Problem Statement

The nonlinear dynamical system is given by

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_u u(t) + \Phi(x,u)\\ y(t) &= Cx(t)\end{aligned}$$

where $x(t) \in \mathcal{R}^{nx}$ is the state vector,$u \in \mathcal{R}^{nu}$ is the control input, $y(t) \in \mathcal{R}^{ny}$ is output vector. $\Phi(x,u)$ represents nonlinearity in the system and is assumed to be locally Lipschitz.

Assumption 1
The nonlinear function $\Phi(x,u)$ is assumed to be one-sided Lipschitz [1] if there exists $\rho \in \mathcal{R}$ such that $\forall\ x, z \in \mathcal{D}$ i.e.,

$$\langle \Phi(x,u) - \Phi(z,u), x - z \rangle \leq \rho |x - z|^2$$

where ρ is one sided Lipschitz constant.

Assumption 2
The nonlinear function $\Phi(x,u)$ is assumed to be quadratically inner boundedness [2] [3] if there exists $\delta, \phi \in \mathcal{R}$ such that $\forall\ x, z \in \mathcal{D}$ i.e.,

$$(\Phi(x,u) - \Phi(z,u))^T(\Phi(x,u) - \Phi(z,u)) \leq \delta \|x - z\|^2 + \phi \langle \Phi(x,u) - \Phi(z,u), x - z \rangle$$

- If a function is Lipschitz it is both one sided Lipschitz and also quadratically inner-bounded.
- Examples of these nonlinearities are omitted here and can be found in the below references.

References

[1] W. Zhang, H. Su, H. Wang, and Z. Han, "Full-order and reduced-order observers for one-sided lipschitz nonlinear systems using riccati equations," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 12, pp. 4968–4977, 2012.

[2] M. Abbaszadeh and H. J. Marquez, "Nonlinear observer design for one-sided lipschitz systems," in *Proceedings of the 2010 American Control Conference*. IEEE, 2010, pp. 5284–5289.

[3] R. Wu, W. Zhang, F. Song, Z. Wu, and W. Guo, "Observer-based stabilization of one-sided lipschitz systems with application to flexible link manipulator," *Advances in Mechanical Engineering*, vol. 7, no. 12, p. 1687814015619555, 2015.