ECS 60 Written Homework #6: Graph Algorithms (30 points) Fall 2015

Due Wednesday, November 18th, 4:00 pm in 2131 Kemper

1. (10 points, 1 point each) For each of the following scenarios, decide which graph algorithms would be best to use: critical path analysis, unweighted shortest path, Dijkstra's, Kruskal's, Ford-Fulkerson, or none of the above.
   a. Determine the number of people to assign to each job in a large project.
   b. Given a map of the power grid, the production of power plants, and the power consumption of the cities, determine how best to transfer electricity from one city to another to minimize the amount of electricity transferred within the country.
   c. Given a road map of a city, determine which bridges must be reinforced because they are the only way to get from some neighborhood to the rest of the city.
   d. Given the road map of a city, for each house in a city, determine their nearest and second nearest elementary school.
   e. Given the population of an endangered area, and a road map that include the number of people each road can carry in an hour, determine how long it would take to evacuate the area.
   f. Given blueprints that show the location of the main electrical panel and power outlets, design the wiring of a building to minimize the amount of wire used.
   g. It is 3am and you are in Manhattan where all the blocks are the same length, and have 25 mph speed limits, determine the quickest route to your destination.
   h. Given the seven recipes, determine the length of time for you to prepare a seven-course dinner.
   i. Given a map of all the blood vessels of the brain, determine how long it will take for blood entering from the carotid artery to reach every parts of the brain.
   j. Given a list of the times flights will arrive at an airport, and their fuel remaining when they arrive, provide an ordering of the flight landings than ensures all will land before they run out of fuel.

2. (2 points) Weiss 9.6
   What is the worst-case running time of Dijkstra's algorithm when implemented with d-heaps (Section 6.5)?

3. (4 points, 2 points each) Weiss 9.38
   You are given a set of N sticks, which are lying on top of each other in some configuration. Each stick is specified by its two endpoints; each endpoint is an ordered triple giving its x, y, and z coordinates; no stick is vertical. A stick may be picked up only if there is no stick on top of it.
   a. Explain how to write a routine that takes two sticks a and b and reports whether a is above, below, or unrelated to b. (This has nothing to do with graph theory.)
   b. Give an algorithm that determines whether it is possible to pick up all the sticks, and if so, provides a sequence of stick pickups that accomplishes this.

4. (6 points, 2 points each) Weiss 9.53
   The object of the Kevin Bacon Game is to link a movie actor to Kevin Bacon via shared movie roles. The minimum number of links is an actor's Bacon number. For instance, Tom Hanks has a Bacon number of 1; he was in *Apollo 13* with Kevin Bacon. Sally Fields has a Bacon number of 2, because she was in *Forrest Gump* with Tom Hanks, who was in *Apollo 13* with Kevin Bacon. Almost all well-known actors have a Bacon number of 1 or 2. Assume that you have a comprehensive list of actors, with roles, and do the following:
   a. Explain how to find an actor's Bacon number.
   b. Explain how to find the actor with the highest Bacon number.
   c. Explain how to find the minimum number of links between two arbitrary actors.

5. (8 points, 2 points each) Given the below set of figures between US airports apply the specified algorithms. Fill in the tables that have the layouts identical to the samples provided below, including airport ordering, so that it is easier for the TA to grade them.

	BOS	BWI	DFW	JFK	LAX	MIA	ORD	PVD	SFO
BOS				187		1258	867		2704
BWI				184		946	621		
DFW				1391	1235	1121	802		1464
JFK	187	184	1391			1090	740	144	
LAX			1235			2342			337
MIA	1258	946	1121	1090	2342				
ORD	867	621	802	740				849	1846
PVD				144			849		
SFO	2704		1464		337		1846		

a. Assuming the numbers are miles, provide a Dijkstra table for the shortest paths from SFO.
b. Assuming the numbers are miles, provide a Prim's table for the minimum spanning tree starting from MIA.
c. Assuming the numbers are miles, provide the Union by Size table for Kruskal's. When two trees have the same size, have the airport with the smaller index be the new root.
d. Assuming the numbers are the number of people possible to carry between cities, apply Ford-Fulkersons to determine the number of people that can get from LAX to JFK

Table format for parts a and b

	Known	d_v	p_v
BOS			
BWI			
DFW			
JFK			
LAX			
MIA			
ORD			
PVD			
SFO			

Table format format for part c, Kruskal's

0	1	2	3	4	5	6	7	8
BOS	BWI	DFW	JFK	LAX	MIA	ORD	PVD	SFO

Table format for Ford-Fulkerson Solution. Number of passengers flying between cities.

		Destinations								
		BOS	BWI	DFW	JFK	LAX	MIA	ORD	PVD	SFO
Origins	BOS	X								
	BWI		X							
	DFW				X					
	JFK	X	X	X	X	X	X	X	X	X
	LAX					X				
	MIA						X			
	ORD							X		
	PVD								X	
	SFO									X