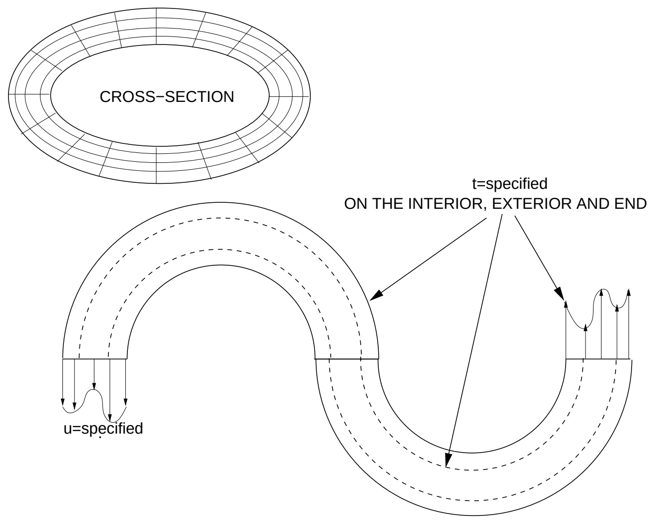
HOMEWORK 7 FOR ME 280-A



You are given a tubular multiphase structure with an elasticity of $\mathbb{E}(x, y, z)$, and with dimensions shown in the figure. It is clamped on one end and externally traction loaded everywhere else, including on the interior surface. The SMALL deformation of the body is governed by (strong form):

$$\nabla \cdot (\mathbb{E} : \nabla \boldsymbol{u}) + \rho \boldsymbol{b} = \boldsymbol{0} \tag{1}$$

where $\mathbb{E}$ and ρ are spatially variable and where $\boldsymbol{b} = \boldsymbol{b}(x, y, z)$ is given data.

- Develop a weak form, providing all the steps and assumptions necessary. Carefully define the spaces of approximation.
- Develop a finite element weak form. Carefully define the spaces of approximation.
- Develop a finite element weak statement using the penalty method. Carefully define the spaces of approximation.
- Derive the equations for element stiffness matrices (be explicit) and load vectors. Thereafter, describe how the global stiffness matrix and load vector are generated, using the penalty method. Use trilinear

subspatial approximations. There are different kinds of loading on the surfaces, so be very explicit as to what each of the individual stiffness matrices and righthandside vectors look like, as well as a generic element that is not on the surface.

- *ASSUME A TUBULAR CIRCULAR CROSS-SECTION OF OUTER RADIUS R_c, THICKNESS t AND A SEMICIRCULAR CROSS-SECTION (ALONG THE LENGTH) OF R_s.* Use $N_r = 5$ elements in the r direction, $N_c = 20$ (circumferantial of the cross-section) elements in the circumferential direction and $N_\theta = 20$ (along the length) elements in the θ direction for each semicircular portion. *DRAW THE MESH-NOT BY HAND.* Explicitly characterize the geometric error for a general number (N_r, N_c, N_θ), defined as the percentage error in the solid volume as a function of the N_r elements in the r direction, the N_c elements in the circumferential direction and the N_θ elements in the θ direction

$$geometric\ error \stackrel{\text{def}}{=} \frac{TRUE\ VOLUME - FEM\ VOLUME}{TRUE\ VOLUME} \tag{2}$$

- If one were to use a Conjugate-Gradient solver, theoretically how many operation counts would be needed to solve this problem for a mesh of N_r elements in the r direction, N_c elements in the circumferential direction and N_θ elements in the θ direction.

- Now consider the time-transient case. The body has the same boundary conditions as before, with the initial condition that $\boldsymbol{u}(t = 0, x, y, z) = \boldsymbol{u}_0(x, y, z)$ and $\dot{\boldsymbol{u}}(t = 0, x, y, z) = \dot{\boldsymbol{u}}_0(x, y, z)$. The governing equation is

$$\nabla \cdot (\boldsymbol{I\!E} : \nabla \boldsymbol{u}) + \rho \boldsymbol{b} = \rho \ddot{\boldsymbol{u}} \tag{3}$$

Develop a finite element weak statement. Carefully define the spaces of approximation. Use the

(1) the IMPLICIT finite difference approximation for the time dependent term

(2) the EXPLICIT finite difference approximation for the time dependent term

(3) the EXPLICIT finite difference approximation for the time dependent term with a lumped mass approximation.