1 Reference Frame of the CMS Experiment

1.1 Rotation SCF $\rightarrow$ CMS

To characterize the rotation of the Earth on itself, two angles in addition to the angular velocity of the Earth are required. The first angle is the latitude λ, an old marine coordinate starting at the equator ($\lambda = 0°$) and ending at the poles ($\lambda = \pm 90°$). The second angle is the azimuth θ at the LHC. The azimuth measures the angle between a tangent vector to the ring (clockwise) and the vector collinear to the Greenwich meridian (oriented North).

A summary of the angles is shown in Table **??**. The goal is to rotate from the SCF frame to the CMS frame. The first step is to define a base at point 5 of the LHC (CMS). Conventionally, the z-axis follows the beam in the clockwise direction and the x-axis is perpendicular to z pointing towards the center of the ring. We then construct the y-axis pointing towards the surface to obtain a right-handed orthonormal reference frame. The second step is to construct the time-dependent rotation matrix R that allows the transition between the SCF and the CMS frame:

$$BCMS(x, y, z) \xrightarrow{R(t)} BSCF(X, Y, Z)$$

where B represents the reference frames. All rotations will be counterclockwise.

1.1.1 Rotation Matrices

First rotation $R_z\left(\frac{\pi}{2}\right)$. This is a rotation around the z-axis which makes the x-axis normal to the plane of the LHC. This allows the x-axis to be normal to the tangent plane of the Earth at the LHC location. The new base is given by $BCMS(x', y', z)$.

$$R_z\left(\frac{\pi}{2}\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Second rotation $R_{x'}(\pi - \theta)$. We want to orient the z-axis towards the North. We rotate counterclockwise around the x' axis with an angle $\pi - \theta$ (co-azimuth). The new base is given by $BCMS(x', y'', z'')$.

$$R_{x'}(-(\pi - \theta)) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\cos(\theta) & \sin(\theta) \\ 0 & 0 & -\sin(\theta) & -\cos(\theta) \end{pmatrix}$$

Third rotation $R_{y''}(\lambda)$. Rotation around the y'' axis to align the z-axis with the Z-axis of the SCF. The new base is given by $BCMS(x'', y'', Z)$.

$$
R_{y''}(\lambda) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\lambda) & 0 & \sin(\lambda) \\ 0 & 0 & 1 & 0 \\ 0 & -\sin(\lambda) & 0 & \cos(\lambda) \end{pmatrix}
$$

Fourth rotation $R_Z(\Omega t)$. A final rotation around the Z-axis has two purposes: to follow the rotation of the Earth over time and to synchronize with the SCF:

$$
R_Z(\Omega t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\Omega t) & -\sin(\Omega t) & 0 \\ 0 & \sin(\Omega t) & \cos(\Omega t) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

With $\Omega t = \Omega_{\mathrm{UTC}} t_{\mathrm{CMS}} + \phi_{\mathrm{Unix}} + \phi_{\mathrm{longitude}}$.

The rotation matrix SCF $\rightarrow$ CMS:

In summary:

$$
R(\lambda, \theta) = R_{y''}(\lambda) R_{x'}(-(\pi - \theta)) R_z \left(\frac{\pi}{2}\right) R_Z(\Omega t)
$$

$$
R(\lambda, \theta, t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\cos(\Omega t) s_\lambda c_\theta + \sin(\Omega t) c_\theta & -\cos(\Omega t) c_\lambda - \cos(\Omega t) s_\lambda c_\theta & -\sin(\Omega t) s_\theta \\ 0 & -\sin(\Omega t) s_\lambda c_\theta - \cos(\Omega t) c_\theta & -\sin(\Omega t) c_\lambda - \sin(\Omega t) s_\lambda c_\theta + \cos(\Omega t) s_\theta & \\ 0 & -c_\lambda s_\theta & -s_\lambda & -c_\lambda c_\theta \end{pmatrix}
$$

1.2 Quantities $A_{\mu\nu}$

To calculate the quantities $A_{\mu\nu}$ introduced in (2.17) in the CMS reference frame, simulations for the process $tt \rightarrow b\ell^+\nu_\ell b\ell^-\nu_\ell$ were performed by generating events at the parton level with MadGraph_aMC@NLO at leading order with the PDF NNPDF2.3 LO in the Standard Model.