -33 & 2\end{bmatrix} \qquad X = \begin{bmatrix}5 & -33\\ -3 & 2\end{bmatrix} : \text{Solution}$$

7. a) $(A-AX)^{-1} = X^{-1}B \longrightarrow X(A-AX)^{-1} = B$

Since $A-AX$ and X is invertible and $X(A-AX)^{-1}$ is the product of B then B is invertible. $X(A-AX)^{-1} = B$

b) Solve for x

$(A-Ax)^{-1} = x^{-1}B$

$x(A-Ax)^{-1} = \underbrace{x(x^{-1}}_{I}B)$ multiply by x to get I

$x\underbrace{(A-AX)^{-1}(A-Ax)}_{I} = B(A-Ax)$ multiply both sides by $(A-AX)$ to get I

$x = B(A-Ax)$

8. $A = n \times n$ matrix, columns are L.I.
show rows of A^{-1} spans $\mathbb{R}^n$

$$AI = IA^{-1}$$

$$\begin{array}{cccc} A & I & I & A^{-1} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$n \times n$ is same number of columns & rows. Since it's linearly independent that means there is a pivot position in every column. 3 columns and 3 rows means there's a pivot in every row as well. Since theres a pivot in every row A^{-1} lives in $\mathbb{R}^3$. Also since the
[redacted] columns are L.I. that means the transpose of A will have L.I. rows.

9. a) True consider matrices $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$$\begin{array}{ccc} A^2 & - & B^2 \end{array}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\underbrace{\left(\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right)}_{(A-B)}\underbrace{\left(\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right)}_{(A+B)} =$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & 0 \end{bmatrix}$$

b) False. consider matrices $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

c) False, When you take the transpose of a 2×2 matrix the first row becomes the first column. This means the first number in the first row won't change positions thus resulting in no way for it to be a negative.

d) False, if a matrix is not invertible it has at most one solution

e) True,

$$\left[(A^{-1}B^{T}C)^{-1}\right]^{T} = \left[(A^{-1})^{-1}(B^{T})^{-1}C^{-1}\right]^{T} = \left[A(B^{-1})^{T}(C^{-1})\right]^{T}$$

$$= A^{T}B^{-1}(C^{T})^{-1}$$

f) True, using IMT if the linear transformation defined by $T(x) = Ax$. If T is onto then $(A^{T})^{5}$ or whatever power in inertible based on IMT9 & IMT 12