Homework 1

The pdf you submit must look exactly like this with the answers and all supporting works shown on the the page with the question.

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1. (5 points) Use Boolean Algebra to prove that

$$(\bar{A}*B*\bar{C})+(\bar{A}*B*C)+(A*\bar{B}*C)+(A*B*\bar{C})+(A*B*C)=(A+B)*(B+C)$$

$$(\bar{A}\cdot B\cdot \bar{C})+(\bar{A}\cdot B\cdot C)+(A\cdot \bar{B}\cdot C)+(A\cdot B\cdot \bar{C})+(A\cdot B\cdot C)=(A+B)\cdot(B+C)$$

Taking $(\bar{A}\cdot B)$ common

$$(\bar{A}\cdot B)(\bar{C}+C)+(A\cdot\bar{B}\cdot C)+(A\cdot B\cdot\bar{C})+(A\cdot B\cdot C)=(A+B)\cdot(B+C)$$

Taking $(A\cdot B)$ common and putting $(C+\bar{C})=1$

$$(\bar{A}\cdot B)(1)+(A\cdot\bar{B}\cdot C)+(A\cdot B)(\bar{C}+C)+(A\cdot B\cdot C)=(A+B)\cdot(B+C)$$

putting $(\bar{C}+C)=1$

$$(\bar{A}\cdot B)(1)+(A\cdot\bar{B}\cdot C)+(A\cdot B)(1)+(A\cdot B\cdot C)=(A+B)\cdot(B+C)$$

$$(\bar{A}\cdot B)+(A\cdot B)+(A\cdot\bar{B}\cdot C)+(A\cdot B\cdot C)=(A+B)\cdot(B+C)$$

taking C and A common

$$(\bar{A}\cdot B)+(A\cdot B)+(A\cdot C)(\bar{B}+B)=(A+B)\cdot(B+C)$$

setting $(B+\bar{B}=1)$

$$(\bar{A}\cdot B)+(A\cdot B)+(A\cdot C)=(A+B)\cdot(B+C)$$

Taking B common

$$B(\bar{A}+A)+(A\cdot C)=(A+B)\cdot(B+C)$$

By $\bar{A}+A=1$

$$B+(A\cdot C)=(A+B)\cdot(B+C)$$

$$(B+A)\cdot(B+C)=(A+B)\cdot(B+C)$$

Rearranging

$$(A+B)\cdot(B+C)=(A+B)\cdot(B+C)$$

LHS = RHS

2. (3 points) Prove that A XOR B = $A * \bar{B} + \bar{A} * B$

Truth table for A XOR B

A	B	$A \oplus B$
0	1	0
0	0	1
1	1	1
1	0	0

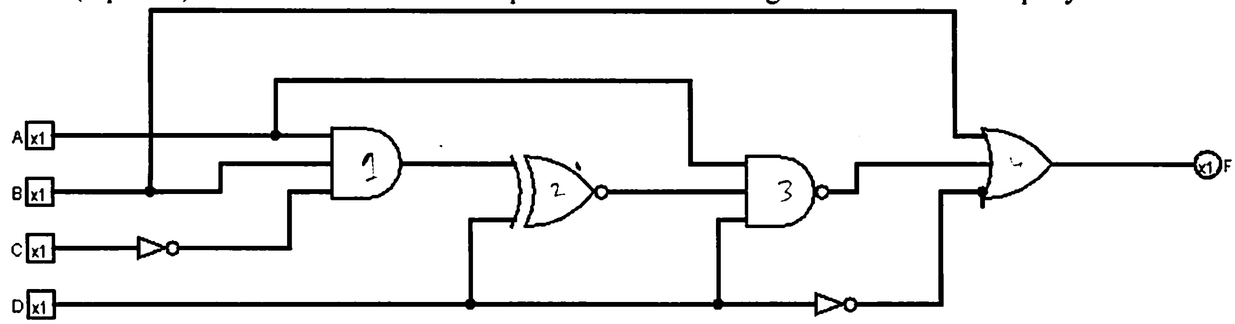
Truth table for expression $A \cdot \bar{B} + \bar{A} \cdot B$

A	B	$\bar{A}$	$\bar{B}$	$A \cdot \bar{B}$	$\bar{A} \cdot B$	$A \cdot \bar{B} + \bar{A} \cdot B$
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

Both sides are equal

$\therefore$, A XOR B $= A \cdot \bar{B} + \bar{A} \cdot B$

3. (3 points) Write the function that represents the following circuit. Do not simplify.



$1 = A \cdot B \cdot \bar{C}$

XOR $\oplus$

$2 = D$

AND ~

$3 = (A * D)$ ~

$4 = B + \bar{D}$

Solution:

$[(((A \cdot B \cdot \bar{C}) \oplus D)\sim * A * D)\sim B + \bar{D}]$

4. Given the following truth table

A	B	C	Output
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

1. (3 points) Write a function in SOP form that behaves according to the truth table. Do not simplify.

$$= (\bar{A}\cdot\bar{B}\cdot\bar{C}) + (\bar{A}\cdot\bar{B}\cdot C) + (\bar{A}\cdot B\cdot\bar{C}) + (A\cdot\bar{B}\cdot C) + (A\cdot B\cdot\bar{C})$$

2. (3 points) Write a function in POS form that behaves according to the truth table. Do not simplify.

$$= (A+\bar{B}+\bar{C})\cdot(\bar{A}+B+C)\cdot(\bar{A}+\bar{B}+\bar{C})$$

5. (3 points each) For each of the following problems assume that the variables are $x_0 - x_{N-1}$, with x_0 representing the least significant bit and x_{N-1} the most significant. For example if we had an equation of 3 variables, $m_1 = \bar{x}_2 * \bar{x}_1 * x_0$ and $m_6 = x_2 * x_1 * \bar{x}_0$. For each of the following problems write each function in **both** its most simplified SOP and POS form. There are a total of **5** subquestions

1. $m_0 + m_1 + m_2$

X_0	X_1	output
0	0	1
0	1	1
1	0	1
1	1	0

$x_1 \backslash x_0$	0	1
0	1	1
1	1	0

POS: $\bar{X}_1 + \bar{X}_0$

SOP: $\bar{X}_1 + \bar{X}_0$

2. $M_0 * M_3 * M_4 * M_7$

x_0	x_1	x_2	Out
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

x_2 \ x_0x_1	00	01	11	10
0	0	1	1	0
1	1	0	0	1

SOP: $(x_1 \cdot \bar{x}_2) + (\bar{x}_1 \cdot x_2)$

POS: $(x_1 + x_2) \cdot (\bar{x}_1 + \bar{x}_2)$

3. $m_4+m_5+m_7+m_{12}+m_{13}+m_{15}$

	x_0	x_1	x_2	x_3	out
1	0	0	0	0	0
2	0	0	0	1	0
3	0	0	1	0	0
4	0	0	1	1	0
5	0	1	0	0	1
6	0	1	0	1	1
7	0	1	1	0	0
8	0	1	1	1	1
9	1	0	0	0	0
10	1	0	0	1	0
11	1	0	1	0	0
12	1	0	1	1	0
13	1	1	0	0	1
14	1	1	0	1	1
15	1	1	1	0	0
16	1	1	1	1	1

x_0x_1 \ x_2x_3	00	01	11	10
00	0	0	0	0
01	1	1	1	0
11	1	1	1	0
10	0	0	0	0

POS: $(\bar{x}_2+x_3)\cdot x_1$

SOP: $(x_1\cdot\bar{x}_2)+(x_1\cdot x_3)$

4. $m_0 + m_3 + m_4 + m_8 + D_2 + D_5 + D_7 + D_{10} + D_{13} + D_{15}$

	X_0	X_1	X_2	X_3	OUT
1	0	0	0	0	1
2	0	0	0	1	0
3	0	0	1	0	d
4	0	0	1	1	1
5	0	1	0	0	1
6	0	1	0	1	d
7	0	1	1	0	0
8	0	1	1	1	d
9	1	0	0	0	1
10	1	0	0	1	0
11	1	0	1	0	d
12	1	0	1	1	0
13	1	1	0	0	0
14	1	1	0	1	d
15	1	1	1	0	0
16	1	1	1	1	d

X_0X_1 \ X_2X_3	00	01	11	10
00	1	0	1	d
01	1	d	d	0
11	0	d	d	0
10	1	0	0	d

SOP: $(\bar{X}_0 \cdot \bar{X}_2 \cdot \bar{X}_3) + (X_2 \cdot \bar{X}_0 \cdot \bar{X}_1) + (\bar{X}_2 \cdot \bar{X}_3 \cdot \bar{X}_1)$

POS: $(\bar{X}_3 + X_2) \cdot (\bar{X}_3 + \bar{X}_2) \cdot (\bar{X}_2 + X_3) \cdot (\bar{X}_1 + \bar{X}_3)$

5. $m_1 + m_3 + m_7 + m_9 + m_{11} + m_{15} + m_{17} + m_{19} + m_{25} + m_{27} + D_4 + D_6 + D_{12} + D_{14} + D_{16} + D_{18} + D_{20} + D_{22} + D_{24} + D_{26} + D_{28} + D_{30}$

	X_0	X_1	X_2	X_3	X_4	Out
1	0	0	0	0	0	
2	0	0	0	0	1	
3	0	0	0	1	0	
4	0	0	0	1	1	
5	0	0	1	0	0	
6	0	0	1	0	1	
7	0	0	1	1	0	
8	0	0	1	1	1	
9	0	1	0	0	0	
10	0	1	0	0	1	
11	0	1	0	1	0	
12	0	1	0	1	1	
13	0	1	1	0	0	
14	0	1	1	0	1	
15	0	1	1	1	0	
16	0	1	1	1	1	
17	1	0	0	0	0	
18	1	0	0	0	1	
19	1	0	0	1	0	
20	1	0	0	1	1	
21	1	0	1	0	0	
22	1	0	1	0	1	
23	1	0	1	1	0	
24	1	0	1	1	1	
25	1	1	0	0	0	
26	1	1	0	0	1	
27	1	1	0	1	0	
28	1	1	0	1	1	
29	1	1	1	0	0	
30	1	1	1	0	1	
31	1	1	1	1	0	
32	1	1	1	1	1	

$x_0x_1x_2 \backslash x_3x_4$	00	01	11	10
000	0	1	1	0
001	d	0	1	d
011	d	0	1	d
010	0	1	1	0

$x_0x_1x_2 \backslash x_3x_4$	00	01	11	10
100	X	1	1	X
101	X	0	0	X
111	X	0	0	X
110	X	1	1	X

$x_0x_1x_2 \backslash x_3x_4$	00	01	11	10
000	0	1	1	0
001	X	0	1	X
011	X	0	1	X
010	0	1	1	0

$x_0x_1x_2 \backslash x_3x_4$	00	01	11	10
100	X	1	1	X
101	X	0	0	X
111	X	0	0	X
110	X	1	1	X

SOP: $x_4 \cdot \overline{x_2} + x_3 \cdot x_2 \cdot \overline{x_0}$

POS: $x_4 \cdot (x_3 + \overline{x_2})(x_0 + x_2)$