ME280A Fall 2014
Homework 6: Time Dependent Problem

Formulation of Time Dependent Problem

Consider the time-dependent heat equation problem

$$\begin{aligned} \nabla \cdot (k \nabla T) + z &= \rho c_p \dot{T} \,\mathrm{in}\, \Omega \,\forall t > 0 \\ T &= T_0 \,\mathrm{in}\, \Omega \,\mathrm{at}\, t = 0 \\ T &= \bar{T} \,\mathrm{on}\, \Gamma_T \,\forall t \geq 0 \\ (k \nabla T) \cdot \mathbf{n} &= \bar{q}_n \,\mathrm{on}\, \Gamma_q \,\forall t \geq 0 \end{aligned}$$

for $t \geq 0$ with a constant-in-time conductivity k, heat capacitance c_p, density ρ and source term z.

1. **Derive the weak form of this problem for the discrete approximation.** State the expressions of each of the matrices involved.

2. **Derive the Forward Euler with lumped mass approximation, Forward Euler with a mass matrix, and Backward Euler time stepping schemes.** Comment on the differences between these formulations. Do not use a lumped mass approximation for the Backward Euler scheme.

Radially Symmetric Problem

Recall the simple domain from Homework 5 which is reproduced in Figure 1. Again, let $\bar{T}_i = 100$, $k = 0.04$, $r_i = 0.1$, $r_o = 0.25$, $z = 500$, $\bar{q}_n = -25$. Use the initial value $T\,(t = 0) = 100 \,\mathrm{in}\, \Omega$ and $\rho = c_p = 2$. Solve the system from $t_i = 0$ to $t_f = 100$.

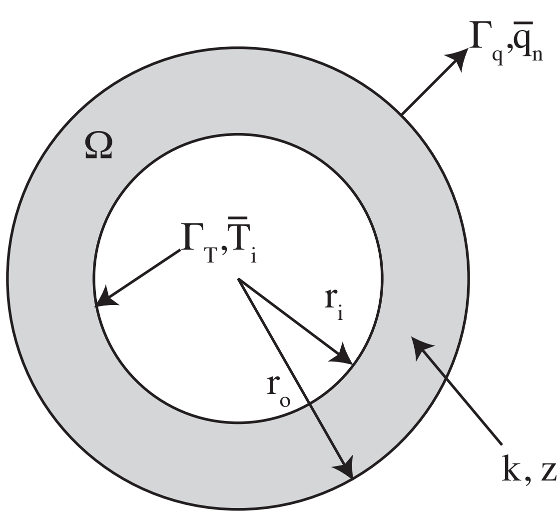

Figure 1: Radially symmetric annulus

1. **Modify your code from Homework 5 to solve the time dependent problem.** Use Forward Euler with a lumped-mass approximation for a variable timestep size δt. Use $NE_\theta = 20$ and $NE_r = 10$ for your mesh. Plot the radial temperature distribution at multiple points in time to demonstrate the evolution of the problem with $\delta t = t_f/10,000$. Also plot the temperature at the outer surface against time for this time step. Include the steady-state analytical solution from Homework 5 on the plots (the time-dependent solution will converge to it.)

2. **Provide a detailed description of your algorithm.** Detail how you enforce the boundary conditions.

3. **Comment on the effect of timestep size.** Experiment with timesteps that smaller and larger than the one stated above. Make a plot of the outer surface temperature versus time with the curves for multiple timestep sizes on the same plot.

Hints

1. **Post-processing is expensive.** Plotting the solution with patch will be slower than doing a timestep, so avoid plotting every iteration. Do something along the lines of

```
if mod(ti,numti/5)==0
    %post processing here
        patch(...)
        plot(...)
end
```

   where *ti* is the current timestep number, and *numti* is how many total number of timesteps to take.

2. **This is a linear problem.** You only need to compute the matrix K once, and then it can be used in every timestep. The routines for computing K and R will be unchanged from Homework, except the Dirichlet boundary condition will not be applied the same way: remove the penalty calculations.) The timestep loop will just be applying K once when evaluating the derivative and then enforcing the Dirichlet condition.