ECS 60 **Written Homework #1 (46 points)** **Fall 2015**

Due Friday, October 2, 4:00 pm in 2131 Kemper

1. (2 points) Explain the meaning of: $f(n)$ is $\Theta(1)$.

2. (8 points, 2 points each) Assuming that $f_1(n)$ is $O(g_1(n))$ and $f_2(n)$ is $O(g_2(n))$:
   a. Prove that $f_1(n) + f_2(n)$ is $O(\max(g_1(n), g_2(n)))$.
   b. Prove that $f_1(n) * f_2(n)$ is $O(g_1(n) * g_2(n))$.
   c. Find a counterexample to refute that $f_1(n) - f_2(n)$ is $O(g_1(n) - g_2(n))$.
   d. Find a counterexample to refute that $f_1(n) / f_2(n)$ is $O(g_1(n) / g_2(n))$.
   e.

3. (4 points) Prove that 2^n is $O(n!)$, and $n!$ is not $O(2^n)$.

4. (2 points) Find functions f_1 and f_2 such that both f_1 and f_2 are $O(g(n))$, but $f_1(n)$ is not $O(f_2)$.

5. (2 points) Find the complexity of the function used to find the *k*th integer in an unordered array of integers:

```
int selectKth(int a[], int k, int n) {

  for(int i = 0; i < k; i++) {
    int minI = i;
    for(int j = i + 1; j < n; j++)
      if(a[j] < a[minI])
        minI = j;
    tmp = a[i];
    a[i] = a[minI];
    a[minI] = tmp;
  } // for i

  return a[k - 1];
} // selectKth()
```

6. (14 points, 2 points each) Find the computational complexity for the following code fragments:

```
a. for(int count = 0, i = 0; i < n; i++)
     for(int j = 0; j < n; j++)
       count++;

b. for(int count = 0, i = 0; i < n; i++)
     for(int j = 0; j < i; j++)
       count++;

c. for(int count = 0, i = 1; i < n; i *= 2)
     for(int j = 0; j < n; j++)
       count++;

d. for(int count = 0, i = 1; i < n; i * = 2)
     for(int j = 0; j < i; j++)
       count++;

e. for(int count = 0, i = 0; i < n * n; i++)
     for(int j = 0; j < n; j++)
       count++;

f. for(int count = 0, i = 0; i < n * n; i++)
     for(int j = 0; j < i; j++)
       count++;
```

```
g. for(int count = 0, i = 0; i < n * n; i++)
     if( i % n == 0)
       for(int j = 0; j < i; j++)
         count++;
```

7. (2 points) Find the average case complexity of sequential search in an array if the probability of accessing the last cell equals 1/2, the probability of the next to last cell equals ¼, and the probability of locating a number in any of the remaining cells is the same and equal to $\frac{1}{4(n-2)}$.

8. (2 points) Prove by induction that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$. You must state when you use your inductive hypothesis in your proof.

9. (2 points) Prove by induction that the sum of the first n odd positive integers is n^2, i.e., $1 + 3 + 5 + \ldots + (2n - 1) = n^2$. You must state when you use your inductive hypothesis in your proof.

10. (2 points) What is the big-O of the following power() function? (Hint: it is in terms of the exponent's value)

```
int power(int x, int exponent)
{
  if(exponent == 0)
    return 1;

  if(exponent == 1)
    return x;

  if(exponent mod 2 == 0) // exponent is even
    return power(x * x, exponent / 2);
  else
     return x * power(x * x, exponent / 2);
}
```

11. (6 points, 2 points each) Evaluate the following sums:

a. $\sum_{i=0}^{\infty} \frac{1}{4^i}$

b. $\sum_{i=0}^{\infty} \frac{i}{4^i}$

c. $\sum_{i=0}^{\infty} \frac{i^2}{4^i}$