ME 280A Fall 2014

HOMEWORK 2: HIGHER ORDER ELEMENTS

- Solve the following boundary value problem, with domain $\Omega = (0, L)$, analytically. You should use appropriate boundary conditions and interface conditions to derive your answer:

$$\boxed{\begin{array}{l} \frac{d}{dx}\left(A_1 \frac{du}{dx}\right) = 256 \sin\left(\frac{3}{4}\pi x\right) \cos(16\pi x) \\ A_1 = 0.2 \\ L = 1 \\ u(0) = \Delta_1 = given\ constant = 0 \\ A_1(L)\frac{du}{dx}(L) = 1 \end{array}} \tag{1}$$

- Solve this with the finite element method using order p equal-sized elements. In order to achieve

$$\boxed{\begin{array}{l} e^N \stackrel{\text{def}}{=} \frac{||u-u^N||_{A_1(\Omega)}}{||u||_{A_1(\Omega)}} \leq TOL = 0.01, \\ \\ ||u||_{A_1(\Omega)} \stackrel{\text{def}}{=} \sqrt{\int_\Omega \frac{du}{dx} A_1 \frac{du}{dx}\, dx} \end{array}} \tag{2}$$

how many finite elements (N) are needed for

$$\boxed{\begin{array}{l} p = 1 \Rightarrow N =? \\ p = 2 \Rightarrow N =? \\ p = 3 \Rightarrow N =? \end{array}} \tag{3}$$

- Plot the numerical solutions for several values of N, for each p, along with the exact solution
- Plot e^N as a function of the element size h for each p
- Plot e^N as a function of the number of degrees of freedom for each p
- Determine the relationship between the error and the element size for each p ($e^N = f(h)$)

Notes:

- It might help to test your code from the previous assignment on the new forcing function and before implementing anything new. Then, add the Neumann boundary condition and test that component independently.
- Modify your code to perform Gaussian quadrature for the integrals. Please be careful with the quadrature order for each integral as you will need higher order Gauss rules for quadratic and cubic elements. You should do this before implementing the higher order elements, since it will let you integrate them easily.

- The FEM solution is not piecewise linear for $p > 1$, so plotting the solution vector a_i against the nodal positions x_i is not an adequate depiction of the solution. Present the solution taking into account the higher order elements.

Report: Your report should reference your first homework report in terms of the common content (weak form etc). The homework 2 report should focus on how you extended your work to higher-order elements from a linear element code.