Association Rule Learning

From Frequent Patterns to Business Insights

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Outline

Introduction

Motivation

- Data often comes as **transactions** or **baskets**:
  - Shopping carts, web sessions, app clicks, medical events.
- Goal: discover patterns of the form
  $$X \Rightarrow Y$$
  meaning "when X occurs, Y tends to occur as well" [1, 4].
- Classic use case: **market basket analysis** in retail.

Market Basket Example

- Suppose many customers buying {Bread, Butter} also buy {Milk}.
- Association rule:

$$\{\text{Bread}, \text{Butter}\} \Rightarrow \{\text{Milk}\}$$

- Applications:
  - Product placement and promotions.
  - Recommendation and cross-selling.

Key Concepts

Basic Definitions

- Item set:
$$\mathcal{I} = \{i_1, i_2, \ldots, i_m\}$$
- Dataset of transactions:
$$\mathcal{D} = \{T_1, \ldots, T_N\}, \quad T_j \subseteq \mathcal{I}$$
- **Itemset** X: any subset $X \subseteq \mathcal{I}$.
- Size of X: its cardinality $|X|$ (2-itemset, 3-itemset, etc.).

Association Rule

Definition

An association rule is an implication

$$X \Rightarrow Y$$

where $X, Y \subseteq \mathcal{I}$, $X \cap Y = \emptyset$, $X \neq \emptyset$, $Y \neq \emptyset$.

- X: **antecedent** (left-hand side).
- Y: **consequent** (right-hand side) [4].
- Objective: find rules that are both **frequent** and **strong**.

Metrics: Support, Confidence, Lift

Support

Support of an Itemset

$$\text{supp\_count}(X) = |\{T_j \in \mathcal{D} : X \subseteq T_j\}|$$

$$\text{supp}(X) = \frac{\text{supp\_count}(X)}{N}$$

- Measures how **frequently** X occurs in the dataset.
- For a rule $X \Rightarrow Y$, support is $\text{supp}(X \cup Y)$.

Confidence

Confidence of a Rule

$$\text{conf}(X \Rightarrow Y) = \frac{\text{supp}(X \cup Y)}{\text{supp}(X)} = P(Y \mid X)$$

- Measures how often Y appears in transactions containing X.
- Directional: $\text{conf}(X \Rightarrow Y) \neq \text{conf}(Y \Rightarrow X)$ [4].

Lift

Lift of a Rule

$$\mathrm{lift}(X \Rightarrow Y) = \frac{\mathrm{conf}(X \Rightarrow Y)}{\mathrm{supp}(Y)} = \frac{\mathrm{supp}(X \cup Y)}{\mathrm{supp}(X)\,\mathrm{supp}(Y)}$$

- Compares observed co-occurrence to what is expected under independence [4].
- Interpretation:
  - > 1: positive association.
  - $= 1$: independence.
  - < 1: negative association.

Example Rules

Rule	Support	Confidence	Lift
Bread $\Rightarrow$ Butter	0.20	0.50	1.25
Butter $\Rightarrow$ Bread	0.20	1.00	2.50
Bread $\Rightarrow$ Milk	0.30	0.75	1.50

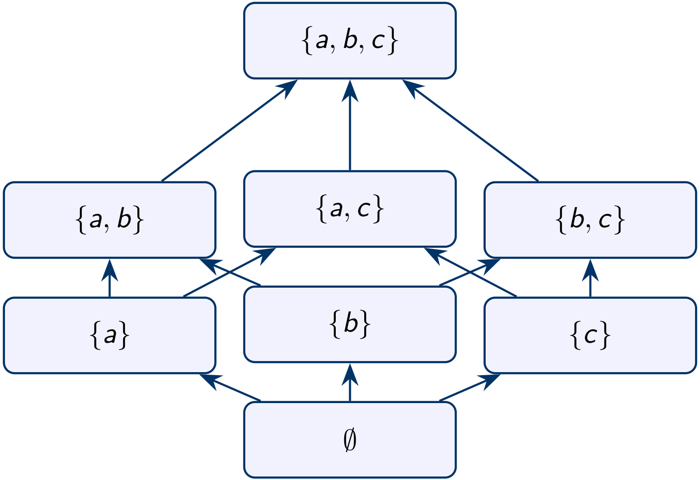
Table: Illustrative rule metrics.

Frequent Itemset Mining

From Itemsets to Rules

- Most algorithms operate in two phases [1, 4]:
  1. **Frequent itemset mining**: find all X with $\text{supp}(X) \geq \text{min\_supp}$.
  2. **Rule generation**: from each frequent itemset L, generate rules $X \Rightarrow L \setminus X$ meeting min_conf (and possibly lift).
- Challenge: there are 2^m possible itemsets.

Search Space



Lattice of itemsets; algorithms must explore or prune this space.

Apriori Algorithm

Apriori Property

Key Property [1, 4]

If an itemset is **frequent**, all of its subsets are frequent.

If an itemset is **not** frequent, no superset can be frequent.

- Anti-monotonicity of support.
- Enables strong pruning of the search space.

Apriori: Level-wise Procedure

1. Choose minimum support min_supp.
2. Find frequent 1-itemsets L_1.
3. For $k = 2, 3, \ldots$:
   - Generate candidate k-itemsets C_k from L_{k-1}.
   - Prune candidates whose $(k-1)$-subsets are not all in L_{k-1}.
   - Scan database, compute support of candidates.
   - Form $L_k = \{c \in C_k \mid \text{supp}(c) \geq \text{min\_supp}\}$.
4. Stop when L_k becomes empty; output $\bigcup_k L_k$.

Apriori: Toy Illustration

- Start with frequent 1-itemsets: $\{Bread\}$, $\{Butter\}$, $\{Milk\}$.
- Generate candidates:
  - 2-itemsets: $\{Bread, Butter\}$, $\{Bread, Milk\}$, $\{Butter, Milk\}$.
  - Keep only those above min_supp.
- Continue to 3-itemsets if any 2-itemsets are frequent.
- Stop when no larger frequent itemsets exist.

FP-Growth & Eclat

FP-Growth: Motivation

- Apriori may generate many candidates and scan the database multiple times.
- FP-Growth avoids explicit candidate generation using an **FP-tree** [2, 4].
- Typically needs only two passes over the data.

FP-Growth: Main Steps

1. First pass: compute frequent items and their supports.
2. Reorder items in each transaction by descending support, discard infrequent items.
3. Build FP-tree: insert each ordered transaction, sharing common prefixes.
4. Recursively mine the FP-tree:
    - For each item, build a conditional pattern base.
    - Construct conditional FP-tree and mine for patterns containing that item.

Eclat: Vertical Representation

- Eclat stores data in **vertical** form: TIDLists [3].
- For each item i:

$$\mathrm{TIDList}(i) = \{j \mid i \in T_j\}$$

- For itemset $X = \{i_1, \ldots, i_k\}$:

$$\mathrm{TIDList}(X) = \bigcap_{\ell=1}^{k} \mathrm{TIDList}(i_\ell)$$

- Support count is $|\mathrm{TIDList}(X)|$.

Eclat: Depth-First Mining

- Start from frequent 1-itemsets with their TIDLists.
- Extend an itemset X by intersecting its TIDList with those of following items.
- Depth-first exploration of the itemset lattice.
- Efficient when TIDLists are relatively small (sparse data).

Algorithm Comparison

Algorithm	Search Strategy	Structure	Key Strength
Apriori	Level-wise, breadth-first	None (flat)	Simple; strong pruning with Apriori property
FP-Growth	Pattern growth, divide-and-conquer	FP-tree	Few scans; efficient on dense data
Eclat	Depth-first	TIDLists (vertical)	Fast intersections on sparse data

Table: Frequent itemset mining algorithms [1, 2, 3].

Rule Generation & Applications

From Frequent Itemsets to Rules

- For each frequent itemset L ($|L| \geq 2$) [1, 4]:
  1. Generate all non-empty proper subsets $X \subset L$.
  2. For each X, form rule $X \Rightarrow L \setminus X$.
  3. Compute
  $$\text{conf}(X \Rightarrow Y) = \frac{\text{supp}(L)}{\text{supp}(X)}.$$
  4. Keep rules with confidence (and optionally lift) above thresholds.
- Post-processing: remove redundant or uninteresting rules.

Applications

- **E-commerce & retail**:
  - Market basket analysis, cross-selling, product placement.
- **Food delivery platforms**:
  - Suggesting combos (burger + fries, pizza + drink).
- **Streaming services**:
  - Co-consumption patterns across movies, series, or songs.
- **Finance, healthcare, fitness**:
  - Spending patterns, co-occurring diagnoses or treatments [4].

Practical Considerations

- **Threshold tuning**:
  - Low thresholds $\Rightarrow$ too many, noisy rules.
  - High thresholds $\Rightarrow$ missed interesting patterns.
- **Redundancy**:
  - Many rules may convey similar information.
  - Need ranking and pruning.
- **Domain validation**:
  - Statistical significance $\neq$ business value.
  - Expert review and A/B testing often required [4].

Conclusion

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- Association rule learning discovers interpretable co-occurrence patterns in transactional data.
- Metrics: **support**, **confidence**, **lift** quantify rule quality.
- Algorithms Apriori, FP-Growth, and Eclat offer different trade-offs for frequent itemset mining [1, 2, 3, 4].
- With appropriate thresholds and domain expertise, association rules power recommendation, marketing analytics, and exploratory data mining.

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