

1 Rotations

$$R(\lambda, \theta, \alpha) = R_Z(\Omega t) \cdot R_{y''}(\lambda) \cdot R_{x'}(-(\pi - \theta)) \cdot R_z\left(\frac{\pi}{2}\right) \cdot R_x(\alpha) \quad (1)$$

With:

$$\begin{aligned} c_\lambda &\equiv \cos(\lambda) & s_\lambda &\equiv \sin(\lambda) \\ c_\theta &\equiv \cos(\theta) & s_\theta &\equiv \sin(\theta) \\ c_\alpha &\equiv \cos(\alpha) & s_\alpha &\equiv \sin(\alpha) \end{aligned}$$

We can develop the rotation with tilt:

$$R(\lambda, \theta, \alpha, t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sin(\Omega t)c_\theta - \cos(\Omega t)s_\theta s_\lambda & \cos(\Omega t)(-s_\alpha c_\theta s_\lambda - c_\alpha c_\lambda) - \sin(\Omega t)s_\alpha s_\theta & \cos(\Omega t)(s_\alpha c_\lambda - c_\alpha c_\theta s_\lambda) - \sin(\Omega t)c_\alpha s_\theta \\ 0 & -\sin(\Omega t)s_\theta s_\lambda - \cos(\Omega t)c_\theta & \sin(\Omega t)(-s_\alpha c_\theta s_\lambda - c_\alpha c_\lambda) + \cos(\Omega t)s_\alpha s_\theta & \sin(\Omega t)(s_\alpha c_\lambda - c_\alpha c_\theta s_\lambda) + \cos(\Omega t)c_\alpha s_\theta \\ 0 & -s_\theta c_\lambda & c_\alpha s_\lambda - s_\alpha c_\theta c_\lambda & -s_\alpha s_\lambda - c_\alpha c_\theta c_\lambda \end{pmatrix} \quad (2)$$

There were 3 errors (Rxx, Ryx which have the same coefficients except for time variations, and Rzy which you had already noted).

Verification: If $\alpha = 0$, we verify the thesis expression (after corrections).

$$R(\lambda, \theta, 0, t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sin(\Omega t)c_\theta - \cos(\Omega t)s_\theta s_\lambda & -\cos(\Omega t)c_\lambda & -\cos(\Omega t)c_\theta s_\lambda - \sin(\Omega t)s_\theta \\ 0 & -\sin(\Omega t)s_\theta s_\lambda - \cos(\Omega t)c_\theta & -\sin(\Omega t)c_\lambda & -\sin(\Omega t)c_\theta s_\lambda + \cos(\Omega t)s_\theta \\ 0 & -s_\theta c_\lambda & s_\lambda & -c_\theta c_\lambda \end{pmatrix} \quad (3)$$

2 The $A^{\mu\nu}$

In fact, the $A^{\mu\nu}$ are calculated completely independently of the rotations. Thus, we always have completely diagonal matrices (within uncertainties).

At the LHC at 13 TeV:

$$\langle A_{\mu\nu}^{Pq\bar{q}} \rangle = \begin{pmatrix} 1.178 \pm 0.007 & 0.000 \pm 0.001 & 0.000 \pm 0.001 & 0.004 \pm 0.008 \\ 0.000 \pm 0.001 & 0.195 \pm 0.001 & 0.000 \pm 0.001 & 0.000 \pm 0.001 \\ 0.000 \pm 0.001 & 0.000 \pm 0.001 & 0.195 \pm 0.001 & 0.000 \pm 0.001 \\ 0.004 \pm 0.008 & 0.000 \pm 0.001 & 0.000 \pm 0.001 & 2.890 \pm 0.007 \end{pmatrix} \quad (4)$$

with $\Gamma_{q\bar{q}} = 0.114$

$$\langle A_{\mu\nu}^{Pgg} \rangle = \begin{pmatrix} 13.55 \pm 0.02 & 0.000 \pm 0.001 & 0.000 \pm 0.001 & 0.00 \pm 0.02 \\ 0.000 \pm 0.001 & 0.144 \pm 0.001 & 0.000 \pm 0.001 & 0.000 \pm 0.001 \\ 0.000 \pm 0.001 & 0.000 \pm 0.001 & 0.143 \pm 0.001 & 0.000 \pm 0.001 \\ 0.00 \pm 0.02 & 0.000 \pm 0.001 & 0.000 \pm 0.001 & 9.42 \pm 0.02 \end{pmatrix} \quad (5)$$

with $\Gamma_{gg} = 0.886$

$$\langle A_{\mu\nu}^F \rangle = \begin{pmatrix} -31.2 \pm 0.3 & 0.04 \pm 0.05 & 0.00 \pm 0.05 & 0.00 \pm 0.3 \\ 0.04 \pm 0.05 & -1.798 \pm 0.02 & 0.00 \pm 0.02 & -0.01 \pm 0.04 \\ 0.00 \pm 0.05 & 0.00 \pm 0.02 & -1.81 \pm 0.02 & 0.05 \pm 0.04 \\ 0.00 \pm 0.3 & -0.01 \pm 0.04 & 0.05 \pm 0.04 & -20.4 \pm 0.2 \end{pmatrix} \quad (6)$$

3 Modulation Equations

As we saw previously, the $A^{\mu\nu}$ are diagonal, so only the α coefficients need to be updated. The strategy is always to develop the calculation and isolate the temporal sine and cosine dependencies.

3.1 $c_{XX} = -c_{YY}$

Considering that only $c_{XX} = -c_{YY}$ are non-zero, we have:

$$f_{SME}^{(XX)}(t) \cdot c_{XX} = ((R_X^X R_X^X + R_X^Y R_X^Y) \langle A_{XX} \rangle + R_X^Z R_X^Z \langle A_{ZZ} \rangle) \quad (7)$$

$$= a_1 \cos^2(\Omega t) + a_2 \sin^2(\Omega t) + 2a_3 \sin(\Omega t) \cos(\Omega t) \quad (8)$$

$$= \frac{a_1}{2}(\cos(2\Omega t) + 1) + \frac{a_2}{2}(1 - \cos(2\Omega t)) + a_3 \sin(2\Omega t) \quad (9)$$

$$= \left(\frac{a_1 + a_2}{2} \right) + \left(\frac{a_1 - a_2}{2} \right) \cos(2\Omega t) + a_3 \sin(2\Omega t) \quad (10)$$

$$f_{SME}^{(YY)}(t) \cdot c_{YY} = ((R_Y^X R_Y^X + R_Y^Y R_Y^Y) \langle A_{XX} \rangle + R_Y^Z R_Y^Z \langle A_{ZZ} \rangle) \quad (11)$$

$$= a_1 \sin^2(\Omega t) + a_2 \cos^2(\Omega t) - 2a_3 \sin(\Omega t) \cos(\Omega t) \quad (12)$$

$$= \frac{a_1}{2}(1 - \cos(2\Omega t)) + \frac{a_2}{2}(\cos(2\Omega t) + 1) - a_3 \sin(2\Omega t) \quad (13)$$

$$= \left(\frac{a_1 + a_2}{2} \right) - \left(\frac{a_1 - a_2}{2} \right) \cos(2\Omega t) - a_3 \sin(2\Omega t) \quad (14)$$