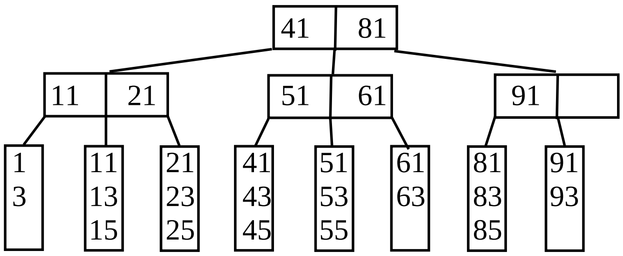
ECS 60 **Written Homework #3: Trees (26 points)** **Fall 2015**

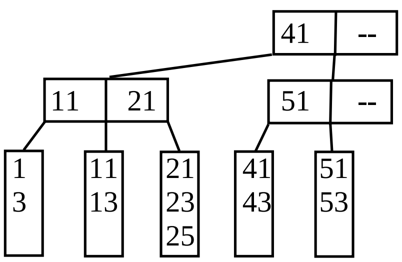
Due Friday, October 23rd, 4:00 pm in 2131 Kemper

1. (5 points, 1 point each) For the following M = 3 and L = 3 BTree as the starting point for each part, draw the BTree that would result after the specified insertion. Sean's Rule for Insertion: Give left, give right, else split.



   a. 95
   b. 84
   c. 44
   d. 54
   e. 22

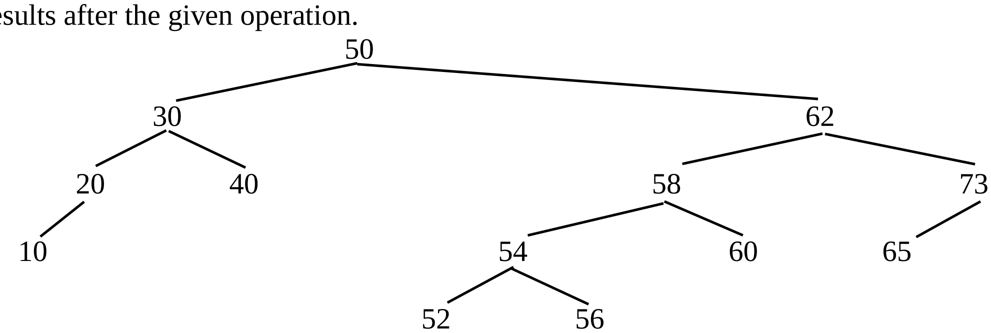
2. (5 points, 1 point each) For the following M = 3 and L = 3 BTree as the starting point for each part, draw the BTree that would result after the specified series of deletion. Sean's Rule for Deletion: Borrow left, merge left, borrow right, merge right.



   a. 21
   b. 43
   c. 51
   d. 3, 11
   e. 3, 53

3. (4 points) Describe a modification to the binary search tree data structure that would allow you to find the median entry, that is the entry with rank $\lfloor n/2 \rfloor$, in a binary search tree. Describe both the modification and the algorithm for finding the median assuming all keys are distinct. (Goodrich, p. 494).

4. (4 points, 1 point each) Using the following binary search tree as the starting point for each part, draw the tree that results after the given operation.



   a. Assuming it is an AVL tree, delete 40.
   b. Assuming it is an AVL tree, insert 57

c. Assuming it is a splay tree, delete 10.
d. Assuming it is a splay tree, insert 55.

5. (2 points) Construct a Huffman trie based on the following distribution of letters. When combining two trees, always place the smaller on the left.

```
A
BB
CCCC
DDDDD
EEEEEE
FFFFFFFF
GGGGGGGGG
HHHHHHHHHH
JJJJJJJJJJJJJ
KKKKKKKKKKKKKK
LLLLLLLLLLLLLLLL
MMMMMMMMMMMMMMMMM
```

6. (2 points) Many vectors are designed to double in size whenever a new element is inserted into a full vector. This operation may entail allocating a new array that is twice the size of the original, and then copying the original elements into the new array before appending the new element. Use one of the three methods taught in class to show that the push_back() operation of such a vector is amortized O(1).

7. (2 points) Use induction to show that the maximum number of nodes in a binary tree of height h is $2^{h+1} - 1$.

8. (2 points) A *full node* is a node with two children. Prove that the number of full nodes plus one is equal to the number of leaves in a nonempty binary tree. (Weiss, p.183).

Sources of questions:

Michael T. Goodrich, Roberto Tamassia, and David Mount, *Data Structures & Algorithms, Second Edition*, Hoboken, NJ, John Wiley & Sons, 2011.

Mark Weiss, *Data Structures and Algorithm Analysis in C++, Fourth Edition,* New York, NY, Pearson Education, 2014.