Clipping Line on Circle

If both the starting point and ending point of the line are outside the circle the no clipping is necessary.

Otherwise we will run intersection to find the points.

- No points or one point -> no clipping
- Two points. Only care about the points that are on the actual segment.

Point inside circle test:

To test if a point is inside the circle, we use the equation of the circle.

$$(x - x_{org})^2 + (y - y_{org})^2 \leq R^2$$

Where (x_{org}, y_{org}) is the origin of the circle, R is the radius.

For simplicity, we will consider the origin of the circle as $(0,0)$ and the line coordinates as relative to the original origin.

$$x^2 + y^2 \leq R^2$$

Simply plug in the coordinates and return the inequality result.

Intersection between line and circle

Circle equation: $x^2 + y^2 = R^2$.

Line equation: $ax + by + c = 0$.

To find the values of the constants, we will use the original line equation using the two points defining the line:

$$\text{L: } \Delta y(x - x_1) = \Delta x(y - y_1)$$

$$\text{L: } \Delta y\ x - \Delta y\ x_1 = \Delta x\ y - \Delta x\ y_1$$

$$\text{L: } \Delta y\ x - \Delta x\ y + (\Delta x\ y_1 - \Delta y\ x_1)$$

$$(\Delta x\ y_1 - \Delta y\ x_1) = x_2y_1 - x_1y_1 - (y_2x_1 - y_1x_1) = x_2y_1 - x_1y_2$$

$$\text{L: } \Delta y\ x - \Delta x\ y + (x_2y_1 - x_1y_2)$$

So $a = \Delta y, b = -\Delta x$, and $c = x_2y_1 - y_2x_1$

To find the intersection point, we will solve the two equations together.

$$ax + by + c = 0$$

$$ax = -by - c$$

$$a^2x^2 = (by + c)^2$$

$$a^2x^2 = b^2y^2 + 2bcy + c^2$$

$$a^2(R^2 - y^2) = b^2y^2 + 2bcy + c^2$$

$$a^2R^2 - a^2y^2 = b^2y^2 + 2bcy + c^2$$

$$b^2y^2 + a^2y^2 + 2bcy + c^2 - a^2R^2 = 0$$

$$(a^2 + b^2)y^2 + (2bc)y + (c^2 - a^2R^2) = 0$$

As a reminder, the solutions for a general equation of the form $a't^2 + b't + c' = 0$

$$t = \frac{-b' \pm \sqrt{b'^2 - 4ac}}{2a}$$

If we plug in the expressions into the above formula we get:

$$y = \frac{-2bc \pm \sqrt{(2bc)^2 - 4(a^2 + b^2)(c^2 - a^2R^2)}}{2(a^2 + b^2)}$$

$$y = \frac{-bc \pm \sqrt{(bc)^2 + (a^2 + b^2)(a^2R^2 - c^2)}}{(a^2 + b^2)}$$

$$y = \frac{-bc \pm \sqrt{b^2c^2 + (a^2 + b^2)a^2R^2 - a^2c^2 - b^2c^2}}{(a^2 + b^2)}$$

$$y = \frac{-bc \pm a\sqrt{(a^2 + b^2)R^2 - c^2}}{(a^2 + b^2)}$$

We will assign $q = a^2 + b^2, sq = \sqrt{qR^2 - c^2}$.

$$y = \frac{-bc \pm a \cdot sq}{q}$$

Substituting in the line equation we get:

$$x = \frac{-ac \mp b \cdot sq}{q}$$

Notice that the line doesn't intersect the circle if $qR^2 - c^2 \le 0$

Point on segment

If we know that a point is on the line, to verify that it is on the segment, we calculate $\overrightarrow{AB} \cdot \overrightarrow{AC}$ and $\overrightarrow{BA} \cdot \overrightarrow{BC}$. The dot product indicates the type of angle. If the angle is acute, it is positive. If both angles are non-negative, then the point must be on the line.

Clipping Circle on Circle

Intersection between circle and circle

$$x^2 + y^2 = R^2$$

$$(x - x_s)^2 + (y - y_s)^2 = r^2$$

$$x^2 + y^2 - 2xx_s - 2yy_s + y_s^2 + x_s^2 = r^2$$

Subtracting gets us:

$$x^2 + y^2 - (x^2 + y^2 - 2xx_s - 2yy_s + y_s^2 + x_s^2) = R^2 - r^2$$

$$2xx_s + 2yy_s = R^2 - r^2 + y_s^2 + x_s^2$$

$q = y_s^2 + x_s^2$

$c = R^2 - r^2 + q$

$$2xx_s = c - 2yy_s$$

$$4x^2x_s^2 = c^2 - 4cyy_s + 2y^2y_s^2$$

$$4(R^2 - y^2)x_s^2 = c^2 - 4cyy_s + 2y^2y_s^2$$

$$4(y_s^2 + x_s^2)y^2 - 4cyy_s + c^2 - 4R^2x_s^2 = 0$$

$$4qy^2 - 4cy_sy + c^2 - 4R^2x_s^2 = 0$$

$$y = \frac{4cy_s \pm \sqrt{16c^2y_s^2 - 16q(c^2 - 4R^2x_s^2)}}{8q}$$

$$y = \frac{cy_s \pm \sqrt{c^2y_s^2 - q(c^2 - 4R^2x_s^2)}}{2q}$$

$$y = \frac{cy_s \pm \sqrt{c^2y_s^2 - qc^2 + 4qR^2x_s^2}}{2q}$$

$$y = \frac{cy_s \pm \sqrt{4qR^2x_s^2 - c^2x_s^2}}{2q}$$

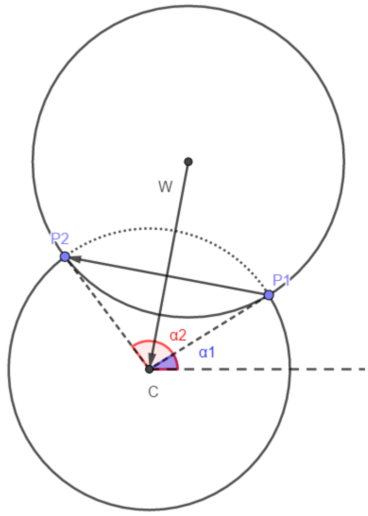
$$y = \frac{cy_s \pm x_s\sqrt{4qR^2 - c^2}}{2q}$$

$$y = \frac{cy_s \pm x_s s}{2q}$$

Substituting in the circle equation we get:

$$x = \frac{cx_s \mp y_s s}{2q}$$

These two intersection points split the circle into two arcs. One of the two arcs is inside the clipping window, and the other is outside. To figure out which is which we will start by defining α_1, α_2 as the angles that the intersection points p_1 and p_2 make with the circle center(with respect to positive $x - axis$). W is the window, while C is the circle we are clipping (see figure.)



The combined direction between $\overrightarrow{WC}$ and $\overrightarrow{p_1p_2}$ dictates whether the counterclockwise curve between p_1 and p_2 is inside the clipping window or not. If we organize rename the intersection points such that $|\overrightarrow{WC} \times \overrightarrow{p_1p_2}| > 0$. We guarantee that the curve is inside the circle.

$$p_1 = \left(\frac{cx_s \pm y_s s}{2q}, \frac{cy_s \mp x_s s}{2q}\right), p_2 = \left(\frac{cx_s \mp y_s s}{2q}, \frac{cy_s \pm x_s s}{2q}\right)$$

$$\overrightarrow{p_1p_2} = p_2 - p_1 = \frac{s}{q}(y_s, -x_s)$$

$$\overrightarrow{WC} = \vec{C} = (x_s, y_s)$$

$$|\overrightarrow{WC} \times \overrightarrow{p_1p_2}| = \frac{s}{q}\left(y_s y_s - (-x_s x_s)\right) = s$$

Because s is guaranteed to be positive (it is a square root), we guarantee that calculating the points in the given order will ensure that the clipped curve is the clockwise curve between $p_1$and p_2

Clipping Bezier on Circle

Since a cubic curve can intersect with a circle at 6 points, the degree of the equation could be of the 6th degree. So, I will opt in for the easy method for Bezier curve.

Simply, for each pixel, if the pixel is inside the circle (check the test above), then it is draw, otherwise it is “clipped”.