Results of tests performed on MEBDF.m

We follow the same testing as was done for the original MEBDF code, further details of these problems may be found at www.imperial.ac.uk/~jcash. Among these problems are some selected DAEs from the Amsterdam Test set (http://pitagora.dm.uniba.it/~testset//).

Here the computed solution y^{comp}, the true solution y^{true}, the error $abs(y^{comp} - y^{true})$ and a plot of the numerical solution (where appropriate) are given. We will work our way through the test set beginning first with the Van Der Pol equation.

1.1 I.V.P Tests.

Problem 1: Van Der Pol equation of dimension 2.

Numerical results at the end of range of integration ([0,11]).

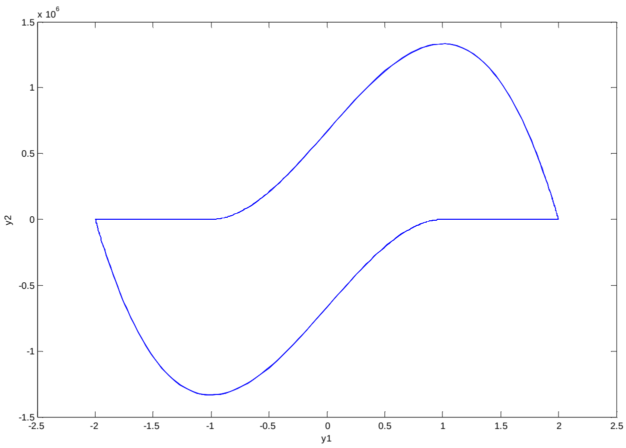
h_0 = *1E-8.*

RTOL = ATOL = 1E-7.

CPU Time: 11.5867s

n	y_n^{comp}	y_n^{true}	*error*
1	-1.590154210734571	-1.590150544829062	3.665906E-6
2	1.0402738448255782	1.040279389212485	5.544387E-6

Plot:



Problem 2:The Robertson problem of Dimension 3.

Numerical results at the end of range of integration ([0,1E11]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

*ATOL =10E-4*RTOL*

CPU Time: 1.5022s

n	y_n^{comp}	y_n^{true}	*error*
1	.20833423783730993e-7	0.2083340149701284E-7	3.962506E-017
2	.83333696850181525e-13	0.8333360770334744E-13	1.585015E-022
3	.99999997916650452	0.9999999791665152	2.309264E-014

Problem 3: Hires problem of dimension 8.

Numerical results at the end of the range of integration ([0, 421.8122]).

$h_0 =$ 1E-8.

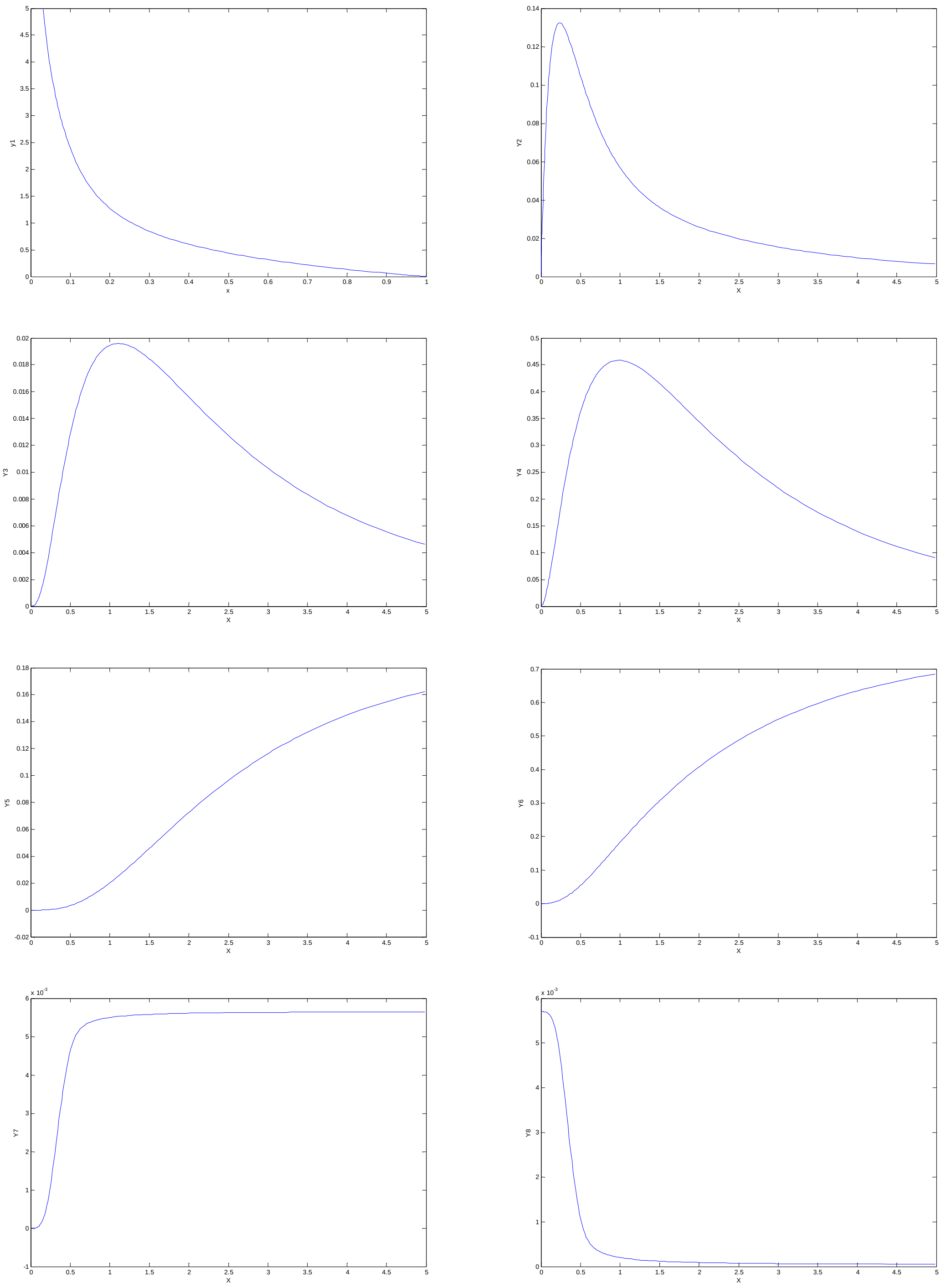
RTOL = 1E-7.

*ATOL =10E-3*RTOL*

CPU Time: 1.0215s

n	y_n^{comp}	y_n^{true}	*error*
1	.67030550358598677e-3	.670305503581864E-3	4.469011E-12
2	.13099684698717200e-3	.130996846986347E-13	8.943264E-13
3	.46862231598435660e-4	.46862231597733E-4	7.621378E-13
4	.10446680205605289e-2	.1044668020551705E-2	9.549031E-12
5	.59488383104042413e-3	.594883830951485E-3	9.661749E-11
6	.13996288341347543e-2	.1399628833942774Ee-2	2.085274E-10
7	.10144927578447629e-2	.1014492757718480e-2	1.370960E-10
8	.46855072421554071e-2	.4685507242281520e-2	1.370960E-10

__Plots on the rang [0,5]:__



Problem 4:Cusp catastrophe model of dimension 96.

Selected numerical results at the end of range of integration ([0,1.1]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

ATOL =RTOL

CPU Time: 7.1703s

n	y_n^{comp}	y_n^{true}	*error*
1	*-1.3350382458480710*	*-1.335038235173363825*	1.067471E-8
13	*-.92255149594056229*	*-0.922551477213655165*	1.872691E-8
25	*-.19267159762159494*	*-0.192671593795137051*	3.826458E-9
38	*1.3798048023657081*	*1.379804789392094260*	1.297361E-8
50	*.16998127433462953*	*0.169981336507011738*	6.217238E-8
62	*-1.2692402304318338*	*-1.269240297385074114*	6.695324E-8
74	*-1.6950676417198314*	*-1.695067644864453216*	3.144622E-9
86	*-1.4558605334458385*	*-1.455860565434940201*	3.198910E-8

Problem 5: Elastic beam problem of dimension 80.

Selected numerical results at the end of range of integration ([0,5]).

$h_0 =$ 1E-8.

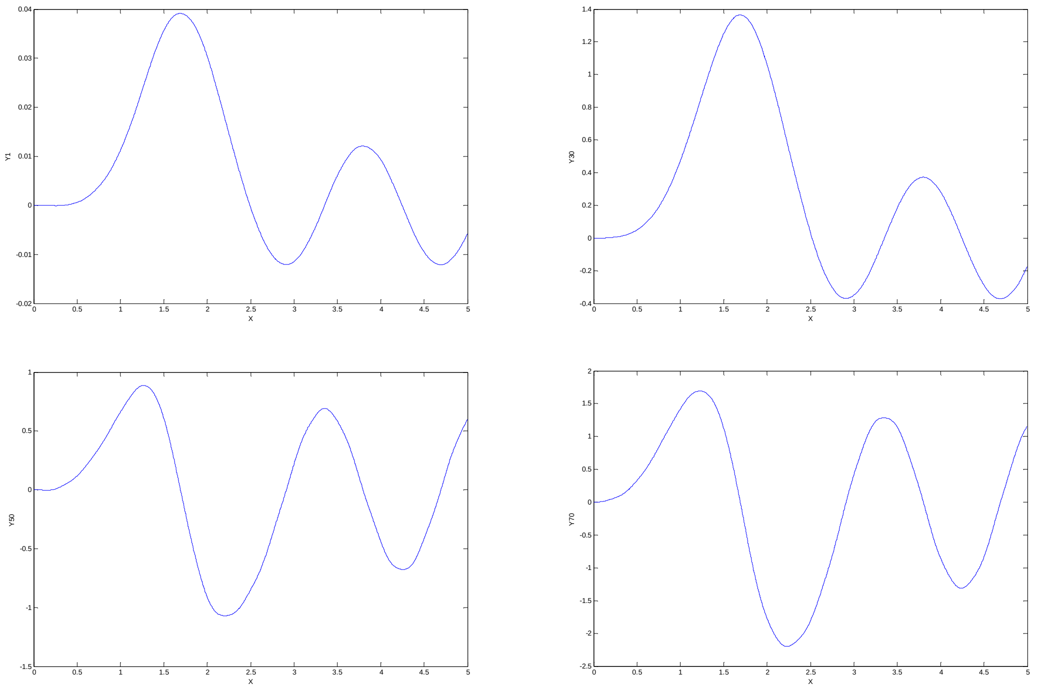
RTOL = 1E-7.

ATOL =RTOL

CPU Time: 22.8428s

n	y_n^{comp}	y_n^{true}	*error*
1	*-.57923576629327405e-2*	*-1.335038235173363825*	8.928362E-9
10	*-.91191340397807130e-1*	*-0.922551477213655165*	6.148639E-9
20	*-.14835962931091259*	*-0.192671593795137051*	8.206466E-8
30	*-.17258205517919159*	*1.379804789392094260*	1.295670E-7
40	*-.17672038610916155*	*0.169981336507011738*	2.100017E-7
50	*.59874648570712630*	*-1.269240297385074114*	1.448454E-5
60	*.98688427446686833*	*-1.695067644864453216*	9.492317E-6
70	*1.1567853566355060*	*-1.455860565434940201*	6.775331E-6

Plots:



Problem 6: Movement of a rectangular plate of dimension 80.

Selected numerical results at the end of range of integration ([0,7]).

h_0 = 1E-8.

$RTOL$ = 1E-7.

$ATOL$ =$RTOL$

CPU Time: 1.1116s

n	y_n^{comp}	y_n^{true}	*error*
1	.49014379881524373e-3	0.000490143813851336	1.503609E-11
10	.16978849525463367e-2	0.001697885005625757	5.307942E-11
20	.38316448519552034e-2	0.003831644928823870	7.686867E-11
30	.43202316301815718e-2	0.004320231736082990	1.059014E-10
40	.14383125580403806e-2	0.001438312591933600	3.389322E-11
50	-.41748293677851102e-2	-0.004174829877953482	5.101684E-10
60	-.10157559985999431e-1	-0.010157560112096413	1.260970E-10
70	-.12352388017072103e-1	-0.012352389570141255	1.553069E-9

Problem 7: Brusselator problem with 1D diffusion of dimension 1000.
Selected numerical results at the end of range of integration ([0,10]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

ATOL =RTOL

CPU Time: 16.8642s

n	y_n^{comp}	y_n^{true}	*error*
1	*.99491970009316177*	*0.9949197002317599*	1.385981E-10
204	*3.4157106717145100*	*3.4157099395342736*	7.321802E-7
400	*3.5175260238761230*	*3.5175249049438184*	1.118932E-6
603	*.45630388178269371*	*0.4563039400688689*	5.828618E-8
806	*3.4187569653313088*	*3.4187562033108012*	7.620205E-7
995	*.98483673026147256*	*0.9848367306701233*	4.086508E-10

Problem 8: Oregonator, Belusov-Zhabotinskii reaction of dimension 3.
Numerical results at the end of range of integration ([0,360]).

$h_0 =$ 1E-8.

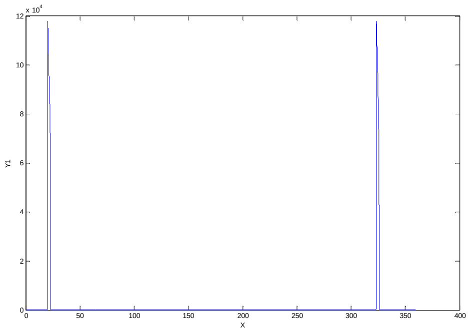
RTOL = 1E-7.

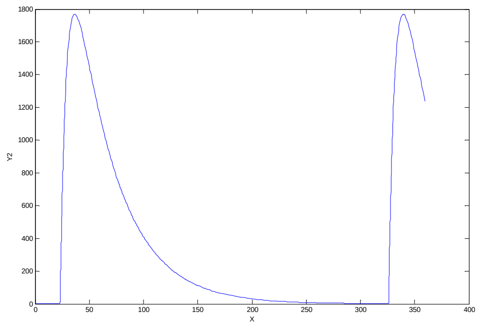
*ATOL =10^-4*RTOL*

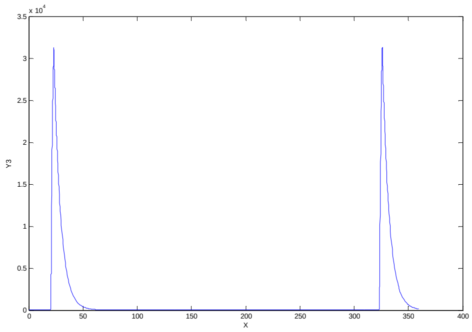
CPU Time: 3.3648s

n	y_n^{comp}	y_n^{true}	*error*
1	*1.0008148702143438*	*1.000814870318523*	1.041791E-010
2	*1228.1785946893915*	*1228.178521549889*	7.313950E-005
3	*132.05553211189832*	*132.0554942846513*	3.782725E-005

Plots:







<u>Problem 9: E5 of dimension 4.</u>

Numerical results at the end of range of integration ([0,1E13]).

h_0 = 1E-8.

RTOL = 1E-7.

ATOL =1.7E-4

CPU Time: 2.4035s

n	y_n^{comp}	y_n^{true}	*error*
1	*.83187938179125541316e-58*	*0*	8.318794E-059
2	*.88356773097319198572e-22*	*8.86123349762637834 20E-23*	2.555619E-025
3	*.88842038543680930950e-22*	*8.8612334976263783421E-23*	2.297036E-025
4	*.50343603019680743340e-71*	*0*	5.034360E-072

Problem 10: The B5 (50, 200, 500, 800, 1000) equations.

B550 equation:

Numerical results at the end of range of integration ([0, 20.5108]).

$h_0 =$ 1E-8.

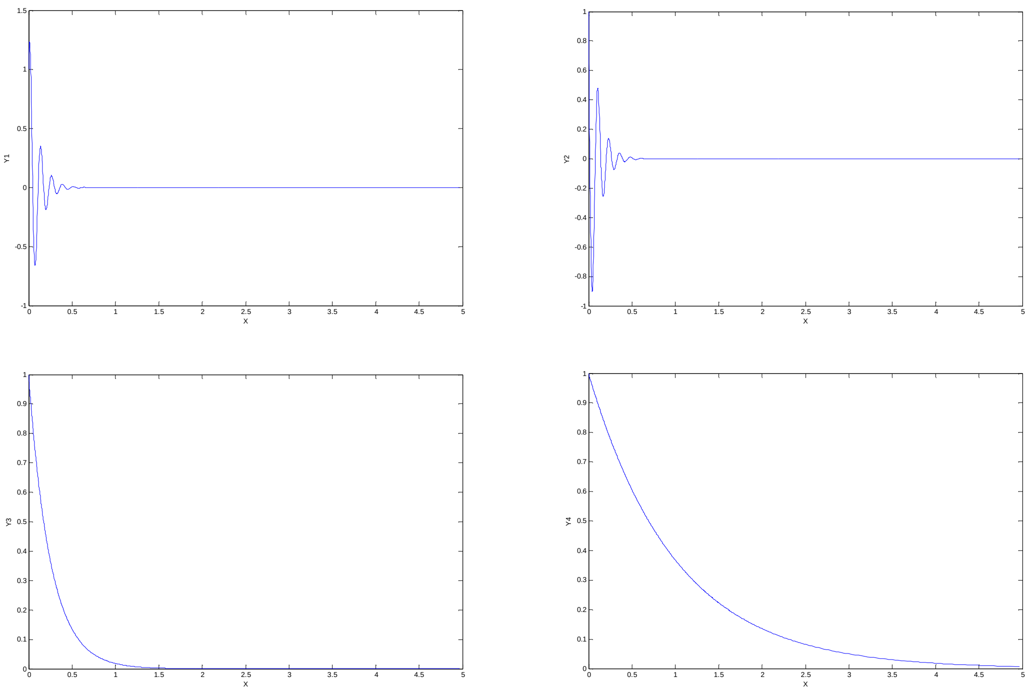
RTOL = 1E-7.

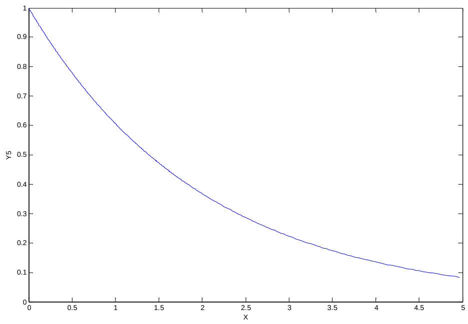
ATOL =RTOL.

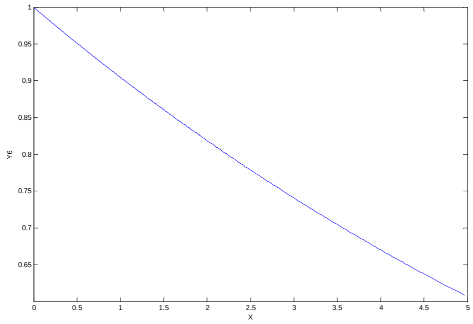
CPU Time: 0.8012s.

n	y_n^{comp}	y_n^{true}	*error*
1	*.10934913663539676e-19*	*.97879969570047458e-89*	*1.093491E-20*
2	*-.10716885037297267e-21*	*-.66506274552447790e-89*	*1.071689E-22*
3	*.76010191183159531e-15*	*.23390622114965014e-35*	*7.601019E-16*
4	*.12544492784601787e-8*	*.12366888442931234e-8*	*1.776043E-11*
5	*.35167029604437110e-4*	*.35166587043571965e-4*	*4.425609E-10*
6	*.12859558081464298*	*.12859558081436012*	*2.828571E-13*

Plots:







B5200 equation:

Numerical results at the end of range of integration ([0, 20.0569]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

ATOL =RTOL.

CPU Time: 2.1331s.

n	y_n^{comp}	y_n^{true}	*error*
1	*.68729964784255835e-33*	*-.37890586590763557e-87*	*6.872996E-034*
2	*.37983687258443411e-34*	*-.10412565145233292e-86*	*3.798369E-035*
3	*-.48325577705061128e-14*	*.14375350757823458e-34*	*4.832558E-015*
4	*.19630270224630655e-8*	*.19471733303215571e-8*	*1.585369E-011*
5	*.44127148821387145e-4*	*.44126786992954255e-4*	*3.618284E-010*
6	*.13456758586381776*	*.13456758586375031*	*6.744605E-014*

B5500 equation:

Numerical results at the end of range of integration ([0, 20.1286]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

ATOL =RTOL.

CPU Time: 4.9872s.

n	y_n^{comp}	y_n^{true}	*error*
1	*.57053138551032796e-40*	*-.29585464813493674e-87*	*5.705314E-41*
2	*.43271824592207074e-40*	*.45283123549263000e-87*	*4.327182E-41*
3	*.17694775140229357e-13*	*.10790578762337460e-34*	*1.769478E-14*
4	*.18436053422768319e-8*	*.18124297604681010e-8*	*3.117558E-11*
5	*.42572945926378166e-4*	*.42572640985357028e-4*	*3.049410E-10*
6	*.13360604815525093*	*.13360604815545910*	*2.081668E-13*

B5800 equation:

Numerical results at the end of range of integration ([0, 20.5243]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

ATOL =RTOL.

CPU Time: 7.6180s.

n	y_n^{comp}	y_n^{true}	*error*
1	*-.10502687569879855e-41*	*.80013811820795525e-89*	*1.050269E-042*
2	*-.66478283238687150e-42*	*-.65538203269721579e-89*	*6.647828E-043*
3	*-.63371678994333107e-14*	*.22164104300721623e-35*	*6.337168E-015*
4	*.12564521304486922e-8*	*.12201480952966386e-8*	*3.630404E-011*
5	*.34931337028875465e-4*	*.34930618306818423e-4*	*7.187221E-010*
6	*.12842253979000834*	*.12842253978978405*	*2.242928E-013*

B51000 equation:

Numerical results at the end of range of integration ([0, 20.3574]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

ATOL =RTOL.

CPU Time: 9.3935s.

n	y_n^{comp}	y_n^{true}	*error*
1	*.12979403074040256e-46*	*.32632621594482051e-88*	*1.297940E-047*
2	*-.22043916594925028e-46*	*.44149098021382395e-88*	*2.204392E-047*
3	*-.71957691733912372e-14*	*.43213907646656742e-35*	*7.195769E-015*
4	*.14813075555073207e-8*	*.14418028672951908e-8*	*3.950469E-011*
5	*.37971798504003979e-4*	*.37971079353834423e-4*	*7.191502E-010*
6	*.13058418668522187*	*.13058418668348123*	*1.740635E-012*

Problem 11: Ring Modulator problem with CS=1e-9.

Selected numerical results at the end of range of integration ([0,1E-3]).

$h_0 =$ 1E-8.

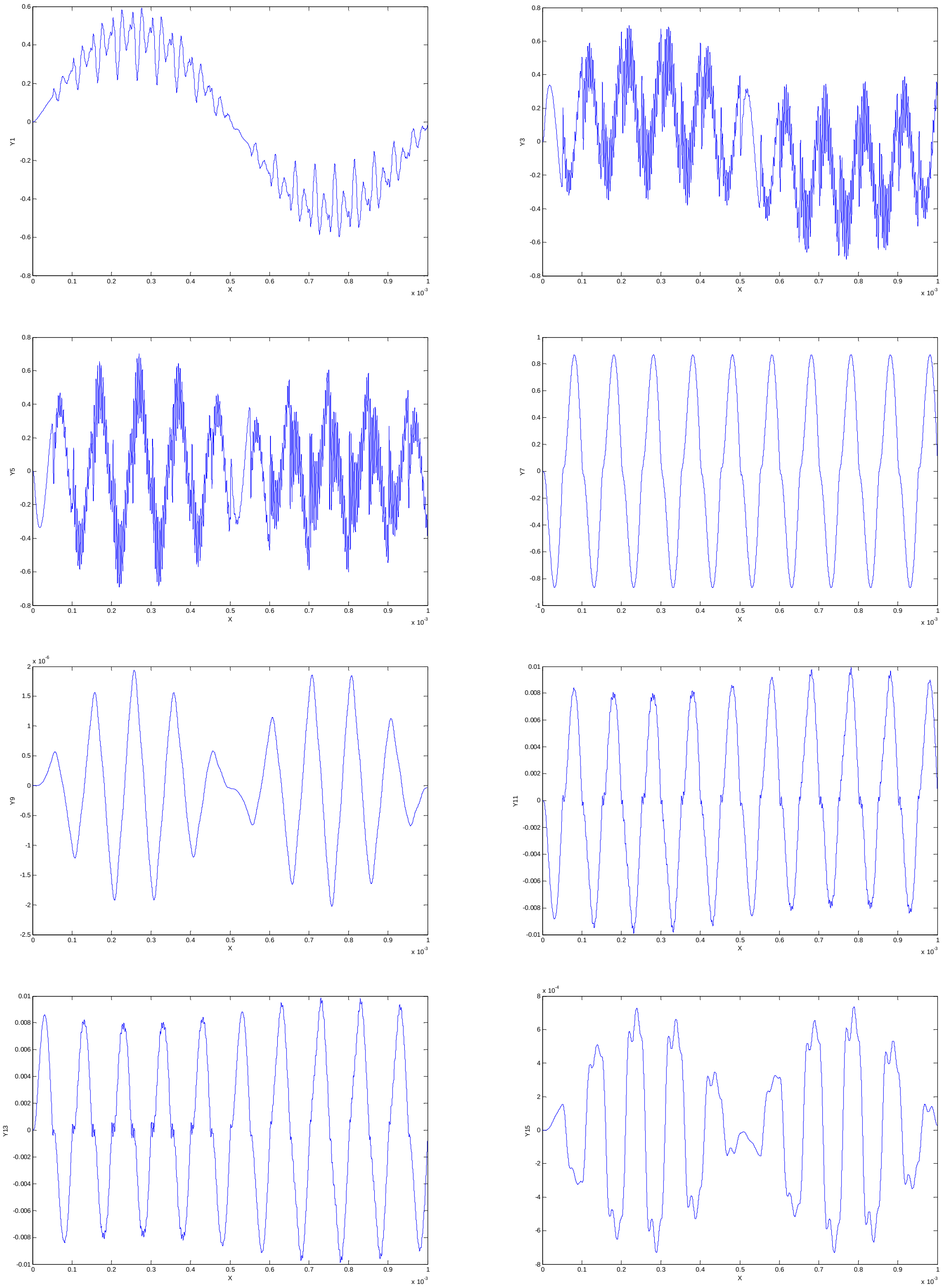
RTOL = 1E-7.

ATOL =RTOL.

CPU Time: 22.2720s.

n	y_n^{comp}	y_n^{true}	*error*
1	*-.170799076e-1*	*-0.170799033E-1*	4.302054E-009
3	*.275319290*	*0.27531919*	1.000075E-007
5	*-.388517208*	*-0.38851731*	1.021335E-007
7	*.111460030*	*0.1114600281*	1.463807E-009
9	*-.314277291e-7*	*-0.31427403E-7*	3.261369E-013
11	*.852075230e-3*	*0.85207538E-3*	1.496000E-010
13	*-.776319784e-3*	*-0.77631967E-3*	1.141251E-010
15	*.252322745e-4*	*0.252322784E-4*	3.860995E-012

Plots:



Problem 12: Ring Modulator problem with CS=2e-12.

Selected numerical results at the end of range of integration ([0,1E-3]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

ATOL =RTOL.

CPU Time: 598.6809s.

n	y_n^{comp}	y_n^{true}	*error*
1	-.233905741736391663e-1	-0.233905735842070174E-01	5.894321E-010
3	.258300694357122107	0.258295670891733276	5.023465E-006
5	-.403940542939079550	-0.403945566551075719	5.023612E-006
7	.110676186098624132	0.110676186126208484	2.758435E-011
9	-.284001803414159398e-7	-0.284002993248608379E-07	1.189834E-013
11	.792948883138238562e-3	0.792948719688146885E-03	1.634501E-010
13	-.794140033064962914e-3	-0.794140196849026266E-03	1.637841E-010
15	.239005908164041839e-4	0.239005907525775679E-04	6.382662E-014

1.2 DAE Tests.

Problem 13: The pendulum problems with index 0,1,2,3.

Pendulum index 0:

Numerical results at the end of range of integration ([0, 1]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

ATOL =RTOL.

CPU Time: 0.1402s.

n	y_n^{comp}	y_n^{true}	*error*
1	.86734863936873641	0.867348640604551746	1.235815E-9
2	.49770105161037403	0.497701050476373918	1.134000E-9
3	-.33748020028971148e-1	-0.337480180329866458E-1	1.995985E-9
4	.58813013428716400e-1	0.588130114660715686E-1	1.962645E-9
5	-.49310315258438886	-0.493103151528274442	1.056114E-9

Pendulum index 1:

Numerical results at the end of range of integration ([0, 1]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

ATOL =RTOL.

CPU Time: 0.2103s.

n	y_n^{comp}	y_n^{true}	*error*
1	.86734864007552426	0.867348640604551746	5.290275E-10
2	.49770105032220918	0.497701050476373918	1.541647E-10
3	-.33748013494383701e-1	-0.337480180329866458E-1	4.538603E-9
4	.58813010360391126e-1	0.588130114660715686E-1	1.105680E-9
5	-.49310314335838673	-0.493103151528274442	8.169888E-9

Pendulum index 2:

Numerical results at the end of range of integration ([0, 1]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

ATOL =RTOL.

CPU Time: 0.1903s.

n	y_n^{comp}	y_n^{true}	*error*
1	*.86734862981951477*	*0.867348640604551746*	*1.078504E-008*
2	*.49770107507424621*	*0.497701050476373918*	*2.459787E-008*
3	*-.33748042763088071e-1*	*-0.337480180329866458E-1*	*2.473010E-008*
4	*.58813051113324410e-1*	*0.588130114660715686E-1*	*3.964725E-008*
5	*-.49310307500331796*	*-0.493103151528274442*	*7.652496E-008*

Pendulum index 3:

Numerical results at the end of range of integration ([0, 1]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

ATOL =RTOL.

CPU Time: 0.1702s.

n	y_n^{comp}	y_n^{true}	*error*
1	*.86735096133969158*	*0.867348640604551746*	*2.320735E-006*
2	*.49769701430385954*	*0.497701050476373918*	*4.036173E-006*
3	*-.33744202174102567e-1*	*-0.337480180329866458E-1*	*3.815859E-006*
4	*.58808191999897015e-1*	*0.588130114660715686E-1*	*4.819466E-006*
5	*-.49308579822923737*	*-0.493103151528274442*	*1.735330E-005*

Problem 14: The stiff pendulum problem of index 3.

Numerical results at the end of range of integration ([0, 10]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

ATOL =RTOL.

CPU Time: 0.8713s.

n	y_n^{comp}	y_n^{true}	*error*
1	*-.81152787673403559*	*-0.811576367657398601*	*4.849092E-005*
2	*-.58431672138550228*	*-0.584249351678194584*	*6.736971E-005*
3	*-.63165535463144651*	*-0.631552659557417462*	*1.026951E-004*
4	*.87728019110692801*	*0.877289510510234183*	*9.319403E-006*
5	*1.7530861750015949*	*1.75273357180425515*	*3.526032E-004*

Problem 15: Chemical Akzo Nobel problem of index 1.

Numerical results at the end of range of integration ([0, 180]).

$h_0 =$ 1E-8.

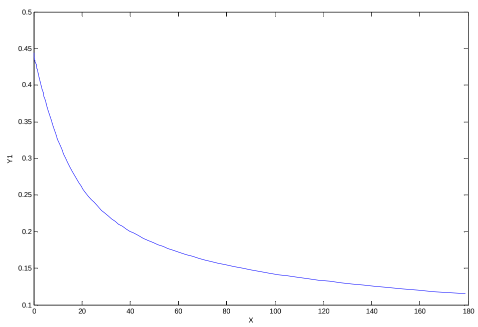
RTOL = 1E-7.

ATOL =RTOL.

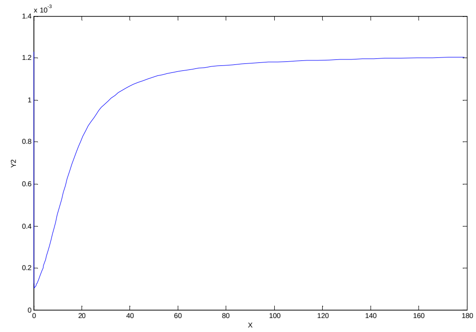
CPU Time: 0.3405s.

n	y_n^{comp}	y_n^{true}	*error*
1	*.11507949065400548*	*0.1150794920661702*	*1.412165E-009*
2	*.12038314735231025e-2*	*0.1203831471567715E-2*	*1.955387E-012*
3	*.16115628941475876*	*0.1611562887407974*	*6.739614E-010*
4	*.36561562029902386e-3*	*0.3656156421249283E-3*	*2.182590E-011*
5	*.17080110187003967e-1*	*0.1708010885264404E-1*	*1.334360E-009*
6	*.48735302093874805e-2*	*0.4873531310307455E-2*	*1.100920E-009*

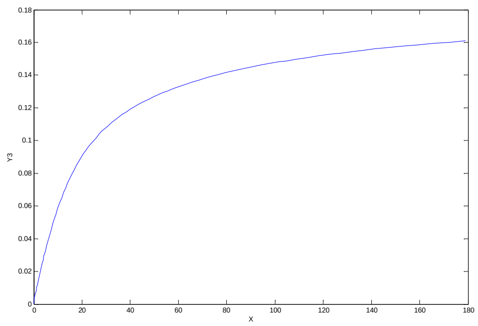
Plots:



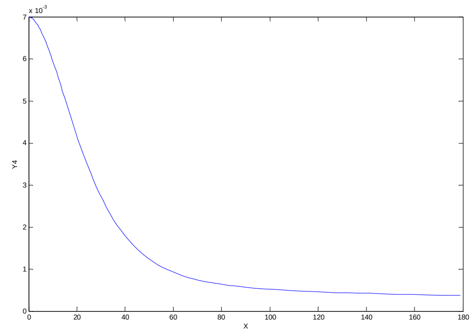
Y1
X
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0 20 40 60 80 100 120 140 160 180



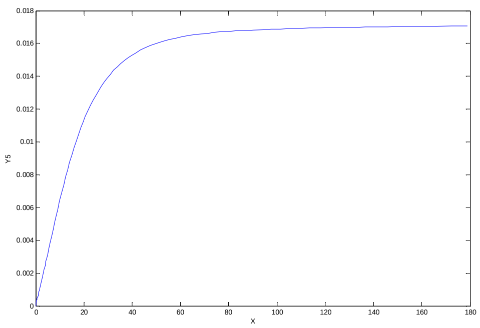
x 10-3
Y2
X
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0 20 40 60 80 100 120 140 160 180



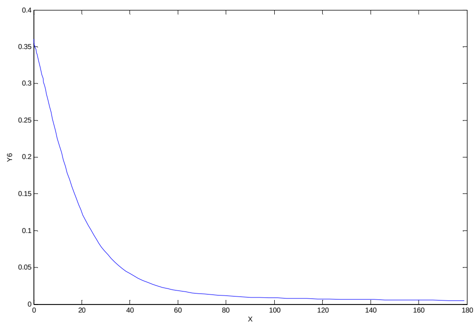
Y3
X
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0 20 40 60 80 100 120 140 160 180



x 10-3
Y4
X
7
6
5
4
3
2
1
0
0 20 40 60 80 100 120 140 160 180



Y5
X
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0 20 40 60 80 100 120 140 160 180



Y6
X
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 20 40 60 80 100 120 140 160 180

Problem 16: Transistor Amplifier problem of index 1.

Numerical results at the end of range of integration ([0, 0.2]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

ATOL =RTOL.

CPU Time: 11.0259s.

n	y_n^{comp}	y_n^{true}	*error*
1	*-.55621450122049056e-2*	*-0.556214501E-02*	*2.204906E-012*
2	*3.0065224719040282*	*0.3006522473125E+01*	*1.220972E-009*
3	*2.8499587886096669*	*0.284995878984E+01*	*1.230333E-009*
4	*2.9264225356303744*	*0.29264225362E+01*	*5.696257E-010*
5	*2.7046178644491681*	*0.27046178656E+01*	*1.150832E-009*
6	*2.7618377766282980*	*0.2761837776452E+01*	*1.762981E-010*
7	*4.7709275534170743*	*0.4770927631E+01*	*7.758293E-008*
8	*1.2369957885749647*	*0.1236995867E+01*	*7.842504E-008*

Problem 17:Fekete problem of index 2.

Selected numerical results at the end of range of integration ([0, 1E3]).

$h_0 =$ 1E-8.

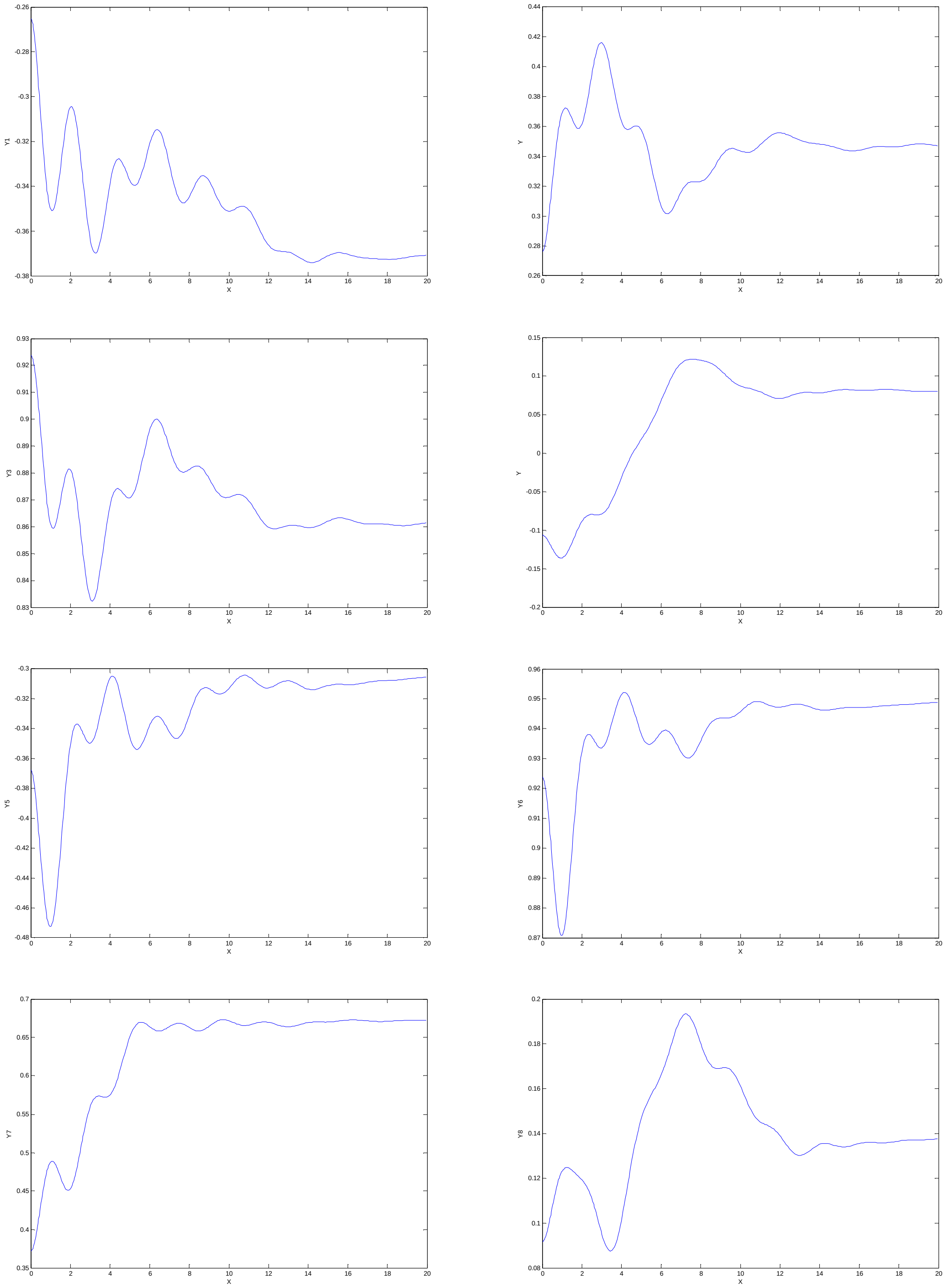
RTOL = 1E-7.

ATOL =RTOL.

CPU Time: 8.5924.

n	y_n^{comp}	y_n^{true}	*error*
20	*-.43108335416872606*	*-0.4310833259390688*	2.822966E-008
40	*-.23478973710756401*	*-0.2347897778700538*	4.076249E-008
60	*-.90828464611594051*	*-0.9082846503446250*	4.228685E-009
80	*.17006645913349788e-13*	*-0.5940734725324115E-16*	1.706605E-014
100	*.21877169993046868e-13*	*-0.2234188557495892E-15*	2.210059E-014
120	*-.18256356708100815e-13*	*-0.1229326332276474E-15*	1.813342E-014
140	*-4.7500000000000000*	*-0.4750000000000000E+01*	8.881784E-016
160	*-.12371556105219603e-18*	*0.6255426995473074E-19*	1.862698E-019

Selected plots on the range [0,20]:



Problem 18: Slider crank problem of index 2.

Selected numerical results at the end of range of integration ([0, 0.1]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

ATOL =RTOL.

CPU Time: 39.5469s.

n	y_n^{comp}	y_n^{true}	*error*
3	.16973733343082337	1.697373326718410E-01	7.589824E-010
6	-.53241711884344371e-5	-5.323907988763734E-06	2.631997E-010
9	60.253463564641756	6.025346682645789E+01	3.261816E-006
12	-.55008185903216034e-2	-5.500488291932075E-03	3.302984E-007
15	-.21185737064506258e-16	0	2.118574E-017
18	33.972121068812768	3.391435115267547E+01	5.776992E-002
21	.72742423075763762	9.903251655235532E-01	2.629009E-001
24	25.297623097210234	2.529853213732781E+01	9.090401E-004

Problem 19: Water tube problem of index 2.

Selected numerical results at the end of range of integration ([0, 43200]).

$h_0 =$ 1E-8.

RTOL = 1E-7.

*ATOL =RTOL and 1E-6*RTOL.*

CPU Time: 17.5853s.

n	y_n^{comp}	y_n^{true}	*error*
7	.34003290756955719e-2	0.3243571480903489e-002	1.567576E-004
14	.23983878778872161e-2	0.2397639310739395e-002	7.485671E-007
21	.47519404529185287e-1	0.4751940452918529e-001	4.320118E-007
28	.47519404529185287e-1	0.4751940452918529e-001	0.000000E+000
35	.47519404529185287e-1	0.4751940452918529e-001	0.000000E+000
42	109604.31689877754	0.1111269322658045e+006	1.522615E+003
49	109607.15338208279	0.1111298338971779e+006	1.522681E+003

Problem 20: Wheel set problem of index 2.

Selected numerical results at the end of range of integration ([0, 10]).

$h_0 = 1E\text{-}8.$

RTOL = 1E-7 and 1E10

ATOL =RTOL and 1E10

CPU Time: 146.2203s.

n	y_n^{comp}	y_n^{true}	*error*
1	.86354905929706890e-2	0.86355386965811E-02	4.810361E-008
8	.20541509116415355e-2	0.20541188079539E-02	3.210369E-008
15	.14173219914692131	0.14173214964613	4.950079E-008
22	-.10124021959238982e-1	-0.10124044903201E-01	2.294396E-008

Problem 21: Andrews squeezing mechanism of index 3.

Selected numerical results at the end of range of integration ([0, 10]).

$h_0 = 1E\text{-}8.$

RTOL = 1E- 7.

ATOL =RTOL.

CPU Time: 1.4821.

n	y_n^{comp}	y_n^{true}	*error*
5	.52440988867555405	0.5244099658805304	7.720498E-008
10	10.988972809781801	0.1103291221937134E+2	4.393941E-002
15	-24805.941152141244	-0.2463176316945196E+5	1.741780E+002
20	-566765.56956600794	-0.5667493645176222E+6	1.620505E+001
25	31.458016800693656	0.3145271365475927E+2	5.303146E-003

Problem 22: Car axis problem of index 3.

Selected numerical results at the end of range of integration ([0, 3]).

h_0 = *1E-8.*

RTOL = 1E- 7.

ATOL =RTOL.

CPU Time: 2.2933s.

n	y_n^{comp}	y_n^{true}	*error*
2	*.49699175326640116*	*0.4969894602303324*	*2.293036E-006*
4	*.37391182580045856*	*0.3739110272652214*	*7.985352E-007*
6	*.72806849513280007e-2*	*0.7446866596327776*	*1.661816E-004*
8	*.77043601732882094*	*0.7703410437794031*	*9.497355E-005*
10	*-.11047622712061936e-2*	*-0.1104680411345730*	*8.185986E-008*

1.3 Work Precision Diagrams.

The following Work Precision diagrams compare the performance of MEBDF.m and ODE15s (the equivalent inbuilt MATLAB solver) on selected problems from the I.V.P tests above. For these experiments we have followed the method from the book by Hairer and Wanner[1]. Each diagram is a plot of CPU time against number of correct digits achieved for the tolerances,

$$RTOL = 10^{-2-m/4}, \quad m = 0,1,2,...,32\,,$$

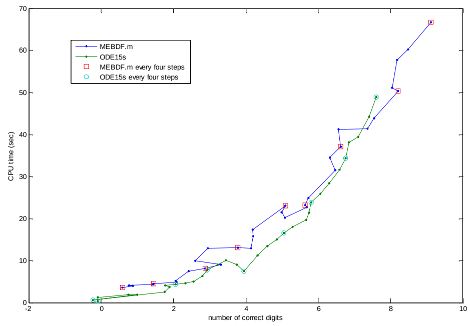
with ATOL set accordingly with the above corresponding experiments with respect to RTOL. We give both the complete graphs and the graphs where only every other four solutions are displayed.

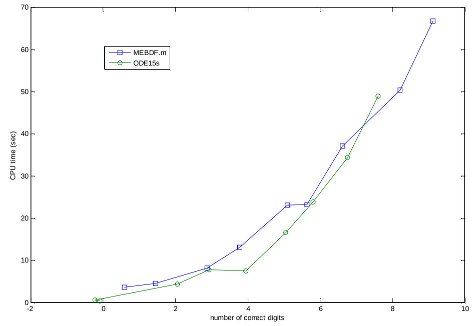
The problems presented here are:

- Van Der Pol equation.
- Robertson problem.
- Hires problem.
- Movement of a rectangular plate.
- Oregonator, Belusov-Zhabotinskii reaction.
- E5 equation.
- Ring Modulator problem with CS=1e-9 (easy ring).
- B5 equations (50, 200 & 500).

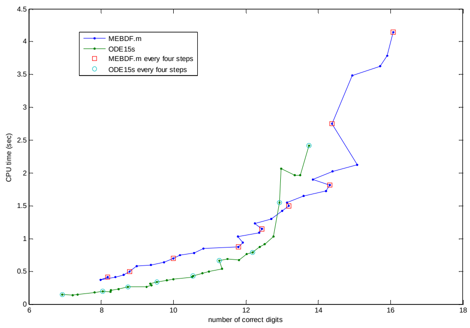
[1] Hairer, E and Wanner, G (1996) *Solving Ordinary Differential Equations 2*. Springer-Verlag Berlin Heidelberg.

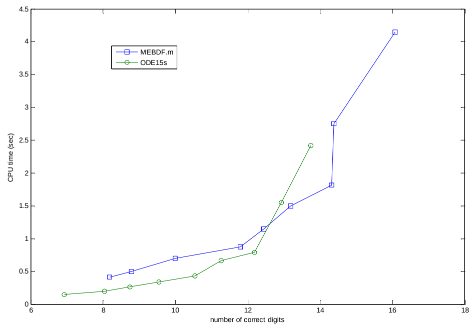
Van Der Pol equation.



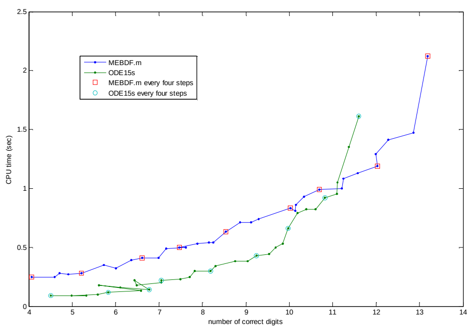


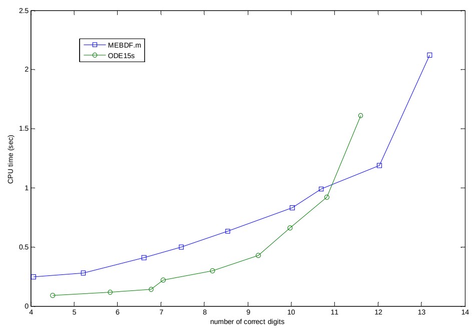
Robertson Problem.



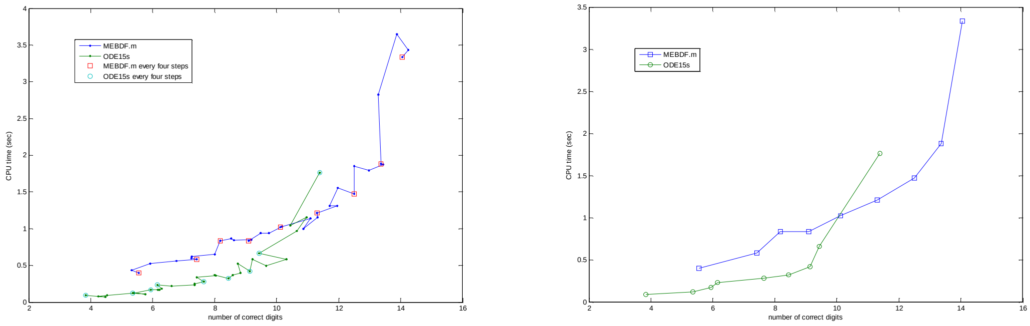


Hire Problem.

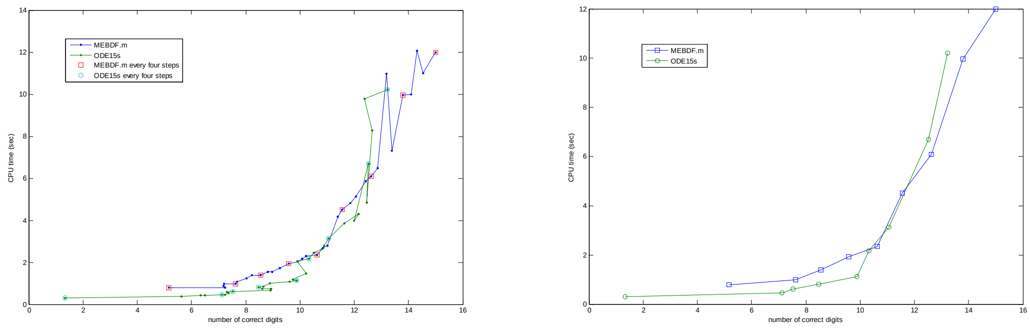




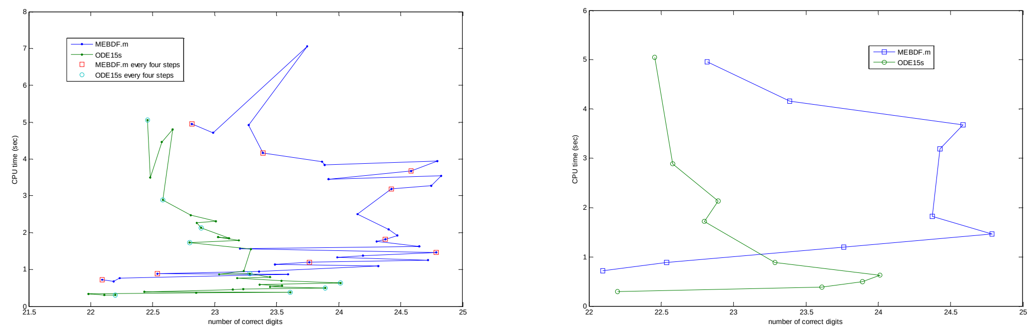
Movement of a rectangular plate.



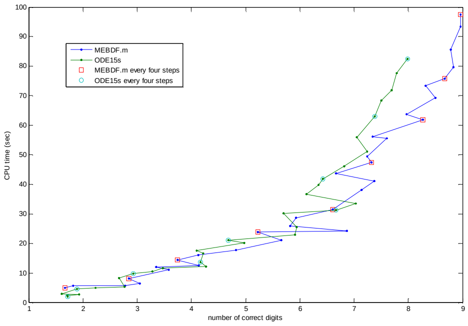
Oregonator, Belusov-Zhabotinskii reaction.

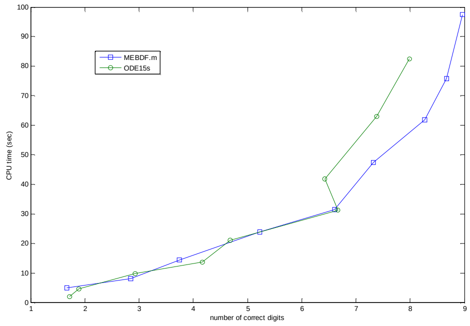


E5 equation.



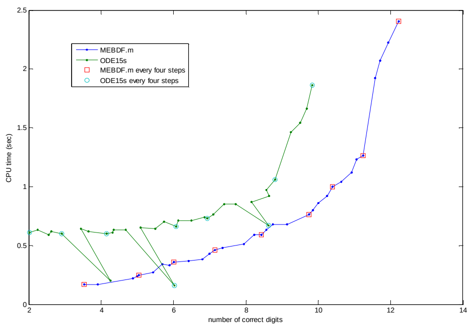
Ring Modulator problem with CS=1e-9 (easy ring).

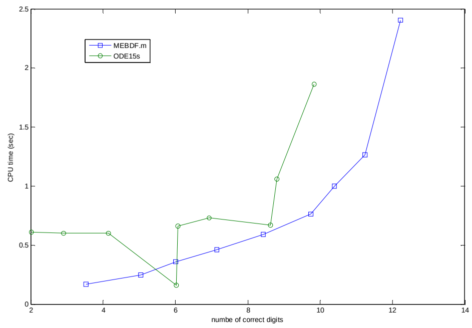




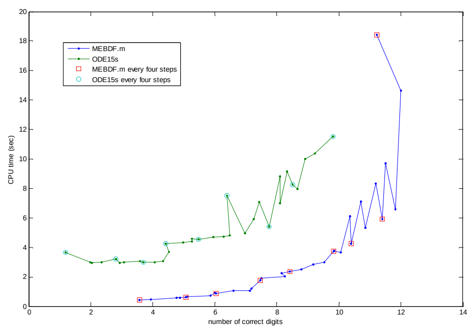
B5 equations.

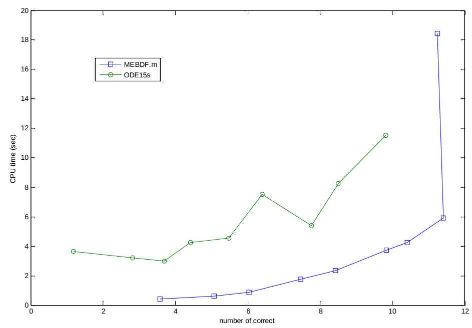
B550



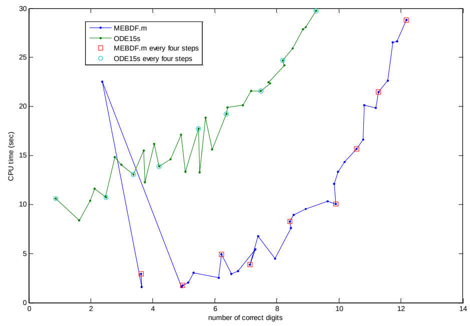


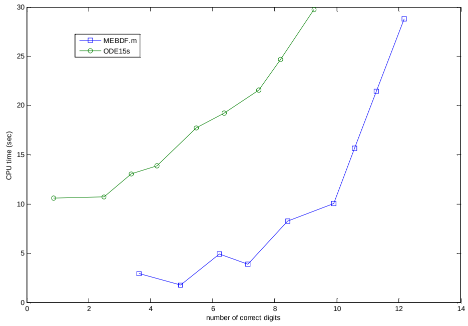
B5200





B5500





1.3 Remarks.

- We notice good, accurate results for the test problems above, but MEBDF.m is a lot slower then MEBDF.f, which was always going to be the case. However more work on MEBDF.m is still possible in order to reduce run time.
- For any given tolerance MEBDF.m consistently achieves more correct digits than ODE15s but generally takes more time to do so.
- For many experiments there exists a ‘cut off’ point where MEBDF.m begins to perform better then ODE15s both in time and number of correct digits. Example: The Hires problem at about 11 correct digits (see above).
- On the B5 equations, which are a severe test for A-Stability, MEBDF.m consistently out performs ODE15s.
- ODE15s cannot solve DAEs with index greater then 1. MEBDF.m can solve DAEs for index up to 3.